q fermions or bosons

$$\hat{H} = \sum_{ijkl\ldots} J_{ijkl\ldots}(c_i^\dagger c_j c_k^\dagger c_l \ldots)$$

drawn from static
random distribution

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TWO-BODY RANDOM HAMILTONIAN AND LEVEL DENSITY

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Received 22 December 1970

Volume 33B, number 7 PHYSICS LETTERS 7 December 1970

VALIDITY OF RANDOM MATRIX THEORIES FOR MANY-PARTICLE SYSTEMS [*]

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Received 19 October 1970

modeling chaos in complex nuclei

Universal Quantum-Critical Dynamics of Two-Dimensional Antiferromagnets

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(Received 13 April 1992)

The universal dynamic and static properties of two-dimensional antiferromagnets in the vicinity of a zero-temperature phase transition from long-range magnetic order to a quantum-disordered phase are studied. Random antiferromagnets with both Néel and spin-glass long-range magnetic order are considered. Explicit quantum-critical dynamic scaling functions are computed in a $1/N$ expansion to two-loop level for certain nonrandom, frustrated square-lattice antiferromagnets. Implications for neutron scattering experiments on the doped cuprates are noted.

PACS numbers: 75.10.Jm, 05.30.Fk, 75.50.Ee

… discovery of conformal symmetries

Sachdev-Ye-Kitaev Model (15)

A model of *N* randomly interacting *Majorana* fermions

$$\hat{H} = \sum_{ijkl}^{N} J_{ijkl}\, \chi_i \chi_j \chi_k \chi_l, \qquad \{\chi_i, \chi_j\} = 2\delta_{ij}$$

SYK model

where the interaction constants are static and random,

$$\langle |J_{ijkl}|^2 \rangle = \frac{6J^2}{N^3}$$

high energy scale

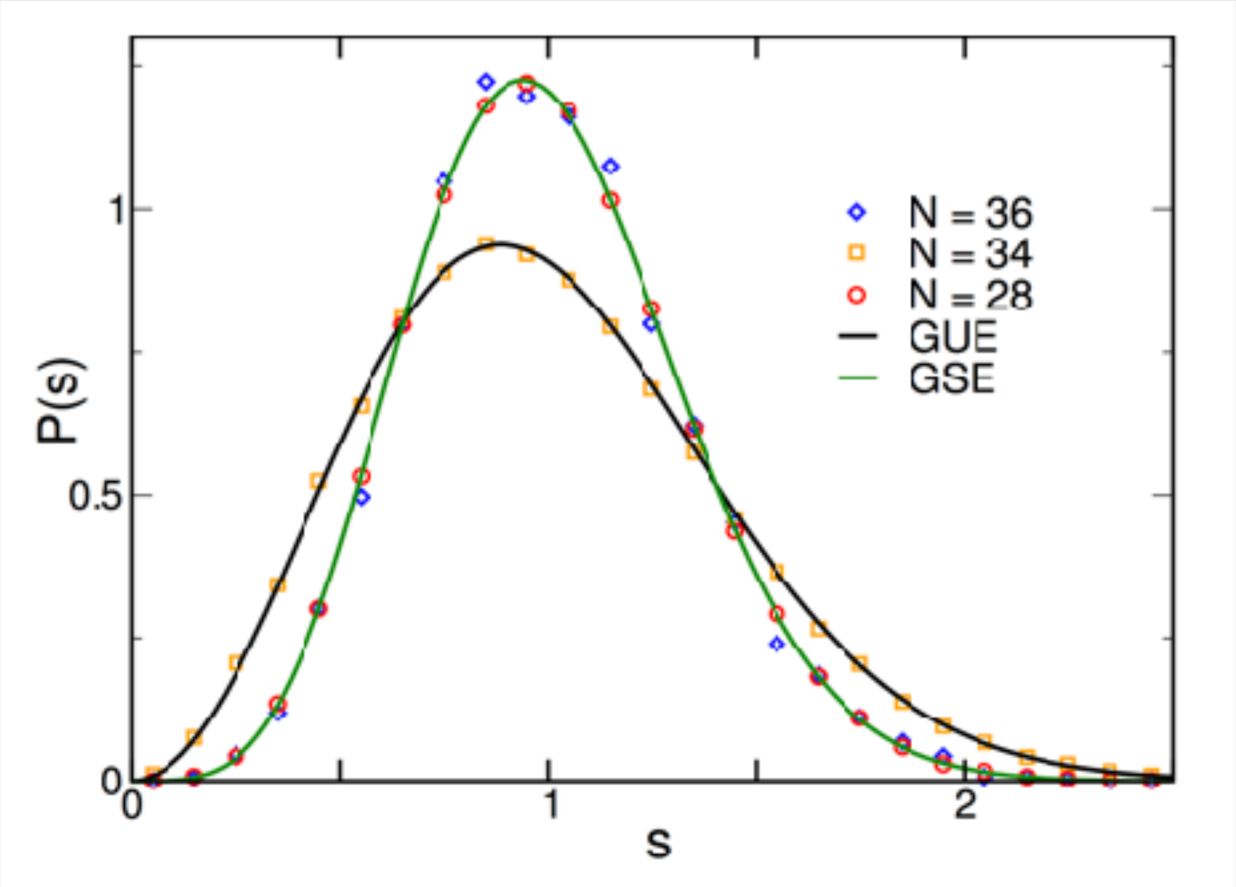
Three perspectives:

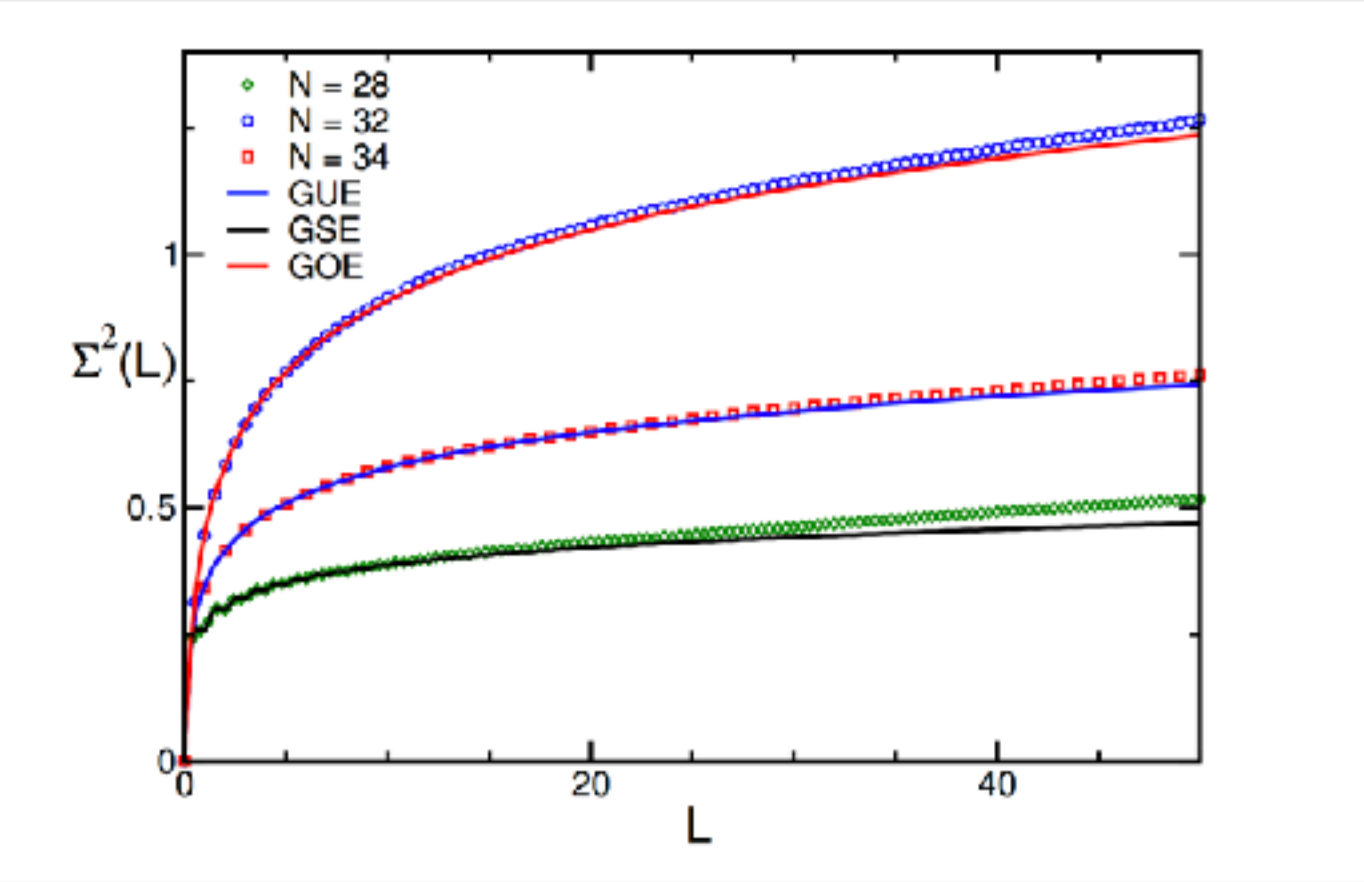
- random matrix theory
- strong correlation physics
- holography

quantum chaos

random matrix correlations:

