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Field-theory aspects of condensed matter physics

Examples II

To be discussed Tuesday 19th November in the examples class

I. **Axial anomaly in $1 + 1$ dimensions**

a) In a 1+1-dimensional theory of free fermions, define the vector current J^V_μ and the axial current J^A_μ. The Ward identity for the axial anomaly reads

$$\partial^\mu {J^A}_\mu = k\, \epsilon^{\mu\nu} F_{\mu\nu}\,, \tag{1}$$

where the vector current is coupled to an external source field A^μ. Explain the significance of each term in this equation. Determine the engineering dimension and the order in $\hbar$ of each term.

b) Take a functional derivative of (1) with respect to $A^\nu(y)$. Take the Fourier transform of the resulting equation. Determine the Feynman diagram necessary to be computed to obtain the value of k to lowest non-trivial order of $\hbar$.

c) Write down the integral corresponding to this Feynman diagram and evaluate it, following pages 245 - 253 in Peskin+Schroeder. Use dimensional regularization. Determine the value of k.

d) Using the relation between the axial current and the currents for left- and right-handed fermions, determine the physical implications of the axial anomaly in terms of fermion number nonconservation. In addition, discuss the physical implications of the topological nature of the axial anomaly.

II. Axial anomaly in $3 + 1$ dimensions

a) Write down the $3 + 1$-dimensional analogue of equation (1). This is the Adler-Bell-Jackiw anomaly.

b) Draw the two Feynman diagrams necessary to calculate the coefficient of this anomaly.

c) Evaluate these diagrams in momentum space (see Peskin+Schroeder p. 661) and obtain the value of the anomaly coefficient.

(continued on next page)

III. Parity anomaly in $2 + 1$ dimensions

a) Discuss the difference of fermions in odd and even dimensions.

b) Explain the significance of the Chern-Simons action (see for instance G. V. Dunne, hep-th/9902115).

c) Explain how to calculate the parity anomaly in 2+1 dimensions.