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Introduction to Gauge/Gravity Duality

Examples VIII

To hand in Tuesday 12th June

I. **DBI action for a Dp–brane**

The Dirac–Born–Infeld (DBI) action for a Dp–brane reads

$$S_{Dp} = -\tau_p \int d^{p+1}\zeta \, e^{-\Phi} \sqrt{-\det(\mathcal{P}[g]_{ab} + 2\pi\alpha' F_{ab})}\,,$$

where Φ is the dilaton, $g_{\mu\nu}$ the metric of the curved spacetime and $\mathcal{P}$ the pullback on the worldvolume (given by coordinates ζ^a). Moreover, F_{ab} is the $U(1)$ field strength tensor. det denotes the determinant of the matrix $\mathcal{P}[g]_{ab} + 2\pi\alpha' F_{ab}$.

a) Simplify the action to a flat target spacetime with a vanishing dilaton. Furthermore, take the Dp–brane to be aligned along the coordinate axis and the embedding functions into the target spacetime (given by coordinates X^m) vanish, i.e. $X^a = \zeta^a$ for $a = 0, \ldots, p$ and $X^m = 0$ for $m = p, \ldots, 9$. (1 point)

b) Show that we can expand $\sqrt{\det(\mathbb{1} + \epsilon M)}$ for small ϵ in the form

$$\sqrt{\det(\mathbb{1} + \epsilon M)} = 1 + \frac{1}{2}\,\epsilon\,\mathrm{tr}\,M + \epsilon^2 \left(\frac{1}{8}\,(\mathrm{tr}\,M)^2 - \frac{1}{4}\mathrm{tr}\,(M^2)\right) + \mathcal{O}(\epsilon^3)$$

What is the expansion for antisymmetric M?

Hint: $\det(A) = \exp(\mathrm{Tr}\,\ln A)$ and expand the right side of the equation. (2 points)

c) Expand the simplified DBI action of exercise a) using the results of b) up to the first non-trivial order of the field strength tensor F. (2 points)

II. **The AdS/CFT duality**

a) State the precise AdS_5/CFT_4 duality in the strongest form, i.e. for general ranks N of the gauge group and for arbitrary 't Hooft coupling constants λ! What is the strong and weak form of the duality? Which limits are taken on both sides of the duality?

Hint: Helpful relations: $g_{YM}^2 = 4\pi g_s$, $\lambda = N g_{YM}^2$ and $L^4 = 4\pi g_s N \alpha'^2$. (3 points)

b) Show that the number of degrees of freedom per site in the 4-dimensional field theory is proportional to the size of the AdS boundary, i.e. show that

$$N^2 \propto \frac{L^3}{G_N^5} \tag{1}$$

with L the AdS radius and G_N^5 is the Newton constant in 5 dimensions. Hint: The Newton constant G_N in 10 dimensions is given by $16\pi G_N = (2\pi)^7 \alpha'^4 g_s^2$ and the five-dimensional Newton constant is obtained by dividing by the volume of S^5.

(4 points)