Testing Models with Higher Dimensional Effective Interactions at the LHC and Dark Matter Experiments

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Kurzzusammenfassung

Dunkle Materie und nichtverschwindende Neutrinomassen sind nur zwei Hinweise auf das mögliche Vorhandensein neuer Physik jenseits des Standardmodells der Teilchenphysik. Solche möglichen Konsequenzen neuer Physik können modellunabhängig mit effektiven Feldtheorien beschrieben werden. Beispielsweise aufgrund zusätzlicher Symmetrien ist es möglich, dass Operatoren mit Dimension $d > 5$ den dominanten Beitrag zu den Effekten neuer Physik bei niedrigen Energieskalen liefern. Da diese stärker unterdrückt sind als die gewöhnlicherweise betrachteten Operatoren niedrigerer Dimension, können sie zu äußerst schwachen Wechselwirkungen führen, selbst wenn neue Physik bereits bei vergleichsweise niedrigen Energien auftritt. Dies ermöglicht unter anderem neue Teilchen mit Massen im Bereich der TeV-Skala mit der Erzeugung der sehr geringen Neutrinomassen in Verbindung zu bringen. Solche Teilchen sind besonders interessant, da sie an Beschleunigerexperimenten wie dem Large Hadron Collider untersucht werden können. Deswegen wird in dieser Arbeit zunächst die Erzeugung von Neutrinomassen durch höherdimensionale effektive Operatoren in supersymmetrischen Modellen rekapituliert. Darüber hinaus sollen mögliche Prozesse zum Nachweis dieser Modelle am Large Hadron Collider anhand eines Beispiels diskutiert werden. Da das Einführen neuer Teilchen das Laufen der Kopplungskonstanten beeinflussen kann, wird ferner betrachtet, inwiefern solche Szenarien vereinbar mit großen vereinheitlichten Theorien *(Grand Unified Theories)* sind. Die entsprechende Erweiterung dieser Modelle kann beispielsweise das Auftreten neuer schwerer Quarks zur Folge haben, die auf ihre Vereinbarkeit mit kosmologischen Beobachtungen untersucht werden. Höherdimensionale Operatoren können jedoch nicht nur sehr kleine Neutrinomassen erzeugen, sondern auch für Experimente zum Nachweis dunkler Materie relevant sein. Daher sollen die zuvor angewandten Methoden zur systematischen Diskussion effektiver Operatoren, die Wechselwirkungen dunkler Materie beschreiben, verwendet werden.

Abstract

Dark matter and non-zero neutrino masses are possible hints for new physics beyond the Standard Model of particle physics. Such potential consequences of new physics can be described by effective field theories in a model independent way. It is possible that the dominant contribution to low-energy effects of new physics is generated by operators of dimension $d > 5$, *e.g.*, due to an additional symmetry. Since these are more suppressed than the usually discussed lower dimensional operators, they can lead to extremly weak interactions even if new physics appears at comparatively low scales. Thus neutrino mass models can be connected to TeV scale physics, for instance. The possible existence of TeV scale particles is interesting, since they can be potentially observed at collider experiments, such as the Large Hadron Collider. Hence, we first recapitulate the generation of neutrino masses by higher dimensional effective operators in a supersymmetric framework. In addition, we discuss processes that can be used to test these models at the Large Hadron Collider. The introduction of new particles can affect the running of gauge couplings. Hence, we study the compatibilty of these models with Grand Unified Theories. The required extension of these models can imply the existence of new heavy quarks, which requires the consideration of cosmological constraints. Finally, higher dimensional effective operators can not only generate small neutrino masses. They also can be used to discuss the interactions relevant for dark matter detection experiments. Thus we apply the methods established for the study of neutrino mass models to the systematic discussion of higher dimensional effective operators generating dark matter interactions.

Contents

1. Introduction

The discovery of a Higgs boson at the Large Hadron Collider (LHC) [1, 2] is the latest success of the Standard Model of particle physics (SM). This was also honored by the Nobel Prize of Physics 2013, which was awarded to François Englert and Peter Higgs for their theoretical work on the mechanism that generates the masses of elementary particles, and that predicts such a boson. Despite its success, the SM leaves many open questions: *Why is the mass of the Higgs at such a small scale? How can the even many orders of magnitude smaller neutrino masses be understood? Why have the different particle families so different masses? Can the fundamental forces be united? What is the origin of dark matter?* Such questions motivate the investigation of new physics beyond the Standard Model (BSM). The most popular framework for BSM physics is supersymmetry (SUSY). In SUSY fermions of the SM have bosonic partners and SM scalars have fermionic ones. Introducing this symmetry between fermions and bosons is not only aesthetically pleasing. SUSY stabilizes the Higgs mass at the electroweak scale, avoiding the so-called hierarchy problem. Other than in the SM, the coupling constants of the fundamental interactions can be united at a high energy scale. Furthermore, the lightest supersymmetric particle is a good candidate for dark matter (DM). It is quite astonishing that just by requiring the existence of this very fundamental theory, so many open issues could be addressed. It is due to this fact that many physicists still believe that SUSY may be realized in nature, although a several decades long hunt for any evidence for SUSY has so far been unsuccessful. The results that a future run of the LHC may produce are therefore highly expected. Since SUSY allows the unification of the SM gauge couplings, it becomes interesting to think about a Grand Unified Theory (GUT). In such a theory the forces of the SM are described by a single gauge group and relations between quarks and leptons can be obtained.

Also in another field of particle physics have been interesting developments in the past. While neutrinos where considered to be massless particles for a long time, the observation of flavor oscillations of neutrinos has established the fact that they have tiny but non vanishing masses. The smallness of these masses can be understood in the framework of the so-called seesaw mechanism. It describes the effect that the introduction of new very massive particles can lead to very small masses for the known neutrinos. These new particles are usually required to be very heavy, far beyond the reach of any current experiments. But new models that extend the generic seesaw mechanism, and which have been recently discussed in literature, make it possible that the new physics scale is of order TeV. Such scenarios are especially interesting since they have phenomenological consequences for colliders, such as the LHC. Besides finite masses, the observation of flavor oscillations requires that the mass eigenstates of neutrinos are a mixture of their flavor eigenstates. For quite some time the paradigm of so-called tri-bimaximal mixing has been very popular since it can be generated by simple symmetry groups such as A_4 and was also in good agreement with experimentally measured parameters, until it was discovered that the reactor mixing angle θ_{13} is significantly different from zero [3, 4]. This rules out pure tri-bimaximal mixing. Which underlying physics is responsible for this behavior is an open issue, that currently is strongly debated. Also in the field of neutrino physics, interesting experimental results are expected to be obtained in the not so far future.

A third field that is currently expecting more input from experiments is the search for dark matter. From cosmological observations, such as the measurement of the Cosmic Microwave Background (CMB) [5, 6], we know that the visible matter is only a small fraction (∼5%) of the total energy-density of the Universe. The largest part is instead composed of dark matter (∼27%) and dark energy (∼69%) [7]. The results from experiments trying to observe dark matter directly have so far been contradictory. While some experiments such as CoGeNT [8] and CDMS [9, 10] have found hints for the existence of DM, other experiments such as XENON100 [11] and recently the LUX experiment [12, 13] have excluded this parameter region.

Usually specific high energy models are used to describe these observations. Alternatively one can work with effective field theories (EFTs), which are parametrizations of new physics effects at lower energies that are independent of the actual implementation of BSM physics. They can be used to systematically study a wide field of different physics scenarios. We will use this “technology” to study several questions related to the topics discussed above.

This thesis is organized as follows: After an introduction to the basic concepts of SUSY in chapter 2 and an overview of neutrino physics in chapter 3, we discuss effective field theory in chapter 4. In this context we will show how neutrino mass can be generated by an effective operator. The connection of neutrino masses to TeV scale physics has been recently discussed in literature [14–30]. In the context of EFTs an interesting scenario is the generation of neutrino masses by a higher dimensional operator [15, 31–42]. The study of higher-dimensional operators has been also applied to other fields, *e.g.*, neutrinoless double beta decay [43] or anomalous Higgs couplings [44]. Here we will discuss higher dimensional operators that generate neutrino mass systematically within the framework of SUSY models. We will illustrate this concept for a specific model that gives rise to neutrino mass via an effective operator of dimension $d = 7$ in chapter 5, and demonstrate how it could be potentially tested at the LHC. Since the UV completion of these operators require new particles, the unification of gauge couplings that arises in SUSY might be spoiled. Therefore we want to study how these neutrino mass models can be embedded into a GUT inspired model. Also Dark matter interactions can be studied using EFTs [45–56]. In chapter 7 we will therefore use the previously established methods to study DM interactions via higher dimensional operators. Finally we will summarize and give our conclusions.

2. Supersymmetry

Supersymmetry (SUSY) has been one of the most popular models for physics beyond the Standard Model (BSM) in the past decades. After its start in 2009, the LHC has begun to test supersymmetric models. For the simplest SUSY models, new consraints on the parameter space could already be obtained. After further data will have been acquired, it will be possible to either strengthen these constraints or to detect first hints of supersymmetric particles. In the following we present an introduction to the basic concepts of SUSY and provide an overview on the current experimental status. A more detailed introduction can be found in Refs. [57, 58].

2.1. One step back: The Standard Model

We assume that the reader is familiar with quantum field theories (QFT) and the basic concepts of particle physics (see, *e.g.*, Ref. [59]). In order to better understand the need for BSM physics, we will, however, briefly recapitulate the main aspects of the Standard Model.[1] The basic building blocks of the SM are scalars, fermions and vector bosons:

Scalars

A non-interacting massive complex scalar field ϕ is described by the Klein-Gordon equation

$$(\partial^\mu \partial_\mu + m^2)\phi = 0\,, \tag{2.1}$$

where m is its mass and $\partial_\mu = \left(\frac{\partial}{\partial t}, \nabla\right)$. The scalar is invariant under Lorentz transformations $\Lambda^{\mu\nu}$ according to $\phi(x) \to \phi'(x) = \phi(\Lambda^{-1}x)$.

Fermions

Fermions instead are Dirac spinors, which are multicomponent fields that transform as $\psi_a \to M_{ab}(\Lambda)\psi_b(\Lambda^{-1}x)$. Their equation of motion is given by the Dirac equation

$$(\mathrm{i}\gamma^\mu \partial_\mu - m)\psi(x) \equiv (\not{\partial} - m)\psi(x) = 0\,, \tag{2.2}$$

[1] We will use this opportunity also to fix some notation conventions. We will use natural units ($c = 1$, $\hbar = 1$) and the signature $(+, -, -, -)$ for the metric $g^{\mu\nu}$ throughout this thesis.

where γ_μ are the Dirac matrices.[2]

Vector bosons

While Eq. (2.1) and Eq. (2.2) are invariant under global symmetry transformations, this is not the case for local gauge transformations. This can be compensated through the interaction with gauge bosons A^μ, which transform as Lorentz vectors $A^\mu(x) \rightarrow (A^\mu)'(x) = \Lambda^{\mu\nu} A^\nu(\Lambda^{-1}x)$. Their equations of motion are the MAXWELL equations

$$\partial^\mu F_{\mu\nu} = 0\,, \tag{2.3}$$

where $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$. The last term containing the structure constant f^{abs} induces a self interaction proportional to a coupling constant g. It is zero for Abelian gauge fields.

Putting all these ingredients together, we can write down a simple gauge invariant Lagrangian

$$\begin{aligned}\mathcal{L} = (D^\mu\phi)^\dagger(D_\mu\phi) + \overline{\psi}\not{D}\psi - \frac{1}{4}(F^a)^{\mu\nu}(F^a)_{\mu\nu} - \overline{\psi}m\psi \\ - \mu\phi^\dagger\phi + \lambda(\phi^\dagger\phi) + y\,(\phi\overline{\psi}\psi + \text{h.c.})\,,\end{aligned} \tag{2.4}$$

where $\overline{\psi} = \psi\gamma^0$ and $D_\mu = \partial_\mu + igA^\mu$ is the covariant derivative (for an Abelian gauge group).

SM particles

The gauge group of the SM is $SU(3)_{\text{color}} \times SU(2)_{\text{L}} \times U(1)_{\text{Y}}$.

- SU(3) is responsible for the strong interaction of colored particles. The corresponding gauge bosons are the 8 *gluons*.
- The SU(2) group acts on left-handed particles and leads to weak interactions. It includes three gauge bosons W_i.
- The hypercharge[3] of U(1) has one associated boson B_0.

[2] We will work in the WEYL basis, *i.e.*, the basis in which $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ is block diagonal. Thus

$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}, \quad \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}, \text{ and } \gamma^5 = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix},$$

where $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ are the Pauli matrices. Using the common convention $\sigma^\mu = (\sigma_0, \sigma_1, \sigma_2, \sigma_3)$, $\bar{\sigma}^\mu = (\sigma_0, -\sigma_1, -\sigma_2, -\sigma_3)$ and $\sigma^0 = \mathbb{1}$ we can also write $\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}$.

[3] We will use the convention $Y = (Q - I_3)$ for the hypercharge of a particle, where Q is its electric charge and I_3 its weak isospin, so that the right handed electron has $Y = -1$.

The fermions of the SM have the following transformation properties under these gauge groups

- The *quarks* are the only particles that carry a color charge. They are therefore triplets of SU(3). The left-handed particles form an SU(2) doublet
$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \qquad \text{with} \qquad Y = +\frac{1}{6}\,.$$
The right handed particles are u_R and d_R with hypercharge $Y = +\frac{2}{3}$ and $Y = -\frac{1}{3}$.
- The *leptons* are color singlets. We have left-handed SU(2) doublet
$$L = \begin{pmatrix} \nu \\ \ell^- \end{pmatrix} \qquad \text{with} \qquad Y = -\frac{1}{2}\,,$$
and a right-handed field e_R with $Y = -1$.

We have three generations for each of these fields, the up-type quarks being the *up*, the *charm* and the *top* quark. The down-type ones are the *down*, the *strange* and the *bottom* quark. The charged leptons are the *electron*, the muon and the tau, and the neutrinos are ν_e, ν_μ and ν_τ.

Electroweak symmetry breaking (EWSB)

Lorentz invariance implies that we can only write down mass terms for a combination of right- and left-handed fields. With the matter content described above no such term can be formed without violating gauge invariance. Also the gauge bosons cannot have an explicit mass term for the same reason. We can circumvent this issue by introducing a scalar field, the Higgs field
$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}\,,$$
which is an SU(2) doublet with $Y = +\frac{1}{2}$. Its potential reads
$$V(H) = -\mu H^\dagger H + \lambda (H^\dagger H)^2\,. \tag{2.5}$$
If one replaces H with a complex scalar field ϕ, the potential $V(\phi)$ can be plotted vs. the real and imaginary component of ϕ and we obtain a surface which at its bottom has the characteristic shape of a "Mexican hat". Also the more complicated potential $V(H)$ has a global minimum different from zero. Due to the freedom of SU(2) we can rewrite the corresponding vacuum expectation value (VEV) of the Higgs field as
$$\langle H \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}\,, \tag{2.6}$$

with $v = \langle H^0 \rangle = \sqrt{\frac{\mu}{2\lambda}} = 174\,\text{GeV}$.[4] One can easily see that $\langle H \rangle$ is not invariant under $SU(2)_L \times U(1)_Y$ transformations. As a consequence the electroweak part $SU(2)_L \times U(1)_Y$ of the SM gauge group will be spontaneously broken down to an $U(1)_{\text{em}}$ which only conserves electromagnetic charge, but not weak isospin and hypercharge. The masses for the gauge bosons are obtained via the covariant derivative $(D^\mu H)^\dagger (D_\mu H)$. This will cause the mixing of the B^0 and W^0 bosons. In the end we will obtain three massive mass eigenstates W^+, W^- and Z^0 as well as a massless *photon* γ. The masses of the fermions will be obtained via Yukawa couplings

$$\mathcal{L}_{\text{Yukawa}} = Y_e \overline{e_R} L H^\dagger + Y_d \overline{d_R} Q_L H^\dagger + Y_u \overline{u_R} Q_L H + \text{h.c.} \tag{2.7}$$

The structure of the Yukawa constants determines the mass hierarchy and mixing behavior of the three fermion generations. Without the introduction of new fields the neutrino is still massless in this picture. The addition of a right-handed neutrino would in principle allow for a Yukawa coupling $Y_N \overline{\nu_R} L H$. This coupling, however, would have to be extraordinarily small. We will discuss this issue in more detail in a later chapter.

So far, the Standard Model has been tested very successfully and with high precision. With the observation of a Higgs boson, the last missing piece has been finally discovered. As we will discuss in the next section, one has, however, reason to believe that this picture of particle physics is not complete.

2.2. Basic concept and motivation of SUSY

The basic idea of SUSY is to relate bosons and fermions via a symmetry. This can be schematically described as

$$Q|\text{fermion}\rangle = |\text{boson}\rangle \quad \text{and} \quad Q^\dagger|\text{boson}\rangle = |\text{fermion}\rangle\,, \tag{2.8}$$

where the operator Q is a generator of SUSY. A pair of SUSY generators obeys the following anticommutation relations

$$\{Q_a; Q_b\} = \{Q_a^\dagger; Q_b^\dagger\} = 0\,, \tag{2.9a}$$

$$\{Q_a; Q_b^\dagger\} = (\sigma_\mu)_{ab} \hat{P}^\mu\,, \tag{2.9b}$$

where $\hat{P}^\mu$ is the 4-momentum operator and σ^μ represents the Pauli matrices. There are several considerations that motivate this approach:

[4] Note that also the convention $\langle H^0 \rangle = \frac{v}{\sqrt{2}}$ with $v = 246\,\text{GeV}$ is commonly used.

Symmetry considerations

As stated by the Coleman-Mandula theorem [60], there exist only two non-trivial types of generators of the Poincaré group. With other words, in a four-dimensional space-time the only non-scalar operators that generate Lorentz-invariant transformations are the momentum operators $\hat{P}_\mu$ and the angular momentum operators $\hat{M}_{\mu\nu}$. If one extends spacetime to a superspace, by allowing for additional fermionic degrees of freedom, the complete set of non-trivial transformations includes also the operators from Eq. (2.9) that transform as spinors [61].

The hierarchy problem

Assuming the SM is only valid up to a cut-off scale Λ, the Higgs mass will obtain next-to leading order corrections, *e.g.*,

$$\delta m_H \propto \lambda \int^{\Lambda} \mathrm{d}^4k \frac{1}{k^2 - m_H^2} \propto \lambda\Lambda^2 \,, \tag{2.10}$$

where λ is the parameter of the quartic Higgs self-coupling (see Eq. (2.5)). Due to the quadratic dependence on the cut-off scale, these corrections are huge. In order to obtain a physical Higgs mass at the electro-weak scale the bare mass and the corrections δm_H must have very precisely agreeing values. Such a fine-tuning is considered unnatural. In SUSY, however, this problem does not appear. The existence of SUSY partners of the Higgs field leads to additional loop-corrections that cancel the corrections from Eq. (2.10). As will be discussed later in this chapter, SUSY is not an exact symmetry. Since SUSY breaking happens typically at the TeV scale, we still need some fine-tuning in order to obtain a Higgs-mass at the EW scale. This is also known as the little hierarchy problem and connected to the naturalness of SUSY (see discussion in section 2.4).

Gauge coupling unification

Grand unified theories (GUTs) assume that the SM gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$ is a subgroup of a single larger gauge group. Typical examples of such groups are $SU(5)$ and $SO(10)$. The unification of the gauge groups implies that also the corresponding gauge couplings must be unified at the GUT scale. If one assumes that the SM is valid up to this scale and no new phyiscs appears, one can convince oneself that the renormalization group running of the couplings does not lead to the required unification. If, however, new SUSY particles appear at the TeV scale, they will affect the running of the gauge couplings. As a consequence they will obtain (nearly) the same value at a high scale [62–68].

Higgs mass and electro-weak symmetry breaking

The discovery of a Higgs particle with a mass $m_H \sim 126\,\text{GeV}$ at the LHC [1, 2] is in accordance with SUSY, which predicts a light Higgs mass. For electro-weak symmetry breaking to happen, the Higgs potential must have its famous Mexican-hat shape. This means that the relative sign between the quadratic and quartic Higgs coupling μ and λ must be negative. In SUSY a positive μ at the GUT scale will automatically become negative at the EW scale due to renormalization group running.

Vacuum stability

The observed values for the masses of the Higgs and the top-quark imply that the SM model ground state is most probably in a meta-stable vacuum [69]. This means that the current vacuum is not the global minimum. The tunneling times to this vacuum are, however, longer than the age of the universe. In SUSY models, on the other hand, a stable vacuum can be more easily obtained.

2.3. SUSY models

It is not possible to combine any scalar and fermionic field into the same supermultiplet, if both are SM fields. The same is true for fermions and gauge bosons of the SM. This is due to the fact that the supermultiplets must not break the SM gauge group. The superpartner of a SM particle must therefore have the same quantum numbers as the particle itself. The construction of SUSY invariant multiplets requires the introduction of additional particles.[5]

2.3.1. The MSSM

The most minimal supersymmetric extension of the SM is the so-called MSSM, the *Minimal Supersymmetric Standard Model*. It adds exactly one SUSY partner for each SM field. There is also a modification in the Higgs-sector. The MSSM is a type-II Two Higgs Doublet Model (2HDM). This means a second Higgs supermultiplet of opposite hypercharge is present. This is necessary in order to generate SUSY invariant Yukawa couplings for up- as well as down-quarks. The full field content of the MSSM is listed in Tab. 2.1. The MSSM is specified by the superpotential

$$W_{\text{MSSM}} = \hat{\bar{u}} Y_u \hat{Q} \hat{H}_u - \hat{\bar{d}} Y_d \hat{Q} \hat{H}_d - \hat{\bar{e}} Y_e \hat{L} \hat{H}_d + \mu \hat{H}_d \hat{H}_u \,. \tag{2.11}$$

[5] If unfamiliar with the formalism used to describe SUSY models, we refer the reader to appendix A.

Chiral super-multiplets					
Name	Spin 0	Spin 1/2	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
squarks, quarks	$\widetilde{Q} = (\widetilde{u}_L, \widetilde{d}_L)$	$Q = (u_L, d_L)$	$\mathbf{3}$	$\mathbf{2}$	$\frac{1}{6}$
	$\widetilde{\bar{u}}_L$	$\bar{u}_L \sim (u_R)^c$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$-\frac{2}{3}$
	$\widetilde{\bar{d}}_L$	$\bar{d}_L \sim (d_R)^c$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$\frac{1}{3}$
sleptons, leptons	$\widetilde{L} = (\widetilde{\nu}, \widetilde{e}_L)$	$L = (\nu, e_L)$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$
	$\widetilde{\bar{e}}_L$	$\bar{e}_L \sim (e_R)^c$	$\mathbf{1}$	$\mathbf{1}$	1
Higgs, Higgsinos	$H_u = (H_u^+, H_u^0)$	$\widetilde{H}_u = (\widetilde{H}_u^+, \widetilde{H}_u^0)$	$\mathbf{1}$	$\mathbf{2}$	$\frac{1}{2}$
	$H_d = (H_d^0, H_d^-)$	$\widetilde{H}_d = (\widetilde{H}_d^0, \widetilde{H}_d^-)$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$
Vector super-multiplets					
Name	Spin 1/2	Spin 1	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
gluino, gluon	$\widetilde{g}$	g	$\mathbf{8}$	$\mathbf{1}$	0
winos, W's	$\widetilde{W}^{\pm}, \widetilde{W}^0$	$W^{\pm}, W^0$	$\mathbf{1}$	$\mathbf{3}$	0
bino, B	$\widetilde{B}$	B	$\mathbf{1}$	$\mathbf{1}$	0

Table 2.1. – The particle content of the MSSM. Besides the SM fermions and gauge bosons, we also have their SUSY partners, the sfermions and gauginos. After EWSB the physical states of the Higgs bosons are h^0, H^0, A and $H^{\pm}$. Their counterparts, the Higgsinos, mix together with the winos and binos resulting in 4 neutralinos $\widetilde{\chi}_i^0$ and 2 charginos $\widetilde{\chi}_i^{\pm}$. W^0 and B mix to Z and the photon, as usual.

With the method described in the last section, a SUSY invariant Lagrangian can be obtained from this superpotential. SUSY is, however, not an exact symmetry. This becomes obvious by considering that no SUSY particles have been discovered so far. But if SUSY were conserved, all particles would have the same mass as their superpartners.

There are different ways how SUSY breaking can occur. For our purposes, however, it is sufficient to parametrize the SUSY breaking. One therefore can add so-called soft-breaking terms to the Lagrangian. These terms have only couplings of positive mass dimension and are therefore renormalizable. The possible terms that are invariant under the SM gauge group are:

Gaugino mass terms

$$-\frac{1}{2}(M_1\widetilde{B}\widetilde{B} + M_2\widetilde{W}^a\widetilde{W}^a + M_3\widetilde{g}^\alpha\widetilde{g}^\alpha + \text{h.c.})\,, \tag{2.12}$$

Sfermion mass terms

$$-m^2_{\widetilde{Q}}\widetilde{Q}^\dagger\widetilde{Q} - m^2_{\widetilde{\bar{u}}}\widetilde{\bar{u}}^\dagger\widetilde{\bar{u}} - m^2_{\widetilde{\bar{d}}}\widetilde{\bar{d}}^\dagger\widetilde{\bar{d}} - m^2_{\widetilde{L}}\widetilde{L}^\dagger\widetilde{L} - m^2_{\widetilde{\bar{e}}}\widetilde{\bar{e}}^\dagger\widetilde{\bar{e}}\,, \tag{2.13}$$

where we have to sum over all families,

Higgs mass terms

$$-m^2_{H_u}H_u^\dagger H_u - m^2_{H_d}H_d^\dagger H_d - (B_H H_u H_d + \text{h.c.})\,, \tag{2.14}$$

and

Trilinear terms

$$-T_u\widetilde{\bar{u}}\widetilde{Q}H_u - T_d\widetilde{\bar{d}}\widetilde{Q}H_d - T_e\widetilde{\bar{e}}\widetilde{L}H_d + \text{h.c.} \tag{2.15}$$

While an unbroken SUSY would not require any more parameters than the SM, these soft-breaking terms introduce over 100 new parameters. Therefore, one usually considers models with an restricted parameter space. In the constrained MSSM (CMSSM) or minimal supergravity (mSUGRA) framework [70–74] the masses of sfermions m_0 and the masses of the gauginos $m_{\frac{1}{2}}$ have a universal value at the GUT scale. Also the trilinear couplings T_i are obtained from an unviersal value A_0 multiplied by the corresponding Yukawa coupling Y_i at the GUT scale. The SUSY sector is then fully determined by the following parameters

$$m_0, \quad m_{\frac{1}{2}}, \quad \tan\beta, \quad A_0, \quad \text{and} \quad \text{sgn}(\mu)\,, \tag{2.16}$$

where $\tan\beta$ is the ratio of the vacuum expectation values $\frac{v_u}{v_d}$, and μ the Higgs self coupling as in Eq. (2.11).

2.3.2. The NMSSM

The *Next-to Minimal Supersymmetric Standard Model* (NMSSM) is the most minimal extension of the MSSM (for a review see, *e.g.*, Ref. [75]). It requires the introduction of an additional superfield $\hat{S}$ that is a singlet under the SM gauge group. It is defined by the following superpotential.

$$W_{\text{NMSSM}} = \hat{\bar{u}} Y_u \hat{Q} \hat{H}_u - \hat{\bar{d}} Y_d \hat{Q} \hat{H}_d - \hat{\bar{e}} Y_e \hat{L} \hat{H}_d + \lambda \hat{S} \hat{H}_d \hat{H}_u + \kappa \hat{S}^3 \,. \tag{2.17}$$

The scalar component S of $\hat{S}$ will obtain a VEV v_S. This will dynamically generate the μ-term of the MSSM (*c.f.*, Eq. (2.11)) with $\mu = \lambda \langle S \rangle$. This is in contrast to the MSSM, where the μ parameter has to be given a value close to the electroweak scale manually, which is a somewhat arbitrary choice.

The addition of the new superfield $\hat{S}$ will lead to a further neutral scalar and pseudoscalar field if compared to the MSSM, which alters the phenomenology of the Higgs sector. Its fermionic component, instead, has an additional neutralino as consequence.

The necessity to include the term $\kappa \hat{S}^3$ in the superpotential is due to the fact that otherwise the NMSSM superpotential would be invariant under an U(1) symmetry the so-called Peccei-Quinn symmetry. The breaking of this symmetry during EWSB would generate a massless Goldstone boson. To avoid experimental exclusion bounds for this so-called axion, significant fine-tuning would be required. Instead, this symmetry is explicitly broken down to a discrete $\mathbb{Z}_3$ symmetry by the κ-term.

During EWSB also the $\mathbb{Z}_3$ will be broken spontaneously. The spontaneous breaking of a discrete symmetry leads to the domain wall problem: The vacuum in neighboring regions of the universe is in different degenerate minima of the NMSSM potential. Between these regions is a so-called domain wall. This wall would contribute to the energy-density of the universe and alter cosmic evolution in a way that is contradictory to observations. To avoid this, one can assume that the discrete symmetry is broken by a small amount, *e.g.*, by Planck scale suppressed effective operators [76–79].

2.4. Current status

Recently, experimental data has put pressure on SUSY models (see, *e.g.* [81, 82] and Refs. therein). Most prominently, the data the LHC has collected in the past years, has by now already excluded large areas in the parameter space of the most simple SUSY models. In Fig. 2.1 we show, for instance, some recent results of the Atlas collaboration. For a recent review on the current status of SUSY searches and models see, *e.g.*, Ref. [83]. There are various ways in which SUSY can be tested:

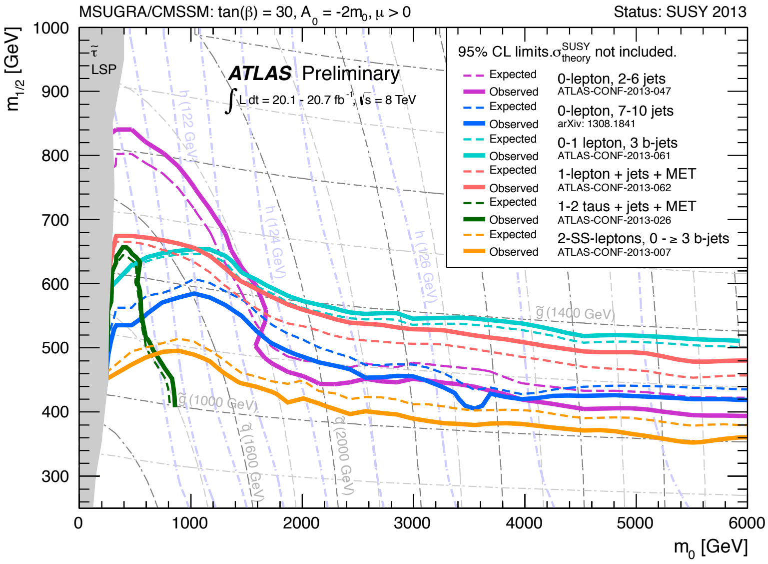

Figure 2.1. – Recent exclusion limits from the ATLAS collaboration for 8 TeV analyses in the $m_0 - m_{\frac{1}{2}}$ plane in the CMSSM for different search channels, showing the strong constraints for light squarks and gluinos [80].

