Diplomarbeit

Electroweak Constraints on split Universal Extra Dimensions

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Zusammenfassung

In dieser Arbeit beschäftigen wir uns mit dem Modell mit split Universal Extra Dimensions (sUED). Nach einer kurzen Einführung in die grundlegenden Konzepte von Modellen mit Universellen Extradimensionen und des sUED Modells wenden wir uns zunächst dem Fermionensektor zu. Ausgehend von der Lagrangedichte leiten wir die Bewegungsgleichungen ab, lösen diese und bestimmen das Fermionen-Kaluza-Klein-Massenspektrum. Im Anschluss werden Vertizes zwischen zwei Nullmoden Fermionen und n-Moden Eichbosonen betrachtet. Aufgrund der veränderten Form der Fermion-Nullmoden erhalten wir in der effektiven Lagrangedichte modifizierte effektive Kopplungen mit den darin enthaltenen Überlappintegralen. Diese Überlappintegrale sind dabei Funktionen des 5D Fermion-Massenparameters μ, welcher in sUED eingeführt wird, sowie des (inversen) Kompaktifizierungsradius R^{-1}. Um nun abschließend die theoretischen Vorhersagen mit den aktuellen Messdaten zu vergleichen, werden - ausgehend von den Peskin-Takeuchi Parametern - die effektiven S, T und U Parameter eingeführt. Diese berücksichtigen neben den Korrekturen der Eichbosonen-Selbstenergien auch die Vertexkorrekturen, die im sUED Modell nicht vernachlässigbar sind. Der Vergleich mit den elektroschwachen Messdaten liefert somit Beschränkungen auf den μ-R^{-1} Parameterraum des sUED Modells. Die hier berechneten Beschränkungen sind stärker als die bisher in der Literatur bekannten [1], und schließen einen Großteil des Parameterraumes aus, für den sUED in Di-Lepton-Signalen am LHC nachweisbar sind [1].

Abstract

In this thesis we concentrate on the model with split Universal Extra Dimensions (sUED). After a short introduction of the basic concepts of models with Universal Extra Dimensions and the description of the sUED Model we focus on the fermion sector. Starting from the Lagrangian we derive the equations of motion and their solutions. Afterwards we calculate the fermion Kaluza-Klein mass spectrum. Because of the non-flat profile of the fermion zero modes in the extra dimension, the vertices between two zero mode fermions and n-mode gauge bosons obtain a modified effective coupling constant due to the overlap integrals in the effective action. These overlap integrals are functions of the 5D fermion mass-parameter μ - which is introduced in the sUED Model - as well as the (inverse) compactification radius R^{-1}. In order to compare the theoretical predictions of the sUED Model with the current measured data we introduce the modified effective S, T and U parameter (based on the Peskin-Takeuchi parameters). These parameters describe the oblique corrections as well as the vertex corrections which can not be neglected in sUED. Consequently, the comparison with the electroweak measurements yield restrictions on the μ-R^{-1} parameter space of the sUED Model. The resulting restrictions we find are stronger than the constraints known in the literature, so far [1]. A large part of parameter space, for which one can verify sUED in dilepton signals at LHC [1], is excluded.

Contents

Chapter 1

Introduction

The Standard Model (SM) is up to date the most successful theory in particle physics, and there are only very small deviations between the predictions of the SM and the current measured data. However, the SM does not solve all problems in particle physics. For example there is no dark matter candidate, it does not solve the hierarchy problem, the strong CP problem and the flavor problem and the SM does not include the neutrino masses [2]. Many approaches were made to answer these questions and to solve these problems by embedding the Standard Model in a larger theory. Each of these extensions have to be in accord with the measurements and can also yield new predictions for prospective collider experiments as the LHC.

One possibility to extend the SM is the addition of space-like dimensions to the so far used four dimensional space-time. There are two classes of models which differ in the choice of the applied metric. *Randall-Sundrum models* (introduced in [3]) use an anti-deSitter metric. Other models use a flat five dimensional metric. Inside this class one has to distinguish between ADD models where only gravity can enter into the bulk and *Universal Extra Dimension models* (UED) in which all particles propagate in the bulk [4].

In this thesis, we concentrate on UED models. Here, we assume a flat metric with one additional compact dimension. The simplest choice would be the compactification on a S^1 (a circle). But for $d > 4$ it would follow that the fermions are non-chiral [5] and we get an additional scalar gauge field for each of the vector fields. In order to solve these problems we choose a $S^1/\mathbb{Z}_2$ orbifold for compactification. Therewith it is possible to project out half of the fermions by employing boundary conditions on the fermion fields. Furthermore, the additional scalar fields will be projected out by the use of boundary conditions on the gauge fields. A compactification on a $S^1/\mathbb{Z}_2$

orbifold effectively is a projection of a circle (S^1) on a line with two fix points.

In chapter 2 we explain these fact in more detail. We introduce the minimal extension of the SM to five dimensions, the so-called mUED Model. After that we extend this model to a so-called sUED Model with an additional fermion mass term. We discuss the main differences between mUED and sUED and give a short outlook of the expected differences.

In chapter 3 we use the fermion Lagrangian of the sUED model (introduced in chapter 2) to derive the equations of motion. Afterwards we solve these equations first for the zero modes and finally for an arbitrary nth KK mode. A summary of these solutions is also given in appendix B.

In chapter 4 we specify the full Lagrangian of the sUED Model and give a detailed explanation of the several parts and terms. Then we concentrate on the lepton Lagrangian to compute the vertices of two zero mode fermions and one nth KK-mode gauge boson. There, we obtain a general expression for the modified couplings. These vertex corrections will be used in the following chapter for introducing the effective parameters.

In chapter 5 we give an introduction of the Peskin-Takeuchi parameters and an explanation why and how we have to extend these parameters for the sUED Model. Following [6], we use the effective parameters S_{eff}, T_{eff} and U_{eff}, which include the oblique and the non-oblique corrections, to compare the theoretical predictions with the current experimental bounds as given in [7]. This comparison allows to exclude part of the μ-R^{-1} parameter space beyond the bounds known from the literature, so far [1].

Chapter 2

The UED Model

2.1 The basic setup

In this section we explain the basic assumptions of models with universal extra dimensions. We use the minimal UED Model (mUED) which is the simplest extension of the SM to five dimensions. In UED models, all Standard Model fields are promoted to 5D fields, propagating on the background geometry. The additional fifth dimension is compactified on a $S^1/\mathbb{Z}_2$ orbifold with radius R. This is shown in Fig. 2.1. The metric of the 5D space $\mathbb{M} \times S^1/\mathbb{Z}_2$ is assumed to be flat.

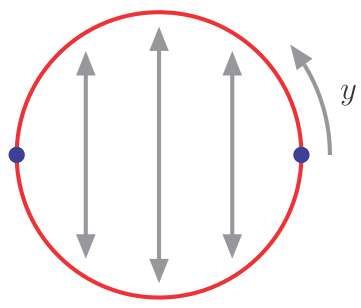

Figure 2.1: The compactification of the fifth dimension on a $S^1/\mathbb{Z}_2$ orbifold. The gray arrows describe the points on a circle which are identified with each other. The blue points show the fixed points at which later the boundary conditions for fields propagating on this space are applied. The picture is taken from Ref. [8].

Because of the additional $\mathbb{Z}_2$ symmetry, an orbifold with two fixed points is created, so that two opposite points on a circle are identified with each other. The existence of such fixed point breaks the 5D Lorentz invariance. From this it follows, that the 5D momentum conservation and therefore KK-number conservation, is violated. Only a parity called 'KK-parity' remains unbroken. Due to this fact, the lightest particle at the first KK-level (LKP) is a stable particle. In mUED, this stable particle is a good dark matter candidate. This candidate is the first KK-mode of the $U(1)$ gauge boson. Its dark matter phenomenology is reviewed in detail in [9]. The full Lagrangian of the mUED Model is given in [8].

In comparison to the SM, UED models have two additional parameters: the campactification radius R and the cut-off scale Λ. These models predict for each SM particle a whole so-called KK-tower. All particles of one tower have the identical quantum numbers but different masses. The SM particles are identified with the zero modes (KK-level $n = 0$) of these towers. In Fig. 2.2 the mass spectrum of the first KK-level in the mUED Model is shown as an example.

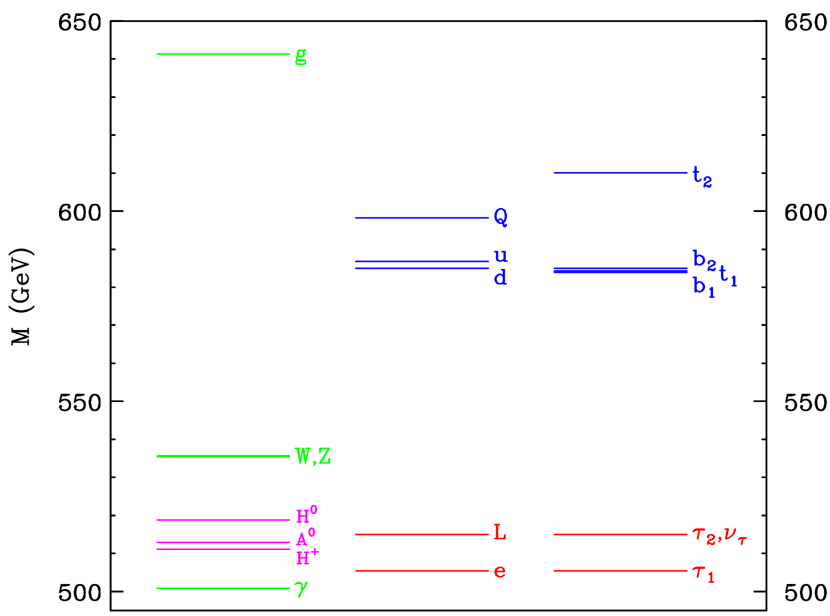

Figure 2.2: The mass spectrum of the first KK-level at one-loop with compactification radius $R^{-1} = 500$ GeV, cutoff scale $\Lambda = 20\ R^{-1}$ and Higgs mass $m_H = 120$ GeV. From [10].

Now we introduce the sUED Model on which we focus in this thesis.

2.2 The sUED Model

The sUED Model was introduced 2009 by Park and Shu in [11, 12]. Their main motivation was to modify the mUED Model to explain the observation of a cosmic positron excess by PAMELA [13]. For more details see [11]. The main feature of the sUED Model is an additional Dirac fermion mass parameter $m_5(y)$[1]. This mass parameter violates KK-number conservation. From this it follows that in sUED vertices are possible which are zero at tree-level in mUED. Consequently LHC signatures change [1].

The mass parameter $m_5(y)$ must have a KK-parity odd profile in order to maintain the KK-parity is a good symmetry of the Lagrangian. This means that for the mass parameter applies: $m_5(y) = -m_5(-y)$. In section 3.1 we establish in detail how the additional Dirac fermion mass term is implemented in the fermion Lagrangian. We will also see that this implies a change of the whole fermion mass spectrum. The shape of the fermion zero modes changes from a flat profile to an exponential one (section 3.2) and the fermion KK-modes get a m_5-dependence (section 3.3).

[1] This idea to add a 5D fermion mass term to the Lagrangian was also used before in RS models [14].

Chapter 3

The sUED fermion spectrum

3.1 The KK-decomposition of fermions

In order to get the equations of motions (EOM) we start with a setup with 5D fermions. These fermions are on a $S^1/\mathbb{Z}_2$ orbifold along the fifth dimension. This orbifold has the radius R and the two boundary points at y=-L and y=L where L=πR/2. The bulk action is

$$S = \int d^4x \int_{-L}^{+L} dy \big[\mathrm{i}\bar{\Psi}\Gamma^M \partial_M \Psi - m_5(y)\bar{\Psi}\Psi) \big] \ , \tag{3.1}$$

where M=0,1,2,3,5, the metric is $(+----)$ and the gamma matrices in 5D are $\Gamma^M = (\gamma^\mu, \mathrm{i}\gamma^5)$ (for details see appendix A). The bulk mass $m_5(y)$ is odd under inversion of the coordinate y so that

$$m_5(y) = -m_5(-y) \ . \tag{3.2}$$

The simplest choice to fulfill (3.2) is the following

$$m_5(y) = \mu \cdot \theta(y) \ , \tag{3.3}$$

with $\theta(y < 0) = -1$ and $\theta(y > 0) = 1$. Now we define the left-handed and the right-handed fermions as:

$$\Psi_L = P_L \Psi = \frac{1-\gamma^5}{2}\Psi \quad \text{and} \quad \Psi_R = P_R \Psi = \frac{1+\gamma^5}{2}\Psi \ . \tag{3.4}$$

This implies the following relation

$$\gamma_5 \Psi_{L/R} = \mp \Psi_{L/R} \tag{3.5}$$

and by using $\Psi = \Psi_L + \Psi_R$ we obtain from (3.1):

$$S = \int d^4x \int_{-L}^{+L} dy \Big[\mathrm{i}\bar{\Psi}_L\gamma^\mu\partial_\mu\Psi_L + \mathrm{i}\bar{\Psi}_R\gamma^\mu\partial_\mu\Psi_R - \bar{\Psi}_L\gamma^5\partial_5\Psi_R - \bar{\Psi}_R\gamma^5\partial_5\Psi_L - m_5(\bar{\Psi}_R\Psi_L + \bar{\Psi}_L\Psi_R)\Big] \ . \tag{3.6}$$

By doing the variation[1] of (3.6) we obtain the equations of motion

$$\begin{aligned} &\mathrm{i}\gamma^\mu\partial_\mu\Psi_R - \gamma^5\partial_5\Psi_L - m_5(y)\Psi_L = 0 \ , \\ &\mathrm{i}\gamma^\mu\partial_\mu\Psi_L - \gamma^5\partial_5\Psi_R - m_5(y)\Psi_R = 0 \ , \end{aligned} \tag{3.7}$$

which should be splitted in different domains of y

$$\textbf{for y > 0:} \qquad \begin{aligned} &\mathrm{i}\gamma^\mu\partial_\mu\Psi_R + \partial_5\Psi_L - \mu\Psi_L = 0 \ , \\ &\mathrm{i}\gamma^\mu\partial_\mu\Psi_L - \partial_5\Psi_R - \mu\Psi_R = 0 \ , \end{aligned} \tag{3.8}$$

$$\textbf{for y < 0:} \qquad \begin{aligned} &\mathrm{i}\gamma^\mu\partial_\mu\Psi_R + \partial_5\Psi_L + \mu\Psi_L = 0 \ , \\ &\mathrm{i}\gamma^\mu\partial_\mu\Psi_L - \partial_5\Psi_R + \mu\Psi_R = 0 \ . \end{aligned} \tag{3.9}$$

