

Integrability, One-point Functions and Tests of AdS/dCFT

Charlotte Kristjansen

Niels Bohr Institute

Based on:

- M. de Leeuw, C.K., G. Linardopoulos, ArXiv:1802.01258[hep-th], Phys.Lett. B781 (2018) 238
- I. Buhl Mortensen, M. de Leeuw, A. Ipsen, C.K., M. Wilhelm, ArXiv: 1704.07386 [hep-th], Phys.Rev.Lett. 119 (26) (2017) 261604
- Previous work involving the same authors plus G. Semenoff & K. Zarembo as well as ongoing work involving A.G. Grau & M.Volk

Gauge/Gravity Duality 2018

Universität Würzburg,

August 1st, 2018

One-point functions --- Motivation

Possibly the simplest observables beyond the spectrum

Strong signs of integrability, closed determinant formulas
(Similarities with three-point functions)

Lead to a positive test of AdS/CFT in a situation where conformal symmetry is partially broken and susy symmetry is partially or fully broken.

Provide input for the boundary conformal bootstrap program
 $1+2=3$

Reveal interesting connections to statistical physics
(Quenched action approach to non-equilibrium stat. phys.)

Plan of the talk

- I. The AdS/dCFT set-up and its extra parameter(s)
- II. Gauge theory results on one-point functions, featuring integrability.
- III. String theory predictions for one-point functions, involving double scaling limit.
- IV. Confirming predictions (conf. symmetry is partially broken and susy partially or completely broken.)
- V. Integrability at finite g ?
- VI. Summary/Open problems

Part I.

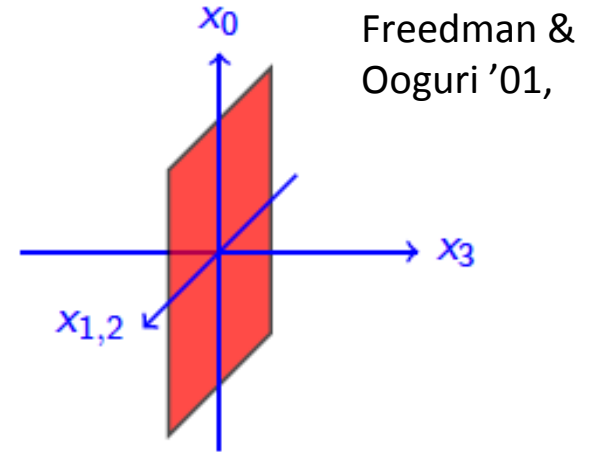
The AdS/dCFT set-up and its parameters

One-point functions in AdS/CFT

A defect CFT ---- with string theory dual

$\mathcal{N} = 4$ SYM with a co-dimension one defect

$$S = \underbrace{S_{\mathcal{N}=4}}_{\text{bulk}} + \underbrace{S_{D=3}}_{\text{defect}}, \quad \text{Certain b.c. on bulk fields}$$



Novel features

1. One-point functions

$$\langle \mathcal{O}_{\Delta}^{\text{bulk}}(x) \rangle = \frac{C}{|x_3|^{\Delta}}$$

Cardy '84
McAvity & Osborn '95

Normalization given by: $\lim_{x_3 \rightarrow \infty} \langle \mathcal{O}_{\Delta}^{\text{bulk}}(y+x) \mathcal{O}_{\Delta'}^{\text{bulk}}(z+x) \rangle = \frac{\delta_{\Delta\Delta'}}{|y-z|^{2\Delta}}$

2. Two-point functions between op's with different conf. dims.

3. Mixed correlators involving bulk and defect fields

AdS/dCFT (D3-D5 or D3-D7 case)

String theory:

		x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
N	$D3$	×	×	×	×						
Probe	$D5$	×	×	×		×	×	×			
or Probe	$D7$	×	×	×		×	×	×	×	×	

Brane Geometry:

Karch & Randall '01,

Geometry of D5 brane: $AdS_4 \times S^2$ (1/2 susy)

Geometry of D7 brane: $AdS_4 \times S^2 \times S^2$ or $AdS_4 \times S^4$ (No susy)

Precise embedding of D-brane determined by $S = S_{DBI} + S_{WZ}$

Parameters: λ, N

Issues

One-point functions at leading order in QFT involve a high number of loops (both 3D and 4D).