- *Collider searches for superpartners* – As mentioned above, the most stringent constraints from direct searches for SUSY partners of the SM particles come from the LHC. Most colored superpartners are now required to have masses larger than 1 TeV. But, as we will discuss below, in certain models these constraints can be evaded. There exists a multitude of different search strategies for different SUSY particles, depending on the assumed model properties. In general, SUSY particles, if they are produced by proton-proton collisions, will decay in characteristic ways and produce signatures in the detector that have to be distinguished from the SM background in dedicated analyses. A typical signature is missing transverse energy. This is due to the LSP, which is stable, if R-parity is conserved, and thus will not decay inside the detector. The LHC has the highest sensitivity for squarks and gluinos. The experimental details are beyond the scope of this thesis and will not be discussed here.

- *Higgs physics* – The properties of the Higgs boson discovered at the LHC are in agreement with a SM like Higgs. While this does not imply that there is no physics beyond the Standard Model, it requires that new physics may not alter the Higgs decays in a significant way and must predict the correct Higgs mass. In models with additional scalars, such as the NMSSM, it is easier to obtain acceptable Higgs masses [84–89], although the MSSM can also accomadate a mass of 126 GeV [90–96].

- *Indirect Constraints* – Generic SUSY models contain CP and flavor violating currents. Low energy observables such as, *e.g.*, $\mu \to e\gamma$, put strict limits on flavor breaking effects. Furthermore, if CP is broken, SUSY particles contribute to the strongly constrained electric dipole moment of the electron and neutron via penguin diagrams.

All these constraints indicate that superpartners are very heavy, at the TeV or multi-TeV range. This, however, is in contradiction to naturalness: Higher-order-corrections to the Higgs mass depend on the SUSY breaking scale. The higher this scale, the more fine-tuning is required. How much fine-tuning can still be considered natural is of course very subjective, but models with minimal fine-tuning, maximally at the percent level, are generally preferred. Some, on the other hand, might object naturalness as criteria at all. Another concern is obviously that too heavy superpartners cannot be detected at the LHC. There are several classes of models that are currently discussed in literature, that are not bound by these constraints. Here, we want to present two examples:

- *Natural SUSY* – In natural or effective SUSY, one assumes a SUSY spectrum, where sbottoms, stops and gluinos are light, whereas the first and second generation quarks are heavy. This scenario makes it possible to avoid experimental constraints, which are strongest for the first and second generation. At the same time the third-generation squarks, which couple strongest to the Higgs, have lighter masses and thus reduce the necessary fine-tuning. Such a spectrum might be motivated by an additional symmetry, *e.g.*, an U(1) (see, *e.g.*,Refs. [97–99]).

- *Compressed Spectra* – If the masses of the SUSY particles are highly degenerate, the gluino and squark cascade decays will produce less missing energy than the corresponding decays in generic SUSY models. This feature makes it more difficult to identify the corresponding signals in a detector. For this reason, compressed SUSY models can avoid detection more easily resulting in weaker constraints on the masses of the SUSY particles [100–107]. Those compressed spectra can also be theoretically motivated. In Pati-Salam models, for example, a larger gauge group is broken down to the SM symmetries in several steps. A subgroup of this symmetry is thus conserved down to a rather low scale, which prevents a too large splitting of the masses [108, 109].

If one of these models is realized in nature, might become clearer when the LHC will deliver more data in the future.

3. Neutrino Physics

Neutrinos have been first postulated in 1930 by Pauli, since they allow for a consistent description of the kinematics of β-decay. Their first experimental observation was the discovery of the electron anti-neutrino $\overline{\nu}_e$ in 1956 by Cowan and Rines, which earned them the 1995 Nobel prize. Later also the other neutrino flavors the μ and τ neutrino, ν_μ and ν_τ have been found by experiments.

For a long time neutrinos have been considered massless. This changed, however, when among other experimental indications, the solar neutrino anomaly [110] was finally better understood: Knowing the nuclear processes that are the energy source of the Sun [111], one can use solar models to predict the production rate of electron neutrinos at its core. But when the Homestake experiment [112] started measuring the flux of electron neutrinos originating at the Sun on Earth in the 1970s, only a fraction of the expected flux was observed. As we know today, the solution is that neutrinos oscillate, as will be discussed in the next section. This oscillation can be enhanced by matter effects. As a consequence a sizable percentage of electron neutrinos is converted to other flavors on their way from the core of the Sun to its surface. Later the SNO experiment, which used flavor-blind neutral current interactions, could indeed observe a total neutrino flux of all three neutrino species that was consistent with predictions [113]. One condition to observe flavor oscillations is that neutrinos must have non-zero masses and their flavor states are a mixture of distinct mass eigenstates. The smallness of these masses is an indication of new physics.

In the following we will therefore first discuss the formalism of neutrino oscillations and mixing. Later we will focus on the basic models that can generate small neutrino masses, such as the default seesaw mechanism. See, *e.g.*, Refs. [114–116] for more details.

3.1. Flavor oscillations of neutrinos

The relation between flavor states and mass eigenstates of neutrinos is given by a unitary mixing matrix U, so that

$$|\nu_\alpha\rangle = U_{\alpha k}|\nu_k\rangle \qquad \text{and} \qquad |\nu_k\rangle = U^{-1}_{k\alpha}|\nu_\alpha\rangle \,, \tag{3.1}$$

where Greek indices denote flavor eigenstates ($\alpha = e, \mu, \tau$) and Latin indices mass eigenstates ($i = 1, 2, 3$). A similar matrix can be defined for the charged lepton sector. In the following we will assume a basis for the charged leptons where their mass matrix is flavor diagonal. The total mixing matrix in the lepton sector is also known as PONTECORVO-MAKI-NAKAGAWA-SAKATA or PMNS matrix. The PMNS matrix is the analogue to the CKM matrix of the quark sector. It is usually parametrized as

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}\mathrm{e}^{\mathrm{i}\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13}\mathrm{e}^{-\mathrm{i}\delta_{CP}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{3.2}$$

here $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$ and δ_{CP} is a CP phase.

In nature neutrinos should be described by Quantum Field Theory. To understand the basic principle of neutrino oscillations it is, however, sufficient to describe them as plane waves. The time evolution of their mass eigenstates is then given by

$$|\nu_k(t)\rangle = \exp(-\mathrm{i}E_k t)|\nu_k\rangle\,. \tag{3.3}$$

We can now calculate the transition probability $P_{\alpha\beta}$ that expresses with what probability the time evolution $\nu_\alpha(t)$ of an initial state ν_α will be observed as state ν_β. First we obtain the transition amplitude

$$A_{\alpha\to\beta} = \langle\nu_\beta|\nu_\alpha(t)\rangle = \langle\nu_k|U_{k\beta}U^{-1}_{\alpha k}|\nu_k(t)\rangle = U^{-1}_{\alpha k}U_{k\beta}\exp(-\mathrm{i}E_k t)\,. \tag{3.4}$$

The probability reads then

$$P_{\alpha\beta} = |A_{\alpha\to\beta}|^2 = U^{-1}_{\alpha k}U_{k\beta}U^{-1}_{\alpha l}U_{l\beta}\exp\left[-\mathrm{i}(E_k - E_l)t\right]\,. \tag{3.5}$$

Using the ultra-relativistic limit $E_k = \sqrt{\vec{p}^{\,2} + m_k^2} \approx E + \frac{m_k^2}{2E}$ we obtain

$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4\,\mathrm{Re}\left(J^{\alpha\beta}_{kl}\right)\sin^2\left(\frac{\Delta m^2_{kl}L}{2E}\right) + 2\,\mathrm{Im}\left(J^{\alpha\beta}_{kl}\right)\sin^2\left(\frac{\Delta m^2_{kl}L}{2E}\right)\,, \tag{3.6}$$

where $J^{\alpha\beta}_{kl} = U^{-1}_{\alpha k}U_{k\beta}U^{-1}_{\alpha l}U_{l\beta}$ and $\Delta m^2_{kl} = m_k^2 - m_j^2$. Its imaginary part leads to CP violation. As can be seen from the formula, by observation of neutrino oscillations in vacuum only information about the difference of the squared masses can be obtained. So far, the ordering of the mass eigenstates has not been established. Two scenarios are possible, the normal ordering ($m_1 < m_2 < m_3$) and the inverted ordering ($m_3 < m_1 < m_2$). If all three masses are close to the upper bound their relative mass differences are small and we therefore speak of a degenerate spectrum. If the lightest neutrino mass is instead close to zero the spectrum is called hierarchical.

Parameter	Best-fit value
$\Delta m^2_{21}/10^{-5}\,\mathrm{eV}^2$	$7.50^{+0.18}_{-0.19}$
$\Delta m^2_{31}/10^{-3}\,\mathrm{eV}^2$	$2.473^{+0.070}_{-0.067}$
$\sin^2\theta_{12}$	$0.302^{+0.013}_{-0.012}$
$\sin^2\theta_{23}$	$0.413^{+0.037}_{-0.025} \oplus 0.594^{+0.021}_{-0.022}$
$\sin^2\theta_{13}$	$0.0227^{+0.0023}_{-0.0024}$

Table 3.1. – Values for neutrino oscillation parameters obtained from a global fit [117]. The presented errors are the 1σ range. In case of $\sin^2\theta_{23}$ two intervalls around 0.5 are possible. The lower limit for θ_{13} is $7.19°$ at 3σ.

Experimental results and current status

In Tab. 3.1 we list the current experimental values for the neutrino oscillation parameters and their uncertainties, which have been obtained from a global fit to the available experimental data [117]. The assumption of a tri-bimaximal mixing pattern

$$U = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \tag{3.7}$$

where $U_{31} = 0$ implies a vanishing reactor angle, is quite popular. This is due to the fact that it can be elegently obtained from a flavor symmetry such as A_4, the symmetry group of a tetrahedron [118]. Tri-bimaximal mixing is also an approximation of experimental results. Recently it was discoverd that θ_{13} is significantly different from zero [3, 4] and that also θ_{12} might deviate from $\frac{\pi}{4}$. As a consequence there is considerable tension between experiment and theory in this case. Tri-bimaximal mixing might, however, still be realized at leading order, while higher-order corrections give rise to a non-vanishing reactor angle (see, *e.g.*, Refs. [119, 120]). One possibility discussed in literature is that corrections are obtained via contribtuions from the charged lepton mixing matrix, see, *e.g.*, Refs. [121, 122].

Data from current and future experiments will help to clarify further open issues, for instance the determination of the octant of θ_{23}. Experiements such as T2K [123] and Noνa [124] will allow to further constrain the possible range of the oscillation parameters, and can potentially provide information about the mass ordering. Even more precise data could be obtained by accelerator based neutrino experiments, see, *e.g.*, Refs. [125, 126]. One additional task is to measure the CP violating phase δ_{CP} from

Eq. (3.2) (see, *e.g.*, Ref. [127]). Finally, it is expected to obtain information about the absolute mass scale of neutrinos from experiments such as KATRIN [128]. The potential observation of neutrinoless double β-decay could provide further information on the scale and the nature of neutrino masses (see, *e.g.*, Ref. [129] for a recent review).

3.2. The concept of the seesaw mechanism

As discussed in the last section, the observed neutrino masses are orders of magnitude smaller then the ones of all other known particles. This smallness is commonly explained by the so-called seesaw mechanism, which we want to discuss in the following.

To generate masses for neutrinos we have to consider left- and right-handed neutrinos ν_L and ν_R, in general. Then we can obtain a Dirac mass term

$$\mathcal{L}_{\text{Dirac}} = m_D \overline{\nu_R} \nu_L + \text{h.c.} \tag{3.8}$$

as well as the Majorana Mass terms

$$\mathcal{L}_{\text{Majorana}} = \frac{1}{2} \left(m_L \overline{\nu_L^c} \nu_L + m_R \overline{\nu_R^c} \nu_R \right) + \text{h.c.} \tag{3.9}$$

Since m_L explicitly breaks the SM symmetry group it must be zero. The Dirac mass m_D can be generated via a Yukawa coupling $Y_N \overline{\nu_R} L H$, where $L = (\nu_L, e_L)^\mathsf{T}$ is the left-handed SM lepton doublet. In the basis $n_L = (\nu_L, \nu_R^c)^\mathsf{T}$ we obtain mass terms of the form:

$$\mathcal{L}_{\text{mass}} = \mathcal{L}_{\text{Dirac}} + \mathcal{L}_{\text{Majorana}} = \frac{1}{2} \overline{n_L^c} M n_L \,, \tag{3.10}$$

so that

$$M = \begin{pmatrix} 0 & m_D \\ m_D^T & m_R \end{pmatrix} . \tag{3.11}$$

Assuming that $m_R \gg m_D$ this matrix has the eigenvalues

$$m_1 \approx \frac{m_D^2}{m_R} \,, \qquad \text{and} \qquad m_2 \approx m_R \,. \tag{3.12}$$

This means we obtain a heavy mass eigenstate at the new physics scale m_R and a light mass eigenstate that is suppressed by the heavy mass. Hence the name “seesaw” mechanism: The heavier the right-handed neutrino, the lighter the left-handed neutrino. If we make the assumption that the Dirac mass is of the order of the EWSB scale, the Majorana mass of the right-handed neutrinos must be close to the GUT scale in order to obtain a mass m_1 that is in agreement with the observational bounds on the light neutrino masses. Since we have three generations of neutrinos in the SM, the mass terms in the above equations have to be matrices in flavor space. In the most minimal model agreeing with data we require at least two generations of right handed neutrinos. In this scenario we obtain a strictly hierarchical spectrum for the three light neutrinos, where ν_1 is massless at leading order.

4. Effective Field Theories

Since in the SM neutrinos are massless, the observation of neutrino oscillation is possible hint for the existence of yet unknown new physics. To describe the low energy effects of such new physics at a higher scale, it is common to use an effective field theory. In this picture, non-renormalizable effective operators are added to the SM Lagrangian:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda}\mathcal{L}^{d=5} + \frac{1}{\Lambda^2}\mathcal{L}^{d=6} + \frac{1}{\Lambda^3}\mathcal{L}^{d=7} + \cdots \tag{4.1}$$

These operators have dimension $n \geq 5$ and are suppressed by powers $n-4$ of the new physics scale Λ, since the Lagrangian has dimension $d = 4$.

4.1. Overview

The EFT can be obtained from a known fundamental theory by integrating out the heavy fields of this theory. The principal idea is briefly described as follows (for technical details please refer to Refs. [130–132]): The action $S = \int \mathrm{d}^4 x \mathcal{L}$ of a specific model with light fields ϕ and heavy fields Φ can be written as an effective action

$$\mathrm{e}^{\mathrm{i}S_{\text{eff}}[\phi]} = \mathrm{e}^{\mathrm{i}S'[\phi]} \int \mathcal{D}\Phi\, \mathrm{e}^{\mathrm{i}S''[\phi,\Phi]} \,. \tag{4.2}$$

In order to integrate out the heavy fields we have to compute the integral over Φ. At energies much lower than the mass of the heavy field we can expand Φ_0 around its stationary configuration (defined by $\delta S[\Phi_0] = 0$) and obtain

$$\int \mathcal{D}\Phi\, \mathrm{e}^{\mathrm{i}S''[\phi,\Phi]} = \int \mathcal{D}\Phi\, \mathrm{e}^{\mathrm{i}(S''[\phi,\Phi_0]+\delta S''[\phi,\Phi_0]+\delta^2 S''[\phi,\Phi_0]+\cdots)} \approx \mathrm{e}^{\mathrm{i}S''[\phi,\Phi_0]} \,. \tag{4.3}$$

From this approximation we obtain an effective Lagrangian

$$\mathcal{L}_{\text{eff}}(\phi) = \mathcal{L}'(\phi) + \mathcal{L}''(\phi, \Phi_0) \,. \tag{4.4}$$

We can determine Φ_0 by using the equations of motions, which for the stationary field are simply

$$\left[\frac{\partial \mathcal{L}}{\partial \Phi} - \partial^\mu \frac{\partial \mathcal{L}}{\partial(\partial^\mu \Phi)}\right]_{\Phi=\Phi_0} = \left[\frac{\partial \mathcal{L}}{\partial \Phi}\right]_{\Phi=\Phi_0} = 0 \,. \tag{4.5}$$

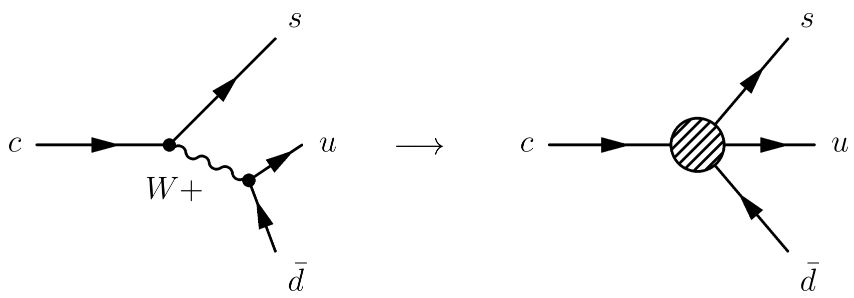

Figure 4.1. – Schematic illustration of the Fermi theory as EFT. After integrating out the SM vector bosons, effective operators proportional to the Fermi constant G_F are obtained. (The according calculations for this example can be found in Ref. [131])

We can interpret this result in the way that at low energies the kinetic terms of a heavy fields are negligible as compared to its mass. But how do we use this to obtain an effective theory from a given model? The practical procedure is as follows

- We write down the complete Lagrangian of the fundamental theory.
- We identify the heavy field with the largest mass and solve Eq. (4.5). We will obtain an expression for Φ_0 in terms of other fields.
- We replace Φ with the expression for Φ_0 in the original Lagrangian and take care of all cancellations.
- We then have obtained a new Lagrangian that does not depend on Φ anymore, but instead contains higher dimensional operators of the remaining fields.
- We repeat this procedure until all heavy fields have disappeared.

A well known example of an EFT is the Fermi theory. It describes weak interactions by effective 4-vertices of fermions proportional to a coupling constant G_F, the Fermi constant. As mentioned above, such a theory is non-renormalizable and therefore only valid up to a certain energy scale. To describe physics above this scale an *UV completion* of the EFT is necessary. In the case of the Fermi theory the UV completion is the electro-weak theory of the SM. On the other hand one can start with the SM and then integrate out the vector bosons in order to obtain the Fermi interactions. This is illustrated in Fig. 4.1.

4.2. The seesaw mechanism as EFT

Also the seesaw mechanism can be described as an EFT. The lowest-dimensional effective operator that is invariant under the SM gauge group and contains only SM fields is the

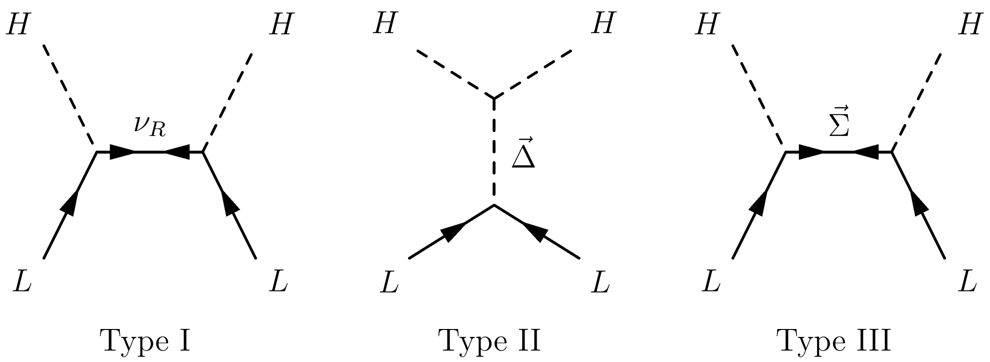

Figure 4.2. – The three types of the seesaw mechanism. ν_R is an SU(2) singlet fermion, $\vec{\Delta}$ is a scalar SU(2) triplet and $\vec{\Sigma}$ is a fermionic triplet.

Weinberg operator [133]

$$\mathcal{L}^{d=5} = -\frac{1}{2} c_{\alpha\beta}^{d=5} (\overline{L_\alpha^c} \mathrm{i}\tau_2 H)(H^\mathsf{T} \mathrm{i}\tau_2 L_\beta) + \text{h.c.}\,. \tag{4.6}$$

After EWSB it will lead to the Majorana mass term

$$m_\nu^{\text{eff}}\, \overline{\nu_L^c} \nu_L = \frac{1}{2} c_{\alpha\beta}^{d=5} v^2 \overline{\nu_L^c} \nu_L + \text{h.c.} \qquad \text{with} \qquad c_{\alpha\beta}^{d=5} \propto \frac{1}{\Lambda}\,. \tag{4.7}$$

Using Fierz identities one can rewrite Eq. (4.6) as

$$\mathcal{L}_{II}^{d=5} = -\frac{1}{2} c_{\alpha\beta}^{d=5} (\overline{L_\alpha^c} \mathrm{i}\tau_2 \vec{\tau} L_\beta)(H^\mathsf{T} \mathrm{i}\tau_2 \vec{\tau} H) + \text{h.c.}\,, \quad \text{and} \tag{4.8a}$$

$$\mathcal{L}_{III}^{d=5} = -\frac{1}{2} c_{\alpha\beta}^{d=5} (\overline{L_\alpha^c} \mathrm{i}\tau_2 \vec{\tau} H)(H^\mathsf{T} \mathrm{i}\tau_2 \vec{\tau} L_\beta) + \text{h.c.} \tag{4.8b}$$

These three configurations correspond to the only possible three UV completions of the Weinberg operator [14] as shown in Fig. 4.2. In general, there are always several possible fundamental theories that after integrating out the heavy fields lead to the same effective operator. We also speak of the *decompositions* of an effective operator. The new physics fields that appear in the decompositions are referred to as *mediators* (sometimes also as messengers). The parts of the Lagrangians of these models different from the SM read as follows[1]:

Type I seesaw

$$\mathcal{L}_I \supset \mathrm{i}\overline{\nu_R} \partial\!\!\!/ \nu_R - \overline{L} \widetilde{H} Y_\nu^\dagger \nu_R - \frac{1}{2} \overline{\nu_R} M_N \nu_R^c + \text{h.c.}\,, \tag{4.9}$$

[1] In Refs. [134, 135] the authors show, how the effective operator Eq. (4.6) can be obtained from these Lagrangians.

where ν_R is a right-handed fermion that is invariant under the SM gauge group. We use the short-hand notation $\widetilde{H} = \mathrm{i}\tau_2 H^*$ and $\overline{\widetilde{L}} = \overline{L^c}\mathrm{i}\tau_2$. The couplings and masses are matrices in flavor space.

Type II seesaw

$$\begin{aligned}\mathcal{L}_{II} \supset &\Delta_a^\dagger (D^\mu)^2_{ab}\Delta_b + \left(\overline{\widetilde{L}} Y_\Delta (\tau_a \Delta_a) L + \mu_\Delta \widetilde{H}^\dagger (\Delta_a^\dagger \tau_a) H + \text{h.c.}\right) \\ &- \left[\Delta_a^\dagger {M_\Delta}^2 \delta_{ab}\Delta_b + \lambda_3 \left(H^\dagger H\right)\left(\Delta_a^\dagger \delta_{ab}\Delta_b\right) + \lambda_5 \left(\Delta_a^\dagger T^i_{ab}\Delta_b\right) H^\dagger \tau^i H\right] ,\end{aligned} \tag{4.10}$$

where $\vec{\Delta}$ is a scalar SU(2) triplet.

Type III seesaw

$$\mathcal{L}_{III} \supset \mathrm{i}\,\overline{\vec{\Sigma}_R} \not{D}\, \vec{\Sigma}_R - \left[\frac{1}{2}\overline{\vec{\Sigma}_R} M_\Sigma \vec{\Sigma}^c_R + \overline{\vec{\Sigma}}_R Y_\Sigma (\widetilde{H}^\dagger \vec{\tau} L) + \text{h.c.}\right] , \tag{4.11}$$

where $\vec{\Sigma}$ is a fermionic triplet.

4.3. TeV scale neutrino mass models

As discussed in section 3.2 the default seesaw mechanism hints to new physics close to the GUT scale. Recently, however, models where neutrino mass is generated by new physics at the TeV scale have been discussed in literature. They can be categorized as follows:

Radiative mass generation

In this scenario, neutrino mass is not generated at tree-level. Instead, an effective mass term is generated via a loop diagram. The loop factors provide then an additional suppression so that the new physics scale can be lower (Examples can be found in Refs. [14–19]). Models where neutrino mass first appears at higher orders have been recently studied up to 3-loop level (see, *e.g.*, Ref. [136]).

Minimal lepton number violation

The small size of a parameter is considered T'HOOFT natural, if a symmetry of the model is restored in the limit where this parameter becomes zero. In certain models the suppression of neutrino mass is then a consequence of the smallness of such a parameter. Well known examples are, *e.g.*, the inverse seesaw mechanism [20] or SUSY with R-parity violation[21–30].

Higher dimensional operators

In these models one assumes that the WEINBERG operator is forbidden, *e.g.*, by an additional symmetry. Neutrino mass is then generated by higher dimensional operators [15, 31–42, 137]. In the following we want to discuss this option further and recapitulate how this approach allows for new physics at the TeV scale [138].

As already mentioned in section 4.2 the WEINBERG operator is the lowest dimensional operator with $d > 4$ that is in agreement with the SM symmetries. If one goes to higher dimensions there are, however, more possible operators that contribute to neutrino masses. In models with an SM-like scalar sector they have the generic form

$$\mathcal{O}^{d=2n+4} = \frac{1}{\Lambda^{2n+1}} LLHH(H^\dagger H)^n\,, \qquad n \in \{1,2,3,\dots\}\,. \tag{4.12}$$

After EWSB they generate an effective neutrino mass

$$m_{\text{eff}}^{d=2n+4} \propto \frac{v^{2(n+1)}}{\Lambda^{2n+1}}\,. \tag{4.13}$$

From this equation one can immediatly see that the higher-dimensional operators will generate only *subdominant* contributions to neutrino mass, since they are suppressed by higher powers of the new physics scale Λ . This means the next-to leading operator would be suppressed by v^2/Λ^2 compared to the WEINBERG operator. In the standard seesaw scenario with new physics around 10^{14} GeV this would be a suppression of 24 orders of magnitude, which clearly renders any higher-dimensional contribution to neutrino mass absolutely negligible.

This picture changes, however, if we assume that the Weinberg operator is not allowed for some reason in a specific model and hence one of the higher-dimensional operators becomes the *leading contribution* to neutrino mass [36, 137]. If this is, *e.g.*, the $d = 7$ operator, neutrino mass becomes

$$m_\nu^{\text{eff}} \propto \frac{v^4}{\Lambda^3} + \mathcal{O}\left(\frac{v^6}{\Lambda^5}\right)\,. \tag{4.14}$$

As a consequence, a much smaller Λ will be sufficient to obtain the necessary suppression for neutrino masses.

In a model with only SM content in the scalar sector, *i.e.*, a model with only a single Higgs-doublet, we face, however, a substantial problem: As discussed in Ref. [36], an operator such as in Eq. (4.12) will induce a lower dimensional operator with a loop suppression factor:

$$\frac{1}{\Lambda^{2n+1}} LLHH(H^\dagger H)^n \qquad \to \qquad \frac{1}{16\pi^2}\frac{1}{\Lambda^{2n-1}} LLHH(H^\dagger H)^{n-1} \tag{4.15}$$

So, again, we have a (substantial) contribution to neutrino mass from a lower dimensional operator, which in turn might even induce again a lower dimensional operator by closing $(H^\dagger H)$ in a loop.

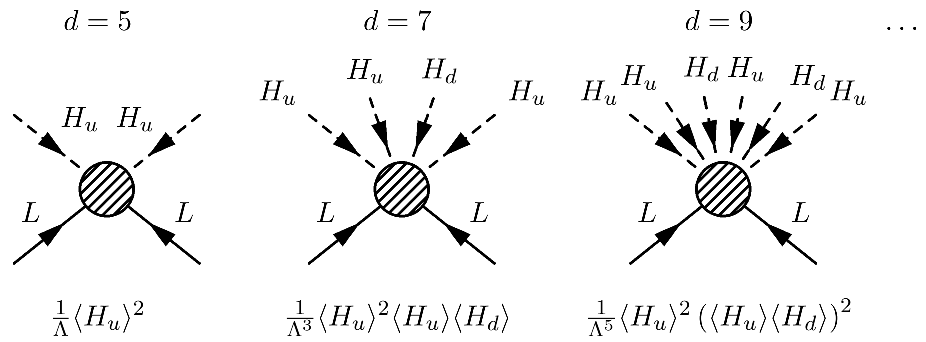

Figure 4.3. – Higher-dimensional effective operators contributing to neutrino mass in models with two Higgs-doublets and an additional scalar.

How can this issue be avoided? If the lower dimensional operator is genuinely forbidden by a symmetry, it will not be reintroduced at any order of perturbation. We can immediatly see that if the operator $LLHH$ is forbidden by such a symmetry, also any operators of the type $LLHH(H^\dagger H)^n$ will be forbidden, since $H^\dagger H$ is always invariant under the according symmetry transformations. But this is not the case in models with an enlarged scalar sector. The most minimal extension would be the introduction of an additional scalar singlet S. Many new physics models, however, require the introduction of an additional SU(2) doublet, as is the case in SUSY. We therefore want to consider operators of the type

$$\mathcal{O}^{d=2k+l+4} = \frac{1}{\Lambda^{2k+l+1}} LLH_uH_u(H_uH_d)^k S^l\,, \qquad k,l \in \{0,1,2,3,\dots\}\,. \tag{4.16}$$

where S is a scalar singlet and H_u and H_d are Higgs-like SU(2) doublets with hypercharge $Y=+\frac{1}{2}$ and $Y=-\frac{1}{2}$ respectively. All these operators can be present in the NMSSM. The subset with $l=0$ can be generated in the framework of the MSSM. In principle we also have to consider operators where the conjugates of S^*, $H_u^\dagger$ and $H_d^\dagger$ can appear. As is the case in a generic (type II) Two Higgs Doublet Model (2HDM), for example. This has been studied in detail in Ref. [36]. Here we want to restrict ourselves to SUSY scenarios [137]. In this case only operators of the type shown in Eq. (4.16) are allowed, due to the requirement of invariance under supersymmetric transformations. The operators realized in the MSSM up to $d=9$ are depicted in Fig. 4.3. The ones that are additionally possible in the NMSSM are shown in Fig. 4.4.