Let us apply a separation ansatz of the form

$$\Psi_{L/R}(x^\mu, y) = \sum_n \Psi^{(n)}_{L/R}(x^\mu) \cdot f^{(n)}_{L/R}(y) \tag{3.10}$$

in (3.8) and (3.9). Now, we can separate the 4D part from the y-dependent fermion wave function

$$\textbf{for y > 0:} \qquad \begin{aligned} &\frac{\mathrm{i}\partial\!\!\!/\Psi_R(x^\mu)}{\Psi_L(x^\mu)} = -\frac{(\partial_5 - \mu) f^{(n)}_L(y)}{f^{(n)}_R(y)} = m_n \ , \\ &\frac{\mathrm{i}\partial\!\!\!/\Psi_L(x^\mu)}{\Psi_R(x^\mu)} = \frac{(\partial_5 + \mu) f^{(n)}_R(y)}{f^{(n)}_L(y)} = m_n \ , \end{aligned} \tag{3.11a}$$

$$\textbf{for y < 0:} \qquad \begin{aligned} &\frac{\mathrm{i}\partial\!\!\!/\Psi_R(x^\mu)}{\Psi_L(x^\mu)} = -\frac{(\partial_5 + \mu) f^{(n)}_L(y)}{f^{(n)}_R(y)} = m_n \ , \\ &\frac{\mathrm{i}\partial\!\!\!/\Psi_L(x^\mu)}{\Psi_R(x^\mu)} = \frac{(\partial_5 - \mu) f^{(n)}_R(y)}{f^{(n)}_L(y)} = m_n \ , \end{aligned} \tag{3.11b}$$

[1]Inside the EOMs there are four different fields $\bar{\Psi}_L$, $\bar{\Psi}_R$, Ψ_L and Ψ_R. So we should expect four different EOMs. The two EOMs not given follow from (3.7) by hermitian conjugation.

with the abbreviation $\partial\!\!\!/ = \gamma^\mu \partial_\mu$ and the separation constant m_n. So (3.8) and (3.9) can be written as

$$\textbf{for y} > \textbf{0:} \qquad m_n f_R^{(n)}(y) + (\partial_5 - \mu) f_L^{(n)}(y) = 0 \ , \tag{3.12a}$$
$$m_n f_L^{(n)}(y) - (\partial_5 + \mu) f_R^{(n)}(y) = 0 \ , \tag{3.12b}$$

$$\textbf{for y} < \textbf{0:} \qquad m_n f_R^{(n)}(y) + (\partial_5 + \mu) f_L^{(n)}(y) = 0 \ , \tag{3.12c}$$
$$m_n f_L^{(n)}(y) - (\partial_5 - \mu) f_R^{(n)}(y) = 0 \ . \tag{3.12d}$$

These are two coupled first order differential equations for the two domains of y which can be rewritten as two decoupled differential equations[2] of second order

$$\begin{aligned} \partial_5^2 f_R^{(n)}(y) + (m_n^2 - \mu^2) f_R^{(n)}(y) = 0 \ , \\ \partial_5^2 f_L^{(n)}(y) + (m_n^2 - \mu^2) f_L^{(n)}(y) = 0 \ . \end{aligned} \tag{3.13}$$

(3.13) can be used to determine the solutions $f_{L/R}^{(n)}(y)$, but in the end, these solutions must be verified to satisfy the equations (3.12).

3.2 Fermion zero mode solution

In accordance to the SM we want chiral fermions for the zero modes as they are identified with the SM particles. A left-handed zero mode can be realized by choosing the boundary conditions (BC) to be Dirichlet-BCs $f_R(-L) = f_R(L) = 0$ for the right-handed mode. The two other boundary conditions (we need four BCs because of the four differential equations in (3.12)) follow from the coupled differential equations (3.12). These are modified Neumann-BCs.

$$\partial_5 f_L^{(n)}(L) - \mu f_L^{(n)}(L) = 0 \quad ; \quad \partial_5 f_L^{(n)}(-L) + \mu f_L^{(n)}(-L) = 0 \tag{3.14}$$

Before we determine the general solutions of (3.13) we have a look at the zero mode which can be readily found. For the massless zero mode ($n = 0, m_n = 0$) the equations (3.12) decouple and simplify to

$$\textbf{for y} > \textbf{0:} \qquad \begin{aligned} (\partial_5 - \mu) f_L^{(0)}(y) = 0 \ , \\ (\partial_5 + \mu) f_R^{(0)}(y) = 0 \ , \end{aligned} \tag{3.15}$$

[2]The second order differential equations are identical for y > 0 and y < 0. The reason for this is that $\mu^2 = (-\mu)^2$ and that there are only quadratic terms of μ in (3.13).

$$\textbf{for y} < \textbf{0:} \qquad (\partial_5 + \mu) f_L^{(0)}(y) = 0 \ ,$$
$$(\partial_5 - \mu) f_R^{(0)}(y) = 0 \ . \tag{3.16}$$

We can summarize the two solutions for $y > 0$ and $y < 0$ and use the fact, that $\int_{-L}^{L} dy |f^{(0)}|^2 = 1$ to compute the normalization constant N_L:

$$f_L^{(0)} = N_L \cdot \mathrm{e}^{\mu|y|} \ , \tag{3.17}$$

with the zero mode normalization

$$N_L = \sqrt{\frac{\mu}{\mathrm{e}^{2\mu L} - 1}} \ , \tag{3.18}$$

while $N_R = 0$ for the above boundary conditions. No other N_R can be found for the lower equations in (3.15) and (3.16) and so we can conclude: there is no right-handed zero mode. If we instead choose Dirichlet-BC on the left-handed wave function $f_L(-L) = f_L(L) = 0$, we get only a right-handed zero mode

$$f_R^{(0)} = N_R \cdot \mathrm{e}^{-\mu|y|} \ , \tag{3.19}$$

with the zero mode normalization

$$N_R = \sqrt{\frac{-\mu}{\mathrm{e}^{-2\mu L} - 1}} \tag{3.20}$$

and no left-handed one because of $N_L = 0$.

3.3 Fermion KK-mode solution

If we solve the equations (3.13) we expect two different types of solutions depending on $k_n^2 = m_n^2 - \mu^2$. It can be $k_n^2 < 0$ or $k_n^2 > 0$. Here we concentrate on the case where $k_n^2 > 0$. The reason for that is, that we find only in this area a region where the lightest fermionic KK-particle has a mass >1/R. The lightest KK-particle (LKP) with mass 1/R remains the KK-partner of a gauge boson. This is important, because the LKP provides a dark matter candidate, and KK-partners of the SM fermions are experimentally excluded as DM (see e.g. [15]). Some further comments will be given in section 3.4.

In order to find the solution of (3.13) we use the ansatz:

$$\textbf{for y} > \textbf{0:} \quad f_{L>}^{(n)}(y) = \alpha_{L>}^{(n)} \cdot \cos(k_n y) + \beta_{L>}^{(n)} \cdot \sin(k_n y) \ ,$$
$$f_{R>}^{(n)}(y) = \alpha_{R>}^{(n)} \cdot \cos(k_n y) + \beta_{R>}^{(n)} \cdot \sin(k_n y) \ , \tag{3.21}$$

$$\textbf{for y} < \textbf{0:}\quad f_{L<}^{(n)}(y) = \alpha_{L<}^{(n)} \cdot \cos(k_n y) + \beta_{L<}^{(n)} \cdot \sin(k_n y) \ ,$$
$$f_{R<}^{(n)}(y) = \alpha_{R<}^{(n)} \cdot \cos(k_n y) + \beta_{R<}^{(n)} \cdot \sin(k_n y) \ . \tag{3.22}$$

The wave functions $f_L^{(n)}(y)$ and $f_R^{(n)}(y)$ should be continuous at y = 0. So we have to check the limits $\lim_{\epsilon\to 0} f_{L/R}(\epsilon)$ for (3.21) and $\lim_{\epsilon\to 0} f_{L/R}(-\epsilon)$ for (3.22) at this point. From this it follows that

$$\alpha_{R<}^{(n)} = \alpha_{R>}^{(n)} = \alpha_R^{(n)} \quad \text{and} \quad \alpha_{L<}^{(n)} = \alpha_{L>}^{(n)} = \alpha_L^{(n)} \ . \tag{3.23}$$

We can use (3.21) and (3.22) inside the boundary conditions $f_R(-L) = f_R(L) = 0$ and we obtain the following relation:

$$-2\alpha_R^{(n)} \cdot \cos(k_n L) = \left(\beta_{R>}^{(n)} - \beta_{R<}^{(n)}\right) \cdot \sin(k_n L) \ . \tag{3.24}$$

From this equation we see that we can distinguish two different cases: $\alpha_R^{(n)} = 0$ (case 1) or $\alpha_R^{(n)} \neq 0$ (case 2). If $\alpha_R^{(n)} = 0$ the lower equations of (3.21) and (3.22) simplify to

$$f_{R>}^{(n)}(y) = \beta_{R>}^{(n)} \cdot \sin(k_n y) \ ,$$
$$f_{R<}^{(n)}(y) = \beta_{R<}^{(n)} \cdot \sin(k_n y) \ . \tag{3.25}$$

Therewith the boundary conditions are:

$$f_{R>}^{(n)}(L) = 0 = \beta_{R>}^{(n)} \cdot \sin(k_n L) \ ,$$
$$f_{R<}^{(n)}(-L) = 0 = \beta_{R<}^{(n)} \cdot \sin(k_n L) \ . \tag{3.26}$$

From this we obtain a condition for k_n:

$$k_n = \frac{n\pi}{L} \ . \qquad \textbf{case 1} \tag{3.27}$$

With the assumption, that $\alpha_R^{(n)} \neq 0$ we go back to (3.24) and we obtain a first important relation:

$$\frac{\beta_{R>}^{(n)} - \beta_{R<}^{(n)}}{2\alpha_R^{(n)}} = -\cot(k_n L) \ . \tag{3.28}$$

Now we have a look at (3.12b) and (3.12d). If we use the continuity condition again (by checking the limits at the origin) we find

$$m_n \alpha_L^{(n)} - k_n \beta_{R>}^{(n)} - \mu \alpha_R^{(n)} = 0 \ ,$$
$$m_n \alpha_L^{(n)} - k_n \beta_{R<}^{(n)} + \mu \alpha_R^{(n)} = 0 \ , \tag{3.29}$$

which leads to the second important relation

$$\frac{\beta_{R>}^{(n)} - \beta_{R<}^{(n)}}{2\alpha_R^{(n)}} = -\frac{\mu}{k_n} \ . \tag{3.30}$$

So we can combine (3.28) and (3.30) to get an equation for case 2 which implicitly determines k_n in terms of μ:

$$\mu = k_n \cot(k_n L) \ . \qquad \textbf{case 2} \tag{3.31}$$

After the determination of the quantization conditions for the two cases $\alpha_R^{(n)} = 0$ (case 1) and $\alpha_R^{(n)} \neq 0$ (case 2) we have to specify the coefficients $\alpha_L^{(n)}$, $\alpha_R^{(n)}$, $\beta_{L>}^{(n)}$, $\beta_{L<}^{(n)}$, $\beta_{R>}^{(n)}$ and $\beta_{R<}^{(n)}$ of the wave function for both cases.

3.3.1 Case 1: $\alpha_R^{(n)} = 0$

If we choose $\alpha_R^{(n)} = 0$ and insert this in (3.29), we obtain two relations

$$\begin{aligned} m_n\alpha_L^{(n)} - k_n\beta_{R>}^{(n)} &= 0 \ , \\ m_n\alpha_L^{(n)} - k_n\beta_{R<}^{(n)} &= 0 \ , \end{aligned} \tag{3.32}$$

and from this we see that $\beta_{R>}^{(n)} = \beta_{R<}^{(n)} = \beta_R^{(n)}$. So we can write the right-handed wave function as one function over the whole y range:

$$f_R^{(n)}(y) = \beta_R^{(n)} \cdot \sin(k_n y) \tag{3.33}$$

and the coefficients $\beta_R^{(n)}$ can be determined by the normalization

$$\int_{-L}^{L} dy |f_R^{(n)}(y)|^2 = 1 \tag{3.34}$$

implying

$$\beta_R^{(n)} = \pm\frac{1}{\sqrt{L - \frac{\sin(2k_nL)}{2k_n}}} \ . \tag{3.35}$$

If we use the quantization condition (3.27), the above equation (3.35) simplifies to $\beta_R^{(n)} = \pm 1/\sqrt{L}$. Furthermore, we get from (3.32) a relation between $\beta_R^{(n)}$ and $\alpha_L^{(n)}$:

$$\alpha_L^{(n)} = \frac{k_n}{m_n} \cdot \beta_R^{(n)} \tag{3.36}$$

and for the positive choice in (3.35), it is $\alpha_L^{(n)} = \frac{k_n}{m_n\sqrt{L}}$. Now this can be used in (3.12a) and (3.12c) at y = 0. As a result we obtain for the $\beta_L^{(n)}$'s:

$$\beta_{L>}^{(n)} = \frac{\mu}{k_n} \cdot \alpha_L^{(n)} \quad ; \quad \beta_{L<}^{(n)} = -\frac{\mu}{k_n} \cdot \alpha_L^{(n)} \tag{3.37}$$

and explicitly $\beta_{L>}^{(n)} = \frac{\mu}{m_n\sqrt{L}}$ and $\beta_{L<}^{(n)} = -\frac{\mu}{m_n\sqrt{L}}$. So we receive the following solutions of the differential equations (3.12):

$$\textbf{for y > 0:}\ f_{L>}^{(n)}(y) = \frac{k_n}{m_n\sqrt{L}} \cdot \cos(k_n y) + \frac{\mu}{m_n\sqrt{L}} \cdot \sin(k_n y) \ ,$$
$$f_{R>}^{(n)}(y) = \frac{1}{\sqrt{L}} \cdot \sin(k_n y) \ , \tag{3.38}$$
$$\textbf{for y < 0:}\ f_{L<}^{(n)}(y) = \frac{k_n}{m_n\sqrt{L}} \cdot \cos(k_n y) - \frac{\mu}{m_n\sqrt{L}} \cdot \sin(k_n y) \ ,$$
$$f_{R<}^{(n)}(y) = \frac{1}{\sqrt{L}} \cdot \sin(k_n y) \ , \tag{3.39}$$

which can be summarized for the whole domain $y \in [-L, L]$ as

$$f_L^{(n)}(y) = \frac{k_n}{m_n\sqrt{L}} \cdot \cos(k_n y) + \frac{\mu}{m_n\sqrt{L}} \cdot \sin(k_n|y|) \ ,$$
$$f_R^{(n)}(y) = \frac{1}{\sqrt{L}} \cdot \sin(k_n y) \ . \tag{3.40}$$

3.3.2 Case 2: $\alpha_R^{(n)} \neq 0$

Now we have a look at (3.12b) and (3.12d) again and use the boundary conditions on the right-handed mode $f_R(-L) = f_R(L) = 0$. So we get the following equations:

$$0 = m_n f_{L>}^{(n)}(L) - \partial_5 f_{R>}^{(n)}(L) \ ,$$
$$0 = m_n f_{L<}^{(n)}(-L) - \partial_5 f_{R<}^{(n)}(-L) \ . \tag{3.41}$$

If we insert the ansatz (3.21) and (3.22) in (3.41), we obtain an important relation between the coefficients $\alpha_L^{(n)}$, $\beta_{L>}^{(n)}$, $\beta_{L<}^{(n)}$, $\beta_{R>}^{(n)}$ and $\beta_{R<}^{(n)}$:

$$2\alpha_L^{(n)} + (\beta_{L>}^{(n)} - \beta_{L<}^{(n)}) \cdot \tan(k_n L) = \frac{k_n}{m_n}(\beta_{R>}^{(n)} + \beta_{R<}^{(n)}) \ . \tag{3.42}$$

From (3.29) we get another relation between $\alpha_L^{(n)}$, $\beta_{R>}^{(n)}$ and $\beta_{R<}^{(n)}$

$$2\alpha_L^{(n)} = \frac{k_n}{m_n}(\beta_{R>}^{(n)} + \beta_{R<}^{(n)}) \ . \tag{3.43}$$

Combining (3.42) and (3.43) yields:

$$0 = (\beta_{L>}^{(n)} - \beta_{L<}^{(n)}) \cdot \tan(k_n L) \ . \tag{3.44}$$

Here, $\tan(k_n L)$ can not be equal to zero because this would be in contradiction with the quantization condition (3.31). So it must be $\beta_{L>}^{(n)} - \beta_{L<}^{(n)} = 0$ and, accordingly, $\beta_{L>}^{(n)} = \beta_{L<}^{(n)} = \beta_L^{(n)}$.

