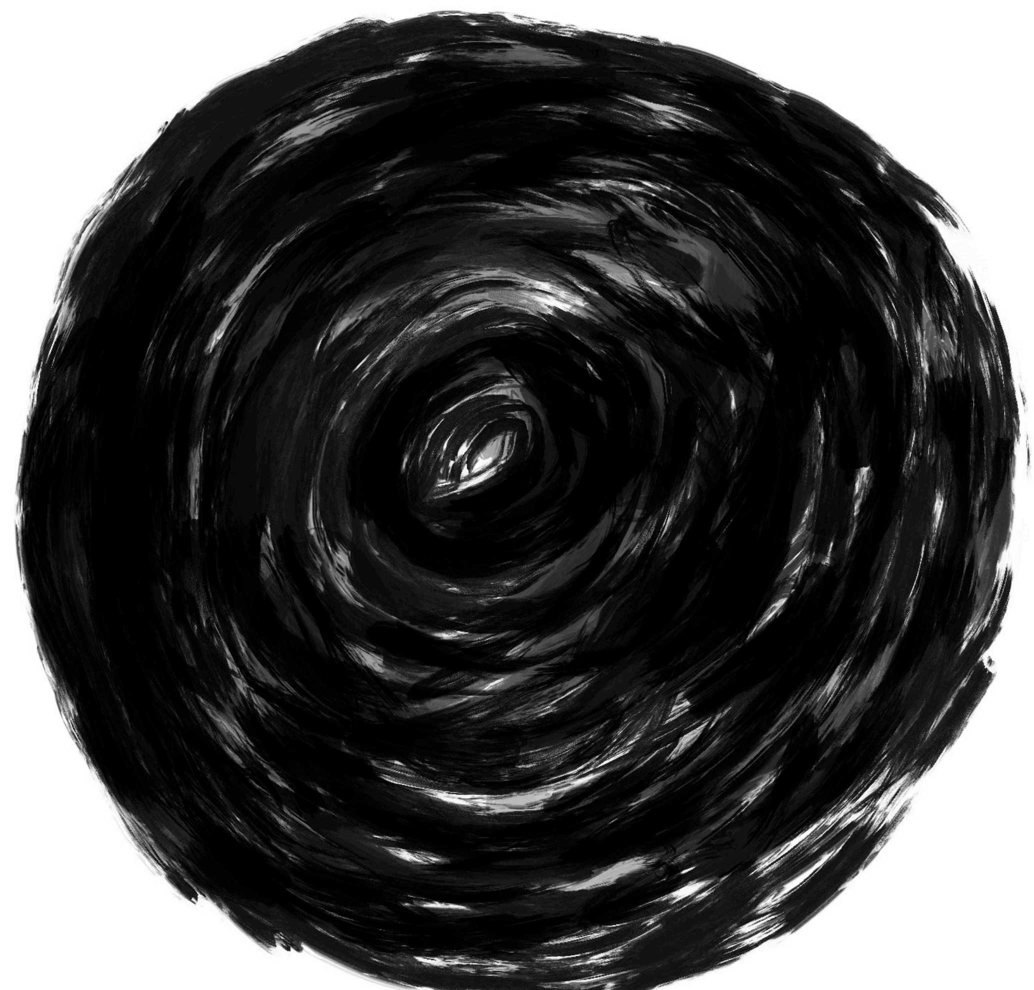
BLACK HOLES IN 3D HIGHER SPIN GRAVITY

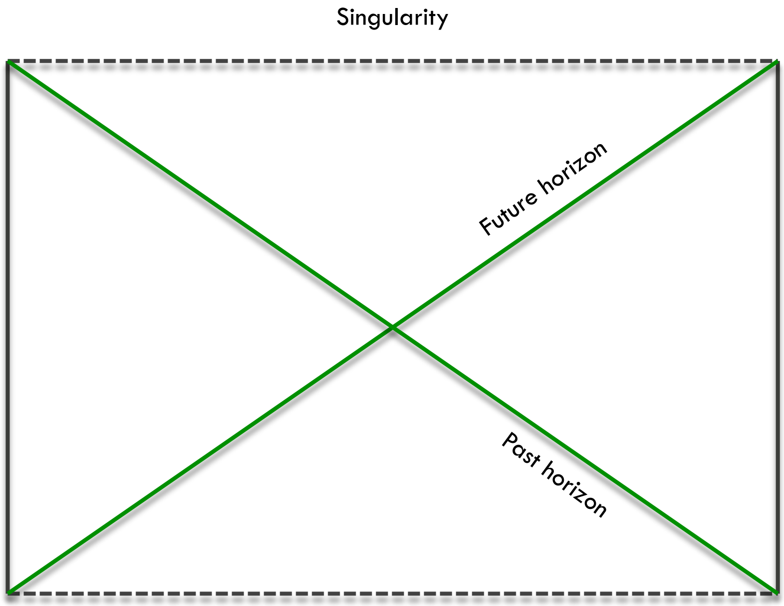
Gauge/Gravity Duality 2018, Würzburg

What is a Black Hole?



What is a Black Hole?

In General Relativity (and its cousins):



What is a Black Hole?

In General Relativity (and its cousins):

- Causality: Horizon & Singularity
- Thermodynamics: Entropy
- Response: Quasi-normal modes
- Quantum Information: Excellent Scramblers

Not every theory of gravity has a classical geometrical description.

What is a Black Hole
in Higher Spin Gravity?

- Causality: Horizon & Singularity
- Thermodynamics: Entropy
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Which of these features implies the rest?
Are they always interconnected?

What is a Black Hole in Higher Spin Gravity?

The goal is to give a definition that

- does not require a geometric description,
- and is fully compatible with the gauge symmetries of the theory.

What is a Black Hole in Higher Spin Gravity?

Along the way, we will

- challenge the holographic dictionary,
- and challenge the intuition we exploit from general relativity.

Outline

1. HS Gravity & Chern-Simons Theory

2. Euclidean Black Holes

3. Extremal Black Holes

4. Eternal Black Holes

5. Outlook

3d Higher Spin Gravity

Chern-Simons formulation

3d Gravity

In 2+1 dimensions, we have the **luxury** of casting general relativity in terms of:

[Acucharro & Townsend; Witten]

Einstein-Hilbert: Metric, curvature

OR

Chern-Simons: Gauge connections

3d Gravity

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OR

Chern-Simons: Gauge connections → Gauge Theory. Topological nature is explicit.

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[Acucharro & Townsend; Witten]

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↓

Inclusion of massless higher spin fields is straightforward!