These operators are also listed in Tab. 4.1 for the NMSSM case. If we want to charge the fields under a discrete symmetry now, we have to make sure that the NMSSM terms $\lambda S H_u H_d$ and κS^3 are still allowed. This means that their charges have to fulfill the

	Op.#	Effective interaction	Charge	Same as
$d = 5$	1	LLH_uH_u	$2q_L + 2q_{H_u}$	
$d = 6$	2	LLH_uH_uS	$2q_L + q_{H_u} - q_{H_d}$	
$d = 7$	3	$LLH_uH_uH_dH_u$	$2q_L + 3q_{H_u} + q_{H_d}$	
	4	LLH_uH_uSS	$2q_L - 2q_{H_d}$	
$d = 8$	5	$LLH_uH_uH_dH_uS$	$2q_L + 2q_{H_u}$	#1
	6	LLH_uH_uSSS	$2q_L + 2q_{H_u}$	#1
$d = 9$	7	$LLH_uH_uH_dH_uH_dH_u$	$2q_L + 4q_{H_u} + 2q_{H_d}$	
	8	$LLH_uH_uH_dH_uSS$	$2q_L + q_{H_u} - q_{H_d}$	#2
	9	LLH_uH_uSSSS	$2q_L + q_{H_u} - q_{H_d}$	#2

Table 4.1. – Effective operators generating neutrino mass in the NMSSM up to $d = 9$. In the column "charge" we show the total charge of the according effective operator in terms of the charges of its component fields. Here we take the conditions from Eq. (4.17) into account. In the column "same as" we indicate if the total charge of the operator is identical to the charge of a lower dimensional one. *Taken from Ref. [137].*

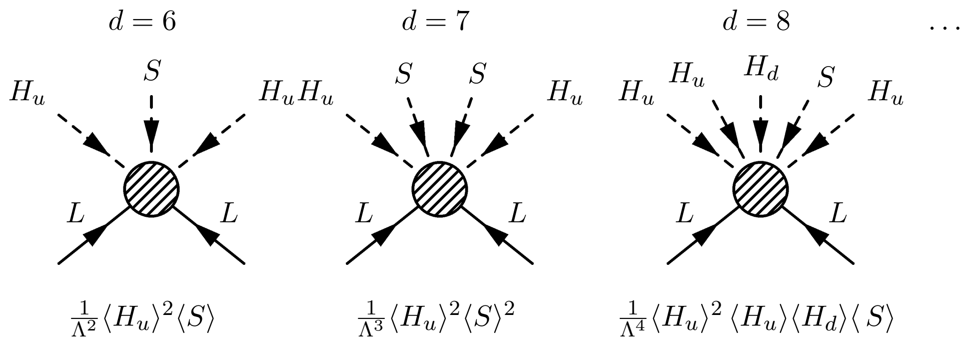

Figure 4.4. – Higher-dimensional effective operators contributing to neutrino mass in models with two Higgs-doublets and an additional scalar.

conditions

$$q_S + q_{H_u} + q_{H_d} = 0\,, \quad \text{and} \quad 3q_S = 0\,. \tag{4.17}$$

In the column “Charge” of Tab. 4.1 we show the total charge of each operator, taking above equations into account. As indicated in the column “Same as” the total charge of two different operators can be the same. Operator #5, for example, has the same total charge as the $d = 5$ operator #1. This means that we never can forbid operator #1 and allow operator #5 at the same time by a discrete symmetry. Therefore, operator #5 can never be a leading contribution to neutrino mass. The same applies to operators #6, #8 and #9. Since all operators of the type $LLH_uH_u(H_dH_u)^nS^k$ contain always products of fields such as H_uH_dS or S^3 for $n \geq 1$, $k \geq 1$ or $n = 0$, $k \geq 3$, and Eq. (4.17) implies that these terms are invariant under the discrete symmetry, they will always have the same charge as a lower-dimensional operator. For operators with $d > 9$ this can only be avoided if no scalars are part of the operators ($k = 0$). Operators with only leptons and Higgs doublets, instead, can also be the leading contributions at even higher dimensions.

The operators in Tab. 4.1 that are not affected by this are #2, #3, #4 and #7. Some of these have already been studied elsewhere [42]. In the following we want to discuss the operator #3 in more detail. The smallest discrete symmetry group we can use is $\mathbb{Z}_3$. We choose the following charges for the fields

$$q_{H_u} = 0,\; q_{H_d} = 1,\; q_L = 1,\; (q_S = 2)\,. \tag{4.18}$$

Note that in the case of the MSSM, this charge assignment implies that the term $\mu H_u H_d$ of the MSSM superpotential explicitly breaks the discrete symmetry (see also Ref. [36] for a related discussion in a 2HDM). In the case of the NMSSM, instead, the whole superpotential is invariant under the discrete symmetry and the μ term is reproduced, after S obtains a VEV.

In the same way that the Weinberg operator can be decomposed into the type-I, type-II and type-III seesaw, also the $d = 7$ operator has several possible decompositions. Some of these have already been studied in Ref. [138] in the context of SUSY. One important conclusion for the following discussion was that operators that contain a scalar singlet are potentially problematic. The reason is, that they can obtain a VEV which induces an operator of lower dimension (see also Refs. [137, 139, 140]). In the next chapter we therefore want to discuss an example of a decomposition of the $d = 7$ operator $(LLH_uH_u)(H_uH_d)$ where only fermionic mediators appear.

5. A d=7 Example and its Phenomenology at the LHC

The generation of neutrino masses via higher dimesnional effective operators implies the existence of new physics at energy scales that are accesible by collider experiments. In the following we therefore want to discuss an example for the decomposition of the $d = 7$ operator $(LLH_uH_u)(H_uH_d)$ and the phenomenological implications of this model at the LHC. The following discussion is based on Ref. [137]. Some early results[1] have also been discussed in Ref. [138].

5.1. The Model

The model we want to discuss is a decompositon of $(LLH_uH_u)(H_uH_d)$ that contains only fermionic mediators, due to the problems that scalar mediators introduce, as we discussed in the previous chapter. The corresponding Feynman diagram is shown in Fig. 5.1. This scenario is an extension of the (N)MSSM, where two SM singlet superfields $\hat{N}$ and $\hat{N}'$ and a vector like pair of SU(2) doublets $\hat{\xi}$ and $\hat{\xi}'$ are added. The doublets carry the hypercharges $Y(\hat{\xi}) = +\frac{1}{2}$ and $Y(\hat{\xi}') = -\frac{1}{2}$. The model is then specified by the superpotential

$$\begin{aligned} W = W_{\text{quarks}} &+ Y_e \hat{e}^c \hat{L} \cdot \hat{H}_d - Y_N \hat{N} \hat{L} \cdot \hat{H}_u + \kappa_1 \hat{N}' \hat{\xi} \cdot \hat{H}_d - \kappa_2 \hat{N}' \hat{\xi}' \cdot \hat{H}_u + m_N \hat{N} \hat{N}' \\ &+ m_\xi \hat{\xi}' \cdot \hat{\xi} + \mu \hat{H}_u \cdot \hat{H}_d \,. \end{aligned} \tag{5.1}$$

For now we will ignore the flavor structure of these fields, for reason of simplicity. Since we want to generate Majorana masses, lepton number is broken in our model. In the

[1] In Ref. [138] we discussed the same example as in this chapter and already described how to obtain the mass matrix and corresponding eigenstates and also implemented the same flavor structure. With an earlier version of the software implementation used here (see appendix E) we obtained the decay width of the mass eigenstates and studied the production of the new particles for a different set of parameters. Furthermore the process $pp \to W^+ \nu W^\pm \ell^\mp$ was studied. Here we present numerical results for two not yet studied processes obtained by a revised software implementation and another choice of parameters (most notably a larger mass for the heaviest particles).

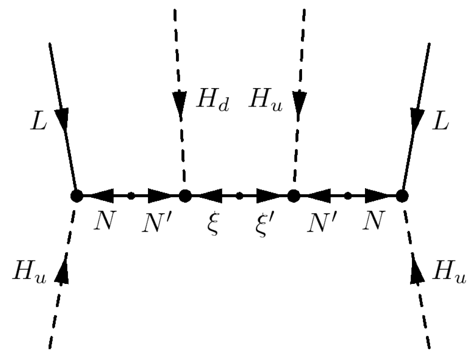

Figure 5.1. – Decomposition of the $d=7$ operator $(LLH_uH_u)(H_uH_d)$ that is studied in this section. *Taken from Ref. [137].*

limit where κ_1 or κ_2 become zero, or where m_ξ becomes infinite, lepton number is restored as an accidental symmetry. In the limit of vanishing κ_2 we obtain the following lepton numbers

$$L(\hat{N}) = -1\,,\ L(\hat{N}') = +1\,,\ L(\hat{\xi}) = -1\,,\ L(\hat{\xi}') = +1\,, \tag{5.2}$$

so that the lepton number is broken at the κ_2 term. A different assignment obtained in the other limits is also possible, but does not change the following discussion. For the superpotential we can derive the Lagrangian of this model. The part for the fermions carrying lepton number that is relevant for us reads using Weyl-spinors

$$\begin{aligned}
&\mathcal{L}^{\text{fermionic}} = \\
&\quad - Y_e(e^cL\cdot H_d + \widetilde{e}_R^*L\cdot \widetilde{H}_d + e^c\widetilde{L}\cdot\widetilde{H}_d) + Y_N(NL\cdot H_u + \widetilde{N}L\cdot\widetilde{H}_u + N\widetilde{L}\widetilde{H}_u) \\
&\quad - \kappa_1(N'\xi\cdot H_d + \widetilde{N}'\xi\cdot\widetilde{H}_d + N'\widetilde{\xi}\cdot\widetilde{H}_d) + \kappa_2(N'\xi'\cdot H_u + \widetilde{N}'\xi'\cdot\widetilde{H}_u + N'\widetilde{\xi}'\cdot\widetilde{H}_u) \\
&\quad - m_N N'N - m_\xi\xi'\cdot\xi + \text{h.c.}\,.
\end{aligned} \tag{5.3}$$

After electroweak symmetry breaking, when the Higgs fields obtain a VEV, we will generate masses for the mediator fields, in the same way as the SM fermions become massive via their Yukawa coupling. If we now define a basis $f^0 = (\nu, N, N', \xi^0, \xi'^0)$ for the neutral fermions, we can write down their mass terms as a matrix

$$M_f^0 = \begin{pmatrix} 0 & Y_N v_u & 0 & 0 & 0 \\ Y_N^\intercal v_u & 0 & m_N^\intercal & 0 & 0 \\ 0 & m_N & 0 & \kappa_1 v_d & \kappa_2 v_u \\ 0 & 0 & \kappa_1^\intercal v_d & 0 & -m_\xi \\ 0 & 0 & \kappa_2^\intercal v_u & -m_\xi & 0 \end{pmatrix}. \tag{5.4}$$

We can diagonalize this matrix, so that we will obtain several mass eigenstates. The lightest eigenstates correspond to the observed neutrinos, as is the case with the usual seesaw, but we will have additional heavy states. We will denote these mass eigenstates with n_i ordered by their masses, with n_1 being the lightest one. Accordingly we can also obtain the mass terms for the charged fermions, which are

$$-v_d e^c Y_e e_L - m_\xi \xi^+ \xi'^- \,. \tag{5.5}$$

5.2. Integrating out the heavy fields

To obtain the low energy effects of this model, *i.e.*, the neutrino masses via the effective $d = 7$ operator, we have to integrate out the heavy fields. Depending on the spectrum of those, we can in principal distinguish two scenarios:

Inverse seesaw scenario ($\mathbf{m_\xi > m_N}$)

If the doublets are heavier then the singlet mediators, we can integrate out the former as a first step and obtain a mass matrix

$$M_f^{0\prime} = \begin{pmatrix} 0 & Y_N\, v_u & 0 \\ Y_N\, v_u & 0 & m_N \\ 0 & m_N & \hat{\mu} \end{pmatrix} \tag{5.6}$$

in the basis (ν, N, N') of the remaining neutral fermions. The structure of this matrix corresponds to an inverse seesaw [20]. In the generic inverse seesaw the lightness of the neutrino masses is attributed to the smallness of a lepton number violating coupling. In our scenario, where the inverse seesaw is induced by a $d = 7$ operator as an intermediate step, we have an effective coupling $\hat{\mu} = v_u v_d\,(2\kappa_1\kappa_2)/m_\xi$, which is small ($\mathcal{O}(10^{-7})$), even for comparatively large couplings $\kappa_{1/2}$ ($\geq \mathcal{O}(10^{-3})$). At the scale where the ξ fields appear, our model will be phenomenologically distinguishable from the inverse seesaw. Both scenarios, however – our model and the usual inverse seesaw – have the same implications for low energy observables. Such implications include, *e.g.*, non-unitarity and its CP violation, which can be tested at possible long-baseline neutrino oscillation experiments (see, *e.g.*, Refs. [141, 142]). Another possibility is the observation of flavor violating processes. Effects of the new physics on the observation of lepton number violating processes at low energies, such as neutrinoless double β-decay, will be small, since the heavy Majorana neutrinos form pseudo-Dirac pairs, hiding their Majorana nature (see, *e.g.*, Ref. [143]).

In a final step, we now have to integrate out the remaining heavy fields N and N'.

We then obtain an effective mass for the light neutrinos

$$m_\nu = v_u^3 v_d Y_N^2 \frac{\kappa_1 \kappa_2}{m_\xi m_N^2} \,. \tag{5.7}$$

As expected from the general discussion of effective operators, we see that the effective mass is proportional to $\approx v^4/\Lambda^3$, where the new physics scale Λ is the mass scale of the mediators. For neutrino masses $m_\nu \approx 1\,\mathrm{eV}$ and mediator masses of about $1\,\mathrm{TeV}$ we require the couplings Y_N and $\kappa_{1/2}$ to be of the order 10^{-3}, which is in the range of the SM Yukawa couplings, and therefore not unreasonably small.

Linear seesaw scenario ($\mathbf{m_N > m_\xi}$)
If, instead, the singlets are heavier than the doublets, we obtain the mass matrix

$$M_f^{0\prime\prime} = \begin{pmatrix} 0 & \tilde{\kappa}_1\, v_d & \tilde{\kappa}_2\, v_u \\ \tilde{\kappa}_1\, v_d & 0 & -m_\xi \\ \tilde{\kappa}_2\, v_u & -m_\xi & 0 \end{pmatrix} , \tag{5.8}$$

in the basis (ν, ξ^0, ξ'^0) and with the effective couplings $\tilde{\kappa}_{1/2} = \kappa_{1/2}\, Y_N^2/m_N$. This structure of the mass matrix is identical to a linear seesaw [144]. After integrating out all fields, we of course arrive again at the effective neutrino mass term of Eq. (5.7).

5.3. The flavor structure of our model

From the discussion in section 3.1 in chapter 3 we know that the light neutrinos must have three distinct masses. Our decomposition should reproduce this mass hierarchy and also the correct mixing between flavor and mass eigenstates. Therefore, we must implement a flavor structure in our model. There are several possible approaches to accomplish this. The most simple idea is to require three generations of each mediator. This ansatz, however, introduces a large number of new parameters, which can not all be constrained by neutrino observables. To reduce this number one can assume a strictly hierarchical neutrino mass spectrum, where the lightest mass m_1 is zero. One way to generate the remaining two masses is to have two different mechanisms leading to m_2 and m_3. One is generated via a tree level effective operator, such as the inverse seesaw or the model discussed above, the other one radiatively at one-loop level [145]. Another way, which we will adopt in the following, is based on the minimal inverse seesaw scenario [146]: The lightest neutrino is again (almost) massless and the other two masses are generated via an effective operator with a flavor structure. Since only two masses need to be considered, the number of free parameters can be reduced.

The latter approach can be applied to our framework if we require two generations for the singlet mediators N and N'. This implies in total nine neutral fermion mass eigenstates, which we will denote n_1 to n_9, with n_1 being the lightest one. For the doublet mediators one generation is sufficient. As a consequence, we have to promote the couplings to matrices in flavor space. In analogy to Eq. (5.7) we obtain then an effective mass matrix for the light neutrinos, which reads

$$(m_\nu)_{\alpha\beta} = v_u^3 v_d (Y_N)_{\alpha i} (m_N^{-1})_{ij} \mu_{jk} (m_N^{-1,\mathsf{T}})_{kl} (Y_N^{\mathsf{T}})_{l\beta} \,, \tag{5.9}$$

where

$$\mu_{jk} = \frac{1}{m_\xi} \left((\kappa_1)_j (\kappa_2)_k + (\kappa_2)_j (\kappa_1)_k \right) \,. \tag{5.10}$$

We can choose an arbitrary basis for the singlet fields, without loss of generality. For reasons of simplicity, we choose one where M_N is flavor diagonal. (Which implies also M_N^{-1} being diagonal.)

We still have more parameters than observables. Necessarily there is no unique way to implement a flavor structure here. We will use the following parametrization:

$$Y_N = y_N \begin{pmatrix} \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} , \quad \kappa_1 = k_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} , \quad \kappa_2 = k_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} , \quad M_N = m_N \begin{pmatrix} 1 & 0 \\ 0 & \rho \end{pmatrix} , \tag{5.11}$$

where

$$\rho = \sqrt{m_2/m_3} \,, \qquad 2 v_u^3 v_d y_N^2 k_1 k_2 / (m_N^2 m_\xi) \stackrel{!}{=} m_2 \,. \tag{5.12}$$

The numerical factors of the couplings can be chosen as $y_N = \frac{1}{3} \cdot 10^{-3}$, $k_1 = k_2 = 10^{-2}$, $m_N = 1070\,\mathrm{GeV}$, and $m_\xi = 200\,\mathrm{GeV}$. Exemplarily we will use $\tan\beta = 10$. As this parameters are not fixed by observations we can also adopt different values by changing the couplings accordingly. This choice of parameters will lead to a mass matrix, that implies tri-bimaximal mixing. Due to the observation of a non-zero θ_{13} this scenario is obviously ruled out. Tri-bimaximal mixing can still be seen as an approximation to the data, where non-zero θ_{13} can be introduced by additional corrections. We will, indeed, discuss such an approach in the next chapter. Another possibility is that corrections are obtained due to the mixing matrix of the charged leptons. In general we have also the possibility to chose different parameters in Eq. (5.11) that allow us to obtain non-zero θ_{13} directly, although this option might be considered less attractive if one assumes the structure of the coupling constants is the consequence of a possible flavor symmetry. In summary, there are options to obtain a more realistic mixing behaviour but for the sake of simplicity we will keep the tri-bimaximal structure for now.

This parametrization will generate the normal hierarchy for neutrino masses. An inverted hierarchy can be achieved by using the parametrization

$$Y_N = y_N \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{pmatrix} , \quad \kappa_1 = k_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} , \quad \kappa_2 = k_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} , \quad M_N = m_N \begin{pmatrix} 1 & 0 \\ 0 & \rho \end{pmatrix} , \tag{5.13}$$

where $\rho = \sqrt{m_1/m_2}$ and $2v_u^3 v_d y_N^2 k_1 k_2/(m_N^2 m_\xi) \stackrel{!}{=} m_1$. In the following we will, however, restrict the discussion to the normal hierarchy scenario.

5.4. LHC phenomenology

One of the motivations to study higher-dimensional operators in the context of neutrino physics, was the possibility for mechanisms generating neutrino masses that imply new physics at the TeV scale. New particles appearing at this scale can then potentially be observed in experiments, such as the LHC. Therefore we are interested, how the model we discussed in this section can be tested at the LHC. Other studies discussing the collider phenomenology of supersymmetric neutrino mass models are presented, *e.g.*, in Refs. [109, 147–158]. Observing the decays of the new particles of our model might also help to reduce the number of free-parameters. As pointed out before, low energy-observables extracted from neutrino oscillation experiments are insufficient to constrain all the parameters of our model.

The relevant couplings and masses are Y_N, κ_i, m_N and m_ξ. Assuming a flavor diagonal mass matrix for the charged leptons and CP conservation, we have 13 real parameters in our model. Taking the low-energy data into account we are left with 7 (18 in the case of non-zero CP phases). There are 18 possible decays of the six heavy neutral states into the three charged leptons of the SM. Their observation would be principally sufficient to determine all remaining particles. For a numerical studies of these processes we have used the software tools Whizard [159] and Sarah [160, 161] (see appendix B for details).

5.4.1. Production of the new particles

First we are interested in how the particles of our model can be produced at the LHC. Since we work in a SUSY scenario, we in principal also have to take the superpartners of the mediators into account. But since the fermionic components of the mediator superfields are the ones connected to the low-energy neutrino phenomenology we will focus on those. Hence, we will assume at this point that their scalar partners have much larger masses. If we now look at the mass matrix in Eq. (5.4), we conclude that the heavy mass eigenstates n_4 to n_9 are composed mainly of either the SU(2) doublet states ξ and ξ' or the two generations of N and N'. The admixture of doublets with singlets is suppressed since the corresponding entries in the mass matrix, proportional to a coupling times the Higgs VEV, are small compared to the mediator masses. The mass eigenstates of the neutral doublet components are then approximately

$$n_{4,5} \simeq (\xi^0 \pm \xi^{0\prime})/\sqrt{2}\,, \tag{5.14a}$$

where we assume the "linear seesaw hierarchy" $m_\xi < m_N$ (see section 5.2).

The smallness of the couplings also has as a consequence that the singlet fields can only be produced in rare cascade decays. For the doublets, however, the situation is quite different: Since they are multiplets of SU(2), they couple to the corresponding gauge bosons, the W and Z particles. Thus, they can be produced directly in Drell-Yan processes. (This is similar to the production of certain MSSM particles, such as sleptons or neutralinos and charginos [162].) The numerical cross section is shown in Fig. 5.2, for a center of mass energy of $\sqrt{s} = 7\,\text{TeV}$ and $\sqrt{s} = 14\,\text{TeV}$. Note that 7 TeV is the center of mass energy used for the initial physics run of the LHC, which was later upgraded to 8 TeV. In 2014 the next run of the LHC is planned with 13 TeV, which might later be upgraded to the maximal design energy of 14 TeV. For the following discussion, we will use the parameter point specified in Eq. (5.11). We will further assume a mass of 200 GeV for the ξ fields. The total production cross section for the ξ particles is then

$$\sigma(pp \to \xi^\pm \xi^0) = 122\,\text{fb} \qquad \text{at } \sqrt{s} = 7\,\text{TeV} \tag{5.15a}$$

$$\text{and} \quad \sigma(pp \to \xi^\pm \xi^0) = 417\,\text{fb} \qquad \text{at } \sqrt{s} = 14\,\text{TeV}\,. \tag{5.15b}$$

5.4.2. Decay modes

Since R-parity is conserved in our model and the ξ fields are the lightest BSM particles with $R = +1$ their principal decay modes are into SM particles or the fields of the enlarged SUSY Higgs sector. They can decay into the following final states

- Charged and neutral leptons $\ell_j = e, \mu, \tau$ and $\nu_\alpha = \nu_e, \nu_\mu, \nu_\tau$, which are composed of $\nu_1, \nu_2, \nu_3 \equiv n_1, n_2, n_3$.
- Higgs bosons $h^0, H^0, H^\pm, A^0$.

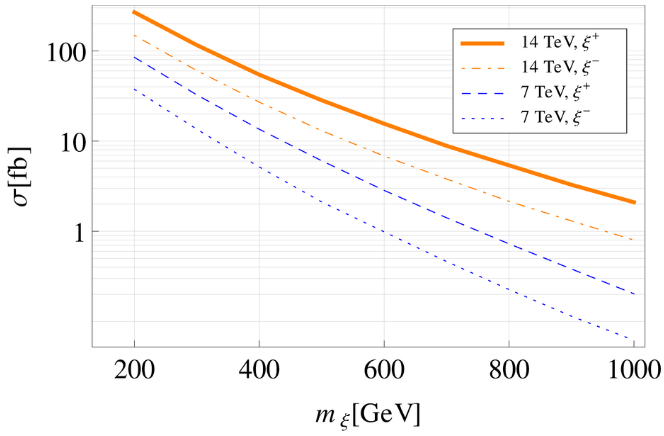

Figure 5.2. – Total cross section $\sigma(pp \to \xi^{\pm}\xi^{0})$ as a function of the mass m_{ξ}. The cross sections are shown for 7 and 14 TeV, corresponding to the initial and maximal design center of mass energies of the LHC. *Taken from Ref. [137].*

- Vector bosons $W^{\pm}, Z$.

Since they do not carry color or baryon number, they cannot decay into quarks or gluons. Since the charged heavy states do not mix with the *charged* leptons there will also be no decays into Z or h^0 (at tree-level). The neutral mediator components, instead, mix with the SM neutrinos due to the off-diagonal elements in the mass matrix in Eq. (5.4). Hence the most important decay modes are

$$\xi^{+} \to W^{+}\nu_k\,,\ \ H^{+}\nu_k \tag{5.16}$$

for ξ^+, which is the Dirac fermion composed of the charged components of ξ and ξ'. We obtain for its total decay width

$$\Gamma(\xi^{+}) = 1.42 \cdot 10^{-5}\,\mathrm{keV}\,. \tag{5.17}$$

The smallness of this value implies a long lifetime of the heavy states. The reason for this, is that the mixing of the light and heavy states is protected by the SM gauge group in combination with the discrete symmetry. Only after EWSB mixing occurs via the Yukawa couplings, which are small. In other words, the symmetry that guarantees the smallness of neutrino masses stabilizes the heavy states to a certain extent. This is no surprise, since a larger mixing of heavy and light states would automatically imply larger neutrino masses due to the nature of the seesaw mechanism. In analogy also the neutral mediator states n_i will have suppressed decay rates. Their decay channels are

$$n_i \ \to \ W^{\pm}l_j^{\mp}\,,\ \ H^{\pm}l_j^{\mp} \tag{5.18a}$$

$$n_i \ \to \ Z\nu_k\,,\ \ h^0\nu_k\,,\ \ H^0\nu_k\,,\ \ A^0\nu_k\,. \tag{5.18b}$$

Particle	Γ[keV]	BR($W^\pm e^\mp$)	BR($W^\pm \mu^\mp$)	BR($W^\pm \tau^\mp$)	BR($Z\nu$)	BR($h^0\nu$)
n_4	$2.3 \cdot 10^{-5}$	$6.6 \cdot 10^{-3}$	$7.0 \cdot 10^{-2}$	0.18	0.36	0.38
n_5	$1.9 \cdot 10^{-5}$	$1.2 \cdot 10^{-2}$	$0.41 \cdot 10^{-2}$	0.18	0.42	0.34
n_6	1.2	$1.2 \cdot 10^{-11}$	0.21	0.21	0.21	0.37
n_7	1.2	$1.2 \cdot 10^{-11}$	0.21	0.21	0.21	0.37
n_8	2.9	0.14	0.14	0.14	0.20	0.38
n_9	2.9	0.14	0.14	0.14	0.20	0.38

Table 5.1. – Total decay widths of the neutral mass eigenstates and branching ratios into the possible final states (where ν is the sum over the three light neutrino mass eigenstates). *Taken from Ref. [137].*

The corresponding decay width are listed in Tab. 5.1.

A consequence of the resulting long life-times is very interesting from an experimental point of view: The observation of displaced vertices. The lifetime in the rest frame of a particle is the inverse of its decay width $\tau = \Gamma^{-1}$. This is why small width lead to long life-times. Since the particles produced in the detector move with relativistic velocities, their life-times will be even larger in the detector reference frame. The expected decay length at the LHC are therefore between 100 μm and several mm. This displaced vertex is a distinguishing feature of our model that helps suppress the background from SM particle decays or cascade decays of SUSY particles. We will discuss possible backgrounds for a specific process in the next section.

The smallness of the Yukawa-like interactions, including the lepton number violating κ_2, as compared to the masses m_N and m_ξ amounts to another characteristic of our model: Pairs of heavy fields will form pseudo-Dirac particles. These are pairs of 2-component Majorana particles that almost behave like a 4-component Dirac spinor. This behavior becomes also apparent in Tab. 5.1. If one compares the branching ratios of n_6 and n_7 one finds that they are almost identical. The same can be said about n_8 and n_9 and to a lesser extend about n_4 and n_5. In the latter case the difference between the two states is due to the parameter ρ in the mass matrix. The different branching ratios for the decays into charged leptons can be attributed to the matrix structure of the coupling Y_N. We also note that these are lepton flavor violating (LFV) processes, meaning they are different for each lepton generation. This similarity makes it difficult to distinguish the branching ratios of the various states, so that we can use maximally nine of them to establish a connection to neutrino physics.

