Diplomarbeit

Flavor Physics in Universal Extra Dimension Models with Brane Kinetic Localized Terms

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To all who seek more beyond our realms

Zusammenfassung

Das Standard Modell (SM) ist eine der am besten getesteten Theorien der Physik. Jedoch sind bis heute einige Fragen offen, die durch Erweiterungen des SM beantwortet werden sollen. Eine Möglichkeit besteht darin, eine zusätzliche, raumartige Dimension mit flacher Metrik anzunehmen und die Auswirkungen auf die effektive 4D Theorie zu berechnen. In dieser Arbeit soll unter der Annahme einer flachen fünf-dimensionalen Raumzeit die Veränderung des Fermionspektrums durch randlokalisierte, kinetische Terme berechnet werden. Zunächst werden die Wellenfunktionen und das Massenspektrum der links- und rechtshändigen Fermionen einer Familie berechnet. Danach werden die Wilsonkoeffizienten in einer effektiven Vier-Fermion-Kontaktwechselwirkung bestimmt, die von den Überlappintegralen der Fermionen abhängen. Beim Übergang von der Eich- in die Masseneigenbasis der Quarks werden flavorverletzende neutrale Ströme induziert (FCNCs), die mit Hilfe der Wilsonkoeffizienten quantifiziert werden können. Diese Transformationen sind ad hoc nicht festgelegt und werden in zwei speziellen Fällen untersucht. Abschließend werden diese Koeffizienten mit Modell-unabhängigen Beschränkungen von $\Delta F=2$ Prozessen verglichen um das Massespektrum der Fermionen einzuschränken.

Abstract

The Standard Model (SM) is one of the best tested theories in physics. However, until today some questions remain unanswered while extensions to the SM try to provide a solution. One possibility is to assume an additional spatial dimension with flat metric. In this diploma thesis we want to calculate the alteration to the fermion spectrum due to boundary localized kinetic terms under the assumption of a flat 5D space-time. At first we calculate the wave functions and the mass spectrum of the left- and right handed fermions for one family. After this we determine the Wilson coefficients in an effective four Fermi interaction which depend significantly on the the overlap integrals from the fermion sector. Going from the gauge eigenbasis to the mass eigenbasis of the quarks, we get flavor changing neutral currents (FCNCs) which can be quantified with the Wilson coefficients. Ad hoc these transformations are not determined and are examined in two special cases. We conclude by comparing the Wilson coefficients to model independent constraints for $\Delta F=2$ processes to constrain the mass spectrum of the fermions.

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1. Introduction

The Standard Model (SM) is one of the best tested theories in physics which shows only small deviations between the theoretically calculated and experimentally measured parameters. But even though the SM is describing nature so successfully, there are several questions left to answer like the hierarchy problem, the strong CP problem, the flavor problem, neutrino masses and the nature of dark matter [1]. There are many ideas to solve these problems and try to embed the SM in larger theories. One of these is to introduce further dimension(s) in addition to our four dimensional space-time. The proposition that space-time has more than 3 spatial dimensions was first proposed by Kaluza [2] and Klein [3] in the attempt to unify electromagnetic forces with gravity. Up to now, there are several main branches in the physics of extra dimensions which try to extend the SM and solve the mentioned problems and furthermore make predictions for colliders like the Large Hadron Collider.

The first type of model which we want to mention are models with large extra dimensions [4], where a n dimensional compact manifold with radius R is attached to every 4D space-time point. In the simplest case, the metric of this extra dimension is flat. In these types of models only the gravitational force is promoted to the extra dimension(s) whereas all other SM fields are localized on a 3-brane. One of the consequences is a deviation in the measurement of Newtons gravitational law which depends on the number n of the extra dimension(s) and the compactification radius R

$$V(r) \sim \frac{m_1 m_2}{M^{n+2}_{\mathrm{Pl(4+n)}}} \frac{1}{r^{n+1}} \ , \tag{1.1}$$

where m_1 and m_2 are two test masses with distance $r << R$ and $M^{n+2}_{\mathrm{Pl(4+n)}}$ is the Planck scale. This has not been found for sizes of the compactification radius $R > 37\ \mu m$ [5].
In so-called "Randall-Sundrum-Models" or models with warped extra dimensions [6], the assumption of a flat metric is dropped and instead an Anti-DeSitter metric is used:

$$\mathrm{d}s^2 = e^{-2kR\phi} g_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu + R^2 \mathrm{d}y^2 \ , \tag{1.2}$$

where k is the curvature and y is the coordinate of the extra dimension with size R. The introduction of a term $e^{-2kR\phi}$ into the metric is used to find a connection between the electroweak and the Planck scale and explain their large hierarchy.
The group of extra dimensional models which is used in this thesis are the universal extra dimensional (UED) models, also called ACD-models introduced by Appelquist, Cheng

and Dobrescu [7]. In contrast to the models we described before, these ACD-models promote every SM field into a (or many) flat extra dimension(s) which is in the simplest case a $S^1/\mathbb{Z}_2$-orbifold. In chapter 2 we will see that a $S^1/\mathbb{Z}_2$-orbifold has several advantages like getting chiral fermions in the 4D SM theory or preserving a stable lightest Kaluza-Klein (KK) particle (LKP) which serves as a viable dark matter candidate [8]. The UED model has been advertised as a model with only three undetermined parameters: the compactification radius R, the cut-off scale Λ and the Higgs mass m_h. It is therefore called the minimal universal extra dimension (mUED) model but as we show in this thesis it is not the complete set of possible parameters for such models.
A typical mUED mass spectrum calculated to one loop level is shown in Fig. 1.1.
The Lagrangian for the 5D mUED model is [9]:

$$\mathcal{L}_{UED} = \mathcal{L}_{Gauge} + \mathcal{L}_{GF} + \mathcal{L}_{Leptons} + \mathcal{L}_{Quarks} + \mathcal{L}_{Yukawa} + \mathcal{L}_{Higgs} \ , \tag{1.3}$$

where the several Lagrangians are:

$$\mathcal{L}_{Gauge} = \int_{-\frac{\pi R}{2}}^{\frac{\pi R}{2}} \left\{ -\frac{1}{4} B_{MN} B^{MN} - \frac{1}{4} W^a_{MN} W^{aMN} - \frac{1}{4} G^A_{MN} G^{AMN} \right\} , \tag{1.4a}$$

$$\begin{aligned} \mathcal{L}_{GF} = \int_{-\frac{\pi R}{2}}^{\frac{\pi R}{2}} \Big\{ & -\frac{1}{2\xi} \left(\partial^\mu B_\mu - \xi \partial_5 B_5\right)^2 - \frac{1}{2\xi} \left(\partial^\mu W^a_\mu - \xi \partial_5 W^a_5\right)^2 \\ & -\frac{1}{2\xi} \left(\partial^\mu G^A_\mu - \xi \partial_5 G^G_5\right)^2 \Big\} , \end{aligned} \tag{1.4b}$$

$$\mathcal{L}_{Leptons} = \int_{-\frac{\pi R}{2}}^{\frac{\pi R}{2}} \left\{ i\bar{L}(x,y)\Gamma^M D_M L(x,y) + i\bar{E}(x,y)\Gamma^M D_M E(x,y) \right\} , \tag{1.4c}$$

$$\begin{aligned} \mathcal{L}_{Quarks} = \int_{-\frac{\pi R}{2}}^{\frac{\pi R}{2}} \Big\{ & i\bar{Q}(x,y)\Gamma^M D_M Q(x,y) \\ & + i\bar{U}(x,y)\Gamma^M D_M U(x,y) + i\bar{D}(x,y)\Gamma^M D_M D(x,y) \Big\} , \end{aligned} \tag{1.4d}$$

$$\begin{aligned} \mathcal{L}_{Yukawa} = \int_{-\frac{\pi R}{2}}^{\frac{\pi R}{2}} \Big\{ & \lambda^U \bar{Q}(x,y) U(x,y) \tilde{H}(x,y) + \lambda^D \bar{Q}(x,y) D(x,y) H(x,y) \\ & + \lambda^E \bar{L}(x,y) E(x,y) H(x,y) \Big\} , \end{aligned} \tag{1.4e}$$

$$\mathcal{L}_{Higgs} = \int\limits_{-\frac{\pi R}{2}}^{\frac{\pi R}{2}} \Big[\left(D_M H(x,y)\right)^\dagger \left(D^M H(x,y)\right) + \mu^2 H^\dagger(x,y)\, H(x,y) \\ -\lambda \left(H^\dagger(x,y) H(x,y)\right)^2 \Big] \,. \tag{1.4f}$$

Here $H(x,y)$ is the 5D Higgs scalar field and $\tilde{H}(x,y) = i\tau_2\, H^*(x,y)$ is its charge conjugate, $(B_\mu(x,y), B_5(x,y))$, $(W_\mu(x,y), W_5(x,y))$ and $(G_\mu(x,y), G_5(x,y))$ are the 5D gauge fields B_M, W_M and G_M for $U(1)_Y$, $SU(2)_W$ and $SU(3)_C$, respectively. The capital index M runs over $M = \mu, 5$, where the greek index is $\mu = 0,1,2,3$. The 5D field strength tensors are defined as follows:

$$\begin{aligned} B_{MN} &= \partial_M B_N - \partial_N B_M \,, \\ W^a_{MN} &= \partial_M W^a_N - \partial_N W^a_M + \hat{g}\epsilon^{abc} W^b_M W^c_N \,, \\ G^A_{MN} &= \partial_M G^A_N - \partial_N G^A_M + \hat{g}_s f^{ABC} G^B_M G^C_N \,, \end{aligned} \tag{1.5}$$

where ϵ^{abc} and f^{ABC} are the structure constants for $SU(2)_W$ and $SU(3)_C$, respectively. The 5D coupling constants $\hat{g}'$ and $\hat{g}_s$ are related to the 4D couplings for the $SU(2)_W$ and $SU(3)_C$ via

$$g = \frac{\hat{g}}{\sqrt{\pi R}} \;; \qquad g_s = \frac{\hat{g}_s}{\sqrt{\pi R}} \,. \tag{1.6}$$

The parameter ξ in (1.4b) is the gauge fixing parameter in a R_ξ gauge. The definition and explanation for components of $\mathcal{L}_{Leptons}$, $\mathcal{L}_{Quarks}$ and $\mathcal{L}_{Yukawa}$ are moved to chapter 3 and 4.

Now we have to ask ourselves if we can extend the Lagrangian in eq. (1.3) and add terms which are not forbidden by gauge or 4D Lorentz invariance since 5D Lorentz invariance is explicitly broken due to working on a $S^1/\mathbb{Z}_2$-orbifold (see chapter 2). One possibility is to add mass terms which depend on the extra dimensional coordinate for the fermions to the Lagrangians $\mathcal{L}_{Leptons,Quarks}$ in eq. (1.4c) and/or eq. (1.4d) what is done in the so-called split-UED models [11]. Another possibility is to add boundary localized kinetic terms for the scalar, gauge and/or fermion fields. These δ-localized kinetic terms are induced radiatively [12] and therefore should be accounted for in the non minimal UED (nUED) Lagrangian. To the three parameters R, Λ and m_h in the mUED case, we then get another set of parameters, the boundary kinetic localized term (BKLT) parameters $a_h^{\Psi_i\Psi_j}$ which play the central role in this thesis. Previously, calculations for nUED where boundary localized kinetic terms are added for scalar and gauge fields were done in [13]. Here we want to concentrate on BKLT for fermions, especially quarks, and study how these terms alter the quark mass spectrum.

This thesis is organized as follows: In chapter 2 the structure of the $S^1/\mathbb{Z}_2$-orbifold and its consequences for the theory are examined more closely. Moreover we introduce the $\mathbb{Z}_2$- and KK parity and the gauge field decomposition which will be needed throughout

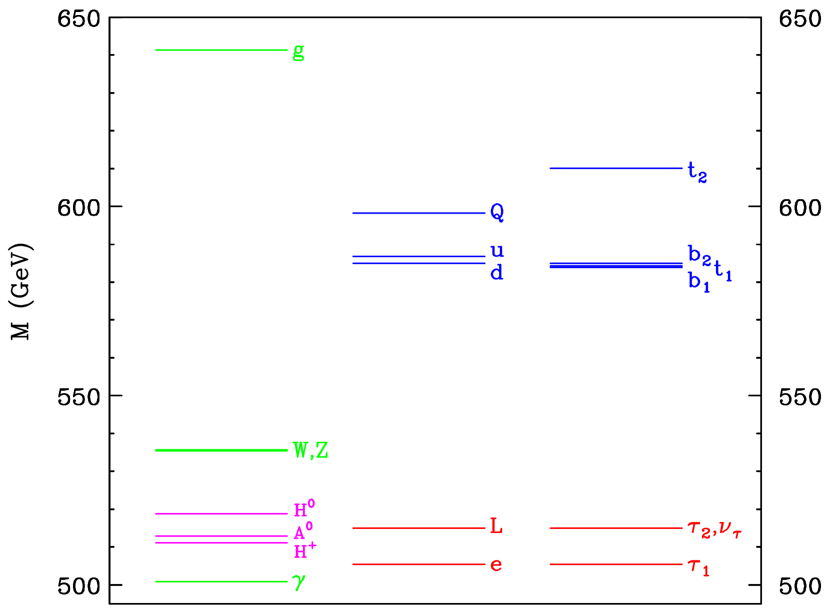


Figure 1.1.: One-loop corrected mass spectrum of the first Kaluza-Klein level in mUED for R^{-1}= 500GeV, ΛR=20 and m_h=120 GeV (from [10]).

this thesis. In chapter 3 we will decompose the fermion Lagrangian which is altered by BKLT and calculate the subsequent fermionic wave functions and the fermion mass spectrum. In chapter 4 we will use an effective theory approach to calculate a low energy limit of the nUED model in 4D and derive analytically the Wilson coefficients for four fermion interactions. We show that the Wilson coefficients of flavor changing neutral currents (FCNCs) four fermion operators are non-vanishing in nUED. Their dominant contribution arises due to the exchange of the KK gauge bosons with even KK modes with the KK zero modes of the fermions which are modified by the BKLT. In chapter 5 we will use these results to examine the bounds we get from FCNC processes with ΔF=2 operators and how these will constrain the fermion mass spectrum. To do this we assume how the Yukawa diagonalizing matrices are oriented in the $SU(3)$ flavor space and calculate these constraints with two special cases of these transformation matrices. Additionally, the Yukawa contributions to the fermion masses are calculated and it is shown how these shift the quark mass spectrum and finally if it is possible to lift mass degeneracies between the fermion families.

2. Compactification and Orbifolds

The KK decomposition for all SM fields contains many possibilities to modify the spectrum and the form of the 5D wave functions. In this diploma thesis only the fermion spectrum is significantly altered; all scalar and gauge fields will be decomposed as it is usually done in mUED. For a detailed discussion of the decomposition of the scalar or gauge fields see [7, 14].
We now shortly summarize the results which are needed throughout this thesis. First of all one has to find a way to incorporate further space-time dimensions since our observations are consistent with three spatial and one time-like dimension for distances larger than 37 μm [5]. One possibility to "hide" the extra dimensions is using compactification, i.e. we use compact manifolds that are attached to every four dimensional space-time point. We only want to consider one extra dimension while it is possible to extend this argument to more than one. The fifth dimension is assumed to be spatial [15]. We also work on a $S^1/\mathbb{Z}_2$-orbifold; this orbifold is obtained by reflecting one half of the circle S^1 on the other. This can be interpreted as an interval with orbifold fixed points at the end as shown in Fig. 2.1.

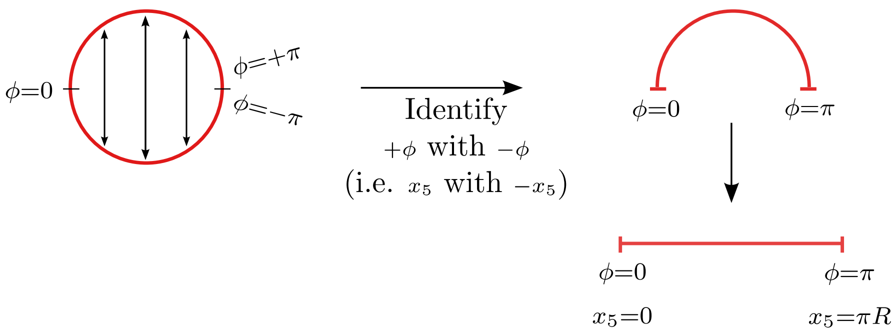


Figure 2.1.: Orbifolding the circle S^1 to an interval or $S^1/\mathbb{Z}_2$-orbifold. From [16].

Introducing an orbifold like this breaks 5D Lorentz invariance which automatically leads to a violation of 5D-momentum-conservation. Therefore, KK number is no longer a good quantum number [17]. A remnant from KK number conservation is KK parity which is still conserved in a $S^1/\mathbb{Z}_2$ compactification. This KK parity is responsible for the fact that only an even number of KK parity odd particles can interact with each other; so the first KK excitation of a particle is stable and cannot decay into SM particles. This lightest KK particle (LKP) is a possible dark matter candidate in UED which usually is the first KK excitation of the $U(1)_Y$ gauge field B^M [8].
The $S^1/\mathbb{Z}_2$ orbifold has further crucial advantages: It is possible to get chiral fermions in our 4D theory, which is not the case on a S^1 manifold [16]. The boundary conditions at the orbifold fixed points can be used to eliminate one chiral zero mode. So the fermion sector of the SM can be restored via setting the fermion which should not have a zero mode under a Dirichlet boundary condition:

$$\Psi_h(x,-L) = 0 = \Psi_h(x,L) \ , \tag{2.1}$$

where $\Psi_h(x,y)$ is a chiral Dirac fermion with h=right or h=left (see chapter 3) and -L,L are the orbifold fixed points. The fundamental domain is chosen symmetrically between L and -L with $L = \frac{\pi R}{2}$, where R is the compactification radius of the extra dimension.
Furthermore, one can introduce two symmetries which leave the 5D action invariant. This invariance will be shown for a arbitrary scalar field $\phi(x,y)$ and used for the KK decomposition of the gauge fields $A^M(x,y)$ in chapter 2.2.

2.1. $\mathbb{Z}_2$-Parity and KK Parity

First of all, we fix our notation: greek letters denote the Lorentz-indices in 4D space-time ($\mu = 0,1,2,3$), capital letters in 5D space-time ($M = 0,1,2,3,5$) where $x^5 \equiv y$ is the coordinate of the extra dimension. The metric g^{MN} in 5D is defined by:

$$g^{MN} = \begin{pmatrix} g^{\mu\nu} & \\ & -1 \end{pmatrix} \ , \tag{2.2}$$

where $g^{\mu\nu}$=diag(+1,-1,-1,-1).
The scalar action is given by:

$$S_\phi = \int\limits_{\mathbb{M}} \int\limits_{S^1/\mathbb{Z}_2} \mathrm{d}^5x \ \frac{1}{2}\partial_M\phi(x,y)\partial^M\phi(x,y) - \frac{m^2}{2}\phi(x,y)^2 - \frac{\lambda}{4!}\phi(x,y)^4 \ , \tag{2.3}$$

where $\phi(x,y)$ is a real scalar field, ∂^M is the derivative in 5D

$$\partial^M = \begin{pmatrix} \partial^\mu \\ \partial^5 \end{pmatrix} \quad \text{and} \quad \partial_M = \begin{pmatrix} \partial_\mu \\ \partial_5 \end{pmatrix} \ , \tag{2.4}$$

and m and λ are two real constants. $x^\mu \equiv x$ is the 4D space-time coordinate with Lorentz-index μ suppressed. We see that eq. (2.3) is invariant under

$$\begin{aligned} y-\frac{\pi R}{2} &\to -(y-\frac{\pi R}{2}) \quad \text{and} \\ \phi(x,y-\frac{\pi R}{2}) &\to \phi(x,-(y-\frac{\pi R}{2})) = \pm\phi(x,y-\frac{\pi R}{2})\ , \end{aligned} \tag{2.5}$$

so that

$$\begin{aligned} S_\phi \to S'_\phi = \int\limits_{\mathbb{M}}\int\limits_{S^1/\mathbb{Z}_2} & \mathrm{d}^5\mathrm{x}\ \frac{1}{2}(-\partial_M)(\pm\phi(x,y-\frac{\pi R}{2}))(-\partial^M)(\pm\phi(x,y-\frac{\pi R}{2})) \\ & -\frac{m^2}{2}(\pm\phi(x,y-\frac{\pi R}{2}))^2-\frac{\lambda}{4!}(\pm\phi(x,y-\frac{\pi R}{2}))^4 = S_\phi\ . \end{aligned} \tag{2.6}$$

This symmetry is called $\mathbb{Z}_2$-parity and is a direct consequence of the orbifolding process. We call a field *even* under $\mathbb{Z}_2$-parity if $\phi(x,-\tilde{y}) = \phi(x,\tilde{y})$ and *odd* under $\mathbb{Z}_2$-parity if $\phi(x,-\tilde{y}) = -\phi(x,\tilde{y})$ for $\tilde{y} = y - \frac{\pi R}{2}$. This can be used to eliminate the zero mode of fields which are not present in the SM (see chapter 2.2).
Another symmetry which leaves eq. (2.3) invariant is the KK parity. The corresponding transformations are

$$\begin{aligned} y &\to -y\ , \\ \phi(x,y) &\to \phi(x,-y) = \pm\phi(x,y)\ . \end{aligned} \tag{2.7}$$

We call a field *even* under KK parity if $\phi(x,-y) = \phi(x,y)$ and *odd* under KK parity if $\phi(x,-y) = -\phi(x,y)$. This parity is not a special feature of the $S^1/\mathbb{Z}_2$-orbifold but it is relevant for further investigation. The $\mathbb{Z}_2$-parity and KK parity are depicted in Fig. 2.2.

As an example we will do the KK decomposition for the scalar field. To get the action for a free and massive scalar field, we set λ=0 in eq. (2.3)[1]. The action now is:

$$S_\phi = \int\limits_{\mathbb{M}}\int\limits_{S^1/\mathbb{Z}_2} \mathrm{d}^5x\ \frac{1}{2}\partial_M\phi(x,y)\partial^M\phi(x,y) - \frac{m^2}{2}\phi(x,y)^2\ . \tag{2.8}$$

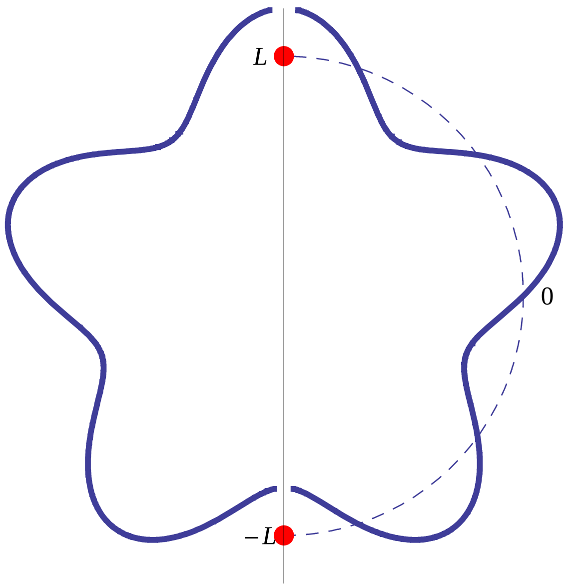
Using the Euler-Lagrange equation [18]

$$\frac{\partial\mathcal{L}}{\partial\phi} - \partial^M\frac{\partial\mathcal{L}}{\partial\left(\partial^M\phi\right)} = 0\ , \tag{2.9}$$

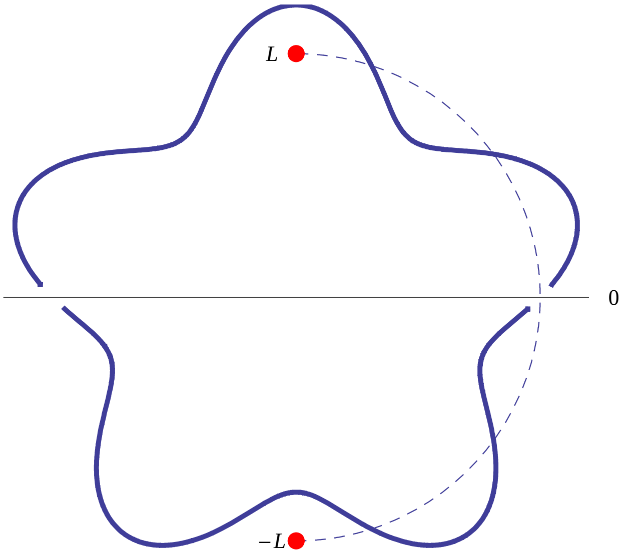
we get the following differential equation:

$$\Box_5\phi(x,y) + m^2\phi(x,y) = (\partial^\mu\partial_\mu + \partial^5\partial_5)\phi(x,y) + m^2\phi(x,y) = 0\ , \tag{2.10}$$

[1]The KK decomposition for the gauge fields can be done in an analogue calculation.



(a) $\mathbb{Z}_2$-parity even wave function.



(b) KK parity odd wave function.