Higher Spin Theories

How to interpret Chern-Simons theory as a theory of gravity?

$$S_{CS}[\mathcal{A}] = \frac{k}{4\pi}\int_{\mathcal{M}} \mathrm{Tr}\left(\mathcal{A}\wedge d\mathcal{A} + \frac{2}{3}\mathcal{A}\wedge\mathcal{A}\wedge\mathcal{A}\right)$$

It is not just a matter of actions and equations of motion.
Other important **INPUTS** are:

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Other important **INPUTS** are:

1. Gauge Group:
Organization of the massless modes

$$\mathcal{A} \in G = \underbrace{G_L \times G_R}_{SL(2,\mathbb{R})\times SL(2,\mathbb{R})}$$

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Organization of the massless modes

$$\mathcal{A} \in G = \underbrace{G_L \times G_R}_{SL(2,\mathbb{R}) \times SL(2,\mathbb{R})}$$

2. Boundary Conditions:
Setup the AdS/CFT dictionary

$$\mathcal{A} - \mathcal{A}_{\mathrm{AdS}} = O(1)$$

Perturbative Aspects

Asymptotic Symmetry Group and Ward Identities

$$SL(2,\mathbb{R}) \times SL(2,\mathbb{R}) \longrightarrow \mathrm{Vir} \times \mathrm{Vir}$$

[Brown & Henneaux]

$$SL(N,\mathbb{R}) \times SL(N,\mathbb{R}) \longrightarrow \mathcal{W}_N \times \mathcal{W}_N$$

[Campoleoni et al]

$$\mathrm{hs}[\lambda] \times \mathrm{hs}[\lambda] \longrightarrow \mathcal{W}_\infty[\lambda] \times \mathcal{W}_\infty[\lambda]$$

[Henneaux & Rey; Gaberdiel & Hartman]

With central charge: $$\boxed{c = \frac{3\ell}{2G_3} = 6k}$$

Strategy

- Work with a Chern-Simons formulation of higher spin theories.
- Emphasis on SL(N), and SUSY cousins, i.e. a finite number of higher spin fields.
- Exploit holography: comparison with dual W_N theories when possible.
- I'll never ever involve a metric in the subsequent definitions.

Euclidean Black Holes

Thermal Properties

Characterizing Solutions

Theory

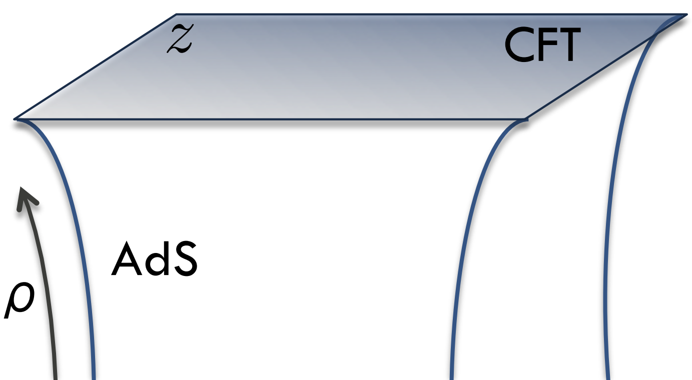
$$S_{CS}[\mathcal{A}] = S_{CS}[A] - S_{CS}[\bar{A}]$$

$$SL(N) \times SL(N)$$

Solutions

$$A(\rho, z, \bar{z}) = b^{-1}(\rho)\Big(a(z,\bar{z}) + d\Big)b(\rho)$$

$$\bar{A}(\rho, z, \bar{z}) = b(\rho)\Big(\bar{a}(z,\bar{z}) + d\Big)b^{-1}(\rho)$$



Physical content in the connection:

$$A(\rho, z, \bar{z}) = b^{-1}(\rho)\Big(a(z,\bar{z}) + d\Big)b(\rho)$$

$$\boxed{a(z,\bar{z}) = a_\phi d\phi + a_{t_E} dt_E}$$

[Gutperle & Kraus; de Boer & Jottar; Bunster et al]

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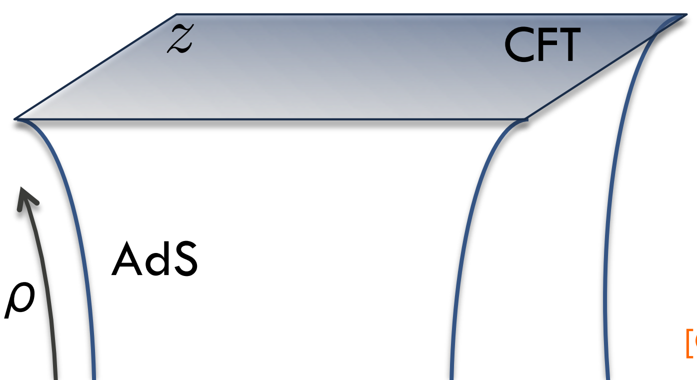
VEV conserved charges

Sources



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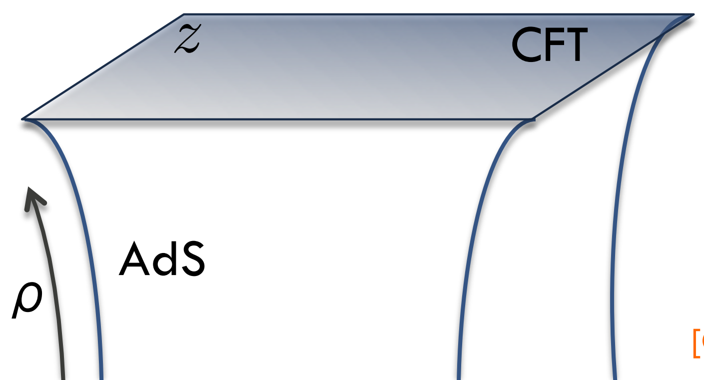
$$\boxed{a(z,\bar{z}) = \underbrace{a_\phi}_{\text{VEV conserved charges}} d\phi + \underbrace{a_{t_E}}_{\text{Sources}} dt_E}$$

From the perspective of CFT dual,

$$H = H_{\mathrm{CFT}} + \oint d\phi \sum_s \mu_s J_s + \oint d\phi \sum_s \underbrace{\bar{\mu}_s}_{\text{sources}} \underbrace{\bar{J}_s}_{\text{vev}}$$



[Gutperle & Kraus; de Boer & Jottar; Bunster et al]

$$A(\rho, z, \bar{z}) = b^{-1}(\rho)\Big(a(z,\bar{z}) + d\Big)b(\rho)$$

$$\boxed{a(z,\bar{z}) = a_\phi d\phi + a_{t_E} dt_E}$$

Consider N=3, the explicit form of the connection is

$$a_\phi = \begin{pmatrix} 0 & \frac{1}{2}\mathcal{L} & -2W \\ 1 & 0 & \frac{1}{2}\mathcal{L} \\ 0 & 1 & 0 \end{pmatrix} \longrightarrow$$

