String Theory II - Exercise Sheet 6

Due to Thu Feb 8 2018 10-12 Uhr M1.03.020

Problem 6.1: Stress Energy Tensor of the RNS String

The goal of this exercise is to calculate the stress energy tensor of the RNS string in the Vielbein formalism. In order to do this, first show that

$$T_{\alpha\beta} = \frac{4\pi}{\sqrt{-g}} \frac{\delta S_{RNS}}{\delta g^{\alpha\beta}} = \frac{\pi}{e} \frac{\delta S_{RNS}}{\delta e_c^\gamma} \left(\delta_\alpha^\gamma e_{c\beta} + \delta_\beta^\gamma e_{c\alpha} \right) . \tag{1}$$

Then use the partially gauge fixed $N = (1,1)$ supergravity action with $\chi_\alpha = 0$,

$$S = -\frac{1}{4\pi\alpha'} \int d^2\sigma \left[(\partial_\alpha X^\mu)(\partial_\beta X_\mu) e_a^\alpha e^{\beta a} + i\alpha' \bar{\psi}^\mu e_b^\alpha \rho^b \partial_\alpha \psi_\mu \right] \tag{2}$$

to show that in superconformal gauge $e_\alpha^a = \delta_\alpha^a$, the energy-momentum tensor takes the form

$$T_{\alpha\beta} = -\frac{1}{\alpha'} \partial_\alpha X^\mu \partial_\beta X_\mu - \frac{i}{4} \left(\bar{\psi}^\mu \rho_\alpha \partial_\beta \psi_\mu + \bar{\psi}^\mu \rho_\beta \partial_\alpha \psi_\mu \right) , \tag{3}$$

where the fermionic part is automatically trace free upon using the Dirac equation $\rho^\alpha \partial_\alpha \psi^\mu = 0$.

Problem 6.2: Dirac Brackets and classical Super Virasoro Algebra

Using the fundamental equal-time Dirac brackets (nonvanishing ones only)

$$\{X^\mu(\sigma), \partial_\tau X^\nu(\sigma')\} = 2\pi\alpha' \eta^{\mu\nu} \delta(\sigma - \sigma') , \tag{4}$$

$$\{\psi_A^\mu(\sigma), \psi_B^\nu(\sigma')\} = -2\pi i \eta^{\mu\nu} \delta_{AB} \delta(\sigma - \sigma') , \tag{5}$$

and the expressions for the stress energy tensor and the supercurrent in light cone coordinates

$$T_{\pm\pm} = -\frac{1}{\alpha'} \partial_\pm X^\mu \partial_\pm X_\mu - \frac{i}{2} \psi_\pm^\mu \partial_\pm \psi_{\mu\pm} \tag{6}$$

$$J_{F\pm} = -\frac{1}{2} \sqrt{\frac{2}{\alpha'}} \psi_\pm^\mu \partial_\pm X_\mu , \tag{7}$$

show the following expressions:

$$\{J_{F\pm}(\sigma), X^\mu(\sigma')\} = \frac{1}{2}\sqrt{\frac{\alpha'}{2}}\psi^\mu_\pm\ 2\pi\delta(\sigma-\sigma')\,, \tag{8}$$

$$\{J_{F\pm}(\sigma), \psi^\mu_\pm(\sigma')\} = \frac{i}{2}\sqrt{\frac{2}{\alpha'}}\partial_\pm X^\mu\ 2\pi\delta(\sigma-\sigma')\,, \tag{9}$$

$$\{T_{\pm\pm}(\sigma), \psi^\mu_\pm(\sigma')\} = \pm\left(\frac{1}{2}\psi^\mu_\pm(\sigma')\partial' + \partial'\psi_\pm(\sigma')\right)\ 2\pi\delta(\sigma-\sigma') \tag{10}$$

$$\{T_{\pm\pm}(\sigma), T_{\pm\pm}(\sigma')\} = \pm\left(2\,T_{\pm\pm}(\sigma')\partial' + \partial' T_{\pm\pm}(\sigma')\right)\ 2\pi\delta(\sigma-\sigma') \tag{11}$$

$$\{T_{\pm\pm}(\sigma), J_{F\pm}(\sigma')\} = \pm\left(\frac{3}{2}\,J_{F\pm}(\sigma')\partial' + \partial' J_{F\pm}(\sigma')\right)\ 2\pi\delta(\sigma-\sigma') \tag{12}$$

$$\{J_\pm(\sigma), J_{F\pm}(\sigma')\} = \frac{i}{2}T_{\pm\pm}(\sigma')\,2\pi\delta(\sigma-\sigma')\,. \tag{13}$$

In doing so, discard all boundary terms arising from partial integrations. The last relation was already derived in the lecture, but it may be useful to review that calculation first.