Now we insert the ansatz (3.21) and (3.22) into the boundary condition for the left-handed wave function (3.14), and we obtain the following relation:

$$-2\alpha_L^{(n)} \Big[k_n \sin(k_n L) + \mu \cos(k_n L) \Big] = 0 \ . \tag{3.45}$$

Therein, the term in the bracket can not be equal to zero because this would be inconsistent with (3.31). So the conclusion is, that it must be $\alpha_L^{(n)} = 0$. Now the left-handed wave function has a very simple structure and can be written for the whole y range as

$$f_L^{(n)}(y) = \beta_L^{(n)} \cdot \sin(k_n y) \tag{3.46}$$

and the coefficient

$$\beta_L^{(n)} = \pm \frac{1}{\sqrt{L - \frac{\sin(2k_n L)}{2k_n}}} \ , \tag{3.47}$$

follows again from the normalization

$$\int_{-L}^{L} dy |f_L^{(n)}(y)|^2 = 1 \ . \tag{3.48}$$

Now we go back to (3.43) and we use the fact, that $\alpha_L^{(n)} = 0$. This gives rise to a relation for the $\beta_R^{(n)}$'s:

$$\beta_{R>}^{(n)} = -\beta_{R<}^{(n)} = \beta_R^{(n)} \ . \tag{3.49}$$

This can be used in (3.30) to obtain a connection between $\alpha_R^{(n)}$ and $\beta_R^{(n)}$ with

$$\beta_R^{(n)} = -\frac{\mu}{k_n} \cdot \alpha_R^{(n)} \ . \tag{3.50}$$

At the end, we put together (3.49) and (3.50), and the right-handed wave function can be written as

$$\begin{aligned} f_{R>}^{(n)}(y) &= \alpha_R^{(n)} \Big(\cos(k_n y) - \frac{\mu}{k_n} \sin(k_n y) \Big) \ , \\ f_{R<}^{(n)}(y) &= \alpha_R^{(n)} \Big(\cos(k_n y) + \frac{\mu}{k_n} \sin(k_n y) \Big) \ . \end{aligned} \tag{3.51}$$

In order to specify $\alpha_R^{(n)}$, we use the following normalizations

$$\frac{1}{2} = \int_0^L dy |f_{R>}^{(n)}(y)|^2 \quad ; \quad \frac{1}{2} = \int_{-L}^0 dy |f_{R<}^{(n)}(y)|^2 \ , \tag{3.52}$$

and we get

$$\alpha_R^{(n)} = \pm \frac{1}{\sqrt{L(1+\frac{\mu^2}{k_n^2}) + \frac{\sin(2k_nL)}{k_n}(1-\frac{\mu^2}{k_n^2}) - \frac{2\mu}{k_n^2}\sin^2(k_nL)}} \ . \tag{3.53}$$

Now we can write the solutions of the differential equations (3.12) for the positive choice of $\beta_L^{(n)}$ and $\alpha_R^{(n)}$ for both domains of y together:

$$\begin{aligned} f_L^{(n)}(y) &= \frac{1}{\sqrt{L - \frac{\sin(2k_nL)}{2k_n}}} \sin(k_n y) \ , \\ f_R^{(n)}(y) &= \frac{1}{\sqrt{L(1+\frac{\mu^2}{k_n^2}) + \frac{\sin(2k_nL)}{k_n}(1-\frac{\mu^2}{k_n^2}) - \frac{2\mu}{k_n^2}\sin^2(k_nL)}} \\ &\quad \cdot \Big(\cos(k_n y) - \frac{\mu}{k_n}\sin(k_n|y|) \Big) \ . \end{aligned} \tag{3.54}$$

So what changes when we choose the Dirichlet-BC on the left-handed modes and only a right-handed zero mode exists?

3.3.3 Right-handed zero mode

Of course we start with the identical ansatz as in(3.21) and (3.22). The boundary conditions are $f_{L>}^{(n)}(L) = f_{L<}^{(n)}(-L) = 0$ and

$$\partial_5 f_R^{(n)}(L) + \mu f_R^{(n)}(L) = 0 \quad ; \quad \partial_5 f_R^{(n)}(-L) - \mu f_R^{(n)}(-L) = 0 \ . \tag{3.55}$$

The considered calculation is absolutely analogous to the sections 3.3.1 and 3.3.2. The equations (3.23) and (3.24) hold also for this case. Now we get for case 1 the same quantization condition as in (3.27), however, not from the right-handed but from the left-handed wave function. The analogous relations to (3.28) and (3.30) are:

$$\begin{aligned} \frac{\beta_{L>}^{(n)} - \beta_{L<}^{(n)}}{2\alpha_L^{(n)}} &= -\cot(k_nL) \ , \\ \frac{\beta_{L>}^{(n)} - \beta_{L<}^{(n)}}{2\alpha_L^{(n)}} &= \frac{\mu}{k_n} \ , \end{aligned} \tag{3.56}$$

and so in the quantization condition for case 2, the sign changes:

$$\mu = -k_n \cot(k_n L) \ . \tag{3.57}$$

At the end, we obtain the following solutions of the coupled differential equations (3.12) for the first case with the quantization condition (3.27)

$$\begin{aligned} f_L^{(n)}(y) &= \frac{1}{\sqrt{L}} \cdot \sin(k_n y) \ , \\ f_R^{(n)}(y) &= -\frac{k_n}{m_n\sqrt{L}} \cdot \cos(k_n y) + \frac{\mu}{m_n\sqrt{L}} \cdot \sin(k_n|y|) \ , \end{aligned} \tag{3.58}$$

and for the second case with the changed condition (3.57)

$$\begin{aligned} f_L^{(n)}(y) =& \frac{1}{\sqrt{L(1+\frac{\mu^2}{k_n^2}) + \frac{\sin(2k_nL)}{k_n}(1-\frac{\mu^2}{k_n^2}) + \frac{2\mu}{k_n^2}\sin^2(k_nL)}} \\ & \cdot \Big(\cos(k_n y) + \frac{\mu}{k_n}\sin(k_n|y|) \Big) \ , \\ f_R^{(n)}(y) =& \frac{1}{\sqrt{L - \frac{\sin(2k_nL)}{2k_n}}} \sin(k_n y) \ . \end{aligned} \tag{3.59}$$

3.4 Fermion KK spectrum

In order to obtain the KK mass spectrum we first plot the quantization conditions (see Fig. 3.1). For case 1 we have the same quantization condition (3.27) for both choices - Dirichlet boundary conditions on the right-handed mode or on the left-handed mode. This gives us the vertical lines at π, 2π, 3π, ... For case 2 we have to distinguish. For Dirichlet on right it is (3.31) and for Dirichlet on left we have to use (3.57). The solutions of (3.31) and (3.57) can be read off graphically, and the masses can be calculated with $m_n^2 = k_n^2 + \mu^2$.

As can be seen from the plots in Fig. 3.1, the points of intersection inside the KK-tower came alternately from case 2 and from case 1. We can enumerate the masses with n = 1, 2, 3, ... So we obtain red point $\rightarrow$ n=1, blue point $\rightarrow$ n=2, red point $\rightarrow$ n=3, ... In this enumeration the k_n or m_n with an odd number n always came from case 2 ((3.31) for (a) and (3.57) for (b)) and the masses with an even number n always came from case 1 ((3.27) for (a) and (b)).

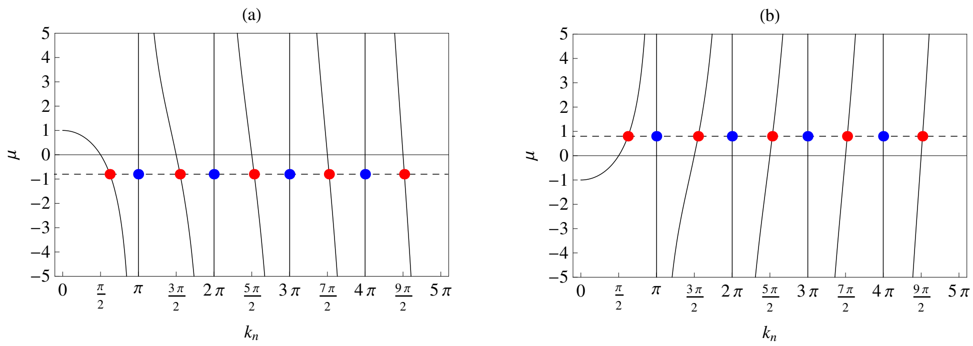

Figure 3.1: Kaluza-Klein spectrum for Dirichlet boundary condition on the right-handed (a) or on the left-handed (b) mode. It is chosen L = 1 and $|\mu|$ = 0.8 to solve the transcendental quantization conditions. The red points are the solutions for case 2 and the blue points are the solutions for case 1.

Now we use the numerically determined k_n's to calculate m_n depending on μ. The results are shown in Fig. 3.2. There we see, that for the minimal

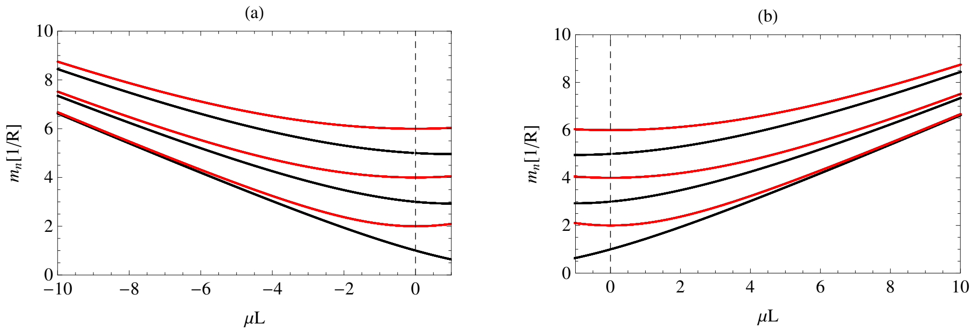

Figure 3.2: Kaluza-Klein masses for Dirichlet boundary condition on the right-handed (a) and the left-handed (b) mode. The black lines came from an odd number of n, here ascending $n = 1$, 3, 5 and the red lines came from even numbers of $n = 2$, 4, 6. In the whole area which is plotted it applies (3.31) for odd n. The dashed lines at $\mu\text{L} = 0$ show the minimal UED case.

UED case ($\mu = 0$) m_n has the well-known form

$$m_n = \frac{n}{R} \qquad\qquad n = 1, 2, 3, 4, \ldots \ . \tag{3.60}$$

The calculation of m_n is only possible with the equations of section 3.3 for restricted μL with:

$$-\infty < \mu L \leq 1 \quad \text{for (a)} \quad \text{and} \tag{3.61a}$$

$$-1 \leq \mu L < \infty \quad \text{for (b)} \ . \tag{3.61b}$$

The reason for this is the change of the quantization conditions (3.31) and (3.57). In the areas which are not plotted in Fig. 3.1 it is: $k_n^2 < 0$. This entails that the general ansatz for the wave function (3.21) and (3.22) do not hold anymore. For the case of negative k_n^2 we would have to do an ansatz with hyperbolic functions [16]. The consequence is that also the quantization conditions (3.31) and (3.57) change, there would be a *coth* instead of a *cot* inside. We can see from Fig. 3.2, that the condition $m_1 \geq \frac{1}{R}$ (which was mentioned in the introduction of section 3.3) is fulfilled if we have

$$\mu L \leq 0 \quad \text{for (a)} \;, \tag{3.62a}$$

$$\mu L \geq 0 \quad \text{for (b)} \tag{3.62b}$$

and this justifies our choice at the beginning of section 3.3 with $k_n^2 > 0$. The ranges $0 \leq \mu L \leq 1$ for (a) and $-1 \leq \mu L \leq 0$ for (b) as well as the whole area where $k_n^2 < 0$ would imply a first KK-mode of the fermion with $m_1 \leq \frac{1}{R}$. This would be give a fermion as a dark matter candidate which is in contradiction with experimental bounds, see for example [15].

Chapter 4

effective 4D Lagrangian

In this chapter we discuss the calculation of the couplings of two zero mode fermions and one KK-level gauge boson (vector component) for the 5D sUED Model introduced in chapter 2. First, we describe this model, and we mention the differences in comparison with the mUED Model. We start with the five dimensional Lagrangian and specify all the used definitions. In order to get the vertices, we take the fermion Lagrangian and multiply it out. Afterwards, we integrate over the fifth dimension y to obtain the effective 4D Lagrangian. Here, we have to use the wave functions of the fermions which were calculated in the previous chapter and the wave functions of the gauge bosons which have the same structure as in mUED [8]. After determining these wave functions we get the couplings with an additional factor inside, the overlap integrals. For these factors we give a general formula.

