String Theory II - Exercise Sheet 5

Due to Thu Feb 1 2018 10-12 Uhr M1.03.020

Problem 5.1: Residual supersymmetries in superconformal gauge

In superconformal gauge $e_a^\alpha = \delta_a^\alpha$, $\chi_\alpha = 0$ the action of the RNS superstring reduces to

$$S = -\frac{1}{4\pi\alpha'} \int d^2\sigma \left[(\partial_\alpha X^\mu)(\partial^\alpha X_\mu) + i\alpha' \bar{\psi}^\mu \rho^\alpha \partial_\alpha \psi_\mu \right] \tag{1}$$

Show that (1) is invariant up to a boundary term under residual local supersymmetry transformations $\epsilon(\tau, \sigma)$

$$\delta_\epsilon X^\mu = i\sqrt{\frac{\alpha'}{2}} \bar{\epsilon} \psi^\mu \,, \tag{2}$$

$$\delta_\epsilon \psi^\mu = \frac{1}{\sqrt{2\alpha'}} \rho^\alpha \epsilon \partial_\alpha X^\mu \,, \tag{3}$$

as long as ϵ fulfills

$$\rho^\alpha \rho^\beta \partial_\alpha \epsilon = 0 \,. \tag{4}$$

Show furthermore that in light cone coordinates $\sigma^\pm = \tau \pm \sigma$ and with the definitions

$$\psi^\mu = \begin{pmatrix} \psi_+^\mu \\ \psi_-^\mu \end{pmatrix} , \qquad \epsilon = \begin{pmatrix} \epsilon_+ \\ \epsilon_- \end{pmatrix} = \epsilon_{AB} \epsilon^B = \begin{pmatrix} \epsilon^- \\ -\epsilon^+ \end{pmatrix} \tag{5}$$

(4) reduces to

$$\partial_\mp \epsilon^\pm = 0 \,. \tag{6}$$

Work out the form of the supersymmetry transformations (2), (3) on the left and right moving sector of the RNS string.