Formeln zur Vorlesung: Theoretische Physik I: Mechanik

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1. Vektor: $\vec{a} = (a_1, a_2, a_3) = a_1\vec{e}_1 + a_2\vec{e}_2 + a_3\vec{e}_3$
2. Ortsvektor: $\vec{r} = (x, y, z)$
3. Skalarprodukt: $\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3$
4. Vektorprodukt: $\vec{a} \times \vec{b} = (a_2b_3 - a_3b_2,\ a_3b_1 - a_1b_3,\ a_1b_2 - a_2b_1)$
5. Gradient: $\operatorname{grad} U(x, y, z) = \nabla U = (\frac{\partial U}{\partial x}, \frac{\partial U}{\partial x}, \frac{\partial U}{\partial y});\ \nabla U(r) = \frac{dU}{dr}\ \frac{\vec{r}}{r}$
6. Rotation: $\operatorname{rot}\vec{A}(x, y, z) = \nabla \times \vec{A} = (\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}, \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}, \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y})$
7. Differential: $dU = \nabla U \cdot d\vec{r} = \frac{\partial U}{\partial x}dx + \frac{\partial U}{\partial y}dy + \frac{\partial U}{\partial z}dz$
8. Kettenregel: $\frac{d}{dt}U(\vec{r}(t)) = \nabla U \cdot \dot{\vec{r}} = \left[\frac{\partial U}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial U}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial U}{\partial z} \cdot \frac{dz}{dt}\right]$
   $= \left(\frac{\partial U}{\partial x}, \frac{\partial U}{\partial y}, \frac{\partial U}{\partial z}\right) \cdot \begin{pmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \\ \frac{dz}{dt} \end{pmatrix}$
9. Produktregel: $\frac{d}{dt}(\vec{a}(t) \cdot \vec{b}(t)) = \dot{\vec{a}} \cdot \vec{b} + \vec{a} \cdot \dot{\vec{b}};\ \frac{d}{dt}(\vec{a}(t) \times \vec{b}(t)) = \dot{\vec{a}} \times \vec{b} + \vec{a} \times \dot{\vec{b}}$
10. Wegintegral: $\int\limits_C \vec{F} \cdot d\vec{r} = \int\limits_{t_a}^{t_e} \vec{F}(\vec{r}(t)) \cdot \dot{\vec{r}}(t)\, dt;\ \int\limits_C \nabla U \cdot d\vec{r} = U(\vec{r}_e) - U(\vec{r}_a)$
11. Leistung: $\frac{dT}{dt} = P = \frac{dW}{dt}$
12. Gesamtimpuls: $\vec{P}_g = \sum_\nu m_\nu \dot{\vec{r}}_\nu$
13. Polarkoordinaten: $d\vec{r} = dr\, \vec{e}_r + r\, d\varphi\, \vec{e}_\varphi$
14. Zylinderkoordinaten: $d\vec{r} = d\rho\, \vec{e}_\rho + \rho\, d\varphi\, \vec{e}_\varphi + dz\, \vec{e}_z$
15. Kugelkoordinaten: $d\vec{r} = dr\, \vec{e}_r + r d\theta\, \vec{e}_\theta + r \sin\theta\, d\varphi\, \vec{e}_\varphi$
16. Quadrat der Geschwindigkeit: $\vec{v}^2 = \dot{x}^2 + \dot{y}^2 + \dot{z}^2 = \dot{\rho}^2 + \rho^2\dot{\varphi}^2 + \dot{z}^2 = \dot{r}^2 + r^2\dot{\theta}^2 + r^2 \sin^2\theta\ \dot{\varphi}^2;\quad \vec{v}^2 = \dot{r}^2 + r^2\dot{\varphi}^2$

17. Schwingung: $\exp(i\omega t) = \cos(\omega t) + i\sin(\omega t)$, $\sin x = \frac{1}{2i}(e^{ix} - e^{-ix}), \cos x = \frac{1}{2}(e^{ix} + e^{-ix})$, $\sinh x = \frac{1}{2}(e^{x} - e^{-x}), \cosh x = \frac{1}{2}(e^{x} + e^{-x})$

18. Gedrehtes BS: $\left(\frac{d\vec{G}}{dt}\right)_{IS} = \left(\frac{d\vec{G}}{dt}\right)_{KS'} + \vec{\omega} \times \vec{G}$

19. Gedrehtes BS: $m\ddot{\vec{r}} = \vec{F} - 2m(\vec{\omega} \times \dot{\vec{r}}) - m\vec{\omega} \times (\vec{\omega} \times \vec{r})$

20. Lagrangefunktion: $\mathcal{L}(q_1, \ldots, q_f, \dot{q}_1, \ldots, \dot{q}_f, t) = T(q, \dot{q}, t) - U(q, t)$

21. Lagrangegleichungen: $\dfrac{d}{dt}\dfrac{\partial\mathcal{L}}{\partial\dot{q}_k} = \dfrac{\partial\mathcal{L}}{\partial q_k} \quad (k = 1, \ldots, f)$

22. Wirkung: $S[q] = \int_{t_1}^{t_2} dt\, \mathcal{L}(q, \dot{q}, t)$

23. Hamiltonsche Prinzip $\delta S = 0 \Leftrightarrow \frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{q}_k} = \frac{\partial\mathcal{L}}{\partial q_k}$ (Wirkung S für die physikal. Bahn $q(t)$ minimal)

24. Mechanische Ähnlichkeit: $U(\alpha\vec{r}) = \alpha^k U(\vec{r}) \Rightarrow (t/t') = (l/l')^{1-\frac{k}{2}}$

25. Zentralpotential: $E = \frac{\mu}{2}\dot{r}^2 + \frac{l^2}{2\mu r^2} + U(r);\ \mu = \frac{m_1 m_2}{m_1 + m_2}$

26. Kegelschnitte: $r = p/(1 + \varepsilon\cos\varphi)$

27. Raumzeit: (ct, x, y, z)

28. Invariantes Wegelement ds: $ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$

29. Lorentztransformation:
    $x' = \gamma(x - vt), y' = y, z' = z, t' = \gamma(t - xv/c^2), \gamma = 1/\sqrt{(1 - v^2/c^2)}$

30. Relativistische Masse: $m = \gamma m_0 = E/c^2 = |\vec{p}|/|\vec{v}|$

31. Impuls: $\vec{p} = \gamma m_0\,\vec{v};\ \frac{d\vec{p}}{dt} = \vec{F}$

32. Hamiltonfunktion $\mathcal{H}(q, \dot{q}, t) = \sum_k \dot{q}_k \frac{\partial\mathcal{L}}{\partial\dot{q}_k} - \mathcal{L}$

33. Hamiltongleichungen: $\dot{q}_k = \frac{\partial\mathcal{H}}{\partial p_k},\ \dot{p}_k = -\frac{\partial\mathcal{H}}{\partial q_k}$