The D3-D7 probe brane set-up is unstable to fluctuations

AdS/dCFT a strong-weak coupling duality (eternal problem)

Resolution

Consider a non-trivial vacuum --- non-trivial fluxes on the
of the gauge theory string theory side.

Non-trivial vacua of the gauge theory

$$\phi_i^{\text{cl}}(x) = \phi_i^{\text{cl}}(x_3), \quad i = 1, \dots, 6, \quad \psi_\alpha^{\text{cl}} = 0, \quad A_\mu^{\text{cl}} = 0,$$

$$\frac{d^2 \phi_i^{\text{cl}}}{dx_3^2} = [\phi_j^{\text{cl}}, [\phi_j^{\text{cl}}, \phi_i^{\text{cl}}]]$$

D3-D5 brane solution:

For $x_3 > 0$

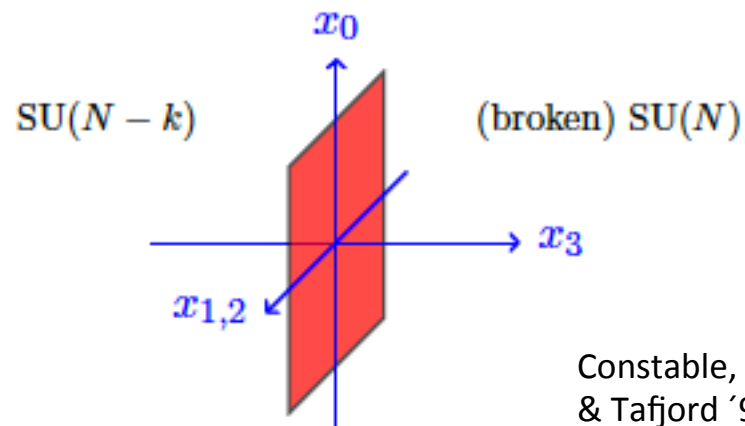
$$\phi_i^{\text{cl}} = \frac{1}{x_3} \begin{pmatrix} (t_i)_{k \times k} & 0 \\ 0 & 0 \end{pmatrix}, \quad i = 1, 2, 3$$

$$\phi_i^{\text{cl}} = 0, \quad i = 4, 5, 6$$

where t_i constitute a k -dimensional irreducible representation of $SU(2)$, in particular $[t_i, t_j] = \epsilon_{ijk} t_k$,

For $x_3 < 0$

$$\phi_i^{\text{cl}} = 0 \in \mathfrak{su}(N - k)$$



Constable, Myers
& Tafford '99

k is an additional tunable parameter (a flux)

Two D3-D7 brane solutions

$SO(3) \times SO(3)$ symmetric solution

For $x_3 > 0$

$$\phi_i^{\text{cl}} = \frac{1}{x_3} \begin{pmatrix} (t_i)_{k_1 \times k_1} \otimes 1_{k_2 \times k_2} & 0 \\ 0 & 0 \end{pmatrix}, \quad i = 1, 2, 3$$

$$\phi_i^{\text{cl}} = \frac{1}{x_3} \begin{pmatrix} 1_{k_1 \times k_1} \otimes (t_i)_{k_2 \times k_2} & 0 \\ 0 & 0 \end{pmatrix}, \quad i = 4, 5, 6$$

Broken $SU(N)$ invariance

For $x_3 < 0$

$$\phi_i^{\text{cl}} = 0 \in \mathfrak{su}(N - k_1 k_2), \quad i = 1, \dots, 6$$

$SU(N - k_1 k_2)$ invariance

$SO(5)$ symmetric solution

For $x_3 > 0$

$$\phi_i^{\text{cl}} = \frac{1}{\sqrt{8}x_3} \begin{pmatrix} (G_i)_{d_G \times d_G} & 0 \\ 0 & 0 \end{pmatrix}, \quad i = 1, 2, 3, 4, 5$$

$$\phi_6^{\text{cl}} = 0,$$

where $G_{ij} = [G_i, G_j]$ generate a d_G -dimensional irreducible representation of $SO(5)$

Broken $SU(N)$ invariance

For $x_3 < 0$

$$\phi_i^{\text{cl}} = 0 \in \mathfrak{su}(N - d_G), \quad i = 1, \dots, 6$$

$SU(N - d_G)$ invariance

OBS: AdS/CFT with additional parameters --- The dimension of the repr. (or fluxes), i.e.

k or (k_1, k_2) or d_G

Part II.