Equations of motion
+
Boundary conditions

$\mathcal{L}$: vev spin-2 current (energy-momentum tensor)

W : vev spin-3 current

$$A(\rho, z, \bar{z}) = b^{-1}(\rho)\Big(a(z,\bar{z}) + d\Big)b(\rho)$$

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$$ia_{t_E} + a_\phi = -\frac{\mu}{2}\begin{pmatrix} -\frac{1}{6}\mathcal{L} & -2W & \frac{1}{4}\mathcal{L}^2 \\ 0 & \frac{1}{3}\mathcal{L} & -2W \\ 1 & 0 & -\frac{1}{6}\mathcal{L} \end{pmatrix}$$

$$[a_{t_E}, a_\phi] = 0$$

τ : source spin-2 current (temperature)

μ : source spin-3 current

$$z \simeq z + 2\pi \simeq z + 2\pi\tau$$

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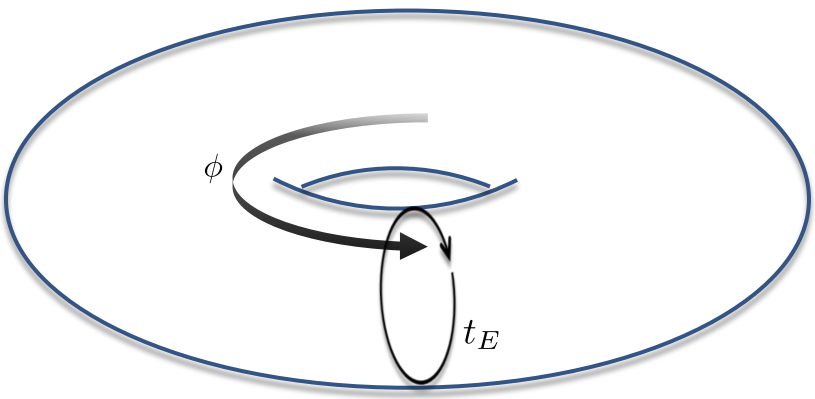
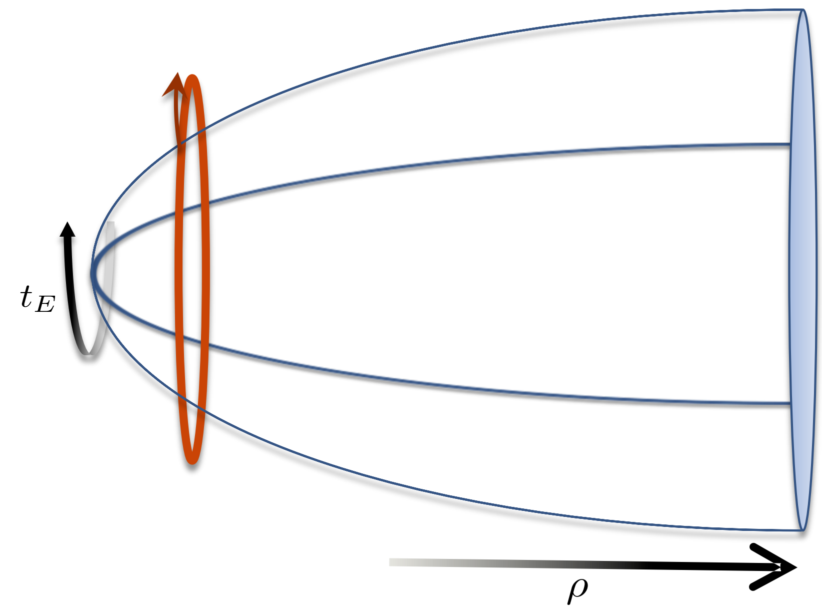
Consistency Check: Chern-Simons eoms map to Ward identities of the CFT

Smoothness Conditions

$$z \simeq z + 2\pi \simeq z + 2\pi\tau$$

$$z = \phi + it_E$$





Connection should support the topology underneath!

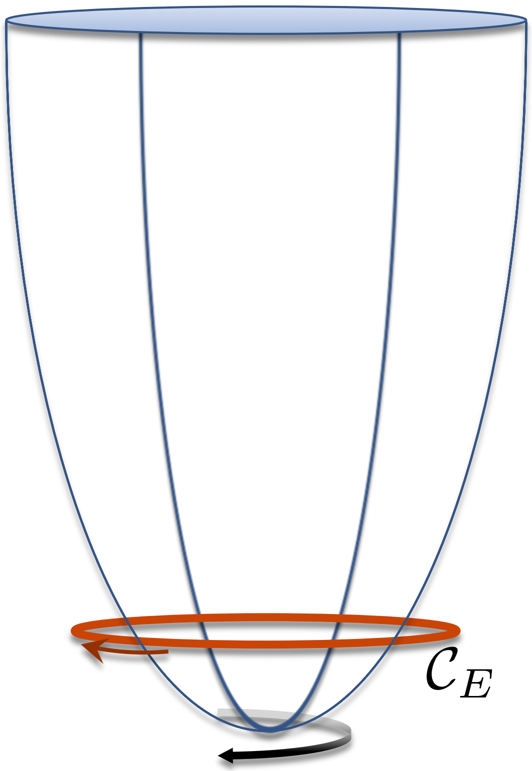
$$A(\rho, z, \bar{z}) = b^{-1}(\rho)\Big(a(z,\bar{z}) + d\Big)b(\rho)$$

$$\bar{A}(\rho, z, \bar{z}) = b(\rho)\Big(\bar{a}(z,\bar{z}) + d\Big)b^{-1}(\rho)$$

[Gutperle & Kraus]

Smoothness Conditions

$$\boxed{\mathrm{Hol}_{\mathcal{C}_E}(A) = \text{“trivial”}}$$



[Gutperle & Kraus]

Smoothness Conditions

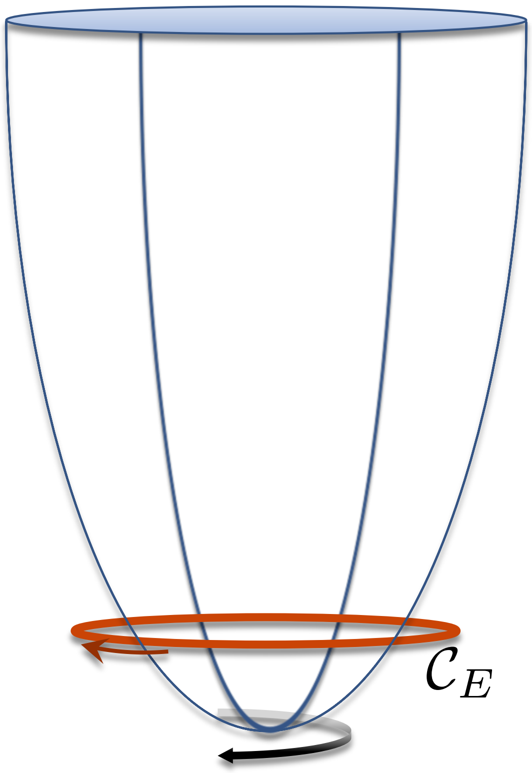
$$\boxed{\mathrm{Hol}_{\mathcal{C}_E}(A) = \text{“trivial”}}$$

