Aspects of Gauge-Strings Duality

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Outline

1. I will discuss some work in progress on AdS/CFT. The focus will be on the general ideas and outcomes.
2. The knowledge of field theory results at strong coupling allows us to say things about a geometry (ranges of coordinates, smoothing-out of singularities, various CFT quantities, etc).
3. The particular example I will discuss today will be in the context of an $N = 2$ SCFT in four dimensions. The same logic applies to various other cases.
4. This talk mostly based on work in preparation, but it uses material from various papers I wrote with: S. Zacarías, G. Itsios, J. Montero, D. Roychowdhury, J. van Gorsel, J. M. Penin, K. Sfetsos, D. Thompson, Y. Lozano, H. Nastase.

SCFTs in diverse dimensions (16 SUSY). An *incomplete* picture.

- d=6:Hanany-Zaffaroni, Brunner-Karch —D6-D8-NS5— Gaiotto-Tomasielo, Apruzzi, Fazzi, Rosa, Passias, Cremonesi.
- d=5:Aharony-Hanany-Kol —D5-D7-NS5– D'Hoker, Gutperle, Uhlemann, Trivella.
- d=4:Gaiotto —D4-D6-NS5— Gaiotto, Maldacena; Aharony, Berkooz, Berdichevsky; Stefanski, Reid-Edwards.
- d=3: Gaiotto-Witten —D3-D5-NS5— D'Hoker, Estes, Gutperle; Assel, Bachas, Gomis.
- d=2:(0,4) SCFT —D2-D4-NS5.
- d=1 (not a SCFT): Lin, Lunin, Maldacena —D0-D2-NS5— Lin, Maldacena.

In all these examples, there is (at least) an $SU(2)$ R-symmetry. The dual backgrounds have the form

$$ds^2 \sim f_1 AdS_{d+1} + f_2 d\Omega_2 + f_3 d\Omega_{5-d} + f_4 d\eta^2 + f_5 d\sigma^2; \quad f_i(\sigma,\eta).$$

There are also NS B_2, Φ and RR fields respecting the isometries above.

In particular, for four dimensional $\mathcal{N}=2$ CFTs, Lin, Lunin and Maldacena wrote in 2005 the Type IIA backgrounds ($\alpha' = g_s = 1$)

$$ds_{10}^2 = 4f_1 ds_{AdS_5}^2 + f_2(d\sigma^2 + d\eta^2) + f_3 d\Omega_2(\chi,\xi) + f_4 d\beta^2.$$
$$B_2 = f_5 d\Omega_2(\chi,\xi), \;\; C_1 = f_6 d\beta, \; A_3 = f_7 d\beta \wedge d\Omega_2, \; e^{2\phi} = f_8.$$

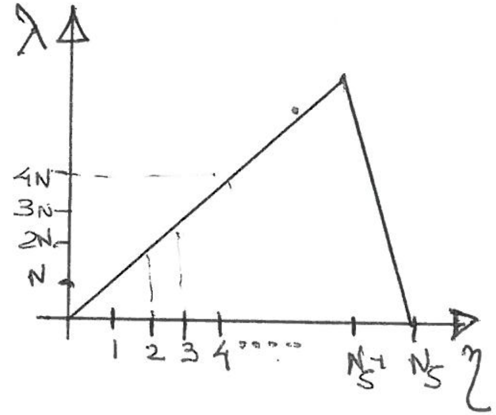
The functions $f_i(\sigma,\eta)$ can be all written in terms of a function $V(\sigma,\eta)$ and its derivatives, $f_i \sim f_i(V, \partial_\sigma V, \partial_\eta V)$

The function $V(\sigma,\eta)$ satisfies a Laplace-like equation with certain given boundary conditions