Figure 2.2.: A polar plot for $f(y) = \sin(\frac{ny}{R})$ on a S^1. We set the compactification radius $R = 1\text{TeV}^{-1}$ and the KK mode number $n = 5$. The dashed line shows the $S^1/\mathbb{Z}_2$ orbifold, the big dots are the orbifold fixed points and the continuous line represents the symmetry axis. a) Depiction of the $\mathbb{Z}_2$-parity. b) Depiction of the KK parity

where $\Box_5$ is the d'Alembert operator in 5D

$$\Box_5 = \partial^M \partial_M = \partial^\mu \partial_\mu + \partial^5 \partial_5 = \Box_4 + \partial^5 \partial_5 \ . \tag{2.11}$$

Eq. (2.10) is the 5D Klein-Gordon equation for a scalar field $\phi(x, y)$ with mass m. To solve this partial differential equation we will use a separation ansatz of the form

$$\phi(x, y) = \sum_{n=0}^{\infty} \phi^{(n)}(x) f^{(n)}(y) \ . \tag{2.12}$$

Inserting the separation ansatz in eq. (2.10), we get:

$$\sum_{n=0}^{\infty} \left(f^{(n)}(y)\, \Box_4 \phi^{(n)}(x) - \phi^{(n)}(x)\, \partial_5^2 f^{(n)}(y) + m^2\, \phi^{(n)}(x) f^{(n)}(y) \right) = 0 \ . \tag{2.13a}$$

Dividing by $\phi^{(n)}(x)f(y)^{(n)}$, it holds:

$$\sum_{n=0}^{\infty}\left(\frac{\Box_4\phi^{(n)}(x)}{\phi^{(n)}(x)} - \underbrace{\frac{\partial_5^2 f^{(n)}(y)}{f^{(n)}(y)} + m^2}_{m_n^2}\right) = 0 \ . \tag{2.13b}$$

Since we can vary x and y independently, the fractions must be constant and therefore m_n has to be constant, too. Now we reformulate eq. (2.13b) and get a differential equation for the 5D part of the scalar field $f^{(n)}(y)$:

$$-\partial_5^2 f^{(n)}(y) = (m_n^2 - m^2)f^{(n)}(y) \equiv M_n^2 f^{(n)}(y) \ . \tag{2.14}$$

This is a linear second order differential equation. For $m_n^2 > m^2$ we choose the ansatz:

$$f^{(n)}(y) = A\sin(M_n y) + B\cos(M_n y) \ , \tag{2.15}$$

where M_n is defined in eq. (2.14) and A and B are integration constants. Additionally, the solutions must have a certain $\mathbb{Z}_2$-parity (see eq. (2.5)) so that either A or B must be zero. This results in a Dirichlet and Neumann condition for the 5D wave functions on the orbifold fixed points, respectively [19]:

$$\left.\begin{array}{rl} \partial_5\phi = 0 & \text{fields with even } \mathbb{Z}_2\text{-parity} \\ \phi = 0 & \text{fields with odd } \mathbb{Z}_2\text{-parity} \end{array}\right\} \quad \text{at } y = \frac{-\pi R}{2}, \ \frac{\pi R}{2} \ , \tag{2.16}$$

which gives us an expression for M_n in eq. (2.14)

$$M_n = \frac{n}{R} \ , \tag{2.17}$$

so that the mass m_n of the scalar field $\phi(x,y)$ of the nth KK mode can be written as:

$$m_n = \sqrt{m^2 + \left(\frac{n}{R}\right)^2} \ . \tag{2.18}$$

Finally the solutions for the scalar 5D wave functions are:

$$f^{(n)}(y) = \begin{cases} \frac{1}{\sqrt{\pi R}} & \text{for } n = 0 \\ \sqrt{\frac{2}{\pi R}}\cos(\frac{ny}{R}) & \text{for } n \ \in \{2,4,6,\cdots\} \\ \sqrt{\frac{2}{\pi R}}\sin(\frac{ny}{R}) & \text{for } n \ \in \{1,3,5,7,\cdots\} \ , \end{cases} \tag{2.19}$$

where the normalization is determined by

$$\int_{-\frac{\pi R}{2}}^{\frac{\pi R}{2}} f^{(n)}(y)\, f^{(m)}(y) = \delta_{m,n} \ . \tag{2.20}$$

2.2. Gauge Field Kaluza-Klein Decomposition

In this section we want to show how one can use the orbifolding mechanism to eliminate certain zero modes and how the KK decomposition of the gauge fields looks like. The action in eq. (1.3) is invariant under a gauge transformation

$$A_\mu(x,y) \to A_\mu(x,y) + \partial_\mu \Theta(x,y) \ , \tag{2.21a}$$

$$A_5(x,y) \to A_5(x,y) + \partial_5 \Theta(x,y) \ , \tag{2.21b}$$

where the 4D vector $A_\mu(x,y)$ is the 4D part, the 4D scalar $A_5(x,y)$ is the 5D part of the gauge field and $\Theta(x,y)$ is an arbitrary scalar gauge transformation function. The $\mathbb{Z}_2$-parity of $\Theta(x,y)$ determines the $\mathbb{Z}_2$-parity of the gauge fields $A_\mu(x,y)$ and $A_5(x,y)$

$$A_\mu(x,\tilde{y}) + \partial_\mu \Theta(x,\tilde{y}) \to \ (\pm A_\mu(x,\tilde{y})) + \partial_\mu(\pm\Theta(x,\tilde{y})) \ , \tag{2.22a}$$

$$A_5(x,\tilde{y}) + \partial_5 \Theta(x,\tilde{y}) \to \ (\mp A_5(x,\tilde{y})) + (-\partial_5)(\pm\Theta(x,\tilde{y})) \ , \tag{2.22b}$$

with $\tilde{y} = y - \frac{\pi R}{2}$. We see that A_μ and A_5 always have opposite $\mathbb{Z}_2$-parity. If $\Theta(x,y)$ has positive $\mathbb{Z}_2$-parity, $A_\mu(x,y)$ and $A_5(x,y)$ have positive and negative $\mathbb{Z}_2$-parity, respectively, and thus, $A_5(x,y)$ cannot have a zero mode. We can see this calculating the gauge field zero mode wave function $f(y)$=const of a gauge field with odd $\mathbb{Z}_2$-parity. The gauge fields can now be decomposed in the following way:

$$A^\mu(x) = \frac{1}{\sqrt{\pi R}} A^\mu_{(0)}(x^\mu) + \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} A^\mu_{(n)}(x^\mu) f^{(n)}(y) \ , \tag{2.23a}$$

$$A^5(x) = \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} A^5_{(n)}(x^\mu) f_5^{(n)}(y) \ , \tag{2.23b}$$

where the 5D wave functions can be calculated as shown in the previous section or in [7, 14]. We get:

$$f^{(n)}(y) = \begin{cases} \cos(\frac{ny}{R}) & \text{for } n \ \in \{0,2,4,6,\cdots\} \\ \sin(\frac{ny}{R}) & \text{for } n \ \in \{1,3,5,7,\cdots\} \ , \end{cases} \tag{2.24a}$$

$$f_5^{(n)}(y) = \begin{cases} \sin(\frac{ny}{R}) & \text{for } n \ \in \{0,2,4,6,\cdots\} \\ \cos(\frac{ny}{R}) & \text{for } n \ \in \{1,3,5,7,\cdots\} \ . \end{cases} \tag{2.24b}$$

We also see here that the KK parity alternates for the individual gauge fields $A^\mu(x, y)$ and $A^5(x, y)$ for different KK mode numbers n and how this determines the wave functions when we fix the $\mathbb{Z}_2$-parity of a field. We will encounter this behavior again in the next chapter when we derive the fermion wave functions (see Fig. 3.2). For electroweak symmetry breaking and gauge fixing in 5D theories see [14].

3. Fermion Kaluza-Klein Decomposition

3.1. nUED Lagrangian in a Brane Kinetic Localized Term Setup

In the last chapter the $^{S^1}/_{\mathbb{Z}_2}$-orbifold was introduced which ensures that we can remove zero modes of unwanted fields and formulate a chiral theory. Furthermore we reviewed the KK decomposition of the scalar and the gauge fields. In this chapter the fermion wave function and mass spectrum will be derived from the fermion Lagrangian with brane kinetic localized terms (BKLT) for one fermion family.
The fermion Lagrangian in a BKLT setup has the form:

$$\begin{aligned}\mathcal{S} &= \int\limits_{\mathbb{M}} \int\limits_{S^1/\mathbb{Z}_2} \mathrm{d}^5x \; \frac{i}{2} \left(\overline{\Psi} \; \Gamma^M \partial_M \Psi - \partial_M \overline{\Psi} \; \Gamma^M \Psi \right) + \mathcal{L}_{BKLT} \\ &= \mathcal{S}_{Bulk} + \mathcal{S}_{BKLT} \; , \end{aligned} \tag{3.1}$$

where $\mathcal{S}_{Bulk}$ is the fermionic UED-action, $\mathcal{S}_{BKLT}$ denotes the BKLT action, Ψ and $\overline{\Psi}$ are 5D Dirac fermions and ∂_M is the 5D partial derivative defined in eq. (2.4). Γ^M is the extended gamma matrix in order to fulfill the Clifford algebra in higher dimensional theories [20]. We extend the four dimensional basis of gamma matrices by using the four $\Gamma^\mu = \gamma^\mu$ matrices and $\Gamma^5 = i\gamma^5$ to close the 5D Clifford algebra [19, 21]:

$$\{\Gamma^M, \Gamma^N\} = 2g^{MN} \; , \tag{3.2}$$

with the metric defined in eq. (2.2), the anti-commutator {a,b}=ab+ba and where

$$\Gamma^M = \begin{pmatrix} \gamma^\mu \\ i\gamma^5 \end{pmatrix} \; . \tag{3.3}$$

The Lagrangian for the BKLT is:

$$\mathcal{L}_{BKLT} = i\overline{\Psi}_h \partial\!\!\!/ \Psi_h \left(a_{h,1} \delta \left(y - \frac{\pi R}{2} \right) + a_{h,2} \delta \left(y + \frac{\pi R}{2} \right) \right) \; , \tag{3.4}$$

where $a_{h,1}$ and $a_{h,2}$ are two additional real parameters with mass dimension -1 and $\delta(y)$ is the Dirac-Delta-distribution

$$\int\limits_{-L}^{L} \mathrm{d}y\; f(y)\;\delta(y-c) = \begin{cases} f(c) & \text{for} \quad c \in [-L, L] \\ 0 & \text{else} \end{cases} ,$$

with $c \in \mathbb{R}$. The chiral fermions Ψ_h with h=R,L are defined as

$$\Psi_R = P_R \cdot \Psi = \frac{1+\gamma_5}{2}\Psi; \quad \Psi_L = P_L \cdot \Psi = \frac{1-\gamma_5}{2}\Psi \ , \tag{3.5}$$

so that $\Psi = \Psi_R + \Psi_L$, $\gamma_5\ \Psi_{R,L} = \pm\Psi_{R,L}$ and $P_R P_L \Psi$=0. For the calculation in this section we choose left chiral BKLT (h = L). The reasons for choosing a BKLT of certain chirality and the results for a right chiral BKLT are given in section 3.3.
The action in eq. (3.1) is invariant under the following symmetry which will be given for completeness:

$$\Psi(x,y) \to \Psi(x,-y) = \mathcal{Z}\Psi(x,y) \ , \tag{3.6}$$

if $\mathcal{Z}$ fulfills the following conditions:

$$\gamma^0 \mathcal{Z}^\dagger \gamma^0 \Gamma^M \mathcal{Z} = \Gamma^M \ , \tag{3.7a}$$

$$\mathcal{Z}\left(\mathcal{Z}\Psi(x,y)\right) = \Psi(x,y) \ . \tag{3.7b}$$

This can be accomplished by choosing $\mathcal{Z} = \pm\gamma^5$.

3.2. The Equations of Motion

After integrating $\mathcal{S}$ from eq.(3.1) by parts and rewrite the fermions into their chiral constituents, the action takes the following form:

$$\begin{aligned} \mathcal{S} = \int\limits_{\mathbb{M}} \int\limits_{S^1/\mathbb{Z}_2} & \mathrm{d}^5x\ i\ (\overline{\Psi}_R + \overline{\Psi}_L)\ \left(\partial_\mu, \partial_5\right) \cdot \begin{pmatrix} \gamma^\mu \\ i\gamma^5 \end{pmatrix} (\Psi_R + \Psi_L) \\ & + i\ \overline{\Psi}_L \partial\!\!\!/ \Psi_L \left(a_{1,L}\delta\left(y - \frac{\pi R}{2}\right) + a_{2,L}\delta\left(y + \frac{\pi R}{2}\right)\right) \\ + \int\limits_{\mathbb{M}} \mathrm{d}^4x\ & \underbrace{\frac{1}{2}(\overline{\Psi}_R + \overline{\Psi}_L)\gamma^5(\Psi_R + \Psi_L)\big|_{-\frac{\pi R}{2}}^{\frac{\pi R}{2}}}_{\text{boundary term}} , \end{aligned} \tag{3.8}$$

with $f(y)|_b^a \equiv f(a) - f(b)$. Using the Clifford algebra in eq. (3.2), eq. (3.8) simplifies to

$$\begin{aligned}\int\limits_{\mathbb{M}}\int\limits_{S^1/\mathbb{Z}_2} \mathrm{d}^5x \;\; & i\,\overline{\Psi}_R \partial\!\!\!/ \Psi_R + i\,\overline{\Psi}_L \partial\!\!\!/ \Psi_L \\ & + \overline{\Psi}_R \partial_5 \gamma_5 \Psi_L + \overline{\Psi}_L \partial_5 \gamma_5 \Psi_R \\ & + i\,\overline{\Psi}_L \partial\!\!\!/ \Psi_L \left(a_{1,L}\delta\left(y - \frac{\pi R}{2}\right) + a_{2,L}\delta\left(y + \frac{\pi R}{2}\right)\right) \\ - \int\limits_{\mathbb{M}} \mathrm{d}^4x \;\; & \underbrace{\frac{1}{2}(\overline{\Psi}_R\,\gamma_5\Psi_L - \overline{\Psi}_L\,\gamma_5\Psi_R|_{-\frac{\pi R}{2}}^{\frac{\pi R}{2}}}_{\text{boundary term}} \,,\end{aligned} \tag{3.9}$$

where we used $\gamma^5 = -\gamma_5$ to get the common notation of the 5D terms. It is assumed that 4D fields vanish at infinity, but the integration by parts generates a boundary term at $\frac{\pi R}{2}$ and $-\frac{\pi R}{2}$ which does not vanish since the fifth dimension is finite.

The variation of eq. (3.9) yields:

$$\begin{aligned}\delta\overline{\Psi}_L : \quad 0 =& i\partial\!\!\!/\Psi_L + \partial_5\Psi_R + i\partial\!\!\!/\Psi_L(a_{1,L}\delta(y - \frac{\pi R}{2}) + a_{2,h}\delta(y + \frac{\pi R}{2})) \\ & - \frac{1}{2}\Psi_R|_{-\frac{\pi R}{2}}^{\frac{\pi R}{2}} \,, \end{aligned} \tag{3.10a}$$

$$\delta\overline{\Psi}_R : \quad 0 = i\partial\!\!\!/\Psi_R - \partial_5\Psi_L + \frac{1}{2}\Psi_L|_{-\frac{\pi R}{2}}^{\frac{\pi R}{2}} \,. \tag{3.10b}$$

The variation for Ψ_R and Ψ_L is redundant since $\overline{\Psi}_h = \Psi_h^\dagger\gamma^0$. We perform the KK decomposition with the following ansatz:

$$\Psi_R(x,y) = \sum_{n=0}^{\infty} \Psi_R^{(n)}(x)\; f_R^{(n)}(y) \,; \qquad \Psi_L(x,y) = \sum_{n=0}^{\infty} \Psi_L^{(n)}(x)\; f_L^{(n)}(y) \,, \tag{3.11}$$

where n is the KK mode number, $\Psi_{R/L}^{(n)}(x)$ are the nth KK excitations of the 4D fermions and $f_{R/L}^{(n)}(y)$ are the to the right/left chiral fermion corresponding 5D wave functions. Inserting the KK decomposition into eq. (3.10), we get:

$$\begin{aligned} 0 = \sum_{n=0} & i\partial\!\!\!/\Psi_L^{(n)}(x) f_L^{(n)}(y) + \partial_5\Psi_R^{(n)}(x) f_R^{(n)}(y) \\ & + i\partial\!\!\!/\Psi_L^{(n)}(x) f_L^{(n)}(y)(a_{1,L}\delta(y - \frac{\pi R}{2}) + a_{2,L}\delta(y + \frac{\pi R}{2})) \\ & - \frac{1}{2}\Psi_R^{(n)}(x) f_R^{(n)}(y)|_{-\frac{\pi R}{2}}^{\frac{\pi R}{2}} \,, \end{aligned} \tag{3.12a}$$

$$0 = \sum_{n=0} i\partial\!\!\!/\Psi_R^{(n)}(x) f_R^{(n)}(y) - \partial_5\Psi_L^{(n)}(x) f_L^{(n)}(y) + \frac{1}{2}\Psi_L^{(n)}(x) f_L^{(n)}(y)|_{-\frac{\pi R}{2}}^{\frac{\pi R}{2}} \,. \tag{3.12b}$$

We can rewrite the equations in the bulk (i.e. at y $\neq \pm\frac{\pi R}{2}$) in the following form introducing a separation constant m_n:

$$\frac{i\partial\!\!\!/\Psi_R^{(n)}(x)}{\Psi_L^{(n)}(x)} = m_n = \frac{\partial_5 f_L^{(n)}(y)}{f_R^{(n)}(y)}; \qquad \frac{i\partial\!\!\!/\Psi_L^{(n)}(x)}{\Psi_R^{(n)}(x)} = m_n = -\frac{\partial_5 f_R^{(n)}(y)}{f_L^{(n)}(y)} \; . \tag{3.13}$$

The separation constant m_n will be identified as the KK mass of the nth KK mode. The 4D fields $\Psi_{R,L}^{(n)}(x)$ satisfy the 4D Dirac equation if we set $m_{n=0} = 0$:

$$0 = i\,\partial\!\!\!/\Psi_R^{(n)}(x) - m_n\Psi_L^{(n)}(x)\;, \tag{3.14a}$$
$$0 = i\,\partial\!\!\!/\Psi_L^{(n)}(x) - m_n\Psi_R^{(n)}(x)\;, \tag{3.14b}$$

thus m_n must be determined from the set of differential equations in y. The set of equations in eq. (3.12) become in the bulk

$$0 = \partial_5 f_L^{(n)}(y) - m_n f_R^{(n)}(y)\;, \tag{3.15a}$$
$$0 = \partial_5 f_R^{(n)}(y) + m_n f_L^{(n)}(y)\;, \tag{3.15b}$$

or formulated as second order differential equations:

$$(\partial_5^2 + m_n^2) f_L^{(n)}(y) = 0\;, \tag{3.16a}$$
$$(\partial_5^2 + m_n^2) f_R^{(n)}(y) = 0\;. \tag{3.16b}$$

Before solving the partial differential equations in eq. (3.15) completely, we concern ourselves with the zero modes. One condition to restore the SM is that the fermion zero mode is chiral. This can be accomplished by imposing a Dirichlet boundary condition (see eq. (2.1)) on the fermion wave function which we want to have a vanishing zero mode. In this case, a Dirichlet boundary condition is imposed on the right handed fermion wave function:

$$f_R^{(n)}(\pm\frac{\pi R}{2}) = 0\;. \tag{3.17}$$

All boundary terms we get because of the integration by parts (see eq. (3.12)) will therefore vanish, too [21]. For a massless fermion zero mode, we set $m_{n=0} = 0$ in eq. (3.15)

$$\partial_y f_L^{(0)}(y) = 0\;, \tag{3.18a}$$
$$\partial_y f_R^{(0)}(y) = 0\;. \tag{3.18b}$$

The solution is a constant wave function for the left chiral part and a vanishing wave function for the right one since eq. (3.17) and eq. (3.18b) hold:

$$f_L^0(y) = \text{constant}\;, \tag{3.19a}$$
$$f_R^0(y) = 0 \tag{3.19b}$$

The value of $f_L^0(y)$ will be only determined by the modified scalar product introduced in eq. (3.28).
Now we use the following ansatz to solve the differential equation in eq. (3.15):

$$f_L^{(n)}(y) = A_L \sin(k_n\ y) + B_L \cos(k_n\ y)\ , \tag{3.20a}$$
$$f_R^{(n)}(y) = A_R \sin(k_n\ y) + B_R \cos(k_n\ y)\ , \tag{3.20b}$$

where A_L, A_R, B_L, B_R and k_n are complex coefficients. Inserting this ansatz in eq. (3.15) and eq. (3.16), we get $k_n = m_n$, $B_R = A_L$ and $B_L = -A_R$.
The variation is a crucial point when working with BKLT. We work here with a scheme introduced in [21]. The BKLT are shifted away from the boundary by ϵ, which will be taken to zero at the end of the calculation. Now eq. (3.12a) and eq. (3.12b) have the form:

$$\begin{aligned} 0 = &\ \partial_y f_R^{(n)}(y) + m_n f_L^{(n)}(y) \\ &+ m_n f_L^{(n)}(a_{1,L}\delta(y - (\frac{\pi R}{2} - \epsilon)) + a_{2,L}\delta(y + (\frac{\pi R}{2} - \epsilon)))\ , \end{aligned} \tag{3.21a}$$
$$0 = \partial_y f_L^{(n)}(y) - m_n f_R^{(n)}(y)\ . \tag{3.21b}$$

We integrate eq. (3.21a) over a small ϵ dependent region:

$$0 = \int\limits_{-\frac{\pi R}{2}}^{-\frac{\pi R}{2}+2\epsilon} \mathrm{d}y\ m_n f_L^{(n)}(y) + \partial_5 f_R^{(n)} + m_n f_L^{(n)} a_{2,L}\delta(y + (\frac{\pi R}{2} - \epsilon))\ , \tag{3.22a}$$
$$0 = \int\limits_{\frac{\pi R}{2}-2\epsilon}^{\frac{\pi R}{2}} \mathrm{d}y\ m_n f_L^{(n)}(y) + \partial_5 f_R^{(n)} + m_n f_L^{(n)} a_{1,L}\delta(y - (\frac{\pi R}{2} - \epsilon))\ , \tag{3.22b}$$

where we dropped the Dirac-Delta distribution parts which will be zero due to the integration regions we have chosen. To simplify the notation, we set

$$\pm \frac{\pi R}{2} \mp b\epsilon \equiv \pm(\frac{\pi R}{2})_{b\epsilon}\ , \tag{3.23}$$

where $b \in \mathbb{N}$. Evaluating the integrals and inserting the ansatz from eq. (3.20a) and eq. (3.20b), we get:

$$\begin{aligned} 0 =& A_R \sin[-(\frac{\pi R}{2})_{2\epsilon} m_n] + A_L \cos[-(\frac{\pi R}{2})_{2\epsilon} m_n] + \\ & m_n a_{L,2} \left(A_L \sin[-(\frac{\pi R}{2})_{1\epsilon} m_n] - A_R \cos[-(\frac{\pi R}{2})_{1\epsilon} m_n] \right) , \end{aligned} \tag{3.24a}$$

$$\begin{aligned} 0 =& - A_R \sin[(\frac{\pi R}{2})_{2\epsilon} m_n] - A_L \cos[(\frac{\pi R}{2})_{2\epsilon} m_n] + \\ & m_n a_{L,1} \left(A_L \sin[(\frac{\pi R}{2})_{1\epsilon} m_n] - A_R \cos[(\frac{\pi R}{2})_{1\epsilon} m_n] \right) . \end{aligned} \tag{3.24b}$$

We again dropped the terms which will vanish in the limit of $\epsilon \to 0$ and used eq. (2.1). By applying KK parity on eq. (3.15) we see:

$$\partial_5 \to -\partial_5 \; ; \qquad f_L^{(n)} \to \pm f_L^{(n)} \; ; \qquad f_R^{(n)} \to \mp f_R^{(n)} \; . \tag{3.25}$$

This shows that the wave functions have always opposite KK parity. To fulfill eq. (3.25), one of the two remaining parameters A_L and A_R in eq. (3.20) has to be zero. We now take a closer look on these two cases:

3.2.1. Case 1: $A_L = 0$

Setting A_L to zero, $f_L^{(n)}$ has positive and $f_R^{(n)}$ has negative KK parity. Eq. (3.20) and eq. (3.24) simplify to:

$$f_L^{(n)}(y) = -A_R \cos(m_n \, y) \; , \tag{3.26a}$$

$$f_R^{(n)}(y) = A_R \sin(m_n \, y) \; , \tag{3.26b}$$

$$\sin(-(\frac{\pi R}{2})_{2\epsilon} m_n) = m_n a_{2,L} \cos(-(\frac{\pi R}{2})_{1\epsilon} m_n) \; , \tag{3.26c}$$

$$-\sin((\frac{\pi R}{2})_{2\epsilon} m_n) = m_n a_{1,L} \cos((\frac{\pi R}{2})_{1\epsilon} m_n) \; . \tag{3.26d}$$

We see that the wave functions are now only determined by the same normalization constant (see eq.(3.28)) and that the BKLT parameter $a_{1,L} = a_{2,L} = a_L$ have to be the same after $\epsilon \to 0$.