4.1 The model

We start with a five dimensional sUED Model. The general full Lagrangian can be written as

$$\mathcal{L} = \mathcal{L}_{gauge} + \mathcal{L}_{GF} + \mathcal{L}_{fermion} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa} \ , \tag{4.1}$$

where the individual parts for our model are analogous to the parts in [8]. The only differences concern $\mathcal{L}_{fermion}$:

$$\mathcal{L}_{gauge} = -\frac{1}{4}B_{MN}B^{MN} - \frac{1}{4}W^a_{MN}W^{aMN} - \frac{1}{4}G^A_{MN}G^{AMN} \ , \tag{4.2a}$$

$$\mathcal{L}_{GF} = -\frac{1}{2\xi}F(B)^2 - \frac{1}{2\xi}F(W^a)^2 - \frac{1}{2\xi}F(G^A)^2 \ , \tag{4.2b}$$

$$\mathcal{L}_{fermion} = \mathrm{i}\bar{\Psi}\Gamma^M D_M \Psi - \mu\theta(y)\bar{\Psi}\Psi \ , \tag{4.2c}$$

$$\mathcal{L}_{Higgs} = (D_M\phi)^\dagger (D^M\phi) - V(\phi) \ , \tag{4.2d}$$

$$\mathcal{L}_{Yukawa} = \lambda_u \bar{Q} u \mathrm{i}\tau_2 \phi^* + \lambda_d \bar{Q} d\phi + \lambda_e \bar{L} e\phi \ . \tag{4.2e}$$

Therein, B_M, W_M^a and G_M^A are the gauge fields from the $U(1)_Y$, $SU(2)_W$ and $SU(3)_c$ gauge groups. The indices a and A have the ranges $a = 1, 2, 3$ and $A = 1, 2, ..., 8$. The 5D field strengths B_{MN}, W_{MN}^a and G_{MN}^A are defined as

$$\begin{aligned} B_{MN} &= \partial_M B_N - \partial_N B_M \ , \\ W_{MN}^a &= \partial_M W_N^a - \partial_N W_M^a + g_2^{(5)} \epsilon^{abc} W_M^b W_N^c \ , \\ G_{MN}^A &= \partial_M W_N^A - \partial_N W_M^A + g_3^{(5)} f^{ABC} W_M^B W_N^C \ , \end{aligned} \tag{4.3}$$

where ϵ^{abc} and f^{ABC} are the structure constants of $SU(2)_W$ and $SU(3)_c$. In (4.3) we introduce $g_2^{(5)}$ and $g_3^{(5)}$. These are two of the three 5D coupling constants $g_i^{(5)}$ (one for each gauge group):

$$g_1^{(5)} \Leftrightarrow U(1)_Y \ ; \ g_2^{(5)} \Leftrightarrow SU(2)_W \ ; \ g_3^{(5)} \Leftrightarrow SU(3)_c \ . \tag{4.4}$$

From them we obtain the effective 4D couplings with

$$g_i = \frac{g_i^{(5)}}{\sqrt{2L}} \ . \tag{4.5}$$

We define the necessary gauge fixing functions in the same way as it was done in [17] to compensate the mixing terms between the vector and the scalar components of the 5D gauge fields

$$\begin{aligned} F(B) &= \partial_\mu B^\mu - \xi\Big(\partial_5 B_5 - \mathrm{i}g_1^{(5)}\left(\phi^\dagger\phi_0 - \phi_0^\dagger\phi\right)\Big) \ , \\ F(W^a) &= \partial_\mu W^{a\mu} - \xi\Big(\partial_5 W_5^a - \mathrm{i}g_2^{(5)}\left(\phi^\dagger\tau_a\phi_0 - \phi_0^\dagger\tau_a\phi\right)\Big) \ , \\ F(G^A) &= \partial_\mu G^{A\mu} - \xi\partial_5 G_5^A \ , \end{aligned} \tag{4.6}$$

where ξ is the gauge fixing parameter and $\tau_a = \frac{\sigma_a}{2}$, where σ_a are the Pauli matrices[1]. ϕ is the standard Higgs field

$$\phi(x^\mu, y) = \frac{1}{\sqrt{2}}\left[\begin{pmatrix} 0 \\ v_5 \end{pmatrix} + \begin{pmatrix} \chi^1(x^\mu, y) + \mathrm{i}\chi^2(x^\mu, y) \\ h(x^\mu, y) + \mathrm{i}\chi^3(x^\mu, y) \end{pmatrix}\right] \tag{4.7}$$

[1] The Pauli matrices are defined as $\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_2 = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}$ and $\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

and the Higgs potential is defined as

$$V(\phi) = -|\mu_5|^2(\phi^\dagger\phi) + \lambda_5(\phi^\dagger\phi)^2 \ . \tag{4.8}$$

D_M which appears in $\mathcal{L}_{fermion}$ and $\mathcal{L}_{Higgs}$ is the covariant derivative. It acts as follows on the different fields [8]:

$$D_M Q = \Big(\partial_M - \mathrm{i}\frac{g_3^{(5)}}{2} G_M^A \lambda_A - \mathrm{i}\frac{g_2^{(5)}}{2} W_M^a \sigma_a - \mathrm{i}\frac{g_1^{(5)}}{2} B_M Y_W^Q \mathbb{1}_{2\times 2}\Big) Q \ , \tag{4.9a}$$

$$D_M U = \Big(\partial_M - \mathrm{i}\frac{g_3^{(5)}}{2} G_M^A \lambda_A - \mathrm{i}\frac{g_1^{(5)}}{2} B_M Y_W^U\Big) U \ , \tag{4.9b}$$

$$D_M D = \Big(\partial_M - \mathrm{i}\frac{g_3^{(5)}}{2} G_M^A \lambda_A - \mathrm{i}\frac{g_1^{(5)}}{2} B_M Y_W^D\Big) D \ , \tag{4.9c}$$

$$D_M L = \Big(\partial_M - \mathrm{i}\frac{g_2^{(5)}}{2} W_M^a \sigma_a - \mathrm{i}\frac{g_1^{(5)}}{2} B_M Y_W^L \mathbb{1}_{2\times 2}\Big) L \ , \tag{4.9d}$$

$$D_M E = \Big(\partial_M - \mathrm{i}\frac{g_1^{(5)}}{2} B_M Y_W^E\Big) E \ , \tag{4.9e}$$

$$D_M \phi = \Big(\partial_M - \mathrm{i}\frac{g_2^{(5)}}{2} W_M^a \sigma_a - \mathrm{i}\frac{g_1^{(5)}}{2} B_M Y_W^\phi \mathbb{1}_{2\times 2}\Big) \phi \ . \tag{4.9f}$$

Here every field depends on x^μ and y. Furthermore, λ_A are the SU(3)-generators and σ_a are again the Pauli matrices. The hypercharges of the fields are $Y_W^Q = \frac{1}{3}$, $Y_W^U = \frac{4}{3}$, $Y_W^D = -\frac{2}{3}$, $Y_W^L = -1$, $Y_W^E = -2$ and $Y_W^\phi = 1$. $\mathbb{1}_{2\times 2}$ is a (2×2) identity matrix in the isospin space.

The only changes from the mUED scenario concern the fermion sector because there is an additional mass term which is characteristic for the sUED Model. Inside the fermion Lagrangian there is $\Psi = (Q, U, D, L, E)$ where Q is a quark doublet, U and D are quark singlets, L is a lepton doublet and E is the a lepton singlet[2]. The fermion fields can be written with the separation ansatz in (3.10) as:

$$Q(x^\mu, y) = Q_L^{(0)}(x^\mu) f_L^{Q(0)}(y) + \sum_{n=1}^{\infty} \Big[Q_L^{(n)}(x^\mu) f_L^{Q(n)}(y) + Q_R^{(n)}(x^\mu) f_R^{Q(n)}(y)\Big] \ ,$$

$$U(x^\mu, y) = U_R^{(0)}(x^\mu) f_R^{U(0)}(y) + \sum_{n=1}^{\infty} \Big[U_L^{(n)}(x^\mu) f_L^{U(n)}(y) + U_R^{(n)}(x^\mu) f_R^{U(n)}(y)\Big] \ ,$$

[2]In UED, these multiplets have a different chiral structure than in the SM. For example inside the quark doublet Q, there are parts which are left-handed and there are also parts which are right-handed.

$$
\begin{aligned}
D(x^\mu, y) &= D_R^{(0)}(x^\mu) f_R^{D(0)}(y) + \sum_{n=1}^{\infty} \left[D_L^{(n)}(x^\mu) f_L^{D(n)}(y) + D_R^{(n)}(x^\mu) f_R^{D(n)}(y) \right] \ , \\
L(x^\mu, y) &= L_L^{(0)}(x^\mu) f_L^{L(0)}(y) + \sum_{n=1}^{\infty} \left[L_L^{(n)}(x^\mu) f_L^{L(n)}(y) + L_R^{(n)}(x^\mu) f_R^{L(n)}(y) \right] \ , \\
E(x^\mu, y) &= E_R^{(0)}(x^\mu) f_R^{E(0)}(y) + \sum_{n=1}^{\infty} \left[E_L^{(n)}(x^\mu) f_L^{E(n)}(y) + E_R^{(n)}(x^\mu) f_R^{E(n)}(y) \right] \ .
\end{aligned}
\tag{4.10}
$$

Therein, $Q^n_{L/R}$, $U^n_{L/R}$, $D^n_{L/R}$, $L^n_{L/R}$ and $E^n_{L/R}$ are the 4D fields depending on x^μ. The zero modes of the fermions which are separated from the other modes in (4.10) have the same chirality as in the SM. The 5D parts of the fermion wave functions $f^{(n)}_{L/R}$ were calculated in chapter 3, the zero modes in section 3.2 and the higher KK-modes in section 3.3. A summary is also given in appendix B.

We are in particular interested in vertex terms with two lepton zero modes and one arbitrary KK-mode of a gauge boson so that in an effective low energy theory the outer fermion lines of a Feynman diagram are only SM particles. On this account the interesting part is the fermion Lagrangian $\mathcal{L}_{fermion}$ and we can write the fermion multiplets as in the SM:

$$Q_L^0 = \begin{pmatrix} u_L^0 \\ d_L^0 \end{pmatrix} \ ; \ U_R^0 = (u_R^0) \ ; \ D_R^0 = (d_R^0) \ , \tag{4.11a}$$

$$L_L^0 = \begin{pmatrix} \nu_L^0 \\ e_L^0 \end{pmatrix} \ ; \ E_R^0 = (e_R^0) \ . \tag{4.11b}$$

In order to get the vertices of the physical fields directly from the Lagrangian it is useful to write D_M in the mass eigenbasis. Therefore we use the following relations for the gauge fields

$$W_M^+ = \frac{1}{\sqrt{2}}(W_M^1 + \mathrm{i}W_M^2) \ , \tag{4.12a}$$

$$W_M^- = \frac{1}{\sqrt{2}}(W_M^1 - \mathrm{i}W_M^2) \ , \tag{4.12b}$$

$$Z_M = W_M^3 \cos\theta - B_M \sin\theta \ , \tag{4.12c}$$

$$A_M = W_M^3 \sin\theta + B_M \cos\theta \ , \tag{4.12d}$$

where θ is the Weinberg angle[3]. By using $\sigma_\pm = \sigma_1 \pm \mathrm{i}\sigma_2$ and $Q = \frac{\sigma_3}{2} + \frac{Y_W}{2} \cdot \mathbb{1}_{2\times 2}$ we receive:

$$D_M L(x^\mu, y) = \left[\partial_M - \mathrm{i}\frac{g_2^{(5)}}{2\sqrt{2}} \Big(W_M^+ \sigma_- + W_M^- \sigma_+ \Big) - \mathrm{i}e^{(5)} Q A_M \right.$$
$$\left. - \mathrm{i}\frac{g_2^{(5)}}{\cos\theta} \Big(\frac{\sigma_3}{2} - Q \cdot \sin^2\theta \Big) Z_M \right] L(x^\mu, y) \ , \tag{4.13a}$$

$$D_M E(x^\mu, y) = \left[\partial_M + \mathrm{i}e^{(5)} A_M - \mathrm{i}\frac{g_2^{(5)}}{\cos\theta} \sin^2\theta Z_M \right] E(x^\mu, y) \ . \tag{4.13b}$$

From the above definition it follows for the L leptons

$$Q = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \ . \tag{4.14}$$

For the next step, we split up the fermion Lagrangian into a quark and a lepton part.

$$\mathcal{L}_{fermion} = \mathcal{L}_{quark} + \mathcal{L}_{lepton} \ . \tag{4.15}$$

In our later analysis we focus on the lepton-gaugefield interactions. With the results of the last section we multiply out all interacting terms of $\mathcal{L}_{lepton}$. From these terms we can directly read off the Feynman rules for the vertices.

4.2 Lepton part of the fermion Lagrangian

We start with the kinetic part of the lepton Lagrangian for the sUED Model

$$\mathcal{L}_{lepton} = \mathrm{i}\overline{L}(x^\mu, y)\Gamma^M D_M L(x^\mu, y) + \mathrm{i}\overline{E}(x^\mu, y)\Gamma^M D_M E(x^\mu, y)$$
$$- m_5(y)\overline{L}L - m_5(y)\overline{E}E \ . \tag{4.16}$$

So we use $D_M = (D_\mu, D_5)$ and $\Gamma^M = (\gamma^\mu, \mathrm{i}\gamma^5)$. We obtain from the upper equation:

$$\mathcal{L}_{lepton} = \mathrm{i}\overline{L}\gamma^\mu D_\mu L + \mathrm{i}\overline{E}\gamma^\mu D_\mu E - \overline{L}\gamma^5 D_5 L - \overline{E}\gamma^5 D_5 E$$
$$- m_5(y)\overline{L}L - m_5(y)\overline{E}E \ . \tag{4.17}$$

[3]Note that these relations hold for the 5D fields $\phi_M(x^\mu, y)$ as well as for each KK-mode $\phi_M^{(n)}(x^\mu)$ for itself. The reason for that is, that the mass matrix can be written as $M = (\frac{n}{R})^2 \cdot \mathbb{1}_{2\times 2} + M_{SM}$ and so the rotation matrix which diagonalizes M_{SM} also diagonalizes M.

Now we choose the gauge $\Phi_5 = 0$, where Φ_5 is the 5D scalar component of the five dimensional gauge field Φ_M. It stands for $\Phi_M = W_M^+, W_M^-, Z_M, A_M$. By using the covariant derivatives from (4.13) we obtain for the lepton Lagrangian:

$$\begin{aligned}\mathcal{L}_{lepton} &= \mathrm{i}\overline{L}\partial\!\!\!/ L + \mathrm{i}\overline{E}\partial\!\!\!/ E - \overline{L}\gamma^5\partial_5 L - \overline{E}\gamma^5\partial_5 E - m_5(y)\overline{L}L - m_5(y)\overline{E}E \\ &+ \frac{g_2^{(5)}}{2\sqrt{2}}\overline{L}\gamma^\mu\left(W_\mu^+\sigma_- + W_\mu^-\sigma_+\right)L \\ &+ \frac{g_2^{(5)}}{\cos\theta}\overline{L}\gamma^\mu\left(\frac{\sigma_3}{2} - Q\cdot\sin^2\theta\right)Z_\mu L + \frac{g_2^{(5)}}{\cos\theta}\overline{E}\gamma^\mu\sin^2\theta Z_\mu E \\ &+ e^{(5)}\overline{L}\gamma^\mu Q A_\mu L - +e^{(5)}\overline{E}\gamma^\mu Q A_\mu E \ . \end{aligned} \tag{4.18}$$

In (4.18) the first line is the kinetic part $\mathcal{L}_{kin}$ and the other lines are the interaction part $\mathcal{L}_{WW}$ so that we can write:

$$\mathcal{L}_{lepton} = \mathcal{L}_{kin} + \mathcal{L}_{WW} \ . \tag{4.19}$$

The kinetic Lagrangian $\mathcal{L}_{kin}$ can be expanded as[4]

$$\begin{aligned}\mathcal{L}_{kin} &= \mathrm{i}\overline{L}^{(0)}\partial\!\!\!/ L^{(0)} + \mathrm{i}\overline{E}^{(0)}\partial\!\!\!/ E^{(0)} \qquad (4.20)\\ &+ \sum_{n,m=1}^{\infty}\left[\overline{L}^{(n)}\left(\mathrm{i}\partial\!\!\!/ - \gamma^5\partial_5 - m_5(y)\right)L^{(m)} + \overline{E}^{(n)}\left(\mathrm{i}\partial\!\!\!/ - \gamma^5\partial_5 - m_5(y)\right)E^{(m)}\right]\end{aligned}$$

and therein $L^{(n)}(x^\mu, y)$ and $E^{(n)}(x^\mu, y)$ for $n > 0$ can be extracted from (4.10). It applies

$$\begin{aligned}L^{(n)}(x^\mu, y) &= L_L^{(n)}(x^\mu)f_L^{L(n)}(y) + L_R^{(n)}(x^\mu)f_R^{L(n)}(y) \ , \\ E^{(n)}(x^\mu, y) &= E_L^{(n)}(x^\mu)f_L^{E(n)}(y) + E_R^{(n)}(x^\mu)f_R^{E(n)}(y) \ . \end{aligned} \tag{4.21}$$