One-point functions in gauge theory

One-point functions in gauge theory

Scalar operators can have non-zero 1-pt fcts already at tree-level

Wish: A Systematic approach to the computation of 1-pt functions of *conformal* scalar operators using the tools of integrability

Consider operators built from

$$X = \phi_1 + i\phi_4, \quad W = \phi_2 + i\phi_5, \quad Y = \phi_3 + i\phi_6$$

$$\bar{X} = \phi_1 - i\phi_4, \quad \bar{W} = \phi_2 - i\phi_5, \quad \bar{Y} = \phi_3 - i\phi_6$$

Conformal scalar operators=Eigenstates of integrable SO(6) spin chain

Eigenstates of length L: $|u_i, v_j^+, v_k^- \rangle_L$

characterized by three sets of rapidities $\{u_i\}_{i=1}^M, \{v_j^+\}_{i=1}^{N^+}, \{v_j^-\}_{i=1}^{N^-}$

Corresponding operator: $L - M = \#X - \#\bar{X},$
 $M - N_+ = \#W + \#\bar{X} + \#\bar{Y},$
 $N_+ - N_- = \#Y - \#\bar{Y},$
 $N_- = \# \text{ barred fields}$

One-point functions in gauge theory

$$\langle \mathcal{O}_\Delta(x) \rangle = (\text{Tr}(\phi_{i_1} \dots \phi_{i_\Delta}) + \dots) |_{\phi_{i_a} \rightarrow \phi_{i_a}^{\text{cl}}} \equiv \frac{C(\{u_i, v_j^+, v_l^-\})}{x_3^\Delta}$$

Matrix Product State associated with the defect:
(D3-D5 case for simplicity)

deLeeuw, C.K.
& Zarembo '15,

$$|\text{MPS}_k\rangle = \sum_{\vec{i}} \text{tr}[t_{i_1} \dots t_{i_L}] |\phi_{i_1} \dots \phi_{i_L}\rangle,$$

Object to calculate:

$$C_k(\{u_i, v_j^+, v_l^-\}) = \frac{\langle \text{MPS}_k | \{u_i, v_j^+, v_l^-\} \rangle_L}{\langle \{u_i, v_j^+, v_l^-\} | \{u_i, v_j^+, v_l^-\} \rangle^{\frac{1}{2}}}$$

NB: Parameters:

$$L = \Delta,$$

M, N_+, N_- (number of Bethe roots/fields of various types),

k (representation label)

Solution D3-D5 case

Selection rules:

de Leeuw, C.K &
Linardopoulos, 18.

- Momentum of Bethe state equal to zero
- M and $L + N_+ + N_-$ even
- The rapidities come in pairs, i.e. $\{u_i\}, \{v_j^+\}, \{v_l^-\} = \{-u_i\}, \{-v_j^+\}, \{-v_l^-\}$
Follows from the fact that $Q_{2n+1}|\text{MPS}_k\rangle = 0$ for all $n \in N_0$

Result for C_k :

- Exact formula valid for any, L, M, N^+, N^- and k
- Expressed in terms of objects well known from integrability
 - Baxter polynomials $Q(u) = \prod_{i=1}^M (u - u_i)$, $Q_+(v)$, $Q_-(v)$
 - Determinant of Gaudin matrix, G
 $\langle \{u_i, v_j^+, v_l^-\} | \{u_i, v_j^+, v_l^-\} \rangle = \det G = \det G_+ \det G_-$
 - Transfer matrix in higher reps (or some projection thereof).

Solution D3-D5 case

de Leeuw, C.K &
Linardopoulos, 18.