3.2.2. Case 2: A_R= 0

Setting A_R to zero, $f_L^{(n)}$ has negative and $f_R^{(n)}$ has positive parity. Eq. (3.20) and eq. (3.24) simplify to:

$$f_L^{(n)}(y) = A_L \sin(m_n\ y)\ , \tag{3.27a}$$

$$f_R^{(n)}(y) = A_L \cos(m_n\ y)\ , \tag{3.27b}$$

$$\cos(-(\frac{\pi R}{2})_{2\epsilon} m_n) = m_n a_{2,L} \sin(-(\frac{\pi R}{2})_{1\epsilon} m_n)\ , \tag{3.27c}$$

$$\cos((\frac{\pi R}{2})_{2\epsilon} m_n) = m_n a_{1,L} \sin((\frac{\pi R}{2})_{1\epsilon} m_n)\ . \tag{3.27d}$$

Using the results from eq. (3.26) and eq. (3.27) we can calculate the mass spectrum of the fermions. The masses of the first three KK excitations are plotted in Fig. 3.1.

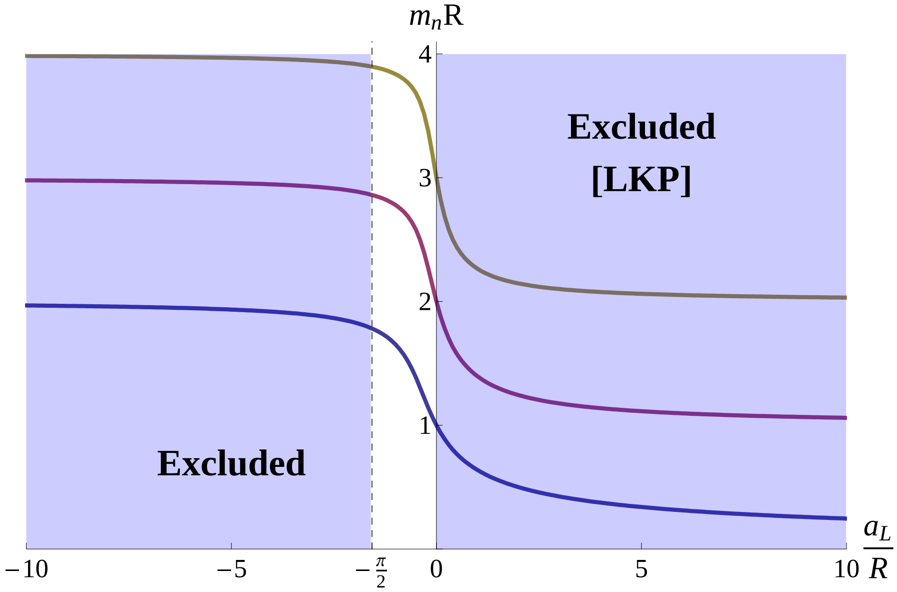


Figure 3.1.: The fermion mass spectrum $m_n\ R$ for the first three KK modes where the transcendental equations eq. (3.26c) and eq. (3.27c) were solved numerically for different $\frac{a_L}{R}$. The zeroth mode is massless. The blue(gray) colored regions $\frac{a_L}{R} < -\frac{\pi}{2}$ and $0 < \frac{a_L}{R}$ are excluded (see text).

We now have a set of wave functions which characterize the KK decomposition of the right handed and left handed fermions. The final step is to clarify how to enumerate the KK-modes. In Fig. 3.2 the two quantization conditions from eq. (3.26c) and eq. (3.27c) are plotted. We see that the solutions of the different quantization conditions alternate. Giving the solution corresponding to $m_{n=0} = 0$ the number $n = 0$ and count forth, we can assign the even numbers to eq. (3.26c) and the odd numbers to eq. (3.27c). Hence we define Case 1 in section 3.2.1 as the even-numbered solution having even mode numbers and Case 2 as the odd-numbered solution having odd mode numbers.

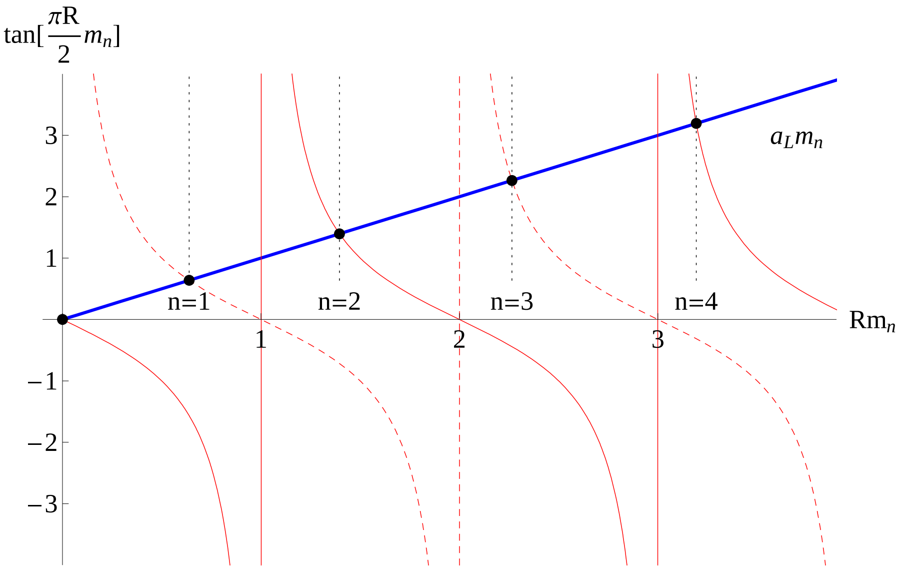


Figure 3.2.: Solution to the transcendental equations eq. (3.26c)(continuous line) and eq. (3.27c)(dashed line) for $\frac{a_L}{R} = \frac{\pi}{2}$. The solutions to both equations, represented through the intersection labeled with different numbers n, take turns and define the even and odd numbered KK modes.

Now we check the orthonormality of the wave functions under the following scalar products.

$$\delta_{m,n} = \int_{-\frac{\pi R}{2}}^{\frac{\pi R}{2}} \mathrm{d}y \; f_L^{(n)}(y) f_L^{(m)}(y) \left(1 + a_L[\delta(y - \frac{\pi R}{2}) + \delta(y + \frac{\pi R}{2})]\right) \; , \tag{3.28a}$$

$$\delta_{m,n} = \int_{-\frac{\pi R}{2}}^{\frac{\pi R}{2}} \mathrm{d}y \; f_R^{(n)}(y) f_R^{(m)}(y) \; . \tag{3.28b}$$

The wave functions furthermore satisfy:

$$m_n \; \delta_{n,m} = \int_{-\frac{\pi R}{2}}^{\frac{\pi R}{2}} \mathrm{d}y \; f_R^{(n)}(y) \partial_y f_L^{(m)}(y) + \frac{1}{2} f_R^{(n)}(y) f_L^{(m)}(y)|_{-\frac{\pi R}{2}}^{\frac{\pi R}{2}} \; . \tag{3.28c}$$

These scalar products are the conditions to restore the SM and they occur when the KK decomposition in the Lagrangian eq. (3.8) is done. From the Sturm-Liouville theory we know that such a scalar product must exist and that the solutions from the differential equation in eq. (3.15) form an orthonormal basis under these scalar products. The weight function in this Sturm-Liouville problem is the boundary kinetic localized term which appears in eq. (3.28a) and has a significant impact on the wave functions and later on the the couplings in the 4D effective theory (cf. chapter 4). For detailed explanation see Appendix A. Tab. 3.1 shows the wave functions for the left and right fermions and the quantization conditions which determine the masses of the fermions for a BKLT setup with a left zero mode.

KK zero modes	even numbered KK-modes	odd numbered KK-modes
$f_L^{(0)}(y) = \frac{1}{\sqrt{2a_L+\pi R}}$	$f_L^{(n)}(y) = -N\cos(m_n y)$	$f_L^{(n)}(y) = N\sin(m_n y)$
$f_R^{(0)}(y) = 0$	$f_R^{(n)}(y) = N\sin(m_n y)$	$f_R^{(n)}(y) = N\cos(m_n y)$
$m_0 = 0$	$\tan(\frac{\pi R}{2}m_n) = -a_L m_n$	$\cot(\frac{\pi R}{2}m_n) = a_L m_n$

Table 3.1.: The solutions for the 5D fermion wave functions and their masses with a left handed zero mode. N is the normalization constant from eq. (3.29) and a_R is the BKLT parameter.

N is the normalization constant which depends on the mass of the nth mode:

$$N = \left(\frac{\pi R}{2} + \frac{\sin(\frac{\pi R}{2}m_n)\cos(\frac{\pi R}{2}m_n)}{m_n}\right)^{-\frac{1}{2}} . \tag{3.29}$$

Now we will constrain the BKLT parameter a_L. On the one hand we want a real zero mode and therefore set $a_L \in (-\frac{\pi R}{2}, \infty)$ or the zero mode wave function in Tab. 3.1 gets complex which can lead to anomalies in the theory. On the other hand, we exclude the region $0 \leq a_L < \infty$ for phenomenological reasons. As seen in Fig. 3.1, the fermion masses get lighter than $\frac{n}{R}$ which are the masses of the gauge bosons in this model. If we look at the full UED mass spectrum and choose a BKLT parameter a_L bigger than zero, we would get a fermion as the lightest KK particle (LKP). This would mean our dark matter candidate could be charged[1] (or even colored) what is in contrast to its characteristics demanded by astrophysical observation. So the allowed region for the BKLT parameter is

$$-\frac{\pi R}{2} < a_L < 0 . \tag{3.30}$$

The wave functions for four different BKLT parameters a_L are plotted for the first three KK mode in Fig. 3.3 and Fig. 3.4.

[1]Calculation in [22] showed that the first KK excitation of the KK neutrino $\nu^{(1)}$ cannot be the LKP.

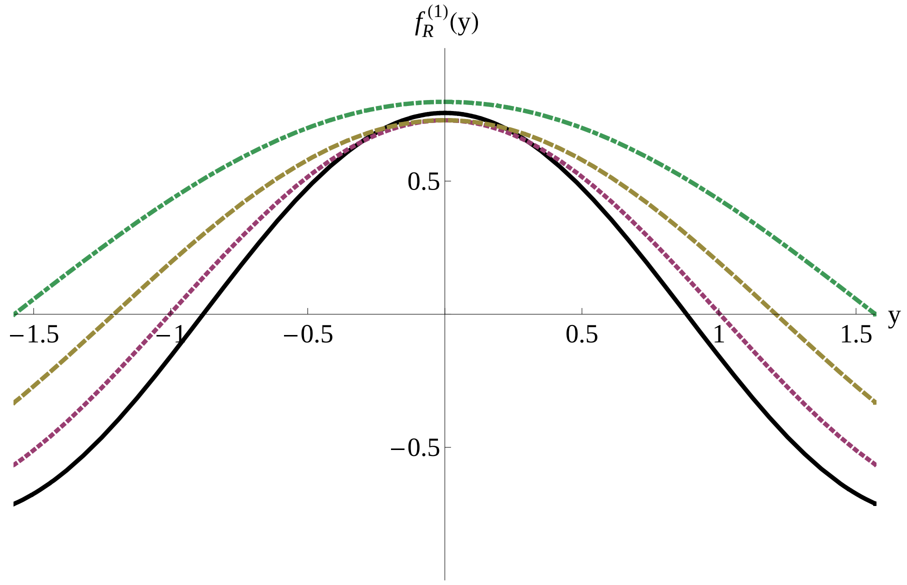


Figure 3.3.: The right handed wave functions of the first KK mode plotted for several BKLT parameters $\frac{a_L}{R} = \{-\frac{\pi}{2}, -\frac{\pi}{4}, -\frac{\pi}{8}, 0\}$ from bottom (continuous line) to top (dot-dashed line) in a setup with a left chiral zero mode and a compactification radius $R = 1\text{TeV}^{-1}$.

3.3. Form of the Brane Kinetic Localized Terms

In the beginning we have chosen brane kinetic terms of the form:

$$\mathcal{L}_{BKLT} = ia_L \overline{\Psi}_L \partial\!\!\!/ \Psi_L \left(\delta(y - \frac{\pi R}{2}) + \delta(y + \frac{\pi R}{2}) \right) , \tag{3.31}$$

i.e. only for the left handed fermions and imposing a Dirichlet condition on $f_R^{(n)}(y)$. If we would impose a Dirichlet condition on the left fermions in the same setup, we would arrive again at a normal UED theory, because the relevant modification with the BKLT parameter a_L would vanish. To get a spectrum with right chiral zero modes, the BKLT must be chosen to

$$\mathcal{L}_{BKLT,R} = ia_R \overline{\Psi}_R \partial\!\!\!/ \Psi_R \left(\delta(y - \frac{\pi R}{2}) + \delta(y + \frac{\pi R}{2}) \right) , \tag{3.32}$$

and the left chiral fermion must obey a Dirichlet boundary condition:

$$f_L^{(n)} \left(\pm \frac{\pi R}{2} \right) = 0 \, . \tag{3.33}$$

Then the solution for a right chiral zero mode can be obtained by exchanging $f_R^{(n)}(y) \leftrightarrow f_L^{(n)}(y)$ and $a_L \leftrightarrow a_R$ in the solution for a left chiral zero mode. The solutions for a setup with a right chiral zero mode are shown in Tab. 3.2.

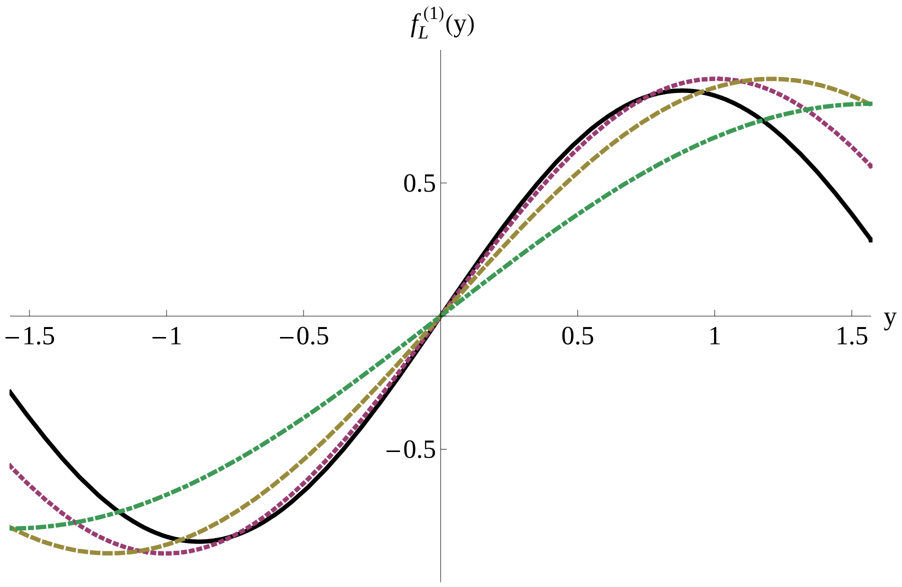


Figure 3.4.: The left handed wave functions of the first KK mode plotted for several BKLT parameters $\frac{a_L}{R} = \{-\frac{\pi}{2}, -\frac{\pi}{4}, -\frac{\pi}{8}, 0\}$ from bottom (dot-dashed line) to top (continuous line) in a setup with a left chiral zero mode and a compactification radius $R = 1\text{TeV}^{-1}$.

KK zero mode	even numbered KK modes	odd numbered KK modes
$f_R^{(0)}(y) = \frac{1}{\sqrt{2a_R+\pi R}}$	$f_R^{(n)}(y) = -N\cos(m_n y)$	$f_R^{(n)}(y) = N\sin(m_n y)$
$f_L^{(0)}(y) = 0$	$f_L^{(n)}(y) = N\sin(m_n y)$	$f_L^{(n)}(y) = N\cos(m_n y)$
$m_0 = 0$	$\tan(\frac{\pi R}{2} m_n) = -a_R m_n$	$\cot(\frac{\pi R}{2} m_n) = a_R m_n$

Table 3.2.: The solutions for the 5D fermion wave functions and their masses with a right handed zero mode. N is the normalization constant from eq. (3.29) and a_R is the BKLT parameter. These are obtained using Tab. 3.1 and the index substitution $L \leftrightarrow R$.

4. Effective Description of Fermion Interactions

In the last section we calculated the fermion wave functions and the mass spectrum from a Lagrangian with brane kinetic localized terms (see eq. (3.1)). Now we use these wave functions to determine the overlap integrals which appear when we go from a 5D theory to an effective 4D theory. After a transformation into a basis where the Yukawa matrices are diagonal, we can derive the Wilson coefficients for ΔF=2 processes, that means, processes where the flavor of a particle is changed by two. For example a hadron with one strange quark having strangeness $s = 1$ going to a state with an anti-strange $\bar{s}$ with strangeness $s = -1$.[1] This can be used to constrain the BKLT parameters due to bounds from flavor changing neutral currents (FCNCs).
The action for the fermion and the Yukawa sector is

$$\mathcal{S} = \int \mathrm{d}^5x \, (\mathcal{L}_F + \mathcal{L}_Y) \,, \tag{4.1}$$

with

$$\mathcal{L}_F = \sum_{i,j,h} \delta_{ij} \frac{i}{2} \left(\overline{\Psi}_i D_M \Gamma^M \Psi_j - D_M \overline{\Psi}_i \Gamma^M \Psi_j + a_h^{\Psi_i,\Psi_j} \overline{\Psi}_{i,h} \not{D} \Psi_{j,h} b(y) \right) \,, \tag{4.2a}$$

$$\mathcal{L}_Y = \sum_{i,j} \left(\lambda_{ij}^U \overline{Q}_i \tilde{H} U_j + \lambda_{ij}^D \overline{Q}_i H D_j + \lambda_{ij}^E \overline{L}_i H E_j \right) + \text{h.c.} \,. \tag{4.2b}$$

Q_i is a SU(2) quark doublet $(u_i(x,y), d_i(x,y))^T$ with a left handed zero mode, U_i and D_i are up-type and down-type SU(2) quark singlets with a right handed zero mode (cf. Tab. 3.1 and Tab. 3.2). L_i is a SU(2) lepton doublet $(\nu_i(x,y), e_i(x,y))^T$ with a left handed zero mode and E_i is a SU(2) lepton singlet with a right handed zero mode. $\overline{\Psi} = \Psi^\dagger \gamma^0$ are the corresponding antifermions. H is the Higgs field, $\tilde{H} = i\, \tau^2 H^*$ its charge-conjugate and $\lambda_{ij}^{\Psi_i}$ are the Yukawa matrices for the quark and lepton fields. $a_h^{\Psi_i \Psi_j}$ is the hermitian BKLT-parameter matrix for the different fermions with chirality h=R,L, depending on which fermion should get modified by the BKLT and the function $b(y)$ is defined by $b(y) = \left(\delta\left(y - \frac{\pi R}{2}\right) + \delta\left(y + \frac{\pi R}{2}\right)\right)$. For the fermion Lagrangian $\mathcal{L}_F$ we choose a basis in which the BKLT matrix $a_h^{\Psi_i \Psi_j}$ is diagonal so that there is no mixing between the different fermions and between different KK modes from the brane kinetic localized

[1] An example for such a process in the SM would be neutral Kaon mixing $K_0 - \bar{K}_0$ [1].

terms.
D_M is the covariant derivative defined as:

$$D_M = \partial_M - \frac{\hat{g}'}{2} Y B_M - \sum_{a=1}^{3} \frac{\hat{g}}{2} \tau^a W^a_M - \sum_{a=1}^{8} \frac{\hat{g}_s}{2} T^a G^a_M \ , \tag{4.3a}$$

$$\begin{aligned} \hat{g}' &: \ 5D\ U(1)_Y \text{ coupling} ; && Y : \ hypercharge ; \\ \hat{g} &: \ 5D\ SU(2)_L \text{ coupling} ; && \tau^a : \ Pauli\ matrices ; \\ \hat{g}_s &: \ 5D\ SU(3)_C \text{ coupling} ; && T^a : \ Gell-Mann\ matrices . \end{aligned}$$

G^a_M are the gluon fields, B_M and W^a_M are the $W^\pm_M$, Z_M and A_M fields in the flavor eigenbasis and the $SU(3)_C \times SU(2)_L \times U(1)_Y$ charges of the fermions are [23, 24]:

$$\Psi_i(x,y) = (Q_i, U_i, D_i, L_i, E_i) = \Big((3,2)_{\frac{1}{3}}, (\bar{3},1)_{\frac{4}{3}}, (\bar{3},1)_{-\frac{2}{3}}, (1,2)_{-1}, (1,1)_{-2} \Big) \ , \tag{4.4}$$

where the first number in the parenthesis is the $SU(3)$ representation, the second number the $SU(2)$ representation and the subscript the $U(1)_Y$ charge of the fermion. The expression $\Gamma^M D_M$ in eq. (4.2a) can be rewritten into a kinetic and in a gauge part:

$$\Gamma^M D_M = \gamma^\mu D_\mu + i\gamma^5 D_5 \tag{4.5a}$$

$$= \partial\!\!\!/ - \gamma^\mu \sum_{d,a} \frac{\hat{g}_d}{2} t^a_d A^a_{d,\mu} + i\gamma^5 \partial_5 - i\gamma^5 \sum_{d,a} \frac{\hat{g}_d}{2} t^a_d A^a_{d,5}$$

$$= \underbrace{\partial\!\!\!/ + i\gamma^5 \partial_5}_{\text{kinetic part}} - \underbrace{\sum_{d,a} \left(\gamma^\mu \frac{\hat{g}_d}{2} t^a_d A^a_{d,\mu} + i\gamma^5 \frac{\hat{g}_d}{2} t^a_d A^a_{d,5} \right)}_{\text{gauge part}} \ , \tag{4.5b}$$

where d runs over the three gauge groups (eq. (4.3a)) and a over the corresponding number of generators. A_M represents the three different gauge fields B_M, W_M and G_M and $\partial\!\!\!/ = \gamma^\mu \partial_\mu$ as defined as in Feynman slash notation. The fermionic Lagrangian in eq. (4.2a) can be rewritten in a kinetic part only containing derivatives and a gauge part which gathers up all the components with gauge fields

$$\mathcal{L}_F = \mathcal{L}_{kin} + \mathcal{L}_g \ , \tag{4.6}$$

with

$$\begin{aligned}\mathcal{L}_{kin} =& i\ \delta_{ij} \sum_{i,j} [\overline{\Psi}_i (\not{\partial} + i\gamma^5 \partial_5)\Psi_j + a_h^{\Psi_i\Psi_j} \overline{\Psi}_{i,h} \not{\partial} \Psi_{j,h} b(y)] \\ &+ \frac{1}{2} [\overline{\Psi}_{i,L}\Psi_{j,R} - \overline{\Psi}_{i,R}\Psi_{j,L}]_{y=-\frac{\pi R}{2}}^{y=\frac{\pi R}{2}} , \end{aligned} \tag{4.7a}$$

$$\begin{aligned}\mathcal{L}_g =& -i\ \delta_{ij} \sum_{i,j,d,a} (\overline{\Psi}_i [\underbrace{\gamma^\mu \frac{\hat{g}_d}{2} t_d^a A_{d,\mu}^a}_{\text{4D}} - i \underbrace{\gamma^5 \frac{\hat{g}_d}{2} t_d^a A_{d,5}^a}_{\text{5D}}] \Psi_j \\ &\underbrace{+ a_h^{\Psi_i\Psi_j} \overline{\Psi}_{i,h} [\gamma^\mu \frac{\hat{g}_d}{2} t_d^a A_{d,\mu}^a] \Psi_{j,h} b(y))}_{\text{gauge BKLT}} , \end{aligned} \tag{4.7b}$$

where the kinetic part was integrated by parts. The 5D part of $\mathcal{L}_g$ includes the 5D components of the gauge fields $A_5^{a,(n)}$. Those are needed to give the higher KK modes of the eight gluons $G_\mu^{a,(n)}$ at each non zero KK level their mass and, together with the four 5D components of the Higgs fields, generate four physical fields and four goldstone bosons for the massive $SU(2)\times U(1)$ gauge field KK modes [14,19]. For further discussion the 4D components of the gauge fields of $\mathcal{L}_g$ remain:

$$\mathcal{L}_g = -i \sum_{i,d,a} i \overline{\Psi}_{i,h} \left(\mathbb{1} + a_h^{\Psi_i\Psi_i} b(y)\right) \gamma^\mu \frac{\hat{g}_d}{2} t_d^a A_{d,\mu}^a \Psi_{i,h} , \tag{4.8}$$

with the Kronecker symbol δ_{ij} evaluated in eq. (4.7b) and combining the gauge fields with the BKLT part. Here we see the modification in comparison to normal UED since the BKLT induce deviations from the unity matrix $\mathbb{1}$. In what follows we will neglect the summation sign for the gauge fields, generators and flavor in the equations for clearing up the notation. Inserting the KK decomposition for the different gauge fields as defined in eq. (2.23), eq. (4.7a) and eq. (4.7b), we get:

$$\begin{aligned}\mathcal{L}_{kin} = i \sum_{m,n=0}^{\infty} & [\overline{\Psi}_i^{(n)}(x) f_i^{(n)}(y) (\not{\partial} + i\gamma^5\partial_5) \Psi_i^{(m)}(x) f_i^{(m)}(y) \\ &+ a_h^{\Psi_i\Psi_i} \overline{\Psi}_{i,h}^{(n)}(x) f_{i,h}^{(n)}(y) \not{\partial} \Psi_{i,h}^{(m)} f_{i,h}^{(m)} b(y)] \\ &+ \frac{1}{2} [\overline{\Psi}_{i,L}^{(n)}(x) f_{i,L}^{(n)}(y) \Psi_{i,R}^{(m)}(x) f_{i,R}^{(m)}(y) - \overline{\Psi}_{i,R}^{(n)}(x) f_{i,R}^{(n)}(y) \Psi_{i,L}^{(m)}(x) f_{i,L}^{(m)}(y)]_{y=-\frac{\pi R}{2}}^{y=\frac{\pi R}{2}} , \end{aligned} \tag{4.9a}$$

$$\begin{aligned}\mathcal{L}_g =& -i \sum_{k,m,n=0}^{\infty} \overline{\Psi}_{i,h}^{(n)}(x) f_{i,h}^{(n)}(y) \left(\mathbb{1} + a_h^{\Psi_i\Psi_i} b(y)\right) \cdot \\ & \cdot \gamma^\mu \frac{\hat{g}_d}{2} t_d^a A_{d,\mu}^{a,(k)}(x) f_d^{(k)}(y) \Psi_{i,h}^{(m)}(x) f_{i,h}^{(m)}(y) . \end{aligned} \tag{4.9b}$$

