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Introduction to Gauge/Gravity Duality

Examples VII

To hand in Tuesday 25th July in the examples class

I. **The AdS/CFT duality**

a) State the precise AdS_5/CFT_4 duality in the strongest form, i.e. for general ranks N of the gauge group and for arbitrary 't Hooft coupling constants λ! What is the strong and weak form of the duality? Which limits are taken on both sides of the duality?

Hint: Helpful relations: $g_{YM}^2 = 4\pi g_s$, $\lambda = N g_{YM}^2$ and $L^4 = 4\pi g_s N \alpha'^2$. (4 points)

b) Show that the number of degrees of freedom per site in the d-dimensional field theory is proportional to the size of the AdS boundary, i.e. show that

$$N^2 \propto \frac{L^{d-1}}{G_N} \tag{1}$$

with L the AdS radius and G_N is the Newton constant in $d+1$ dimensions.
(4 points)

c) What is the field-operator map? What are normalizable and non–normalizable modes and what is their meaning on the field theory side? (2 points)

II. **Fefferman-Graham expansion**

Consider the $(d+1)$-dimensional AdS metric in the form

$$ds^2 = L^2 \left(\frac{d\rho^2}{4\rho^2} + \frac{1}{\rho} g_{ij}(x,\rho) dx^i dx^j \right) . \tag{2}$$

Consider a scalar field in this space with boundary expansion

$$\Phi(x,\rho) = \rho^{(d-\Delta)/2} \phi(x,\rho) , \tag{3}$$

$$\phi(x,\rho) = \phi_{(0)}(x) + \rho \phi_{(2)}(x) + \rho^2 \phi_{(4)}(x) + \ldots . \tag{4}$$

a) Derive the equation of motion for the scalar $\phi(x,\rho)$. (4 points)

b) Using the equation of motion, show that

$$\phi_{(2)}(x) = \frac{1}{2(2\Delta - d - s)} \Box_{(0)} \phi_{(0)}(x) . \tag{5}$$

(6 points)