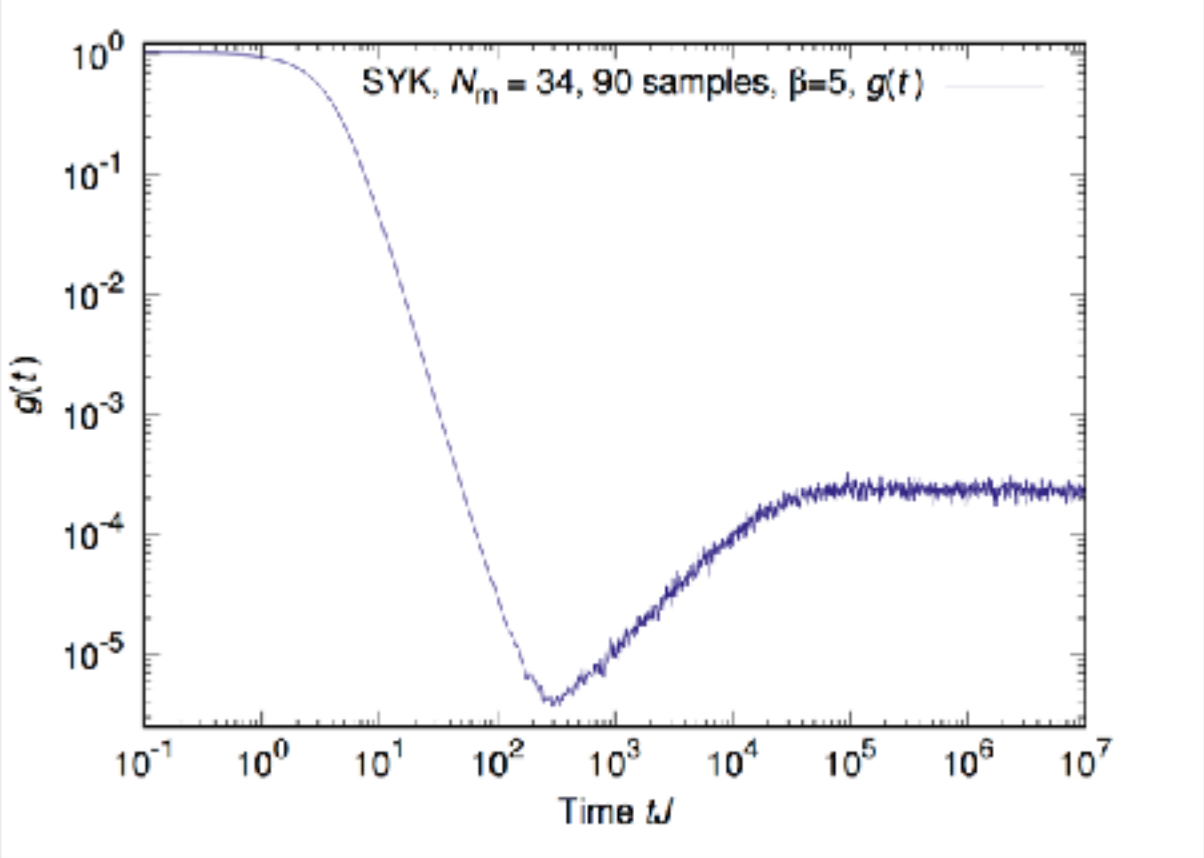
Verbaarschot, Garcia-Garcia, 16





Note: depending on the value of *N* mod 8 the model realizes different symmetries

Cotler et al. 17



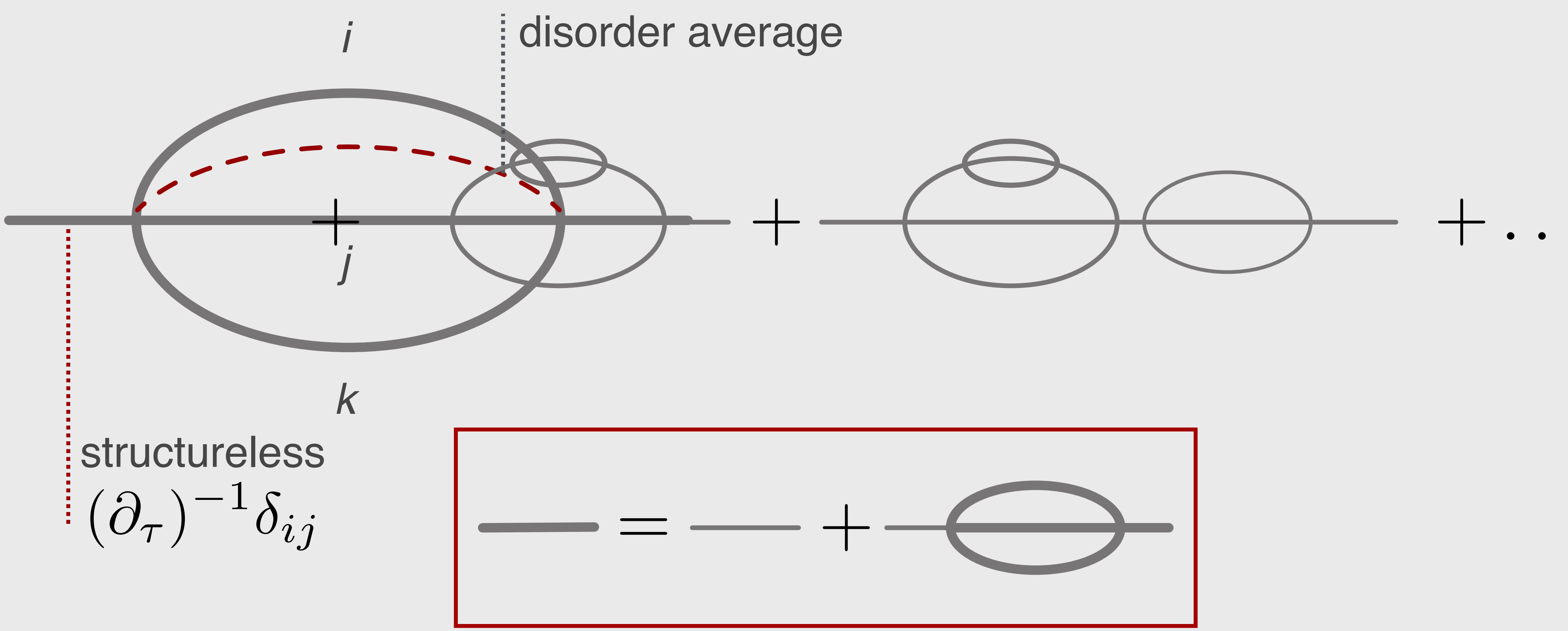
strong correlations

Strong interactions:

$$\hat{H} = \sum_{ijkl}^{N} J_{ijkl}\, \chi_i \chi_j \chi_k \chi_l$$

'infinite range', strong, chaotic: amenable to large *N* mean field methods

diagrammatic expansion of Majorana propagator



Solutions of mean field equation

For $\partial_\tau \ll J$

numerical factor

$$G(\tau,\tau') = -\frac{b}{J^{1/2}} \frac{\text{sgn}(\tau-\tau')}{|\tau-\tau'|^{1/2}}$$

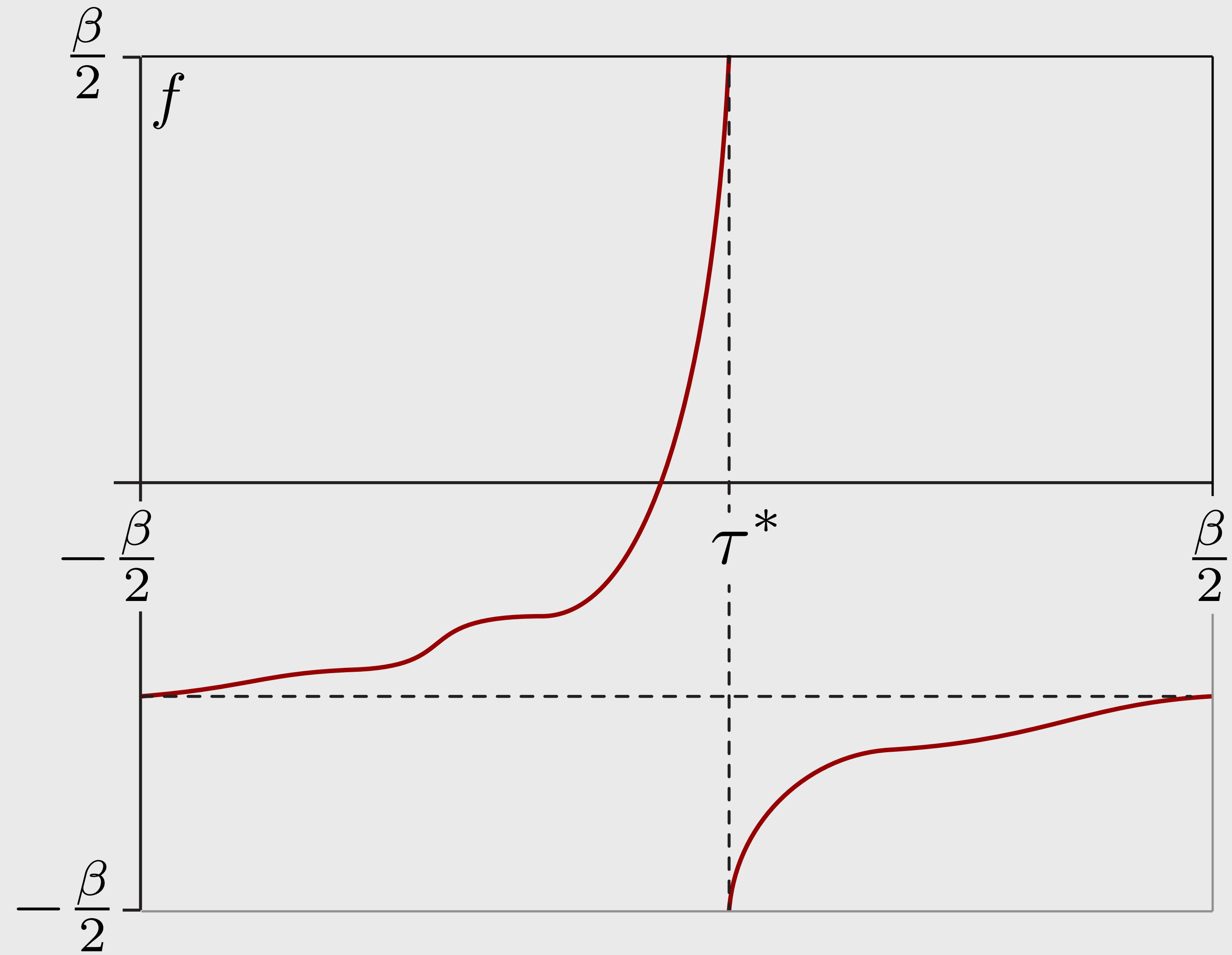
$$\Sigma(\tau,\tau') = -\frac{b^3}{J^{1/2}} \frac{\text{sgn}(\tau-\tau')}{|\tau-\tau'|^{3/2}}$$

Non Fermi liquid mean field Green functions

Symmetries

Hamiltonian action (neglecting time derivatives) invariant under reparameterization of time

$$f : S^1 \to S^1, \tau \mapsto f(\tau),$$
$$f \in \mathrm{Diff}(S^1)$$



$$G(\tau, \tau') \to f'(\tau)^{1/4} G\big(f(\tau) - f(\tau')\big) f'(\tau')^{1/4},$$
$$\Sigma(\tau, \tau') \to f'(\tau)^{3/4} \Sigma\big(f(\tau) - f(\tau')\big) f'(\tau')^{3/4}$$

Elements of the diffeomorphism manifold describe reparameterizations of time. Infinitesimally: generated by Virasoro algebra. Weakly broken by time derivatives — problem has NCFT_1 symmetry (Maldacena and Stanford, 15).