$\Big\downarrow$ Use $A = b(a+d)b^{-1}$

$$\mathcal{P}\exp\left(\oint_{\mathcal{C}_E} a\right) = e^{2\pi(\tau a_z + \bar{\tau} a_{\bar{z}})} = e^{2\pi i L_0}$$

$\Big\downarrow$ Diagonalize

$$\boxed{\mathrm{Eigen}(\tau a_z + \bar{\tau} a_{\bar{z}}) = \mathrm{Eigen}(i L_0)}$$



Elegant condition that only uses natural variables of Chern-Simons theory.

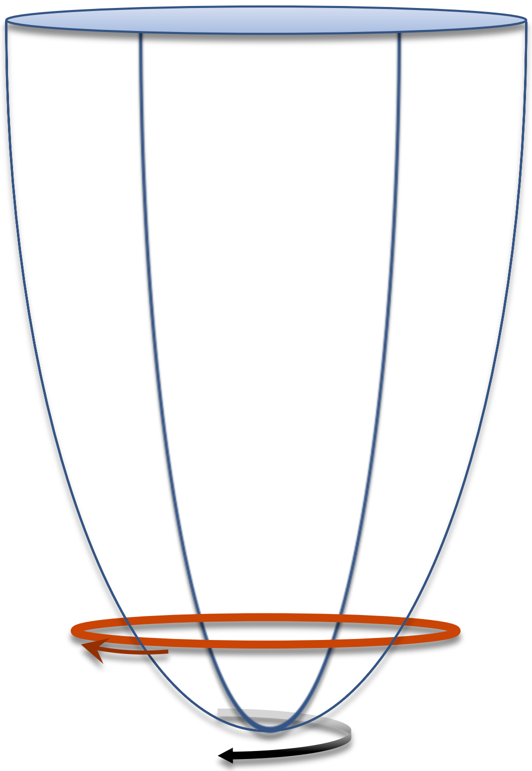
[Gutperle & Kraus]

Thermodynamics

Holonomy condition leads to thermodynamics!

$$\mathrm{Eigen}\big(\tau a_z + \bar{\tau} a_{\bar{z}}\big) = \mathrm{Eigen}\big(iL_0\big)$$

$$\xrightarrow{\text{Solve}} \quad \boxed{\tau(\mathcal{L}, J_s) \quad \& \quad \mu_s(\mathcal{L}, J_s)}$$



[Gutperle & Kraus; de Boer & Jottar]

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Solve

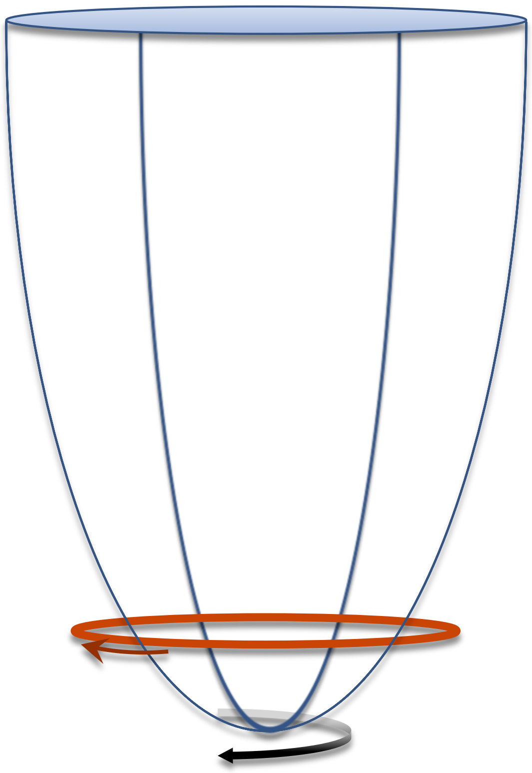
$$\boxed{\tau(\mathcal{L}, J_s) \quad \& \quad \mu_s(\mathcal{L}, J_s)}$$

Combined with the CS variational principle, one can show

$$\delta S = \tau \delta \mathcal{L} - \bar{\tau} \delta \bar{\mathcal{L}} + \sum_{s=3}^{N} (\mu_s \delta J_s - \bar{\mu}_s \delta \bar{J}_s)$$

$$\boxed{S = 2\pi k \mathrm{Tr}\left[(\lambda_\phi - \bar{\lambda}_\phi) L_0\right]}$$ Entropy of HSBH

$$\mathrm{Eigen}(a_\phi) \equiv \lambda_\phi \quad \& \quad \mathrm{Eigen}(\bar{a}_\phi) \equiv \bar{\lambda}_\phi$$



[Gutperle & Kraus; de Boer & Jottar]

What is an Euclidean Black Hole in Higher Spin Gravity?

- Causality: Horizon & Singularity
- Thermodynamics: Entropy
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Lesson:
Euclidean regularity implies thermodynamic relations

Extremal Black Holes

And their awkward SUSY features

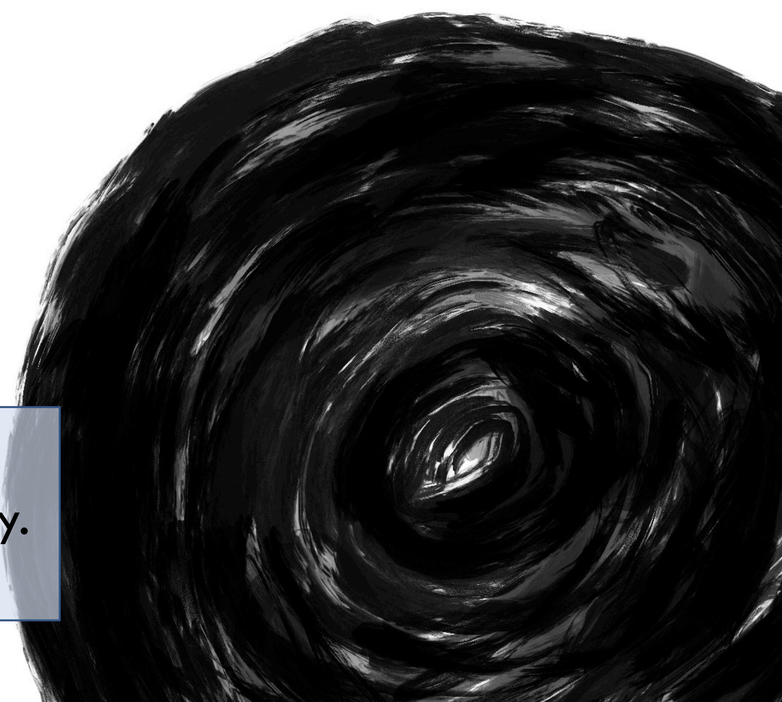
Based on 1512.00073 with Bañados, Faraggi and Jottar

What defines an extremal black hole?