$\mathcal{L}_{kin}$ determines the equations of motion and the shape of the fermion wave functions as shown in chapter 3. All terms containing a y-dependence in $\mathcal{L}_g$ are collected to define an overlap matrix which contains all the information from the extra dimension. After integrating over y, the gauge interactions of the fermion KK modes are described by:

$$\mathcal{L}_g = -i \sum_{k,m,n=0}^{\infty} \overline{\Psi}_{i,h}^{(n)} \gamma^\mu \frac{1}{2} \underbrace{\frac{\hat{g}_d}{\sqrt{\pi R}}}_{g_d} t_d^a A_{d,\mu}^{a,(k)} \mathcal{F}_{d,ii}^{\Psi,[nkm]} \Psi_{i,h}^{(m)} \ , \tag{4.10}$$

where we used eq. (1.6) and define an overlap matrix with

$$\begin{aligned} \mathcal{F}_{d,ii}^{\Psi,[nkm]} &= \int_{-\frac{\pi R}{2}}^{\frac{\pi R}{2}} \mathrm{d}y \ \tilde{\mathcal{F}}_{d,ii}^{\Psi,[nkm]} \\ &= \sqrt{\pi R} \int_{-\frac{\pi R}{2}}^{\frac{\pi R}{2}} \mathrm{d}y f_{i,h}^{(n)}(y) f_d^{(k)}(y) f_{i,h}^{(m)}(y) \left(\mathbb{1} + a_h^{\Psi_i \Psi_i} b(y)\right) \ . \end{aligned} \tag{4.11}$$

The indices $\{k, m, n\}$ are the mode numbers of the gauge field and the fermions, i is the flavor index and h the chirality of the fermion, d is the gauge field index introduced in eq. (4.3a) and the superscript Ψ in the overlap matrix $\mathcal{F}_{d,ii}^{\Psi,[nkm]}$ shows which BKLT parameter for the fermions Q,U,D,L,E it contains. The factor of $\sqrt{\pi R}$ has been introduced in order to express $\mathcal{L}_g$ in terms of 4D couplings (cf. chapter 1). We are interested in constraints on the BKLT by tree-level FCNC processes. These will appear after rotating the overlap matrix in eq. (4.11) into a basis where the quark mass matrices in the Yukawa sector are diagonal (see eq. (4.20)). The off-diagonal elements now induce flavor violation. On a closer look, only the coupling between the even KK modes of the gauge bosons $k = \{2, 4, 6, \cdots\}$ and the zero modes of the fermions $m = 0$ and $n = 0$ have a significant contribution whereas other contributions can be neglected. This is called the zero mode approximation [25, 26]. This approximation is valid since any contribution to flavor changing neutral currents comes from off-diagonal matrix elements. If we look at couplings between the KK zero mode $k = 0$ of the gauge bosons or the Higgs and any arbitrary KK mode of the fermions $\{m, n\} > 0$, it holds:

$$\mathcal{F}_{\Psi_i \Psi_j}^{[n0m]} = \delta_{nm} \delta_{\Psi_i \Psi_j} \ , \tag{4.12}$$

since all fermion wave functions form a orthonormal basis (see eq. (3.28) and appendix A) and the gauge boson and Higgs have a flat zero mode profile in this setup. Further we see in eq. (4.2) that we have chosen a basis where the BKLT matrix $a_h^{\Psi_i \Psi_i}$ is diagonal and so there is no mixing between different KK fermion modes which then can superpose to the lightest possible mode and therefore the zero mode. So the zero mode of the fermion is the actual lightest fermionic mode. Every other overlap matrix containing KK fermion or KK gauge modes with $k \neq 0$, $m \neq 0$ and $n \neq 0$ is not relevant for a tree-level process where only SM particles (KK zero mode particles) are in the initial and the final state of the process. Integrating over the $S^1/\mathbb{Z}_2$-orbifold, eq. (4.11) becomes:

$$\mathcal{F}^{\Psi,[k]}_{d,ii} \equiv \int\limits_{-\frac{\pi R}{2}}^{\frac{\pi R}{2}} \mathrm{d}y\ \tilde{\mathcal{F}}^{\Psi,[0k0]}_{d,ii} \tag{4.13a}$$

$$= \begin{cases} \int\limits_{-\frac{\pi R}{2}}^{\frac{\pi R}{2}} \mathrm{d}y\ \dfrac{1+a_h^{\Psi_i\Psi_i} b(y)}{(2a_h^{\Psi_i\Psi_i}+\pi R)} & \text{for } k\ = 0 \\ \int\limits_{-\frac{\pi R}{2}}^{\frac{\pi R}{2}} \mathrm{d}y\ \sqrt{2}\sin(\dfrac{ny}{R})\dfrac{1+a_h^{\Psi_i\Psi_i} b(y)}{(2a_h^{\Psi_i\Psi_i}+\pi R)} & \text{for } k\ \in\{1,3,5,\cdots\} \\ \int\limits_{-\frac{\pi R}{2}}^{\frac{\pi R}{2}} \mathrm{d}y\ \sqrt{2}\cos(\dfrac{ny}{R})\dfrac{1+a_h^{\Psi_i\Psi_i} b(y)}{(2a_h^{\Psi_i\Psi_i}+\pi R)} & \text{for } k\ \in\{2,4,6,\cdots\} \end{cases} \tag{4.13b}$$

$$= \begin{cases} 1 & \text{for } k\ = 0 \\ 0 & \text{for } k\ \in\{1,3,5,\cdots\} \\ \sqrt{8}(-1)^{\frac{k}{2}}\dfrac{a_h^{\Psi_i\Psi_i}}{(2a_h^{\Psi_i\Psi_i}+\pi R)} & \text{for } k\ \in\{2,4,6,\cdots\} \end{cases}, \tag{4.13c}$$

with $b(y)$ defined in eq. (4.2). The overlap matrices $\mathcal{F}^{[k]}_d$ are the same for every gauge field since only the fermion spectrum is altered by the BKLT so that

$$\mathcal{F}^{[k]}_B \equiv \mathcal{F}^{[k]}_{W^a} \equiv \mathcal{F}^{[k]}_{G^a} \equiv \mathcal{F}^{[k]}\ . \tag{4.14}$$

We are only interested in the quark sector and therefore neglect the leptonic parts.[2] The Lagrangian $\mathcal{L}_\mathrm{g}$ in eq. (4.9b) will be decomposed further and after integrating out all y-dependence in the zero mode approximation it becomes:

$$\mathcal{L}_g = -i\ q^{(0)}_{i,h}\gamma^\mu\left(\frac{g'}{2}YB^{(k)}_\mu\mathcal{F}^{\Psi,[k]}_{ii} + \frac{g}{2}\tau^a W^{a,(k)}_\mu\mathcal{F}^{\Psi,[k]}_{ii} + \frac{g_s}{2}T^a G^{a,(k)}_\mu\mathcal{F}^{\Psi,[k]}_{ii}\right) q^{(0)}_{i,h}\ , \tag{4.15}$$

where the notation is defined in eq. (4.3a) and $q^{(0)}_{i,h}$ stands for either the left handed $SU(2)$ doublet quark zero mode or the right handed $SU(2)$ singlet quark zero mode. One has to notice that the W^a_μ fields only couple to fields with left chirality (h=L) whereas B_μ and G^a_μ couple to both chiralities with their respective charges.
In an analogous calculation, the KK decomposition of the Yukawa-Lagrangian in eq. (4.2)

[2]The calculation would be analogous except that the leptons do not couple to gluons.

can be done. First of all we choose the vacuum expectation value (vev) of the Higgs-doublet and its conjugate to be:

$$\langle H \rangle = \begin{pmatrix} 0 \\ \hat{v}_5(y) \end{pmatrix} \neq 0 \; ; \qquad \left\langle \tilde{H} \right\rangle = \begin{pmatrix} \hat{v}_5(y) \\ 0 \end{pmatrix} \neq 0 \; , \tag{4.16}$$

where the 5D wave function of the Higgs-doublets are the same as in chapter 2.2. Using the Higgs vev in eq. (4.2b), we get:

$$\mathcal{L}_Y = \sum_{i,j} \hat{v}_5(y) \left(\overline{u}_{L,i}(x,y) \lambda^U_{ij} u_{R,j}(x,y) + \overline{d}_{L,i}(x,y) \lambda^D_{ij} d_{R,j}(x,y) \right) + \text{h.c.} \; . \tag{4.17}$$

The KK decomposition now yields:

$$\mathcal{L}_Y = \sum_{i,j} \sum_{n,m,k=0}^{\infty} v \left(\overline{u}^{(n)}_{L,i}(x) \lambda^U_{ij} \tilde{\mathcal{F}}^{[nkm]}_{Y_u,ij} u^{(m)}_{R,j}(x) + \overline{d}^{(n)}_{L,i}(x) \lambda^D_{ij} \tilde{\mathcal{F}}^{[nkm]}_{Y_d,ij} d^{(m)}_{R,j}(x) \right) + \text{h.c.} \; , \tag{4.18a}$$

with

$$\tilde{\mathcal{F}}^{[nkm]}_{Y_q,ij} = \sqrt{\pi R}\; f^{Q,(n)}_i(y) f^{(k)}(y) f^{u/d,(m)}_j(y) \; , \tag{4.18b}$$

where we used $\hat{v}_5 = \sqrt{\pi R}\; v$. We are using again the zero mode approximation and integrate the overlap matrix over the $S^1/\mathbb{Z}_2$-orbifold:

$$\int_{-\frac{\pi R}{2}}^{\frac{\pi R}{2}} \mathrm{d}y \; \tilde{\mathcal{F}}^{[0k0]}_{Y_q,ij} \equiv \mathcal{F}^{[k]}_{Y_q,ij} = \begin{cases} \dfrac{\pi R}{\sqrt{2a^Q_L + \pi R}\sqrt{2a^q_L \; + \pi R}} & \text{for } k \; = 0 \\ 0 & \text{for } k \text{ else} \end{cases} \; . \tag{4.19}$$

We now transform the Yukawa Lagrangian $\mathcal{L}_Y$ in eq. (4.18a) with a bi-unitary transformation:

$$(S^\dagger_u)_{ij} u^{(0)}_{L,j} = u_{L,i} \; ; \qquad (S^\dagger_d)_{ij} d^{(0)}_{L,j} = d_{L,i} \; ; \tag{4.20a}$$

$$(T^\dagger_u)_{ij} u^{(0)}_{R,j} = u_{L,i} \; ; \qquad (T^\dagger_d)_{ij} d^{(0)}_{R,j} = d_{R,i} \; , \tag{4.20b}$$

which diagonalizes the quark mass matrices in the Yukawa sector [11]. These transformations will alter the overlap matrices in the gauge sector and give rise to FCNCs since the rotated $\mathcal{F}^{\Psi,[k]}$ in eq. (4.11) are ad hoc not diagonal anymore. The Yukawa Lagrangian then becomes:

$$\begin{aligned} \mathcal{L}_Y =& v \sum_{i,j,k} \Big(\underbrace{\overline{u}^{(0)}_{L,i} (S_u)_{ik}} \underbrace{(S^\dagger_u)_{ki} \lambda_{ij} \mathcal{F}^{[0]}_{Y_u,ij} (T_u)_{jk}}_{\text{diagonal}} \underbrace{(T^\dagger_u)_{kj} u^{(0)}_{R,j}} \\ & + \underbrace{\overline{d}^{(0)}_{L,i} (S_d)_{ik}} \underbrace{(S^\dagger_d)_{ki} \lambda_{ij} \mathcal{F}^{[0]}_{Y_d,ij} (T_d)_{jk}}_{\text{diagonal}} \underbrace{(T^\dagger_d)_{kj} d^{(0)}_{R,j}} \Big) + \text{h.c.} \\ =& v \sum_{k} \left(\overline{u}_{L,k} \lambda'_{kk} u_{R,k} + \overline{d}_{L,k} \lambda'_{kk} d_{R,k} \right) + \text{h.c.} \; . \end{aligned} \tag{4.21}$$

The Yukawa matrices now depend on the overlap matrices. Here we see that the introduction of an extra dimension has effects on the masses in the fermion sector in the effective 4D theory.

4.1. Neutral Currents

The main contribution to the neutral currents are generated by the gluons since the $SU(2)$ coupling g and $U(1)$ g' coupling are much smaller than the strong coupling g_s as shown in Tab. 4.1. Therefore only the gluon Wilson coefficients will be used when we calculate the flavor bounds on the nUED model.

$SU(3)$ coupling	$SU(2)$ coupling	$U(1)$ coupling
$g_s = \sqrt{4\pi\alpha_s}$	$g = \frac{\sqrt{4\pi\alpha}}{\sin(\Theta_W)}$	$g' = \frac{\sqrt{4\pi\alpha}}{\cos(\Theta_W)}$

Table 4.1.: List of the coupling constants for the $SU(3)$, $SU(2)$ and $U(1)$ groups. The values for their calculation are given in eq. (B.3) in appendix B.

The fermion-gluon Lagrangian can be written as:

$$\mathcal{L}_{\mathrm{g,gl}} = -\, i\frac{g_s}{2}\left(\overline{q}^{(0)}_{L,i}\gamma^\mu T^a \mathcal{F}^{Q,[k]}_{ii} q^{(0)}_{L,i} + \overline{q}^{(0)}_{R,i}\gamma^\mu T^a \mathcal{F}^{q,[k]}_{ii} q^{(0)}_{R,i}\right) G^{(k)}_\mu \,, \tag{4.22}$$

with the definition for the generators from eq. (4.3a) and where the superscript of the overlap matrices Q and $q = u, d$ must be matched to the left quark doublet and right quark singlet. Now we use the transformation in eq. (4.20) to rotate all the fermions in the gauge Lagrangian into the Yukawa diagonal basis. The gluonic part $\mathcal{L}_{gl,g}$ gets

$$\begin{aligned}
\mathcal{L}_{g,gl} =& -\, i\frac{g_s}{2}\Big(\underbrace{\overline{q}^{(0)}_{L,i}(S_q)_{im}}\,\gamma^\mu T^a\,\underbrace{(S^\dagger)_{mj}\mathcal{F}^{Q,[k]}_{jj}(S_q)_{jn}}\,\underbrace{(S^\dagger_q)_{ni}q^{(0)}_{L,i}} \\
&+ \underbrace{\overline{q}^{(0)}_{R,i}(T_q)_{im}}\,\gamma^\mu T^a\,\underbrace{(T^\dagger_q)_{mj}\mathcal{F}^{q,[k]}_{jj}(T_q)_{jn}}\,\underbrace{(T^\dagger_q)_{ni}q^{(0)}_{R,i}}\Big)\, G^{(k)}_\mu \\
=& -\, i\frac{g_s}{2}\left(\overline{q}_{L,m}\gamma^\mu T^a V^{Q,[k]}_{mn} q_{L,n} + \overline{q}_{R,m}\gamma^\mu T^a V^{q,[k]}_{mn} q_{R,n}\right) G^{(k)}_\mu \qquad (4.23) \\
=& \frac{g_s}{2} J^{(k),\mu}_G G^{(k)}_\mu \,, \qquad (4.24)
\end{aligned}$$

where $J^{(k),\mu}_G$ ist the gluon current and $V^{Q/q,[k]}_{mn}$ are the overlap matrices rotated by the bi-unitary transformation

$$V^{Q,[k]}_{mn} = (S^\dagger_q)_{mj}\mathcal{F}^{Q,[k]}_{jj}(S_q)_{jn}\,; \qquad V^{q,[k]}_{mn} = (T^\dagger_q)_{mj}\mathcal{F}^{u/d,[k]}_{jj}(T_q)_{jn}\,. \tag{4.25}$$

The off-diagonal elements of these matrices induce FCNCs. The indices m and n denote the particular flavor of the quarks. The $SU(3)$ generators T^a are not affected by this rotation and should not be confused with the rotation matrices T_q.

For completeness, we calculate the Zdish (because we want to distinguish between the three neutral currents and the current belonging to the $Z_\mu^{(k)}$ field) and the electromagnetic currents. We know from electroweak symmetry breaking that the $B_\mu^{(k)}$ and $W_\mu^{3,(k)}$ are not in mass eigenbasis. To rotate between flavor and mass eigenbasis, we need the transformation [23]:

$$\begin{aligned} W_\mu^{3,(k)} &= \sin(\theta_W) A_\mu^{(k)} + \cos(\theta_W) Z_\mu^{(k)} \ , \\ B_\mu^{(k)} &= \cos(\theta_W) A_\mu^{(k)} - \sin(\theta_W) Z_\mu^{(k)} \ . \end{aligned} \tag{4.26}$$

The $U(1)_Y$ coupling g', $SU(2)_L$ coupling g and electromagnetic coupling e are related by:

$$g' = g\tan(\theta_W) \quad \text{and} \quad e = g\sin(\theta_W) \ , \tag{4.27}$$

where θ_W is the Weinberg-angle. We recall that only the left $SU(2)$ doublets couple to the W^3 field whereas the B field couples to both. The left $SU(2)$ doublet and right $SU(2)$ singlet become

$$\begin{aligned} \mathcal{L}_{\text{g,nc}} =& - i \left[\overline{q}_{L,i}^{(0)} \gamma^\mu \left(\frac{g'}{2} Y B_\mu^{(k)} \mathbb{1}_{2\times 2} \mathcal{F}_{ii}^{Q,[k]} + \frac{g}{2} \tau^3 W_\mu^{3,(k)} \mathcal{F}_{ii}^{Q,[k]} \right) q_{L,i}^{(0)} \right. \\ & \left. + \overline{q}_{R,i}^{(0)} \frac{g'}{2} Y \gamma^\mu B_\mu^{(k)} \mathcal{F}_{ii}^{q,[k]} q_{R,i}^{(0)} \right] \end{aligned} \tag{4.28}$$

$$\begin{aligned} =& - \frac{i}{2} \left[(\overline{u}_{L,i}^{(0)}, \overline{d}_{L,i}^{(0)}) \gamma^\mu \begin{pmatrix} g' Y B_\mu^{(k)} + g W_\mu^{3,(k)} & 0 \\ 0 & g' Y B_\mu^{(k)} - g W_\mu^{3,(k)} \end{pmatrix} \begin{pmatrix} u_{L,i}^{(0)} \\ d_{L,i}^{(0)} \end{pmatrix} \mathcal{F}_{ii}^{Q,[k]} \right. \\ & \left. + \overline{q}_{R,i}^{(0)} \frac{g'}{2} Y \gamma^\mu B_\mu^{(k)} q_{R,i}^{(0)} \mathcal{F}_{ii}^{q,[k]} \right] \ , \end{aligned} \tag{4.29}$$

where $\mathcal{F}_{ii}^{Q,[k]}$ are the overlap matrices for the left chiral quarks, $\mathcal{F}_{ii}^{q,[k]}$ for the right chiral quarks and τ^3 is one of the Pauli matrices defined in eq. (4.40). Using eq. (4.26), we get for the left handed fields:

$$\begin{aligned} & \mathcal{L}_{g,nc} \supset \\ & \frac{i}{2} \left[\overline{u}_{L,i}^{(0)} \gamma^\mu \left(g' Y [\cos(\theta_W) A_\mu^{(k)} - \sin(\theta_W) Z_\mu^{(k)}] + g [\sin(\theta_W) A_\mu^{(k)} + \cos(\theta_W) Z_\mu^{(k)}] \right) u_{L,i}^{(0)} \mathcal{F}_{ii}^{Q,[k]} \right. \\ & \left. + \overline{d}_{L,i}^{(0)} \gamma^\mu \left(g' Y [\cos(\theta_W) A_\mu^{(k)} - \sin(\theta_W) Z_\mu^{(k)}] - g [\sin(\theta_W) A_\mu^{(k)} + \cos(\theta_W) Z_\mu^{(k)}] \right) d_{L,i}^{(0)} \mathcal{F}_{ii}^{Q,[k]} \right] \ . \end{aligned} \tag{4.30}$$

Collecting all the terms with the same gauge field and using the substitution for the coupling constants from eq. (4.27), we get

$$
\begin{aligned}
\mathcal{L}_{g,nc} \supset\ & i \left(\overline{u}^{(0)}_{L,i}\gamma^{\mu}\frac{[Y+1]}{2}u^{(0)}_{L,i} + \overline{d}^{(0)}_{L,i}\gamma^{\mu}\frac{[Y-1]}{2}d^{(0)}_{L,i}\right)\mathcal{F}^{Q,[k]}_{ii}\ eA^{(k)}_{\mu} \\
& - i \left(\overline{u}^{(0)}_{L,i}\gamma^{\mu}\frac{[(Y+1)\sin^2(\theta_W)-1]}{2}u^{(0)}_{L,i} \right. \\
& + \left. \overline{d}^{(0)}_{L,i}\gamma^{\mu}\frac{[(Y-1)\sin^2(\theta_W)+1]}{2}d^{(0)}_{L,i}\right)\mathcal{F}^{Q,[k]}_{ii}\frac{g}{\cos(\theta_W)}Z^{(k)}_{\mu}\ .
\end{aligned}
\tag{4.31}
$$

In an analogous procedure, it holds for the right singlets:

$$
\begin{aligned}
\mathcal{L}_{g,nc} \supset\ & i\left[\overline{q}^{(0)}_{R,i}\frac{g'}{2}Y\gamma^{\mu}q^{(0)}_{R,i}\mathcal{F}^{q,[k]}_{ii}B^{(k)}_{\mu}\right] \\
=\ & i\left[e\frac{Y}{2}\overline{q}^{(0)}_{R,i}\gamma^{\mu}q^{(0)}_{R,i}\mathcal{F}^{q,[k]}_{ii}A^{(k)}_{\mu} - \frac{g}{\cos(\theta_W)}\frac{Y\sin^2(\theta_W)}{2}\overline{q}^{(0)}_{R,i}\gamma_{\mu}q^{(0)}_{R,i}\mathcal{F}^{q,[k]}_{ii}Z^{(k)}_{\mu}\right]\ .
\end{aligned}
\tag{4.32}
$$