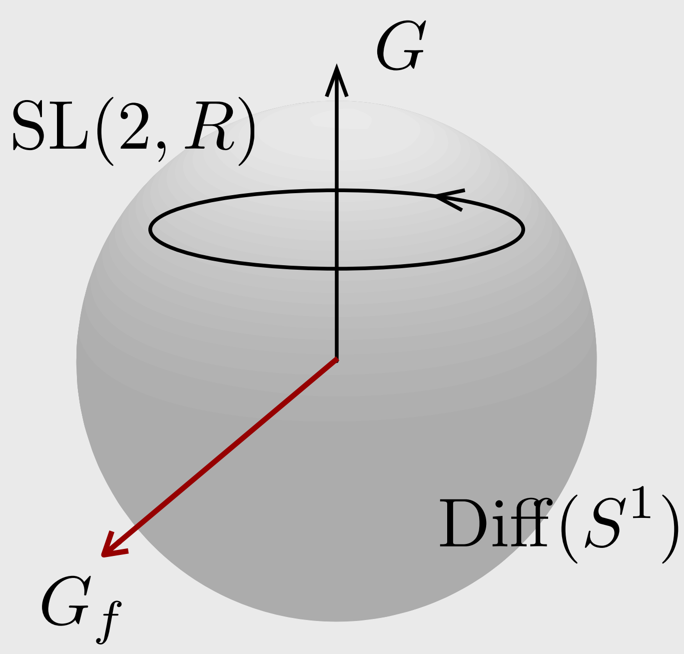
Symmetry of the mean field

$$G(\tau,\tau') \sim \frac{\mathrm{sgn}(\tau-\tau')}{|\tau-\tau'|^{1/2}}$$

invariance under conformal transformations $\tau \to \dfrac{a\tau+b}{d\tau+c}$

each $f: S^1 \to S^1, \tau \mapsto f(\tau), \quad f \in \mathrm{Diff}(S^1)/\mathrm{SL}(2,R)$ generates new solution

$$G(\tau,\tau') \to G_f(\tau,\tau') = f'(\tau)^{1/4} f'(\tau')^{1/4}\, G(f(\tau), f(\tau'))$$



emergence of infinite dimensional
Goldstone mode manifold

$$\boxed{\mathrm{Diff}(S^1)/\mathrm{SL}(2,R)}$$

Quantum fluctuations & ergodicity in the SYK model

Würzburg, Aug. 2nd 2018

Alexander Altland, Dmitry Bagrets (Cologne), Alex Kamenev (Minnesota)

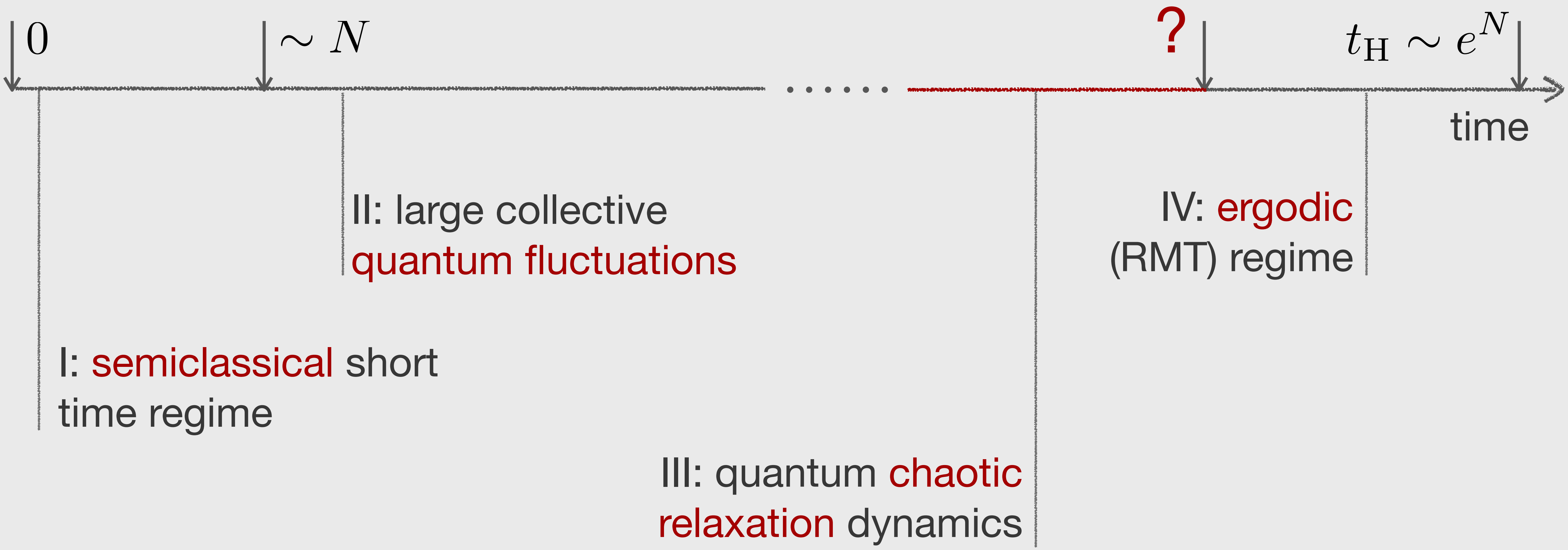
conformal symmetry breaking, chaos and short time dynamics

quantum ergodicity and long time dynamics

Nucl. Phys. B **911**, 191 (2016)
Nucl. Phys. B **921**, 727 (2017)
Nucl. Phys. B **930**, 45 (2018)

stages of SYK dynamics

SYK timeline



conformal symmetry &
Liouville quantum mechanics

Conformal Goldstone mode action

$$Z = \int \mathcal{D}\varphi \exp(-S[\varphi]), \qquad S[\varphi] = M \int d\tau \left(\frac{1}{2}(\varphi')^2 + 2e^{-\varphi} \right)$$

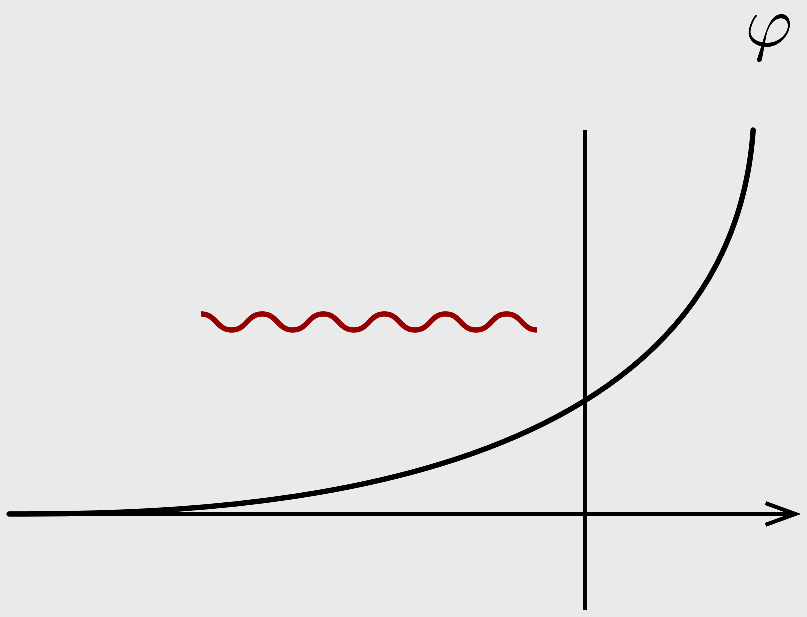
flat measure

action of Liouville QM

aa, Bagrets, Kamenev, 16

effect of low energy Goldstone mode fluctuations encapsulated in Liouville QM. Universal feature (Shelton, Tsvelik 98): all operator correlation functions decay as

$$\langle \mathcal{O}(\tau)\mathcal{O}(\tau') \rangle \sim |\tau - \tau'|^{-3/2}$$