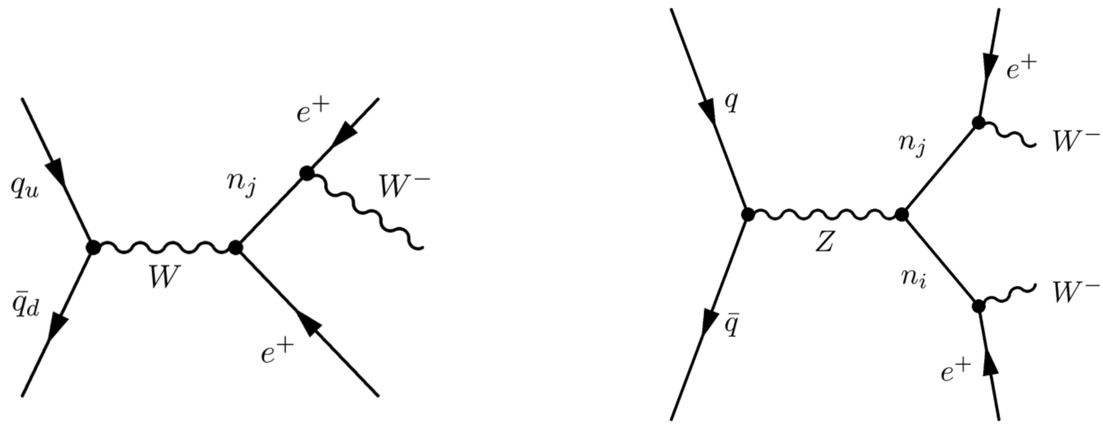

Figure 5.3. – Dominant contribution to the LNV processes $u\bar{d} \to W^- e^+ e^+$ and $q\bar{q} \to W^- W^- e^+ e^+$. *Taken from Ref. [137].*

5.4.3. Possible Observation of lepton number violating processes

Another way to make this connection, is via lepton number violating (LNV) processes. Observing lepton number violation by two units can be attributed to the Majorana nature of neutrinos. In our case we can use the decay of the heavy particles into charged leptons plus vector bosons to observe LNV. Therefore we want to consider the following processes

$$q_u \bar{q}_d \to l^+ l'^+ W^- \tag{5.19a}$$

$$q_u \bar{q}_d \to l^+ l'^+ W^- Z \tag{5.19b}$$

$$q\bar{q} \to l^+ l'^+ W^- W^-, l^- l'^- W^+ W^+ , \tag{5.19c}$$

where in each case all three generations of quarks are meant (accordingly u and d signify all three up- and down-like quarks). These quarks are partons of the protons colliding in the detector. The final states violate lepton number and also flavor. The dominant processes that lead to these decays are shown in Fig. 5.3 for the example of $W^- e^+ e^+$ and $W^- W^- e^+ e^+$ as final state. To understand these decays we must first understand the coupling structure of the particles. For the calculation of the Feynman amplitude we must of course use the mass eigenstates and sum over all possible intermediate particles. The couplings of the n_i with the W boson

$$\mathcal{L} \supset -\frac{g}{\sqrt{2}} \bar{n}_i \gamma^\mu (a_{ij} P_L + b_{ij} P_R) l_j W_\mu^+ - \frac{g}{\sqrt{2}} \bar{l}_j \gamma^\mu (a_{ij}^* P_L + b_{ij}^* P_R) n_i W_\mu^- \tag{5.20}$$

are therefore proportional to the mixing matrix elements

$$a_{ij} = U_{ij} , \qquad b_{ij} = 0 \qquad j = e, \mu, \tau . \tag{5.21}$$

These coefficients can be understood in the following way: The left handed couplings originate from the gauge couplings of the SM leptons. The term $-\frac{g}{\sqrt{2}}\bar{\nu}_j\gamma^\mu P_L l_j W^+_\mu$ for example becomes $-\frac{g}{\sqrt{2}}\bar{n}_i\gamma^\mu P_L U_{ij} l_j W^+_\mu$ since $\bar{\nu}_i = \bar{n}_j U_{ij}$ is a mixture of the mass eigenstates. Right handed couplings, however, arise in our model only from the gauge interaction of ξ^-. But since the charged heavy fields do not mix with the SM leptons, b_{ij} is zero. From these interactions we obtain instead the couplings

$$\mathcal{L} \supset -\frac{g}{\sqrt{2}}\bar{n}_i\gamma^\mu(c_i P_L + d_i P_R)\xi^- W^+_\mu - \frac{g}{\sqrt{2}}\bar{\xi}^-\gamma^\mu(c_i^* P_L + d_i^* P_R)n_j W^-_\mu \,. \tag{5.22}$$

where

$$c_i = U_{i\xi'}\,, \qquad d_i = -U^*_{i\xi}\,. \tag{5.23}$$

The process $q_u\bar{q}_d \to l^+ l'^+ W^-$

Do these right handed couplings indeed lead to the observation of lepton number violation? Therefore we have first computed the cross-sections of the processes $q_u\bar{q}_d \to l^+l'^+W^-$. The results are presented in Tab. 5.2. This processes are mainly the result of the production of a charged lepton and a neutral fermion

$$u\bar{d} \to l^+ n_i^* \ (i = 1, \ldots 5) \tag{5.24}$$

The absence of the heaviest states n_{6-9} is due to the mass hierarchy we have chosen. Their large mass has the consequence that they only can be produced off-shell and therefore are suppressed. The states n_4 and n_5, instead, can have off-shell contributions. They are only suppressed to a certain extend since the relevant coupling a_{ij} is proportional of the small mixing between ξ^0 and $\xi^{0\prime}$ to the light neutrinos. The n_i will then decay into a lepton and W boson, again via the coupling a_{ij}. Since this coupling is proportional to the mixing matrix, which can have large off-diagonal entries in the lepton sector, we obtain sizable cross sections of the order of 1 fb. Such cross-sections are in principle observable at the LHC for sufficiently high luminosity. The LNV processes, however, are strongly suppressed. The main source for LNV would be the mixing between leptons and the ξ fields. This is because the lepton number violating coupling κ_2 appears in the mixing matrix. But here we have to consider that we have to sum over all intermediate states. Since n_4 and n_5 form a pseudo-Dirac pair as noted before. Due to the projectors at the W couplings we obtain an amplitude that is proportional to the masses of the intermediate states. The reason is that we will obtain terms in the amplitude that have, *e.g.*, the structure $P_L(\not{p} - m)P_L = mP_L$. After summing over the mass states we will then obtain a cross-section proportional to $m^2_{n5} - m^2_{n4}$ so that both contributions of

Process	σ [fb] (7 TeV)	σ [fb] (14 TeV)
$pp \to W^+e^+e^-$	$(1.651 \pm 0.024) \cdot 10^2$	$(4.161 \pm 0.023) \cdot 10^2$
$pp \to W^-e^+e^-$	$(9.240 \pm 0.033) \cdot 10$	$(2.671 \pm 0.042) \cdot 10^2$
$pp \to W^+e^+\mu^-$	(1.068 ± 0.099)	(2.848 ± 0.011)
$pp \to W^+e^-\mu^+$	(1.057 ± 0.013)	(2.871 ± 0.012)
$pp \to W^-e^+\mu^-$	$(5.748 \pm 0.015) \cdot 10^{-1}$	(1.742 ± 0.015)
$pp \to W^-e^-\mu^+$	$(5.755 \pm 0.015) \cdot 10^{-1}$	(1.753 ± 0.017)
$pp \to W^+e^+\tau^-$	(1.058 ± 0.096)	(2.861 ± 0.011)
$pp \to W^+e^-\tau^+$	(1.056 ± 0.095)	(2.854 ± 0.011)
$pp \to W^-e^+\tau^-$	$(5.714 \pm 0.015) \cdot 10^{-1}$	(1.754 ± 0.015)
$pp \to W^-e^-\tau^+$	$(5.750 \pm 0.015) \cdot 10^{-1}$	(1.744 ± 0.019)
$pp \to W^+\mu^+\mu^-$	$(1.676 \pm 0.014) \cdot 10^2$	$(4.116 \pm 0.023) \cdot 10^2$
$pp \to W^-\mu^+\mu^-$	$(9.242 \pm 0.033) \cdot 10$	$(2.677 \pm 0.035) \cdot 10^2$
$pp \to W^+\mu^+\tau^-$	$(2.668 \pm 0.024) \cdot 10^{-1}$	$(7.092 \pm 0.028) \cdot 10^{-1}$
$pp \to W^+\mu^-\tau^+$	$(2.652 \pm 0.026) \cdot 10^{-1}$	$(7.187 \pm 0.029) \cdot 10^{-1}$
$pp \to W^-\mu^+\tau^-$	$(1.432 \pm 0.006) \cdot 10^{-1}$	$(4.424 \pm 0.038) \cdot 10^{-1}$
$pp \to W^-\mu^-\tau^+$	$(1.439 \pm 0.004) \cdot 10^{-1}$	$(4.433 \pm 0.037) \cdot 10^{-1}$
$pp \to W^+\tau^+\tau^-$	$(1.665 \pm 0.023) \cdot 10^2$	$(4.138 \pm 0.063) \cdot 10^2$
$pp \to W^-\tau^+\tau^-$	$(9.265 \pm 0.034) \cdot 10$	$(2.652 \pm 0.035) \cdot 10^2$
$pp \to W^-e^+e^+$	$(4.711 \pm 0.069) \cdot 10^{-12}$	$(4.847 \pm 0.030) \cdot 10^{-11}$
$pp \to W^+e^-e^-$	$(1.423 \pm 0.008) \cdot 10^{-12}$	$(1.818 \pm 0.071) \cdot 10^{-11}$
$pp \to W^-e^+\mu^+$	$(1.017 \pm 0.014) \cdot 10^{-11}$	$(9.869 \pm 0.054) \cdot 10^{-11}$
$pp \to W^+e^-\mu^-$	$(3.184 \pm 0.015) \cdot 10^{-12}$	$(3.22 \pm 0.15) \cdot 10^{-11}$
$pp \to W^-e^+\tau^+$	$(1.169 \pm 0.015) \cdot 10^{-11}$	$(1.050 \pm 0.054) \cdot 10^{-10}$
$pp \to W^+e^-\tau^-$	$(4.173 \pm 0.020) \cdot 10^{-12}$	$(4.12 \pm 0.28) \cdot 10^{-11}$
$pp \to W^-\mu^+\mu^+$	$(5.861 \pm 0.082) \cdot 10^{-9}$	$(2.278 \pm 0.013) \cdot 10^{-8}$
$pp \to W^+\mu^-\mu^-$	$(2.377 \pm 0.010) \cdot 10^{-9}$	$(1.153 \pm 0.017) \cdot 10^{-8}$
$pp \to W^-\mu^+\tau^+$	$(1.184 \pm 0.013) \cdot 10^{-8}$	$(4.584 \pm 0.023) \cdot 10^{-8}$
$pp \to W^+\mu^-\tau^-$	$(4.788 \pm 0.018) \cdot 10^{-9}$	$(2.363 \pm 0.039) \cdot 10^{-8}$
$pp \to W^-\tau^+\tau^+$	$(5.956 \pm 0.080) \cdot 10^{-9}$	$(2.292 \pm 0.031) \cdot 10^{-8}$
$pp \to W^+\tau^-\tau^-$	$(2.383 \pm 0.010) \cdot 10^{-9}$	$(1.120 \pm 0.014) \cdot 10^{-8}$

Table 5.2. – Cross-sections for the processes with $W^\pm\ell^\pm\ell^\pm$ as final states (lepton number violating processes in lower section). A cut on the invariant lepton mass of 10 GeV has been assumed. *Taken from Ref. [137].*

the pair will cancel each other almost exactly. The only remaining effect is due to the small mass difference between the two states. This mass difference is an artifact of the mixing with the light neutrinos and therefore of the order of m_ν, which is extremely small.

For experimental reasons we have to include some additional considerations. First, we require a cut of 10 GeV on the invariant mass of the leptons. If we would not apply this cut, another diagram with a nearly on-shell photon becomes dominant in the case of e^+e^- final states leading to several orders of magnitude larger cross-sections. Second, we have to take into account that the emitted W boson will decay further in the detector into leptons and quarks. In the former case it would be more difficult to identify the LFV of the primary leptons. This will reduce the effective cross-section by a factor proportional to the branching ratio of W into quarks. Since LFV does not occur in the SM it will be easy to suppress the background for these signals. In the case of flavor conserving final states the main background comes from multi W-production in association with a Z-boson or off-shell photon. This background can be reduced by cuts on the invariant mass of the leptons.

The process $q\bar{q} \to l^+l'^+W^-W^-, l^-l'^-W^+W^+$

We will now consider the $2 \to 4$ processes. We do not consider the ones with a Z boson in the final state. These can in principle be obtained by attaching a Z at any line in the diagram of Fig. 5.3. Instead we focus on the ones with two final state W bosons. In this case we obtain for the lepton number conserving processes a similar picture as in the $2 \to 3$ case, with cross-sections roughly two orders of magnitude smaller. We again observe sizable LFV. It might be unexpected that also the cross-sections of the LNV processes are quite large. One might anticipate a similar suppression as in the case of the $2 \to 3$ process, since we again have pseudo-Dirac particles as intermediate states. But this time we do not only have contributions proportional to the masses. We can also obtain terms in the amplitude such as $P_R(\not{p} - m)P_L = \not{p}P_L$. The cancellation of the squared mass differences of the pseudo-Dirac pairs does not affect this part of the cross-section. These contributions are proportional to products of LNV couplings $|c_i d_i|^2$ (see Eq. (5.22)). We remember that d_i was the only source of right handed couplings to the W bosons. This combination of couplings can be approximated as (see appendix C)

$$\frac{(a_i\kappa_1 + b_i\kappa_2)^4}{M_N^4 m_\xi^4}\,. \tag{5.25}$$

We observe that contrary to the $2 \to 3$ case, the source for the lepton number violation is not proportional to the light neutrino mass. It is only vanishing if the mediator masses become infinite or one of the couplings goes to zero. This is a consequence of the

Process	σ [fb] (7 TeV)	σ [fb] (14 TeV)
$pp \to W^+e^-W^-e^+$	$(3.447 \pm 0.87) \cdot 10^{-1}$	(1.277 ± 0.66)
$pp \to W^+e^-W^-\mu^+$	$(7.06 \pm 0.15) \cdot 10^{-3}$	$(3.141 \pm 0.027) \cdot 10^{-2}$
$pp \to W^+e^+W^-\mu^-$	$(6.99 \pm 0.16) \cdot 10^{-3}$	$(3.206 \pm 0.027) \cdot 10^{-2}$
$pp \to W^+e^-W^-\tau^+$	$(1.037 \pm 0.020) \cdot 10^{-2}$	$(4.293 \pm 0.036) \cdot 10^{-2}$
$pp \to W^+e^+W^-\tau^-$	$(1.015 \pm 0.021) \cdot 10^{-2}$	$(4.411 \pm 0.036) \cdot 10^{-2}$
$pp \to W^+\mu^-W^-\mu^+$	$(3.74 \pm 0.10) \cdot 10^{-1}$	(1.279 ± 0.017)
$pp \to W^+\mu^-W^-\tau^+$	$(2.913 \pm 0.048) \cdot 10^{-3}$	$(1.096 \pm 0.007) \cdot 10^{-1}$
$pp \to W^+\mu^+W^-\tau^-$	$(2.990 \pm 0.042) \cdot 10^{-2}$	$(1.139 \pm 0.007) \cdot 10^{-1}$
$pp \to W^+\tau^-W^-\tau^+$	$(4.27 \pm 0.10) \cdot 10^{-1}$	(1.606 ± 0.017)
$pp \to W^+e^-W^+e^-$	$(1.112 \pm 0.013) \cdot 10^{-4}$	$(4.261 \pm 0.028) \cdot 10^{-4}$
$pp \to W^+e^-W^+\mu^-$	$(1.537 \pm 0.023) \cdot 10^{-3}$	$(5.810 \pm 0.050) \cdot 10^{-3}$
$pp \to W^+e^-W^+\tau^-$	$(4.721 \pm 0.055) \cdot 10^{-3}$	$(1.761 \pm 0.016) \cdot 10^{-2}$
$pp \to W^+\mu^-W^+\mu^-$	$(4.099 \pm 0.052) \cdot 10^{-3}$	$(1.514 \pm 0.013) \cdot 10^{-2}$
$pp \to W^+\mu^-W^+\tau^-$	$(2.704 \pm 0.036) \cdot 10^{-2}$	$(1.062 \pm 0.093) \cdot 10^{-1}$
$pp \to W^+\tau^-W^+\tau^-$	$(4.614 \pm 0.065) \cdot 10^{-2}$	$(1.729 \pm 0.016) \cdot 10^{-1}$

Table 5.3. – Cross-sections for the processes with $W^+\ell^-W^\pm\ell^\mp$ as final states (lepton number violating processes in lower section). A cut on the invariant lepton mass of 10 GeV has been assumed. *Taken from Ref. [137].*

vector-like SU(2) nature of the ξ doublets: In our model we have a mass term for the two doublets ξ and ξ'. This leads to the right-handed couplings of the corresponding spinors with the W bosons, which as discussed above induces the LNV processes. If we want to observe those states at the LHC we would require luminosities of $\mathcal{O}(\text{ab}^{-1})$.

6. GUT Inspired Extension of the Model

The introduction of new particles in the model discussed in the previous chapter, as well as in other potential UV completions of effective operators, modifies the running of gauge couplings and therefore spoils unification. Hence we want to discuss in the following how this model can be embedded in the framework of SUSY GUTs.

6.1. Introduction to GUT theories

A detailed introduction to the vast field of GUT physics is far beyond the scope of this thesis. We will, however, present a short overview of the basic concepts relevant for the following discussion. For more details see, *e.g.*, Ref. [163].

In the past, physicists were successful in uniting forces into a common framework that previously were considered distinct phenomena. A well known example from electrodynamics is the unification of the electric and magnetic force as described by the MAXWELL equations. Another one is the unification of the electromagnetic and the weak sector in the SM. How the electroweak and strong force as well as gravity can be described in a common framework is so far unknown, although these issues are addressed in the fields of string theory or quantum loop gravity. GUT theories, instead, can give a coherent description of all three forces of the SM. This is done by introducing a new gauge group for instance SU(5) or SO(10), which contains the SM gauge group $SU(3) \times SU(2) \times U(1)$ as subgroup. The GUT group is assumed to be broken down to the SM group by a spontaneous symmetry breaking mechanism. The unification of the SM forces requires that the according gauge coupling constants meet all at the same scale. As pointed out in chapter 2, this is the case in SUSY models.

In the following we want to focus on GUT models with an SU(5) gauge group, or a gauge group that contains an SU(5) subgroup. The fundamental representation of SU(5) is a 5-plet. The matter content of the SM is usually described by a $\bar{5}$ and 10 representation of SU(5) in the following way:

$$\bar{5}_M = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e^- \\ -\nu_e \end{pmatrix}, \qquad \text{and} \quad 10 = \begin{pmatrix} 0 & u_3^c & -u_2^c & -u_1 & -d_1 \\ -u_3^c & 0 & u_1^c & -u_2 & -d_2 \\ u_2^c & -u_1^c & 0 & -u_3 & -d_3 \\ u_1 & u_2 & u_3 & 0 & -e^+ \\ d_1 & d_2 & d_3 & e^+ & 0 \end{pmatrix} \tag{6.1}$$

The Higgs doublets H_u and H_d are the two lower components of a 5 and $\bar{5}$ representation. The upper components of these 5-plets are so called colored Higgs fields, which are assumed to be heavy and therefore have negligible effects at scales much smaller than the GUT scale. The gauge fields are part of the adjoint representation of SU(5), which is a 24-plet. This 24-plet can be decomposed in the following way

$$\hat{A}_\mu = \frac{1}{2}\sum_{a=1}^{24} A_\mu^a \tilde{\lambda}_a = \frac{1}{2}\left[\sum_{a=1}^{8} G_a^\mu \tilde{\lambda}_a + \sum_{a=9}^{20} A_\mu^a \tilde{\lambda}_a + \sum_{a=21}^{23} A_\mu^a \tilde{\lambda}_a + B_\mu \tilde{\lambda}_{24}\right]. \tag{6.2}$$

The first sum in the equation above corresponds to the 8 gluons of $SU(3)$. The last two terms represent the W and B gauge fields of $SU(2)$ and $U(1)$ that become the W, Z and γ bosons after EWSB. The second term contains so called leptoquarks that mediate interactions between quarks and leptons – as implied by the name.

Such minimal GUT models as presented here face several problems. Leptoquark interactions, *e.g.*, imply the decay of the proton, which has an experimentally confirmed lifetime of at least 10^{33} years [164], so that these leptoquarks must be extremely heavy. Furthermore, it is particularly difficult to obtain the so-called doublet triplet splitting between the heavy colored Higgs components and the SM Higgs components, with masses at the electroweak scale. To achieve this, one usually has to accept a large amount of fine-tuning. The breaking of the GUT group also requires the presence of additional fields. A more general issue is the lack of unification of gauge interactions with gravity and the difference between the GUT and the Planck scale. Models that provide solutions to at least some of these problems are discussed in literature, see, *e.g.*, Ref. [165]. How this is done in detail cannot be discussed at this place and is not relevant for the following discussion.

6.2. GUT completion for the decomposition of a d=7 neutrino mass operator.

If additional fields are present that are charged under at least one of the SM symmetry groups, the running of the coupling constants will be modified. As a consequence, one can spoil the unification of the coupling constants. This is for example the case for the type II and type III seesaw models [152, 154, 156, 166–172]. Another example is the addition of the SU(2) doublets ξ and ξ' in the model discussed in chapter 5. To avoid these problems one can embed these doublets into complete representations of SU(5) [139]. The field content of our GUT extended model is then, besides the matter fields from Eq. (6.1),

$$\bar{5}_{\xi'} = \begin{pmatrix} d'^c \\ \xi' \end{pmatrix}, \quad 5_\xi = \begin{pmatrix} d'' \\ \xi \end{pmatrix}, \quad H_5 = \begin{pmatrix} H_{\rm col} \\ H_u \end{pmatrix}, \quad H_{\bar{5}} = \begin{pmatrix} H'_{\rm col} \\ H_d \end{pmatrix}, \tag{6.3}$$

where the upper component corresponds to three colored states and the lower one to the SU(2) doublets. Furthermore we have the SM-singlet fields N and N', which are also singlets under SU(5). As mentioned above, additional fields are usually required in these SUSY GUT models but are not relevant for this study.

The group properties of SU(5) now tell us, in which way these fields can be combined. The possible SU(5) invariant terms are

- Terms from $\bar{\mathbf{5}} \otimes \mathbf{5} (\otimes \mathbf{1})$

$$N \left(5_\xi\, H_{\bar{5}} + \bar{5}_{\xi'}\, H_5 + \bar{5}_M\, H_5\right) + (N \leftrightarrow N') \tag{6.4a}$$

$$m_{\xi'}\, \bar{5}_M\, 5_\xi \tag{6.4b}$$

$$m_\xi\, \bar{5}_{\xi'}\, 5_\xi \tag{6.4c}$$

- Terms from $\bar{\mathbf{5}} \otimes \mathbf{10} \otimes \bar{\mathbf{5}}$

$$\bar{5}_M\, 10\, H_{\bar{5}} \tag{6.5a}$$

$$\text{and} \qquad \bar{5}_{\xi'}\, 10\, H_{\bar{5}} \tag{6.5b}$$

- Terms from $\mathbf{10} \otimes \mathbf{10} \otimes \mathbf{5}$

$$10\, 10\, H_5 \tag{6.6}$$

With these terms we can now construct the most general superpotential that is invariant under SU(5) and has the same field content as our model:

$$\begin{aligned} W \;=\; & y_1\, N\, 5_\xi\, H_{\bar{5}} + y_2\, N\, \bar{5}_{\xi'}\, H_5 + y_3\, N\, \bar{5}_M\, H_5 + \\ & y'_1, N'\, 5_\xi\, H_{\bar{5}} + y'_2\, N'\, \bar{5}_{\xi'}\, H_5 + y'_3\, N'\, \bar{5}_M\, H_5 + \\ & m_{\xi'}\, \bar{5}_M\, 5_\xi + m_\xi\, \bar{5}_{\xi'}\, 5_\xi + m_N N' N + \\ & m_{NN} N N + m_{N'N'} N' N' + y_d\, \bar{5}_M\, 10\, H_{\bar{5}} + \\ & y'_d\, \bar{5}_{\xi'}\, 10\, H_{\bar{5}} + y_u\, 10\, 10\, H_5 - \mu H_{\bar{5}} H_5\,. \end{aligned} \tag{6.7}$$

Multiplet	$\bar{5}_M$	H_5	$H_{\bar{5}}$	N	N'	5_ξ	$\bar{5}_{\xi'}$	10
$\mathbb{Z}_3$ **charge**	1	1	1	1	2	0	0	1

Table 6.1. – Possible $\mathbb{Z}_3$ assignments to forbid the Weinberg operator. *Taken from Ref. [139].*

This superpotential, however, leads not to a $d = 7$ operator as leading contribution to neutrino mass. If one integrates out the fields N and N' one obtains a $d = 5$ operator, generating an effective Majorana neutrino mass $m_\nu \propto v_u^2 y_3 y_3'/m_N$. This means that one has to forbid one of the couplings y_3 or y_3' to avoid the WEINBERG operator. This aim can be accomplished with the same method we used in the previous chapter: We will assume that the fields are charged under an additional discrete symmetry. This symmetry will forbid the $d = 5$ operator, if, *e.g.*, the coupling proportional to y_3' is not invariant under this symmetry. One possible realization is an additional $\mathbb{Z}_3$ symmetry under which the fields are charged as listed in Tab. 6.1. The most general superpotential that is invariant under SU(5) as well as the $\mathbb{Z}_3$ reads then

$$\begin{aligned} W &= y_3\, N\, \bar{5}_M\, H_5 + y_1'\, N'\, 5_\xi\, H_{\bar{5}} + y_2'\, N'\, \bar{5}_{\xi'}\, H_5 + \\ &\quad m_\xi\, \bar{5}_{\xi'}\, 5_\xi + m_N N' N \\ &\quad y_d\, \bar{5}_M\, 10\, H_{\bar{5}} + y_u\, 10\, 10\, H_5 - \mu H_{\bar{5}} H_5 \,. \end{aligned} \tag{6.8}$$

If we replace y_3 with Y_N and $y'_{1/2}$ with $\kappa_{1/2}$, we now can reproduce the superpotential from Eq. (5.1), but obtain additional terms for d' and d''. In the superpotential shown above we also see that the term $\mu H_{\bar{5}} H_5$ explicitly breaks the discrete symmetry.

6.3. The model as an extension of the NMSSM

As discussed in subsection 2.3.2 of chapter 2 the value of the μ term at the electroweak scale seems somewhat unnatural in a GUT theory, with a typical mass scale several orders of magnitude higher. In the NMSSM this issue is avoided since instead of the μ-term one has the coupling $\lambda S H_u H_d$ that dynamically generates $\mu = \lambda \langle S \rangle$. A straight forward implementation of our model as a GUT extension of the NMSSM could then read

$$\begin{aligned} W &= y_3\, N\, \bar{5}_M\, H_5 + y_1',\, N'\, 5_\xi\, H_{\bar{5}} + y_2'\, N'\, \bar{5}_{\xi'}\, H_5 + \\ &\quad m_\xi\, \bar{5}_{\xi'}\, 5_\xi + m_N N' N \\ &\quad y_d\, \bar{5}_M\, 10\, H_{\bar{5}} + y_u\, 10\, 10\, H_5 - \lambda S H_{\bar{5}} H_5 + \kappa S^3 \,. \end{aligned} \tag{6.9}$$

With the same charges as before, but $q(S) = 1$. We can also avoid the explicit breaking of the discrete symmetry, this way. This scenario is, however, not distinct from the one discussed in the previous section after S obtains a VEV. The only differences are the ones that usually appear by promoting the MSSM to the NMSSM.

A further issue has to be considered with respect to the model of Eq. (6.8). Also the masses of the mediators could be considered more natural, if they were of the order of the GUT scale. Here, instead, they have to be set manually to the TeV scale. We will therefore consider an NMSSM based model where all masses are generated dynamically. The most general superpotential that fulfills these assumption and is SU(5) invariant reads

$$\begin{aligned} W &= y_1\, N\, 5_\xi\, H_{\bar{5}} + y_2\, N\, \bar{5}_{\xi'}\, H_5 + y_3\, N\, \bar{5}_M\, H_5 + \\ &\quad y_1',\, N'\, 5_\xi\, H_{\bar{5}} + y_2'\, N'\, \bar{5}_{\xi'}\, H_5 + y_3'\, N'\, \bar{5}_M\, H_5 + \\ &\quad \lambda_{\xi'}\, S\, \bar{5}_M\, 5_\xi + \lambda_\xi\, S\, \bar{5}_{\xi'}\, 5_\xi + \lambda_N S\, N' N + \\ &\quad \lambda_{NN} S\, NN + \lambda_{N'N'} S\, N' N' + \\ &\quad y_d\, \bar{5}_M\, 10\, H_{\bar{5}} + y_d'\, \bar{5}_{\xi'}\, 10\, H_{\bar{5}} + y_u\, 10\, 10\, H_5 \,. \end{aligned} \tag{6.10}$$

If we want to derive the operators that generate neutrino masses from this potential now, we observe that they become

$$\frac{1}{\langle S \rangle} LLH_uH_u, \qquad\qquad \frac{1}{\langle S \rangle^3} (LLH_uH_d)(H_uH_d) \,.$$

Since $\langle S \rangle$ breaks any discrete symmetry under which it is charged, we have to be more careful in order to find a discrete symmetry that leads to neutrino masses generated by a higher-dimensional operator. This symmetry and the according charges of the fields have to be chosen in a way that fulfills the following criteria:

- All terms that are necessary to reproduce the generic NMSSM sector of our model must be allowed.
- Terms that lead to the Weinberg operator must be forbidden.
- A higher-dimensional operator generating neutrino masses must be allowed.

Multiplet	$\bar{5}_M$	H_5	$H_{\bar{5}}$	N	N'	5_ξ	$\bar{5}_{\xi'}$	10	S	S'
$\mathbb{Z}_3$ **charge**	1	1	1	1	2	0	0	1	1	0

Table 6.2. – Charges for the fields of the model defined in Eq. (6.12). *Taken from Ref. [139].*

As we show in appendix D it is not possible to fulfill all these requirements simultaneously. Instead the existence of any higher dimensional operator of the type

$$\mathcal{O}^d = \frac{1}{\langle S \rangle^{1+2k+l}} (LLH_uH_u)(H_uH_d)^k S^l \tag{6.11}$$

also implies unavoidably the existence of the $d = 5$ operator. We conclude therefore that in order to obtain an NMSSM-like model that generates all masses dynamically and produces neutrino masses by a higher dimensional operator we have to think of a further extension. The most simplistic strategy, is to add just another singlet S', which also must obtain a VEV. One can convince oneself that this is not possible without introducing new symmetry breaking terms. But if we break the symmetry, it is again not possible to forbid the $d = 5$ operator. We can, however, assume that the symmetry breaking couplings are very small and in this sense can be considered T'HOOFT natural. If they are sufficiently small, the symmetry breaking $d = 5$ operator will be only a subdominat contribution to neutrino mass. We will now present a model that exemplifies these arguments by introducing a singlet S' and implying the charges as specified in Tab. 6.2. The according superpotential is then

$$\begin{aligned} W = \; & y_3\, N\, \overline{5}_M\, H_5 + y_1',\, N'\, 5_\xi\, H_{\overline{5}} + y_2'\, N'\, \overline{5}_{\xi'}\, H_5 + \\ & \lambda_\xi\, S'\, \overline{5}_{\xi'}\, 5_\xi + \lambda_N S'\, N' N + y_d\, \overline{5}_M\, 10\, H_{\overline{5}} + \\ & y_u\, 10\, 10\, H_5 + \lambda_S S H_{\overline{5}} H_5 + \kappa S^3 + \kappa' S'^3 + \\ & \lambda_S' S' H_{\overline{5}} H_5 + y_3'\, N'\, \overline{5}_M\, H_5 + y_d'\, \overline{5}_\xi\, 10\, H_{\overline{5}} + \cdots \,, \end{aligned} \tag{6.12}$$

The terms that break the symmetry are the ones in the last line. S' can obtain a VEV via the therm $\lambda_S' S' H_{\overline{5}} H_5$. Even if λ_S' is very small, it is possible to obtain a VEV of the order of the TeV scale. Due to the term $y_3'\, N'\, \overline{5}_M\, H_5$ we will obtain via a $d = 5$ operator an effective neutrino mass term that reads

$$m_\nu^{d=5} = \frac{y_3 y_3' v_u^2}{\langle S' \rangle} \,. \tag{6.13}$$

Since we want that this mass is only a subdominant contribution to neutrino mass we require

$$m_\nu^{d=5} \ll m_\nu^{d=7} = \frac{y_1 y_2 y_3^2 v_u^3 v_d}{\langle S' \rangle^3}\,, \tag{6.14}$$

If we assume that the symmetry conserving couplings are of the order 10^{-2} as used in the previous chapter and $\langle S' \rangle$ is of order TeV, we find that this condition is fulfilled if all symmetry breaking couplings, including y_3' are smaller than 10^{-8}.

6.4. A possible origin of $\theta_{13} > 0$

It is interesting to observe that in the scenario described in the previous section we have two contributions to neutrino mass. The leading one is induced by a $d = 7$ operator and conserves a discrete symmetry. The second contribution is introduced by a $d = 5$ operator, that is additionally suppressed, since it is generated by symmetry breaking couplings. If the size of these couplings is close to the upper limit of 10^{-8} derived above, also this second contribution will be significant for the effective neutrino mass. It is possible that the discrete symmetry we use here might be the remnant of a larger flavor symmetry. If this is the case, one can think of a scenario where the leading $d = 7$ operator leads to a tri-bimaximal mass matrix, whereas the $d = 5$ contribution of the symmetry breaking introduces corrections that lead to a non-zero θ_{13}. We will now discuss a parametrization of our model, where this behavior is indeed realized. This is similar to the parametrization shown in Eq. (5.11), but here we require three generations of the singlet fields N and N'. The flavor structure of the coupling constants is given by

$$\begin{aligned} y_1' = \widetilde{y}_1' \begin{pmatrix} 0 \\ 1 \\ \rho \end{pmatrix}, \quad y_2' = \widetilde{y}_2' \begin{pmatrix} 0 \\ -1 \\ \rho \end{pmatrix}, \\ y_3 = \widetilde{y}_3 \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}, \end{aligned} \tag{6.15}$$

where $\widetilde{y}_1$, $\widetilde{y}_2$ and $\widetilde{y}_3$ are numerical parameters, $\rho = \sqrt{m_3/m_2}$ and the mass matrices for N and N' are diagonal. One can now convince oneself that the $d = 7$ operator generates indeed a tri-bimaximal mass matrix

$$\begin{aligned} m_\nu^{d=7} &= \frac{v_u^3 v_d}{\langle S \rangle^3} \cdot y_3 \left[y_1' (y_2')^\mathsf{T} + y_2' (y_1')^\mathsf{T} \right] y_3^\mathsf{T} \\ &= U_\mathrm{TB} \cdot \mathrm{diag}(0, m_2, -m_3) \cdot U_\mathrm{TB}^\mathsf{T}\,, \end{aligned} \tag{6.16}$$

where U_{TB} is the tri-bimaximal mixing matrix. As we now from experiments, the actual mixing matrix U_{PMNS} deviates from the tri-bimaximal structure. This can be parametrized as [173]

$$U_{\text{PMNS}} = \begin{pmatrix} \sqrt{\frac{2}{3}(1-\frac{1}{2}s)} & \frac{1}{\sqrt{3}}(1+s) & \frac{1}{\sqrt{2}}r \\ -\frac{1}{\sqrt{6}}(1+s-a-r) & \frac{1}{\sqrt{3}}(1-\frac{1}{2}s-a+r) & -\frac{1}{2}(1+a) \\ -\frac{1}{\sqrt{6}}(1+s+a+r) & \frac{1}{\sqrt{3}}(1-\frac{1}{2}s+a-r) & \frac{1}{2}(1-a) \end{pmatrix}, \tag{6.17}$$

where a, r and s are small linear deviations from the tri-bimaximal structure, with $\sin\theta_{13} = r/\sqrt{2}$, $\sin\theta_{12} = 1/\sqrt{3}(1+s)$, and $\sin\theta_{23} = 1/\sqrt{2}(1+a)$. In our case the total neutrino mass is given by

$$m_\nu = m_\nu^{d=7} + m_\nu^{d=5}\,, \tag{6.18}$$

which implies

$$U_{\text{PMNS}} \cdot \text{diag}(0, m_2, -m_3) \cdot U_{\text{PMNS}}^{\mathsf{T}} \tag{6.19}$$

$$= U_{\text{TB}} \cdot \text{diag}(0, m_2, -m_3) \cdot U_{\text{TB}}^{\mathsf{T}} + \frac{v_u^2}{\langle S \rangle}(y_3(y_3')^{\mathsf{T}} + y_3'(y_3)^{\mathsf{T}})\,. \tag{6.20}$$

For simplicity, let us assume only corrections to θ_{13}. From Eq. (6.19) we obtain then a condition on the symmetry breaking coupling

$$y_3' \simeq \tilde{y}_3'\, r \left[\begin{pmatrix} 0 & 0 & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{2\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & -\frac{1}{2\sqrt{3}} & 0 \end{pmatrix} \rho^2 - \begin{pmatrix} 0 & 0 & \frac{1}{3\sqrt{2}} \\ 0 & \frac{1}{2\sqrt{3}} & \frac{1}{3\sqrt{2}} \\ 0 & \frac{1}{2\sqrt{3}} & \frac{1}{3\sqrt{2}} \end{pmatrix} \right] \tag{6.21}$$

With this exemplary parametrization, we now indeed achieved that the total neutrino mass matrix is tri-bimaximal with deviations leading to a sizable θ_{13}. The structure of y_3' that is shown above may be caused by a subgroup of the original flavor symmetry that is still unbroken.

