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Introduction to Gauge/Gravity Duality

Examples VII

To hand in Tuesday 5th June

I. Near-Horizon limit of M2–branes

Let us consider the near horizon limit of *M2–branes* in 11-dimensional supergravity. The supergravity solution of M2–branes reads

$$\begin{aligned} ds^2 &= H(r)^{-2/3}\left(-dt^2+dx^2+dy^2\right)+H(r)^{1/3}\left(dr^2+r^2 d\Omega_7\right)\,, \\ F_{(4)} &= dt\wedge dx\wedge dy\wedge dH^{-1}\,, \end{aligned}$$

where $H(r)$ is given by

$$H(r)=1+\frac{L^6}{r^6}\,, \quad \text{where} \quad L^6=32\pi^2 N l_p^6$$

and $F_{(4)}$ is a four-form.

a) Take the near-horizon limit $r\to 0$ and calculate the metric and the four-form $F_{(4)}$ in this limit. (6 points)

b) Use the coordinate transformation $z=\frac{L^3}{2r^2}$ and compute the metric as well as the four-form $F_{(4)}$ in the coordinates (z,t,x,y,Ω_7). Which manifold is described by this metric?

(4 points)

II. Symmetries of AdS/CFT

a) Consider $\mathcal{N}=4$ $SU(N)$ Super-Yang-Mills theory. Describe the (bosonic) symmetries of this theory, i.e. those with bosonic symmetry generators. Which group is formed by these symmetries? Furthermore, what is the field content of the theory? Explain what it means that this theory is maximally supersymmetric.

(4 points).

b) Consider the ten-dimensional space $\mathrm{AdS}_5\times S^5$, which is the direct product of five-dimensional Anti-de Sitter space and of the five-sphere S^5. Define both of AdS_5 and S^5 as hypersurfaces in six-dimensional spaces. Which signature do these spaces have? The space $\mathrm{AdS}_5\times S^5$ inherits its symmetries from these embeddings. What are these symmetries?

(4 points).

c) Compare the symmetries discussed in parts a) and b) and comment on their relation.
(2 points).