chaos and OTO correlation functions

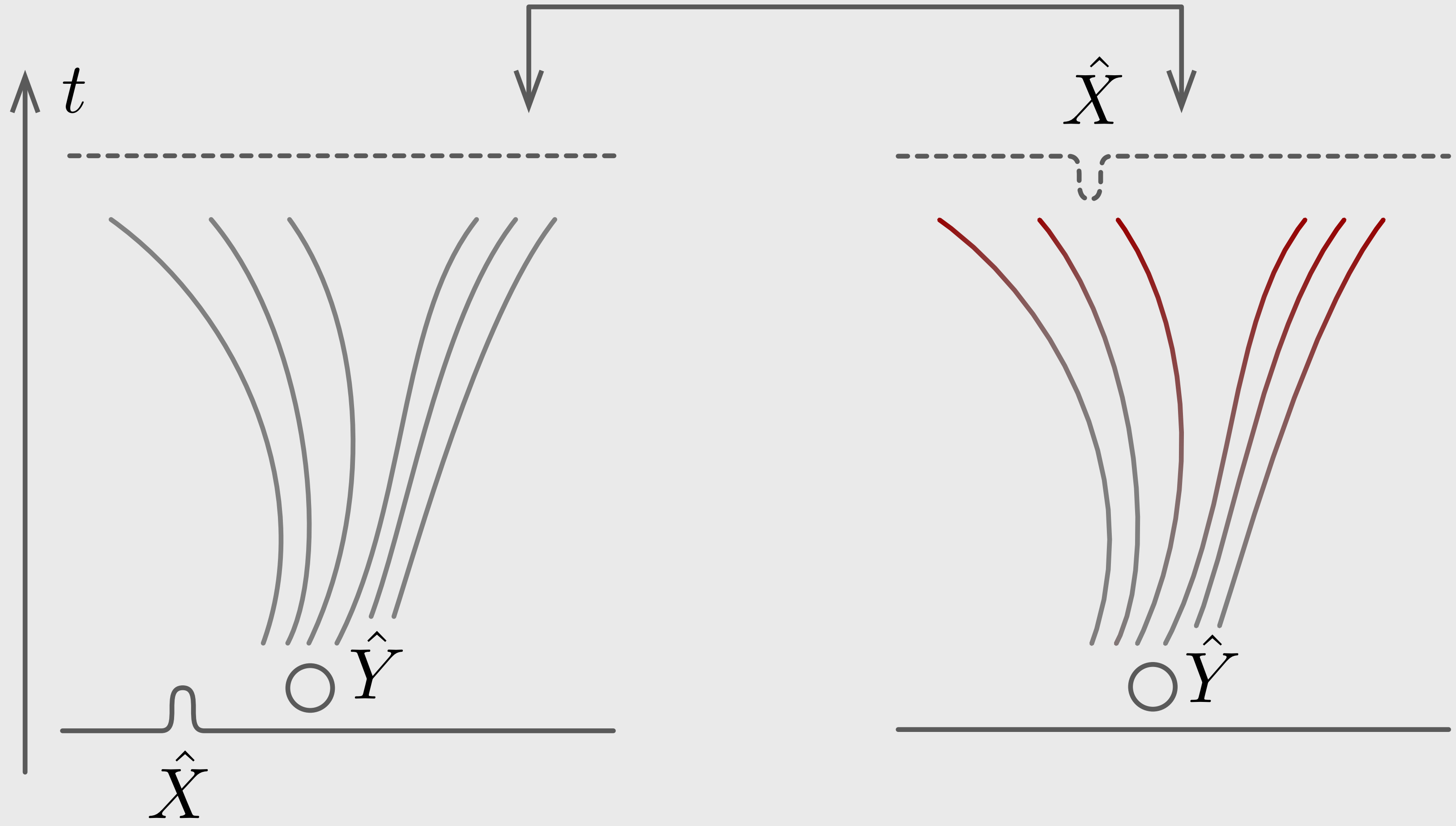
OTO correlation function

Out of time order (OTO) correlation function: a tool for diagnosing early stages of quantum chaotic dynamics (Larkin, Ovchinikov 69):

$$F(t) = \mathrm{tr}\left(e^{-\beta \hat{H}} \hat{X}\hat{Y}(t)\hat{X}\hat{Y}(t)\right)$$

X, *Y* one-body operators in many body context.

Interpretation: quantum butterfly effect



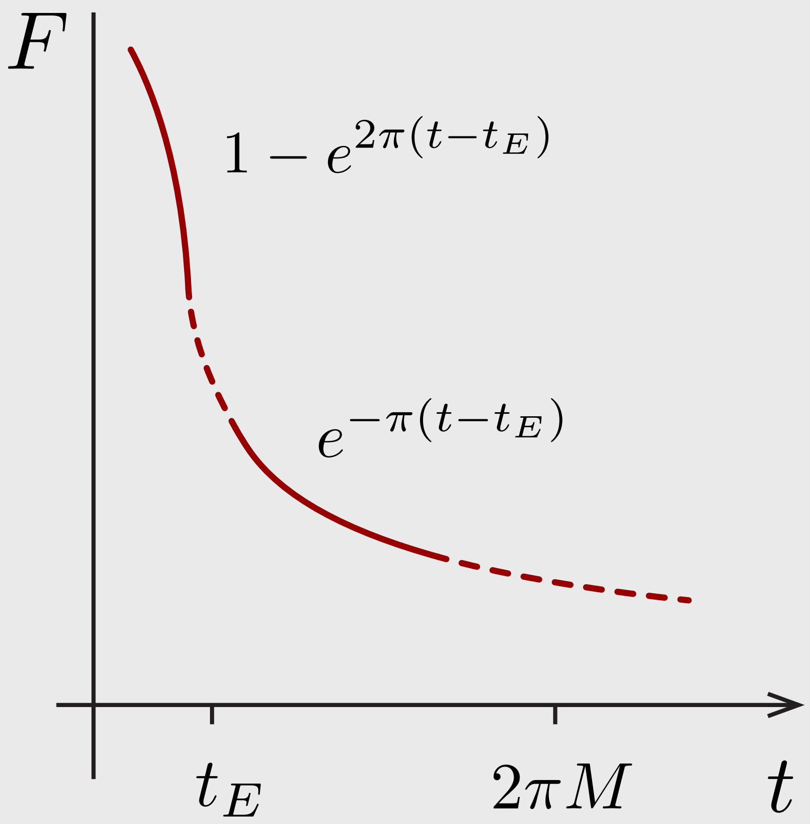
Short time OTO: stationary phase

At short times stationary phase analysis leads to (Maldacena 16)

$$F(t) = 1 - \frac{\beta e^{2\pi t/\beta}}{64\pi M} + \mathcal{O}(e^{\pi t/\beta}/M)$$

Result can be trusted up to effective Ehrenfest time (chaos bound maxed out!)

$$t \sim t_E \equiv \frac{\ln(MT)}{2\pi T}$$

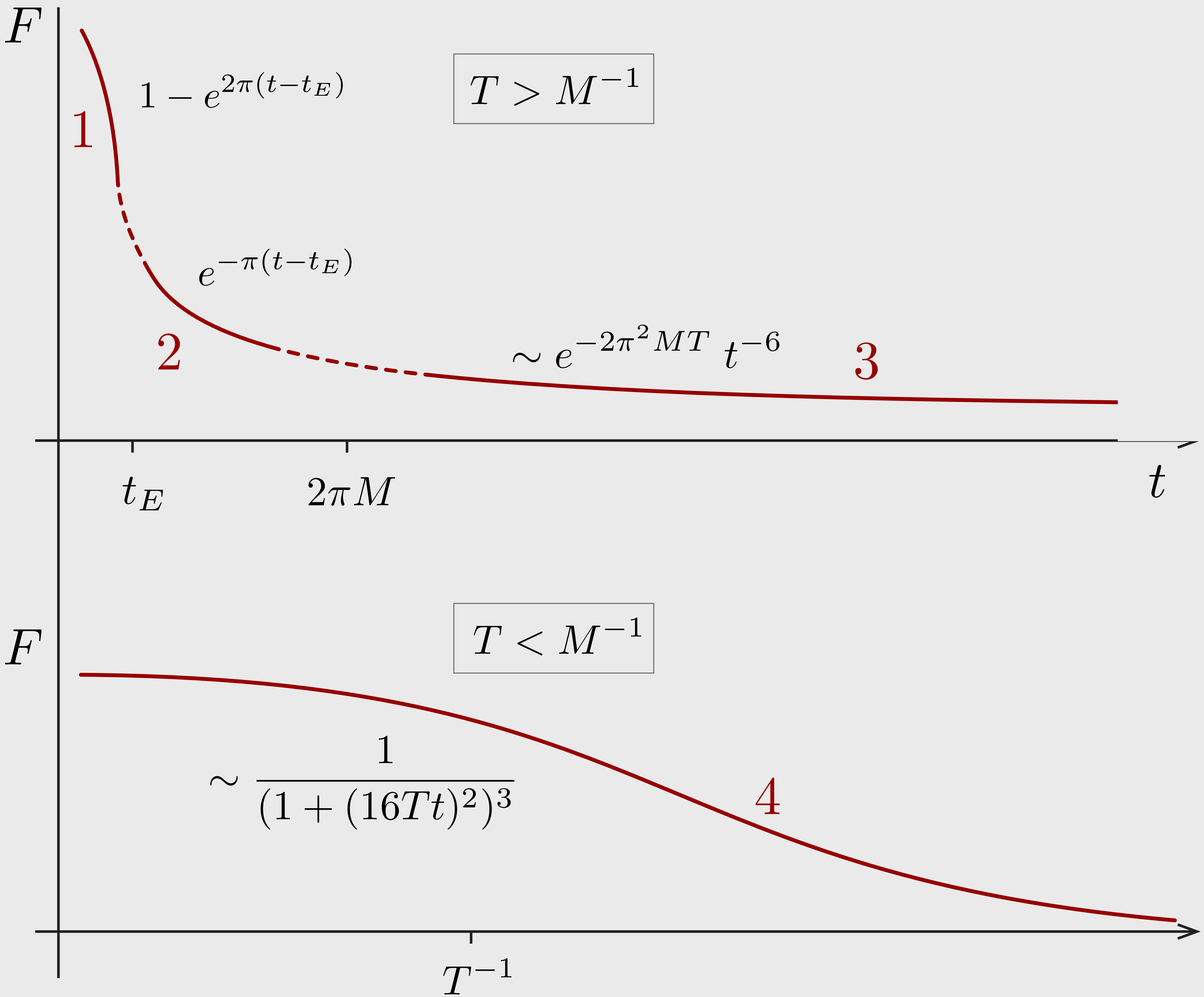


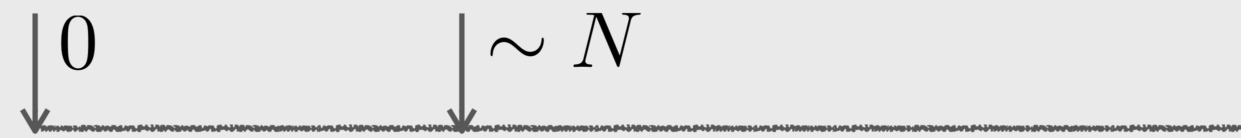
At intermediate times $t_E < t < M$

$$F(t) = \ln(MT)\, e^{-\pi T(t-t_E)}$$

Long time OTO: conformal Goldstone modes

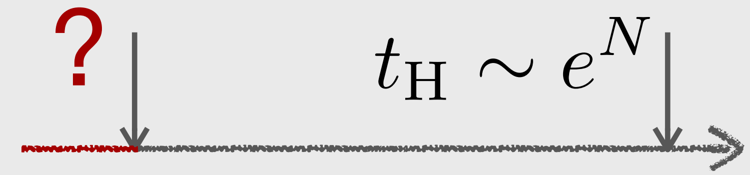
At long times large Goldstone mode fluctuations generate power law tail