6.5. Phenomenology of the additional d-quarks

The extension of our original model to a GUT theory, required the introduction of additional d-quarks (see Eq. (6.3) and the following discussion). These colored states are part of the same 5-plet as ξ and ξ', respectively. Their according mass eigenstates D', composed of $d^{c\prime}$ and d'', and L', composed of ξ and ξ', therefore have a common mass $m_{\xi,\mathrm{GUT}}$ at the unification scale. Renormalization group effects, however, will lead to a splitting between the masses of the colored and the doublet states. As a consequence we will have different masses $m_{L'}$ and $m_{D'}$ at lower energies. The RGEs leading to this splitting are at one-loop order expressed by

$$\frac{d}{dt}m_k = \sum_i \frac{c_i^k \alpha_i(t)}{4\pi} m_k \qquad\qquad (k = L', D') \tag{6.22}$$

where the index i runs over the different gauge groups, $t = \ln(Q^2/M_{GUT}^2)$, $c_i^{D'} = (-4/15, 0, -16/3)$ and $c_i^{\xi} = (-3/5, -3, 0)$ for $i = U(1)_Y, SU(2)_L, SU(3)_C$ and the Yukawa couplings are assumed to be negligible compared to the gauge interactions. The solution of the RGEs yields

$$m_k(t) = \prod_i \left(\frac{\alpha_i(t)}{\alpha_{GUT}}\right)^{\frac{c_i^k}{b_i}} m_{\xi,GUT} \tag{6.23}$$

with $b_i = (38/5, 2, -2)$. Assuming that no further particles are present between the electroweak an the GUT scale we obtain $m_{D'}/m_\xi \simeq 5$ at $Q = 1$ TeV. As discussed in chapter 5, these leptons can be observed for masses of up to 800 GeV at the LHC, implying that the D' can be as heavy as 4 TeV. They can be potentially observed for masses of maximal 3 TeV [174].

6.5.1. Stability of the heavy d-quarks

One could now assume that the heavy d-quarks – due to their large mass – can easily decay. This is, however, not the case (if we, for now, ignore the symmetry breaking NMSSM scenario). This might be unexpected since d' and d'' are part of the same 5-plet as ξ and ξ' and the latter decay to SM particles. As we can read of Eq. (6.12), the only coupling of the heavy d-quarks to other particles is via the terms $y_1' N' 5_\xi H_{\bar{5}}$ and $y_2' N' \bar{5}_{\xi'} H_5$. If we expand these terms now in their SU(5) components we can see that the colored components of the 5_ξ and $\bar{5}_{\xi'}$ interact only with the colored components of the Higgs multiplets, as is required by SU(5) invariance. These interactions therefore are negligible after integrating out the colored Higgs components. Any other possible interactions in the superpotential including the heavy d-quarks are prohibited by exactly the symmetry that we need to forbid the Weinberg operator. We conclude that the additional d-quarks are stabilized by the discrete symmetry of our model. The presence of heavy stable quarks, however, is in conflict with cosmological constraints. To understand this better, we will discuss shortly the thermal evolution of these particles in the early universe.

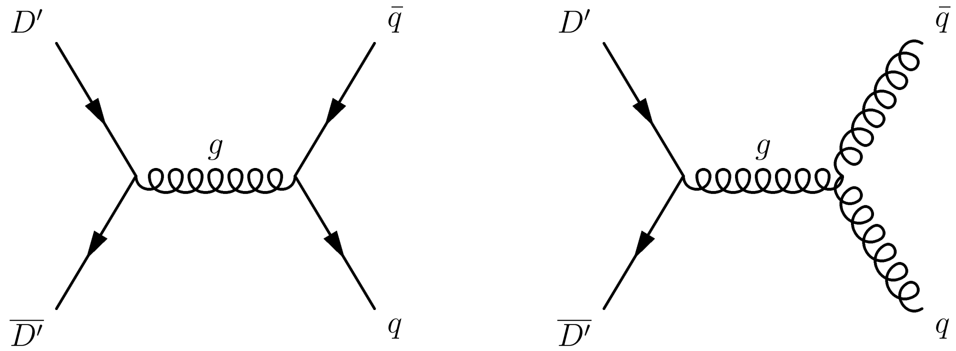

Figure 6.1. – Annihilation of D' via strong interaction. Left: SM quarks as end-states. Right: gluon end-states. (There is a further contribution from t-channel diagram).

6.5.2. Thermal evolution of heavy d-quarks in the early universe

As discussed above, the superpotential of our model does not allow for interactions of the heavy quarks with lighter particles. But since they are colored states, they have gauge interactions leading to the annihilation processes depicted in Fig. 6.1. Despite their heavier mass, the additional d-quarks behave like their SM counterparts in these processes. Therefore one can use the standard QCD cross-sections of SM quarks. For $\overline{D}'D' \to q\bar{q}$ we have

$$\frac{d\sigma}{d\hat{t}}(\overline{D}'D' \to q\bar{q}) = \frac{4\pi\alpha_s^2}{9\hat{s}^2}\left(\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\right) \tag{6.24}$$

and for gluon end-states

$$\frac{d\sigma}{d\hat{t}}(\overline{D}'D' \to gg) = \frac{32\pi\alpha_s^2}{27\hat{s}^2}\left(\frac{\hat{u}}{\hat{t}}+\frac{\hat{t}}{\hat{u}}-\frac{9}{4}\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\right), \tag{6.25}$$

where $\hat{t}$, $\hat{u}$ and $\hat{s}$ are Mandelstam variables. We assume now the following scenarios: The D' quarks will be present in the quark-gluon plasma and in thermal equilibrium with the SM quarks. At some point ($\Gamma < \dot{R}/R$) they will freeze-out, i.e. leave thermal equilibrium. (The cosmology parameters we use here are described in appendix E.) The number density in thermal equilibrium is given by a Fermi distribution

$$n_0(T) = \frac{2}{(2\pi)^3}\int_0^\infty 4\pi p^2 dp\left[\exp\left(\frac{\sqrt{m_{D'}^2+p^2}}{kT}\right)\right] \tag{6.26}$$

and their evolution by the Boltzmann equation

$$\frac{dn}{dt} = -\frac{3\dot{R}}{R}n - \langle\sigma n\rangle n^2 + \langle\sigma n\rangle n_0^2 \tag{6.27}$$

Later, after chiral symmetry breaking, the quarks will be confined. Using the particle yield $Y = n/s$ the Boltzmann equation can be rewritten as

$$\frac{dY}{dx} = -xs\frac{\langle\sigma|v|\rangle}{H(m_{D'})}(Y^2 - Y_0^2) \tag{6.28}$$

with

$$Y_0(x) = \frac{45}{2\pi^4}\left(\frac{\pi}{8}\right)\frac{g}{g_{*S}}x^{3/2}e^{-x} \tag{6.29}$$

in the non relativistic limit ($x \gg 3$). For Cold Relics ($x_f \gtrsim 3$) we can use an approximate formula to calculate particle yield and freeze-out point (see Ref. [175] for more details).

$$\begin{aligned} x_f &= \ln[0.038(n+1)(g/\sqrt{g_*})m_{\text{Pl}}m_{D'}\sigma_0] \\ &\quad - \left(n+\frac{1}{2}\right)\ln\left[\ln[0.038(n+1)(g/\sqrt{g_*})m_{\text{Pl}}m_{D'}\sigma_0]\right] \end{aligned} \tag{6.30}$$

$$Y_\infty = \frac{3.79(n+1)x_f^{n+1}}{g_{*S}/\sqrt{g_*}} \tag{6.31}$$

where $n = 0$ for s-wave annihilation and

$$\langle\sigma|v|\rangle \equiv \sigma_0 x^{-n} \tag{6.32}$$

For one heavy down-type quark of $m = 1\,\text{TeV}$ and neglecting the mass of the light quarks we find that freeze out takes place at

$$x_f = 26\,, \qquad\qquad T_f = 38\,\text{GeV}\,. \tag{6.33}$$

Since this is long before BBN we have

$$Y(T_{\text{BBN}}) \approx Y_\infty = 10^{-14} \tag{6.34}$$

and the corresponding number density is

$$n(T_{\text{BBN}}) \approx Y_\infty s = 10^{-26}\,\text{GeV}^3\ (= 10^{21}\,\text{m}^{-3})\,. \tag{6.35}$$

As we will discuss in the next section, such number densities imply that we must consider cosmological consraints.

6.5.3. Cosmological constraints

Effects due to the presence of heavy particles ($m_X \gg m_e$) during BBN have been discussed in the literature. This is motivated by the possible presence of dark matter during BBN or (charged) parent particles thereof. (See Sec. 9 of Ref. [176].) In this context DM has no gravitational effect since we are in the radiation dominated period of the universe. But massive particles at the EW scale can be easily produced in the thermal bath and are often stable due to additional symmetries introduced in DM scenarios (for instance SUSY with R-parity protecting the LSP from decaying). Whereas any DM candidate should be uncharged and weakly interacting, their parent particles can have different properties, such that nuclear processes during BBN can be influenced. The following two scenarios have been discussed in literature:

Neutral Particles (Cascade Nucleosynthesis)

Neutral massive particles will have different effects depending on possible end-states (electroweak or hadronic). The nuclear reaction chain during nucleosynthesis can be quite complicated. But some general possible effects are a change in the initial n/p ratio, to which BBN is quite sensitive. Another effect is, that by inelastic scattering of heavy nucleons a large number of lighter nuclei can be produced, altering the abundances of light isotopes (^{2}H, ^{3}H, ^{3}He,...).

Charged Particles (Catalyzed Nucleosynthesis)

In this scenario, heavy strongly interacting particles which carry charge can bind to baryons. These new states will alter the reaction chains of BBN and thus affect the yield of light elements.

Also the D' quarks have to build bound states as heavy hadrons, such as dominantly heavy protons ($p' = uuD'$) and heavy pions ($\pi' = u\bar{D}', d\bar{D}', ...$). These heavy neutral ($X^0$) and charged particles ($X^\pm$) can be identified with the heavy particles for which the BBN constraints are studied in Literature.

Thus their presence during Big Bang Nucleosynthesis (BBN) would alter the observed abundances of the light elements in the universe (see, *e.g.*, Ref. [176] for a review). Further bounds come, *e.g.*, from direct heavy element searches in water [177]. More detailed studies [178, 179] taking also effects due to confinement into account, have found that the annihilation rate for these quarks is not sufficient to lower the yield below the experimental limits [178, 179] if their masses are below 2.5 TeV, *i.e.*, the range interesting for the LHC. We conclude therefore that a stable D' can not accomadate cosmological requirements.

6.5.4. A way out

From the discussion above we know that heavy stable d-quarks must have life-times much smaller than the age of the universe to avoid constraints from direct searches, and annihilation processes that are efficient enough to lower their abundance below the bounds from BBN. These bounds do not affect particles that decay before BBN ($\tau \ll 1\,\mathrm{s}$).

As discussed before, the model specified in Eq. (6.12) has symmetry violating couplings. This symmetry breaking is a potential way to avoid the stability of the D'. Indeed we have symmmetry breaking couplings in this scenario that lead to the two-body decays

$$D' \to H^- u \,,\; H^0 d \,. \tag{6.36}$$

If we assume that the symmetry violation is of the same order for all couplings, the upper bound of 10^{-8}, obtained from the neutrino interactions, restricts also the decay rates of above processes. Due to these considerations, we can obtain lifetimes for the D' as small as $10^{-10}\,\mathrm{s}$ to $10^{-13}\,\mathrm{s}$, depending on $m_{D'}$. This is sufficiently short to make decays before BBN possible. If these decays happen at the LHC, one will also observe displaced vertices.

6.6. Systematic review of the decompositions of the d=7 operator in an SU(5) GUT

As already discussed in the previous sections, the $d = 7$ operator can be decomposed in different ways. In Tab. 6.3 we list these possibilities and show how these mediators can be embedded into SU(5) multiplets. How they are embedded depends on the SM quantum numbers of these operators. Operators that are singlets under the SM gauge group remain also singlets under the GUT group. SU(2) triplets 3_0 with zero hypercharge become 24-plets under SU(5). This can be understood since a coupling of the external fields $H_u \mathrm{i}\tau_2 \vec{\tau} L$ to the triplet mediator, becomes the SU(5) invariant coupling $H_5 \bar{5}_M 24$. If SU(2) singlet and triplet mediators are present at the same time, it may be possible to combine them in a single 24-plet. SU(2) triplets with hypercharge $+1$ (-1) are embedded into a 15 $(\bar{15})$ representation. The coupling $\overline{L^c} \mathrm{i}\tau_2 \vec{\tau} L$ to 3_{+1}, *e.g.*, becomes $\bar{5}_M \bar{5}_M 15$ in the GUT superpotential. Finally, doublets with hypercharge $\frac{3}{2}$ are embedded into 40-plets (with couplings, such as $15 \bar{5}_M 40$ to one external field and another mediator).

In order to conserve gauge coupling unification all component fields of these multiplets must have similar masses, which are at the TeV scale. While we can avoid to spoil the unification of the gauge couplings this way, their running will still be modified. The contribution of the additional particles to the MSSM beta functions, calculated using Ref. [180], are listed in Tab. 6.4. The unmodified MSSM beta functions are $b_i = (33/5, 1, -3)$ for $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$. If we compare these values to the

#	Operator	Mediators	SU(5) multiplets
1	$(H_u \mathrm{i}\tau^2 \overline{L^c})(H_u \mathrm{i}\tau^2 L)(H_d \mathrm{i}\tau^2 H_u)$	$\mathbf{1}_0^R,\ \mathbf{1}_0^L,\ \mathbf{1}_0^s$	$\mathbf{1}, \mathbf{1}, \mathbf{1}$
2	$(H_u \mathrm{i}\tau^2 \vec{\tau} \overline{L^c})(H_u \mathrm{i}\tau^2 L)(H_d \mathrm{i}\tau^2 \vec{\tau} H_u)$	$\mathbf{3}_0^R,\ \mathbf{3}_0^L\ \mathbf{1}_0^R,\ \mathbf{1}_0^L,\ \mathbf{3}_0^s$	$\mathbf{24}, \mathbf{24}, (\mathbf{1}), (\mathbf{1}), \mathbf{24}$
3	$(H_u \mathrm{i}\tau^2 \vec{\tau} \overline{L^c})(H_u \mathrm{i}\tau^2 \vec{\tau} L)(H_d \mathrm{i}\tau^2 H_u)$	$\mathbf{3}_0^R,\ \mathbf{3}_0^L,\ \mathbf{1}_0^s$	$\mathbf{24}, \mathbf{24}, \mathbf{1}$
4	$(-\mathrm{i}\epsilon^{abc})(H_u \mathrm{i}\tau^2 \tau^a \overline{L^c})(H_u \mathrm{i}\tau^2 \tau^b L)(H_d \mathrm{i}\tau^2 \tau^c H_u)$	$\mathbf{3}_0^R,\ \mathbf{3}_0^L,\ \mathbf{3}_0^s$	$\mathbf{24}, \mathbf{24}, \mathbf{24}$
5	$(\overline{L^c} \mathrm{i}\tau^2 \vec{\tau} L)(H_d \mathrm{i}\tau^2 H_u)(H_u \mathrm{i}\tau^2 \vec{\tau} H_u)$	$\mathbf{3}_{+1}^s,\ \mathbf{3}_{+1}^s,\ \mathbf{1}_0^s$	$\mathbf{15}, \mathbf{15}, \mathbf{1}$
6	$(-\mathrm{i}\epsilon_{abc})(\overline{L^c} \mathrm{i}\tau^2 \tau_a L)(H_d \mathrm{i}\tau^2 \tau_b H_u)(H_u \mathrm{i}\tau^2 \tau_c H_u)$	$\mathbf{3}_{+1}^s,\ \mathbf{3}_{+1}^s,\ \mathbf{3}_0^s$	$\mathbf{15}, \mathbf{15}, \mathbf{24}$
7	$(H_u \mathrm{i}\tau^2 \overline{L^c})(L \mathrm{i}\tau^2 \vec{\tau} H_d)(H_u \mathrm{i}\tau^2 \vec{\tau} H_u)$	$\mathbf{1}_0^R,\ \mathbf{1}_0^L,\ \mathbf{3}_{-1}^R,\ \mathbf{3}_{-1}^L,\ \mathbf{3}_{+1}^s$	$\mathbf{1}, \mathbf{1}, \mathbf{15}, \overline{\mathbf{15}}, \mathbf{15}$
8	$(-\mathrm{i}\epsilon^{abc})(H_u \mathrm{i}\tau^2 \tau^a \overline{L^c})(L \mathrm{i}\tau^2 \tau^b H_d)(H_u \mathrm{i}\tau^2 \tau^c H_u)$	$\mathbf{3}_0^R,\ \mathbf{3}_0^L,\ \mathbf{3}_{-1}^R,\ \mathbf{3}_{-1}^L,\ \mathbf{3}_{+1}^s$	$\mathbf{24}, \mathbf{24}, \mathbf{15}, \overline{\mathbf{15}}, \mathbf{15}$
9	$(H_u \mathrm{i}\tau^2 \overline{L^c})(\mathrm{i}\tau^2 H_u)(L)(H_d \mathrm{i}\tau^2 H_u)$	$\mathbf{1}_0^R,\ \mathbf{1}_0^L,\ \mathbf{2}_{-1/2}^R,\ \mathbf{2}_{-1/2}^L,\ \mathbf{1}_0^s$	$\mathbf{1}, \mathbf{1}, \mathbf{5}, \bar{\mathbf{5}}, \mathbf{1}$
10	$(H_u \mathrm{i}\tau^2 \vec{\tau} \overline{L^c})(\mathrm{i}\tau^2 \vec{\tau} H_u)(L)(H_d \mathrm{i}\tau^2 H_u)$	$\mathbf{3}_0^R,\ \mathbf{3}_0^L,\ \mathbf{2}_{-1/2}^R,\ \mathbf{2}_{-1/2}^L,\ \mathbf{1}_0^s$	$\mathbf{24}, \mathbf{24}, \mathbf{5}, \bar{\mathbf{5}}, \mathbf{1}$
11	$(H_u \mathrm{i}\tau^2 \overline{L^c})(\mathrm{i}\tau^2 H_u)(\vec{\tau} L)(H_d \mathrm{i}\tau^2 \vec{\tau} H_u)$	$\mathbf{1}_0^R,\ \mathbf{1}_0^L,\ \mathbf{2}_{-1/2}^R,\ \mathbf{2}_{-1/2}^L,\ \mathbf{3}_0^s$	$\mathbf{1}, \mathbf{1}, \mathbf{5}, \bar{\mathbf{5}}, \mathbf{24}$
12	$(H_u \mathrm{i}\tau^2 \tau^a \overline{L^c})(\mathrm{i}\tau^2 \tau^a H_u)(\tau^b L)(H_d \mathrm{i}\tau^2 \tau^b H_u)$	$\mathbf{3}_0^R,\ \mathbf{3}_0^L,\ \mathbf{2}_{-1/2}^R,\ \mathbf{2}_{-1/2}^L,\ \mathbf{3}_0^s$	$\mathbf{24}, \mathbf{24}, \mathbf{5}, \bar{\mathbf{5}}, \mathbf{24}$
13	$(H_u \mathrm{i}\tau^2 \overline{L^c})(L)(\mathrm{i}\tau^2 H_u)(H_d \mathrm{i}\tau^2 H_u)$	$\mathbf{1}_0^R,\ \mathbf{1}_0^L,\ \mathbf{2}_{+1/2}^s,\ \mathbf{1}_0^s$	$\mathbf{1}, \mathbf{1}, \mathbf{5}, \mathbf{1}$
14	$(H_u \mathrm{i}\tau^2 \vec{\tau} \overline{L^c})(\vec{\tau} L)(\mathrm{i}\tau^2 H_u)(H_d \mathrm{i}\tau^2 H_u)$	$\mathbf{3}_0^R,\ \mathbf{3}_0^L,\ \mathbf{2}_{+1/2}^s,\ \mathbf{1}_0^s$	$\mathbf{24}, \mathbf{24}, \mathbf{5}, \mathbf{1}$
15	$(H_u \mathrm{i}\tau^2 \overline{L^c})(L)(\mathrm{i}\tau^2 \vec{\tau} H_u)(H_d \mathrm{i}\tau^2 \vec{\tau} H_u)$	$\mathbf{1}_0^R,\ \mathbf{1}_0^L,\ \mathbf{2}_{+1/2}^s,\ \mathbf{3}_0^s$	$\mathbf{1}, \mathbf{1}, \mathbf{5}, \mathbf{24}$
16	$(H_u \mathrm{i}\tau^2 \tau^a \overline{L^c})(\tau^a L)(\mathrm{i}\tau^2 \tau^b H_u)(H_d \mathrm{i}\tau^2 \tau^b H_u)$	$\mathbf{3}_0^R,\ \mathbf{3}_0^L,\ \mathbf{2}_{+1/2}^s,\ \mathbf{3}_0^s$	$\mathbf{24}, \mathbf{24}, \mathbf{5}, \mathbf{24}$
17	$(H_u \mathrm{i}\tau^2 \overline{L^c})(H_d)(\mathrm{i}\tau^2 H_u)(H_u \mathrm{i}\tau^2 L)$	$\mathbf{1}_0^R,\ \mathbf{1}_0^L,\ \mathbf{2}_{-1/2}^R,\ \mathbf{2}_{-1/2}^L$	$\mathbf{1}, \mathbf{1}, \mathbf{5}, \bar{\mathbf{5}}$
18	$(H_u \mathrm{i}\tau^2 \vec{\tau} \overline{L^c})(\vec{\tau} H_d)(\mathrm{i}\tau^2 H_u)(H_u \mathrm{i}\tau^2 L)$	$\mathbf{3}_0^R,\ \mathbf{3}_0^L,\ \mathbf{2}_{-1/2}^R,\ \mathbf{2}_{-1/2}^L,\ \mathbf{1}_0^R,\ \mathbf{1}_0^L$	$\mathbf{24}, \mathbf{24}, \mathbf{5}, \bar{\mathbf{5}}, (\mathbf{1}), (\mathbf{1})$
19	$(H_u \mathrm{i}\tau^2 \overline{L^c})(H_d)(\mathrm{i}\tau^2 \vec{\tau} H_u)(H_u \mathrm{i}\tau^2 \vec{\tau} L)$	$\mathbf{1}_0^R,\ \mathbf{1}_0^L,\ \mathbf{2}_{-1/2}^R,\ \mathbf{2}_{-1/2}^L,\ \mathbf{3}_0^R,\ \mathbf{3}_0^L$	$(\mathbf{1}), (\mathbf{1}), \mathbf{5}, \bar{\mathbf{5}}, \mathbf{24}, \mathbf{24}$
20	$(H_u \mathrm{i}\tau^2 \tau^a \overline{L^c})(\tau^a H_d)(\mathrm{i}\tau^2 \tau^b H_u)(H_u \mathrm{i}\tau^2 \tau^b L)$	$\mathbf{3}_0^R,\ \mathbf{3}_0^L,\ \mathbf{2}_{-1/2}^R,\ \mathbf{2}_{-1/2}^L,$	$\mathbf{24}, \mathbf{24}, \mathbf{5}, \bar{\mathbf{5}}$
21	$(\overline{L^c} \mathrm{i}\tau^2 \tau^a L)(H_u \mathrm{i}\tau^2 \tau^a)(\tau^b H_d)(H_u \mathrm{i}\tau^2 \tau^b H_u)$	$\mathbf{3}_{+1}^s,\ \mathbf{2}_{+1/2}^s\ ,\ \mathbf{3}_{+1}^s$	$\mathbf{15}, \mathbf{5}, \mathbf{15}$
22	$(\overline{L^c} \mathrm{i}\tau^2 \tau^a L)(H_d \mathrm{i}\tau^2 \tau^a)(\tau^b H_u)(H_u \mathrm{i}\tau^2 \tau^b H_u)$	$\mathbf{3}_{+1}^s,\ \mathbf{2}_{+3/2}^s,\ \mathbf{3}_{+1}^s$	$\mathbf{15}, \mathbf{40}, \mathbf{15}$
23	$(\overline{L^c} \mathrm{i}\tau^2 \vec{\tau} L)(H_u \mathrm{i}\tau^2 \vec{\tau})(H_u)(H_d \mathrm{i}\tau^2 H_u)$	$\mathbf{3}_{+1}^s,\ \mathbf{2}_{+1/2}^s,\ \mathbf{1}_0^s$	$\mathbf{15}, \mathbf{5}, \mathbf{1}$
24	$(\overline{L^c} \mathrm{i}\tau^2 \tau^a L)(H_u \mathrm{i}\tau^2 \tau^a)(\tau^b H_u)(H_d \mathrm{i}\tau^2 \tau^b H_u)$	$\mathbf{3}_{+1}^s,\ \mathbf{2}_{+1/2}^s,\ \mathbf{3}_0^s$	$\mathbf{15}, \mathbf{5}, \mathbf{24}$
25	$(H_d \mathrm{i}\tau^2 H_u)(\overline{L^c} \mathrm{i}\tau^2)(\vec{\tau} L)(H_u \mathrm{i}\tau^2 \vec{\tau} H_u)$	$\mathbf{1}_0^s,\ \mathbf{2}_{+1/2}^L,\ \mathbf{2}_{+1/2}^R,\ \mathbf{3}_{+1}^s$	$\mathbf{1}, \mathbf{5}, \bar{\mathbf{5}}, \mathbf{15}$
26	$(H_d \mathrm{i}\tau^2 \tau^a H_u)(\overline{L^c} \mathrm{i}\tau^2 \tau^a)(\tau^b L)(H_u \mathrm{i}\tau^2 \tau^b H_u)$	$\mathbf{3}_0^s,\ \mathbf{2}_{+1/2}^L,\ \mathbf{2}_{+1/2}^R,\ \mathbf{3}_{+1}^s$	$\mathbf{24}, \mathbf{5}, \bar{\mathbf{5}}, \mathbf{15}$
27	$(H_u \mathrm{i}\tau^2 \overline{L^c})(\mathrm{i}\tau^2 H_d)(\vec{\tau} L)(H_u \mathrm{i}\tau^2 \vec{\tau} H_u)$	$\mathbf{1}_0^R,\ \mathbf{1}_0^L,\ \mathbf{2}_{+1/2}^R,\ \mathbf{2}_{+1/2}^L,\ \mathbf{3}_{+1}^s$	$\mathbf{1}, \mathbf{1}, \mathbf{5}, \bar{\mathbf{5}}, \mathbf{15}$
28	$(H_u \mathrm{i}\tau^2 \tau^a \overline{L^c})(\mathrm{i}\tau^2 \tau^a H_d)(\tau^b L)(H_u \mathrm{i}\tau^2 \tau^b H_u)$	$\mathbf{3}_0^R,\ \mathbf{3}_0^L,\ \mathbf{2}_{+1/2}^R,\ \mathbf{2}_{+1/2}^L,\ \mathbf{3}_{+1}^s$	$\mathbf{24}, \mathbf{24}, \mathbf{5}, \bar{\mathbf{5}}, \mathbf{15}$
29	$(H_u \mathrm{i}\tau^2 \overline{L^c})(L)(\mathrm{i}\tau^2 \vec{\tau} H_d)(H_u \mathrm{i}\tau^2 \vec{\tau} H_u)$	$\mathbf{1}_0^R,\ \mathbf{1}_0^L,\ \mathbf{2}_{+1/2}^s,\ \mathbf{3}_{+1}^s$	$\mathbf{1}, \mathbf{1}, \mathbf{5}, \mathbf{15}$
30	$(H_u \mathrm{i}\tau^2 \tau^a \overline{L^c})(\tau^a L)(\mathrm{i}\tau^2 \tau^b H_d)(H_u \mathrm{i}\tau^2 \tau^b H_u)$	$\mathbf{3}_0^R,\ \mathbf{3}_0^L,\ \mathbf{2}_{+1/2}^s,\ \mathbf{3}_{+1}^s$	$\mathbf{24}, \mathbf{24}, \mathbf{5}, \mathbf{15}$
31	$(\overline{L^c} \mathrm{i}\tau^2 \tau^a H_d)(\mathrm{i}\tau^2 \tau^a H_u)(\tau^b L)(H_u \mathrm{i}\tau^2 \tau^b H_u)$	$\mathbf{3}_{+1}^L,\ \mathbf{3}_{+1}^R,\ \mathbf{2}_{+1/2}^L,\ \mathbf{2}_{+1/2}^R,\ \mathbf{3}_{+1}^s$	$\mathbf{15}, \overline{\mathbf{15}}, \mathbf{5}, \bar{\mathbf{5}}, \mathbf{15}$
32	$(\overline{L^c} \mathrm{i}\tau^2 \tau^a H_d)(\tau^a L)(\mathrm{i}\tau^2 \tau^b H_u)(H_u \mathrm{i}\tau^2 \tau^b H_u)$	$\mathbf{3}_{+1}^L,\ \mathbf{3}_{+1}^R,\ \mathbf{2}_{+3/2}^s,\ \mathbf{3}_{+1}^s$	$\mathbf{15}, \overline{\mathbf{15}}, \mathbf{40}, \mathbf{15}$
33	$(\overline{L^c} \mathrm{i}\tau^2 \vec{\tau} H_d(\mathrm{i}\tau^2 \vec{\tau} H_u)(H_u)(H_u \mathrm{i}\tau^2 L)$	$\mathbf{3}_{+1}^L,\ \mathbf{3}_{+1}^R,\ \mathbf{2}_{+1/2}^L,\ \mathbf{2}_{+1/2}^R,\ \mathbf{1}_0^L,\ \mathbf{1}_0^R$	$\mathbf{15}, \overline{\mathbf{15}}, \mathbf{5}, \bar{\mathbf{5}}, \mathbf{1}, \mathbf{1}$
34	$(\overline{L^c} \mathrm{i}\tau^2 \tau^a H_d)(\mathrm{i}\tau^2 \tau^a H_u)(\tau^b H_u)(H_u \mathrm{i}\tau^2 \tau^b L)$	$\mathbf{3}_{+1}^L,\ \mathbf{3}_{+1}^R,\ \mathbf{2}_{+1/2}^L,\ \mathbf{2}_{+1/2}^R,\ \mathbf{3}_0^L,\ \mathbf{3}_0^R$	$\mathbf{15}, \overline{\mathbf{15}}, \mathbf{5}, \bar{\mathbf{5}}, \mathbf{24}, \mathbf{24}$

Table 6.3. – Decompositions of the $d = 7$ operator $LLH_uH_uH_dH_u$ at tree level. We use the following notation for the mediators: $\mathbf{X}^{\mathbf{L}}_{\mathbf{Y}}$. The X describes the SU(2) nature, *i.e.*, singlet (1), doublet (2), or triplet (3). The superscript L denotes a left- (L) or right- (R) handed fermion, or a scalar (s). The subscript Y represents the hypercharge $Y \equiv Q - I_3^W$. The SU(5) singlets in parentheses can be already contained in the 24-plets. The operators in the shaded rows guarantee perturbativity up to the GUT scale, operator #17 addtionaly allows for the $d = 7$ operator to be the leading contribution to neutrino mass. *Taken from Ref. [139].*

Multiplet	5	15	24	40
Δb_i	1/2	7/2	5	11

Table 6.4. – Contributions of the various $SU(5)$ multiplets to the MSSM beta functions. *Taken from Ref. [139].*

ones in Tab. 6.4, we find that these additional contributions are large, except for 5-plets. The running of the gauge couplings up to the GUT scale is at one-loop order given by

$$\alpha_{\mathrm{GUT}} = \frac{\alpha_i(Q)}{1 - \frac{b_i}{4\pi}\alpha_i(Q)\ln\frac{M_{\mathrm{GUT}}^2}{Q^2}}\,, \tag{6.37}$$

where Q is the scale at which the additional particles are included and α_{GUT} is the value of the gauge couplings at the GUT scale M_{GUT}. If we require that our theory is perturbative up to the GUT scale, we conclude that we α_G will become larger than one if we add more than five 5, $\bar{5}$ pairs or one 24-plet. For $\alpha_G \leq 1/2$ we are limited to four 5, $\bar{5}$ pairs, not considering additional higher-order effects. The only operators in Tab. 6.3 that fullfill thes requirement are operators #1, #9, #13 and #17.

