Introduction to Gauge/Gravity Duality

Examples VII

To hand in Tuesday 25th July in the examples class

I. The AdS/CFT duality

a) State the precise $\text{AdS}_5/\text{CFT}_4$ duality in the strongest form, i.e. for general ranks $N$ of the gauge group and for arbitrary 't Hooft coupling constants $\lambda$. What is the strong and weak form of the duality? Which limits are taken on both sides of the duality?

Hint: Helpful relations: $g^2_{YM} = 4\pi g_s$, $\lambda = N g^2_{YM}$ and $L^4 = 4\pi g_s N \alpha'^2$. (4 points)

b) Show that the number of degrees of freedom per site in the $d$-dimensional field theory is proportional to the size of the AdS boundary, i.e. show that

\[ N^2 \propto \frac{L^{d-1}}{G_N} \]  

(1)

with $L$ the AdS radius and $G_N$ is the Newton constant in $d + 1$ dimensions.

(4 points)

c) What is the field-operator map? What are normalizable and non–normalizable modes and what is their meaning on the field theory side? (2 points)

II. Fefferman-Graham expansion

Consider the $(d + 1)$-dimensional AdS metric in the form

\[ ds^2 = L^2 \left( \frac{d\rho^2}{4\rho^2} + \frac{1}{\rho} g_{ij}(x,\rho) dx^i dx^j \right). \]  

(2)

Consider a scalar field in this space with boundary expansion

\[ \Phi(x,\rho) = \rho^{(d-\Delta)/2} \phi(x,\rho), \]  

(3)

\[ \phi(x,\rho) = \phi(0)(x) + \rho \phi(2)(x) + \rho^2 \phi(4)(x) + \ldots. \]  

(4)

a) Derive the equation of motion for the scalar $\phi(x,\rho)$. (4 points)

b) Using the equation of motion, show that

\[ \phi(2)(x) = \frac{1}{2(2\Delta - d - s)} \Box_{(0)} \phi(0)(x). \]  

(5)