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Field-theory aspects of condensed matter physics

Examples III

To be discussed Tuesday 3rd December in the examples class

I. Conformal anomaly in $1+1$ dimensions

a) In complex coordinates, the two-point function of the energy-momentum tensor in a 1+1-dimensional field theory is given by

$$\langle T(z)T(w)\rangle = \frac{c/2}{(z-w)^4}\,. \tag{1}$$

Using the fact that the components of the energy-momentum tensor may be expanded in terms of the Virasoro generators,

$$T(z) = \sum_{n\in\mathbb{Z}} z^{-n-2}L_n\,, \qquad L_n = \frac{1}{2\pi i}\oint dz z^{n+1}T(z)\,, \tag{2}$$

show that (1) is compatible with the Virasoro algebra.

b) Show that in real Euclidean coordinates, the expression (1) may be rewritten as

$$\langle T_{\mu\nu}(x)T_{\sigma\rho}(y)\rangle = -\frac{c}{48\pi^2}S^x_{\mu\nu}S^y_{\sigma\rho}\ln((x-y)^2\mu^2)\,, \qquad S_{\mu\nu} = \partial_\mu\partial_\nu - \delta_{\mu\nu}\delta^2\,, \tag{3}$$

with $T(z) = -2\pi T_{zz}(x)$.

c) Take the trace of the expression (3) using the result of Gelfand+Shilov according to which, as a distribution,

$$\frac{1}{2\lambda} \sim \frac{1}{d-2\lambda}\frac{2\pi^{d/2}}{\Gamma(d/2)}\delta^{(d)}(x)\,. \tag{4}$$

Thus confirm that the conformal two-point function for the energy-momentum tensor satisfies the anomalous conformal Ward identity.

II. Berry phase and holonomy

a) Recapitulate the definition of the Berry phase in physics. Moreover, state the mathematical definition of a *holonomy* and discuss the relation between both notions.

b) Calculate the Berry phase for a two-level system, i.e. consider the Hamiltonian

$$H = R_x\sigma_x + R_y\sigma_y + R_z\sigma_z \tag{5}$$

with σ the Pauli matrices and $\vec{R} \in \mathbb{R}^3 \setminus \{0\}$, with eigenstates

$$H|\pm\rangle = \pm|\pm\rangle\,. \tag{6}$$

Write the eigenstates in spherical coordinates and discuss the gauge fixing. Calculate the Berry phase in terms of $\vec{R}$ in any of the gauges.