Considering the similar discussion in previous chapters, we note that the presence of a scalar singlet mediator is dangerous, if this scalar obtains a VEV. Of the remaining decompositions #1, #9, and #13 and #17 only the latter can avoid this effect. Thus of all possible decompositions of the $d = 7$ operator, only the example we have chosen before is realizable in a GUT theory, if it is the dominant contribution to neutrino mass and perturbativity is guaranteed up to the GUT scale.

7. Effective operators and Dark Matter interactions

In the previous chapters we have used higher dimensional operators that appear in EFTs to generate neutrino masses. The generality of EFTs allows for their use not only in neutrino physics, but also in a wide array of other subjects. Model independent studies of DM interactions with the help of EFT [45–53] and the discussion of their UV completions [54–56] can be found in literature. These studies focus, however, on the lowest dimensional leading operators for such interactions. In the following we want to apply the methods for studying higher dimensional operators, as established in the previous chapters, to DM interactions [140] (but not in a SUSY context).

7.1. Current status of Dark matter searches

There is a variety of experimental setups that are used as detectors for direct interactions of Dark Matter. It is of course beyond the scope of this thesis to give a detailed overview. Instead, we want to briefly illustrate the principal of direct detection of dark matter at the example of XENON100 [181], which is a two-phase liquid-gas time projection chamber (TPC). The basic layout of a TPC is the following: The detector contains a volume of liquid Xenon. The reaction of dark matter with one of the nucleons in this volume leads to a nuclear recoil, which in turn produces scintillation photons and ionization electrons, when the affected nucleus is de-exciting. The scintillation photons will be directly detected by photo-multiplier tubes (PMTs) at the top and the bottom of the volume. The electron, instead, is accelerated by an electric field in the detector. When these electrons reach the interface of liquid and gaseous Xenon at the top of the detector, they produce a secondary scintillation signal, which is also recorded by the PMTs. This is schematically illustrated in Fig. 7.1. The delayed signal of the drifting electrons is used to reconstruct the z-coordinate of the initial DM interaction, hence the designation TPC. The other two coordinates are obtained from the signal location within the PMT array. The ratio of the two measured signals can be used to suppress

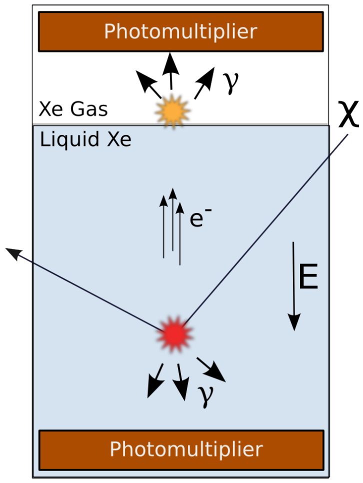

Figure 7.1. – Principal of a two-phase liquid-gas time projection chamber such as XENON100.

background. To further improve the signal, only a *fiducial* volume is used, *i.e.*, signals from the outer detector regions are not considered. This way the detector volume itself can be used as shield against external radiation. The LUX experiment [12] works in a similar way. One-phase detectors instead work without an applied electric field and only observe the primary photons. To resolve the interaction spatially, they usually have a spherical geometry with PMTs covering the whole surface.

Earlier direct detection experiments working on a different principle, such as CoGeNT [8] and CDMS [9, 10], have reported the observation of an potential DM signal in the mass region of 6 to 10 GeV. XENON100 [11] and its predecessor XENON10 [182], however, could not observe such a signal and have instead constrained the corresponding signal region. Most recently also the LUX experiment [13] has published results giving even stronger constraints for light DM. How can this tension be understood? A recent re-evaluation of the XENON10 data [183, 184] came to the conclusion that the obtained bounds are less significant then originally reported. The limits from XENON100, however, can not so easily be avoided, but models have been proposed that avoid these constraints [183, 185, 186]. Besides experimental problems, such as uncertainties about the sensitivity of the XENON100 experiment [187], these models assume that DM interactions are *xenophobic*. This means that interactions with Xenon are suppressed. An example for this scenario is isospin violating DM [188–193]. The other experiments, instead, that are not Xenon based, are not affected by this suppression.

More data on dark matter can also be obtained from collider experiments. At the LHC the strongest bounds come from mono-jet and mono-photon searches [194–198].

Further input stems from indirect searches for DM. These are experiments looking for signals from DM annihilation in the universe. Potential DM signals have been reported by Pamela [199], AMS [200], and Fermi-LAT [201]. If these signals really originate from DM, or are instead caused by systematic uncertainties or astrophysical sources is not yet clear. For more details on DM searches in general see, e.g., Ref. [202].

7.2. Effective dark matter interactions at leading order

We will now discuss, how the interactions of DM relevant for the direct searches presented above can be described by an EFT. We will discuss first the leading order operators, *i.e.*, the ones with the lowest mass dimension. These are well known in literature and have already been systematically discussed, *e.g.*, in Ref. [54].

7.2.1. Overview

We assume that the dark matter particle χ is a fermionic gauge singlet. Hence interactions with the SM gauge bosons are not induced at tree-level, whereas the interaction with SM fermions and the Higgs field is possible. All leading order interactions of DM with SM particles can therefore be described by the following two types of operators

$$\mathcal{O}_1 = \frac{1}{\Lambda^2}\chi\chi f f\,, \tag{7.1}$$

$$\mathcal{O}_H = \frac{1}{\Lambda}\chi\chi H^\dagger H\,. \tag{7.2}$$

Operator $\mathcal{O}_1$ is a dimension six operator describing the interactions among DM fermions and SM fermions directly. Operator $\mathcal{O}_H$ is a dimension five operator describing interactions among DM fermions and the SM Higgs. Both operators are shown in Fig. 7.2. The latter one induces an interaction between SM fermions and χ after EWSB. This mechanism is illustrated in Fig. 7.3. This so called Higgs portal [203–220] interaction can be expressed as

$$\mathcal{O}_2 = \frac{1}{\Lambda m_H^2}\chi\chi f f\langle H\rangle\,. \tag{7.3}$$

The size of this operator depends on the effective coupling λ_{hff} of $\mathcal{O}_H$. Therefore, direct detection experiments, such as XENON100, can constrain this coupling [216, 221, 222]:

$$\frac{\lambda_{hff}}{\Lambda} \lesssim 1\cdot 10^{-3}\,\mathrm{GeV}^{-1}\,. \tag{7.4}$$

Since $\mathcal{O}_2$ is of $d=6$ and $\mathcal{O}_1$ of $d=5$ one might naively assume that the former is sub-leading. This is, however, not always the chase, since the additional suppression of

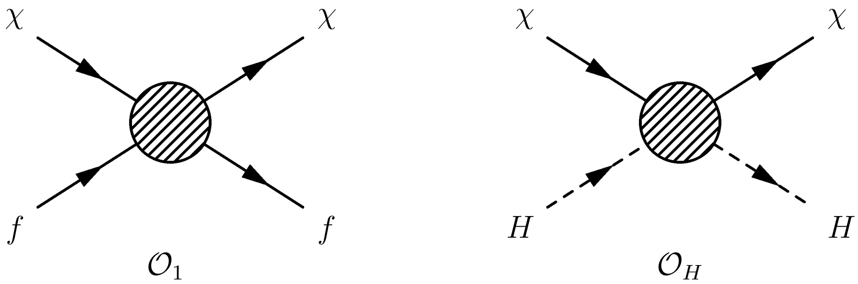

Figure 7.2. – Lowest order dark matter interactions considered in this study. *Taken from Ref. [140].*

$\mathcal{O}_2$ is only by the Higgs mass and not by the mass of a very heavy mediator. Another complication is, that the interactions relevant for direct detection experiments are of course with partons of nucleons. So we parametrize the two operators in terms of their form functions f_N that are effective coefficients for interactions with partons in nucleons. They read then

$$\lambda_{\mathcal{O}_1}^{\mathrm{N,eff}} \chi\chi f f\,, \quad \text{with} \quad \lambda_{\mathcal{O}_1}^{\mathrm{N,eff}} = f_N^{\mathcal{O}_1} \frac{1}{\Lambda^2}\,, \tag{7.5}$$

and

$$\lambda_{\mathcal{O}_2}^{\mathrm{N,eff}} \chi\chi f f\,, \quad \text{with} \quad \lambda_{\mathcal{O}_2}^{\mathrm{N,eff}} = f_N^{H} \frac{\langle H \rangle}{\Lambda m_H^2}\,. \tag{7.6}$$

The form factor of the Higgs to a nucleon is theoretically known [212], since it depends on the quark Yukawa couplings and parton distribution function. It is defined as

$$f_N^H = \sum_q f_{Tq} + \frac{2}{9} f_{TG}\,, \tag{7.7}$$

where m_N is the nucleon mass and f_{Tq} and f_{TG} are the form-factors for the quarks and gluons respectively that constitute the nucleon. In the case of the operator $\mathcal{O}_2$, we do not know the flavor structure of the operator, since it depends on the specifics of its UV completion. We will, for now, assume that all relevant couplings are flavor blind and order one. The form factor is then (*c.f.*, Ref. [223] for the corresponding form factors for neutralino DM, which can be adopted to our more general scenario)

$$f_N^{\mathcal{O}_1} = m_N \left(\sum_{q=u,d,s} \frac{f_{Tq}}{m_q} + \frac{2}{27} f_{TG} \sum_{q=c,b,t} \frac{1}{m_q} \right)\,. \tag{7.8}$$

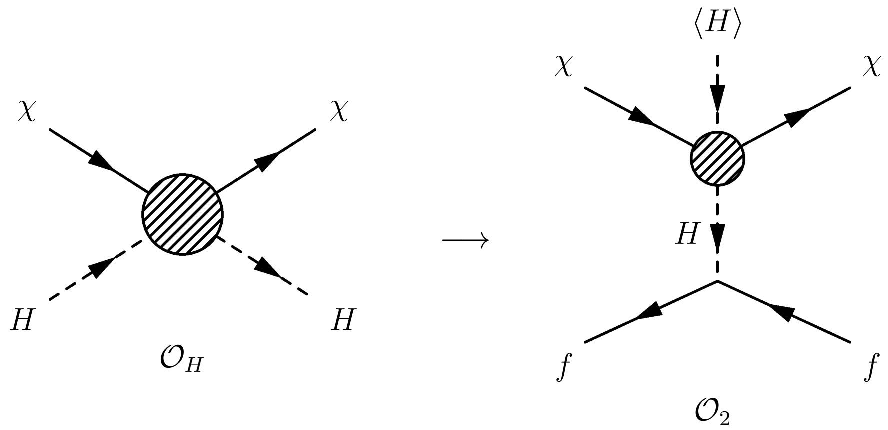

Figure 7.3. – Leading order interactions of fermionic DM with SM particles: Direct detection interaction $\mathcal{O}_2 = 1/(\Lambda m_H^2)\,\chi\chi f f \langle H \rangle$ (r.h.s) generated from the Higgs portal effective operator $\mathcal{O}_H = 1/\Lambda\,\chi\chi H^\dagger H$ (l.h.s) from the SM Yukawa interaction. *Taken from Ref. [140].*

We find that both of these form factors are roughly order one. This means that also the Higgs-portal and the direct interaction operator are of similar order, but $\mathcal{O}_1$ is suppressed by a factor m_H/Λ compared to $\mathcal{O}_2$. What does this mean for us? If we want to suppress the DM interaction to fermions by assuming it is generated by a higher dimensional operator, both of the above contributions must be avoided. How can this be accomplished? We know from the discussion in the earlier chapters that in the case of neutrino mass generation the introduction of a discrete symmetry can achieve this aim. As we will show in the next section this will not be sufficient in the case of DM interactions. We will demonstrate that some realizations of the leading order operator will always be invariant under such a symmetry. It is, however, possible, that in a specific model those operators might be avoided. We will therefore systematically discuss all possible decompositions of the leading and next-to leading order effective operators. Comparing the particle content and the interactions of these decompositions, we will then see in which model a higher-dimensional operator can be the leading contribution to neutrino mass, meaning that in such a model $\mathcal{O}_1$ and $\mathcal{O}_H$ (and therefore also $\mathcal{O}_2$) are not present.

7.2.2. Decomposition of the operator $\chi\chi f f$

We assume to have a DM particle that is a neutral fermion in this discussion. In analogy to the charge-less neutrino, it can therefore be also either of Majorana or Dirac nature.

Dirac particle

In this case, the DM field has 4 independent degrees of freedom which can be expressed either as one Dirac spinor or two Weyl spinors $X = (\chi_R, \chi_L)$. Using the notation $X_{R/L} = (1 \pm \gamma_5)X$ we can rewrite its mass term as

$$m_\chi^{\text{Dirac}} \overline{X_R} X_L = m_\chi^{\text{Dirac}} \chi_R^c \chi_L \,. \tag{7.9}$$

While we are not working in a SUSY framework in this chapter, we will adopt the convention from chapter 2 of using only left-handed fields for reasons of consistency. We will use the same short-hand notation where the product of two left-handed Weyl spinors is assumed to be the Lorentz invariant combination of the two fields. Note that χ_R^c transforms as *left-handed* field.

Majorana particle

For Majorana DM we obtain the mass term

$$m_\chi^{\text{Maj}} \chi\chi \,. \tag{7.10}$$

Chiral structure of the operator

We can now rewrite the operator $\chi\chi f f$ or $\overline{X} X \overline{f} f$, respectively, in terms of its chiral nature using FIERZ identities:

$$(\overline{X_L} f_R)\ (\overline{X_R}(f_R)^c) = \frac{1}{2}(\overline{X_L}\gamma^\mu (f_R)^c)(\overline{X_R}\gamma_\mu f_R) \tag{7.11a}$$

$$\overline{X_L}(f_L)^c\ \overline{X_R} f_L = \frac{1}{2}(\overline{X_L}\gamma^\mu f_L)(\overline{X_R}\gamma_\mu (f_L)^c) \tag{7.11b}$$

$$\overline{X_L} f_R\ \overline{f_R} X_L = \frac{1}{2}(\overline{X_L}\gamma^\mu X_L)(\overline{f_R}\gamma_\mu f_R) \tag{7.11c}$$

$$\overline{X_R} f_L\ \overline{f_L} X_R = \frac{1}{2}(\overline{X_R}\gamma^\mu X_R)(\overline{f_L}\gamma_\mu f_L) \tag{7.11d}$$

$$(\overline{X_L}\gamma^\mu f_L)(\overline{f_L}\gamma_\mu X_L) = -(\overline{X_L}\gamma^\mu X_L)(\overline{f_L}\gamma_\mu f_L) \tag{7.11e}$$

$$(\overline{X_R}\gamma^\mu f_R)(\overline{f_R}\gamma_\mu X_R) = -(\overline{X_R}\gamma^\mu X_R)(\overline{f_R}\gamma_\mu f_R) \tag{7.11f}$$

Here $f_{R/L}$ represents SM model fermions. Due to their behavior under gauge transformations (see section 2.1 of chapter 2), we know that operators such as $\overline{X_R} X_L \overline{f_R} f_L$ are not possible. At this point we can clearly see, why the introduction of a discrete symmetry fails to forbid the operator $\chi\chi f f$: The terms of Eq. (7.11c-f) are neutral under any charge assignment. In the case of Majorana DM this is also true for the operators (7.11a) and (7.11b).

Decompositions

Since we cannot avoid the presence of this operator by an additional symmetry, we have to make sure that none of its decompositions is realized in a model that demands

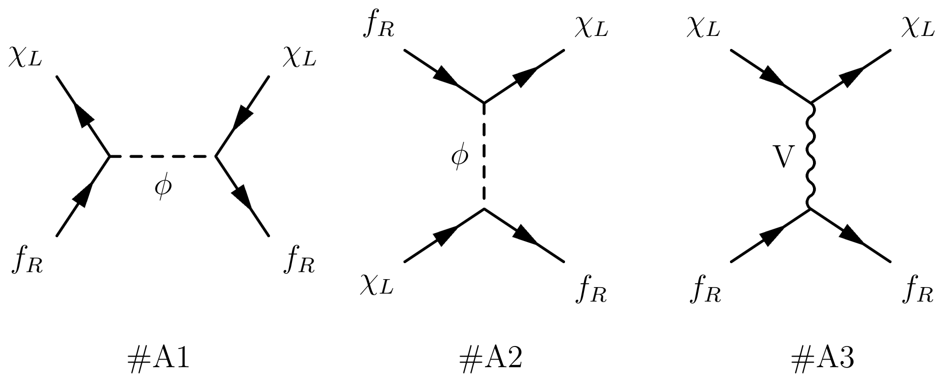

Figure 7.4. – Decompositions of the operator $(\overline{X_L} f_R)\ (\overline{f_R} X_L) = \frac{1}{2}(\overline{X_L}\gamma^\mu X_L)(\overline{f_R}\gamma_\mu f_R)$ from Eq. (7.11). *Taken from Ref. [140].*

only higher dimensional interactions. We therefore have to study how the operator $\chi\chi f f$ can be deconstructed. For the operator $(\overline{X_L} f_R)\ (\overline{f_R} X_L)$ this is shown in Fig. 7.4. (See also Tab. I of Ref. [54]) The decompositions of the other operators have a similar structure. The decomposition with a vector boson as mediator is not realized in our scenario, since χ is assumed to be a gauge singlet. To avoid the other decompositions we must, however, explicitly forbid the presence of scalars with the following charges under $(SU(3)_c, SU(2)_L, U(1)_Y; \mathbb{Z}_2)$:

$$(3, *, *; -)\ , \tag{7.12a}$$

$$\left(1, 2, -\frac{1}{2}; -\right)\ , \tag{7.12b}$$

$$(1, 1, -1; -)\ , \tag{7.12c}$$

or their charge conjugates; the $*$ refers to any possible charge. The $\mathbb{Z}_2$ symmetry here is necessary to obtain a stable DM particle. Hence χ is odd under this parity and the SM particles are even. The possible quantum numbers of the mediators can be deduced by inserting all possible combinations of SM fermions as external fields in Fig. 7.4.

7.2.3. Decomposition of the operator $\chi\chi\, H^\dagger H$

We now have also to discuss the decompositions of the effective operator $\chi\chi\, H^\dagger H$. In this case the only possible chiral combinations are $\chi_L\chi_L H^\dagger H$ and $\chi_R^c\chi_R^c H^\dagger H$ for Dirac and $\chi\chi H^\dagger H$ for Majorana DM. The two possible decompositions are shown in Fig. 7.5.

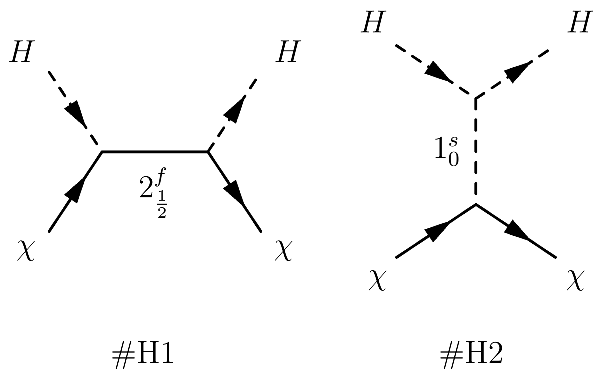

Figure 7.5. – Decompositions of the effective operator $\chi\chi\, H^\dagger H$. *Taken from Ref. [140].*

The corresponding scalars are

$$\left(1, 2, \pm\frac{1}{2}, -\right) \qquad \text{and} \tag{7.13a}$$

$$(1, 1, 0; +) \ , \tag{7.13b}$$

in the notation specified in the previous subsection.

7.3. Higher dimensional operators

In analogy to the discussion of neutrino mass models, we can now systematically study higher-dimensional operators leading to the same interactions of DM with SM fermions as the leading order operators after EWSB.

At next-to leading order we have the $d = 7$ operator $\frac{1}{\Lambda^3}\chi\chi f f H$, which will induce the direct interaction $\frac{v}{\Lambda^3}\chi\chi f f H$. The resulting effective interaction is suppressed by a factor $\frac{v}{\Lambda}$ in comparison to the leading order operator $\frac{1}{\Lambda^2}\chi\chi f f$. In a minimal extension of the SM, where an additional scalar singlet S, which can obtain a VEV v_S, is present, we have to consider also the operator $\frac{1}{\Lambda^3}\chi\chi f f S$. Note that for an appropriate choice of SM fermions, $f f H$ as well as $f f S$ are SU(2) invariant.

If we regard only DM interactions induced via the Higgs portal, the next-to leading operator is $\chi\chi H^\dagger H S$. If only SM particles are considered as external fields it is $\chi\chi(H^\dagger H)^2$, instead. All these operators are listed in Tab. 7.1. For the reason of completeness we also included the $d = 7$ operator $\chi\chi H^\dagger H S^2$, which we will not discuss in detail.

It might seem more economical to just consider SM particles as external fields of the effective operator. As we will see later, however, in many decompositions of operators, such as $\chi\chi H^\dagger H H^\dagger H$, the presence of an additional scalar singlet is required as a

	(a)	(b)
$d = 5$	—	$\chi\chi H^\dagger H$
$d = 6$	$\chi\chi ff$	$\chi\chi H^\dagger HS$
$d = 7$	$\chi\chi ffS$ $\chi\chi ffH$	$\chi\chi(H^\dagger H)^2$ $\chi\chi H^\dagger HS^2$

Table 7.1. – Higher dimensional operators generating dark matter interactions (a) by direct interactions and (b) via the Higgs portal. *Taken from Ref. [140].*

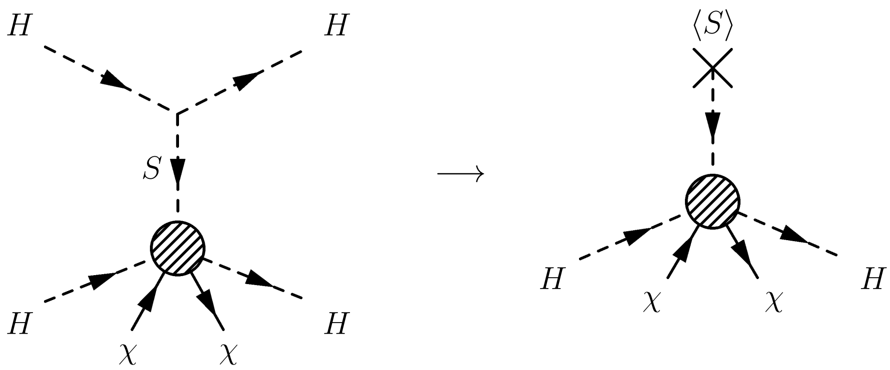

Figure 7.6. – Schematic illustration how decompositions of the type $\chi\chi H^\dagger H H^\dagger H$ with a scalar singlet mediator S will induce operators of the type $\chi\chi H^\dagger HS$ if S obtains a VEV. *Taken from Ref. [140].*

mediator. This mediator usually couples to a pair of Higgs fields. Similar to operators generating neutrino masses (as discussed in the previous chapters) one therefore cannot avoid a non-zero VEV for S, which will induce a lower dimensional operator of the type $\chi\chi H^\dagger HS$. This is schematically illustrated in Fig. 7.6. Due to this reason it is a more consistent approach to include these types of operators from the beginning.

As discussed before, the additional suppression of these higher dimensional operators reduces the direct interaction cross-section and can help to avoid experimental constraints. A reduced cross-section has, however, also cosmological implications. This can be understood as follows: Somewhat similar to the heavy stable quarks discussed in section 6.5 of chapter 6, also the dark matter particles are in thermal equilibrium in the early universe, until their number density becomes too small for annihilation processes to happen and they will freeze-out. The time of this freeze-out determines the DM abundance in the universe. If the interactions are too much suppressed, the resulting abundance will be below experimental bounds. Studies of the leading Higgs portal operator [222] have shown that already current bounds on the DM interaction cross-sections are in conflict with these limits, except for certain parameter points such as $m_\chi = m_H/2$, which leads to resonance enhanced annihilation processes. It is, however, possible that the interactions in the early universe are different from those relevant for direct detection. We will discuss this later for a specific example. Another option would be non-thermal DM production.

We want to discuss now systematically the various possible next-to leading order operators of Tab. 7.1.

7.3.1. The operator $\chi\chi ffS$

All decompositions of this operator that are possible with SM fermions are depicted in Fig. 7.7. One can convince oneself that any of these decompositions requires at least one mediator with the same quantum numbers as specified in Eq. (7.12). We exemplify this for operator #S1: The Lagrangian of a model generating this diagram, must at least contain the terms

$$\mathcal{L}_{\#\mathrm{S1}} = \mathcal{L}_{\mathrm{SM}} + \lambda_{\chi f \phi}\, \chi f \cdot \phi + \lambda_{S\phi\phi}\, S^\dagger \phi \cdot \phi + m_\phi\, \phi^\dagger \phi + m_\chi\, \chi\chi + \cdots + \mathrm{h.c.} \tag{7.14}$$

From this Lagrangian, however, also an effective operator $\chi\chi ff$ can be constructed via the diagrams #A1 and #A2 of Fig. 7.4, since all necessary fields and couplings are present. A similar argument can be made for #S2 and #S3. Therefore, we conclude that none of these decompositions is realizable without introducing the leading order operator. Therefore we must disregard the option of having $\chi\chi ffS$ as dominant contribution to DM interactions. More intuitively this can be understood by the fact that the decompositions #S1-#S3 can be obtained from the diagrams #A1/#A2 by connecting an external S field to any line. Since this scalar does not carry any charge, the quantum numbers of the original fields do not have to be modified, and therefore also the original mediators are present in the new diagram.

7.3.2. The operator $\chi\chi ffH$

The decompositions of the operator $\chi\chi ffH$ are shown in Fig. 7.8. We can make the following observations:

- Operators #B2, #B4 and #B5 suffer from the same problem as #S1-#S3: They also include the mediators of Eq. (7.12).

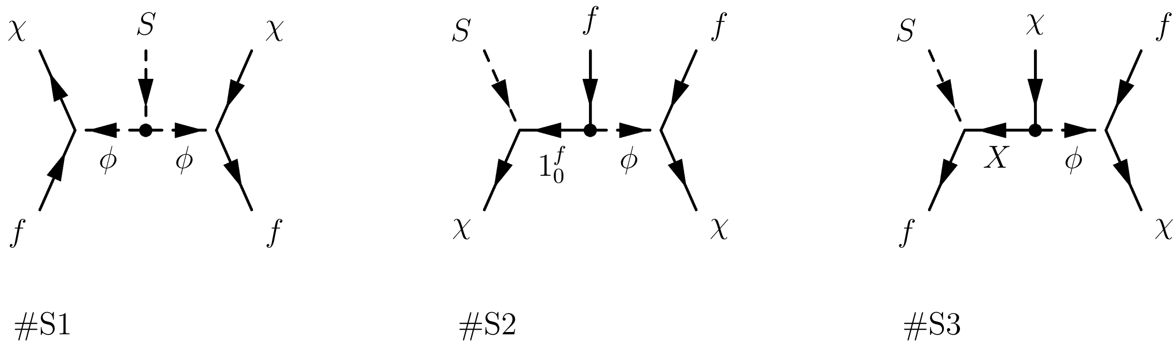

Figure 7.7. – Different decompositions for the operator $\chi\chi f f S$. ϕ is a scalar and X a fermion that has the quantum numbers of the external fermions f. ϕ corresponds to the mediators in Fig. 7.4. *Taken from Ref. [140].*

- Operators #B1 and #B5 contain a mediator $2^s_{\frac{1}{2}}$. We can identify this mediator as the Higgs doublet due to its quantum numbers. By cutting one of these diagrams at the corresponding line we obtain one of the operators from Fig. 7.5.

We conclude that also the operator $\chi\chi\, f f H$ is not a valid possibility as dominant contribution to DM interactions. In the following we want to discuss now the higher dimensional extensions of the Higgs portal operator $\mathcal{O}_H$

7.3.3. The operator $\chi\chi\, H^\dagger H S$

In a simple model with an additional scalar S we cannot avoid the operator #H2 of Fig. 7.5, since S can assume the role of the scalar mediator. We will therefore require an additional $\mathbb{Z}_3$ symmetry under which S is charged as $\omega = \mathrm{e}^{\mathrm{i}\frac{2\pi}{3}}$. The operator $\chi\chi\, HHS$ is then only invariant under this symmetry if also the DM field χ is charged. Hence for Dirac DM we have $q(\chi_L) = q(S) = \omega$ and $q(\chi_R) = \omega^2$, and for Majorana DM we obtain $q(\chi) = \omega$. Obviously, this charge assignment forbids an explicit mass term for the χ field. Instead, the (Majorana or Dirac) mass of the DM particle is generated by a term $\lambda_S S\chi\chi$ after S obtains a VEV. If all other fields are not charged under the $\mathbb{Z}_3$ symmetry, we have now accomplished to forbid the operator $\chi\chi HH$. The possible topologies of the decompositions of the invariant operator $\chi\chi HHS$ are shown in Fig. 7.9 and the decompositions themselves are shown in Fig. 7.10 and also listed in Tab. 7.2.