From general relativity, we know

- Confluence of horizons: inner = outer
- Zero Hawking temperature
- Enhancement of symmetries: e.g. AdS_2
- Saturation of cosmic censorship: $M \geq J$
- BPS conditions (SUSY)
- Reality observables: e.g. $\mathrm{Im}(S)=0$

Task for HS gravity:
Extrapolate one of these conditions in a CS way.
Explore how the rest is interconnected.



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Charges

Sources

Simplistic view: Thermodynamics is an eigenvalue problem

$$\mathrm{Eigen}\big(\tau a_z + \bar{\tau} a_{\bar{z}}\big) = \mathrm{Eigen}\big(i L_0\big)$$: Holonomy condition

$$S = 2\pi k \mathrm{Tr}\left[(\lambda_\phi - \bar{\lambda}_\phi)L_0\right]$$: Entropy of HSBH

Implicit assumption that connections are diagonalizable, which is true of independent values of the charges.

Extreme Limit: our proposal

$$a(z,\bar{z}) = \underbrace{a_\phi}_{\text{Charges}} d\phi + \underbrace{a_{t_E}}_{\text{Sources}} dt_E$$

Extremal = non-diagonalizable a $_\phi$

Extreme Limit: our proposal

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Charges

Sources

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Confluence of eigenvalues

Via $\mathrm{Eigen}\big(\tau a_z + \bar{\tau} a_{\bar{z}}\big) = \mathrm{Eigen}\big(i L_0\big)$

Extremal = zero temperature

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For higher spin gravity, we have

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[Gutperle & Kraus; Henneaux, Perez, Tempo & Troncoso]

BPS Conditions

Embedded in a supersymmetric version of CS, one can ask when

$$a(z,\bar{z}) = a_\phi d\phi + a_{t_E} dt_E$$

is compatible with SUSY and what are the appropriate BPS bounds (`M=Q').

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They match in the large c limit.

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We derived the bounds on both CFT and Gravity.
They match in the large c limit.

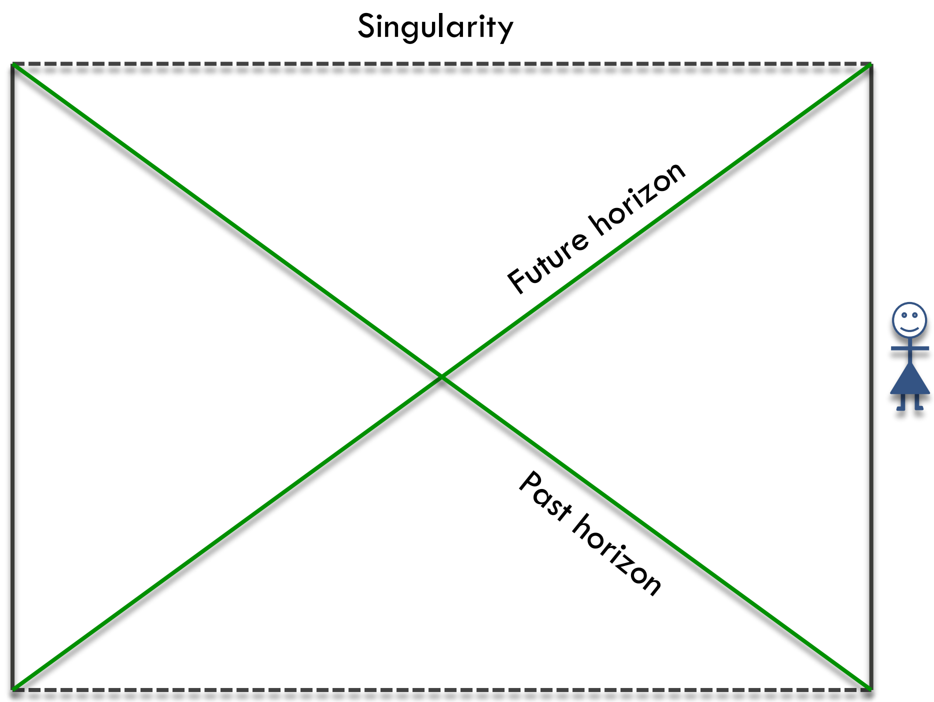
We found that BPS bounds do not imply extremality. In particular, we can construct SUSY HSBH that are at finite temperature!

Eternal Black Holes

Exploring bulk locality in HS gravity

Based on 1306.4338 with Ammon and Iqbal
1602.09057 with Iqbal and Llabres
1805.05398 with Iqbal and Llabres

How to capture casual properties in Higher Spin gravity?
How to probe local bulk physics?



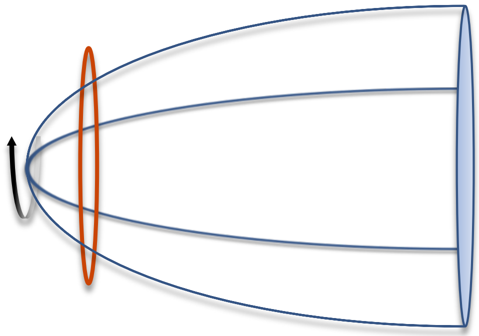
Let’s quantify this diagram in the CFT language.

Eternal HS Black Hole

A possible definition:

An eternal black is a thermo-field double state in the CFT.