In order to compute the $f_0 - f_0 - V_n$ vertices, we separate the zero modes of the fermions inside of $\mathcal{L}_{WW}$ from the higher KK-mode fermions:

$$\begin{aligned}\mathcal{L}_{WW} &= \frac{g_2^{(5)}}{2\sqrt{2}}\overline{L}^{(0)}\gamma^\mu\left(W_\mu^+\sigma_- + W_\mu^-\sigma_+\right)L^{(0)} \\ &+ \frac{g_2^{(5)}}{\cos\theta}\overline{L}^{(0)}\gamma^\mu\left(\frac{\sigma_3}{2} - Q\cdot\sin^2\theta\right)Z_\mu L^{(0)} \\ &+ \frac{g_2^{(5)}}{\cos\theta}\overline{E}^{(0)}\gamma^\mu\sin^2\theta Z_\mu E^{(0)} \\ &+ e^{(5)}\overline{L}^{(0)}\gamma^\mu Q A_\mu L^{(0)} - e^{(5)}\overline{E}^{(0)}\gamma^\mu A_\mu E^{(0)} \\ &+ \text{interactions of higher KK fermions} \ . \end{aligned} \tag{4.22}$$

[4]Note, that the mass terms for the zero modes are neglected. The reason is that the zero modes have to be chiral and so terms like $m\overline{\Psi}_L\Psi_R$ are not allowed

The multiplets for the zero modes of L and E were introduced in (4.11b). We can write with the projectors P_L and P_R defined in (3.4):

$$L^0(x^\mu, y) = \begin{pmatrix} P_L \nu_L^0(x^\mu, y) \\ P_L e_L^0(x^\mu, y) \end{pmatrix} \ ; \ E^0(x^\mu, y) = \begin{pmatrix} P_R e_R^0(x^\mu, y) \end{pmatrix} \ . \tag{4.23}$$

In order to obtain $\overline{L}$ and $\overline{E}$ from (4.23) we have to use the definition of the bar: $\overline{L} = L^\dagger \gamma^0$. We also know that

$$\overline{L^0} = \begin{pmatrix} \overline{\nu_L^0} \\ \overline{e_L^0} \end{pmatrix} \ ; \ \overline{E^0} = (\overline{e_R^0}) \ . \tag{4.24}$$

What is $\overline{\nu_L^0}$, $\overline{e_L^0}$ and $\overline{e_R^0}$? When we use the above definition of the bar for $\overline{\nu_L^0}$ and the anti-commutator $\{\gamma^5, \gamma^0\} = 0$ we obtain

$$\overline{\nu_L^0} = (\nu_L^0)^\dagger \gamma^0 = (\nu^0)^\dagger P_L \gamma^0 = \overline{\nu^0} P_R \tag{4.25}$$

and the analogous equations for $\overline{e_L^0}$ and $\overline{e_R^0}$

$$\overline{e_L^0} = \overline{e^0} P_R \ ; \ \overline{e_R^0} = \overline{e^0} P_L \ . \tag{4.26}$$

Now we go back to (4.22). We can use the definitions of the fermion multiplets (4.11b), (4.24) as well as (4.25) and (4.26) to expand all the terms. The result is the part of the interaction Lagrangian (4.22) with all $f_0 - f_0 - V_n$ vertices. We pick out the first four lines with only fermion zero modes inside:

$$\begin{aligned} \mathcal{L}^0_{WW} = &\frac{g_2^{(5)}}{2\sqrt{2}} \overline{\nu}_L^{(0)} \gamma^\mu e_L^{(0)} W_\mu^+ + \frac{g_2^{(5)}}{2\sqrt{2}} \overline{e}_L^{(0)} \gamma^\mu \nu_L^{(0)} W_\mu^- \\ &+ \frac{g_2^{(5)}}{2\cos\theta} \overline{\nu}_L^{(0)} \gamma^\mu \nu_L^{(0)} Z_\mu - \frac{g_2^{(5)}}{4\cos\theta} \overline{e}^{(0)} \gamma^\mu \left((1 - 4\sin^2\theta) - \gamma^5\right) e^{(0)} Z_\mu \\ &- e^{(5)} \overline{e}^{(0)} \gamma^\mu e^{(0)} A_\mu \ , \end{aligned} \tag{4.27}$$

where $e^{(0)} = e_L^{(0)} + e_R^{(0)}$. Each of the occurring gauge fields $W_\mu^\pm$, Z_μ and A_μ includes the whole KK-tower which will be given in (4.28). The fermion zero modes can be written as $\Psi_{L/R}^{(0)}(x^\mu, y) = \Psi_{L/R}^{(0)}(x^\mu) \cdot f_{L/R}^{(0)}(y)$ with the $f_{L/R}^{(0)}(y)$ from (3.17) and (3.19).

In the sUED Model the gauge boson tower has the same structure as in mUED which is given by [8].

$$W^+_\mu(x^\mu, y) = \frac{1}{\sqrt{2L}}\left\{W^{(0)+}_\mu(x^\mu) + \sqrt{2}\sum_{n=1}^{\infty} W^{(n)+}_\mu(x^\mu) f^{(n)}_V(y)\right\} \ , \quad (4.28a)$$

$$W^-_\mu(x^\mu, y) = \frac{1}{\sqrt{2L}}\left\{W^{(0)-}_\mu(x^\mu) + \sqrt{2}\sum_{n=1}^{\infty} W^{(n)-}_\mu(x^\mu) f^{(n)}_V(y)\right\} \ , \quad (4.28b)$$

$$Z_\mu(x^\mu, y) = \frac{1}{\sqrt{2L}}\left\{Z^{(0)}_\mu(x^\mu) + \sqrt{2}\sum_{n=1}^{\infty} Z^{(n)}_\mu(x^\mu) f^{(n)}_V(y)\right\} \ , \quad (4.28c)$$

$$A_\mu(x^\mu, y) = \frac{1}{\sqrt{2L}}\left\{A^{(0)}_\mu(x^\mu) + \sqrt{2}\sum_{n=1}^{\infty} A^{(n)}_\mu(x^\mu) f^{(n)}_V(y)\right\} \ , \quad (4.28d)$$

where the function $f^{(n)}_V(y)$ is defined as

$$f^{(n)}_V(y) = \begin{cases} \sin\left(\frac{n\pi}{2L}y\right) & n \text{ is odd} \\ \cos\left(\frac{n\pi}{2L}y\right) & n \text{ is even} \ . \end{cases} \quad (4.29)$$

The sUED fermion wave functions are given in (3.10) and there zero mode profiles in (3.17) and (3.19). After using this and inserting (4.28) and (4.29) in (4.27) we have to integrate over the fifth dimension y in order to get the effective 4D Lagrangian from the five dimensional parts because of $\mathcal{L}_{eff} = \int dy \mathcal{L}_{5D}$. It results from (4.27) that

$$\begin{aligned}\mathcal{L}^0_{WWeff} = \sum_{n=0}^{\infty}\Bigg[&\frac{1}{2\sqrt{2}}\frac{g_2^{(5)}}{\sqrt{2L}}\overline{\nu}^{(0)}_L\gamma^\mu e^{(0)}_L W^{(n)+}_\mu(x^\mu) I^n + \frac{g_2^{(5)}}{2\sqrt{2}}\overline{e}^{(0)}_L\gamma^\mu\nu^{(0)}_L W^{(n)-}_\mu(x^\mu) I^n \\ &+ \frac{1}{2\cos\theta}\frac{g_2^{(5)}}{\sqrt{2L}}\overline{\nu}^{(0)}_L\gamma^\mu\nu^{(0)}_L Z^{(n)}_\mu(x^\mu) I^n \\ &- \frac{1}{4\cos\theta}\frac{g_2^{(5)}}{\sqrt{2L}}\overline{e}^{(0)}\gamma^\mu\left((1-4\sin^2\theta)-\gamma^5\right)e^{(0)} Z^{(n)}_\mu(x^\mu) I^n \\ &- \frac{e^{(5)}}{\sqrt{2L}}\overline{e}^{(0)}\gamma^\mu e^{(0)} A^{(n)}_\mu(x^\mu) I^n\Bigg] \ . \end{aligned} \quad (4.30)$$

The integration over y is included in I^n. We obtain in (4.30) two different types of integrals I^n_{odd} if n is an odd number and I^n_{even} if n is an even number.

$$I^n_{odd} = \int_{-L}^{L} dy (f^{L,E(0)})^2 f^{(n)}_V = \frac{\mu}{e^{2\mu L}-1}\int_{-L}^{L} dy \ e^{2\mu|y|}\sin\left(\frac{n\pi}{2L}y\right) , \quad (4.31a)$$

$$I^n_{even} = \int_{-L}^{L} dy (f^{L,E(0)})^2 f^{(n)}_V = \frac{\mu}{e^{2\mu L}-1}\int_{-L}^{L} dy \ e^{2\mu|y|}\cos\left(\frac{n\pi}{2L}y\right) . \quad (4.31b)$$

They are defined, so that the overlap integral for $n = 0$ is equal to one and we can reproduce the SM vertex

$$I^0_{even} = \int_{-L}^{L} dy (f^{(0)})^2 f_V^{(0)} = 1 \ . \tag{4.32}$$

The integrals in (4.31a) and in (4.31b) are analytically solvable and the results are [1]

$$I^n_{odd} = 0 \ , \tag{4.33a}$$

$$I^{2m}_{even} = \frac{2\sqrt{2}\ x^2(\coth x - 1)((-1)^m \mathrm{e}^{2x} - 1)}{m^2\pi^2 + 4x^2} \quad \text{for } m = 1,2,3,4, \ldots \ , \tag{4.33b}$$

where $x = \mu L$, and we used that we can write $n = 2m$ in the even case. Now we can write (4.5) more generally as

$$g^{SM} = g^{eff}_{000} = \frac{g^{(5)}}{\sqrt{2L}} \cdot I^0_{even} \ . \tag{4.34}$$

Thereby g^{eff}_{000} is the effective coupling constant for an interaction between two zero mode fermions and one zero mode gauge boson $f_0 - f_0 - V_0$. In order to obtain the effective coupling constants of the interactions $f_0 - f_0 - V_n$ we have to generalize (4.34) as

$$g^{eff}_{002m} = \frac{g^{(5)}}{\sqrt{2L}} \cdot I^{2m}_{even} = g^{SM} \cdot I^{2m}_{even} \ . \tag{4.35}$$

Because of (4.33a), the coupling constants for odd n are equal to zero. The overlap integrals of (4.33b) are shown in Fig. 4.1. There we can see, that the limits of $I^{2m}_{even}(\mu L)$ are:

$$\lim_{\mu\to 0} I^{2m}_{even}(\mu L) = 0 \ ,$$
$$\lim_{\mu\to\infty} I^{2m}_{even}(\mu L) = (-1)^m\sqrt{2} \ . \tag{4.36}$$

The positive sign of I^{2m}_{even} for even m and the negative one for odd m has its reason in the factor $(-1)^m$.

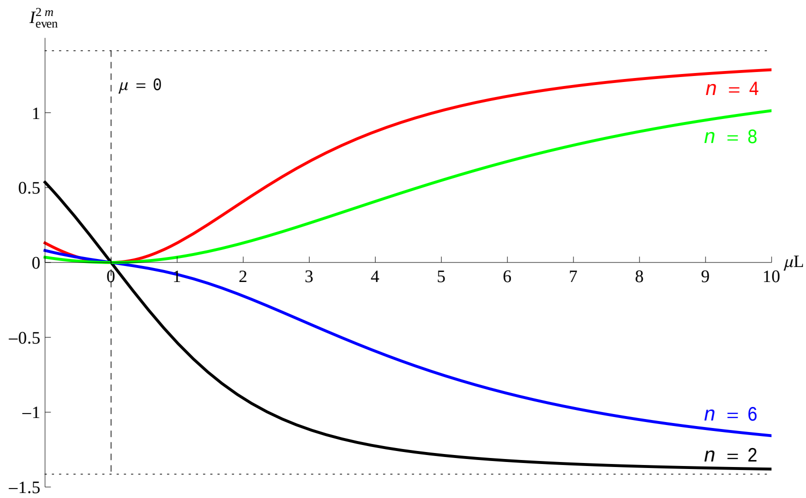

Figure 4.1: The overlap integrals for the vertices of two zero mode fermions with one even KK-level gauge boson as a function of the dimensionless quantity μL. Each line is labeled with the corresponding KK-level of the gauge boson. The dotted lines at $\pm\sqrt{2}$ are the asymptotic limits for $\mu \to \infty$ and the dashed line is the minimal UED case with $\mu = 0$. Couplings of zero mode fermions to odd KK-gaugemodes vanish due to the KK-parity (see text).

Chapter 5

Electroweak Precision Constraints

In this chapter we discuss how the measurements of the Peskin-Takeuchi parameters [18] can supply some restrictions on the parameters μ and R^{-1} of the sUED Model. First we introduce the Peskin-Takeuchi parameters to consider the oblique corrections of the gauge boson propagators. Afterwards we motivate the introduction of effective parameters to include the corrections coming from the additional vertices of zero mode leptons with even numbered KK mode gauge bosons. These non-oblique corrections play an important role as they modify the muon decay rate and thereby the interpretation of the Fermi constant G_μ. We compute the effective Peskin-Takeuchi parameters for the sUED Model and compare them with the current measured values [7]. This results an exclusion plot for the model parameters μ and R^{-1} in the last section of this chapter.