Result for C_k :

- Exact formula valid for any, L, M, N^+, N^- and k

$$C_k^{SO(6)} = \sqrt{\frac{Q(0)Q(\frac{i}{2})Q(\frac{ik}{2})Q(\frac{ik}{2})}{\bar{Q}_+(0)\bar{Q}_+(\frac{i}{2})\bar{Q}_-(0)\bar{Q}_-(\frac{i}{2})}} \cdot \mathbb{T}_{k-1}(0) \cdot \sqrt{\frac{\det G_+}{\det G_-}}$$

$$\mathbb{T}_n(x) = \sum_{a=-\frac{n}{2}}^{\frac{n}{2}} (x+ia)^L \frac{Q_+(x+ia)Q_-(x+ia)}{Q(x+i(a+\frac{1}{2}))Q(x+i(a-\frac{1}{2}))}.$$

- Can be proved analytically for the $SU(2)$ subsector ($N_+ = N_- = 0$).
Here \mathbb{T}_n is the transfer matrix in the $n+1$ dimensional rep.
- Has been checked numerically up to $L=16, k=6$ for $SU(3)$ ($N_- = 0$).
(Involves summing 10^{12} terms.)
- Has been checked numerically up to $L=8, k=6$ for $SO(6)$

Buhl-Mortensen,
de Leeuw, C.K &
Zarembo, 15.

de Leeuw, C.K
& Mori, 16.

SO(5) symmetric the D3-D7 brane set-up

Selection rules:

de Leeuw, C.K &
Linardopoulos, 17.

- Momentum of Bethe state equal to zero
- $(L, M, N_+, N_-) = (L, M, M/2, M/2)$
- The rapidities come in pairs, i.e. $\{u_i\}, \{v_j^+\}, \{v_l^-\} = \{-u_i\}, \{-v_j^+\}, \{-v_l^-\}$
Follows from the fact that $Q_{2p+1}|\text{MPS}_n\rangle = 0$ for all $p \in N_0$

Result for C_n :

- Trivialises for $SU(2)$ sub-sector, $N_- = N_+ = 0$. ($\implies M = 0$)
- Trivialises for $SU(3)$ sub-sector, $N_- = 0$. ($\implies N_+ = M = 0$)
- No closed expression found for the full $SO(6)$ sector (yet)

Should we expect a closed expression?

Integrability of Matrix Product States

In statistical physics: $\langle \text{MPS} | \{u_i, v_j^+, v_l^-\} \rangle \sim \langle \text{Initial} | n \rangle \equiv \langle \Psi_0 | n \rangle$

Time evolution of local observable after a quantum quench:

$$\begin{aligned} \langle \mathcal{O}(t) \rangle &= \langle \Psi_0 | e^{iHt} \mathcal{O} e^{-iHt} | \Psi_0 \rangle \\ &= \sum_{n,m} \langle \Psi_0 | n \rangle \langle m | \Psi_0 \rangle \langle n | \mathcal{O} | m \rangle e^{-i(E_m - E_n)t} \end{aligned}$$

Proposed criterion for integrability of MPS: $\hat{Q}_{2m+1} | \text{MPS} \rangle = 0, \quad m \geq 1$

Piroli, Pozsgay
Vernier '17

NB: Imply pairing of roots for Bethe states in order to have non-vanishing overlap with MPS

- D3-D5 MPS fulfills integrability criterion
- D3-D7 SO(5) symmetric MPS fulfills integrability criterion
- D3-D7 SO(3) x SO(3) symmetric MPS does not fulfil integrability criterion

Puzzle: No closed form for 1-pt functions for the SO(5) symmetric D3-D7 case (yet)

