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Introduction to Gauge/Gravity Duality

Examples IV

To hand in Tuesday 20th June in the examples class

I. Symmetries of AdS/CFT

a) Consider $\mathcal{N} = 4$ $SU(N)$ Super-Yang-Mills theory. Describe the (bosonic) symmetries of this theory, i.e. those with bosonic symmetry generators. Which group is formed by these symmetries? Furthermore, what is the field content of the theory? Explain what it means that this theory is maximally supersymmetric.

(4 points).

b) Consider the ten-dimensional space AdS$_5$ $\times$ S^5, which is the direct product of five-dimensional Anti-de Sitter space and of the five-sphere S^5. Define both of AdS$_5$ and S^5 as hypersurfaces in six-dimensional spaces. Which signature do these spaces have? The space AdS$_5$ $\times$ S^5 inherits its symmetries from these embeddings. What are these symmetries?

(4 points).

c) Compare the symmetries discussed in parts a) and b) and comment on their relation.

(2 points).

II. Ward identity for two-point function

The invariance of a field-theory action under a symmetry leads to the following Ward identity for the two-point function,

$$\langle \delta\phi(x)\ \phi(y)\rangle + \langle \phi(x)\ \delta\phi(y)\rangle = 0, \tag{1}$$

where $\delta\phi$ is the change of the field under an infinitesimal transformation of the symmetry in question. The change of a scalar field under conformal transformation is

$$\delta\phi(x) = -\left(\epsilon^\mu(x)\frac{\partial}{\partial x^\mu} + \frac{\Delta}{d}\frac{\partial \epsilon^\mu}{\partial x^\mu}\right)\phi(x), \tag{2}$$

where Δ is the scaling dimension. This yields constraints for the two-point function $\langle \phi_1(x)\ \phi_2(y)\rangle$ for two scalar fields ϕ_1, ϕ_2 with scaling dimension Δ_1, Δ_2, respectively. In the following, we consider a quantum field theory in Euclidean space.

a) Consider translation symmetry (i.e. $\epsilon^\mu = a^\mu$) and show that the Ward identity implies that the two-point function depends only on $u^\mu = x^\mu - y^\mu$. Furthermore, show that invariance under rotations (i.e. $\epsilon^\mu(x) = \omega^\mu{}_\nu\, x^\nu$ with $\omega_{\mu\nu} = -\omega_{\nu\mu}$) implies that the two-point function is a function of u^2.

(4 points)

b) Assuming that the two-point function only depends on u^2, write down the Ward identity for scale transformation (i.e. $\epsilon^\mu(x) = \lambda x^\mu$) and solve the resulting differential equation.

(3 points)

c) Show that only if $\Delta_1 = \Delta_2 = \Delta$,

$$\langle \phi_1(x)\ \phi_2(y) \rangle = \frac{C}{|x-y|^{-2\Delta}} \tag{3}$$

satisfies the Ward identity for special conformal transformations with

$$\epsilon^\mu(x) = b^\mu x^2 - 2(b \cdot x) x^\mu. \tag{4}$$

C is a real positive constant.

(3 points)