Whereas an Euclidean black holes satisfies:

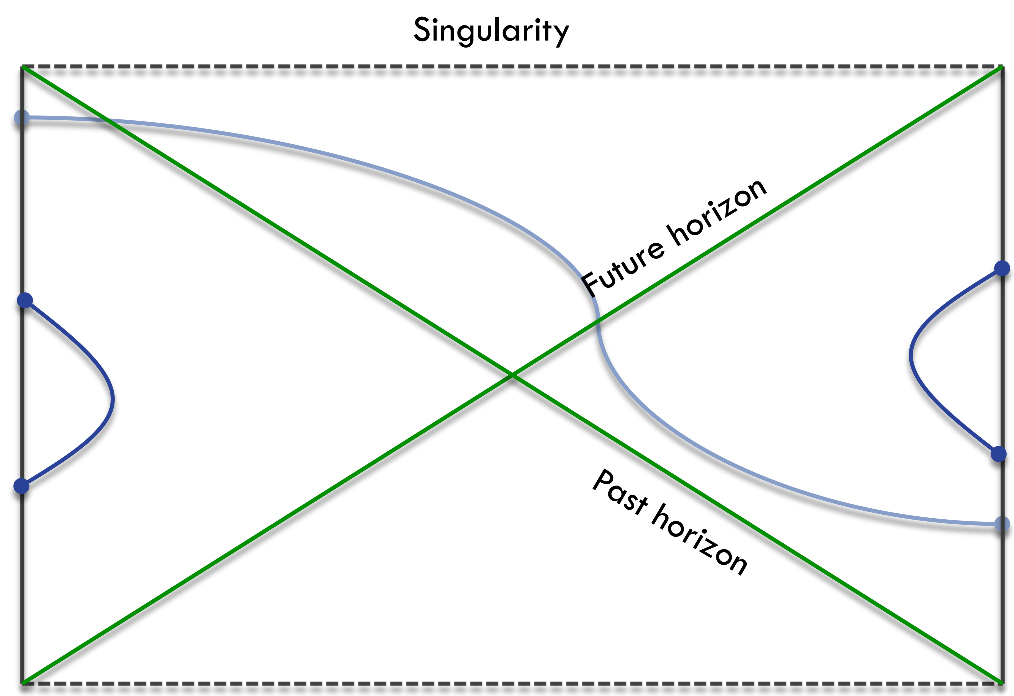


$\mathrm{Hol}_{\mathcal{C}_E}(A) =$ "trivial"

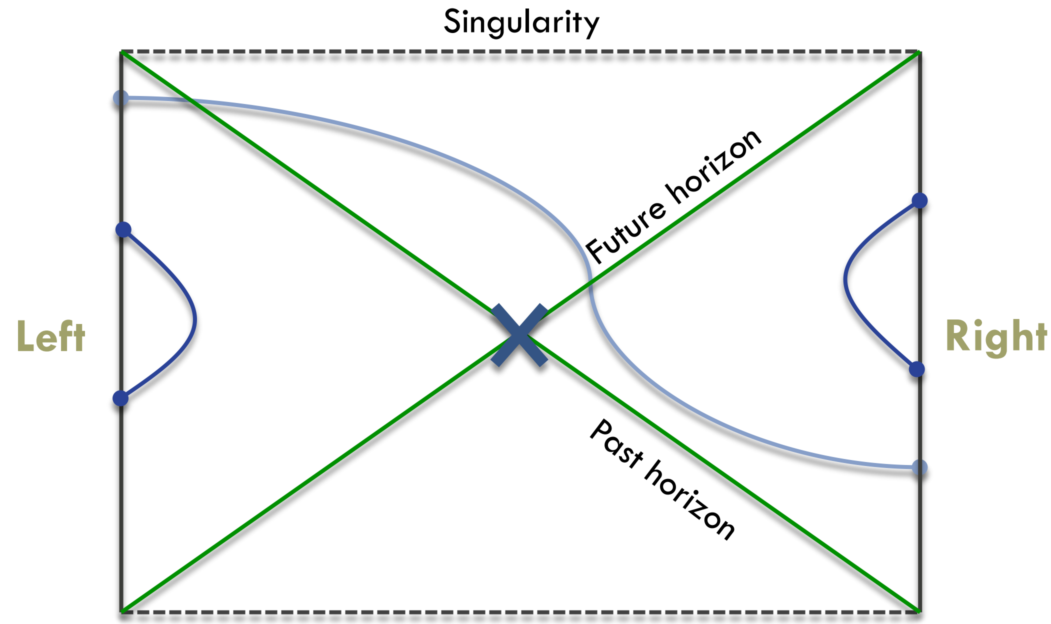
Highly entropic state.

An eternal black is a thermo-field double state in the CFT.

$$|\psi\rangle = \frac{1}{\sqrt{Z}} \sum_n e^{-\frac{\beta}{2}(E_n + \mu Q_n)} |Un\rangle_L |n\rangle_R$$



1. Signal of a bifurcation point.
2. Symmetric and periodic in Euclidean time.
3. Left-Right correlator should obey half-period relation.

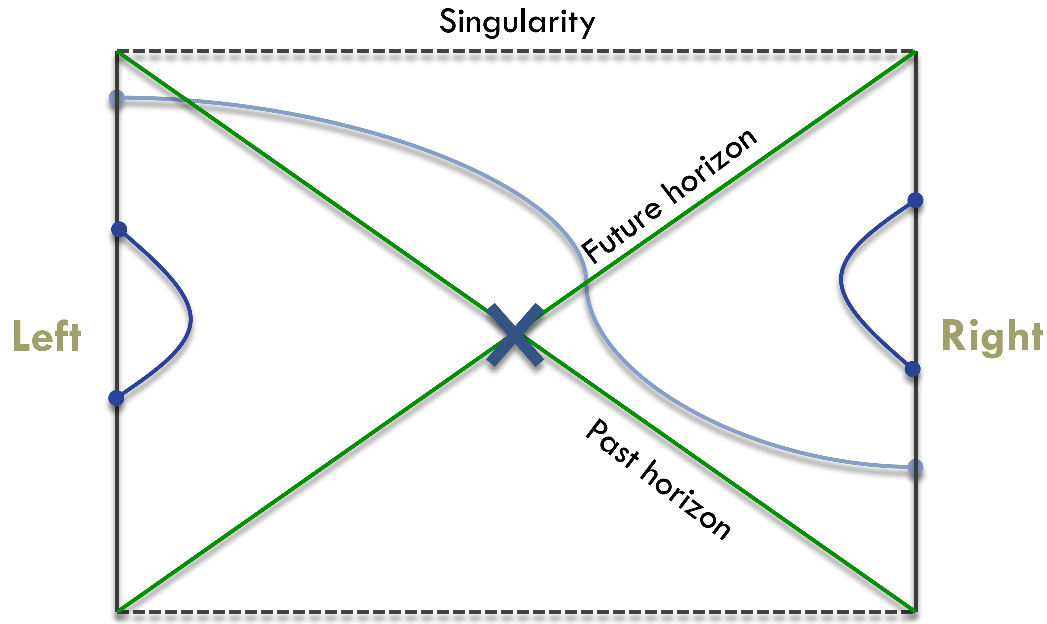


We want:

$$\begin{aligned}\langle\psi|\mathcal{O}_R(t_f)\mathcal{O}_R(t_i)|\psi\rangle &= \langle\psi|\mathcal{O}_R(t_f)\mathcal{O}_R(t_i-i\beta)|\psi\rangle\\ &= \langle\psi|\mathcal{O}_L(-t_f)\mathcal{O}_L(-t_i)|\psi\rangle\\ &= \langle\psi|\mathcal{O}_L(-t_f-i\beta/2)\mathcal{O}_R(t_i)|\psi\rangle\\ &= \langle\psi|\mathcal{O}_R(t_f)\mathcal{O}_L(-t_i-i\beta/2)|\psi\rangle\end{aligned}$$

Testing this proposal is difficult: non-trivial to have test particle. Important prior work made use of Vasiliev scalar field.

