Prof. Dr. Johanna Erdmenger
Nina Miekley

Introduction to Gauge/Gravity Duality

Examples V

To hand in Tuesday 4th July in the examples class

II. **Near-Horizon limit of M2–branes**

Let us consider the near horizon limit of M2–branes in 11-dimensional supergravity. The supergravity solution of M2–branes reads

$$\begin{aligned} ds^2 &= H(r)^{-2/3}\left(-dt^2+dx^2+dy^2\right)+H(r)^{1/3}\left(dr^2+r^2d\Omega_7\right)\,, \\ F_{(4)} &= dt\wedge dx\wedge dy\wedge dH^{-1}\,, \end{aligned}$$

where $H(r)$ is given by

$$H(r)=1+\frac{L^6}{r^6}\,,\quad \text{where}\quad L^6=32\pi^2 N l_p^6$$

and $F_{(4)}$ is a four-form.

a) Take the near-horizon limit $r\to 0$ and calculate the metric and the four-form $F_{(4)}$ in this limit. (6 points)

b) Use the coordinate transformation $z=\frac{L^3}{2r^2}$ and compute the metric as well as the four-form $F_{(4)}$ in the coordinates (z,t,x,y,Ω_7). Which manifold is described by this metric?

(4 points)

II. **Penrose-Brown-Henneaux Transformation**

The AdS metric in Poincaré coordinates may be written as

$$ds^2=L^2\frac{d\rho^2}{4\rho^2}+\frac{1}{\rho}g_{\mu\nu}dx^\mu dx^\nu\,, \tag{1}$$

where the x^μ, $\mu\in\{0,1,2,3\}$, are the coordinates parallel to the boundary of AdS and ρ is the radial direction. We consider coordinate transformations of this metric under the *Penrose-Brown-Henneaux diffeomorphism*, which is given by

$$\rho=\rho'(1-2\sigma(x'))\qquad x^\mu=(x')^\mu+a^\mu(x',\rho')\,, \tag{2}$$

demanding that

$$g'_{55}=g_{55}\ \text{ and }\ g'_{\mu 5}=g_{\mu 5}\,. \tag{3}$$

Note that the index 5 stands for the ρ direction. Show that the conditions (3) imply

$$\partial_5 a^\mu = \frac{L^2}{2} g^{\mu\nu} \partial_\nu \sigma \tag{4}$$

and

$$g_{\mu\nu} \to g_{\mu\nu} + 2\sigma \left(1 - \rho \frac{\partial}{\partial \rho}\right) g_{\mu\nu} + \nabla_\mu a_\nu + \nabla_\mu a_\nu \,. \tag{5}$$

(5 points)

Explain why this transformation induces a conformal transformation $g_{\mu\nu}(x) \to e^{2\sigma(x)} g_{\mu\nu}(x)$ at the boundary.

(5 points)