Taking together the Zdish (Z) and electromagnetic (em) parts of $\mathcal{L}_{g,nc}$ with right and left chirality, we get the electromagnetic Lagrangian $\mathcal{L}_{g,em}$

$$
\begin{aligned}
\mathcal{L}_{\mathrm{g,em}} =\ & i\left(\overline{u}^{(0)}_{L,i}\gamma^{\mu}\frac{[Y+1]}{2}\mathcal{F}^{Q,[k]}_{ii}u^{(0)}_{L,i} + \overline{d}^{(0)}_{L,i}\gamma^{\mu}\frac{[Y-1]}{2}\mathcal{F}^{Q,[k]}_{ii}d^{(0)}_{L,i}\right. \\
& + \left.\frac{Y}{2}\overline{q}^{(0)}_{R,i}\gamma^{\mu}\mathcal{F}^{q,[k]}_{ii}q^{(0)}_{R,i}\right)eA^{(k)}_{\mu} \\
=\ & i\left(\overline{q}^{(0)}_{L,i}\gamma^{\mu}C^{em}\mathcal{F}^{Q,[k]}_{ii}q^{(0)}_{L,i} + \overline{q}^{(0)}_{R,i}\gamma^{\mu}\frac{Y}{2}\mathcal{F}^{q,[k]}_{ii}q^{(0)}_{R,i}\right)eA^{(k)}_{\mu}\ ,
\end{aligned}
\tag{4.33}
$$

and the Zdish Lagrangian $\mathcal{L}_{g,Z}$

$$
\begin{aligned}
\mathcal{L}_{\mathrm{g,Z}} =& - i\left(\overline{u}^{(0)}_{L,i}\gamma^{\mu}\frac{[(Y+1)\sin^2(\theta_W)-1]}{2}\mathcal{F}^{Q,[k]}_{ii}u^{(0)}_{L,i}\right. \\
& + \overline{d}^{(0)}_{L,i}\gamma^{\mu}\frac{[(Y-1)\sin^2(\theta_W)+1]}{2}\mathcal{F}^{Q,[k]}_{ii}d^{(0)}_{L,i} \\
& \left. - \frac{Y\sin^2(\theta_W)}{2}\overline{q}^{(0)}_{R,i}\gamma^{\mu}\mathcal{F}^{q,[k]}_{ii}q^{(0)}_{R,i}\right)\frac{g}{\cos(\theta_W)}Z^{(k)}_{\mu} \\
=& -i\left(\overline{q}^{(0)}_{L,i}\gamma^{\mu}C^{Z}\mathcal{F}^{[k],L}_{ii}q^{(0)}_{L,i} - \frac{Y\sin^2(\theta_W)}{2}\overline{q}^{(0)}_{R,i}\gamma^{\mu}\mathcal{F}^{[k],R}_{ii}q^{(0)}_{R,i}\right)\frac{g}{\cos(\theta_W)}Z^{(k)}_{\mu}\ ,
\end{aligned}
\tag{4.34}
$$

where C^{em} and C^{Z} are defined as follows

$$
C^{em} = \begin{pmatrix} \frac{Y+1}{2} & 0 \\ 0 & \frac{Y-1}{2} \end{pmatrix}\ ; \qquad C^{Z} = \begin{pmatrix} \frac{[(Y+1)\sin^2(\theta_W)-1]}{2} & 0 \\ 0 & \frac{[(Y-1)\sin^2(\theta_W)+1]}{2} \end{pmatrix}\ , \tag{4.35}
$$

and where the fitting hypercharges which are defined in eq. (4.4) must be included. Again, we perform the rotation into the mass eigenbasis and define the remaining neutral currents. The Zdish Lagrangian $\mathcal{L}_{g,Z}$ is:

$$\begin{aligned}
\mathcal{L}_{g,Z} =& -i\,\Big(C^Z_{11}\,\underbrace{\overline{u}^{(0)}_{L,i}(S_u)_{im}}\,\gamma^\mu\,\underbrace{(S_u^\dagger)_{mi}\mathcal{F}^{Q,[k]}_{ii}(S_u)_{in}}\,\underbrace{(S_u^\dagger)_{ni}u^{(0)}_{L,i}} \\
&+ C^Z_{22}\,\underbrace{\overline{d}^{(0)}_{L,i}(S_d)_{im}}\,\gamma^\mu\,\underbrace{(S_d^\dagger)_{mi}\mathcal{F}^{Q,[k]}_{ii}(S_d)_{in}}\,\underbrace{(S_d^\dagger)_{ni}d^{(0)}_{L,i}} \\
&+ \frac{Y\sin^2(\theta_W)}{2}\,\underbrace{\overline{q}^{(0)}_{R,i}(T_q)_{im}}\,\underbrace{(T_q^\dagger)_{mi}\mathcal{F}^{q,[k]}_{ii}(T_q)_{in}}\,\underbrace{(T_q^\dagger)_{ni}q^{(0)}_{R,i}}\Big)\frac{g}{\cos(\theta_W)}Z^{(k)}_\mu \\
=& -i\,\Big(C^Z_{11}\overline{u}_{L,m}\gamma^\mu V^{Q,[k]}_{mn}u_{L,n} + C^Z_{22}\overline{d}_{L,m}\gamma^\mu V^{Q,[k]}_{mn}d_{L,n} \\
&+\frac{Y\sin^2(\theta_w)}{2}\overline{q}_{R,m}V^{q,[k]}_{mn}q_{R,n}\Big)\,\frac{g}{\cos(\theta_W)}Z^{(k)}_\mu \\
=&\frac{g}{\cos(\theta_W)}J_Z^{(k),\mu}Z^{(k)}_\mu\ ,
\end{aligned} \tag{4.36}$$

where C^{em}_{ii} and C^Z_{ii} are the matrix entries of the matrices defined in eq. (4.35) and $J_Z^{(k),\mu}$ is the Zdish current. Furthermore the electromagnetic Lagrangian $\mathcal{L}_{g,em}$ becomes:

$$\begin{aligned}
\mathcal{L}_{g,em} =& -i\,\Big(C^{em}_{11}\,\underbrace{\overline{u}^{(0)}_{L,i}(S_u)_{im}}\,\gamma^\mu\,\underbrace{(S_u^\dagger)_{mi}\mathcal{F}^{Q,[k]}_{ii}(S_u)_{in}}\,\underbrace{(S_u^\dagger)_{ni}u^{(0)}_{L,i}} \\
&+ C^{em}_{22}\,\underbrace{\overline{d}^{(0)}_{L,i}(S_d)_{im}}\,\gamma^\mu\,\underbrace{(S_d^\dagger)_{mi}\mathcal{F}^{Q,[k]}_{ii}(S_d)_{in}}\,\underbrace{(S_d^\dagger)_{ni}d^{(0)}_{L,i}} \\
&+ \frac{Y}{2}\,\underbrace{\overline{q}^{(0)}_{R,i}(T_q)_{im}}\,\underbrace{(T_q^\dagger)_{mi}\mathcal{F}^{q,[k]}_{ii}(T_q)_{in}}\,\underbrace{(T_q^\dagger)_{ni}q^{(0)}_{R,i}}\Big)eA^{(k)}_\mu \\
=& -i\left(C^{em}_{11}\overline{u}_{L,m}\gamma^\mu V^{Q,[k]}_{mn}u_{L,n} + C^{em}_{22}\overline{d}_{L,m}\gamma^\mu V^{Q,[k]}_{mn}d_{L,n} + \frac{Y}{2}\overline{q}_{R,m}V^{q,[k]}_{mn}q_{R,n}\right)eA^{(k)}_\mu \\
=& \,eJ_A^{(k),\mu}A^{(k)}_\mu\ .
\end{aligned} \tag{4.37}$$

where $J_A^{(k),\mu}$ is the electromagnetic current. The matrices $V^{Q/q,[k]}$ we get from the transformation matrices T_q and S_q and the overlap matrix $\mathcal{F}^{Q/q,[k]}$ in eq. (4.13c) are identical to the ones which appear in the gluonic current in eq. (4.24). Since the coupling constant of the $Z^{(k)}_\mu$ and $A^{(k)}_\mu$ fields are much smaller, it is justified to drop these contributions in the calculation of the Wilson coefficients in chapter 4.3.

4.2. Charged Currents

We make the following substitutions which come from the Higgs sector where the gauge boson masses are generated via spontaneous symmetry breaking [14, 18, 23]. The Lagrangian $\mathcal{L}_{g,cc}$ for the charged current can be written as:

$$\mathcal{L}_{g,cc} = -i\,\overline{q}^{(0)}_{L,i}\gamma^\mu\frac{g}{2}\left(\tau^1W^{1,(k)}_\mu\mathcal{F}^{Q,[k]}_{W^+,ii} + \tau^2W^{2,(k)}_\mu\mathcal{F}^{Q,[k]}_{W^-,ii}\right)q^{(0)}_{L,i}\ . \tag{4.38}$$

$W_\mu^{1,(k)}$ and $W_\mu^{2,(k)}$ are redefined such that they generate the charged currents:

$$\begin{aligned} \sqrt{2}\; W_\mu^{\mp,(k)} &= W_\mu^{1,(k)} \pm i\; W_\mu^{2,(k)} \;, \\ 2\;\tau^{\pm} &= \tau^1 \pm i\;\tau^2 \;, \end{aligned} \tag{4.39}$$

where τ^a are the Pauli matrices defined as:

$$\tau^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \;, \qquad \tau^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \;, \qquad \tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \;. \tag{4.40}$$

Under the redefinition in eq. (4.39) the left SU(2)-doublets $q_{L,i}^{(0)} = \begin{pmatrix} u_{L,i}^{(0)} \\ d_{L,i}^{(0)} \end{pmatrix}$ and $\overline{q}_{L,i}^{(0)} = q_{L,i}^{(0)\dagger}\gamma_0$ get:

$$\overline{q}_{L,i}^{(0)}\;\gamma_\mu\;\tau^+\;q_{L,i}^{(0)} = \overline{u}_{L,i}^{(0)}\;\gamma_\mu\;d_{L,i}^{(0)}\;, \tag{4.41}$$

$$\overline{q}_{L,i}^{(0)}\;\gamma_\mu\;\tau^-\;q_{L,i}^{(0)} = \overline{d}_{L,i}^{(0)}\;\gamma_\mu\;u_{L,i}^{(0)}\;, \tag{4.42}$$

so that $\mathcal{L}_{g,cc}$ in eq. (4.38) can be written like

$$\mathcal{L}_{\mathrm{g,cc}} = -i\frac{g}{2\sqrt{2}}\left(\overline{u}_{L,i}^{(0)}\gamma^\mu \mathcal{F}^{Q,[k]}_{W^+,ii} d_{L,i}^{(0)}\;W_\mu^{+,(k)} + \overline{d}_{L,i}^{(0)}\gamma^\mu \mathcal{F}^{Q,[k]}_{W^-,ii} u_{L,i}^{(0)}\;W_\mu^{-,(k)}\right)\;. \tag{4.43}$$

The overlap matrices $\mathcal{F}^{Q,[k]}_{W^+,ii}$ and $\mathcal{F}^{Q,[k]}_{W^-,ii}$ will be identified as the Cabibbo-Kobayashi-Maskawa (CKM) matrix and its conjugate transpose, respectively, after we rotate into the basis where the Yukawa matrices are diagonal (see eq. (4.20)). This fact will be used in chapter 5 to constrain the BKLT parameters $a_h^{\Psi_i\Psi_i}$ and find a parametrization for the transformation matrices. The charged Lagrangian then $\mathcal{L}_{g,cc}$ gets

$$\begin{aligned} \mathcal{L}_{\mathrm{g,cc}} =& -i\frac{g}{2\sqrt{2}}\Big[\underbrace{\overline{u}_{L,i}^{(0)}(S_u)_{im}}\,\gamma^\mu\,\underbrace{(S_u^\dagger)_{mi}\mathcal{F}^{Q,[k]}_{W^+,ii}(S_d)_{in}}\,\underbrace{(S_d^\dagger)_{ni}d_{L,i}^{(0)}}\;W_\mu^{+,(k)} \\ &+\underbrace{\overline{d}_{L,i}^{(0)}(S_d)_{im}}\,\gamma^\mu\,\underbrace{(S_d^\dagger)_{mi}\mathcal{F}^{Q,[k]}_{W^-,ii}(S_u)_{in}}\,\underbrace{(S_u^\dagger)_{ni}u_{L,i}^{(0)}}\;W_\mu^{-,(k)}\Big] \\ =& -i\frac{g}{2\sqrt{2}}\left[\overline{u}_{L,m}\gamma^\mu(V_{ckm})_{mn}d_{L,n}\;W_\mu^{+,(k)} + \overline{d}_{L,m}\gamma^\mu(V_{ckm}^\dagger)_{mn}u_{L,n}\;W_\mu^{-,(k)}\right] && (4.44) \\ =& \frac{g}{2\sqrt{2}}\left(J^{(k),\mu}_{W^+}W_\mu^{+,(k)} + J^{(k),\mu}_{W^-}W_\mu^{-,(k)}\right)\;, && (4.45) \end{aligned}$$

where V_{ckm} has to be the CKM matrix with contributions from the overlap matrices. The CKM matrix in the SM is given by:

$$\begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}\;, \tag{4.46}$$

with its matrix elements in the Wolfenstein parametrization to the order $\mathcal{O}(\lambda^4)$ [27]. The best fit values from the SM are listed in the appendix B. Here, the overlap matrices $\mathcal{F}^{Q,[k]}$ induce corrections to the SM CKM matrix but these are suppressed by $\propto (\frac{M_W}{M_{W^{[k]}}}\mathcal{F}^{Q/q,[k]})^2$ where M_W is the mass of the W-boson and $M_{W^{[k]}}$ are the masses of the higher W-boson KK excitations. We will drop these contributions when we want to determine the transformation matrices T_q and S_q in chapter 5.

4.3. Calculation of the Wilson Coefficients

After the diagonalization of the quark mass matrices we can go to an effective Hamiltonian approach. We assume that the masses of the gauge bosons and their Kaluza-Klein excitations are much bigger than the momentum of all participating particles in the process. Then the propagator of the intermediate vector boson can be approximated by:

$$i\ D^{(k)}_{\mu\nu}(p) = \frac{-i}{p^2 - M^2_{A^{(k)}} + i\epsilon}\left(g^{\mu\nu}\frac{(\xi-1)p_\mu p_\nu}{p^2 - \xi M^2_{A^{(k)}}}\right) \approx -\frac{i\ g^{\mu\nu}}{M^2_{A^{(k)}}} \quad \text{for} \quad \frac{p^2}{M^2_{A^{(k)}}} \ll 1\ , \tag{4.47}$$

where $M_{A^{(k)}}$ is the KK mass of the exchanged gauge boson $A^{(k)}_\mu$, p is the gauge boson momentum and ξ is the gauge fixing parameter which will be set to $\xi = \infty$ (unitary gauge). Using this, we are going to an effective four Fermi interaction [23] (see Fig. 4.1).

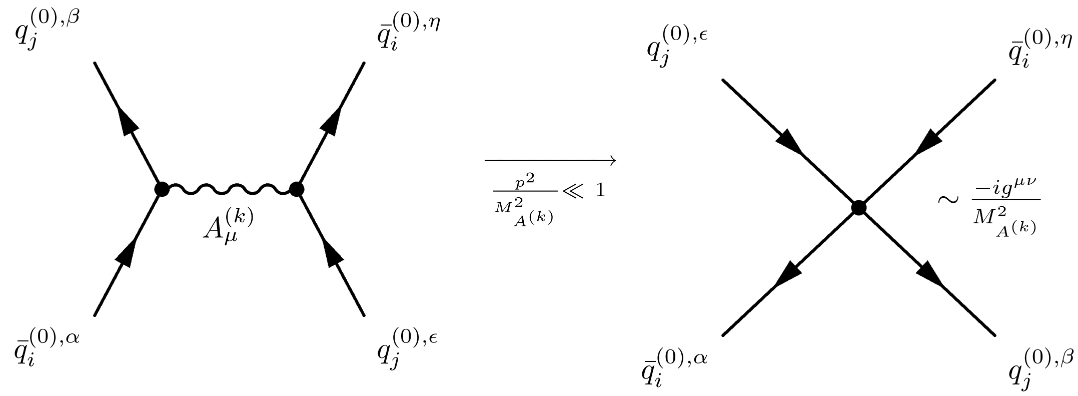


Figure 4.1.: Feynman diagram where the exchanged gauge boson mass $M_{A^{(k)}}$ is much bigger than the momentum of the participating particles; so we receive an effective four-fermion-vertex. The numbers in parenthesis are the KK mode number and the greek indices $\{\alpha, \beta, \epsilon, \eta\}$ are the color of the respective quarks.

The effective Hamiltonian is of the form:

$$\mathcal{H}^{A}_{eff} = \sum_k \frac{g_a^2}{2M^2_{A^{(k)}}} J^{(k)\dagger}_{A,\mu} J^{(k),\mu}_{A} \,, \tag{4.48}$$

where the index A stands for the gauge field, g_a for the corresponding coupling and $J_A^{(k),\mu}$ are the corresponding currents defined in eq. (4.24), eq. (4.36) and eq. (4.37). Additionally, we inserted a factor $\frac{1}{2}$ due to the exchange symmetry present in such current-current interactions [23]. We want to calculate the Wilson coefficients for ΔF=2 processes and try to constrain the BKLT parameters $a_h^{\Psi_i\Psi_j}$ with model independent constraints provided by the UTfit collaboration [28]. To use their data, it is necessary to rewrite the operators in the effective Hamiltonian in the operator basis given in [28]:

$$\begin{aligned}
\mathcal{H}^{\Delta S=2}_{\text{eff}} &= \sum_{i=1}^{5} C_i\, Q_i^{sd} + \sum_{i=1}^{3} \tilde{C}_i\, \tilde{Q}_i^{sd} \,, \\
\mathcal{H}^{\Delta C=2}_{\text{eff}} &= \sum_{i=1}^{5} C_i\, Q_i^{cu} + \sum_{i=1}^{3} \tilde{C}_i\, \tilde{Q}_i^{cu} \,, \\
\mathcal{H}^{\Delta B=2}_{\text{eff}} &= \sum_{i=1}^{5} C_i\, Q_i^{bq} + \sum_{i=1}^{3} \tilde{C}_i\, \tilde{Q}_i^{bq} \,,
\end{aligned}$$

where C_i are the complex valued Wilson coefficients and $q = d(s)$ for $B_{d(s)} - \overline{B}_{d(s)}$ mixing. The operators $Q^{q_i q_j}$ are defined by:

$$\begin{aligned}
Q_1^{q_i q_j} &= (\overline{q}^{\alpha}_{jL}\gamma_\mu q^{\alpha}_{iL})(\overline{q}^{\beta}_{jL}\gamma^\mu q^{\beta}_{iL}) \,, \\
Q_2^{q_i q_j} &= (\overline{q}^{\alpha}_{jR} q^{\alpha}_{iL})(\overline{q}^{\beta}_{jR} q^{\beta}_{iL}) \,, \\
Q_3^{q_i q_j} &= (\overline{q}^{\alpha}_{jR} q^{\beta}_{iL})(\overline{q}^{\beta}_{jR} q^{\alpha}_{iL}) \,, \\
Q_4^{q_i q_j} &= (\overline{q}^{\alpha}_{jR} q^{\alpha}_{iL})(\overline{q}^{\beta}_{jL} q^{\beta}_{iR}) \,, \\
Q_5^{q_i q_j} &= (\overline{q}^{\alpha}_{jR} q^{\beta}_{iL})(\overline{q}^{\beta}_{jL} q^{\alpha}_{iR}) \,.
\end{aligned} \tag{4.49}$$

Here $q_{R,L} = P_{R,L}\, q$ and $\{\alpha, \beta\}$ are color indices which run over the three possible colors. The operators $\tilde{Q}^{q_i q_j}_{1,2,3}$ are obtained from the $Q^{q_i q_j}_{1,2,3}$ by the exchange $L \to R$. We will calculate the gluon Wilson coefficients in detail and give the results for the Zdish and electromagnetic contributions:

$$\begin{aligned}
\mathcal{H}_{\text{eff}}^{G} =& \frac{g_s^2}{2M_{G^{(k)}}^2} J_{G,\mu}^{(k)^\dagger} J_G^{(k),\mu} \\
=& \frac{g_s^2}{2M_{G^{(k)}}^2} \left(\overline{q}_{L,m}^{\alpha} T_{\alpha\epsilon}^a \gamma_\mu V_{mn}^{Q,[k]} q_{L,n}^{\epsilon} + \overline{q}_{R,m}^{\alpha} T_{\alpha\epsilon}^a \gamma_\mu V_{mn}^{q,[k]} q_{R,n}^{\epsilon}\right) \cdot \\
& \left(\overline{q}_{L,k}^{\beta} T_{\beta\eta}^a \gamma^\mu V_{kl}^{Q,[k]} q_{L,l}^{\eta} + \overline{q}_{R,k}^{\beta} T_{\beta\eta}^a \gamma^\mu V_{kl}^{q,[k]} q_{R,l}^{\eta}\right) \\
=& \frac{g_s^2}{2M_{G^{(k)}}^2} \left[\left(\overline{q}_{L,m}^{\alpha} T_{\alpha\epsilon}^a \gamma_\mu V_{mn}^{Q,[k]} q_{L,n}^{\epsilon}\right) \left(\overline{q}_{L,k}^{\beta} T_{\beta\eta}^a \gamma^\mu V_{kl}^{Q,[k]} q_{L,l}^{\eta}\right) + L \to R\right] \\
& + 2\frac{g_s^2}{2M_{G^{(k)}}^2} \left(\overline{q}_{L,m}^{\alpha} T_{\alpha\epsilon}^a \gamma_\mu V_{mn}^{Q,[k]} q_{L,n}^{\epsilon}\right) \left(\overline{q}_{R,k}^{\beta} T_{\beta\eta}^a \gamma^\mu V_{kl}^{q,[k]} q_{R,l}^{\eta}\right) \qquad (4.50) \\
=& \frac{g_s^2}{2M_{G^{(k)}}^2} \left(\mathcal{H}_{pure} + \mathcal{H}_{mixed}\right) \ , \qquad (4.51)
\end{aligned}$$

where $\{\alpha, \beta, \epsilon, \eta\}$ are color indices. We take eq. (4.51) apart into left-left or right-right chiral combinations we call pure part and a left-right chiral combinations of the quarks we call mixed part. We simplify this expression by using a relation for $SU(N)$ representations [18]:

$$(T^a)_{\alpha\epsilon}(T^a)_{\beta\eta} = \frac{1}{2}\left(\delta_{\alpha\eta}\delta_{\beta\epsilon} - \frac{1}{N}\delta_{\alpha\epsilon}\delta_{\beta\eta}\right) \ , \qquad (4.52)$$

where T^a are the generators of the $SU(N)$, N is the dimension of the group and $(T^a)_{\alpha\epsilon}$ and $(T^a)_{\beta\eta}$ are the matrix elements of the $SU(N)$ representation. The pure chiral part $\mathcal{H}_{pure}$ gets:

$$\begin{aligned}
\mathcal{H}_{pure} =& \frac{1}{2}\left(\overline{q}_{L,m}^{\alpha} \gamma_\mu V_{mn}^{Q,[k]} q_{L,n}^{\beta}\right) \left(\overline{q}_{L,k}^{\beta} \gamma^\mu V_{kl}^{Q,[k]} q_{L,l}^{\alpha}\right) \\
& - \frac{1}{6}\left(\overline{q}_{L,m}^{\alpha} \gamma_\mu V_{mn}^{Q,[k]} q_{L,n}^{\alpha}\right) \left(\overline{q}_{L,k}^{\beta} \gamma^\mu V_{kl}^{Q,[k]} q_{L,l}^{\beta}\right) + L \to R \text{ and } Q \to q \ , \qquad (4.53)
\end{aligned}$$

while the mixed part $\mathcal{H}_{mixed}$ gets

$$\begin{aligned}
\mathcal{H}_{mixed} =& \left(\overline{q}_{L,m}^{\alpha} \gamma_\mu V_{mn}^{Q,[k]} q_{L,n}^{\beta}\right) \left(\overline{q}_{R,k}^{\beta} \gamma^\mu V_{kl}^{q,[k]} q_{R,l}^{\alpha}\right) \\
& - \frac{1}{3}\left(\overline{q}_{L,m}^{\alpha} \gamma_\mu V_{mn}^{Q,[k]} q_{L,n}^{\alpha}\right) \left(\overline{q}_{R,k}^{\beta} \gamma^\mu V_{kl}^{q,[k]} q_{R,l}^{\beta}\right) \ . \qquad (4.54)
\end{aligned}$$

We now have to match the expressions to the one in eq. (4.49). Therefore we need Fierz identities so we can exchange the order of the quarks. In [29], we find an algorithm for general Fierz transformations. Using this, we find two Fierz Identities that can be used to transform eq. (4.53) and eq. (4.54):

$$(\underset{1}{\overline{q}_{L,m}^{\alpha}} \ \gamma_\mu \ \underset{2}{q_{L,n}^{\beta}})(\underset{3}{\overline{q}_{L,m}^{\beta}} \ \gamma^\mu \ \underset{4}{q_{L,n}^{\alpha}}) = (\underset{1}{\overline{q}_{L,m}^{\alpha}} \ \gamma_\mu \ \underset{4}{q_{L,n}^{\alpha}})(\underset{3}{\overline{q}_{L,m}^{\beta}} \ \gamma^\mu \ \underset{2}{{q_{L,n}}^{\beta}}) \ , \qquad (4.55)$$

and the analogue relation for $L \leftrightarrow R$, as well as

$$(\underset{1}{\overline{q}^{\alpha}_{L,m}}\,\gamma_\mu\,\underset{2}{q^{\beta}_{L,n}})(\underset{3}{\overline{q}^{\beta}_{R,m}}\,\gamma^\mu\,\underset{4}{q^{\alpha}_{R,n}}) = -2(\underset{1}{\overline{q}^{\alpha}_{L,m}}\,\underset{4}{q^{\alpha}_{R,n}})(\underset{3}{\overline{q}^{\beta}_{R,m}}\,\underset{2}{q^{\beta}_{L,n}})\;, \tag{4.56}$$

where the greek indices $\{\alpha, \beta\}$ denote color, {m,n} indices denote flavor and the numbers enumerate the quarks to show which interchanges we made. After the Fierz transformations, we can calculate the Wilson Coefficients for the gluon current:

$$\begin{aligned}
C_{1,gl} =& \frac{g_s^2}{6M^2_{G^{(k)}}}(V^{Q,[k]}_{mn}V^{Q,[k]}_{kl})\;;\\
\tilde{C}_{1,gl} =& \frac{g_s^2}{6M^2_{G^{(k)}}}(V^{q,[k]}_{mn}V^{q,[k]}_{kl})\;;\\
C_{4,gl} =& -\frac{g_s^2}{M^2_{G^{(k)}}}V^{q,[k]}_{mn}V^{Q,[k]}_{kl}\;;\\
C_{5,gl} =& \frac{g_s^2}{3M^2_{G^{(k)}}}V^{q,[k]}_{mn}V^{Q,[k]}_{kl}\;.
\end{aligned} \tag{4.57}$$