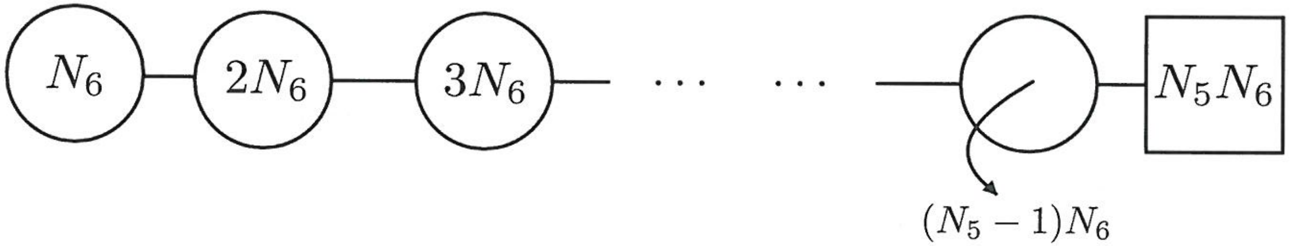
$$\sigma\partial_\sigma\left[\sigma\partial_\sigma V\right] + \sigma^2\partial_\eta^2 V = 0,$$
$$V(\sigma\to\infty,\eta)\to 0, \;\; \sigma\partial_\sigma V(\sigma,\eta)|_{\sigma=0} = \lambda(\eta) \to \lambda(0) = \lambda(N_5) = 0.$$

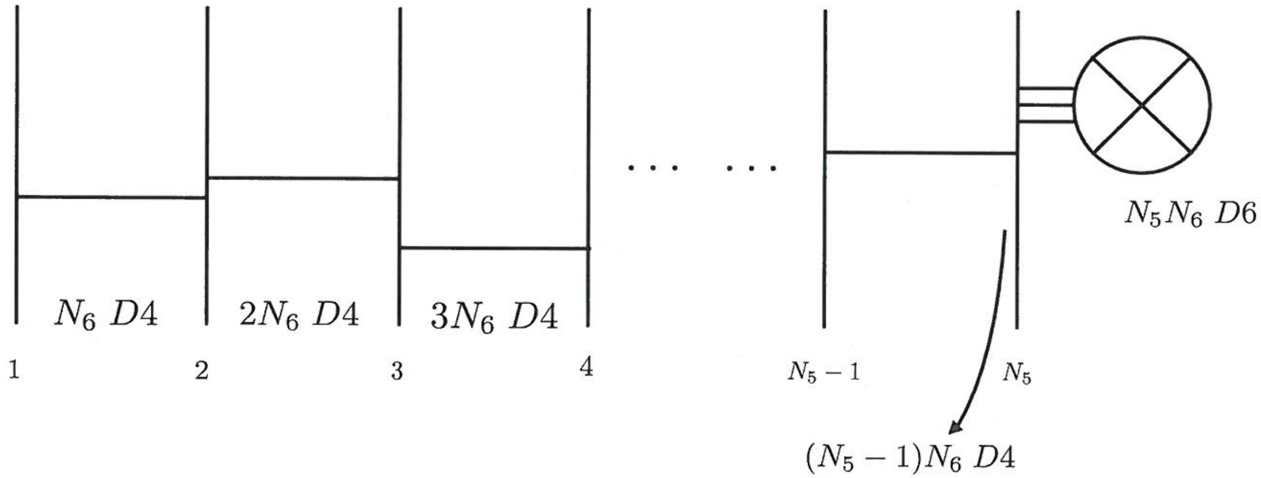
The Physics of the CFT is encoded in the function $\lambda(\eta)$.

A very simple example of an acceptable $\lambda(\eta)$ is,



$$\lambda(\eta) = N_6 \begin{cases} \eta & 0 \leq \eta \leq (N_5 - 1) \\ (N_5 - 1)(N_5 - \eta) & (N_5 - 1) \leq \eta \leq N_5. \end{cases}$$





Given $\lambda(\eta)$, one can write the solution for $V(\sigma,\eta)$ as a Fourier series

$$V_1(\sigma,\eta) = -\sum_{n=1}^{\infty} \frac{c_n}{w_n} K_0(w_n\sigma)\sin(w_n\eta), \qquad w_n = \frac{n\pi}{N_5}.$$

$$c_n = \frac{n\pi}{N_5^2}\int_{-N_5}^{N_5} \lambda(\eta)\sin(w_n\eta)d\eta, \qquad w_n = \frac{n\pi}{N_5}.$$

Using this, one can calculate the Page charges and make a correspondence with the Hanany-Witten brane set-up.

$$\hat{F} = Fe^{-B_2}, Q_{D_p} = \frac{1}{2\kappa_{10}^2 T_{Dp}}\int_\Sigma \hat{F}_{8-p}, \quad 2\kappa_{10}^2 T_{D_p} = g_s(4\pi^2\alpha')^{\frac{7-p}{2}}.$$

$$Q_{NS5} = N_5, \qquad Q_{D6} = \lambda'(0) - \lambda'(N_5), \quad Q_{D4} = \int_0^{N_5} \lambda(\eta)d\eta.$$

One can check that these expressions work for any quiver CFT/Hanany-Witten set-up.

