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Introduction to Gauge/Gravity Duality

Examples VI

To hand in Thursday 13th July in the examples class

I. **Relation between propagators in AdS**

Let us consider Euclidean AdS in the Poincaré patch.

a) Derive the equation of motion for a scalar field with mass m in Euclidean AdS_{d+1}.

(2 points)

b) Use the ansatz $\phi(z) = z^{\Delta}$ for an asymptotic solution of this equation near the boundary $z \to 0$. Determine the two possible values of $\Delta_{\pm}$, where $\Delta_+ > \Delta_-$.

(4 points)

c) The bulk-to-boundary propagator $K(x, z; x')$ is the solution of the equations of motion which is regular in the interior and diverges as

$$\lim_{z \to \epsilon} K(z, x; x') = \epsilon^{\Delta_-} \delta(x - x')$$

near the boundary, i.e. for $\epsilon \ll 1$. The bulk-to-bulk propagator is given by the solution of the equation of motion with a pointlike source term,

$$\left(\Box_{x,z} - m^2\right) G(z, x; z', x') = \frac{1}{\sqrt{g}} \delta(x - x') \delta(z - z') \,,$$

where $\Box_{x,z}$ is the scalar Laplacian in Euclidean AdS_{d+1} acting only on x and z. Moreover the bulk-to-bulk propagator is regular in the interior.

Show that the bulk-to-boundary propagator $K(x, z; x')$ can be calculated from the bulk-to-bulk propagator $G(z, x; z', x')$ by

$$K(z, x; x') = \lim_{z' \to \epsilon} \frac{\Delta_+ - \Delta_-}{\epsilon^{\Delta}} G(z, x; z', x').$$

Hint: Do not use the explicit solution for $G(z, x; z', x')$, but Green's second identity!

(4 points)

(please turn over)

II. **Saturation of unitarity bound**

Consider a free scalar field with mass m in the Poincare patch of Euclidean AdS_{d+1} with radius R.

a) Show that the usual bulk action

$$S_1 = -\frac{1}{2}\int d^d x\, dz \sqrt{g}\left(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + m^2\phi^2\right)\,,$$

evaluated for solutions of the form $\phi(x,z) \sim z^\Delta e^{ikx}$ near $z=0$, is finite if $\Delta > d/2$. This means that the solution with $\Delta = \Delta_+$ (Δ_+ being the larger root of $\Delta(\Delta-d) = m^2R^2$) are normalizable with respect to the action S_1.

(4 points)

b) Consider the bulk action

$$S_2 = -\frac{1}{2}\int d^d x\, dz \sqrt{g}\phi\left(-\Box_g + m^2\right)\phi\,.$$

Show that S_2 can be obtained by adding a boundary term to the action S_1 (hint: integration by parts) and that the equations of motion are the same as for the action S_1.

(2 points)

c) Show, that the action S_2 evaluated for solutions of the form $\phi(x,z) \sim z^\Delta e^{ikx}$ near $z=0$ is finite if $\Delta > (d-2)/2$. Conclude that for

$$-\frac{d^2}{4} < m^2R^2 < -\frac{d^2}{4} + 1$$

both solutions, i.e. $\Delta = \Delta_+$ and $\Delta = \Delta_-$ are normalizable with respect to the action S_2.

(4 points)

For more additional information see arXiv: hep-th/9905104.