For the Zdish and electromagnetic Wilson coefficients, eq. (4.52) cannot be used since the generators in the form of the Pauli matrices where used to formulate the $Z^{(k)}_\mu$ and $A^{(k)}_\mu$ field. Instead we have to think about the color structure of four fermion processes to simplify the expressions. We know that color is a feature of the $SU(3)_C$. The only vector bosons carrying color charge are gluons. Therefore every vertex in a γ-, $W^\pm$- or Z exchange process needs to be color neutral, so that e.g. the effective Hamiltonian corresponding to Zdish currents in eq. (4.36) becomes:

$$\mathcal{H}^Z_{pure} \subset \delta_{\alpha\epsilon}\delta_{\beta\eta}\left(\overline{q}^{\alpha}_{L,m}C^Z\gamma^\mu V^{Q,[k]}_{mn}q^{\epsilon}_{L,n}\right)\left(\overline{q}^{\beta}_{L,m}C^Z\gamma^\mu V^{Q,[k]}_{mn}q^{\eta}_{L,n}\right) + L \to R \text{ and } Q \to q \tag{4.58}$$

and

$$\mathcal{H}^Z_{mixed} \subset 2\delta_{\alpha\epsilon}\delta_{\beta\eta}\left(\overline{q}^{\alpha}_{L,m}C^Z\gamma^\mu V^{Q,[k]}_{mn}q^{\epsilon}_{L,n}\right)\left(\frac{Y\sin^2(\theta_w)}{2}\overline{q}^{\beta}_{R,m}V^{q,[k]}_{mn}q^{\eta}_{R,n}\right)\;. \tag{4.59}$$

The Kronecker deltas show how we have to contract the color indices. These can be read off from Fig. 4.1. The calculation using the Fierz identities eq. (4.55) and eq. (4.56) yield for the Zdish Wilson coefficients:

$$\begin{aligned}
C_{1,Z} =& \frac{g^2}{2\cos^2(\theta_W)M^2_{Z^{(k)}}}(C^Z_{q_iq_i})^2V^{Q,[k]}_{mn}V^{Q,[k]}_{kl}\;;\\
\tilde{C}_{1,Z} =& \frac{g^2}{2\cos^2(\theta_W)M^2_{Z^{(k)}}}\left(\frac{Y\sin^2(\theta_W)}{2}\right)^2V^{q,[k]}_{mn}V^{Q,[k]}_{kl}\;;\\
C_{4,Z} =& \,0\;;\\
C_{5,Z} =& -\frac{g^2}{\cos^2(\theta_W)M^2_{Z^{(k)}}}C^Z_{q_iq_i}\frac{Y\sin^2(\theta_W)}{2}V^{q,[k]}_{mn}V^{Q,[k]}_{kl}\;,
\end{aligned} \tag{4.60}$$

and in an analogous consideration for the electromagnetic Wilson coefficients:

$$
\begin{aligned}
C_{1,em} =& \frac{e^2}{2M_{A^{(k)}}^2}(C_{q_iq_i}^{em})^2 V_{mn}^{Q,[k]} V_{kl}^{Q,[k]} \ ; \\
\tilde{C}_{1,em} =& \frac{e^2}{2M_{A^{(k)}}^2}\left(\frac{Y}{2}\right)^2 V_{mn}^{q,[k]} V_{kl}^{q,[k]} \ ; \\
C_{4,em} =& \ 0 \ ; \\
C_{5,em} =& -\frac{e^2}{M_{A^{(k)}}^2} C_{q_iq_i}^{em} \frac{Y}{2} V_{mn}^{q,[k]} V_{kl}^{Q,[k]} \ .
\end{aligned}
\tag{4.61}
$$

We have now every contribution to the Wilson coefficient from the neutral currents. As we said before, we will only use the gluon coefficients since these have the largest contributions (see Tab. 4.1). If we look at the Wilson coefficients C_1 and $\tilde{C}_1$ in eq. (4.60) and eq. (4.61) we see that they all the same sign as in eq. (4.57). The constraints we will therefore get from the gluonic C_1 solely, neglecting the electromagnetic and Zdish contributions, are an upper bound on the BKLT parameters. Also, we notice that we have no additional contributions to the gluonic coefficients $C_{4,gl}$ since $C_{4,Z}$ and $C_{4,em}$ are zero.

5. Constraints on the BKLT and Implications for the Fermion KK Mass Spectrum

In the last section we calculated the analytic expressions for the neutral current Wilson coefficients C_i and how they depend on the overlap matrices $\mathcal{F}^{Q/q,[k]}$. In Tab. 5.1 are model-independent constraints for ΔF=2 operators on the corresponding Wilson coefficients (from [28]). These will be used to constrain the BKLT parameters and the KK masses of the fermion modes. A full study of the nUED parameter space would require a scan over the 3x3 eigenvalues of the overlap matrices $\mathcal{F}^{Q/q,[k]}$ as well as all possible orientations in flavor space of the BKLT matrices. This would go beyond the scope of this thesis. Instead, we focus on two special cases. This way, we determine the constraints in a generic setup of nUED as well as the minimal constraints which are still left in a maximally aligned setup.

We have to compute off-diagonal elements of the C_i matrix since we are interested in flavor changing processes (ΔF=2). A closer examination shows that we can extract an identity matrix which has only diagonal elements of the same magnitude $\mathcal{F}_{11}^{Q/q,[k]}$ and which does not contribute to the flavor changing neutral processes from the overlap factor matrix $\mathcal{F}^{Q/q,[k]}$. We also see that the BKLT parameters are correlated and only the differences between one BKLT parameter and the other two contribute. It does not make any difference which parameter $\mathcal{F}_{ii}^{Q/q,[k]}$ we choose:

$$\begin{aligned} V^{Q/q,[k]} =& A_q \mathcal{F}^{Q/q,[k]} A_q^\dagger = A_q \left(\begin{pmatrix} \mathcal{F}_{11}^{Q/q,[k]} & 0 & 0 \\ 0 & \mathcal{F}_{11}^{Q/q,[k]} & 0 \\ 0 & 0 & \mathcal{F}_{11}^{Q/q,[k]} \end{pmatrix} \right. \\ & \left. + \begin{pmatrix} 0 & 0 & 0 \\ 0 & \mathcal{F}_{22}^{Q/q,[k]} - \mathcal{F}_{11}^{Q/q,[k]} & 0 \\ 0 & 0 & \mathcal{F}_{33}^{Q/q,[k]} - \mathcal{F}_{11}^{Q/q,[k]} \end{pmatrix} \right) A_q^\dagger \\ =& \mathcal{F}_{11}^{Q/q,[k]} \mathbb{1}_{3\times3} + A_q \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta\mathcal{F}_{22}^{Q/q,[k]} & 0 \\ 0 & 0 & \Delta\mathcal{F}_{33}^{Q/q,[k]} \end{pmatrix} A_q^\dagger \,, \end{aligned} \tag{5.1}$$

Parameter	95% allowed range (TeV^{-2})	Parameter	95% allowed range (TeV^{-2})
ReC_K^1	$[-9.6, 9.6] \cdot 10^{-7}$	$\lvert C_{B_d}^1 \rvert$	$< 2.3 \cdot 10^{-5}$
ReC_K^2	$[-1.8, 1.9] \cdot 10^{-8}$	$\lvert C_{B_d}^2 \rvert$	$< 7.2 \cdot 10^{-7}$
ReC_K^3	$[-6.0, 5.6] \cdot 10^{-8}$	$\lvert C_{B_d}^3 \rvert$	$< 2.8 \cdot 10^{-6}$
ReC_K^4	$[-3.6, 3.6] \cdot 10^{-9}$	$\lvert C_{B_d}^4 \rvert$	$< 2.1 \cdot 10^{-7}$
ReC_K^5	$[-1.0, 1.0] \cdot 10^{-8}$	$\lvert C_{B_d}^5 \rvert$	$< 6.0 \cdot 10^{-7}$
ImC_K^1	$[-4.4, 2.8] \cdot 10^{-9}$	$\lvert C_{B_s}^1 \rvert$	$< 1.1 \cdot 10^{-3}$
ImC_K^2	$[-5.1, 9.3] \cdot 10^{-11}$	$\lvert C_{B_s}^2 \rvert$	$< 5.6 \cdot 10^{-5}$
ImC_K^3	$[-3.1, 1.7] \cdot 10^{-10}$	$\lvert C_{B_s}^3 \rvert$	$< 2.1 \cdot 10^{-4}$
ImC_K^4	$[-1.8, 0.9] \cdot 10^{-11}$	$\lvert C_{B_s}^4 \rvert$	$< 1.6 \cdot 10^{-5}$
ImC_K^5	$[-5.2, 2.8] \cdot 10^{-11}$	$\lvert C_{B_s}^5 \rvert$	$< 4.5 \cdot 10^{-5}$
$\lvert C_D^1 \rvert$	$< 7.2 \cdot 10^{-7}$		
$\lvert C_D^2 \rvert$	$< 1.6 \cdot 10^{-7}$		
$\lvert C_D^3 \rvert$	$< 3.9 \cdot 10^{-6}$		
$\lvert C_D^4 \rvert$	$< 4.8 \cdot 10^{-8}$		
$\lvert C_D^5 \rvert$	$< 4.8 \cdot 10^{-7}$		

Table 5.1.: 95% probability range for C_i for arbitrary new physics flavor structure from [28].

where A_q is a transformation matrix S_q or T_q, respectively, introduced in eq. (4.20) and $\mathcal{F}^{Q/q,[k]}$ is the diagonal overlap factor matrix with:

$$\mathcal{F}_{ii}^{Q/q,[k]} = \sqrt{8}(-1)^{\frac{k}{2}} \frac{a_h^{\Psi_i \Psi_i}}{(2a_h^{\Psi_i \Psi_i} + \pi R)} \;, \tag{5.2}$$

from eq. (4.13c). We see that the only dependence on the mode number k lies in the factor $(-1)^{\frac{k}{2}}$ where k only can take even numbered values. So we can write $\mathcal{F}^{Q/q,[k]} = (-1)^{\frac{k}{2}} \mathcal{F}^{Q/q}$. The only mode number dependent quantity still left is the gauge boson mass $M^2_{A^{[k]}}$. Since we have no mode mixing, the Wilson coefficients contains a sum which can be analytically calculated

$$\sum_{k=1}^{\infty} \frac{(-1)^{\frac{2k}{2}}(-1)^{\frac{2k}{2}}}{M^2_{A^{(2k)}}} \approx \sum_{k=1}^{\infty} \frac{R^2}{(2k)^2} = 2^{-2}\zeta(2)R^2 = \frac{\pi^2}{24}R^2 \;, \tag{5.3}$$

where $\zeta(x)$ is the Riemann ζ function and $M_{A^{(2k)}} = \frac{2n}{R}$. As we know this now, we will drop all superscripts [k] for the overlap matrices $\mathcal{F}^{Q/q,[k]}$ and recall this when it is needed. If we look at FCNCs in the Kaon or D meson sector in Tab. 5.1, we need the matrix elements for $\overline{d}s$ and $\overline{u}c$ and thus we have to calculate the matrix elements $V_{12}^{Q/q}$. The restrictions on $\Delta\mathcal{F}_{ii}^{Q/q}$ will show that $\mathcal{F}_{11}^{Q/q}$ and $\mathcal{F}_{22}^{Q/q}$ are highly correlated, so that we can write $\mathcal{F}_{22}^{Q/q} = \mathcal{F}_{11}^{Q/q} + \Delta F^{Q/q}$ with $\Delta F^{Q/q}$ being a small quantity. So we obtain a matrix structure which looks like:

$$\begin{aligned}
\mathcal{F}^{Q/q} &= \begin{pmatrix} \mathcal{F}_{11}^{Q/q} & 0 & 0 \\ 0 & \mathcal{F}_{22}^{Q/q} & 0 \\ 0 & 0 & \mathcal{F}_{33}^{Q/q} \end{pmatrix} = \begin{pmatrix} \mathcal{F}_{11}^{Q/q} & 0 & 0 \\ 0 & \mathcal{F}_{11}^{Q/q} + \Delta F^{Q/q} & 0 \\ 0 & 0 & \mathcal{F}_{33}^{Q/q} \end{pmatrix} \\
&= -\sqrt{8} \begin{pmatrix} \frac{a_h^{\Psi_1\Psi_1}}{2a_h^{\Psi_1\Psi_1}+\pi R} & 0 & 0 \\ 0 & \frac{a_h^{\Psi_2\Psi_2}}{2a_h^{\Psi_2\Psi_2}+\pi R} & 0 \\ 0 & 0 & \frac{a_h^{\Psi_3\Psi_3}}{2a_h^{\Psi_3\Psi_3}+\pi R} \end{pmatrix} .
\end{aligned} \tag{5.4}$$

We take $\Delta F^{Q/q}$ and calculate the mass differences between the two fermion families. From eq. (5.4), we know that

$$\begin{aligned}
\Delta F^{Q/q} =& \sqrt{8}\frac{a_h^{\Psi_2\Psi_2}}{(2a_h^{\Psi_2\Psi_2}+\pi R)} - \sqrt{8}\frac{a_h^{\Psi_1\Psi_1}}{(2a_h^{\Psi_1\Psi_1}+\pi R)} \\
=& \sqrt{8}\frac{\Delta a \pi R}{(2a_h^{\Psi_1\Psi_1}+\pi R)(2a_h^{\Psi_1\Psi_1}+2\Delta a+\pi R)} \\
\approx& \sqrt{8}\frac{\Delta a \pi R}{(2a_h^{\Psi_1\Psi_1}+\pi R)^2} \\
& \rightarrow \Delta a(\Delta F^{Q/q}) = \frac{\Delta F^{Q/q}(2a_h^{\Psi_1\Psi_1}+\pi R)^2}{\sqrt{8}\pi R} ,
\end{aligned} \tag{5.5}$$

where the difference between the two BKLT parameter $\Delta a = a_h^{\Psi_2\Psi_2} - a_h^{\Psi_1\Psi_1}$ is assumed to be small. Fig. 5.1 shows the comparison between the numerically calculated solution for Δa and the approximation formula in eq. (5.5). This approximation is valid for our purposes.
The masses of the KK fermions with KK mode numbers n are determined by the quantization condition in Tab. 3.1. The mass difference between two fermion families Ψ_1 and Ψ_2 of an odd numbered KK mode corresponding to the BKLT parameter $a_h^{\Psi_1\Psi_1}$ and $a_h^{\Psi_2\Psi_2}$ can be determined via:

$$\begin{aligned}
\cot(\frac{\pi R}{2}m_1^{(n')}) - \cot(\frac{\pi R}{2}m_2^{(n')}) =& a_h^{\Psi_1\Psi_1}m_1^{(n')} - a_h^{\Psi_2\Psi_2}m_2^{(n')} \rightarrow \\
\cot(\frac{\pi R}{2}m_1^{(n')}) - \cot(\frac{\pi R}{2}(m_1^{(n')} - \Delta m)) =& a_h^{\Psi_1\Psi_1}m_1^{(n')} - (a_h^{\Psi_1\Psi_1} + \Delta a)(m_1^{(n')} - \Delta m) ,
\end{aligned} \tag{5.6}$$

where $n' \in \{1, 3, 5, \cdots\}$.
This equation will be solved numerically for Δm in order to obtain predictions for the mass degeneracy of the first KK quark excitation. We now want to analyze how the

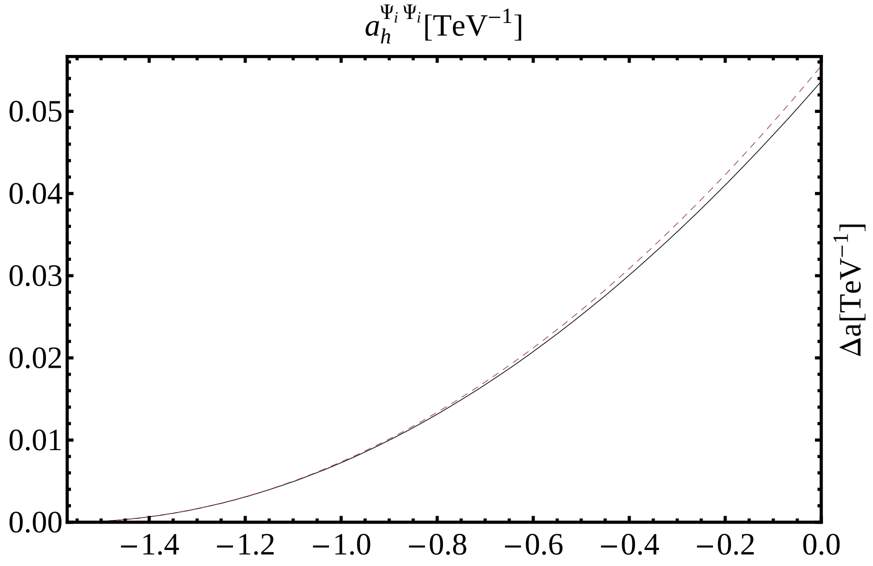


Figure 5.1.: Deviation between the numerical calculation for Δa (continuous line) and the approximated formula (dashed line) in eq. (5.5) with $R = 1\text{TeV}^{-1}$ and $\Delta F^{Q/q} = 0.05$. The deviation between these two is negligible.

constraints are on the BKLT parameters $a_h^{\Psi_i\Psi_i}$. A full analysis would require a variation over all the $a_{R/L}^{\{Q,U,D\}\{Q,U,D\}}$ parameters and the parameters which generate the $SU(3)$ matrices T_q and S_q defined in eq. (4.20). Then we have to find all allowed parameter regions which satisfy the bounds from Tab. 5.1 and reproduce the correct quark masses in the 4D effective theory and the CKM matrix in eq. (4.46). Instead we consider two different scenarios concerning the rotation matrices and derive the constraints on the quark mass spectrum:

Generic rotation matrices All entries in the rotation matrices T_q and S_q are of order one.

Aligned rotation matrices We choose the matrices in such a way, that we will get the weakest constraints from Tab. 5.1 on the mass matrices. These constraints are a prediction for the nUED quark mass spectrum which cannot be avoided by tuning the BKLT parameters $a_{R/L}^{\{Q,U,D\}\{Q,U,D\}}$.

5.1. Generic Rotation Matrices

One possibility to calculate constraints on flavor changing neutral currents in nUED models is to use generic rotation matrices T_q and S_q [30]. We want to estimate how either the KK mass of the second KK gluon mode or the overlap matrix elements $\Delta F^{Q,q}$

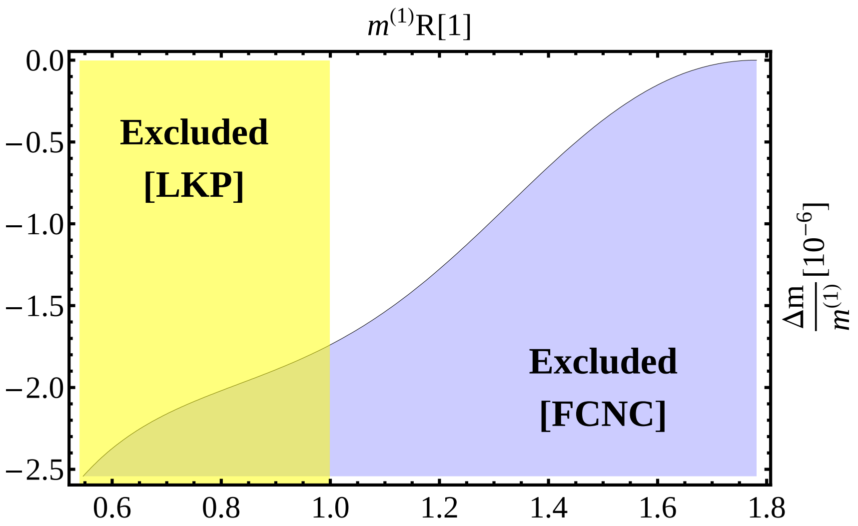


Figure 5.2.: Mass $m^{(1)}$ of the first fermion KK excitation times the compactification radius R vs. the mass degeneracy $\frac{\Delta m}{m^{(1)}}$ between two fermion families for generic matrices. In the region of interest, it is not possible to distinguish between the fermion families. The blue region is excluded because in this area, nUED incorporates too large C_K^4 parameter for generic matrices and the yellow region is excluded due to the demand of a non fermionic LKP.

are constrained in this scenario. Every element in T_q or S_q is assumed to be a complex number of order $\sim \mathcal{O}(1)$. This can be justified by saying that any $SU(3)$ rotation matrix consists of real entries $\sim$1 times a phase like it is realized in the CKM matrix in eq. (4.46). Now the Wilson coefficient $C_{4,gl}$ from eq. (4.57) can be calculated for this case using the constraint from Tab. 5.1 from the Kaon sector since we get the tightest bounds from C_K^4. $C_{4,gl}$ gets

$$|\mathrm{Im}[C_{4,gl}]| \approx 1.5\,\frac{\Delta\mathcal{F}^Q \Delta\mathcal{F}^q}{M_{G^{(2)}}^2} < 0.9 \cdot 10^{-11}\mathrm{TeV}^{-2} = \mathrm{Im}(C_K^4)\;, \tag{5.7}$$

where the numerical value consists of contributions from the transformation matrices and the couplings in the Wilson coefficients defined in Tab. 4.1. Since we used the gluon coefficient C_K^4, we have to make another assumption and set the BKLT for the right fermions equivalent to the left BKLT so that we can estimate the constraints, i.e. $a_L^{\Psi_i\Psi_i} = a_R^{\Psi_i\Psi_i}$. If we vary the right BKLT parameters and therefore the right overlap matrix $\mathcal{F}^q$ independently, we can relax the bounds by turning them down and, finally set them to zero and thus avoid any constraints from C_K^4. If we assume the overlap matrix differences $\Delta F_{ii}^{Q,q}$ are all of order $\mathcal{O}(1)$, we get a lower bound for the mass of the second KK excitation of the gluon to $M_{G^{(2)}} > 8.1 \cdot 10^5$ TeV and therefore a compactification

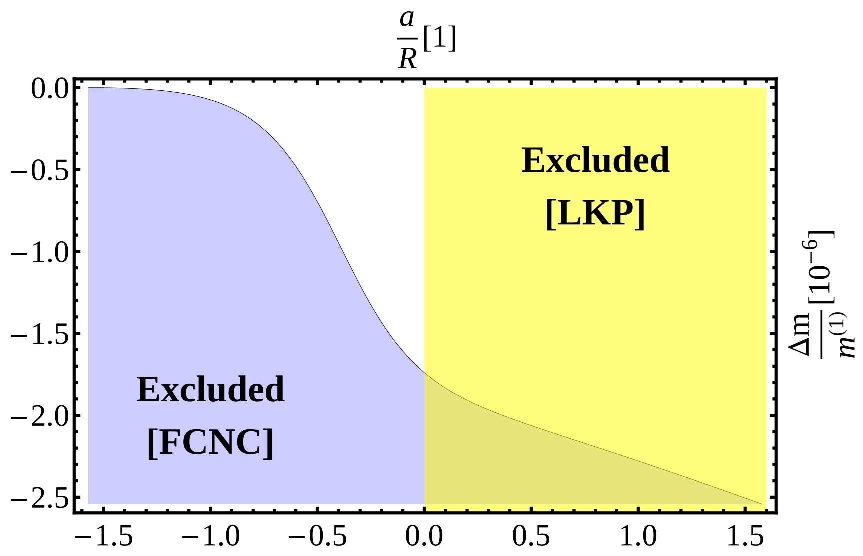


Figure 5.3.: BKLT parameter over compactification radius $\frac{a_h^{\Psi_i \Psi_i}}{R}$ vs. mass degeneracy $\frac{\Delta m}{m^{(1)}}$ between two fermion families for generic matrices. The blue region corresponds to a C_K^4 parameter which is larger than the experimental bound allows and the yellow region is excluded because the viable dark matter candidate would be a fermionic LKP.

radius $R^{-1} \sim 4.1 \cdot 10^5$TeV. These masses are not accessible since this would mean that all KK particles are extremely heavy and can not be detected in colliders like the LHC. The 2nd possibility to satisfy eq. (5.7) is to assume a 1TeV KK scale which then implies bounds on $\Delta F^{Q,q}$. We get $\Delta F^{Q,q} < 2.5 \cdot 10^{-6}$ which shows that we have highly degenerate overlap function eigenvalues.
Using the relation in eq. (5.3) to calculate the KK gluon tower mass, setting the compactification radius to 1TeV^{-1} and the overlap difference to $\Delta F^{Q,q} = 2.5 \cdot 10^{-6}$ from the estimation before, the mass degeneracy with eq. (5.6) can be plotted against the first KK excitation mass $m^{(1)}$ which is shown in Fig. 5.2. Fig. 5.3 shows the mass degeneracy vs. the BKLT parameter $a_h^{\Psi_i \Psi_i}$. In both figures, the blue region is excluded due to FCNC constraints and the yellow area for the demand for a lightest, non fermionic KK particle (LKP) which is a viable dark matter candidate (cf. chapter 3). The bound on the mass degeneracy between the fermion families is $\frac{\Delta m}{m^{(1)}} < 1.8 \cdot 10^{-6}$. This means regarding a compactification scale of $R = 1\text{TeV}^{-1}$ that:

- The masses of the left doublets are degenerate: $m^{Q(1)}_{u,d,c,s,t,b}$ are the same within $\frac{\Delta m^{Q(1)}_{u,d,c,b,t,b}}{m^{Q(1)}_{u,d,c,b,t,b}} < 1.8 \cdot 10^{-6}$ for $R = 1\text{TeV}^{-1}$ (see Fig. 5.3).