[Kraus & Perlmutter]

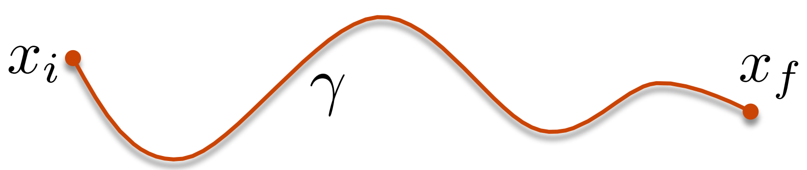


What replaces the notion of distance in HS gravity?
How to probe a solution with local fields?

Wilson lines in CS

Wilson loop encodes the dynamics of a massive particle .
Natural replacement of local probes.

$$W_{\mathcal{R}}(x_i, x_f) = \langle U_f | \mathcal{P} \exp\left(-\int_\gamma A\right) \mathcal{P} \exp\left(-\int_\gamma \bar{A}\right) | U_i \rangle$$



[Ammon, AC, & Iqbal; de Boer & Jottar]

Highlights

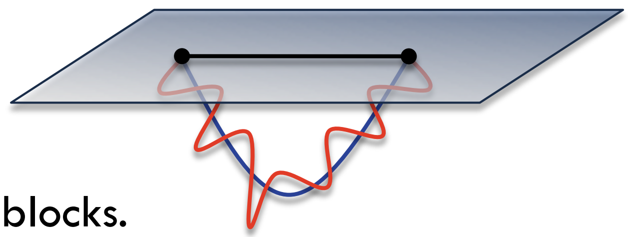
$$W_{\mathcal{R}}(x_i, x_f) = \langle U_f | \mathcal{P} \exp\left(-\int_\gamma A\right) \mathcal{P} \exp\left(-\int_\gamma \bar{A}\right) | U_i \rangle$$

- The representation R will dictate the characteristic of the probe: mass and spin.
- Until now our observables do not connect A and Ā. Probing local physics requires BOTH connections.
- The states U are coherent states in R that combine A and Ā while preserving a diagonal subgroup of SL(N)xSL(N).

Features

1. The Wilson line reproduces boundary correlation functions.

$$W_{\mathcal{R}}(x_i, x_f) \underset{\rho\to\infty}{=} \langle\Psi|\mathcal{O}(z_i)\mathcal{O}(z_f)|\Psi\rangle$$

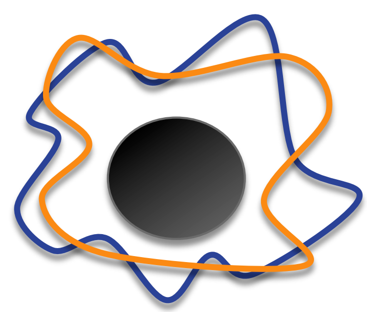


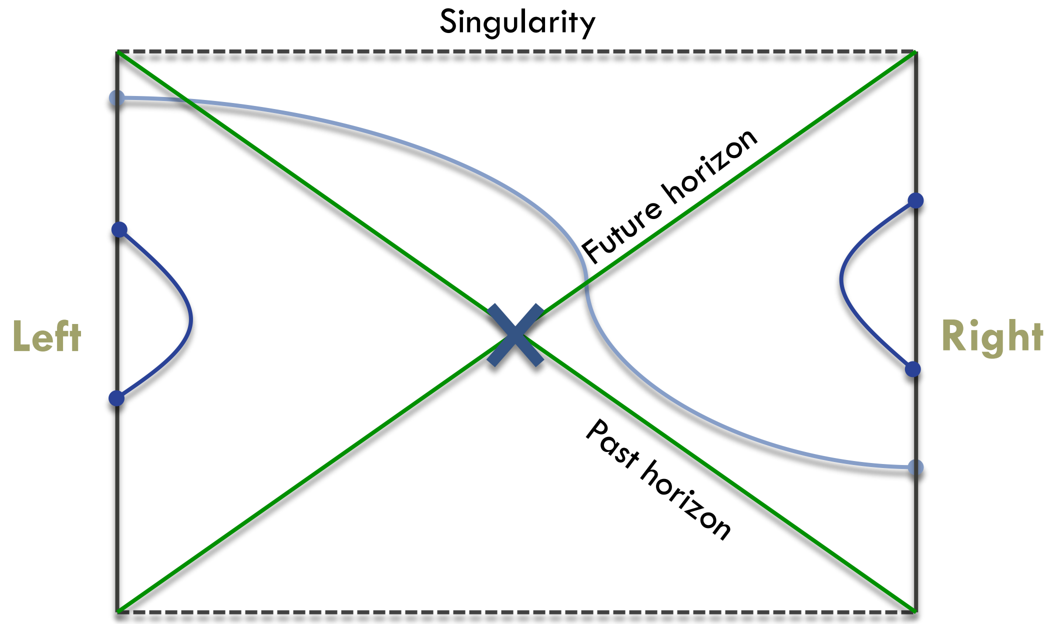
Connections to entanglement entropy and conformal blocks.

2. For HSBH, the Wilson loop gives thermal entropy.

$$\begin{aligned} S &= 2\pi k \mathrm{Tr}[(\lambda_\phi - \bar{\lambda}_\phi)L_0] \\ &= -\log(W_{\mathcal{R}}(C)) \end{aligned}$$

Closed spatial cycle
(non-trivial)





We use

$$W_{\mathcal{R}}(x_i, x_f) \underset{\rho\to\infty}{=} \langle\Psi|\mathcal{O}(z_i)\mathcal{O}(z_f)|\Psi\rangle$$

And answers depend on the radial function!

$$A(\rho, z, \bar{z}) = b^{-1}(\rho)\Big(a(z,\bar{z}) + d\Big)b(\rho)$$

$$\bar{A}(\rho, z, \bar{z}) = b(\rho)\Big(\bar{a}(z,\bar{z}) + d\Big)b^{-1}(\rho)$$

KMS conditions

$$\begin{aligned}\langle\psi|\mathcal{O}_R(t_f)\mathcal{O}_R(t_i)|\psi\rangle &= \langle\psi|\mathcal{O}_R(t_f)\mathcal{O}_R(t_i - i\beta)|\psi\rangle \\ &= \langle\psi|\mathcal{O}_L(-t_f)\mathcal{O}_L(-t_i)|\psi\rangle \\ &= \langle\psi|\mathcal{O}_L(-t_f - i\beta/2)\mathcal{O}_R(t_i)|\psi\rangle \\ &= \langle\psi|\mathcal{O}_R(t_f)\mathcal{O}_L(-t_i - i\beta/2)|\psi\rangle\end{aligned}$$

1. Wormhole gauge

Commonly used by 99.9% of users.