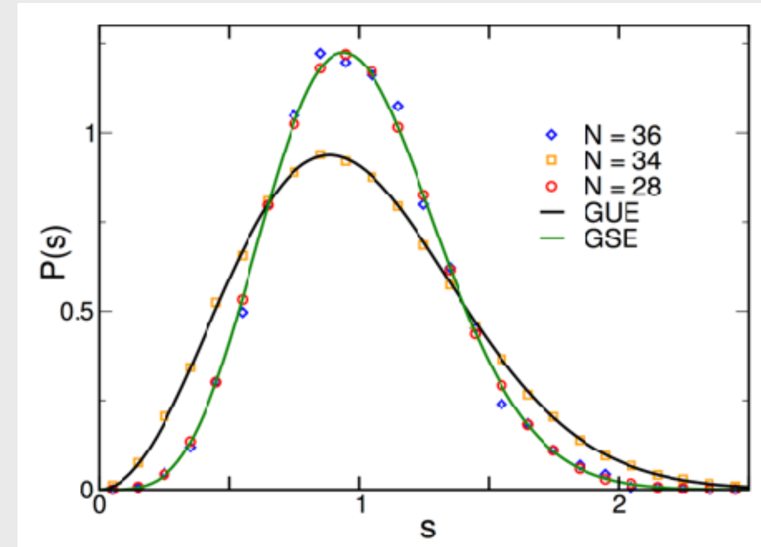
chaos and spectral correlation functions

Ergodic long time regime



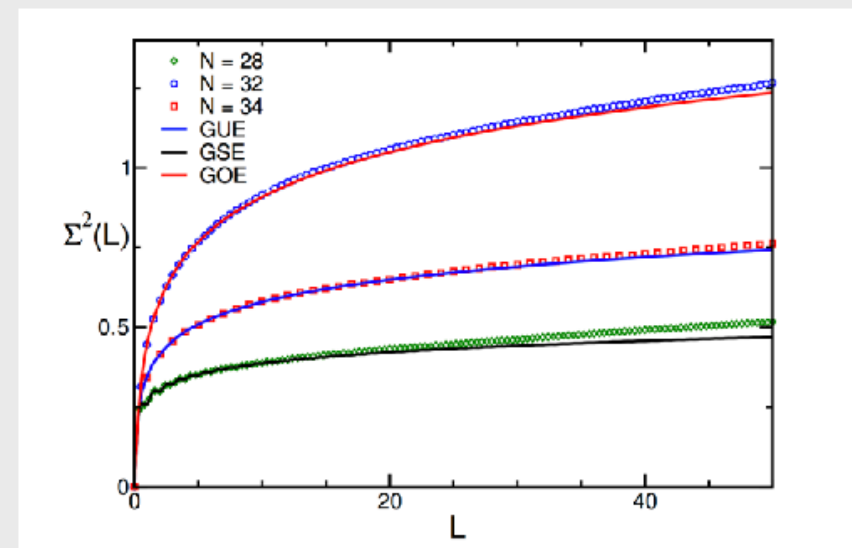
Ergodic long time dynamics diagnosed in universal correlations of many body spectra

Level spacing distribution



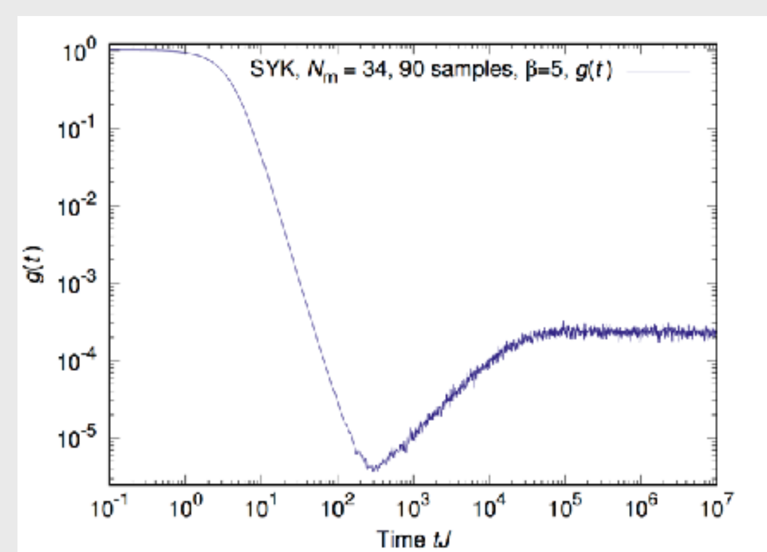
Verbaarschot, Garcia-Garcia, 16

Level number fluctuations



Verbaarschot, Garcia-Garcia, 16

Spectral form factor (Fourier trafo of energy level correlation function)

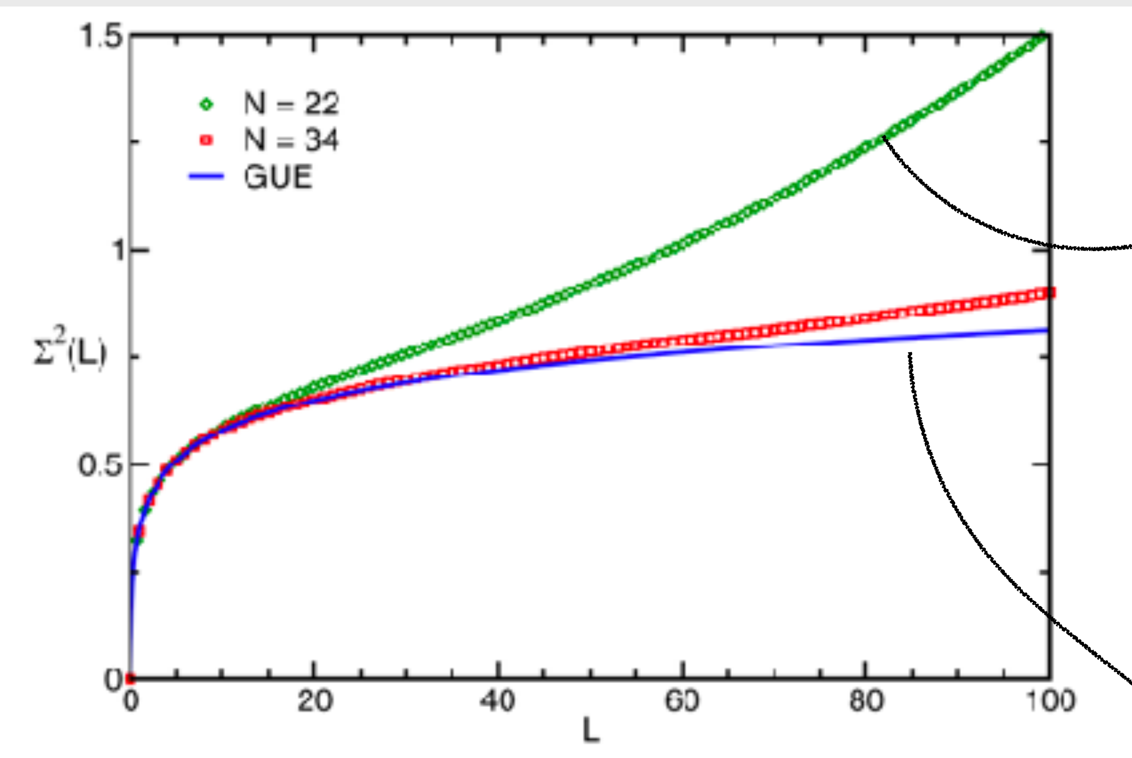


Cotler et al. 17

Pre ergodic regime

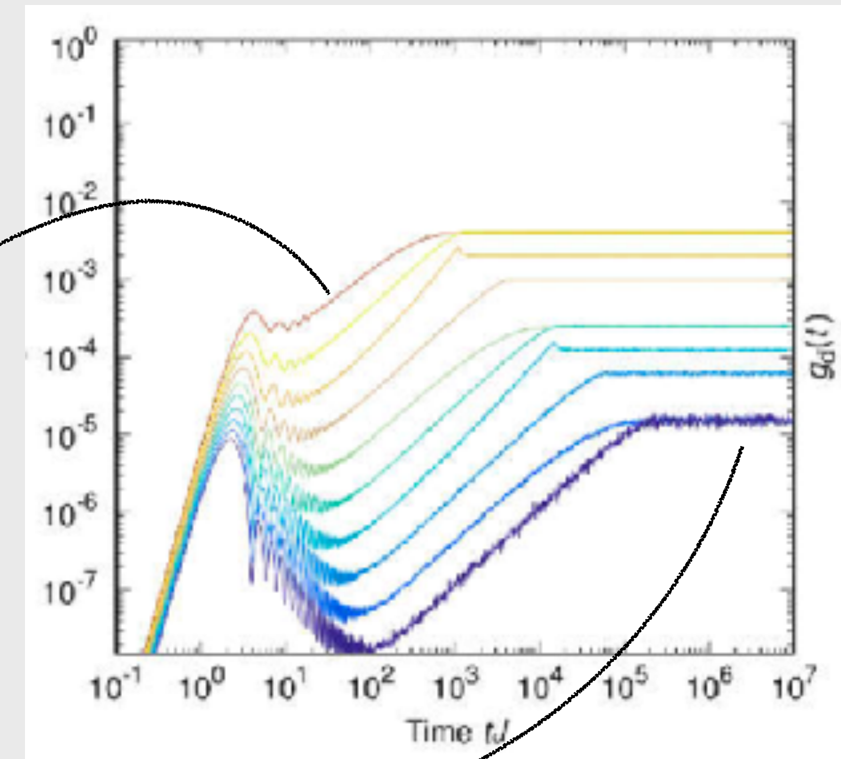
?

For times shorter than an ergodic time $t_{\rm erg}$ universal deviations from RMT behavior are observed.