- The same holds for the right up-type singlets $m^{U^{(1)}}_{u,c,t}$ and the right down-type singlets $m^{D^{(1)}}_{d,s,b}$ that their masses are degenerate within $\frac{\Delta m^{U^{(1)}}_{u,c,t}}{m^{U^{(1)}}_{u,c,t}} < 1.8 \cdot 10^{-6}$ and $\frac{\Delta m^{D^{(1)}}_{d,s,b}}{m^{D^{(1)}}_{d,s,b}} < 1.8 \cdot 10^{-6}$ for $R = 1\text{TeV}^{-1}$ (see Fig. 5.2).

If we take a look again at Fig. 3.1, we see that the constraints on $a_h^{\Psi_i\Psi_i}$ are $-\frac{\pi R}{2} < a_h^{\Psi_i\Psi_i} < 0$ and for the quarks we get masses between $\frac{1}{R} < m^{Q^{(1)},U^{(1)},D^{(1)}} < \frac{2}{R}$ where the upper bound comes from the calculation of the KK-spectrum and the lower bound comes from the demand that we still want to have a non-fermionic LKP. In this scenario we have three sets of degenerate quark families:

- Six degenerate masses $m^{Q^{(1)}}$,
- three degenerate masses $m^{U^{(1)}}$ and
- three degenerate masses $m^{D^{(1)}}$.

It should be pointed out that with our analysis it is not possible to constrain the mass degeneracy between the doublet and singlet quarks i.e. we can make no prediction on the mass differences $m^{Q^{(1)}} - m^{U^{(1)}}$, $m^{Q^{(1)}} - m^{D^{(1)}}$ and $m^{U^{(1)}} - m^{D^{(1)}}$.

5.2. Aligned Rotation Matrices

Now a selective approach is used and we fix the matrices S_q and T_q by hand to avoid as many flavor constraints as possible and only use the relation between the CKM matrix and the transformations in eq. (4.45). To avoid every restriction from the down sector (see Tab. 5.1), we choose the transformation matrices for the left and right d-type quarks, S_d and T_d, as identity matrices $S_d = \mathbb{1}$ and $T_d = \mathbb{1}$ and also set right u-type rotation $T_u = \mathbb{1}$ so the constraints from operators containing right quarks and the Wilson coefficients containing the right handed BKLT parameter $a_R^{\Psi_i\Psi_i}$ like $\tilde{C}_1$, C_4 or C_5 are avoided. $C_{1,gl}$ is the only coefficient that contributes. From the charged sector, we have the condition that $S_u^\dagger \mathcal{F}^{Q,[k]} S_d = V_{ckm}$ (see eq. (4.46)) and therefore $S_u^\dagger \mathcal{F}^{[k]} = V_{ckm}$. Here, the overlap matrices induces corrections to the Standard Model CKM matrix but these are suppressed by $\propto (\frac{M_W}{M_{W^{(k)}}}\mathcal{F}^{Q,[k]})^2$. In this aligned scenario we set $S_u = V_{ckm}^\dagger$.
By using the constraints from Tab. 5.1 from the D meson sector, the following inequality must hold

$$|C_{1,gl}| \approx |\frac{g_s^2 \pi^2}{144} V_{12}^Q V_{12}^Q| < 7.2 \cdot 10^{-7}\text{TeV}^{-2} \ , \tag{5.8}$$

where g_s is defined in eq. (B.3) and the excitations of the gluon masses are substituted with eq. (5.3). $V_{12}^Q = V_{\bar{u}b}^Q$ is the overlap matrix defined in eq. (4.25) for the D^0 meson. The plot in Fig. 5.4 shows the allowed region for the two differences $\Delta\mathcal{F}_{22}^Q$ and $\Delta\mathcal{F}_{33}^Q$.

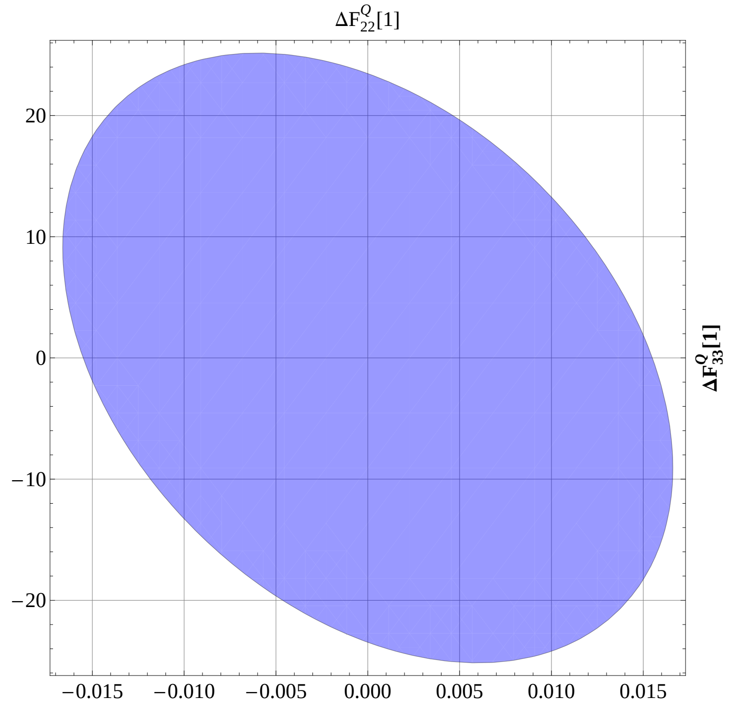


Figure 5.4.: Constraints on the differences $\Delta\mathcal{F}^Q_{22} = \mathcal{F}^Q_{22} - \mathcal{F}^Q_{11}$ and $\Delta\mathcal{F}^Q_{33} = \mathcal{F}^Q_{33} - \mathcal{F}^Q_{11}$ of the eigenvalues of the overlap matrix. The allowed region by the C^D_1 constraints is blue. We see that $\Delta\mathcal{F}^Q_{22}$ is of order $\mathcal{O}(0.01)$ in comparison to $\Delta\mathcal{F}^Q_{33}$, which shows only a small correlation between $\mathcal{F}^Q_{11}$ and $\mathcal{F}^Q_{33}$.

Two of the three overlap matrix elements must be the same up to a deviation in the order of $\mathcal{O}(0.01)$.
Fig. 5.5 shows the ratio $\frac{\Delta m}{m^{(1)}}$ which is a measure for the degeneracy between the first KK mode of the two quark families against $m^{(1)}R$ for different values of R.

Due to this alignment, the degeneracy between two of the three masses $m^{Q^{(1)}}$ can be reduced but not completely avoided. Additionally, the region $a_L^{\Psi_i\Psi_i} > 0$ in Fig. 5.5 corresponding to KK masses lighter than $m^{(1)} = 1$TeV with the largest allowed mass difference is ruled out, if the theory should contain a lightest, non fermionic dark matter candidate. The reason, that the degeneracy rises with higher masses is due to the zero mode of the fermions which are defined in eq. (3.19). The zero mode of the quark which is not zero contains the term $(2a_L^{\Psi_i\Psi_i} + \pi R)^{-\frac{1}{2}}$. This term mainly defines the properties of Δa in eq. (5.5) and therefore in the determination of Δm in eq. (5.6). In the limit for $a_h^{\Psi_i\Psi_i}$ to $\frac{\pi R}{2}$, we see that Δa is zero and eq. (5.6) simplifies to:

$$\cot(\tilde{m} - \Delta\tilde{m}) - \cot(\tilde{m}) = \Delta\tilde{m} \, , \tag{5.9}$$

where $\tilde{m} = \frac{\pi R}{2} m_1^{(n')}$. We see that for $\Delta\tilde{m} = 0$, this equation is fulfilled. This behavior

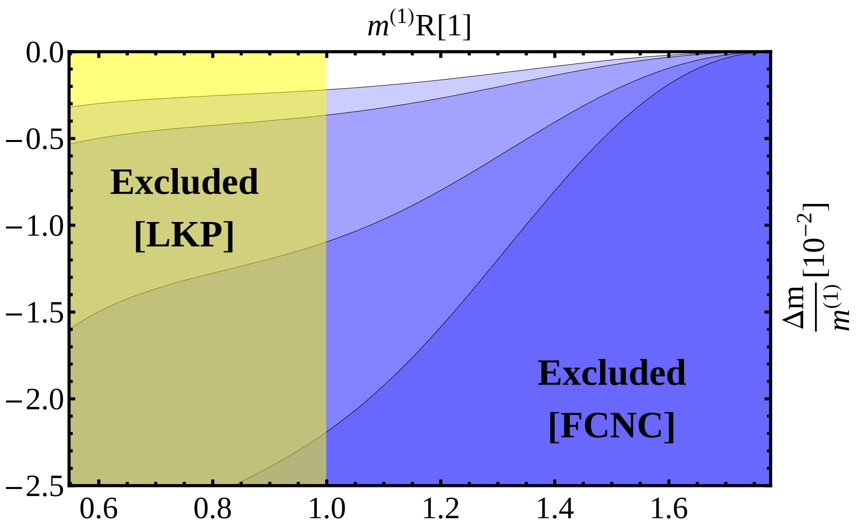


Figure 5.5.: First mode KK mass $m^{(1)}$ times R vs. the mass degeneracy $\frac{\Delta m}{m^{(1)}}$ plotted for different values of the compactification radius $R = \{0.5,1,3,5\}\text{TeV}^{-1}$ from bottom to top in the aligned scenario. The forbidden parameter space due to FCNC constraints for the respective R is blue colored. We see that for higher masses and also for a higher compactification radii R, the difference between the masses $m^{Q_1^{(1)}}$ and $m^{Q_2^{(1)}}$ of two fermion families vanishes. The yellow colored area contains fermionic LKP as possible dark matter candidates and thus is excluded.

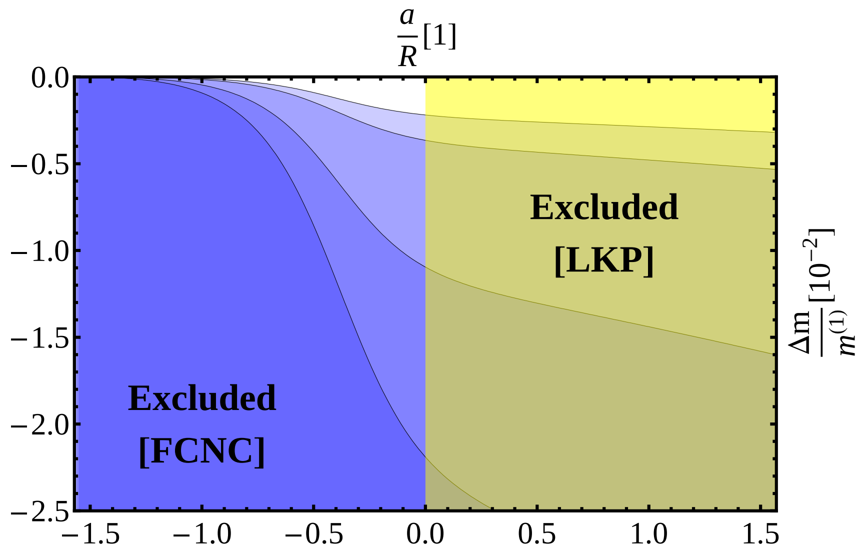


Figure 5.6.: BKLT parameter $a_h^{\Psi_i\Psi_i}$ over R vs. the mass degeneracy $\frac{\Delta m}{m^{(1)}}$ plotted for different values of the compactification radius $R = \{0.5,1,3,5\}\text{TeV}^{-1}$ from bottom to top in the aligned scenario. We see that for smaller BKLT parameters or respectively for a higher compactification radii, the difference between the masses $m^{Q_1^{(1)}}$ and $m^{Q_2^{(1)}}$ of two fermion families decreases. The forbidden parameter space due to FCNC constraints for the respective R is blue colored and because of the demand for a non fermionic LKP as a possible dark matter candidate is yellow colored.

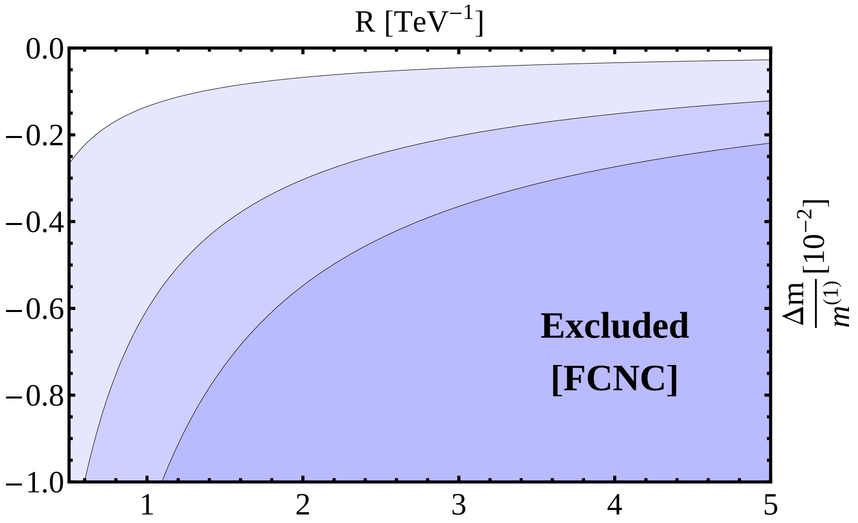


Figure 5.7.: The mass degeneracy $\frac{\Delta m}{m^{(1)}}$ plotted against the compactification Radius R for different values of the BKLT parameter $a_L^{\Psi_i\Psi_i}$. We see that the masses of two fermion families get more and more degenerate for higher compactification scales and smaller BKLT parameter. The values from top to bottom are $\frac{a_L^{\Psi_i\Psi_i}}{R} = \{\frac{-\pi}{4}, \frac{-\pi}{8}, 0\}$. The blue area is excluded since the flavor constraints from the Wilson coefficient C_D^1 are not fulfilled.

can again be seen in Fig. 5.6 where the mass degeneracy is plotted against the BKLT parameter $a_L^{\Psi_i\Psi_i}$ over R. The blue region is again excluded because of FCNC bounds and the yellow region because of the non fermionic LKP argument.
In Fig. 5.7, the mass degeneracy is plotted against the compactification radius R for different values of the BKLT parameter $a_L^{\Psi_i\Psi_j}$. We see two things: on the one hand, that for smaller BKLT parameter $\frac{\Delta m}{m}$ gets smaller, which means the masses are getting more and more degenerate. On the other hand, for higher compactification radii R the mass difference also becomes smaller and so the quark masses become more degenerate. The conclusion of the aligned scenario is, that we can avoid all constraints we get from the overlap matrices $\mathcal{F}^U$ and $\mathcal{F}^D$ by choosing T_u and T_d to be the identity matrix $\mathbb{1}$. The masses of the particles are again located between $\frac{1}{R} < m^{U^{(1)},D^{(1)}} < \frac{2}{R}$ but we can make no statements about the mass differences of the six right singlet masses $m^{U^{(1)}}$ and $m^{D^{(1)}}$. However, we can only set one of the left transformations S_u and S_d to be the identity matrix $\mathbb{1}$ since the CKM matrix correlates these two (see eq. (4.46)). Due to this relation, we get bounds on the overlap matrix $\mathcal{F}^Q$. Fig. 5.4 shows that $\Delta\mathcal{F}_{33}^Q$ is only weakly constrained while the maximal value for $\Delta\mathcal{F}_{22}^Q$ can only lie between ± 0.015. This means for the quark masses of the first KK-excitation that $m_b^{Q(1)}$ and $m_t^{Q(1)}$ are degenerate and due to the bounds on $\mathcal{F}_{22}^L$ the four masses $m_u^{Q(1)}$,$m_d^{Q(1)}$,$m_c^{Q(1)}$

and $m_s^{Q(1)}$ are degenerate with $\frac{\Delta m_{u,d,c,s}^{Q(1)}}{m_{u,d,c,s}^{Q(1)}} < 0.011$ for $R = 1\text{TeV}^{-1}$(see Fig. 5.6) where all the masses are again between $\frac{1}{R} < m^{Q^{(1)}} < \frac{2}{R}$. In contrast to the generic case, there is no requirement that the masses $m_{t,b}^{Q(1)}$ and $m_{u,d,c,s}^{Q(1)}$ are degenerate.
Another approach for calculating the flavor constraints on this model would be to use randomly generated transformation matrices for all the transformations S_q and T_q in eq. (4.20) and take a look at the restrictions they generate. Some attempts showed that parameters $\Delta\mathcal{F}_{ii}$ can be further constrained which seems plausible since we used an aligned approach with minimal flavor violation and specially chosen transformations matrices. For a proper and complete analysis we would need a complete statistical examination of a 6 dimensional parameter space (two BKLT parameter differences for one left doublet and two right singlet field transformations). Further investigations concerning random transformations go beyond the scope of this thesis.
After calculating the flavor restrictions on this nUED theory, we see that that we have two possibilities to lower the bounds on the BKLT parameters so that they do not violate flavor constraints:

- Alignment: If the matrix in eq. (5.1) is of the form $\mathcal{F}^Q = c \times \mathbb{1}_{3\times 3}$ and therefore having three maximally degenerate BKLT parameters $a_h^{\Psi_i\Psi_i}$, they cannot be constrained by this Wilson coefficient approach because they do not induce FCNCs via off-diagonal matrix elements. The extreme case would be setting all BKLT to zero and therefore getting a mUED model.

- Varying the compactification radius R: We can set the size of the extra dimension to 10^5TeV scale and therefore make the KK particles extremely heavy so that we will not detect any effects induced through a higher dimensional theory as discussed here.

Lastly we want to see if the Yukawa contributions to the first KK mode mass together with the contribution Δm of the overlap matrices $\mathcal{F}^{Q/q,[k]}$ are relevant and can lift the mass degeneracy. The transformations in eq. (4.20) were introduced to diagonalize the quark mass matrices and thus the 4D Yukawa matrices, so that e.g. the up sector Yukawa matrices is:

$$v_4 \lambda^{U,4\text{D}} = v_4 M_{diag} = v_4 \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} , \tag{5.10}$$

where the up-type quark masses are given in the appendix B in eq. (B.4). As shown in eq. (4.21) the relation between $\lambda^{U,5\text{D}}$ and $\lambda^{U,4\text{D}}$ is

$$v_4 \lambda^{U,4\text{D}} \equiv v_5 S_u \lambda^{U,5\text{D}} \mathcal{F}_{Y_u}^{[0]} T_u^\dagger \longrightarrow \lambda^{U,5\text{D}} = \frac{1}{\sqrt{\pi R}} S_u^\dagger \lambda^{U,4\text{D}} T_u \mathcal{F}_{Y_u}^{[0]^{-1}} . \tag{5.11}$$

We used the connection between the vacuum expectation value (vev) of the Higgs in 4D and 5D is

$$v_4 = \frac{v_5}{\sqrt{\pi R}} \,. \tag{5.12}$$

Using the bare quark masses as input to formulate the diagonal 4D Yukawa matrix, we can calculate the Yukawa matrices in 5D and derive the deviation of the KK masses with mode number n=1. A crucial point now is the chirality of the quarks with KK mode number $n > 0$. The Dirichlet boundary condition eq. (2.1) only removes one chiral zero mode. On higher KK levels, we have both, left and right handed $SU(2)$ doublets and also left and right handed $SU(2)$ singlets. This fact has to be respected when calculating the KK masses with mode number $n \geq 1$. The first KK excitation masses for the left and right handed up type quarks are:

$$\begin{aligned}\overline{u}_R^{(1)} M u_L^{(1)} =& (\overline{u}_R^{Q,(1)}, \overline{u}_R^{u,(1)}) \begin{pmatrix} M_1 & M_2 \\ M_3 & M_4 \end{pmatrix} \begin{pmatrix} u_L^{Q,(1)} \\ u_L^{u,(1)} \end{pmatrix} \\ =& \overline{u}_R^{Q,(1)} M_1 u_L^{Q,(1)} + \overline{u}_R^{Q,(1)} M_2 u_L^{u,(1)} + \overline{u}_R^{u,(1)} M_3 u_L^{Q,(1)} + \overline{u}_R^{u,(1)} M_4 u_L^{u,(1)} \,.\end{aligned} \tag{5.13}$$

Quarks with a Q superscript are the $SU(2)$ doublets whereas the quarks with a u superscript are $SU(2)$ singlets and the M_i are the different mass matrices. M_1 contains the first mode KK masses of the doublets and M_4 the first mode KK masses of the singlets. They are determined by the quantization condition in Tab. 3.1 and Tab. 3.2. These matrices are diagonal since the left and right quarks are forming a orthogonal basis coming from the KK decomposition eq. (3.28). The matrices M_2 and M_3 are determined by the Yukawa coupling. Those are not diagonal since there is no relation between the chiral $SU(2)$ singlets and doublets. They are determined by

$$M_2 = v_5 \lambda^{U,5\mathrm{D}} \mathcal{F}^{[101]}_{Q_R U_L} \quad \text{and} \quad M_3 = v_5 \lambda^{U,5\mathrm{D}} \mathcal{F}^{[101]}_{Q_L U_R} \,, \tag{5.14}$$

with the overlap factor matrices of the first KK mode quarks and the Higgs zero mode. The matrix M is not hermitian, so we will calculate the eigenvalues of the hermitian matrix $M\ M^\dagger$. The eigenvalues $m^{(1)}_{true,M\ M^\dagger}$ of $M\ M^\dagger$ are real, positive and approximate in a very good manner the squared eigenvalues $m^{(1)}_{true}$ of M

$$m^{(1)}_{true,\ MM^\dagger} \approx (m^{(1)}_{true})^2 \,. \tag{5.15}$$

We calculated the Yukawa contributions to the first KK mode masses numerically to see if it is possible to lift the degeneracy we get due to the flavor constraints in the previous section. First we introduce 3 BKLT parameter for the left handed quarks $a_L^{Q_{1,2,3}}$ and 3 for the right handed up-type quarks $a_R^{U_{1,2,3}}$. Since we know from previous considerations that $\mathcal{F}^Q_{11}$ and $\mathcal{F}^Q_{22}$ are highly correlated in the generic *and* the aligned scenario, we vary $a_L^{Q_1}$ from $-\frac{\pi R}{2}$ to 0 (which is the allowed parameter interval as explained in chapter 3) and calculate $a_L^{Q_2}$ with eq. (5.5). The remaining BKLT parameters are set to a fixed value near $-\frac{\pi R}{2}$. With these BKLT parameters we calculate the corresponding first mode KK masses $m^{Q^{(1)}_{1,2,3}}$ and $m^{U^{(1)}_{1,2,3}}$ of the $SU(2)$ doublets and singlets and their corresponding Yukawa contribution as shown in eq. (5.11) and eq. (5.14) and finally can reconstruct the mass matrix M in eq. (5.13). After diagonalizing the matrix $M\ M^\dagger$ and calculating the eigenvalues $m^{(1)}_{true}$ (see eq. (5.15)), we sort the masses of the first KK excitation without Yukawa contributions $m^{(1)}$ and the eigenvalues of M called here $m^{(1)}_{true}$ so we can assign the correct masses to their corresponding eigenvalues. With these results we were able to make the plots in Fig. 5.8-Fig. 5.11. The rotation matrices of the generic or the aligned scenario affect these calculation when we reconstruct the 5D Yukawa matrix in eq. (5.11) and the bounds from FCNCs affect the BKLT parameters $a_L^{Q_1}$ and $a_L^{Q_2}$ via eq. (5.5).

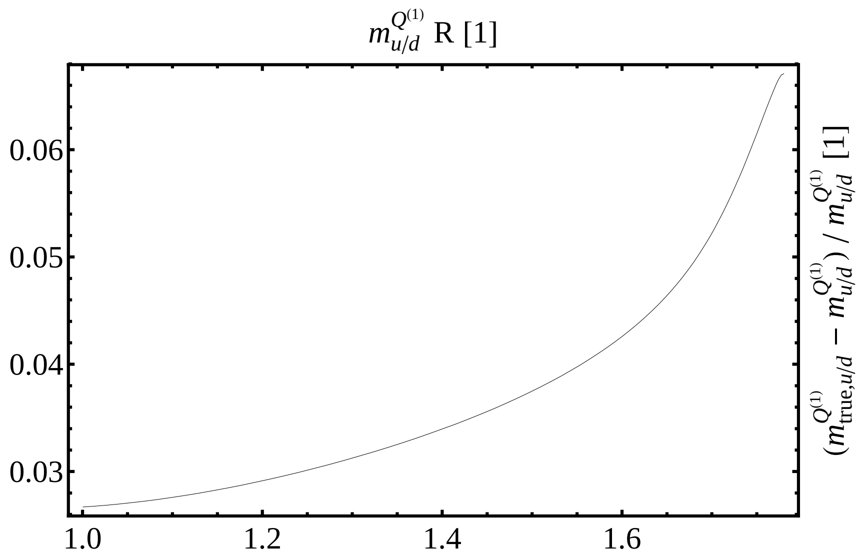


Figure 5.8.: Relative difference of the up-type mass $m^{Q^{(1)}}_{true,u/d}$ including Yukawa contributions and the first fermion KK mode mass $m^{Q^{(1)}}_{u/d}$ ignoring Yukawa contributions plotted against $m^{Q^{(1)}}_{u/d}R$ with compactification radius $R = 1\text{TeV}^{-1}$ for generic rotation matrices.