5.1 Peskin-Takeuchi parameters

The STU parameters got its names *Peskin Takeuchi parameter* after their introduction in [18]. They are used to quantify the electroweak corrections induced by physics beyond the SM which enter solely via the vacuum polarization diagrams (oblique corrections). In many models, including the SM, non-oblique corrections are suppressed. Hence, they are neglected.[1]

In every gauge boson propagator $G^{\mu\nu}_{XY}(q^2)$ we can extract the tensor structure. In this thesis we focus on vertices where the gauge bosons couple to

[1] A complete description of electroweak corrections is given in [19].

fermion lines. From this it follows, that in accordance to the Ward identities all terms q^μ and q^ν will vanish in the S-matrix calculation (see chapter 7 of [20]). We can write:

$$G^{\mu\nu}_{XY}(q^2) = -\mathrm{i}g^{\mu\nu}G_{XY}(q^2) \ . \tag{5.1}$$

These propagators G_{XY}, where the tensor structure is extracted, stand in relation with the one-particle irreducible self energies Π_{XY} through the first-order Dyson-equations [18]:

$$G_{\gamma\gamma} = D_{\gamma\gamma} + D_{\gamma\gamma}\Pi_{\gamma\gamma}D_{\gamma\gamma} \ , \tag{5.2a}$$
$$G_{\gamma Z} = D_{\gamma\gamma}\Pi_{\gamma Z}D_{ZZ} \ , \tag{5.2b}$$
$$G_{ZZ} = D_{ZZ} + D_{ZZ}\Pi_{ZZ}D_{ZZ} \ , \tag{5.2c}$$
$$G_{WW} = D_{WW} + D_{WW}\Pi_{WW}D_{WW} \ . \tag{5.2d}$$

Therein, the variables G_{XY}, D_{XY} and Π_{XY} have no tensor structure inside because of the extraction given in (5.1). D_{XX} are the bare propagators given by

$$D_{XX}(q^2) = \frac{1}{q^2 - m_X^2} \ , \tag{5.3}$$

where m_X is the corresponding bare mass of the gauge boson.
For low energies (in comparison to $q^2 = m_Z^2$) we can do a Taylor expansion of the self energies

$$\Pi_{XY}(q^2) \approx \Pi_{XY}(0) + q^2 \cdot \Pi'_{XY}(q^2)|_{q^2=0} \ , \tag{5.4}$$

with $\Pi'_{XY}(q^2) = d\Pi_{XY}/dq^2$. By using the Ward identities we obtain $\Pi_{\gamma\gamma}(0) = 0$ and $\Pi_{\gamma Z}(0) = 0$. So we write for the explicit possibilities of (5.4):

$$\Pi_{\gamma\gamma}(q^2) = q^2 \cdot \Pi'_{\gamma\gamma}(q^2)|_{q^2=0} \ , \tag{5.5a}$$
$$\Pi_{\gamma Z}(q^2) = q^2 \cdot \Pi'_{\gamma Z}(q^2)|_{q^2=0} \ , \tag{5.5b}$$
$$\Pi_{ZZ}(q^2) = \Pi_{ZZ}(0) + q^2 \cdot \Pi'_{ZZ}(q^2)|_{q^2=0} \ , \tag{5.5c}$$
$$\Pi_{WW}(q^2) = \Pi_{WW}(0) + q^2 \cdot \Pi'_{WW}(q^2)|_{q^2=0} \ . \tag{5.5d}$$

Hence, we have six Taylor series coefficients.

Three very precisely measured variables are: G_μ, m_Z and α. In general we can write these variables in terms of the six coefficients:

$$\begin{aligned} G_\mu &= G_\mu(\Pi'_{\gamma\gamma}, \Pi'_{\gamma Z}, \Pi_{ZZ}, \Pi'_{ZZ}, \Pi_{WW}, \Pi'_{WW}) \ , \\ m_Z &= m_Z(\Pi'_{\gamma\gamma}, \Pi'_{\gamma Z}, \Pi_{ZZ}, \Pi'_{ZZ}, \Pi_{WW}, \Pi'_{WW}) \ , \\ \alpha &= \alpha(\Pi'_{\gamma\gamma}, \Pi'_{\gamma Z}, \Pi_{ZZ}, \Pi'_{ZZ}, \Pi_{WW}, \Pi'_{WW}) \ . \end{aligned} \tag{5.6}$$

In order to fix all six Taylor series coefficients we need three additional functional relation besides the three given with (5.6). Peskin and Takeuchi define in [18] three linear combinations of the self energies and their derivatives. The original definition[2] in [18]

$$\alpha S = 4e^2 \left(\Pi'_{33}(0) - \Pi'_{3Q}(0)\right) \ , \tag{5.7a}$$

$$\alpha T = \frac{e^2}{\hat{s}_Z^2 \hat{c}_Z^2 M_Z^2} \left(\Pi_{11}(0) - \Pi_{33}(0)\right) \ , \tag{5.7b}$$

$$\alpha U = 4e^2 \left(\Pi'_{11}(0) - \Pi'_{33}(0)\right) \tag{5.7c}$$

is given in terms of the self energies for the $SU(2)$ gauge bosons $W^{1,2,3}$ and the photon. We can write S, T and U also in terms of the self energies of the mass eigenstates γ, Z, $W^{\pm}$ by a basis transformation (see appendix C) which yields

$$S = \frac{16\pi \hat{s}_Z^2}{e^2} \left(-\hat{c}_Z^2 \Pi'_{\gamma\gamma}(0) + \left(2\hat{s}_Z^2 \hat{c}_Z^2 - 1\right) \Pi'_{\gamma Z}(0) + \hat{c}_Z^2 \Pi'_{ZZ}(0)\right) \ , \tag{5.8a}$$

$$T = \frac{4\pi}{e^2 \hat{s}_Z^2 \hat{c}_Z^2} \left(\Pi_{WW}(0) - \hat{c}_Z^2 \Pi_{ZZ}(0)\right) \ , \tag{5.8b}$$

$$U = \frac{4\pi}{e^2 \hat{s}_Z^2 \hat{c}_Z^2} \left(\Pi'_{WW}(0) - \hat{c}_Z^2 \Pi'_{ZZ}(0)\right) \ , \tag{5.8c}$$

with the abbreviations $\hat{s}_Z = \sin\theta$ and $\hat{c}_Z = \cos\theta$ where θ is the rotation angle between the gauge and the mass eigenbasis[3].

If oblique corrections dominate in the investigated model, all theoretical predictions for physical observables $\mathcal{O}$ can be calculated as a function

$$\mathcal{O} = \mathcal{O}(\Pi'_{\gamma\gamma}, \Pi'_{\gamma Z}, \Pi_{ZZ}, \Pi'_{ZZ}, \Pi_{WW}, \Pi'_{WW}) \ .$$

By using the inverted relations of (5.6) and (5.8) it is possible to express this observable $\mathcal{O}$ as

$$\mathcal{O} = \mathcal{O}(\alpha(m_Z), G_\mu, m_Z, S, T, U) \ .$$

Before proceeding, we would like to mention two (pseudo) observables. These are $\hat{s}_Z$, the sine of the Weinberg-angle at the Z-pole (in the $\overline{MS}$ scheme), and m_W, the mass of the W boson. $\hat{s}_Z$ can be experimentally determined from the Z-pole asymmetries to be [7] $\hat{s}_Z^2 = 0.2313$. However, as mentioned

[2] Here, we use the abbreviation $\Pi'_{XY}(0) = \frac{d\Pi_{XY}}{dq^2}|_{q^2=0}$.

[3] For our later numerical analysis, we use $\hat{s}_Z^2 = 0.2313$ which is calculated in the $\overline{MS}$ scheme [7]. This value was also used in the electroweak fit to determine the parameters S, T and U [7]

before, it is possible to express all observables in terms of $\alpha(m_Z)$, G_μ , m_Z, S, T and U. The proper relation for $\hat{s}_Z$ is given by [18]

$$\hat{s}_Z^2 - s_0^2 = \frac{\alpha}{\hat{c}_Z^2 - \hat{s}_Z^2}\left(\frac{S}{4} - \hat{s}_Z^2 \hat{c}_Z^2 T\right) \quad , \tag{5.9}$$

where s_0 is defined in terms of $\alpha(m_Z)$, G_μ, m_Z via

$$s_0^2 c_0^2 = \frac{\pi\alpha(m_Z)}{\sqrt{2} G_\mu m_Z^2} \quad . \tag{5.10}$$

The experimental value $s_0 = 0.23108 \pm 0.00005$ [7] implies that $\left(\frac{S}{4} - \hat{s}_Z^2 \hat{c}_Z^2 T\right)$ is small. The second observable we would like to mention is m_W. Its value in terms of α, g_μ, m_Z, S, T, U is given in [18]:

$$\frac{m_W^2}{m_Z^2} - c_0^2 = \frac{\alpha \hat{c}_Z^2}{\hat{c}_Z^2 - \hat{s}_Z^2}\left(-\frac{S}{2} + \hat{c}_Z^2 T + \frac{\hat{c}_Z^2 - \hat{s}_Z^2}{4\hat{s}_Z^2} U\right) \quad . \tag{5.11}$$

The values $m_Z = 91.1876 \pm 0.0021$ GeV and $m_W = 80.420 \pm 0.031$ GeV [7] imply a small value for a linear combination of S, T and U.

We can determine the values S, T and U with a χ^2-fit to $\hat{s}_Z$, m_W and all other electroweak observables measured at LEP. This fit has been done in [7]. Therein the contributions were splitted in a Standard Model part and a Beyond Standard Model part, $S = S_{SM} + S_{BSM}$ and analogous for T and U. The Standard Model part has been taken into account at the two-loop level. The SM contribution depends on the - so far unknown - Higgs mass and the top mass. It was chosen $m_t = 173.0 \pm 1.3$ GeV and two reference Higgs masses for subtracting the SM part in S, T and U in order to obtain the BSM value. These values are

$$\begin{aligned} S_{BSM} &= 0.01 \pm 0.10 \\ T_{BSM} &= 0.03 \pm 0.11 \qquad \text{for } m_H = 117 \text{ GeV} \\ U_{BSM} &= 0.06 \pm 0.10 \end{aligned} \tag{5.12a}$$

$$\begin{aligned} S_{BSM} &= -0.07 \pm 0.10 \\ T_{BSM} &= 0.12 \pm 0.11 \qquad \text{for } m_H = 300 \text{ GeV} \\ U_{BSM} &= 0.07 \pm 0.10 \end{aligned} \tag{5.12b}$$

In order to receive Electroweak constraints on a BSM model which induces only oblique corrections, one can determine S, T and U as a function of the parameters of the BSM model and constrain these by a fit of the experimental values given in (5.12).

5.2 Non-oblique corrections in sUED

We perform the experimental matching between the theoretical variables M_Z, θ, α and the measured data with the following equations

$$m_{Z,exp} = M_Z \qquad \text{at } q^2 = M_Z^2 \ , \tag{5.13a}$$

$$\alpha_{exp}(m_Z) = \frac{g_2^2 \sin^2\theta}{4\pi} \qquad \text{at } q^2 = M_Z^2 \ , \tag{5.13b}$$

$$s_0^2 c_0^2 = \frac{\pi \alpha_{exp}(m_Z)}{\sqrt{2} G_{\mu,exp} m_{Z,exp}^2} \qquad \text{at } q^2 = m_\mu^2 \ . \tag{5.13c}$$

As we can see in (5.13) the measurement of the Fermi constant G_μ is done at a much lower energy $p^2 = m_\mu^2$ than the two measurements for $\alpha(m_Z)$ and m_Z. G_μ is measured from the muon decay. In the SM, there is only one possible diagram at tree-level (shown on the left side of Fig. 5.1) with the SM W boson as the exchanging particle. In theories beyond SM there are in general more candidates for the exchange. For example in the sUED Model all even numbered higher KK-modes of the W contribute to the decay. The corresponding Feynman diagram is shown on the right hand side of Fig. 5.1.

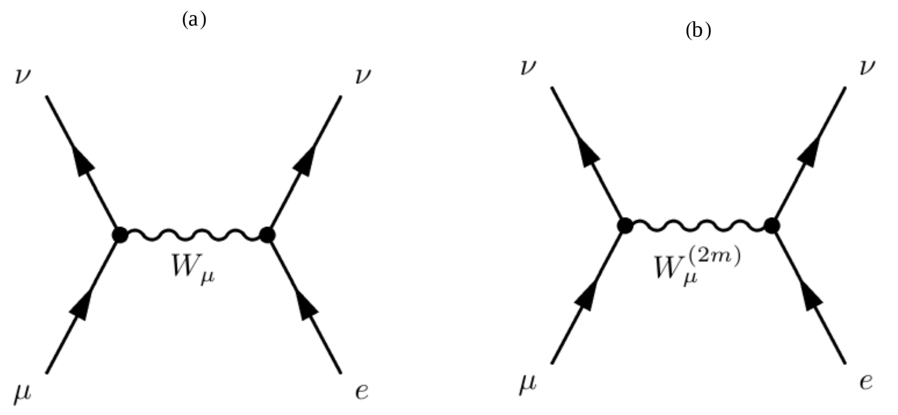

Figure 5.1: The Feynman diagrams of the muon decay. (a) The only possible diagram in the Standard Model. (b) The additional diagrams for sUED where the KK modes of the W boson couple to the muon.

From this it follows, that in sUED we have non-oblique corrections at tree-level because of the additional exchanging particles (see Fig. 5.1 (b)). For measurements at the Z-pole, the W-propagator is near its resonance, and the contributions of the W-KK-modes are therefore suppressed and can be neglected. For the measurement of G_μ, which is performed at $m_\mu \ll m_W$, neither the W-propagator nor the W-KK-mode-propagators are near resonance. Therefore, the contributions of the $2m$th KK-modes to the amplitude are only suppressed by $\mathcal{O}\left[\left(m_W^2/(m_W^{(2m)})^2\right)^2\right]$ and represent the leading non-oblique correction.

In order to determine G_μ from the muon decay we have to introduce an effective W boson propagator $G_{WW}(q^2)$ which include the W boson zero mode as well as the higher KK-modes. We can parametrize these propagator in analogy to [6] as

$$G_{WW}(q^2) = G_{WW}^{obl}(q^2) + G_{WW}^{5D}(q^2) \ , \tag{5.14}$$

where $G_{WW}^{obl}(q^2)$ comes from Fig. 5.1 (a) and describes the W-propagator including oblique corrections, while $G_{WW}^{5D}(q^2)$ comes from Fig. 5.1 (b) which describes the non-oblique corrections. The effective propagator $G_{WW}(q^2)$ of the W boson is related to the Fermi constant in the following equation:

$$-\,G_{WW}(0) = \frac{4\sqrt{2}}{g_2^2} G_\mu \ . \tag{5.15}$$

We can state an analogous relation for the oblique part G_μ^{SM}:

$$-\,G_{WW}^{obl}(0) = \frac{4\sqrt{2}}{g_2^2} G_\mu^{obl} \ . \tag{5.16}$$

With the theoretical value G_μ^{obl} we can also modify the matching relation (5.13c) as follows

$$m_{Z,exp}^2 = \frac{\pi \alpha_{exp}(m_Z)}{\sqrt{2} G_\mu^{obl} (s_0^{obl})^2 (c_0^{obl})^2} \ , \tag{5.17}$$

where G_μ^{obl} is related to the full Fermi constant through $G_\mu = G_\mu^{obl} + \delta G_\mu$. $m_{Z,exp}$, $\alpha_{exp}(m_Z)$ are the fixed experimental values which are the same as in (5.13c). s_0^{obl} and c_0^{obl} then follow from (5.17). The relation between (5.13c) and (5.17) gives us the general values s_0 and c_0 in terms of s_0^{obl} and c_0^{obl}. We obtain:

$$s_0^2 c_0^2 = (s_0^{obl})^2 (c_0^{obl})^2 \left(1 - \frac{\delta G_\mu}{G_\mu^{obl}} + \mathcal{O}\left[(\frac{\delta G_\mu}{G_\mu^{obl}})^2\right]\right) \ . \tag{5.18}$$

Due to the fact that the non-oblique corrections must be small, we express s_0^2 as

$$s_0^2 = (s_0^{obl})^2 + \delta s_0^2 \ . \tag{5.19}$$

We can combine (5.18) and (5.19) in order to determine δs_0^2. The result is:

$$\delta s_0^2 = -\frac{(s_0^{obl})^2 (c_0^{obl})^2}{(c_0^{obl})^2 - (s_0^{obl})^2} \cdot \frac{\delta G_\mu}{G_\mu^{obl}} \tag{5.20}$$