Verbaarschot, Garcia-Garcia, 16

not RMT
(still universal)



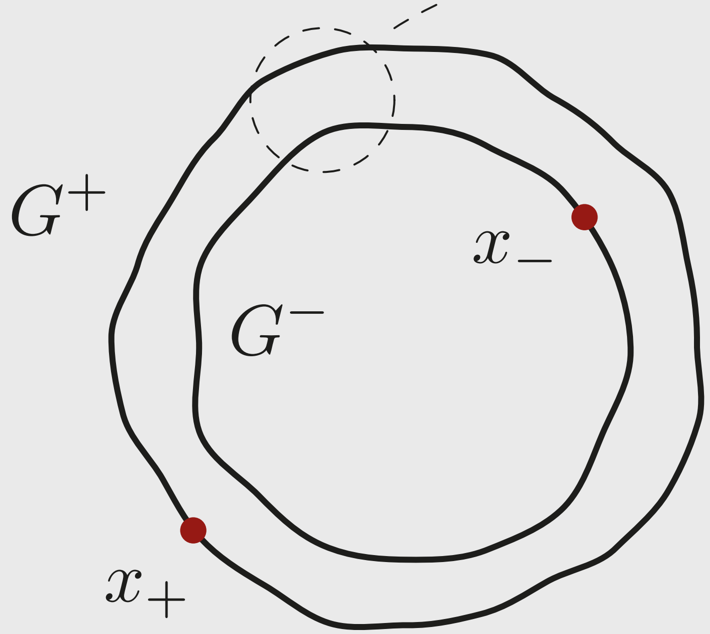
Cotler et al. 17

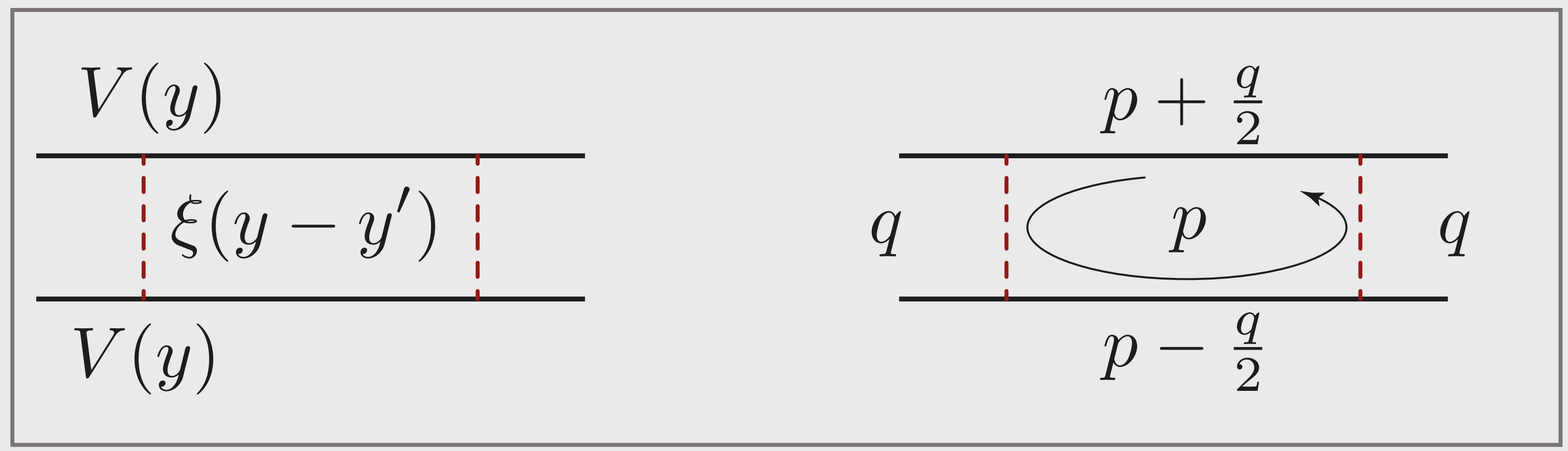
RMT

$$R_2(\omega) \equiv \Delta^2 \left\langle \rho(E+\frac{\omega}{2})\rho(E-\frac{\omega}{2})\right\rangle_c$$

Compare to the physics of dirty metals

Interested in fluctuations of $\rho(E) = -\frac{1}{\pi}\int dx\, \mathrm{Im}(G^+(E,x,x))$



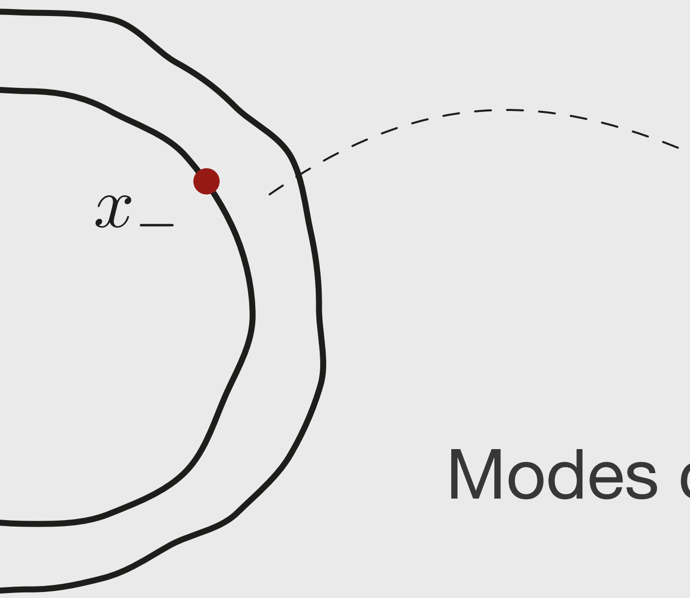


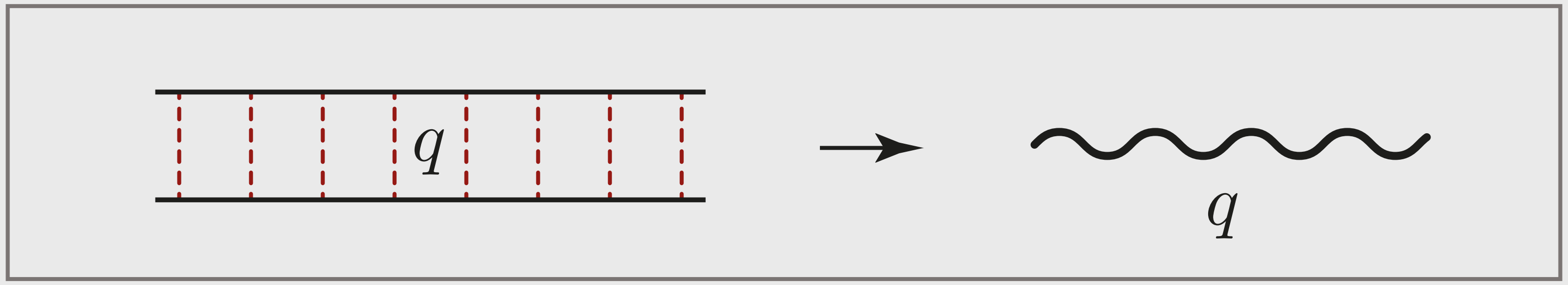
Momentum q difference conserved in scattering.

Idea: (i) basis change $|x\rangle\langle y| \longrightarrow |p+\frac{q}{2}\rangle\langle p-\frac{q}{2}|$

(ii) interpret this as a basis change in the space of Hilbert space *operators.*

dirty metals cont'd





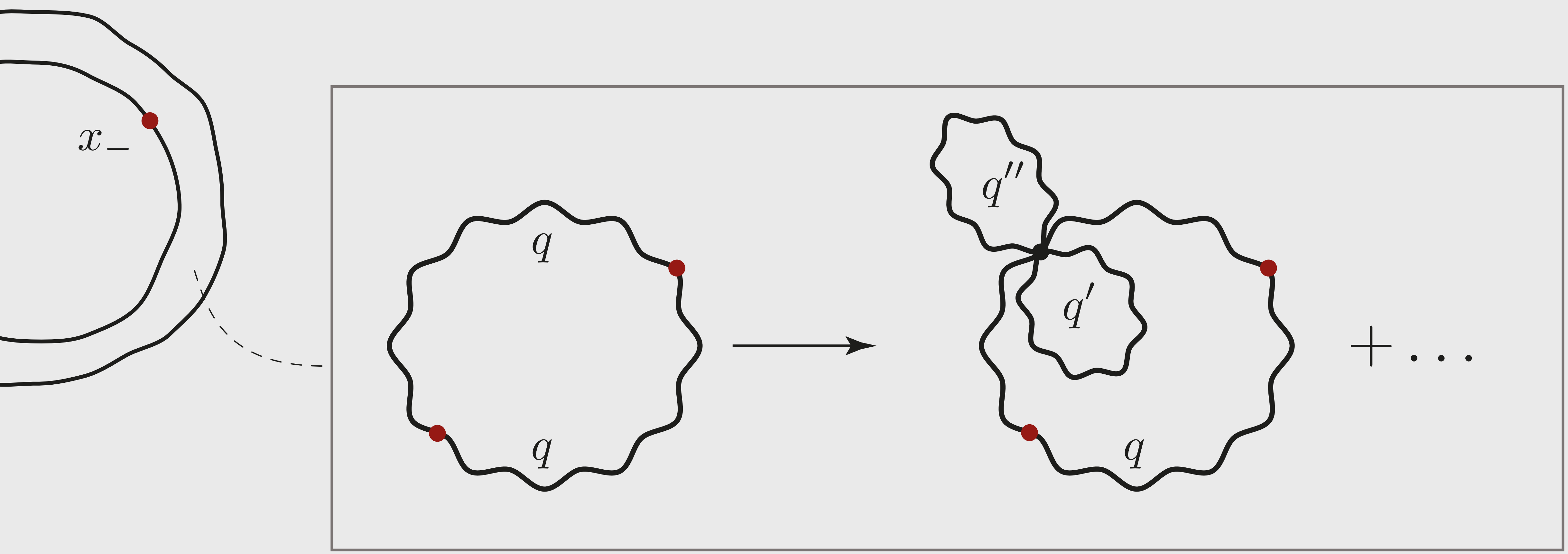
Modes characterized by

(i) 'discrete quantum numbers' $q \in \frac{2\pi}{L}\mathbb{Z}^3$

(ii) decay constants $Dq^2 + i\omega$

(iii) physical interpretation as irreversible relaxation modes

Dirty metals cont'd



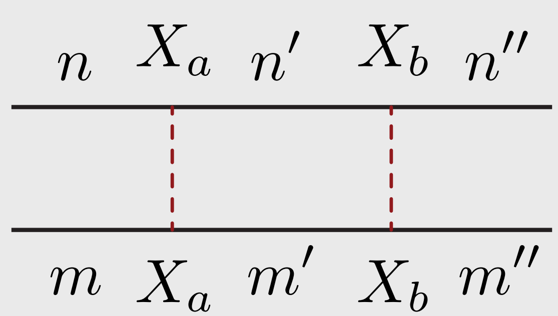
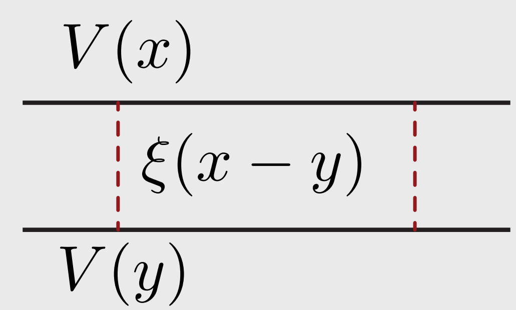
$$R_2(\omega) = R_{2,\mathrm{RMT}}(\omega) + \frac{1}{2}\left(\frac{\Delta}{\pi}\right)^2 \mathrm{Re} \sum_{q \neq 0} \frac{1}{(i\omega - Dq^2)^2}$$

