String Theory II - Exercise Sheet 7

Due to Thu Mar 1 2018 14-16 Uhr M1.03.020

Problem 7.1: Super Virasoro Algebra

Using the fundamental (anti)-commutation relations (nonvanishing ones only and only for the right moving sector)

$$
\begin{aligned}
[x^\mu, p^\nu]_- &= i\eta^{\mu\nu}\,, && (1)\\
[\alpha^\mu_m, \alpha^\nu_n]_- &= m\eta^{\mu\nu}\delta_{m+n,0}\,, && (2)\\
[b^\mu_m, b^\nu_n]_+ &= \eta^{\mu\nu}\delta_{m+n,0} && (3)
\end{aligned}
$$

and the expressions for the modes of the stress energy tensor and the supercurrent in covariant quantization ($\phi = 0$ in R sector and $\phi = \frac{D}{16}$ in NS sector)

$$
\begin{aligned}
L_n &= L^\alpha_n + L^b_n - \frac{D}{8}\phi && (4)\\
L^\alpha_n &= \sum_{m=1}^{\infty} \alpha_{-m}\cdot\alpha_{n+m} + \frac{1}{2}\alpha_0\cdot\alpha_n && (5)\\
L^b_n &= \sum_{r\in\mathbb{Z}+\phi\geq 0} \left(r+\frac{n}{2}\right) b_{-r}\cdot b_{n+r} && (6)\\
G_r &= \sum_{m\in\mathbb{Z}} \alpha_{-m}\cdot b_{r+m}\,, && (7)
\end{aligned}
$$

to show the following algebra:

$$
\begin{aligned}
[L_n, L_m]_- &= (n-m)L_{m+n} + \frac{D}{8}n(n^2-2\phi)\delta_{m+n,0}\,, && (8)\\
[L_n, G_r]_- &= \left(\frac{n}{2}-r\right)G_{n+r}\,, && (9)\\
[G_r, G_s]_+ &= 2L_{r+s} + \frac{D}{2}\left(r^2-\frac{\phi}{2}\right)\delta_{r+s,0}\,. && (10)\\
& && (11)
\end{aligned}
$$

Hint: Evaluate the left hand side of (8) for the fermionic part of the L_m first. Since you know already the bosonic part, and since fermionic and bosonic parts commute with each other adding up the contributions from both sectors should yield the right hand side.