Now we calculate the difference $\hat{s}_Z^2 - s_0^2$. It is:

$$\begin{aligned} \hat{s}_Z^2 - s_0^2 &= \hat{s}_Z^2 - (s_0^{obl})^2 - \delta s_0^2 \\ &= \hat{s}_Z^2 - (s_0^{obl})^2 + \frac{(s_0^{obl})^2 (c_0^{obl})^2}{(c_0^{obl})^2 - (s_0^{obl})^2} \cdot \frac{\delta G_\mu}{G_\mu^{obl}} \end{aligned} \tag{5.21}$$

and with the assumption that the corrections of s are small, we can write:

$$\hat{s}_Z^2 - s_0^2 = \hat{s}_Z^2 - (s_0^{obl})^2 + \frac{\hat{s}_Z^2 \hat{c}_Z^2}{\hat{c}_Z^2 - \hat{s}_Z^2} \cdot \frac{\delta G_\mu}{G_\mu^{obl}} \ . \tag{5.22}$$

Afterwards we can identify the difference $\hat{s}_Z^2 - (s_0^{obl})^2$, which include only oblique corrections, with equation (5.9). It follows

$$\hat{s}_Z^2 - s_0^2 = \frac{\alpha}{\hat{c}_Z^2 - \hat{s}_Z^2} \left(\frac{S}{4} - \hat{s}_Z^2 \hat{c}_Z^2 T \right) + \frac{\hat{s}_Z^2 \hat{c}_Z^2}{\hat{c}_Z^2 - \hat{s}_Z^2} \cdot \frac{\delta G_\mu}{G_\mu^{obl}} \ . \tag{5.23}$$

Carena etal. defined in [6] the effective S, T and U parameters as

$$S_{eff} = S \ , \tag{5.24a}$$
$$T_{eff} = T + \Delta T \ , \tag{5.24b}$$
$$U_{eff} = U + \Delta U \ . \tag{5.24c}$$

If we use these definition of the effective S, T and U parameters from [6] (5.23) can be rewritten as

$$\hat{s}_Z^2 - s_0^2 = \frac{\alpha}{\hat{c}_Z^2 - \hat{s}_Z^2} \left(\frac{S_{eff}}{4} - \hat{s}_Z^2 \hat{c}_Z^2 T_{eff} \right) \ . \tag{5.25}$$

Therein ΔT is defined as:

$$\Delta T = -\frac{1}{\alpha} \frac{\delta G_\mu}{G_\mu^{obl}} \ . \tag{5.26}$$

The non-oblique corrections to G_μ which lead to the introduction of the effective T parameter T_{eff} also affect the relation m_W/m_Z (given for oblique corrections in (5.11)). Therefore we have to express c_0^2 as:

$$c_0^2 = (c_0^{obl})^2 + \delta c_0^2 \quad \text{and} \quad c_0^2 = 1 - s_0^2 \ , \tag{5.27}$$

and from this it follows with (5.19)

$$c_0^2 = (c_0^{obl})^2 - \delta s_0^2 \ . \tag{5.28}$$

This can be used to obtain

$$\frac{m_W^2}{m_Z^2} - c_0^2 = \frac{m_W^2}{m_Z^2} - (c_0^{obl})^2 + \delta s_0^2 \tag{5.29}$$

In analogy to (5.23) there is an additional term inside (because of the non-oblique corrections). The difference $\frac{m_W^2}{m_Z^2} - (c_0^{obl})^2$ can be identified with (5.11) and so we can write:

$$\frac{m_W^2}{m_Z^2} - c_0^2 = \frac{\alpha \hat{c}_Z^2}{\hat{c}_Z^2 - \hat{s}_Z^2}\left(-\frac{S}{2} + \hat{c}_Z^2 T + \frac{\hat{c}_Z^2 - \hat{s}_Z^2}{4\hat{s}_Z^2} U\right) + \frac{\hat{s}_Z^2 \hat{c}_Z^2}{\hat{c}_Z^2 - \hat{s}_Z^2} \cdot \frac{\delta G_\mu}{G_\mu^{obl}} \ . \tag{5.30}$$

By using the effective S, T and U parameters defined in (5.24) and the definition of ΔT in (5.26) we can rewrite (5.30) as

$$\frac{m_W^2}{m_Z^2} - c_0^2 = \frac{\alpha \hat{c}_Z^2}{\hat{c}_Z^2 - \hat{s}_Z^2}\left(-\frac{S_{eff}}{2} + \hat{c}_Z^2 T_{eff} + \frac{\hat{c}_Z^2 - \hat{s}_Z^2}{4\hat{s}_Z^2} U_{eff}\right) \ , \tag{5.31}$$

where ΔU is

$$\Delta U = -4\hat{s}_Z^2 \Delta T = \frac{4\hat{s}_Z^2}{\alpha}\frac{\delta G_\mu}{G_\mu^{obl}} \ . \tag{5.32}$$

Now we are able to relate the non-oblique corrections to G_μ to shift in the T and U parameter. Therewith, every operator in a theory with only oblique corrections and corrections to G_μ can be written as $\mathcal{O}(\alpha, G_\mu, m_Z, \delta G_\mu, S, T, U)$, and via the definition (5.24) and the relations (5.26) and (5.32) it can be expressed as $\mathcal{O}(\alpha, G_\mu, m_Z, S_{eff}, T_{eff}, U_{eff})$ up to corrections $\mathcal{O}(\left[(\frac{\delta G_\mu}{G_\mu^{obl}})^2\right])$ which agree with the expressions for an oblique theory with (S, T, U) $\rightarrow$ (S_{eff}, T_{eff}, U_{eff}). From ref. [7], the fit to the LEP data yielded the values for S, T and U given in (5.12), which for the sUED Model considered in this thesis are to be interpreted as bounds on S_{eff}, T_{eff}, U_{eff}.

In the next section,, we will determine the effective Peskin-Takeuchi parameters as functions of the model parameters R^{-1} and μ and perform a fit to the experimental bounds in (5.12) in order to obtain the R^{-1}-μ parameter space which is in accordance with the electroweak precision measurements at LEP.

5.3 Comparison with data

In order to compute the effective parameters S_{eff}, T_{eff} and U_{eff} for our sUED Model we follow the discussion in [6]. The effective Lagrangian with a parametrization of the oblique corrections is:

$$\begin{aligned}\mathcal{L} = &-\frac{1}{2}(1-\Pi'_{WW})W^{+}_{\mu\nu}W^{\mu\nu}_{-} - \frac{1}{4}(1-\Pi'_{ZZ})Z_{\mu\nu}Z^{\mu\nu} \\ &-\frac{1}{4}(1-\Pi'_{\gamma\gamma})A_{\mu\nu}A^{\mu\nu} - \frac{1}{2}\Pi'_{\gamma Z}A^{\mu\nu}Z_{\mu\nu} \\ &-\Big(\frac{g_2^2\widetilde{v}^2}{4} + \Pi_{WW}(0)\Big)W^{+}_{\mu}W^{\mu}_{-} \\ &-\frac{1}{2}\Big(\frac{(g_1^2+g_2^2)\widetilde{v}^2}{4} + \Pi_{ZZ}(0)\Big)Z_{\mu}Z^{\mu} \\ &+ \text{vertex terms} \ , \end{aligned} \tag{5.33}$$

where $\widetilde{v}$ is the effective vacuum expectation value (vev) given by:

$$-G_{WW}(0) = \frac{4}{g_2^2\widetilde{v}^2} \ . \tag{5.34}$$

The vertex terms are given in (4.27). $\Pi_{XX}(q^2)$ are the self energies of the gauge bosons with there derivatives $\Pi'_{XX} = \mathrm{d}\Pi_{XX}/\mathrm{d}q^2$.
Comparing the equation (5.33) with the Lagrangian of the sUED Model (4.2a) and (4.2d) we find at tree-level:

$$\begin{aligned}&\Pi'_{\gamma\gamma} = 0 \ , \ \Pi'_{WW} = 0 \ , \\ &\Pi'_{\gamma Z} = 0 \ , \ \Pi'_{ZZ} = 0 \ . \end{aligned} \tag{5.35}$$

From this it follows that the Peskin-Takeuchi parameter S and U do not contain tree level contributions (see (5.8)). For computing T with (5.8b) we need the self energies of the lightest mode of the W boson and the Z boson. They can be determined by a consideration analogous to chapter 7 of [20]. The tree-level results are:

$$\Pi_{WW}(0) = -m_W^2 \ , \tag{5.36a}$$

$$\Pi_{ZZ}(0) = -m_Z^2 \ . \tag{5.36b}$$

We find with (5.8b): $T = 0$, so that all Peskin-Takeuchi parameters are equal to zero at tree-level. By using these results we can write the effective parameters given in (5.24) as

$$\begin{aligned} S_{eff} &= 0 \ , \\ T_{eff} &= \Delta T \ , \\ U_{eff} &= \Delta U = -4\hat{s}_Z^2 \Delta T \ , \end{aligned} \tag{5.37}$$

where ΔT is defined in (5.26) and $\hat{s}_Z^2$ is given on page 30.

The full effective W boson propagator G_{WW} (see (5.14)) for the sUED Model is:

$$-G_{WW}(q^2 = 0) = \frac{1}{m_W^2} + \sum_{m=1}^{\infty} \frac{(I^{2m})^2}{m_W^2 + \left(\frac{2m}{R}\right)^2} \ , \tag{5.38}$$

where I^{2m} denotes the overlap integrals calculated in (4.33b). In analogy to (5.14) we can split G_{WW} into two parts, a SM part and an additional 5D part:

$$-G_{WW}^{SM}(0) = \frac{1}{m_W^2} \ , \tag{5.39a}$$

$$-G_{WW}^{5D}(0) = \sum_{m=1}^{\infty} \frac{(I^{2m})^2}{m_W^2 + \left(\frac{2m}{R}\right)^2} \ . \tag{5.39b}$$

From G_{WW}^{SM} and G_{WW}^{5D} we obtain via (5.26) and (5.32)

$$T_{eff} = \Delta T = -\frac{1}{\alpha} \frac{G_\mu^{5D}}{G_\mu^{SM}} = -\frac{m_W^2}{\alpha} \cdot \sum_{m=1}^{\infty} \frac{(I^{2m})^2}{m_W^2 + \left(\frac{2m}{R}\right)^2} \quad \text{and} \tag{5.40a}$$

$$U_{eff} = \Delta U = \frac{4\hat{s}_Z^2}{\alpha} \frac{G_\mu^{5D}}{G_\mu^{SM}} = \frac{4\hat{s}_Z^2 m_W^2}{\alpha} \cdot \sum_{m=1}^{\infty} \frac{(I^{2m})^2}{m_W^2 + \left(\frac{2m}{R}\right)^2} \ . \tag{5.40b}$$

These results depend on R^{-1} and, via the overlap integrals (4.33b), on the 5D fermion mass μ.

By comparing our sUED results (5.40a) and (5.40b) to the experimental constraints on T_{eff} and U_{eff} which are given in (5.12a) for a Higgs mass of 117 GeV and in (5.12b) for $m_H = 300$ GeV, we obtain constraints on the sUED parameter space. Fig. 5.2 (left) shows the constraints from T_{eff} and (right) from U_{eff} on the R^{-1}-μ parameter space. Inside the light gray (dark gray) region T_{eff} and U_{eff} have 1σ - 2σ (2σ - 3σ) discrepancy from the measured values. Inside the black region the parameters have more than 3σ deviation from the measured data.

Comparing the T-plot and the U-plot in Fig. 5.2 we can see that the constraints from ΔT are stronger. This can be understood because

$$\Delta U = -4\hat{s}_Z^2 \Delta T \approx -0.9 \cdot \Delta T \ . \tag{5.41}$$

This means that the theoretical expected values of ΔU and ΔT have different signs, ΔT is negative (see (5.26)) and ΔU is positive. However, the measured T and U in (5.12) have the same signs. So the restrictions inside the R^{-1}-μ parameter space should be smaller in the ΔU-plot. This is exactly what we can see in Fig. 5.2.

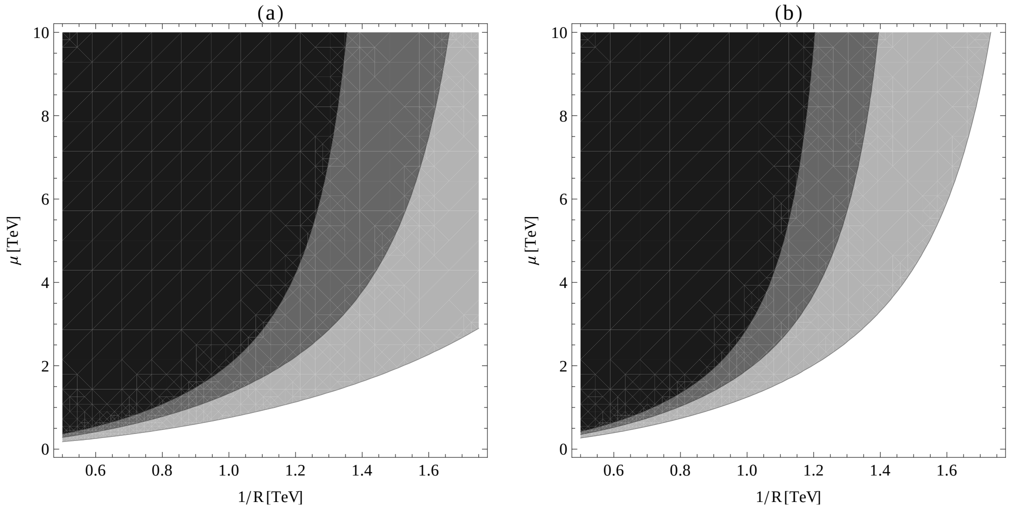

Figure 5.2: (R^{-1},μ) sUED parameter space contraints from (a) T_{eff} and (b) U_{eff} with a chosen Higgs mass $m_H = 117$ GeV. The contour lines are the 1σ (white to light gray boarder), 2σ (light gray to dark gray boarder) and 3σ (dark gray to black boarder) errors for the experimental data from [7]. In the respective areas $|T_{eff} - T_{eff}^{exp}|$ deviates by $< 1\sigma$ (white), 2σ (light gray), 3σ (dark gray).

Fig. 5.3 (left) shows the constraints from T_{eff} and (right) from U_{eff} on the R^{-1}-μ parameter space analogous to Fig. 5.2 but for $m_H = 300$ GeV. Again, the T constraints are stronger, but this time even more. This can be understood because of the fact, that for $m_H = 300$ GeV it is $T_{exp} > U_{exp}$ (see (5.12b)). As mentioned before, the theoretical expected value of ΔT is negative. Accordingly, the constraints from the T-plot are much stronger.