Possible key to progress: A scattering picture with a reflection matrix

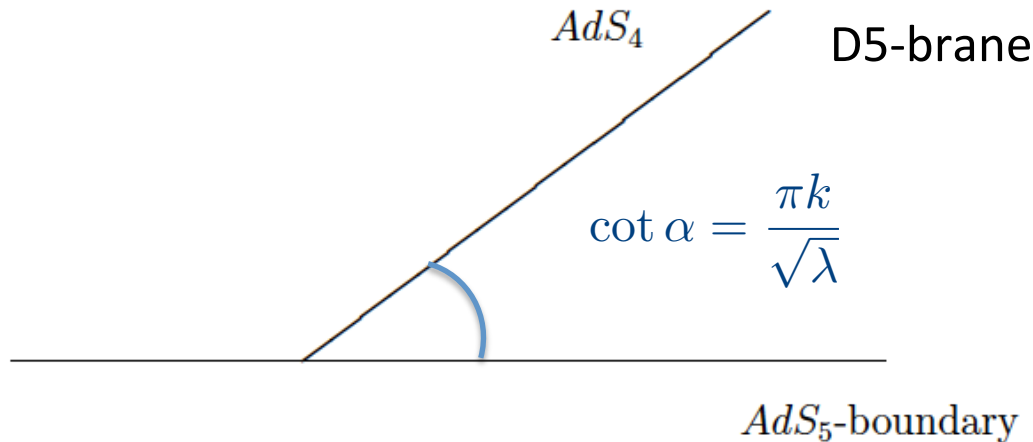
Part III.

One-point functions in string theory

The double scaling limit

D3-D5 probe brane system suggests a new double scaling limit

Nagasaki &
Yamaguchi '12,



$$\lambda \rightarrow \infty, \quad k \rightarrow \infty, \quad \frac{\lambda}{k^2} \text{ finite} \quad (N \rightarrow \infty)$$

One can compare perturbative gauge theory to semi-classical string theory (or sugra).

For D3-D7 with $SO(3) \times SO(3)$ symmetry: $\cot \alpha = \frac{\pi \sqrt{k_1^2 + k_2^2}}{\sqrt{\lambda}}$

C.K., Semenoff &
Young '12,

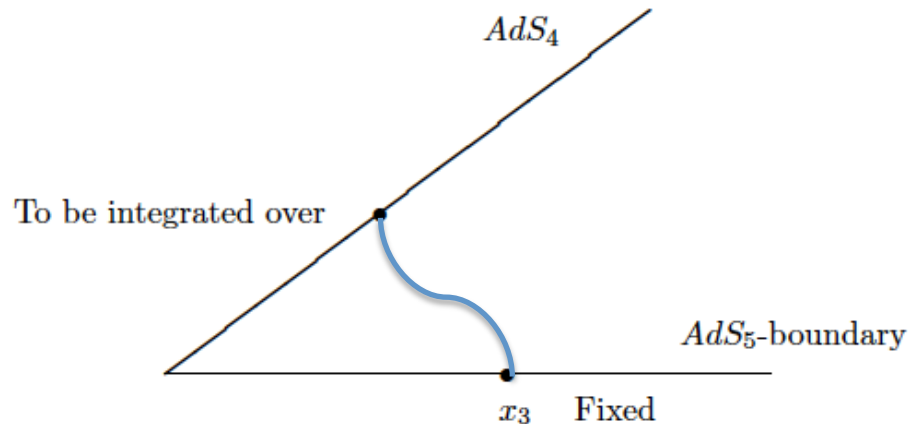
NB: D3-D5 Both susy and conformal symmetry are partially broken
D3-D7 Susy completely broken

One-point functions of chiral primaries --- GKPW method

- Only one chiral primary with the appropriate symmetry for each (even) conf.dim. Δ
- Find the variation $\delta S_E = \delta(S_{DBI} + S_{WZ})$
- Expand fluctuations in terms of spherical harmonics on S^5

$$\delta S_E(X, \Omega) = \sum_{\Delta} \sum_I s_{\Delta I}(X) Y_{\Delta I}(\Omega)$$

- Pick $Y_{\Delta I}(\Omega)$ the wanted unique chiral primary
- Replace $s_{\Delta I}(X)$ with a bulk-to-boundary propagator reaching from a point z on the brane to x_3 at the boundary. Integrate over z .