In the generic case we see that the Yukawa contribution to the uncorrected first KK mode mass of the up-type quarks $m^{Q^{(1)}}_{u/d}$ is about 3%-6% and that the contribution makes the quarks heavier as we expected. This is depicted in Fig. 5.8. More important is the degeneracy of fermions after the Yukawa correction which is shown in Fig. 5.9. The masses of the lightest left handed $SU(2)$ doublets $m^{Q^{(1)}}_{true,u/d}$ and $m^{Q^{(1)}}_{true,c/s}$ indeed differ

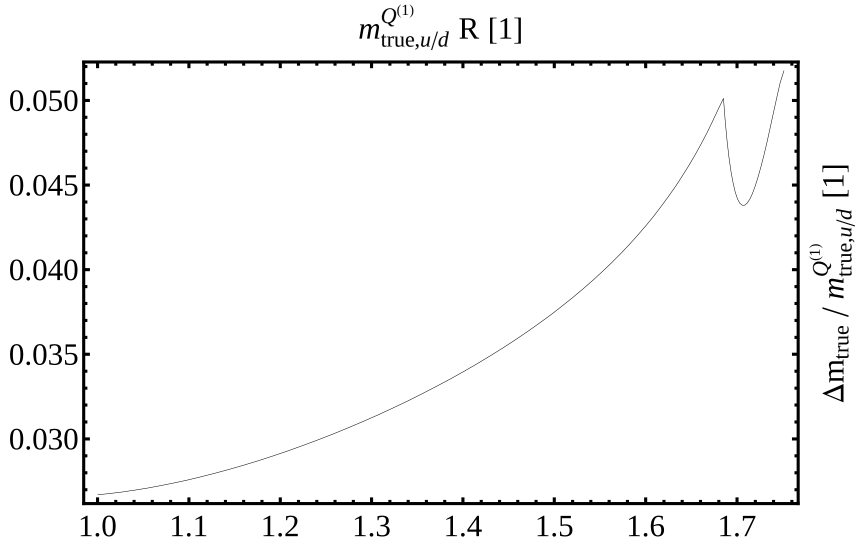


Figure 5.9.: Difference Δm_{true} between the first KK mode mass of left handed doublets $m^{Q^{(1)}}_{true,u/d}$ and $m^{Q^{(1)}}_{true,c/s}$ including Yukawa contributions vs. the Yukawa corrected mass $m^{Q^{(1)}}_{true,u/d}$ in the generic scenario. We see the degeneracy we get from the FCNC bounds cannot be lifted completely due to the Yukawa contributions.

more (up to 5%) than the pure first KK excitation masses $m^{Q^{(1)}}_{u/d}$ and $m^{Q^{(1)}}_{true,c/s}$ but still the degeneracy which is demanded by the FCNC bounds cannot be lifted completely.

In the aligned case a similar behavior as in the generic case can be observed, i.e. that the Yukawa contribution to the uncorrected first KK mode mass of the up-type quarks $m^{Q^{(1)}}_{u/d}$ are about 1%-5% as shown in Fig. 5.10. But in contrast to the generic case the degeneracy of fermion masses $m^{Q^{(1)}}_{true,u/d}$ and $m^{Q^{(1)}}_{true,c/s}$ after the Yukawa correction is smaller (up to 4%) and is of the same order as demanded by flavor bounds (up to 3% in Fig. 5.5). Again the degeneracy which is required by the FCNC constraints between the left handed $SU(2)$ doublet quarks cannot be lifted. Even if the mass difference due to the Yukawa corrections of two fermion families $m^{Q_1^{(1)}}$ and $m^{Q_2^{(1)}}$ is significantly bigger in the generic case than imposed via flavor constraints, it is in both scenarios, generic and aligned, not possible to lift the mass degeneracy completely.

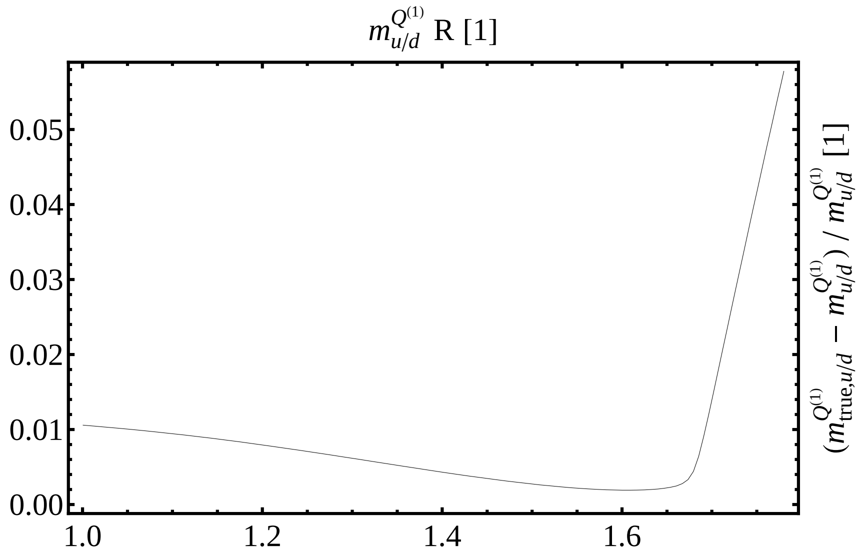


Figure 5.10.: The plot shows the relative difference of the left handed up-type mass $m_{true,u/d}^{Q^{(1)}}$ including Yukawa contributions and the first fermion KK mode mass $m_{u/d}^{Q^{(1)}}$ ignoring Yukawa contributions plotted against $m_{u/d}^{Q^{(1)}} R$ with compactification radius $R = 1\text{TeV}^{-1}$. The Yukawa correction to $m_{u/d}^{Q^{(1)}}$ in this aligned scenario is about the same order as in the generic case.

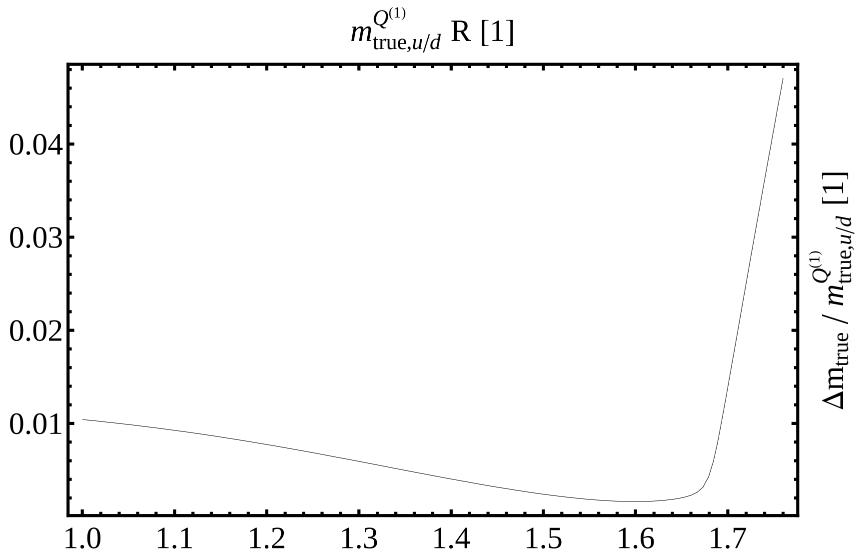


Figure 5.11.: Difference Δm_{true} between the first KK mode mass of left handed doublets $m^{Q^{(1)}}_{true,u/d}$ and $m^{Q^{(1)}}_{true,c/s}$ including Yukawa contributions vs. the Yukawa corrected mass $m^{Q^{(1)}}_{true,u/d}$ in the aligned scenario. We see that the degeneracy in the Yukawa sector is of the same order as the one we get from the FCNC bounds in this aligned scenario.

6. Conclusion

In this thesis we explored the nUED model with boundary kinetic localized terms (BKLT) for the fermions. These BKLT introduced an additional set of parameters $a_h^{\Psi_i\Psi_j}$ (see eq. (3.4)). In chapter 3 the KK decomposition of the fermionic Lagrangian for one family was performed and their wave functions were calculated (Tab. 3.1). One special feature of the fermion sector is that a chiral zero mode is needed which was accomplished by using a Dirichlet condition at the orbifold fixed point of the $S^1/\mathbb{Z}_2$-orbifold. The quantization condition in eq. (3.26c) and in eq. (3.27c) made it possible to calculate the mass spectrum of the fermions. At this point we were able to put first constraints on the BKLT parameters $a_h^{\Psi_i\Psi_j}$ since our dark matter candidate is required to be electrically neutral and colorless which is not possible with fermions lighter than $\frac{n}{R}$ which implied $a_h^{\Psi_i\Psi_j} > 0$. We also saw that using BKLT, a non standard calculation scheme was needed which was introduced in [21] and for the first time was applied to an elaborate treatment of boundary conditions in UED model.
After performing the fermion KK decomposition, we calculated in chapter 4 the 4D effective action and in particular determined the Wilson coefficients for four fermion interactions accounting all three quark families. First of all we integrated out all y-dependence in the Yukawa and the fermion Lagrangian in eq. (4.2b) and defined an overlap matrix $\mathcal{F}^{Q/q,[nkm]}$ which consists of the fermion wave functions and the gauge fields (see eq. (4.13c)). Going to the zero mode approximation we could simplify these overlap matrices to $\mathcal{F}^{Q/q,[0k0]}$ where only the even numbered gauge modes contributed since all other contributions are suppressed or give no rise to flavor violation. Then we made a basis transformation in which the Yukawa matrices in eq. (4.2b) are diagonal. These transformation matrices defined in eq. (4.20) do not commute with the overlap matrices $\mathcal{F}^{Q/q,[0k0]}$ and are a source for FCNCs. Finally in chp. 4.3 we calculated all Wilson coefficients for the neutral currents which are listed in eq. (4.57), eq. (4.60) and eq. (4.61).
As an application we constrained the off-diagonal elements of the Wilson coefficients which induce flavor violation by using model-independent bounds on the Wilson coefficients for ΔF=2 operators from [28] (see Tab. 5.1). To utilize these data Fierz identities were needed (eq. (4.55) and eq. (4.56)) to bring the four fermion operators into a form comparable to the operator basis in [28]. In addition to the BKLT parameters $a_h^{\Psi_i\Psi_j}$ and the mass spectrum of the quarks which we wanted to constrain, we needed the $SU(3)$ flavor transformation matrices T_q and S_q which parametrizes the relative alignment of the BKLT parameters $a_h^{\Psi_i\Psi_j}$ to the 5D Yukawa couplings $\lambda_{U,D}$. Since an analysis of the

full parameter space of the alignments is beyond the scope of this work, we focused on two sample cases, called the generic and the aligned scenario. Additionally, the mass difference between two fermions, which depends on how the BKLT parameter are chosen, was used to calculate the degree of the mass degeneracy.
In the generic ansatz in chapter. 5.1, generic rotation matrices were used for the basis rotation where all entries of the matrices were set to 1. It was shown that we can only make statements on the mass differences of the KK quarks within one $SU(2)$ multiplet. The six quark masses in the left doublet $m_i^{Q^{(1)}}$ and both of the three masses of the right up-type and down-type singlets $m^{(U^{(1)},D^{(1)})}$ are highly degenerate within $\frac{\Delta m^{Q^{(1)}}_{u,d,c,s,t,b}}{m^{Q^{(1)}}_{u,d,c,s,t,b}} < 1.8 \cdot 10^{-5}$ and $\frac{\Delta m^{(U;D)^{(1)}}_{(u,c,t);(d,s,b)}}{m^{(U;D)^{(1)}}_{(u,c,t);(d,s,b)}} < 1.8 \cdot 10^{-5}$. However, our investigation of flavor does not imply constraints on the mass differences $m^{Q^{(1)}} - m^{U^{(1)}}$, $m^{Q^{(1)}} - m^{D^{(1)}}$ and $m^{U^{(1)}} - m^{D^{(1)}}$.
In the aligned approach in chapter 5.2 we tried to avoid as many flavor constraints as possible by choosing the rotation matrices T_u, T_d and S_d as identity matrices to circumvent the constraints from the Kaon sector. The only constraint left was to restore the CKM matrix in eq. (4.46) in the effective 4D theory. This fact determines the last rotation matrix $S_u = V^{\dagger}_{CKM}$. For this case, it was found that the bounds on the overlap matrices and thus on the BKLT parameters $a_h^{\Psi_i \Psi_j}$ were not as severe as in the generic case. Some, but not all of the mass degeneracies could be removed. The bounds on the right handed up-type and down-type quarks vanished and no degeneracy between $m_i^{U^{(1)}}$ and $m_i^{D^{(1)}}$ is required anymore. One degeneracy in the left handed quarks can be lifted, but still the four remaining quark masses $m_u^{Q^{(1)}}$, $m_d^{Q^{(1)}}$, $m_c^{Q^{(1)}}$ and $m_s^{Q^{(1)}}$ are close to degenerate within $\frac{\Delta m^{Q(1)}_{u,d,c,s}}{m^{Q(1)}_{u,d,c,s}} < 0.011$. In both, the generic and the aligned case the fermion masses of the first KK excitation have to lie in between $\frac{1}{R} < m^{Q^{(1)},U^{(1)},D^{(1)}} < \frac{2}{R}$.
Also we calculated the contribution from the Yukawa sector to the masses of the first Kaluza-Klein mode and saw, that in both scenarios the addition to the pure first mode KK masses are not sufficient to lift the degeneracy (Fig. 5.9 and Fig. 5.11).
We conclude, that flavor constraints force the nUED quark spectrum to contain mass degenerate state (typically in sets of 6, 3 and 3 states and with maximal aligning still 4 states) even when BKLTs are included in the UED theory.

A. Sturm-Liouville Theory

The Sturm-Liouville (SL) differential equation in an interval [a,b] is

$$-\frac{d}{dx}\left(p(x)\frac{dy}{dx}(x)\right)+q(x)y(x)=\lambda w(x)y(x)\ , \tag{A.1}$$

$$\text{boundary value conditions}\ \left\{\begin{array}{c}\alpha_1 y(a)+\alpha_2 y'(a)=0\\ \beta_1 y(b)+\beta_2 y'(b)=0\end{array}\right.\ ,$$

with $p(x)>0$, $p'(x)>0$, the weight function $w(x)>0$ and a,b,α_i and $\beta_i\in\mathbb{R}$. SL theory now tries to generalize and examine the properties of the solution to this differential equation. Introducing the operator

$$L\cdot y(x)=\left(\frac{dp}{dx}(x)\frac{d}{dx}+p(x)\frac{d^2}{dx^2}+q(x)\right)y(x)\ , \tag{A.2}$$

you can write the SL equation as an eigenvalue problem:

$$\frac{1}{w(x)}L\cdot y(x)=\lambda_n\ y(x)\ , \tag{A.3}$$

where λ_n is the nth eigenvalue of the operator $\frac{1}{w(x)}L$. The operator L is self adjoint $(L[u],v)=(u,L[v])$, if the so called Lagrange-condition is satisfied, i.e. that the boundary terms that occur in the partial integration vanish. This is always the case when the boundary value conditions are chosen as in eq.(A.1). A direct consequence is that all eigenvalues λ_n are real[1].

Two important theorems are:

Theorem A.0.1 *Every eigenvalue λ_n has a unique eigenfunction $\phi_n(x)$ i.e. there are no degenerate eigenfunctions to one specific eigenvalue.*

The proof will be omitted and can be found in the corresponding mathematical literature as in [31].

Theorem A.0.2 *All eigenfunctions ϕ_n form an orthonormal basis in a Hilbert Space L^2 under the following scalar product:*

$$(\phi_m,\phi_n)=\int_a^b \mathrm{d}x\phi_m\phi_n w(x)=\delta_{m,n}\ , \tag{A.4}$$

[1] It can be shown that the eigenfunctions are also real.

where w(x) is again the weight function of the scalar product. Given two eigenfunctions ϕ_i and ϕ_j and two non-equal eigenvalues λ_i and λ_j, we can write:

$$(L[\phi_i], \phi_j) - (\phi_i, L[\phi_j]) = 0 = (\lambda_i w(x)\phi_i, \phi_j) - (\phi_i, \lambda_j w(x)\phi_j) = (\lambda_i - \lambda_j)(w(x)\phi_i, \phi_j) ,$$

using the self-adjoint operator L from eq.(A.2) and that $w(x) > 0$. Setting $(\phi_i, \phi_i) = 1$, the theorem is proven.

The relevance in the physics of extra dimensions is, that the differential equation of the 5D wave functions will always correspond to a Sturm-Liouville problem. The Dirichlet condition we choose in eq. (2.1) corresponds to a boundary value condition with $\beta_2 = \alpha_2 = 0$, $\beta_1 = \alpha_1 = 1$ and $a = -b = \frac{\pi R}{2}$. The weight function w(x) are the Dirac-Delta functions $b(y) = \left(\delta(y - \frac{\pi R}{2}) + \delta(y + \frac{\pi R}{2})\right)$. Thus we can ensure that the wave functions and eigenvalues which are the masses of the fermions always exist, are always non-degenerate and form an orthonormal basis so that the Standard Model 4D Lagrangian can be restored.

B. Input Parameters

This appendix lists the numerical values of the input parameters used in this thesis to calculate the Wilson coefficients in chapter 4 and both the FCNC constraints and the Yukawa corrections to the first excitation of the KK mass of the fermions $m^{(1)}$. All the values are taken from the Particle Data Group [27].
The values for the Wolfenstein parameter of the CKM matrix are:

$$\lambda = 0.2257^{+0.0009}_{-0.0010} \; ; \qquad A = 0.814^{+0.021}_{-0.022} \; ; \tag{B.1}$$

$$\rho = 0.135^{+0.031}_{-0.016} \; ; \qquad \eta = 0.349^{+0.015}_{-0.017} \; . \tag{B.2}$$

To calculate flavor constraints on the gluon Wilson coefficient, we need the numerical value for g_s (see Tab. 4.1). The constants we need are

$$\alpha_s(m_Z) = 0.1184 \; , \quad \alpha = 0.007297 \quad \text{and} \quad \sin^2(\Theta_W(m_Z)) = 0.2312 \; , \tag{B.3}$$

where $\alpha_s(m_Z)$ and $\sin^2(\Theta_W(m_Z))$ are measured at Z-pole.
In chapter 5 we determine the Yukawa contributions to the first KK excitation of the fermion mass. To do this, we need the up-type quark masses:

$$m_u = 1.7 \cdot 10^{-6} \text{ TeV} \; , \quad m_c = 1.3 \cdot 10^{-3} \text{ TeV} \quad \text{and} \quad m_t = 0.17 \cdot \text{TeV} \; . \tag{B.4}$$

Bibliography

[1] C. Burgess and G. Moore, *The Standard Model: a Primer*. Cambridge University Press, 2007.

[2] T. Kaluza, „Über das Vereinheitslichungsproblem der Physik“, *Sitzungsber.d.Preuss.Akad.d.Wiss.Berlin* no. 966, (1921) .

[3] O. Klein, „Quantentheorie und fünfdimensionale Relativitätstheorie“, *Zeitschrift f. Physik* (1926) .

[4] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, „The Hierarchy problem and new dimensions at a millimeter“, *Phys.Lett.* **B429** (1998) 263–272, `arXiv:hep-ph/9803315 [hep-ph]`.

[5] E. Adelberger, B. R. Heckel, and A. Nelson, „Tests of the gravitational inverse square law“, *Ann.Rev.Nucl.Part.Sci.* **53** (2003) 77–121, `arXiv:hep-ph/0307284 [hep-ph]`.

[6] L. Randall and R. Sundrum, „A Large mass hierarchy from a small extra dimension“, *Phys.Rev.Lett.* **83** (1999) 3370–3373, `arXiv:hep-ph/9905221 [hep-ph]`.

[7] T. Appelquist, H.-C. Cheng, and B. A. Dobrescu, „Bounds on universal extra dimensions“, *Phys.Rev.* **D64** (2001) 035002, `arXiv:hep-ph/0012100 [hep-ph]`.

[8] D. Hooper and S. Profumo, „Dark matter and collider phenomenology of universal extra dimensions“, *Phys.Rept.* **453** (2007) 29–115, `arXiv:hep-ph/0701197 [hep-ph]`.

[9] A. Datta, K. Kong, and K. T. Matchev, „Minimal Universal Extra Dimensions in CalcHEP/CompHEP“, *New J.Phys.* **12** (2010) 075017, `arXiv:1002.4624 [hep-ph]`.

[10] H.-C. Cheng, K. T. Matchev, and M. Schmaltz, „Bosonic supersymmetry? Getting fooled at the CERN LHC“, *Phys.Rev.* **D66** (2002) 056006, `arXiv:hep-ph/0205314 [hep-ph]`.

[11] K. Kong, S. C. Park, and T. G. Rizzo, „A vector-like fourth generation with a discrete symmetry from Split-UED“, *JHEP* **1007** (2010) 059, `arXiv:1004.4635 [hep-ph]`.

[12] H. Georgi, A. K. Grant, and G. Hailu, „Brane couplings from bulk loops“, *Phys.Lett.* **B506** (2001) 207–214, `arXiv:hep-ph/0012379 [hep-ph]`.

[13] T. Flacke, A. Menon, and D. J. Phalen, „Non-minimal universal extra dimensions“, *Phys.Rev.* **D79** (2009) 056009, `arXiv:0811.1598 [hep-ph]`.

[14] A. Mück, A. Pilaftsis, and R. Rückl, „Minimal higher dimensional extensions of the standard model and electroweak observables“, *Phys.Rev.* **D65** (2002) 085037, `arXiv:hep-ph/0110391 [hep-ph]`.

[15] T. Rizzo, „Introduction to Extra Dimensions“, `arXiv:1003.1698v1 [hep-ph]`.

[16] R. Sundrum, „Tasi 2004 lectures: To the fifth dimension and back“, `arXiv:hep-th/0508134 [hep-th]`.

[17] H.-C. Cheng, K. T. Matchev, and M. Schmaltz, „Radiative corrections to Kaluza-Klein masses“, *Phys.Rev.* **D66** (2002) 036005, `arXiv:hep-ph/0204342 [hep-ph]`.

[18] M. E. Peskin and D. V. Schroeder, *Introduction to Quantum Field Theory.* Addison-Wesley Publishing Company, 1995.

[19] A. J. Buras, M. Spranger, and A. Weiler, „The Impact of universal extra dimensions on the unitarity triangle and rare K and B decays“, *Nucl.Phys.* **B660** (2003) 225–268, `arXiv:hep-ph/0212143 [hep-ph]`.

[20] J. Polchinski, *String Theory, Vol.2: Superstring Theory and Beyond.* Cambridge University Press, 1998.

[21] C. Csaki, C. Grojean, J. Hubisz, Y. Shirman, and J. Terning, „Fermions on an interval: Quark and lepton masses without a Higgs“, *Phys.Rev.* **D70** (2004) 015012, `arXiv:hep-ph/0310355 [hep-ph]`.

[22] G. Servant and T. M. Tait, „Is the lightest Kaluza-Klein particle a viable dark matter candidate?“, *Nucl.Phys.* **B650** (2003) 391–419, `arXiv:hep-ph/0206071 [hep-ph]`.

[23] T.-P. Cheng and L.-F. Li, *Gauge theory of elementary particle physics.* Oxford University Press, 2006.

[24] W. Greiner and B. Müller, *Gauge Theory of Weak Interactions.* Springer Verlag, 2009.

[25] M. Bauer, S. Casagrande, U. Haisch, and M. Neubert, „Flavor Physics in the Randall-Sundrum Model: II. Tree-Level Weak-Interaction Processes“, *JHEP* **1009** (2010) 017, `arXiv:0912.1625 [hep-ph]`.

[26] M. Bauer, S. Casagrande, L. Grunder, U. Haisch, and M. Neubert, „Little Randall-Sundrum models: epsilon(K) strikes again", *Phys.Rev.* **D79** (2009) 076001, `arXiv:0811.3678 [hep-ph]`.

[27] **Particle Data Group** Collaboration, K. Nakamura *et al.*, „Review of particle physics", *J.Phys.G* **G37** (2010) 075021.

[28] **UTfit Collaboration** Collaboration, M. Bona *et al.*, „Model-independent constraints on Δ F=2 operators and the scale of new physics", *JHEP* **0803** (2008) 049, `arXiv:0707.0636 [hep-ph]`.

[29] J. F. Nieves and P. B. Pal, „Generalized Fierz identities", *Am.J.Phys.* **72** (2004) 1100–1108, `arXiv:hep-ph/0306087 [hep-ph]`.

[30] C. Csaki, J. Heinonen, J. Hubisz, S. C. Park, and J. Shu, „5D UED: Flat and Flavorless", *JHEP* **1101** (2011) 089, `arXiv:1007.0025 [hep-ph]`.

[31] W. Boyce, R. DiPrima, and D. Mitrea, *Elementary differential equations and boundary value problems.* Wiley New York, 1992.

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Erklärung

Gemäß der allgemeinen Studien- und Prüfungsordung für den Diplomstudiengang Physik an der Julius-Maximilians-Universität Würzburg erkläre ich hiermit, dass ich diese Arbeit selbstständig verfasst, keine anderen als die angegebenen Quellen und Hilfsmittel benutzt habe und die Arbeit bisher keiner anderen Prüfungsbehörde unter Erlangung eines akademischen Grades vorgelegt habe.

Würzburg, den 21. Mai 2011

Daniel Gerstenlauer