The operator $\chi\chi f f$ is not forbidden by the $\mathbb{Z}_3$ symmetry for the reasons discussed in subsection 7.2.2. This is, however, no problem, since none of the mediators of Eq. (7.12) are present. (Note that some of the mediators have indeed the SM quantum numbers as in Eq. (7.12) but are even under the $\mathbb{Z}_2$ parity.)

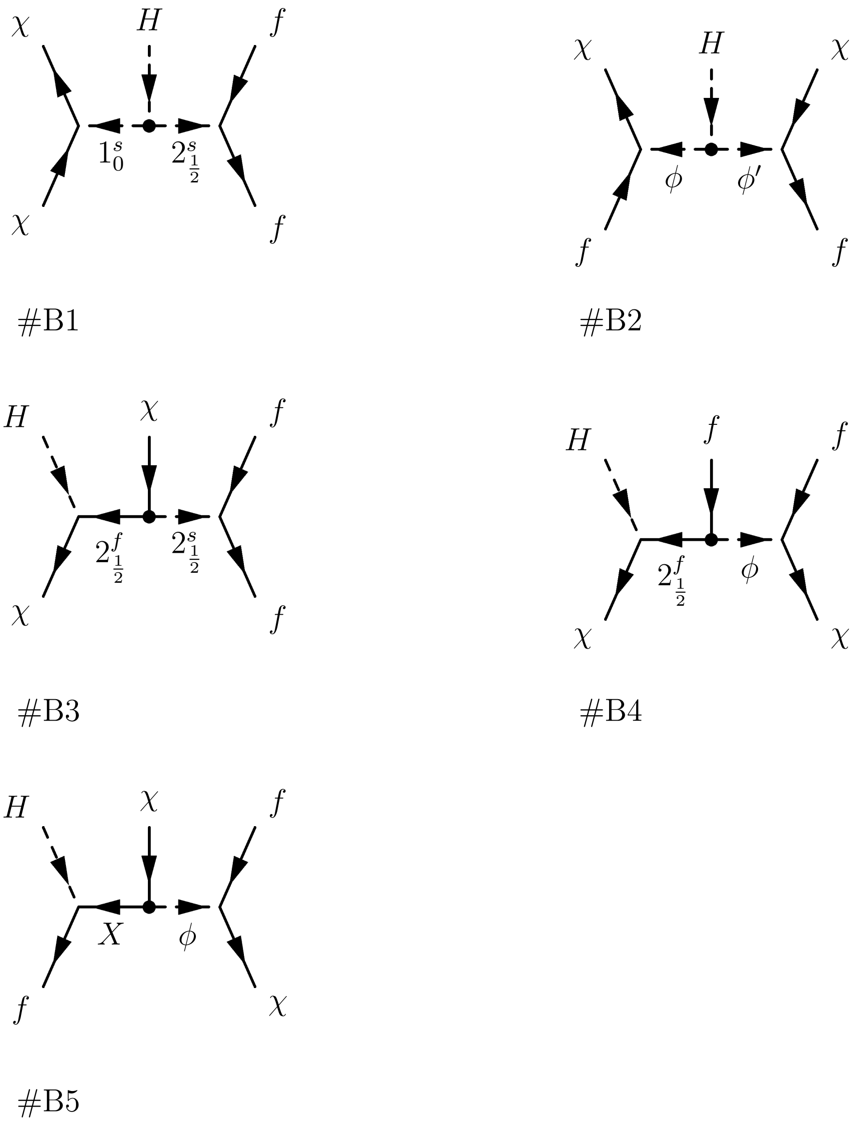

Figure 7.8. – Different decompositions for the operator $\chi\chi f f H$. ϕ is a scalar and X a fermion that has the quantum numbers of the external fermions f. $\phi^{(\prime)}$ corresponds to the mediators in Fig. 7.4. *Taken from Ref. [140].*

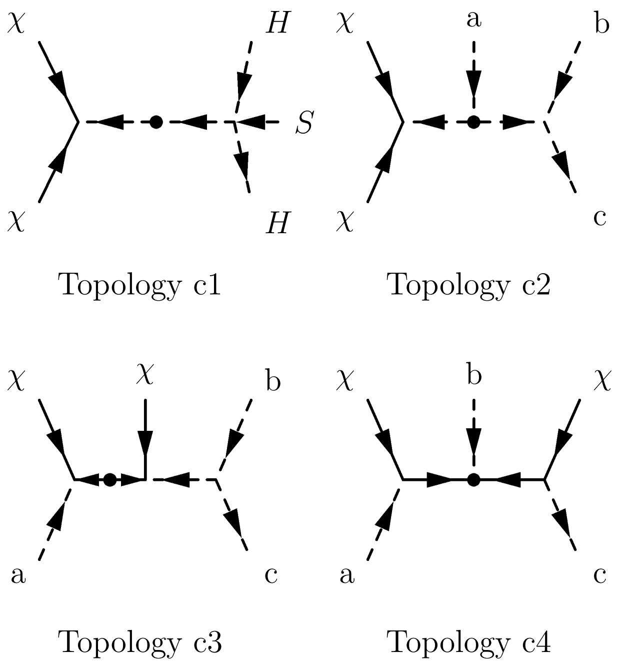

Figure 7.9. – Different topologies for the decomposition of the effective operator $\chi\chi HHS$. *Taken from Ref. [140].*

	Top.	ext. Fields	Mediators
#C1	c1		$1^s_{0,+}$
#C2	c2	$a = S,\ b = H,\ c = H$	$1^s_{0,+}$, $1^s_{0,+}$
#C3	c2	$a = H,\ b = S,\ c = H$	$1^s_{0,+}$, $2^s_{\frac{1}{2},+}$
#C4	c3	$a = S,\ b = H,\ c = H$	$1^f_{0,-}$, $1^s_{0,+}$
#C5	c3	$a = H,\ b = S,\ c = H$	$2^f_{\frac{1}{2},-}$, $2^s_{\frac{1}{2},+}$
#C6	c4	$a = H,\ b = S,\ c = H$	$2^f_{\frac{1}{2},-}$, $2^f_{-\frac{1}{2},-}$
#C7	c4	$a = S,\ b = H,\ c = H$	$1^f_{0,-}$, $2^f_{\frac{1}{2},-}$

Table 7.2. – Decompositions of the operator $\chi\chi HHS$. The numbers in the first column correspond to the decompositions shown in Fig. 7.10. The topologies (Top.) correspond to to Fig. 7.9 where a, b and c are replaced accordingly. The last column lists the new mediator fields that have to be present in a model which generates this specific operator. The second subscript denotes the charge under the $\mathbb{Z}_2$ parity. *Taken from Ref. [140].*

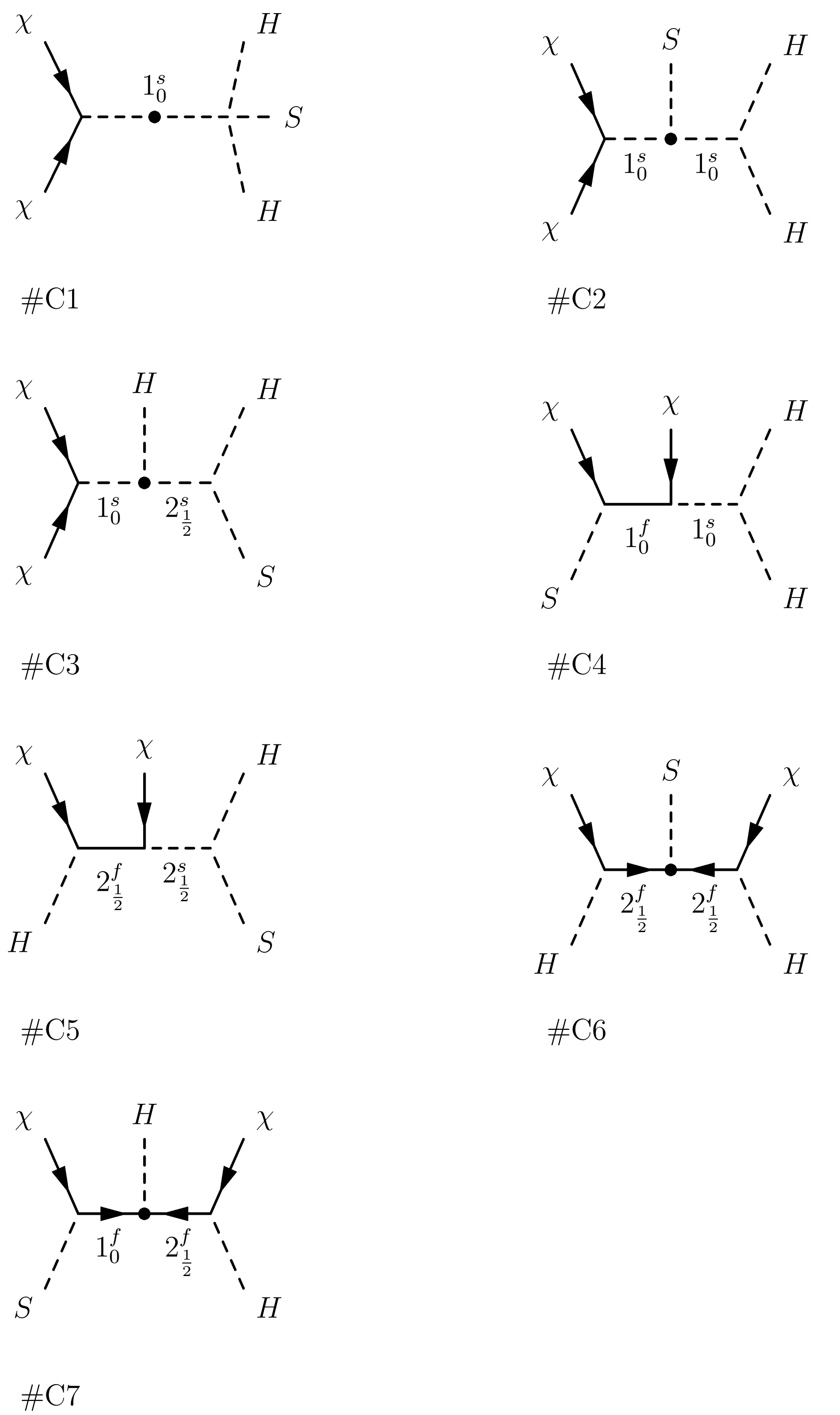

Figure 7.10. – Different decompositions for the operator $\chi\chi HHS$. *Taken from Ref. [140].*

The requirements for having $\chi\chi HHS$ as dominant contribution to DM interactions are therefore

- A symmetry that stabilizes the DM, in our case a $\mathbb{Z}_2$.
- An additional symmetry under which the Higgs portal operator $\mathcal{O}_H$ is not invariant.
- No new particles with the quantum numbers specified in Eq. (7.12) are present.

We make the following additional observations:

- Interestingly, the dominant contribution to direct detection interactions can still be generated by a higher dimensional operator, even if some of the mediators generating #A1 and #A2 are present. This is the case if these mediators do not carry color charge and therefore lead only to interactions with leptons, which are irrelevant for nuclear recoil experiments. The resulting leptonic interactions, however, could accommodate the annihilation cross sections that are required in order to obtain the correct DM relic abundance. Examples for such models can be found, *e.g.*, in Ref. [224, 225].
- For generating the DM mass we need the term $\lambda_\chi\,\chi\chi S$. Furthermore, in general we cannot avoid the terms $m_S S^\dagger S$ and $\lambda_H S^\dagger S H^\dagger H$ whenever a scalar singlet is present. This has the consequence that operator #C1 will be unavoidable. It may, however, not be the only contribution. It is connected to the mass of χ and S in the following way
$$\mathcal{O}^{d=6}_{\#C1} = \lambda_\chi \lambda_H \frac{v_S}{m_S^2} H^\dagger H \chi\chi = \lambda_H \frac{m_\chi}{m_S^2} H^\dagger H \chi\chi\,. \tag{7.15}$$
- Decomposition #C4 is potentially problematic: The mediator $1^f_{0,\omega^2}$ could become an additional DM component χ'. The reason is that the coupling $S\chi\chi'$ induces a mass term $m'_\chi \chi\chi'$ after S obtains a VEV. This means we have a mixing between the states χ and χ'. We then can obtain DM interactions via a $d = 5$ operator $\chi\chi' H^\dagger H$.

7.3.4. The operator $\chi\chi\, H^\dagger H H^\dagger H$

Finally we want to discuss the operator $\chi\chi\, H^\dagger H H^\dagger H$. The possible topologies are presented in Fig. 7.11. None of these topologies can become a dominant contribution to DM interactions, due to the following reasons

- If we have a vertex, where a mediator is connected to two external Higgs fields, it can be a scalar singlet or a triplet. In the former case the mechanism illustrated in Fig. 7.6 automatically induces an effective operator of lower dimension. The same is true for the neutral component of a triplet, which also obtains a VEV via its coupling to the Higgs bosons. Interestingly this triplet would also generate neutrino mass via a type-II seesaw diagram. Therefore it must be either very heavy, or its VEV and couplings must be very small. This argumentation disfavors topologies e2, e3 and e4a-e4d.

- In case of topology e1 the quartic Higgs coupling implies that one of the mediators is also a Higgs field. By cutting the diagram at the corresponding line we obtain then the leading order Higgs-portal operator.

- The topology e4e, which requires only fermionic operators is not affected by those considerations. Here, instead, the coupling of χ and H to one of the mediators implies that this mediator is exactly the one that generates the Higgs-portal operator #H1. The usual ansatz of introducing an additional symmetry is not viable in this case because $H^\dagger H$ is invariant under any such symmetry and therefore if $\chi\chi\, H^\dagger H$ is forbidden, also $\chi\chi\, H^\dagger H H^\dagger H$ will not appear.

We conclude from the discussion in this section that the only attractive option for heaving DM interactions generated by a higher dimensional operator is $\chi\chi H^\dagger H S$. In the following we want to discuss the phenomenology of one particular decomposition of this operator at the LHC.

7.4. LHC phenomenology

The model we exemplarily want to study in this section is the operator #C6. We have listed the fields in Tab. 7.3. The according decomposition is shown in Fig. 7.12 and the corresponding Lagrangian reads

$$\begin{aligned}\mathcal{L}_{\#\mathrm{C6}} = \mathcal{L}_{\mathrm{SM}} + \Big[&\lambda_{\xi_u}\, H\cdot(\xi_{u,R})^c\,\chi + \lambda_{\xi_d}\, H^\dagger\,(\xi_{d,R})^c\,\chi + \lambda_{\xi\xi S}\, S\,\xi_{u,L}\,\cdot\,\xi_{d,L}\\ &+ \lambda'_{\xi\xi S}\, S^*\,(\xi_{u,R})^c\,\cdot\,(\xi_{d,R})^c + m_u\,\xi_{u,L}\cdot(\xi_{u,R})^c + m_d\,(\xi_{d,R})^c\cdot\xi_{d,L}\\ &+\lambda_{S\chi\chi} S\chi\chi + \kappa_S S^3 + \mathrm{h.c.}\Big]\\ &+ \lambda_{SSHH}(S^*S)(H^\dagger H) + m_S S^* S + \lambda_S (S^*S)^2\,,\end{aligned} \tag{7.16}$$

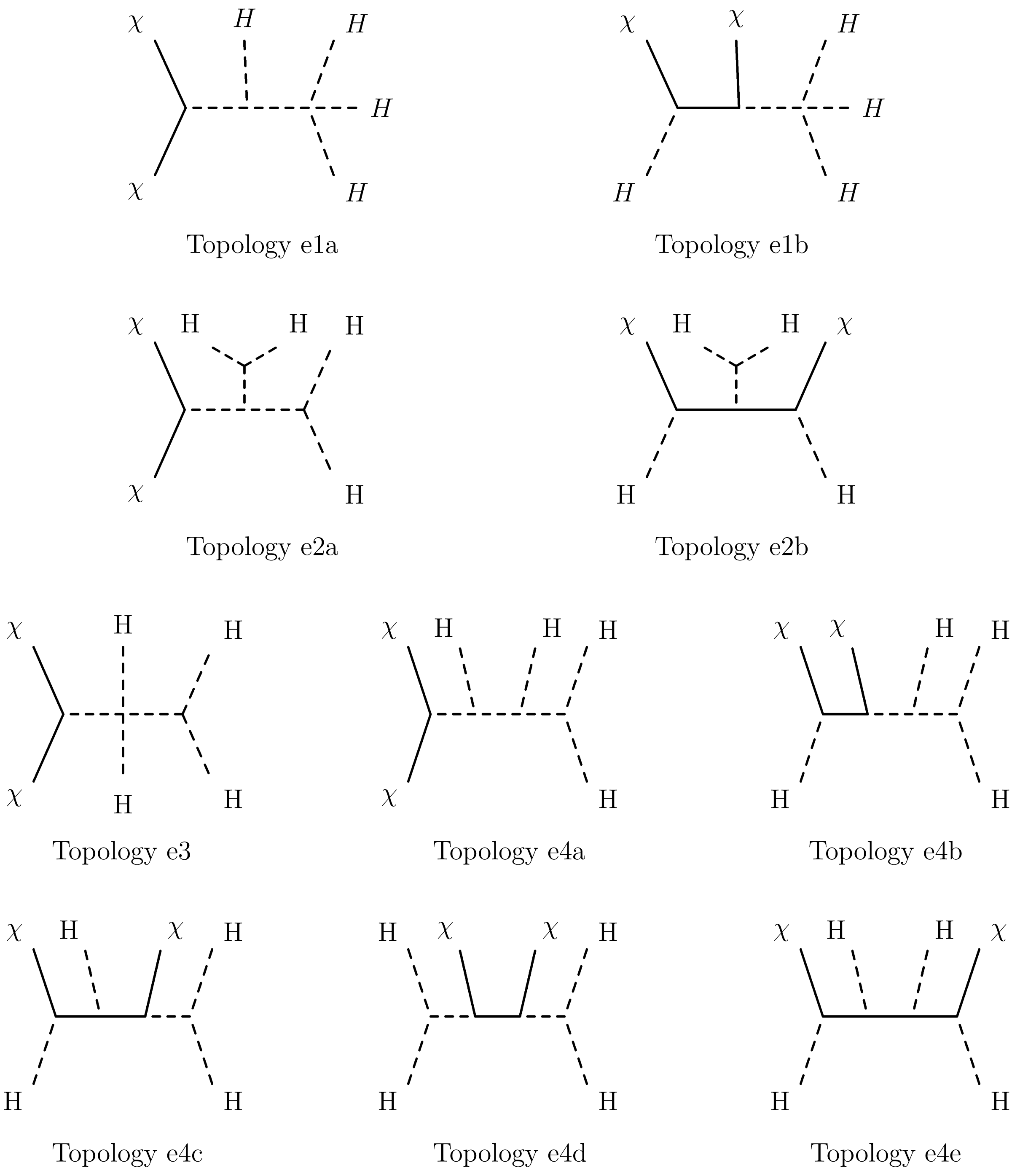

Figure 7.11. – Different topologies for the decomposition of the effective operator $\chi\chi HHHH$. *Taken from Ref. [140].*

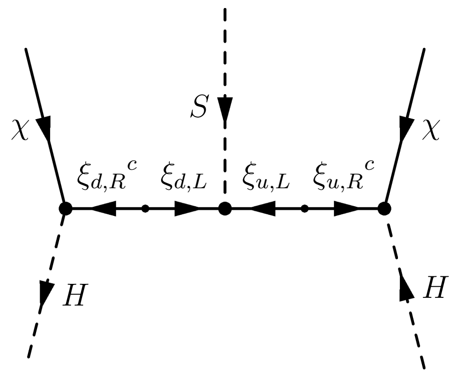

Figure 7.12. – Decomposition of the effective $d = 6$ operator #C6. *Taken from Ref. [140].*

Fields:	SM	Fermions					Scalars
		$\xi_{d,L}$	$\xi_{u,L}$	$(\xi_{u,R})^c$	$(\xi_{d,R})^c$	χ	S
$SU(2)$		2	2	2	2	1	1
$U(1)_Y$		$-\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	0	0
$\mathbb{Z}_3$	1	ω	ω	ω^2	ω^2	ω	ω
$\mathbb{Z}_2$	$+$	-	-	-	-	-	$+$

Table 7.3. – Field charges, with $\omega = \mathrm{e}^{\mathrm{i}\frac{\pi}{3}}$ under $\mathbb{Z}_3$, whereas "+" signifies a phase of 1 and "-" signifies a phase of $\mathrm{e}^{\mathrm{i}\pi}$ under the $\mathbb{Z}_2$. *Taken from Ref. [140].*

where all couplings are assumed to be real, for reasons of simplicity, and the doublet fields can be explicitly expressed as

$$\xi_{d,L} = \begin{pmatrix} \xi^0_{d,L} \\ \xi^-_{d,L} \end{pmatrix} \quad \xi_{u,L} = \begin{pmatrix} \xi^+_{u,L} \\ \xi^0_{u,L} \end{pmatrix} \quad \xi_{u,R}{}^c = \begin{pmatrix} \xi^0_{u,R}{}^c \\ (\xi_{u,R}{}^c)^- \end{pmatrix} \quad \xi_{d,R}{}^c = \begin{pmatrix} (\xi_{d,R}{}^c)^+ \\ \xi^0_{d,R}{}^c \end{pmatrix} . \tag{7.17}$$

As already discussed in chapter 5 for the neutrino mass model, it is necessary to know the mass eigenstates of the fields that are relevant for LHC phenomenology. We have introduced two additional charged fermions. In the basis (ξ_u, ξ_d^c) their mass matrix reads (using the same convention for v and v_S as in the earlier chapters)

$$M^+ = \begin{pmatrix} m_{\xi_u} & -\lambda'_{\xi\xi S}\, v_S \\ \lambda_{\xi\xi S}\, v_S & m_{\xi_d} \end{pmatrix} . \tag{7.18}$$

Furthermore five new neutral fermions $(\xi^0_{u,L}, \xi^0_{u,R}{}^c, \xi^0_{d,R}{}^c, \xi^0_{d,L}, \chi)$ appear, which have

Majorana nature. The corresponding mass matrix is then

$$M^0 = \begin{pmatrix} 0 & -m_{\xi_u} & 0 & -\lambda_{\xi\xi S}\, v_S & 0 \\ -m_{\xi_u} & 0 & \lambda'_{\xi\xi S}\, v_S & 0 & -\lambda_{\xi u}\, v \\ 0 & \lambda'_{\xi\xi S}\, v_S & 0 & -m_{\xi_d} & \lambda_{\xi d}\, v \\ -\lambda_{\xi\xi S}\, v_S & 0 & -m_{\xi_d} & 0 & 0 \\ 0 & -\lambda_{\xi u}\, v & \lambda_{\xi d}\, v & 0 & \lambda_{S\chi\chi}\, v_S \end{pmatrix} . \tag{7.19}$$

Here we will use the notation $\xi^+_{1/2}$ for the mass eigenstates of the charged fermions. For the neutral mass eigenstates we will again use the notation n_i with $i = 1, \ldots, 5$. The additional scalar S has couplings to the SM Higgs field H. This has consequences for the phenomenology of the Higgs sector. First of all we have one massive pseudoscalar P^0. The assumption of real couplings implies that it is purely singlet-like. The pseudoscalar and charged components of the Higgs field are absorbed by the gauge bosons, as usual. The two remaining scalar degrees of freedom, however, will mix. Their mass matrix reads

$$M_H^2 = \begin{pmatrix} 4\lambda v^2 & 2\lambda_{SSHH} v v_S \\ 2\lambda_{SSHH} v v_S & 4\lambda_S v_S^2 + 3\kappa v_S \end{pmatrix} = \begin{pmatrix} 4\lambda v^2 & 2\lambda_{SSHH} v v_S \\ 2\lambda_{SSHH} v v_S & 4\lambda_S v_S^2 - \frac{1}{3} m_P^2 \end{pmatrix} . \tag{7.20}$$

The corresponding mass eigenstates are $h_{1/2}$. The structure of this mass matrix implies $m_P^2 \leq 6\lambda_S v_S^2$. Since the Higgs boson found at the LHC has the characteristics of a SM Higgs, the mixing between the two states must be small. So besides the Higgs-like state we have a second mostly singlet-like state, which is difficult to produce directly. The decay of the pseudoscalar P depends on the mass hierarchy of the new particles. One scenario is that it decays into a pair of the new fermions. Alternatively it can decay into two photons via a loop of ξ^+ fields, where the relevant couplings are $\lambda_{\xi\xi S}$ and $\lambda'_{\xi\xi S}$. In the latter case its life time must be short enough in order to avoid conflicts with the prediction of Big Bang Nucleosynthesis. The relevant partial decay widht is approximately

$$\Gamma(P \to \gamma\gamma) \approx (\lambda_{\xi\xi S} + \lambda'_{\xi\xi S})^2 \times \mathcal{O}(\mathrm{keV}) \,, \tag{7.21}$$

for $m_{X^+} = 500$ GeV.

Also here, the LHC phenomenology depends on the mass hierarchy of the BSM states. The lightest neutral fermion state should mostly correspond to χ in order to obtain a singlet-like DM particle. So, in order to avoid admixtures of the neutral doublet components, we need the off-diagonal entries $\lambda_{\xi u}$ and $\lambda_{\xi d}$ in Eq. (7.19) to be comparatively small. The mass of the singlet-like state is then roughly equivalent to

$\lambda_{S\chi\chi}$ which must be small if this is the lightest mass eigenstate. We distinguish now the following scenarios:

Scenario I ($m_{\xi_d,\xi_u} \simeq \lambda_{\xi\xi S} v_S \simeq \lambda'_{\xi\xi S} v_S$)
In this scenario, the mass terms m_{ξ_d,ξ_u} and the ones induced by v_S are of almost equal size. The charged lepton mass matrix Eq. (7.18) would therefor have one heavy and one rather light eigenstate. Such a light fermion, however, is experimentally excluded [177].

Scenario II ($m_{\xi_d,\xi_u} \gg \lambda_{\xi\xi S} \frac{v_S}{\sqrt{2}}, \lambda'_{\xi\xi S} \frac{v_S}{\sqrt{2}}$)
If the Dirac mass terms of the doublets m_{ξ_d,ξ_u} are significantly larger then their Majorana mass terms induced by v_S, we obtain two pseudo-Dirac fermions. These pseudo-Dirac states, as well as the charged fermions, then have masses close to m_{ξ_d} and m_{ξ_u}. It is reasonable to assume $m_{\xi_d} \simeq m_{\xi_u}$. We then observe the following dominant decay modes.

$$\xi_i^+ \to W^+ n_1^0 \qquad (i = 1, 2) \tag{7.22a}$$
$$n_j^0 \to Z n_1^0 \ , \ h_i n_1^0 \ , \ P n_1^0 \qquad (j = 2, 3, 4, 5) \tag{7.22b}$$

where n_1^0 is the singlet like state, *i.e.*, the DM particle. If, however, the masses m_{ξ_d,ξ_u} are very different, we obtain also the decays

$$\xi_2^+ \to Z \xi_1^+ \ , \ h_i \xi_1^+ \ , \ P \xi_1^+ \ , \ W^+ n_j^0 \qquad (j = 2, 3) \tag{7.22c}$$
$$n_j^0 \to Z n_k^0 \ , \ h_i n_k^0 \ , \ P n_k^0 \ , \ W^\pm \xi_1^\mp \qquad (j = 4, 5 \text{ and } k = 2, 3)\,. \tag{7.22d}$$

The nature of these decays is similar to the ones of charginos and neutralinos in SUSY. The production of this heavy $SU(2)$ doublets is very similar to the one of those discussed in chapter 5. For sufficiently small couplings we can in principal also observe displaced vertices.

Scenario III ($m_{\xi_d,\xi_u} \ll \lambda_{\xi\xi S} v_S, \lambda'_{\xi\xi S} v_S$)
This scenario is similar to Scenario II, with two charged fermions and two neutral quasi-Dirac fermions. In this case, however, their masses will be mostly determined by $\lambda_{\xi\xi S} v_S$. The corresponding decays are the same as above for degenerate masses.

8. Summary and Conclusion

In this thesis we demonstrated that supersymmetric neutrino mass models can be studied with the help of effective field theories. Having neutrino masses generated by higher dimensional effective operators is one possibility to connect neutrino physics to the appearance of new particles at the TeV scale. We discussed these effective operators and their decompositions systematically in the context of the MSSM and NMSSM up to $d = 9$. We illustrated how, due to a discrete symmetry, these higher dimensional operators can become the leading contribution to neutrino mass. It is interesting to note that not all of these operators can become a dominant contribution, due to symmetry considerations. Even if all lower dimensional operators are forbidden at the level of effective operators, they can be re-introduced in specific decompositions with scalars, which obtain a VEV that breaks the discrete symmetry. Therefore only certain decompositions allow for neutrino mass to be generated by a generic higher dimensional operator. If one assumes that such models are embedded in a GUT framework, the requirement of perturbativity up to the unification scale limits the possible implementations even further. For the decompositions of the $d = 7$ operator it is particular interesting that only one possibility remains. If this is only accidental or indeed points to the realization of this specific model in nature has to be seen in the future when new experimental data will be available.

Since the remaining model requires the introduction of new SU(2) singlet and doublet fermions, we studied their possible phenomenology at the LHC. Depending on the hierarchy of these mediators, we will generate the linear or the inverse seesaw mechanism at an intermediate scale. Their decay processes can lead to several characteristic signatures in the detector, such as displaced vertices and lepton flavor and lepton number violating processes. In this case we chose a specific set of parameters, and implemented an approximately tri-bimaximal flavor structure. Prospective new results of neutrino and LHC experiments will show, if this particular choice is indeed the most attractive one. The study of another point in parameter space might be interesting. Further studies can also investigate if the proposed structure of the coupling constants can arise from a flavor symmetry.

We could also illustrate in the framework of the NMSSM with an additonal broken symmetry that a combination of contributions from operators with different dimension can give rise to sizeable corrections from tri-bimaximal mixing leading to a non-zero θ_{13}. It is of course possible to obtain these corrections in other ways, such as from the charged lepton sector. It is, however, an interesting option to study if a mechanism such as described here may be the consequence of a more fundamental theory.

The GUT completion of the doublet fields comes with new heavy d-quarks. Due to the symmetry that protectets the SM neutrino masses, these new quarks are stable. This leads to conflicts with the search for heavy elements and Big Bang Nucleosynthesis. We could demonstrate that in the above mentioned NMSSM implementation of our model, the small symmetry breaking couplings can be sufficient to allow a fast enough decay of these quarks to avoid these cosmological constraints.

Higher dimensional effective operators are not only relevant for neutrino physics. Here we have demonstrated that they can be used to study DM interactions. In order for these operators to become the leading contribution for the interaction of DM with nucleons, it is necessary to avoid the lowest dimensional operators that lead to interactions between DM and SM fermions. This can happen either directly or via the Higgs-portal. Therefore a discrete symmetry and the absence of certain fields in a specific model is required. We have shown that such models fullfilling both conditions are indeed possible if the effective interaction is generated by the extension $\chi\chi H^\dagger HS$ of the Higgs portal operator. We have further shown that such models also have a phenomenology that is interesting for the LHC.