Nagasaki &
Yamaguchi '12,

Results in d.s.l. for chiral primary of length L

Match at leading order in d.s.l. both for D3-D5 and D3-D7

Nagasaki &
Yamaguchi '12,

C.K. , Semenoff,
Young '12,

Next to leading order predictions

D3-D5 set-up:

$$\left. \frac{\langle Y_L(\lambda) \rangle}{\langle Y_L(0) \rangle} \right|_{\text{ugra}} = 1 + \frac{\lambda}{4\pi^2 k^2} \frac{L(L+1)}{4(L-1)} + \mathcal{O} \left(\left(\frac{\lambda}{4\pi^2 k^2} \right)^2 \right)$$

Nagasaki &
Yamaguchi '12,

D3-D7 set-up with SO(3) x SO(3) symmetry

Grau, C.K, Volk &
Wilhelm, to appear

$$\begin{aligned} \left. \frac{\langle Y_L(\lambda) \rangle}{\langle Y_L(0) \rangle} \right|_{\text{ugra}} = & 1 + \frac{\lambda}{4\pi^2(k_1^2 + k_2^2)} \frac{1}{[k_1^2 + k_2^2]^3 (L-1) \sin(L+2)\phi} \Big[\\ & + 4Lk_1k_2 [(k_1)^4 + (k_2)^4 + (k_1k_2)^2(L+1)] \cos L\phi \\ & + [(k_2)^2 - (k_1)^2] [4(k_1k_2)^2(L^2 + L - 1) + ((k_1)^4 + (k_2)^4)(L^2 + 3L - 2)] \sin L\phi \Big] \\ & + \mathcal{O} \left(\left(\frac{\lambda}{4\pi^2(k_1^2 + k_2^2)} \right)^2 \right), \quad \phi = \arctan \left(\frac{k_1}{k_2} \right) \end{aligned}$$

Part IV.

One-loop one-point functions in gauge theory
Matching gauge and string theory

One-loop one-point functions

Motivation:

- I. Setting up the quantum computation is non-trivial
- II. Checking the one-loop predictions from string theory

Choose $\mathcal{O} = \text{Tr} Z^L \in SU(2)$ sector

$$= c_1 Y_L + \dots$$

$$\frac{\langle \text{tr} Z^L(\lambda) \rangle}{\langle \text{tr} Z^L(0) \rangle} = \frac{\langle Y_L(\lambda) \rangle}{\langle Y_L(0) \rangle}$$

- III. Integrability and finite g

The one-loop computation

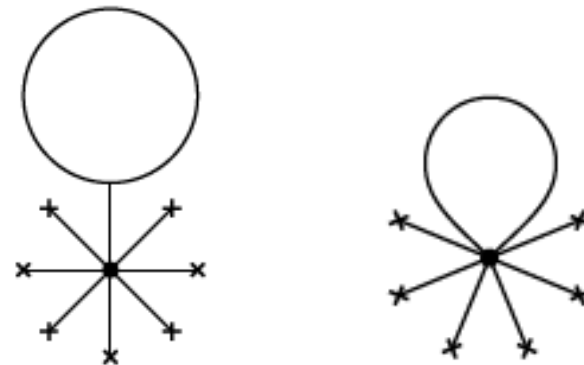
Two types of corrections

1. New one-loop Feynman diagrams
2. Correction of the eigenstate (not for chiral primaries)

1.

Scalars, gauge fields,
fermions, ghosts

Two planar diagrams



2. Can be found using Θ – morphism

Gromov &
Vieira '13, '14

Field theory result, chiral primaries

D3-D5 dCFT (1/2 susy):

$$\frac{\langle \text{tr} Z^L \rangle_{\text{one-loop}}}{\langle \text{tr} Z^L \rangle_{\text{tree level}}} = 1 + \frac{\lambda}{4\pi^2 k^2} \frac{L(L+1)}{4(L-1)}$$

Buhl-Mortensen,
de-Leeuw, Ipsen,
C.K., Wilhelm, '16

D3-D7 dCFT with SO(3) x SO(3) symmetry (no susy):

Grau, C.K, Volk &
Wilhelm, to appear

$$\begin{aligned} \frac{\langle \text{tr} Z^L \rangle_{\text{one loop}}}{\langle \text{tr} Z^L \rangle_{\text{tree-level}}} = 1 + \frac{\lambda}{4\pi^2 (k_1^2 + k_2^2)} \frac{1}{[(k_1)^2 + (k_2)^2]^3 (L-1) \sin(L+2)\phi} \Big[\\ + 4Lk_1k_2 [(k_1)^4 + (k_2)^4 + (k_1k_2)^2(L+1)] \cos L\phi \\ + [(k_2)^2 - (k_1)^2] [4(k_1k_2)^2(L^2 + L - 1) + ((k_1)^4 + (k_2)^4)(L^2 + 3L - 2)] \sin L\phi \Big] \end{aligned}$$

$$\phi = \arctan \left(\frac{k_1}{k_2} \right)$$

Complete match with string theory in both cases

Part V.