Finally, the constraints getting from Fig. 5.2 and Fig. 5.3 on the R^{-1}-μ parameter space with the current electroweak precision measurements at LEP

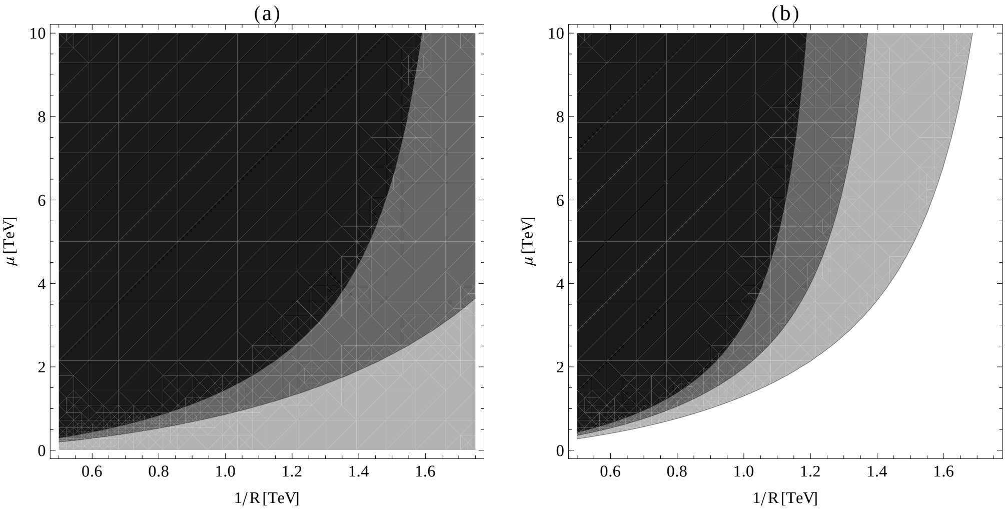

Figure 5.3: (R^{-1},μ) sUED parameter space contraints from (a) T_{eff} and (b) U_{eff} with a chosen Higgs mass $m_H = 300$ GeV. The contour lines are the 1σ (white to light gray boarder), 2σ (light gray to dark gray boarder) and 3σ (dark gray to black boarder) errors for the experimental data from [7]. In the respective areas $|T_{eff} - T_{eff}^{exp}|$ deviates by $< 1\sigma$ (white), 2σ (light gray), 3σ (dark gray).

[7] are much stronger than in the literature, so far [1]. One reason is, that we considered the non-oblique corrections to G_μ caused by additional terms inside the effective W-propagator.

Chapter 6

Conclusions

In this thesis we studied the sUED Model, an extension of the minimal UED Model with one additional uniform fermion mass parameter. The model was introduced by Park and Shu in [11, 12]. A short motivation for this extension is given in section 2.2. In comparison to mUED, only the fermion part of the Lagrangian modified. A detailed description of the full Lagrangian with its several components was given in section 4.1. For the conventions and notations used, see also appendix A.

The corresponding bulk action for the fermions (see (3.1)) was the starting point for derivation of the equations of motion (EOMs). After a variation of the fermion fields we obtained the EOMs in (3.8) and (3.9). These equations of motion are different for the two parts ($y > 0$ and $y < 0$) of the domain because of the KK-odd profile of the fermion mass parameter, defined in (3.3). As a first step we discussed the solution of the EOMs for the zero modes, where $m_n = 0$. Since the zero modes are identified with the SM fields these solutions have to be chiral. Therefore we chose Dirichlet boundary conditions for the right-handed wave function and so we received a left-handed zero mode with the boundary condition given in (3.14). The solution of the differential equations with these BCs is shown in (3.17) and for the opposite case with Dirichlet-BCs on the left-handed mode the result is given in (3.19). Then, we determined the wave function profiles and masses of the KK excitations which are given in (3.40), (3.54) for KK partners of a left-handed zero mode and in (3.58), (3.59) for KK partners of a right-handed zero mode. For convenience, these results are summarized in appendix B.

As the second step we gave a detailed description of the sUED Model Lagrangian (see (4.1) and (4.2)). By using the zero mode profiles of the left-handed fermions (3.17) and the KK towers for the gauge bosons (given in

(4.28)), we computed the effective couplings of the zero mode fermions to KK mode gauge fields which differ from the couplings in the SM as well as in the mUED case. These couplings are defined in (4.35) with the sUED-specific overlap integrals from (4.31) and respectively (4.33). The overlap integrals and thereby also the effective 4D coupling depend on the fermion mass parameter μ as shown in Fig. 4.1.

The final step of this thesis was the calculation of the constraints on the sUED Model which result from electroweak measurements. In section 5.1 we gave a short introduction of the Peskin-Takeuchi parameters, their definition in (5.7) and an explanation why they can be used to parametrize oblique corrections to electroweak measurements. Not all corrections in sUED are oblique, but when the mass parameter μ is chosen equal for all quarks and leptons, they are universal. As discussed in section 5.2 the Peskin-Takeuchi parameters can be extended to effective S, T, U parameters as first proposed in [6]. This was necessary to consider not only the oblique corrections but also the vertex corrections which are not negligible in sUED. The effective Peskin-Takeuchi parameters are defined in (5.24), and the specific expressions for sUED are given in (5.40). Comparing the sUED STU parameters of (5.40) with the experimental values inferred from electroweak measurements at the Z-pole [7], we were able to constrain the μ-R^{-1} parameter space of sUED as shown in Fig. 5.2 and Fig. 5.3. The constraints presented here exceed the bounds known from the literature so far [1]. In particular major parts of the parameter region at which sUED signals would be measurable at LHC in the dilepton channel as studied in [1] are excluded.

Appendix A

Conventions

In this appendix we specify the conventions which are used inside the whole thesis. All equations are given in natural units with $\hbar = c = 1$. Thereby $\hbar$ is the reduced Planck constant and c is the speed of light.

We use two metrics, the Minkowski-metric

$$g^{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} , \tag{A.1}$$

and the metric for mUED and sUED

$$g^{MN} = \begin{pmatrix} 1 & & & & \\ & -1 & & & \\ & & -1 & & \\ & & & -1 & \\ & & & & -1 \end{pmatrix} . \tag{A.2}$$

Furthermore, the Einstein summation convention is used. Capital Arabic letters are running over $M = 0, 1, 2, 3, 5$ and small Greek letters over $\mu = 0, 1, 2, 3$.

The additional fifth dimension inside the UED models is compactified on a $S^1/\mathbb{Z}_2$ orbifold. As the domain we choose the same convention as Park and Shu did in [11]: y $\in$ [-L,L] where L is given by $L = \pi R/2$. R is the so-called compactification radius. For a five dimensional theory we also have to expand the gamma matrices. Therefore we choose $\Gamma^M = (\gamma^\mu, \mathrm{i}\gamma^5)$. These Γ^M satisfy the Clifford algebra in 5D: $\Gamma^M, \Gamma^N = 2g^{MN}$ with g^{MN} given in (A.2).

Appendix B

Fermion solutions

In this chapter we give a short summary of the main results in chapter 3. The bulk action for the sUED model with its domain $-L \leq y \leq L$ (where $L = \frac{\pi R}{2}$) is given by:

$$S = \int d^4x \int_{-L}^{+L} dy \left[\mathrm{i}\bar{\Psi}\Gamma^M \partial_M \Psi - m_5(y)\bar{\Psi}\Psi) \right] \ . \tag{B.1}$$

Hence, the equations of motion (EOM) are

$$\mathrm{i}\gamma^\mu \partial_\mu \Psi_R + \partial_5 \Psi_L - m_5(y)\Psi_L = 0 \ , \tag{B.2a}$$

$$\mathrm{i}\gamma^\mu \partial_\mu \Psi_L - \partial_5 \Psi_R - m_5(y)\Psi_R = 0 \ , \tag{B.2b}$$

where $m_5(y)$ is the fermion mass parameter which is characteristic for sUED. Ψ_L and Ψ_R are the left- and right-handed fermion wave functions depending on x^μ and y.

After performing a separation ansatz we obtain the differential equations for the left- and right-handed functions $f_L^{(n)}$ and $f_R^{(n)}$ which depend only on y:

$$m_n f_R^{(n)}(y) + (\partial_5 - m_5(y)) f_L^{(n)}(y) = 0 \ , \tag{B.3a}$$

$$m_n f_L^{(n)}(y) - (\partial_5 + m_5(y)) f_R^{(n)}(y) = 0 \ . \tag{B.3b}$$

Therein m_n is the separation constant defined as:

$$\frac{\mathrm{i}\partial\!\!\!/ \Psi_R(x^\mu)}{\Psi_L(x^\mu)} = \frac{\mathrm{i}\partial\!\!\!/ \Psi_L(x^\mu)}{\Psi_R(x^\mu)} = m_n \ . \tag{B.4}$$

The solutions of (B.3a) and (B.3b) are depending on the choice of the boundary conditions. The zero modes have a special shape. Furthermore, the solutions of the differential equations for the zero modes are much simpler. For

Dirichlet-BCs on the right-handed mode the zero modes are:

$$f_L^{(0)} = N_L \cdot \mathrm{e}^{\mu|y|} \;\; \text{with} \;\; N_L = \sqrt{\frac{\mu}{\mathrm{e}^{2\mu L} - 1}} \;\; ,$$
$$f_R^{(0)} = 0 \;\; . \tag{B.5}$$

For Dirichlet-BCs on the left-handed mode the zero modes are:

$$f_R^{(0)} = N_R \cdot \mathrm{e}^{-\mu|y|} \;\; \text{with} \;\; N_R = \sqrt{\frac{-\mu}{\mathrm{e}^{-2\mu L} - 1}} \;\; ,$$
$$f_L^{(0)} = 0 \;\; . \tag{B.6}$$

The KK mode wave functions for Dirichlet-BCs on right-handed mode are

$$f_L^{(n)}(y) = \frac{1}{\sqrt{L - \frac{\sin(2k_n L)}{2k_n}}} \sin(k_n y) \;\; ,$$
$$f_R^{(n)}(y) = \frac{1}{\sqrt{L(1+\frac{\mu^2}{k_n^2}) + \frac{\sin(2k_n L)}{k_n}(1-\frac{\mu^2}{k_n^2}) - \frac{2\mu}{k_n^2}\sin^2(k_n L)}} \cdot \Big(\cos(k_n y) - \frac{\mu}{k_n}\sin(k_n|y|)\Big) \tag{B.7}$$

for n odd, and

$$f_L^{(n)}(y) = \frac{k_n}{m_n\sqrt{L}} \cdot \cos(k_n y) + \frac{\mu}{m_n\sqrt{L}} \cdot \sin(k_n|y|) \;\; ,$$
$$f_R^{(n)}(y) = \frac{1}{\sqrt{L}} \cdot \sin(k_n y) \;\; . \tag{B.8}$$

for n even. For Dirichlet-BCs on left-handed mode we obtain for odd n's

$$f_L^{(n)}(y) = \frac{1}{\sqrt{L(1+\frac{\mu^2}{k_n^2}) + \frac{\sin(2k_n L)}{k_n}(1-\frac{\mu^2}{k_n^2}) + \frac{2\mu}{k_n^2}\sin^2(k_n L)}} \cdot \Big(\cos(k_n y) + \frac{\mu}{k_n}\sin(k_n|y|)\Big) \;\; ,$$
$$f_R^{(n)}(y) = \frac{1}{\sqrt{L - \frac{\sin(2k_n L)}{2k_n}}} \sin(k_n y) \;\; . \tag{B.9}$$

and

$$f_L^{(n)}(y) = \frac{1}{\sqrt{L}} \cdot \sin(k_n y) \;\; ,$$
$$f_R^{(n)}(y) = -\frac{k_n}{m_n\sqrt{L}} \cdot \cos(k_n y) + \frac{\mu}{m_n\sqrt{L}} \cdot \sin(k_n|y|) \;\; . \tag{B.10}$$

for n even.

Appendix C

Basis transformation matrix

Peskin and Takeuchi define the S, T and U parameters in [18] as follows:

$$S = 16\pi\left(\Pi'_{33}(0) - \Pi'_{3Q}(0)\right) \quad , \tag{C.1a}$$
$$T = 16\pi\left(\Pi_{11}(0) - \Pi_{33}(0)\right) \quad , \tag{C.1b}$$
$$U = 16\pi\left(\Pi'_{11}(0) - \Pi'_{33}(0)\right) \quad . \tag{C.1c}$$

The chosen basis (XY)=(QQ),(3Q),(33),(11) for the self energies in (C.1) is not very convenient for our calculations. Therefore we can transform the self energies into the mass eigenbasis. It applies:

$$\Pi_{\gamma\gamma} = e^2\Pi_{QQ} \quad , \tag{C.2a}$$
$$\Pi_{\gamma Z} = \frac{e^2}{sc}\Pi_{3Q} - \frac{se^2}{c}\Pi_{QQ} \quad , \tag{C.2b}$$
$$\Pi_{ZZ} = \frac{e^2}{s^2c^2}\Pi_{33} - \frac{2e^2}{c^2}\Pi_{3Q} + \frac{s^2e^2}{c^2}\Pi_{QQ} \quad , \tag{C.2c}$$
$$\Pi_{WW} = \frac{e^2}{s^2}\Pi_{11} \tag{C.2d}$$

and the inverse relations are:

$$\Pi_{QQ} = \frac{1}{e^2}\Pi_{\gamma\gamma} \quad , \tag{C.3a}$$
$$\Pi_{3Q} = \frac{s^2}{e^2}\Pi_{\gamma\gamma} + \frac{sc}{e^2}\Pi_{\gamma Z} \quad , \tag{C.3b}$$
$$\Pi_{33} = \frac{s^4}{e^2}\Pi_{\gamma\gamma} + \frac{2s^3c}{e^2}\Pi_{\gamma Z} + \frac{s^2c^2}{e^2}\Pi_{ZZ} \quad , \tag{C.3c}$$
$$\Pi_{11} = \frac{s^2}{e^2}\Pi_{WW} \quad . \tag{C.3d}$$

From (C.2) and (C.3) we can derive the transformations matrices from 'Peskin-Takeuchi basis' into the mass eigenbasis M_1 and vice versa M_2. These are:

$$M_1 = \begin{pmatrix} e^2 & 0 & 0 & 0 \\ -\frac{se^2}{c} & \frac{e^2}{sc} & 0 & 0 \\ \frac{s^2e^2}{c^2} & -\frac{2e^2}{c^2} & \frac{e^2}{s^2c^2} & 0 \\ 0 & 0 & 0 & \frac{e^2}{s^2} \end{pmatrix} , \tag{C.4a}$$

$$M_2 = \begin{pmatrix} \frac{1}{e^2} & 0 & 0 & 0 \\ \frac{s^2}{e^2} & \frac{sc}{e^2} & 0 & 0 \\ \frac{s^4}{e^2} & \frac{2s^3c}{e^2} & \frac{s^2c^2}{e^2} & 0 \\ 0 & 0 & 0 & \frac{s^2}{e^2} \end{pmatrix} . \tag{C.4b}$$

It is also possible to use the transformation matrices M_1 and M_2 to transform the Π' from Peskin-Takeuchi basis into mass eigenbasis and vice versa. We receive the transformation equations for the Π' from (C.2) and (C.3) by the use of a derivative.

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Erklärung

Gemäß der allgemeinen Studien- und Prüfungsordnung für den Diplomstudiengang Physik an der Julius-Maximilians-Universität Würzburg erkläre ich hiermit, dass ich diese Arbeit selbstständig verfasst, keine anderen als die angegebenen Quellen und Hilfsmittel verwendet habe und die Arbeit bisher keiner anderen Prüfungsbehörde unter Erlangung eines akademischen Grades vorgelegt wurde.

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