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Introduction to Gauge/Gravity Duality

Examples IX

To hand in Tuesday 19th June in the examples class

I. Relation between propagators in AdS

Let us consider Euclidean AdS in the Poincaré patch.

a) Derive the equation of motion for a scalar field with mass m in Euclidean AdS_{d+1} .
(2 points)

b) Use the ansatz $\phi(z) = z^\Delta$ for an asymptotic solution of this equation near the boundary $z \rightarrow 0$. Determine the two possible values of Δ_\pm , where $\Delta_+ > \Delta_-$.
(4 points)

c) The bulk-to-boundary propagator $K(x, z; x')$ is the solution of the equations of motion which is regular in the interior and diverges as

$$\lim_{z \rightarrow \epsilon} K(z, x; x') = \epsilon^{\Delta_-} \delta(x - x')$$

near the boundary, i.e. for $\epsilon \ll 1$. The bulk-to-bulk propagator is given by the solution of the equation of motion with a pointlike source term,

$$\left(\square_{x,z} - m^2\right) G(z, x; z', x') = \frac{1}{\sqrt{g}} \delta(x - x') \delta(z - z'),$$

where $\square_{x,z}$ is the scalar Laplacian in Euclidean AdS_{d+1} acting only on x and z . Moreover the bulk-to-bulk propagator is regular in the interior.

Show that the bulk-to-boundary propagator $K(x, z; x')$ can be calculated from the bulk-to-bulk propagator $G(z, x; z', x')$ by

$$K(z, x; x') = \lim_{z' \rightarrow \epsilon} \frac{\Delta_+ - \Delta_-}{\epsilon^\Delta} G(z, x; z', x').$$

Hint: Do not use the explicit solution for $G(z, x; z', x')$, but Green's second identity!

(4 points)

(please turn over)

II. Saturation of unitarity bound

Consider a free scalar field with mass m in the Poincare patch of Euclidean AdS_{d+1} with radius R .

a) Show that the usual bulk action

$$S_1 = -\frac{1}{2} \int d^d x dz \sqrt{g} \left(g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + m^2 \phi^2 \right),$$

evaluated for solutions of the form $\phi(x, z) \sim z^\Delta e^{ikx}$ near $z = 0$, is finite if $\Delta > d/2$, i.e. the solution with $\Delta = \Delta_+$ (Δ_+ being the larger root of $\Delta(\Delta - d) = m^2 L^2$) are normalizable with respect to the action S_1 .

(4 points)

b) Consider the bulk action

$$S_2 = -\frac{1}{2} \int d^d x dz \sqrt{g} \phi \left(-\square_g + m^2 \right) \phi.$$

Show that S_2 can be obtained by adding a boundary term to the action S_1 (hint: integration by parts) and that the equations of motion are the same as for the action S_1 .

(2 points)

c) Show, that the action S_2 evaluated for solutions of the form $\phi(x, z) \sim z^\Delta e^{ikx}$ near $z = 0$ is finite if $\Delta > (d - 2)/2$. Conclude that for

$$-\frac{d^2}{4} < m^2 L^2 < -\frac{d^2}{4} + 1$$

both solutions, i.e. $\Delta = \Delta_+$ and $\Delta = \Delta_-$ are normalizable with respect to the action S_2 .

(4 points)

For more additional information see arXiv: hep-th/9905104.