Altshuler & Shklovskii, 86
Kravtsov & Mirlin, 94

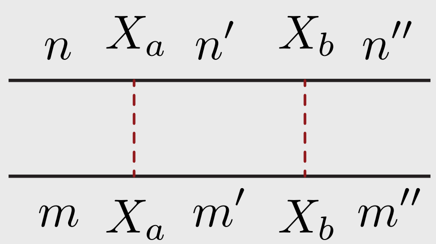
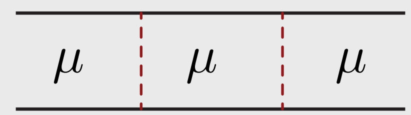
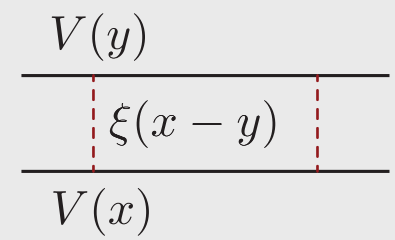
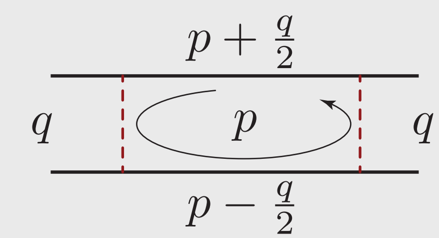
relaxation dynamics in Fock space

The setting

$$\hat{H} = \sum_{ijkl}^{N} J_{ijkl}\chi_i\chi_j\chi_k\chi_l \equiv \sum_a J_a \hat{X}_a, \qquad a = (i,j,k,l)$$

	SYK	dirty metal
Hilbert space	Fock space of *N/2* fermions	function space
dimension	$2^{N/2}$	∞
basis states	$m = (1,0,0,1,\dots)$	$\lvert x\rangle$
scattering vertex		
scattering states	$\lvert n\rangle \otimes \langle m\rvert$	$\lvert x\rangle \otimes \langle y\rvert$
basis of conserved states	$\hat{X}_\mu \equiv \chi_{\mu_1}\chi_{\mu_2}\cdots\chi_{\mu_k}$, $\mu \equiv (\mu_1, \mu_2, \dots, \mu_k)$	$\lvert p+\frac{q}{2}\rangle \otimes \langle p-\frac{q}{2}\rvert$

setting cont'd

	SYK	dirty metal
conserved modes	 	 
eigenvalues	$\epsilon(\lvert\mu\rvert) - i\omega$	$Dq^2 - i\omega$

$\epsilon(\lvert\mu\rvert) - i\omega$

↑ # of Majoranas in state

$$\epsilon(k) \sim 2^{N/2}\Delta \times k$$

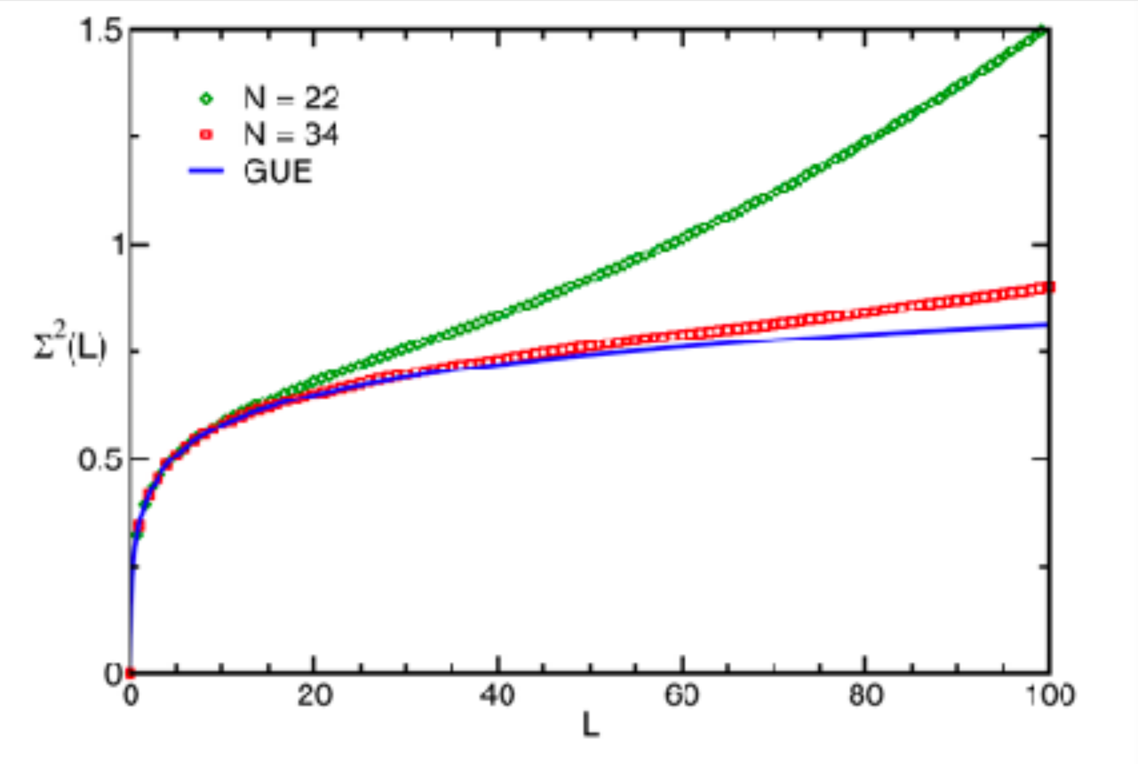
$$\Delta \sim \frac{JN^{1/2}}{2^{N/2}}$$

many body level spacing

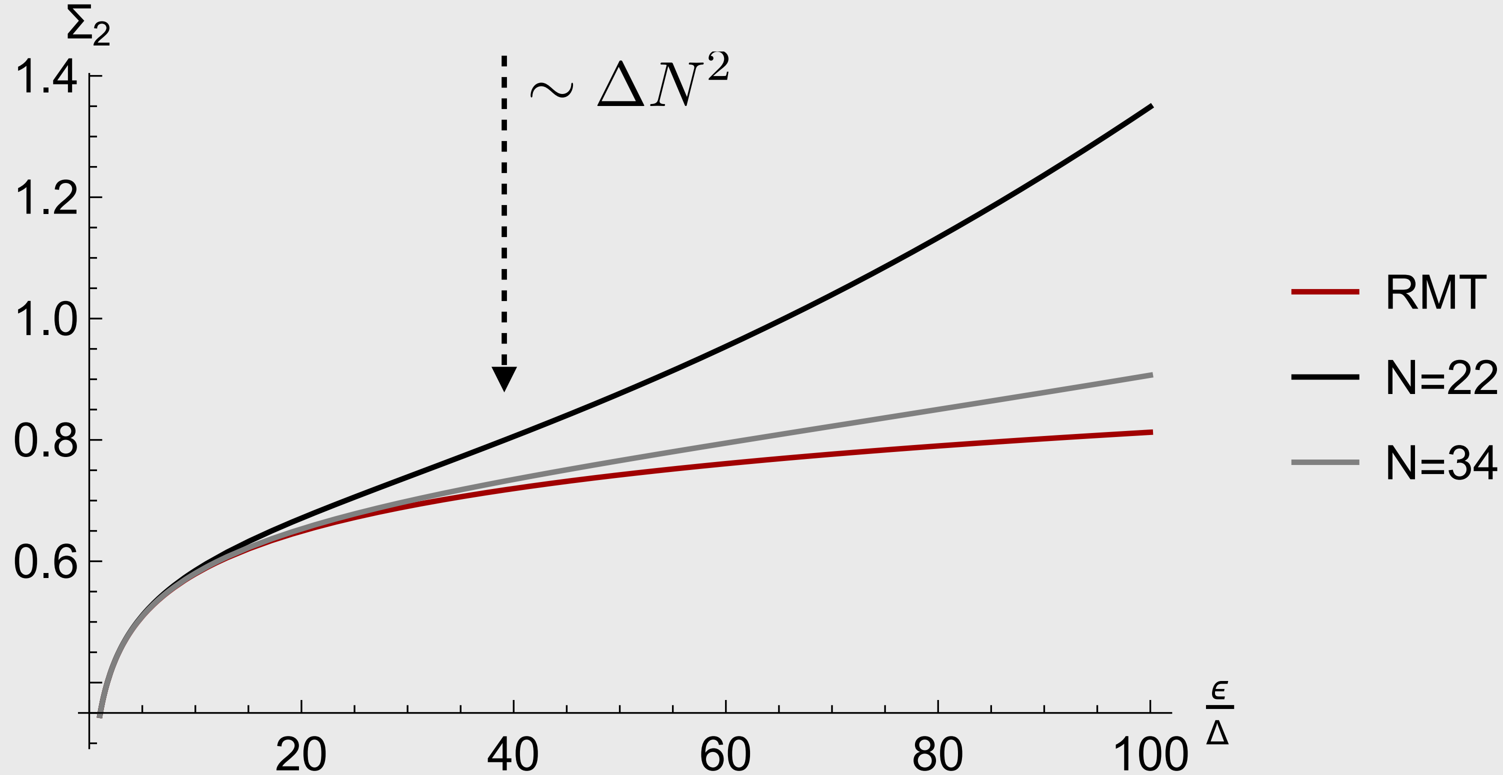
SYK spectral correlation function

$$R_2(\omega) = R_{2,\mathrm{RMT}}(\omega) + \frac{1}{2}\left(\frac{\Delta}{\pi}\right)^2 \mathrm{Re} \sum_{k\neq 0,\mathrm{even}} \binom{N}{k} \frac{1}{(i\omega - \epsilon(k))^2}$$

