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Introduction to Gauge/Gravity Duality

Examples III

To hand in Tuesday 8th May in the examples class

I. **Large N Expansion in Gauge Theories**

Consider a quantum field theory with real scalar field Φ in the adjoint representation of the gauge group $SU(N)$, with Lagrangian given by

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}(D\Phi)(D\Phi) - g\mathrm{Tr}(\Phi^3) - g^2\mathrm{Tr}(\Phi^4)\,, \tag{1}$$

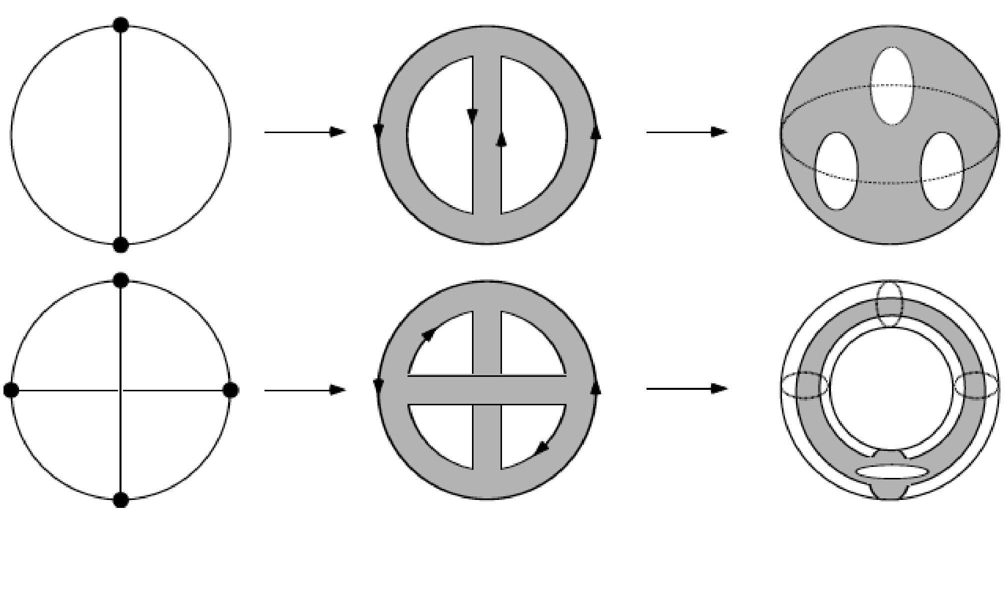
where D is a suitable covariant derivative and ${(\Phi)^i}_j = \Phi^A {(T^A)^i}_j$.

a) Define the *'t Hooft coupling* λ and explain its use in organizing the perturbative expansion. Explain the double line notation for the free field propagator of Φ. Using a counting argument, derive a general formula for the powers of N and λ associated to each Feynman diagram. Organize this perturbative expansion in terms of the Euler characteristic χ. Explain the topological structure of the perturbative expansion.

(6 points)

b) Evaluate the order in N associated with the two diagrams given in the figure and explain the meaning of the three diagrams on each line.

(3 points)



II. **Conformal Symmetry**

a) Show that a special conformal transformation may be written as a combination of an inversion, a translation and another inversion.

(2 points)

b) Consider a curved space with a metric of Euclidean signature. An infinitesimal *Weyl transformation* of the metric is given by

$$\delta g^{\mu\nu}(x) = 2\sigma(x) g^{\mu\nu}(x)\,, \tag{2}$$

with a scalar function $\sigma(x)$. Show that the conformal Killing equation may be obtained by combining an infinitesimal diffeomorphism with an infinitesimal Weyl transformation, and subsequently reducing to flat space.

(5 points)

c) Show that the scalar two-point function on flat Euclidean d-dimensional space,

$$\langle \phi(x)\phi(y)\rangle \;=\; \frac{C}{(x-y)^{2\Delta}} \tag{3}$$

is covariant under conformal transformations.

(3 points)

Special question:

d) Graphical representation of special conformal transformations. Write a computer programme which for a given vector $b_\mu \in \mathbb{R}^2$ generates a graphical representation of the special conformal transformation

$$x'_\mu = \frac{x_\mu + b_\mu x^2}{\Omega^b(x)}\;, \qquad \Omega^b(x) = 1 + 2(b\cdot x) + b^2 x^2\,, \tag{4}$$

of a square lattice.

(3 additional points)