The methods presented in this thesis for studying higher dimensional operators are not only a useful tool in the particular fields mentioned here, but could also be adopted to other areas of phyiscs. Since the properties of the newly discoverd Higgs boson at the LHC will be even more precisely determined in the future, one particular interesting possibility is to study the possible influence of new physics with the help of EFT in this particular case. Also low-energy observables such as neutrinoless double β-decay or flavor physics and CP violation can be potentially studied with these methods. It may be interesting to automatize this type of analyses with the help of programs in the future. As the discussion in this thesis shows, specific models adressing fundamental issues such as the origin of neutrino masses or dark matter usually concern various fields of phyiscs and experiments, here for instance cosmology, dark matter detection, neutrino and collider experiments. It will be interesting to further study the interconnection of these fields in the light of a multitude of current and planned experiments, which presumably will provide unexpected new results.

A. SUSY Formalism

In the following we want to set-up the formalism we use to describe SUSY models and introduce some additional notation.

Chiral supermultiplets

First we want to discuss chiral supermultiplets, which contain the SM fermions and their scalar partners.

Weyl spinors

Since SUSY transforms fermions into bosons and vice versa, one should use representations of scalar and fermionic fields that have both the same (on-shell) degrees of freedom. Scalar particles are therefore complex fields ϕ whereas fermions are represented by Weyl spinors ψ and χ, which are the left- and right-handed components of a Dirac spinor

$$\Psi = \begin{pmatrix} \psi \\ \chi \end{pmatrix} , \tag{A.1}$$

so that

$$\Psi_R \equiv P_R \Psi \equiv \frac{1}{2}(1+\gamma_5)\Psi = \begin{pmatrix} \psi \\ 0 \end{pmatrix} , \tag{A.2a}$$

$$\Psi_L \equiv P_L \Psi \equiv \frac{1}{2}(1-\gamma_5)\Psi = \begin{pmatrix} 0 \\ \chi \end{pmatrix} . \tag{A.2b}$$

The equations of motion for Weyl spinors read

$$\sigma^\mu p_\mu \psi = m\chi \tag{A.3a}$$

$$\bar{\sigma}^\mu p_\mu \chi = m\psi , \tag{A.3b}$$

where $\sigma^\mu = (\sigma^0, \sigma^1, \sigma^2, \sigma^3)$ and $\bar{\sigma}^\mu = (\sigma^0, -\sigma^1, -\sigma^2, -\sigma^3)$, see footnote 2. They can be obtained from the corresponding Lorentz-invariant Lagrangian

$$\mathcal{L} = \psi^\dagger \mathrm{i}\sigma^\mu \partial_\mu \psi + \chi^\dagger \mathrm{i}\bar{\sigma}^\mu \partial_\mu \chi - m(\psi^\dagger \chi + \chi^\dagger \psi) . \tag{A.4}$$

Remarks on notation

One can construct a spinor that transforms as right-handed field ψ from a left handed-field χ

$$\psi_\chi \equiv \mathrm{i}\sigma_2 \chi^* \,. \tag{A.5}$$

In the following we will use the simplified notation $\chi\chi \equiv \psi^\dagger_\chi \chi$, implying that all products of Weyl spinors are to be understood as Lorentz invariants.[1]

We will furthermore introduce the SU(2) invariant product $\chi_A \cdot \chi_B \equiv \chi_A \, \mathrm{i}\tau_2 \chi_B \equiv \psi^\dagger_{\chi_A} \mathrm{i}\tau_2 \chi_B$. Since all right-handed fields can be represented as charge conjugates of left-handed fields and vice versa, we will use only left-handed fields in the following. The right-handed electron e_R, for example, is represented as left-handed positron $(e_R)^c = (e^c)_L$.

SUSY invariant Lagrangian

Infinitesimal SUSY transformations can be described as follows

$$\delta_\xi \phi = \xi\chi \,, \tag{A.6a}$$
$$\delta_\xi \chi = -\mathrm{i}\sigma^\mu \mathrm{i}\sigma_2 \xi^* \partial_\mu \phi^* + \xi F \,, \tag{A.6b}$$
$$\delta_\xi F = -\mathrm{i}\xi^\dagger \bar\sigma^\mu \partial_\mu \chi \,, \tag{A.6c}$$

where ξ is an infinitesimal spinor. F is a so-called auxiliary Field which is necessary to close the SUSY algebra off-shell. With other words the relation

$$(\delta_\eta \delta_\xi) X = (\eta^\dagger \bar\sigma^\mu \xi - \xi^\dagger \bar\sigma^\mu \eta) \mathrm{i}\partial_\mu X \tag{A.7}$$

is true for $X = \phi, \chi, F$. One can say that F compensates the two-additional degrees of freedom that a Weyl spinor has off-shell. We can now write down a Lagrangian that is invariant under SUSY transformations:

$$\mathcal{L} = \partial_\mu \phi^\dagger \partial^\mu \phi + \chi^\dagger \mathrm{i}\bar\sigma_\mu \partial^\mu \chi + F^\dagger F \,. \tag{A.8}$$

Vector multiplets

So far, we have only considered supermultiplets with a scalar and a fermionic part. A theory that contains the SM must of course also allow the description of vector bosons. We start with a generic gauge boson W^a with the field strength tensor

$$F^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g f^{abc} W^b_\mu W^c_\nu \tag{A.9}$$

[1] A more detailed overview on the properties of Weyl-spinors can be found in section 2 of Ref. [57].

and the covariant derivative

$$D_\mu = \partial_\mu + igT^a W_\mu^a \,, \tag{A.10}$$

where the T^a are the generators of the gauge symmetry. The infinitesimal SUSY transformations of the vector multiplets are

$$\delta_\xi W^a = \xi^\dagger \bar{\sigma}_\mu \lambda^a + \text{h.c.}\,, \tag{A.11a}$$

$$\delta_\xi \lambda^a = \frac{1}{2}\sigma^\mu \bar{\sigma}^\nu \xi F_{\mu\nu}^a + \xi D^a \,, \tag{A.11b}$$

$$\delta_\xi D^a = -\mathrm{i}\xi^\dagger \bar{\sigma}^\mu (D_\mu \lambda)^a + \text{h.c.}\,, \tag{A.11c}$$

The λ^a are the so-called gauginos, the spin $\frac{1}{2}$ SUSY partners of the spin-1 gauge bosons. The D^a are again auxiliary fields similar to the F fields of chiral multiplets.

We can now construct a Lagrangian which is invariant under the transformations from Eq. (A.11):

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} + \mathrm{i}\lambda^{a\dagger} \bar{\sigma}^\mu (D_\mu \lambda)^a + \frac{1}{2} D^a D^a \,. \tag{A.12}$$

A SUSY invariant Lagrangian that contains chiral as well as vector multiplets reads

$$\begin{aligned}\mathcal{L} = \partial_\mu \phi^\dagger \partial^\mu \phi + \chi^\dagger \mathrm{i}\bar{\sigma}_\mu \partial^\mu \chi + F^\dagger F - \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} + \mathrm{i}\lambda^{a\dagger} \bar{\sigma}^\mu (D_\mu \lambda)^a + \frac{1}{2} D^a D^a \\ - \sqrt{2} g (\phi^\dagger T^a \chi \lambda^a + \text{h.c.}) - g\phi^\dagger T^a \phi D^a\end{aligned} \tag{A.13}$$

The Lagrangian in Eq. (A.13) is invariant under the SUSY transformations (A.6) and (A.11) if Eq. (A.6c) is replaced by:

$$\delta_\xi F = -\mathrm{i}\xi^\dagger \bar{\sigma}^\mu \partial_\mu \chi - \sqrt{2} q \lambda^\dagger \xi^\dagger \phi \,. \tag{A.14}$$

In a specific model the auxiliary fields can be expressed in terms of the physical fields, by solving their equations of motion.

Superfield formalism

One can describe chiral supermultiplets with so called superfields, which depend on spatial coordinates x and fermionic degrees of freedom. The latter can be expressed using Grassman numbers θ which are anticommuting, *i.e.*, $\{\theta_i; \theta_j\} = 0$. It follows immediatly that $\theta_i^2 = 0$, and the highest non-vanishing product of a two component Grassmann number is $\theta \cdot \theta \equiv \theta \mathrm{i}\sigma_2 \theta = -2\theta_1\theta_2$.

A superfield is then defined in terms of its component fields ϕ, χ and F by

$$\hat{\Phi}(x,\theta) = \phi(x) + \theta\chi(x) + \frac{1}{2}\theta \cdot \theta F(x)\,, \tag{A.15}$$

and transforms as

$$\delta\hat{\Phi} = (-\mathrm{i}\xi Q + \mathrm{i}\xi^* Q^\dagger)\hat{\Phi}\,, \tag{A.16}$$

with

$$Q_a = \mathrm{i}\frac{\partial}{\partial\theta^a}\,. \tag{A.17}$$

One can easily convince oneself that Eq. (A.16) implies the correct transformation of the component fields given in the previous subsections. The following product rules for superfields will be very useful in the following:

Product of two superfields

$$\hat{\Phi}_i\hat{\Phi}_j = \phi_{ij} + \theta\chi_{ij} + \frac{1}{2}\theta\theta F_{ij}\,, \tag{A.18}$$

with

$$\phi_{ij} = \phi_i\phi_j\,, \tag{A.19a}$$
$$\chi_{ij} = \chi_i\phi_j + \phi_i\chi_j\,, \tag{A.19b}$$
$$F_{ij} = \phi_i F_j + \phi_j F_i - \chi_i\chi_j\,. \tag{A.19c}$$

Product of three superfields

$$\hat{\Phi}_i\hat{\Phi}_j\hat{\Phi}_k = \phi_{ijk} + \theta\chi_{ijk} + \frac{1}{2}\theta\theta F_{ijk}\,, \tag{A.20}$$

with

$$\phi_{ijk} = \phi_i\phi_j\phi_k\,, \tag{A.21a}$$
$$\chi_{ijk} = \chi_i\phi_j\phi_k + \phi_i\chi_j\phi_k + \phi_i\phi_j\chi_k\,, \tag{A.21b}$$
$$F_{ijk} = \phi_i\phi_j F_k + \phi_j\phi_k F_i + \phi_k\phi_i F_j - \chi_i\chi_j\phi_k - \chi_j\chi_k\phi_i - \chi_k\chi_i\phi_j\,. \tag{A.21c}$$

The superpotential

A SUSY model is usually described by a so-called superpotential. An example of a simple SUSY invariant model is the Wess-Zumino model [226]. It can be specified by the superpotential

$$W_{\mathrm{WZ}} = \frac{1}{2}M_{ij}\hat{\Phi}_i\hat{\Phi}_j + \frac{1}{6}Y_{ijk}\hat{\Phi}_i\hat{\Phi}_j\hat{\Phi}_k\,. \tag{A.22}$$

To obtain the according Lagrangian, one has to project out the fermionic degrees of freedom by executing the integral $\int \mathrm{d}^2\theta\, W$. Using the integration rules for Grassman numbers will give us the F-terms of the superfields and their products. So for practical use we can compute the products of superfields and obtain SUSY invariant interactions from the F-component of the resulting expression. Applying this procedure to Eq. (A.22) gives us the following SUSY invariant Lagrangian

$$\begin{aligned}\mathcal{L}_{\mathrm{WZ}}^{\mathrm{int}} = \frac{1}{2}M_{ij}(\phi_i F_j + \phi_j F_i - \chi_i\chi_j) + \frac{1}{6}Y_{ijk}(&\phi_i\phi_j F_k + \phi_j\phi_k F_i + \phi_k\phi_i F_j \\ &- \chi_i\chi_j\phi_k - \chi_j\chi_k\phi_i - \chi_k\chi_i\phi_j) + \text{h.c.}\end{aligned} \tag{A.23}$$

For the auxiliary fields we obtain

$$F_i = \frac{1}{2}M_{ij}^\dagger\phi_j^\dagger + \frac{1}{6}Y_{ijk}^\dagger(\phi_j\phi_k)^\dagger\,. \tag{A.24}$$

Similarly the D terms become

$$D^a = g\phi^\dagger T^a\phi\,. \tag{A.25}$$

With the methods presented in this section it is now possible to consistently describe supersymmetric theories. After establishing the formalism, we therefore are now able to discuss its application to more realistic physics models.

B. Programs Used for the Calculation of Cross-Sections

Here we want to outline the principle steps that were necessary to obtain the numerical results for the model discussed in chapter 5. The software we used was WHIZARD [159] and SARAH [160, 161]. The intend of this appendix is not to given a detailed technical description but rather to provide an overview of the relevant features.

An early implementation of the model in WHIZARD 1 was used to obtain the results presented in Ref. [138]. We enlarged and improved the corresponding source code further in order to produce the data presented in Ref. [137] and in this thesis. This data was also cross-checked with an independent implementation in WHIZARD 2 that was automatically generated by SARAH.

More information about WHIZARD as well as download and installation instructions can be found at the URL

http://whizard.hepforge.org/

There it is described as follows:

> *WHIZARD is a program system designed for the efficient calculation of multi-particle scattering cross sections and simulated event samples.*

In principal it has two core components:

- The matrix element generator O'MEGA, which according to an implemented set of fields and interactions ("Feynmanrules") produces tree-level matrix elements for specified initial and final states.
- The other component is Monte-Carlo phase space integrator, which comes with advanced functionality such as the option to use parton distribution functions.

WHIZARD comes pre-bundled with some model files for instance ones for the SM, MSSM and NMSSM. It can, however, also be flexibly used with a large range of uncommon models. We made use of this property in order to realize our TeV scale neutrino mass model. If only a certain number of processes has to be studied it is sufficient to implement only the relevant particles and interactions and not the full model, which would be a much more challenging task. The corresponding code that defines our scenario has not to be build up from scratch. Instead one can build up on pre-existing models such as the SM. (Since we where not interested in the phenomenology of the superpartners of the new physics fields we could avoid the unnecessary overhead of a full SUSY implementation.)

The most important file for our implementation is the models file `models.ml` which is located at `omega-src/bundle/src` within the WHIZARD file-tree. It is written in the functional programming language OCAML. In this file we have to define the new particles besides the SM fields:

```
type matter_field = L of int | U of int | D of int | Xp of int | N1 | N2 | N3 | N4 | N5 | N6 | N7 | N8 |
    N9
type gauge_boson = Ga | Wp | Wm | Z | Gl
type other = Phip | Phim | Phi0 | H
type flavor = M of matter_field | G of gauge_boson | O of other
```

Here `Xp of int` corresponds to the two generations of charged Dirac Fermions and `N1`-`N9` to the nine neutral Majorana fermions. The program has also to be told how the new fields behave under charge conjugation and Lorentz transformations. This can be done in analogy to the SM fields. Some rather technical aspect is the specification of how the names and parameters of the particles have to be translated into FORTRAN code and further aspects regarding the communication between O'MEGA and the main program, which is later automatically produced. The physically more relevant aspect is the definition of the interactions. We will illustrate this for the charged fields and `N1` as example for a neutral state. The interactions of the other neutral states are equivalent. The neutral currents are, *e.g.*, implemented as

```
((N1, Z, N1), FBF (1, Chibar, VLR, Chi), G_Z99);
((N1, Z, N2), FBF (1, Chibar, VLR, Chi), G_Z98);
((N1, Z, N3), FBF (1, Chibar, VLR, Chi), G_Z97);
((N1, Z, N4), FBF (1, Chibar, VLR, Chi), G_Z96);
((N1, Z, N5), FBF (1, Chibar, VLR, Chi), G_Z95);
((N1, Z, N6), FBF (1, Chibar, VLR, Chi), G_Z94);
((N1, Z, N7), FBF (1, Chibar, VLR, Chi), G_Z93);
((N1, Z, N8), FBF (1, Chibar, VLR, Chi), G_Z92);
((N1, Z, N9), FBF (1, Chibar, VLR, Chi), G_Z91);
```

and so on. The first part specifies the participating fields, the middle one the chiral structure and the last one the according coupling which has to be provided as input. The charged currents are in part:

```
((Xp (-1), Wp, N1), FBF (1, Psibar, VLR, Chi), G_U98);
((Xp (-1), Wp, N1), FBF (1, Psibar, VLR, Chi), G_U98);
((Xp (-1), Wp, N1), FBF (1, Psibar, VLR, Chi), G_U98);
((N1, Wm, Xp 1), FBF (1, Chibar, VLR, Psi), CG_U98);
((Xp (-1), Wp, N2), FBF (1, Psibar, VLR, Chi), G_U88);
((N1, Wp, L 1), FBF (1, Chibar, VL, Psi), G_U91);
((L (-1), Wm, N1), FBF (1, Psibar, VL, Chi), CG_U91);
((N1, Wp, L 2), FBF (1, Chibar, VL, Psi), G_U92);
((L (-2), Wm, N1), FBF (1, Psibar, VL, Chi), CG_U92);
((N1, Wp, L 3), FBF (1, Chibar, VL, Psi), G_U93);
((L (-3), Wm, N1), FBF (1, Psibar, VL, Chi), CG_U93);
((N1, Wm, Xp 1), FBF (1, Chibar, VLR, Psi), CG_U98);
((L (-1), Wm, N1), FBF (1, Psibar, VL, Chi), CG_U91);
((N1, Wp, L 2), FBF (1, Chibar, VL, Psi), G_U92);
((L (-2), Wm, N1), FBF (1, Psibar, VL, Chi), CG_U92);
((N1, Wp, L 3), FBF (1, Chibar, VL, Psi), G_U93);
((L (-3), Wm, N1), FBF (1, Psibar, VL, Chi), CG_U93);
```

Similarly we net the Yukawa-like interactions to the neutral Higgs component:

```
((M N1, O H, M N1), FBF (1, Chibar, SLR, Chi), G_H99);
((M N1, O H, M N2), FBF (1, Chibar, SLR, Chi), G_H98);
((M N1, O H, M N3), FBF (1, Chibar, SLR, Chi), G_H97);
((M N1, O H, M N4), FBF (1, Chibar, SLR, Chi), G_H96);
((M N1, O H, M N5), FBF (1, Chibar, SLR, Chi), G_H95);
((M N1, O H, M N6), FBF (1, Chibar, SLR, Chi), G_H94);
((M N1, O H, M N7), FBF (1, Chibar, SLR, Chi), G_H93);
((M N1, O H, M N8), FBF (1, Chibar, SLR, Chi), G_H92);
((M N1, O H, M N9), FBF (1, Chibar, SLR, Chi), G_H91);
```

In the file `conf/models/whizard.mdl` we have to define all constants (including the mixing matrix elements) the particles and their properties and also their vertices, for instance

```
parameter u11 = -0.000102195
parameter u12 = -0.000102195
parameter u13 = -0.000102195
...
particle NEUTRINO_ONE 28
spin 1/2, isospin 1/2
name nu_1, omega:n1, tex:\nu_1
mass mn1
...
particle XI_PLUS 27
spin 1/2,  isospin 1/2, charge 1
name xp, chep:xp1, tex:\xi^+
anti     mad:omega:xpbar, chep:Xp1, tex:\bar\xi^-
mass mxp, width wxp
...
particle N_FIVE 32
spin 1/2
name n5, tex:n_5
mass mn5, width wn5
```

```
...
vertex n1 W- xp1
vertex Xp1 W+ n1
```

For technical reasons we also have to change the files that in our model correspond to `conf/models/parameters.SM.omega.f90` to declare the according Fortran functions and `omega-src/bundle/src/f90_SM.mdl` to enable Majorana particles. The mixing matrix elements are calculated independently and provided as input. The decay width of the particles where first computed with Whizard itself and then added to the parameters.

Sarah is available under

http://sarah.hepforge.org/

and is described as

> *...a Mathematica package for building and analyzing SUSY and non-SUSY models. SARAH just needs the gauge structure, particle content and (super)potential to produce all information about the gauge eigenstates of a model.*

It was possible to generate the above described Model files automatically with Sarah. Therefore one has essentially just to define the fields

```
Fields[[1]] = {{uL,  dL},  3, q,   1/6, 2, 3};
Fields[[2]] = {{vL,  eL},  3, l,  -1/2, 2, 1};
Fields[[3]] = {{Hd0, Hdm}, 1, Hd, -1/2, 2, 1};
Fields[[4]] = {{Hup, Hu0}, 1, Hu,  1/2, 2, 1};

Fields[[5]] = {conj[dR], 3, d,  1/3, 1, -3};
Fields[[6]] = {conj[uR], 3, u, -2/3, 1, -3};
Fields[[7]] = {conj[eR], 3, e,    1, 1,  1};

Fields[[8]] = {N1, 2, n1,    0, 1,  1};
Fields[[9]] = {N2, 2, n2,    0, 1,  1};
Fields[[10]] = {{x10, x1m}, 1, xi1,    -1/2, 2,  1};
Fields[[11]] = {{x2p, x20}, 1, xi2,     1/2, 2,  1};
```

and their superpotential

```
SuperPotential = { {{1, Yu},{u,q,Hu}}, {{-1,Yd},{d,q,Hd}},
                   {{-1,Ye},{e,l,Hd}}, {{1,\[Mu]},{Hu,Hd}},
                   {{1,Yn},{n1,l,Hu}}, {{-1,kappa1}, {n2,xi2,Hd}},
                   {{1,kappa2},{n2,xi1,Hu}}, {{1,mN},{n1,n2}},
                   {{1,mX},{xi2,xi1}}
};
```

We have cross-checked both independent implementations by comparing the generated source code itself to the manually edited files as well as by recalculating some of the previously obtained cross-sections with the automatic code.

C. Approximate Diagonalization of the Neutral Fermion Mass Matrix

Here we want to show how the mass matrix of the model that we studied in chapter 5 can be approximately diagonalized (from appendix B of Ref. [137]):

In our model the complete mass matrix including the flavor structure is given by

$$\begin{pmatrix}
0 & 0 & 0 & v_u Y_{N,11} & v_u Y_{N,12} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & v_u Y_{N,21} & v_u Y_{N,22} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & v_u Y_{N,31} & v_u Y_{N,32} & 0 & 0 & 0 & 0 \\
v_u Y_{N,11} & v_u Y_{N,21} & v_u Y_{N,31} & 0 & 0 & M_N & 0 & 0 & 0 \\
v_u Y_{N,12} & v_u Y_{N,22} & v_u Y_{N,32} & 0 & 0 & 0 & M_N\rho & 0 & 0 \\
0 & 0 & 0 & M_N & 0 & 0 & 0 & -k_1 v_d & k_2 v_u \\
0 & 0 & 0 & 0 & M_N\rho & 0 & 0 & k_1 v_d & k_2 v_u \\
0 & 0 & 0 & 0 & 0 & -k_1 v_d & k_1 v_d & 0 & -m_\xi \\
0 & 0 & 0 & 0 & 0 & k_2 v_u & k_2 v_u & -m_\xi & 0
\end{pmatrix}$$

Using the fact, the the left-handed neutrinos are essentially massless compared to the heavy states we can exploit the usual seesaw formulas to obtain approximate formulas for the entries responsible for the mixing of the light states with the heavy states. The mass matrix of the heavy states is given by

$$M_H = \begin{pmatrix}
0 & 0 & M_N & 0 & 0 & 0 \\
0 & 0 & 0 & M_N\rho & 0 & 0 \\
M_N & 0 & 0 & 0 & -k_1 v_d & k_2 v_u \\
0 & M_N\rho & 0 & 0 & k_1 v_d & k_2 v_u \\
0 & 0 & -k_1 v_d & k_1 v_d & 0 & -m_\xi \\
0 & 0 & k_2 v_u & k_2 v_u & -m_\xi & 0
\end{pmatrix}$$

Neglecting the elements proportional to k_i $(i = 1, 2)$ this matrix is diagonalized by

$$R_H = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

The part of the mixing matrix connecting the heavy states with the light states is given by

$$\begin{aligned} U' &= m M_H^{-1} R_H \\ &= \begin{pmatrix} D_1 Y_{N,11} & D_2 Y_{N,12} & D_2' Y_{N,11} & D_1' Y_{N,12} & \frac{v_u v_d (k_2' Y_{N,12} - k_1' \rho Y_{N,11})}{\sqrt{2} M_N m_\xi \rho} & \frac{v_u v_d (k_2' \rho Y_{N,11} - k_1' Y_{N,12})}{\sqrt{2} M_N m_\xi \rho} \\ D_1 Y_{N,21} & D_2 Y_{N,22} & D_2' Y_{N,21} & D_1' Y_{N,22} & \frac{v_u v_d (k_2' Y_{N,22} - k_1' \rho Y_{N,21})}{\sqrt{2} M_N m_\xi \rho} & \frac{v_u v_d (k_2' \rho Y_{N,21} - k_1' Y_{N,22})}{\sqrt{2} M_N m_\xi \rho} \\ D_1 Y_{N,31} & D_2 Y_{N,32} & D_2' Y_{N,31} & D_1' Y_{N,32} & \frac{v_u v_d (k_2' Y_{N,32} - k_1' \rho Y_{N,31})}{\sqrt{2} M_N m_\xi \rho} & \frac{v_u v_d (k_2' \rho Y_{N,31} - k_1' Y_{N,32})}{\sqrt{2} M_N m_\xi \rho} \end{pmatrix} \end{aligned}$$

with

$$\begin{aligned} m &= \begin{pmatrix} v_u Y_{N,11} & v_u Y_{N,12} & 0 & 0 & 0 & 0 \\ v_u Y_{N,21} & v_u Y_{N,22} & 0 & 0 & 0 & 0 \\ v_u Y_{N,31} & v_u Y_{N,32} & 0 & 0 & 0 & 0 \end{pmatrix} \\ D_1 &= \frac{v_u (M_N m_\xi + 2 k_1 k_2 v_d v_u)}{\sqrt{2} M_N^2 m_\xi} \\ D_2 &= \frac{v_u (\rho M_N m_\xi - 2 k_1 k_2 v_d v_u)}{\sqrt{2} \rho^2 M_N^2 m_\xi} \\ D_1' &= -\frac{v_u (\rho M_N m_\xi + 2 k_1 k_2 v_d v_u)}{\sqrt{2} \rho^2 M_N^2 m_\xi} \\ D_2' &= -\frac{v_u (M_N m_\xi - 2 k_1 k_2 v_d v_u)}{\sqrt{2} M_N^2 m_\xi} \\ k_1' &= k_1 - k_2 \tan\beta \\ k_2' &= k_1 + k_2 \tan\beta \end{aligned}$$

Here we have the following correspondence to the couplings in chapter 5, Eq. (5.22):

$$c_i = U'_{i5} \ , \ \ d_i = (U'_{i6})^*$$

which are the dominating ones for the lepton number violating processes.

D. Charge of Effective Operators in the NMSSM GUT Scenario

We will demonstrate here, that the NMSSM GUT model specified in Eq. (6.12) will always imply the existence of the Weinberg operator. For the convenience of the reader we reprint the according superpotential also at this place:

$$\begin{aligned} W = \quad & y_3\, N\, \overline{5}_M\, H_5 + y_1',\, N'\, 5_\xi\, H_{\overline{5}} + y_2'\, N'\, \overline{5}_{\xi'}\, H_5 + \\ & \lambda_\xi\, S'\, \overline{5}_{\xi'}\, 5_\xi + \lambda_N S'\, N' N + y_d\, \overline{5}_M\, 10\, H_{\overline{5}} + \\ & y_u\, 10\, 10\, H_5 + \lambda_S S H_{\overline{5}} H_5 + \kappa S^3 + \kappa' {S'}^3 + \\ & \lambda_S' S' H_{\overline{5}} H_5 + y_3'\, N'\, \overline{5}_M\, H_5 + y_d'\, \overline{5}_\xi\, 10\, H_{\overline{5}} + \cdots\,, \end{aligned} \tag{D.1}$$

The problematic term here is $\lambda_{\xi'}\, S\, \overline{5}_M\, 5_\xi$, which leads to a mixing between the light and heavy mediators and thus induces a $d = 5$ operator. We will prove now that one cannot forbid this term by a discrete symmetry that fulfills the required conditions (from Ref. [139]):

For this we start with three (yet) unconstrained charges as parameters

$$q_S \equiv s\,, \quad q_{H_5} \equiv 2h'\,, \quad q_N = n\,. \tag{D.2}$$

From the absolutely necessary terms in the superpotential we derive

$$(S H_5 H_{\bar{5}}) \quad \Rightarrow \quad q_{H_{\bar{5}}} = -s - 2h' \tag{D.3a}$$

$$(S N N') \quad \Rightarrow \quad q_{N'} = -s - n \tag{D.3b}$$

$$(N \bar{5}_M H_5) \quad \Rightarrow \quad q_M = -2h' - n \tag{D.3c}$$

$$(\bar{5}_M 10 \bar{H}_{\bar{5}}) \quad \Rightarrow \quad q_{10} = 4h' + n + s\,. \tag{D.3d}$$

From the term $(1010 H_5)$ we obtain

$$n = -s - 5h'\,. \tag{D.4}$$

which leads to the following set of equations

$$q_{H_{\bar{5}}} = -s - 2h'\,,\ q_{N'} = 5h'\,,\ q_M = 3h' + s\,,\ q_{10} = -h'\,. \tag{D.5}$$

As a consequence we derive for the charges of the doublets

$$(N' H_{\bar{5}} 5_\xi) \qquad \Rightarrow \qquad q_\xi = -3h' + s \tag{D.6}$$

$$(N' \bar{5}_{\xi'} H_5) \qquad \Rightarrow \qquad q_{\xi'} = -7h' \tag{D.7}$$

leading to

$$q(S \bar{5}_M 5_\xi) = 3s\,, \tag{D.8}$$

but we know that $3s = 0$ since we need a term S^3 in the NMSSM. This implies that one cannot forbid this unwanted term. Note, that this holds for every Abelian discrete symmetry group. As a consequence one can show along the same lines that every higher dimension operator for neutrino masses

$$\mathcal{O}^d = \frac{1}{\langle S \rangle^{1+2k+l}} (L L H_u H_u)(H_u H_d)^k S^l \tag{D.9}$$

has to have the same charge as the Weinberg operator. We have also checked that similar problems appear if the Abelian symmetry is chosen as a product of two different cyclic groups $Z_N \otimes Z_{N'}$.

E. Definition of Cosmology Parameters

For the discussion of the evolution of stable heavy d-quarks we use the following parameters and definitions [175]:

- R is the scale factor of the universe.
- s is the entropy density. Since s is conserved in a comoving volume ($sR^3 = \text{const}$) we have

$$s\dot{Y} = \dot{n} + 3Hn\,.$$

- The entropy density in the radiation dominated universe is:

$$s = 4\rho_\gamma/(3T) = 4\pi^2/45T^3 = 4\pi^2/45(m/x)^3\,.$$

- The Hubble parameter:

$$H = \sqrt{8\pi^3 g_*/90}\,T^2/m_{\text{Pl}}^2 = \sqrt{8\pi^3 g_*/90}\,(m/m_{\text{Pl}})^2 x^{-2}\,.$$

- The relativistic degrees of freedom at the relevant temperatures are only those of the SM particles so that $g_* \approx g_{*S} = 106.75\,.$
- The photon number density in the radiation dominated epoch is given by $n_\gamma = \zeta(3)/\pi^2 g T^3$ with $g = 2$ the internal d. o. f.

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References

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Erklärung

Hiermit erkläre ich, dass ich die vorliegende Arbeit selbstständig verfasst, keine anderen als die angegebenen Quellen und Hilfsmittel benutzt und die Arbeit bisher oder gleichzeitig keiner anderen Prüfungsbehörde zur Erlangung eines akademischen Grades vorgelegt habe.

Würzburg, den 26. November 2013

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