No signal of a bifurcation point. No KMS relations.
Right side is AAdS.

2. Horizon gauge

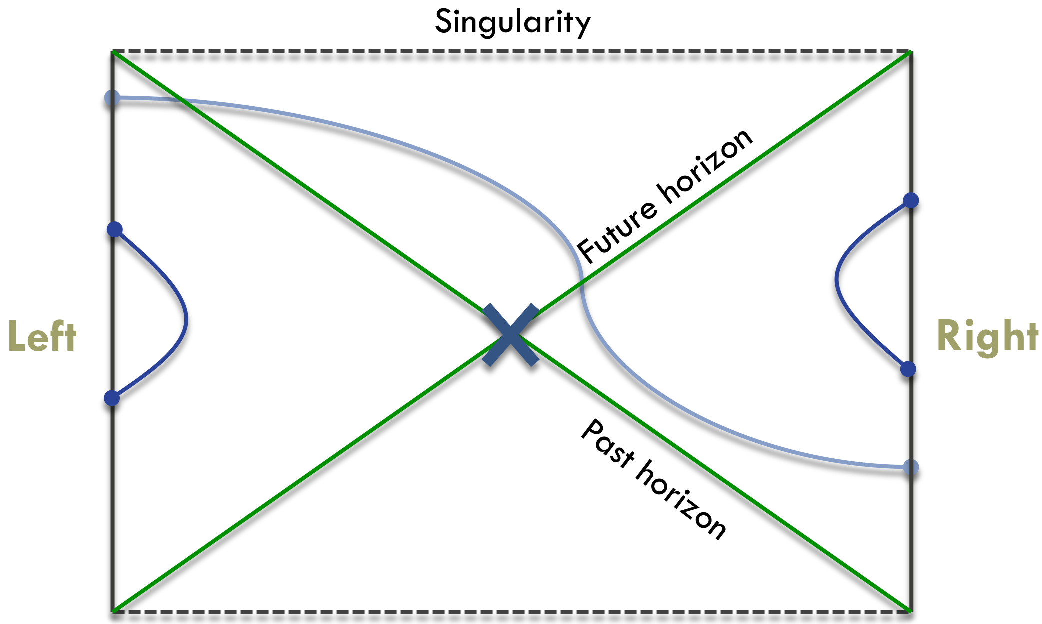
[Ammon, Gutperle, Kraus, Perlmutter]

Radial function adjusted to give a horizon. KMS holds, not AAdS.

3. Kruskal gauge

KMS holds and AAdS. It works!

Different choices of radial function



$$W_{\mathcal{R}}(x_i, x_f) \underset{\rho\to\infty}{=} \langle\Psi|\mathcal{O}(z_i)\mathcal{O}(z_f)|\Psi\rangle$$

And answers depend on the radial function!

$$A(\rho, z, \bar{z}) = b^{-1}(\rho)\Big(a(z,\bar{z}) + d\Big)b(\rho)$$

$$\bar{A}(\rho, z, \bar{z}) = b(\rho)\Big(\bar{a}(z,\bar{z}) + d\Big)b^{-1}(\rho)$$

Lesson:
HS Gravity provides a concrete setup where the bulk reconstruction and dual interpretation is not just dictated by obvious symmetries.

Outlook

There is more to explore…

Results I didn't discuss… but nevertheless important

1. Phase diagram of higher spin black holes

[David, Ferlaino & Kumar; Chen, Long & Wang; Bañados, Düring, Faraggi & Reyes]

2. Partition functions in the CFT

[Kraus & Perlmutter; Gaberdiel, Hartman & Jin]

3. Four point correlation functions

[Perlmutter; de Boer, AC, Hijano, Jottar, Kraus]

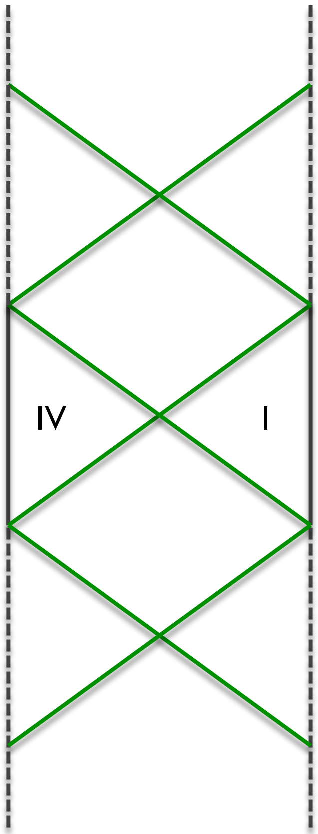
1. CFT counterpart

A derivation of the universal entropy formula for W_N theories.

$$S = 2\pi k \mathrm{Tr}\left[(\lambda_\phi - \bar{\lambda}_\phi) L_0\right]$$

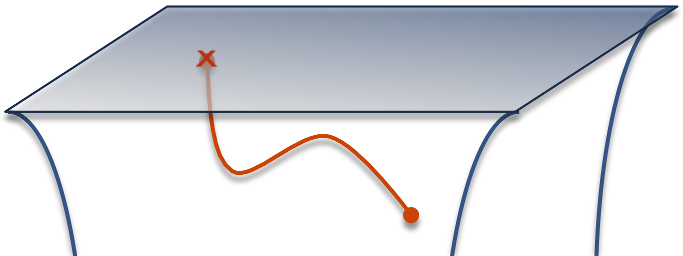
2. Interplay of Euclidean vs Lorentzian

Inner horizons, singularities, wormholes,…. Unexplored!



3. Bulk Locality in Higher Spin Gravity

$$W_{\mathcal{R}}(x_i, x_f) = \langle U_f | \mathcal{P} \exp\left(-\int_\gamma A\right) \mathcal{P} \exp\left(-\int_\gamma \bar{A}\right) | U_i \rangle$$



THANK YOU!