comparison to numerical data I: number variance

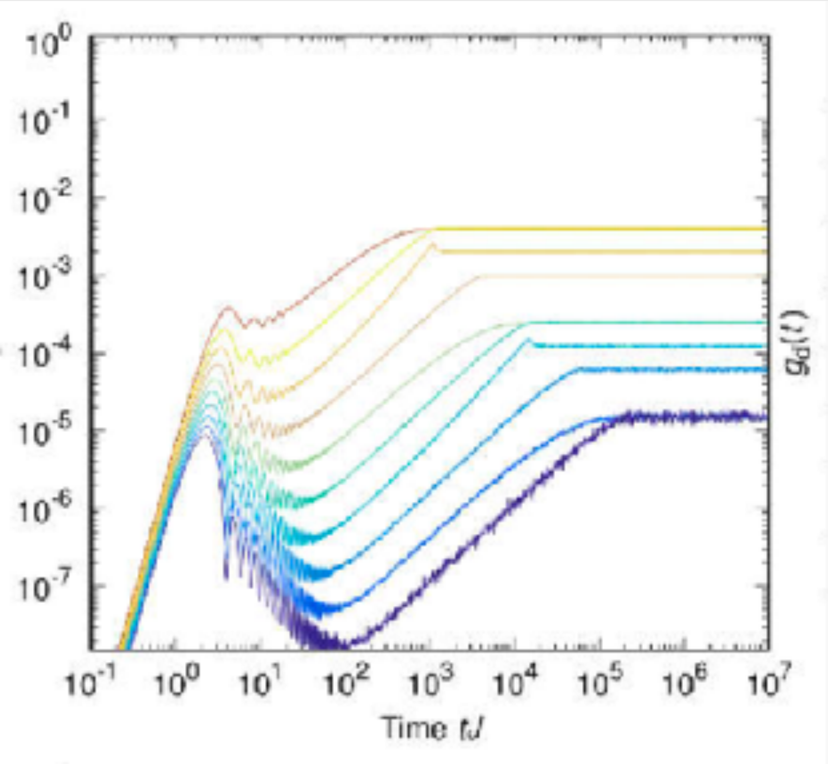


Verbaarschot, Garcia-Garcia, 16

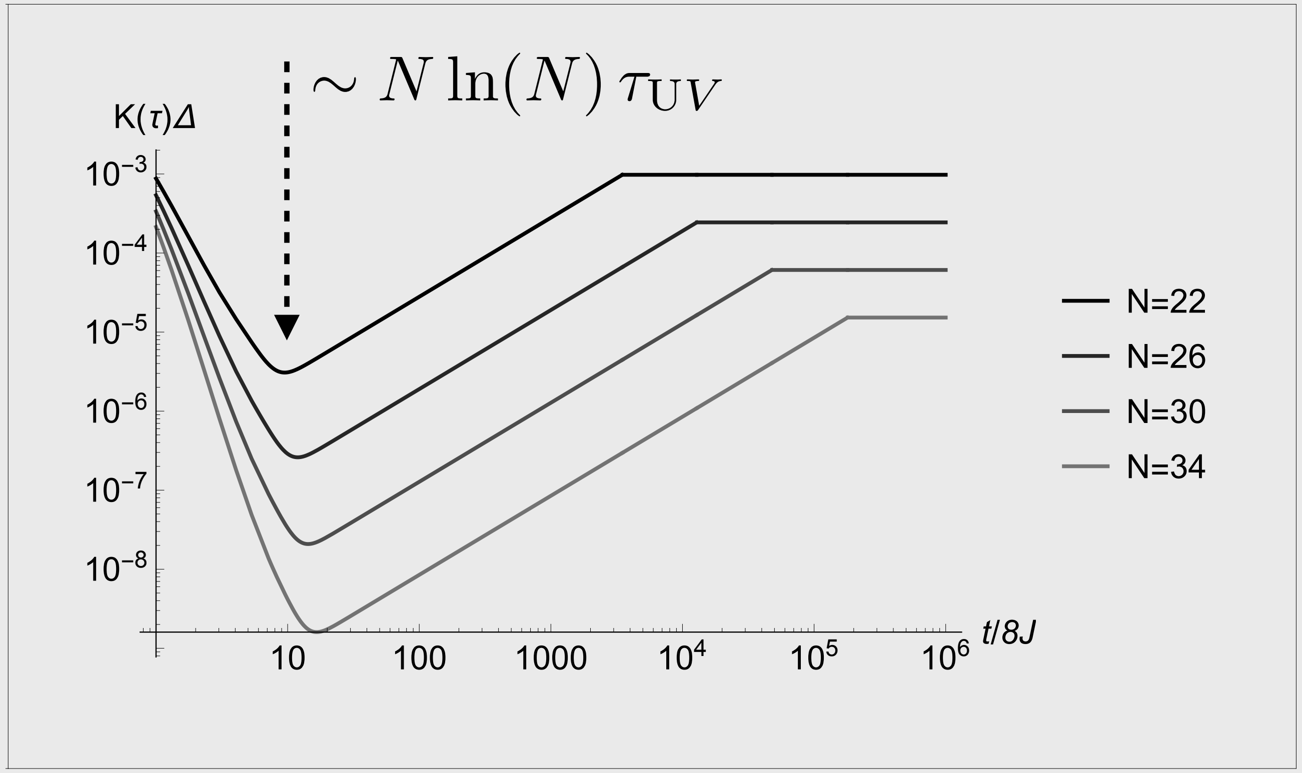


SYK spectral correlation function

comparison to numerical data II: spectral form factor



Cotler et al. 17



all in all: very good parameter free agreement with numerical data.

summary

conformal Goldstone modes cause strong quantum fluctuations at intermediate time scales

identified a high density set of complementary modes dominate fluctuations beyond a non-universal "Thouless energy"

a case study of chaotic relaxation in a strongly nonlinear many body environment

connection between these fluctutations? Ramifications in holography?

Holographic interpretation (Maldacena & Stanford, 16; Almheiri & Polchinksi, 16)

Consider 2d Einstein-Hilbert action

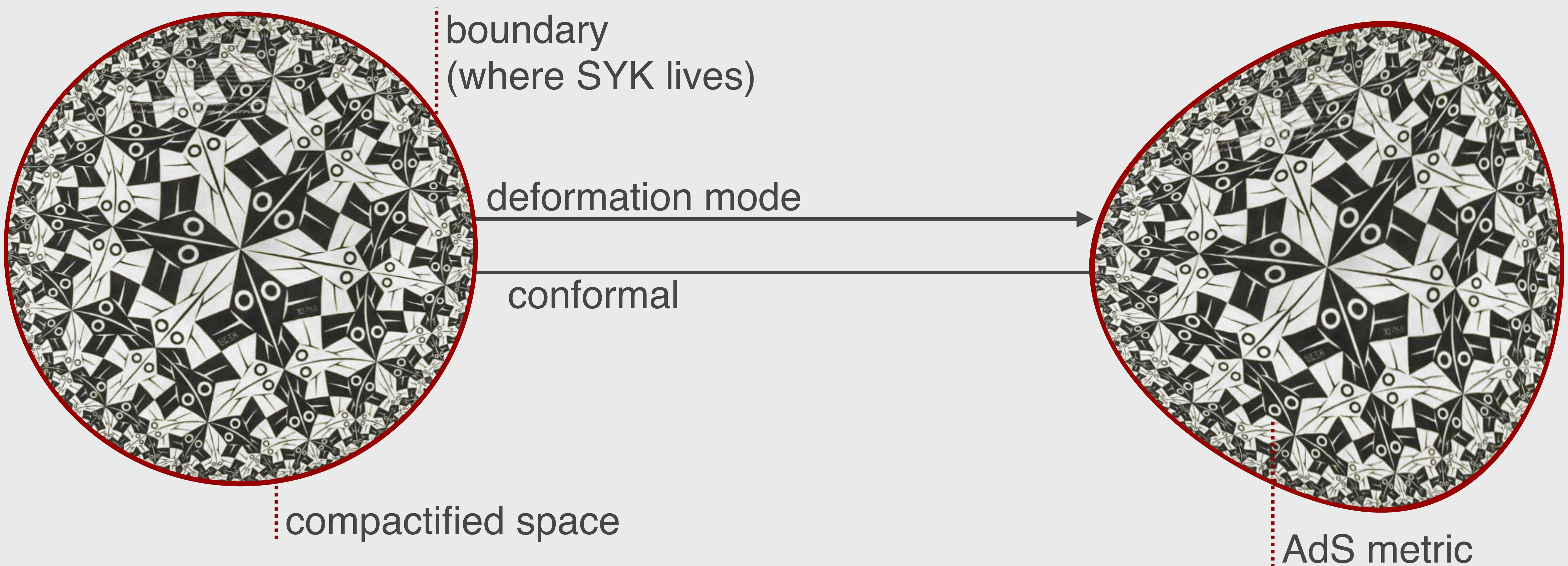
also constant

positive cosmological constant

$$S = \frac{\phi_0}{16\pi G}\int \sqrt{g}(R + \Lambda)$$

gravitational constant

action invariant under conformal deformations of 2d space (because it is topological)



AdS metric (spontaneously) breaks symmetry to *SL(2,R).* Reparameterization Goldstone modes without action.

Holographic interpretation (continued)

Improve situation by upgrading pure gravity action to dilaton action

now a field

$$S = \frac{\phi_0}{16\pi G} \int \sqrt{g}(R + \Lambda) \longrightarrow \frac{1}{16\pi G} \int \sqrt{g}\phi(R + \Lambda) + \ldots$$

Jackiw Teitelboim gravity

This action (i) is non-topological, (ii) fluctuations of the dilaton field weakly break conformal symmetry and (iii) afford physical interpretation if AdS2 action is seen as boundary theory of higher dimensional extremal black hole.

Combination (i-iii) motivates boundary with conformal invariance breaking and signatures of quantum chaos.