Integrability at finite g ?

One point functions at higher loop orders

Buhl-Mortensen,
de-Leeuw, Ipsen,
C.K., Wilhelm ,17

D3-D5 dCFT: Can we guess the result based on integrability ?
For simplicity restrict to SU(2) sub-sector

Idea:

- Make the integrability characteristic replacements in the Bethe eqns.

$$u \pm \frac{i}{2} \longrightarrow x(u \pm \frac{i}{2}), \quad u(x) = x + \frac{g^2}{x}, \quad g^2 = \frac{\lambda}{8\pi^2}$$

(plus dressing phase plus wrapping corrections)

- Make the same replacement in the transfer matrix, i. e.

$$T_n(u) \longrightarrow \tilde{T}_n(u) = g^L \sum_{a=-\frac{n}{2}}^{\frac{n}{2}} x(u + ia)^L \frac{Q(u + \frac{n+1}{2}i)Q(u - \frac{n+1}{2}i)}{Q(u + (a - \frac{1}{2})i)Q(u + (a + \frac{1}{2})i)}$$

- Possible asymptotic (i.e. so far without wrapping) formula

$$C_k = i^L \tilde{T}_{k-1}(0) \sqrt{\frac{Q(\frac{i}{2})Q(0)}{Q^2(\frac{ik}{2})}} \sqrt{\frac{\det \tilde{G}_+}{\det \tilde{G}_-}}$$

Outcome from the one-loop computation

Buhl-Mortensen,
de-Leeuw, Ipsen,
C.K., Wilhelm ,17

$$C_k = i^L \tilde{T}_{k-1}(0) \sqrt{\frac{Q(\frac{i}{2})Q(0)}{Q^2(\frac{ik}{2})}} \sqrt{\frac{\det \tilde{G}_+}{\det \tilde{G}_-}} \mathbb{F}_k \quad (*)$$

$$\mathbb{F}_k = 1 + g^2 \left[\Psi\left(\frac{k+1}{2}\right) + \gamma_E - \log 2 \right] \Delta^{(1)} + O(g^4),$$

OBS: Independent
of rapidities

Checked for: L, M=2, any k (analytical derivation)
L=8, M=4, k=2,3,4,5,6 (numerically)

Exponentialization at higher loop orders ?

$$\mathbb{F}_k \stackrel{?}{=} 2^{L-\Delta} \exp \left[(\Delta - L) \left(\Psi\left(\frac{k+1}{2}\right) + \gamma_E \right) \right]$$

Two loop computation might clarify this (NB Two loops in AdS)

(*) reproduces string theory result in d.s.l. up to wrapping order!

Summary of results

- A closed expression for all tree-level scalar one-point functions in the D3-D5 case for any representation label k .
- Closed one-point function formula in $SU(2)$ sub-sector at one-loop for D3-D5 set-up. (Proposal for asymptotic formula.)
- Match with string theory for D3-D5 to two leading orders in $d.s.l.$ for the one-pt function of a chiral primary (conf. & susy partially broken.)
- Match with string theory for D3-D7 to two leading orders in double scaling parameter for the one-pt function of a chiral primary (susy completely broken).
- Ongoing investigations of the D3-D7 brane system. (Non-protected one-point functions vanish in $SU(2)$ and $SU(3)$ sector in the $SO(5)$ symmetric case, no closed formula for $SO(6)$ (yet) in either case)

Open questions:

- Understanding the integrability of D3-D5 from scattering theory
- Understanding if D3-D7 is integrable or not
- Working out the precise action of the 3D defect field theory
(important for higher loop computations as well as the conf. bootstrap program)
- More elaborate data-mining using one and two-point functions by means of boundary conformal bootstrap eqns.
- More detailed comparisons with string theory for both one- and two-point functions :
f.inst. involving spinning strings (i.e. non-protected operators)
- Moving on to higher loop orders (understand wrapping)

Thank you