

## Introduction to Gauge/Gravity Duality

### Examples VI

To hand in Tuesday 29th May

#### String theory: Target space coordinates $X^\mu$

a) Show that the energy-momentum tensor of the free fields  $X^\mu(\sigma, \tau)$ ,

$$T_{\alpha\beta}(\sigma, z) = \frac{1}{2\pi\alpha'} \partial_\alpha X^\mu \partial_\beta X_\mu - \frac{1}{4\pi\alpha'} \eta_{\alpha\beta} \partial^\gamma X^\mu \partial_\gamma X_\mu \quad (1)$$

where  $\alpha, \beta, \gamma \in \{\sigma, \tau\}$ , is conserved ( $\partial^\alpha T_{\alpha\beta} = 0$ ) and traceless ( $T_a^a = 0$ ).

b) We introduce lightcone coordinates  $\sigma^- = \tau - \sigma$ ,  $\sigma^+ = \tau + \sigma$ . Using that

$$X^\mu(\sigma^+, \sigma^-) = x^\mu + \alpha' p^\mu \tau + i \sqrt{\frac{\alpha'}{2}} \sum_{n \neq 0} \frac{1}{n} (\alpha_n^\mu e^{-in\sigma^-} + \tilde{\alpha}_n^\mu e^{-in\sigma^+}), \quad (2)$$

show that the Poisson bracket

$$\left\{ \frac{\partial}{\partial \tau} X^\mu(\tau, \sigma), X^\nu(\tau, \sigma') \right\} = 2\pi\alpha' \eta^{\mu\nu} \delta(\sigma - \sigma') \quad (3)$$

implies the following Poisson brackets of the modes,

$$\{\alpha_m^\mu, \alpha_n^\nu\} = im\eta^{\mu\nu} \delta_{m+n,0}, \quad \{\tilde{\alpha}_m^\mu, \tilde{\alpha}_n^\nu\} = im\eta^{\mu\nu} \delta_{m+n,0}, \quad \{p^\mu, x^\nu\} = \eta^{\mu\nu}, \quad (4)$$

with all other brackets vanishing. (Note that in this example,  $\{, \}$  denotes a Poisson bracket, not an anticommutator.)

c) We expand the the energy-momentum tensor in modes  $l_n$ , for instance

$$T_{--} = \frac{1}{2\pi} \sum_n l_n e^{-in\sigma^-}. \quad (5)$$

Show that the modes have the following expression in terms of the modes of  $X^\mu$ ,

$$l_n = \frac{1}{2} \sum_{m=-\infty}^{\infty} \alpha_m^\mu \alpha_{n-m}^\nu \eta_{\mu\nu}, \quad (6)$$

with  $\alpha_0^\mu = \sqrt{\frac{\alpha'}{2}} p^\mu$ . .../...

Use the expression for the  $l_n$  to verify that

$$\begin{aligned} \{, l_m, l_n\} &= i(m-n)l_{m+n}, \\ \{l_m, X^\mu(\sigma^-, \sigma^+)\} &= ie^{im\sigma^-} \partial_{\sigma^-} X^\mu(\sigma^-, \sigma^+). \end{aligned} \quad (7)$$

(10 points)