Other quantities can be calculated in terms of $\lambda(\eta)$, for example the linking numbers of the different branes

$$K_i = N_{D4}^{right} - N_{D4}^{left} - N_{D6}^{right}, \quad L_j = N_{D4}^{right} - N_{D4}^{left} + N_{NS}^{left}.$$

$$\sum_{i=1}^{N_5} K_i + \sum_{j=1}^{N_6} L_j = 0.$$

One can find expressions that compute the linking numbers, purely in terms of $\lambda(\eta)$,

$$\sum_{i=1}^{N_5} K_i = \lambda'(N_5)N_5 = \frac{1}{2\kappa_{10}^2 T_{D4}} \int_{\Sigma_4} F_4 - B_2 \wedge F_2.$$

$$\sum_{j=1}^{N_6} L_j = -\sum_{j=1}^{N_6} \lambda'(\eta_j)\eta_j = -\lambda'(N_5)N_5 = \frac{1}{2\kappa_{10}^2 T_{D4}} \int_{\tilde{\Sigma}_4} F_4 + C_1 \wedge H_3.$$

Similarly, the central charge of the CFT can be calculated using the gravity solution. After various manipulations one obtains,

$$c = \frac{\pi^3}{8} \int_0^{N_5} \lambda^2(\eta) d\eta = \frac{\pi N_5^3}{2^6} \sum_{m=1}^{\infty} \frac{c_m^2}{m^2}.$$

One can show, that this result coincides with the field theoretical calculation (in the limit of large N_5),

$$c = \frac{2n_v + n_h}{12\pi}, \qquad a = \frac{5n_v + n_h}{24\pi}.$$

Where we count certain combinations of the number of vector multiplets and hypermultiplets of the $\mathcal{N} = 2$ SCFT. In some sense, the central charge is the 'power' of the function $\lambda(\eta)$.
For any generic quiver CFT or $\lambda(\eta)$, we have shown that the expression above is correct. There are similar expressions for the Entanglement Entropy. Other observables should also be calculable in terms of $\lambda(\eta)$.

One can construct deformations of these backgrounds. These deformations preserve conformality and half of the SUSY. They should be dual to $\mathcal{N}=1$ SCFTs. They depend on a parameter. They should be interpreted as marginal deformations on the CFT superpotential.

The way to construct them is to bring the backgrounds to eleven dimensions and use an $SL(3,R)$ transformation discussed by Gauntlett, Lee, Mateos and Waldram in 2005. One gets a more involved background, in terms of the functions $f_i(\sigma,\eta)$. These are expressed in terms of $V(\sigma,\eta)$ solving the same Laplace-like equation.

We are then constructing an analog of a 'beta deformation' for these $\mathcal{N}=2$ SCFTs

Starting from the IIA Gaiotto-Maldacena backgrounds, written in terms of the $f_i(\sigma,\eta)$ functions, one finds

$$ds^2_{10} = 4f_1 ds^2_{AdS_5} + f_2(d\sigma^2 + d\eta^2) + f_3 d\chi^2 +$$
$$\frac{f_3 \sin^2\chi}{(1+\gamma^2 f_3 f_4 \sin^2\chi)} d\xi^2 + \frac{f_4}{(1+\gamma^2 f_3 f_4 \sin^2\chi)}(d\beta - \gamma f_5 \sin\chi d\chi)^2.$$
$$e^{2\phi} = \frac{f_8}{(1+\gamma^2 f_3 f_4 \sin^2\chi)}, \quad C_1 = f_6 d\beta + \gamma(f_7 - f_5 f_6)\sin\chi d\chi,$$
$$B_2 = \frac{1}{(1+\gamma^2 f_3 f_4 \sin^2\chi)}\left[f_5 d\Omega_2 - \gamma f_3 f_4 \sin^2\chi d\xi \wedge d\beta\right],$$
$$A_3 = \frac{1}{(1+\gamma^2 f_3 f_4 \sin^2\chi)} f_7 d\beta \wedge d\Omega_2.$$

One can study various aspects of the CFT using these backgrounds. The central charge, the Page charges, etc. These seem consistent with a gamma deformation of the superpotential of the $\mathcal{N}=2$ field theory.

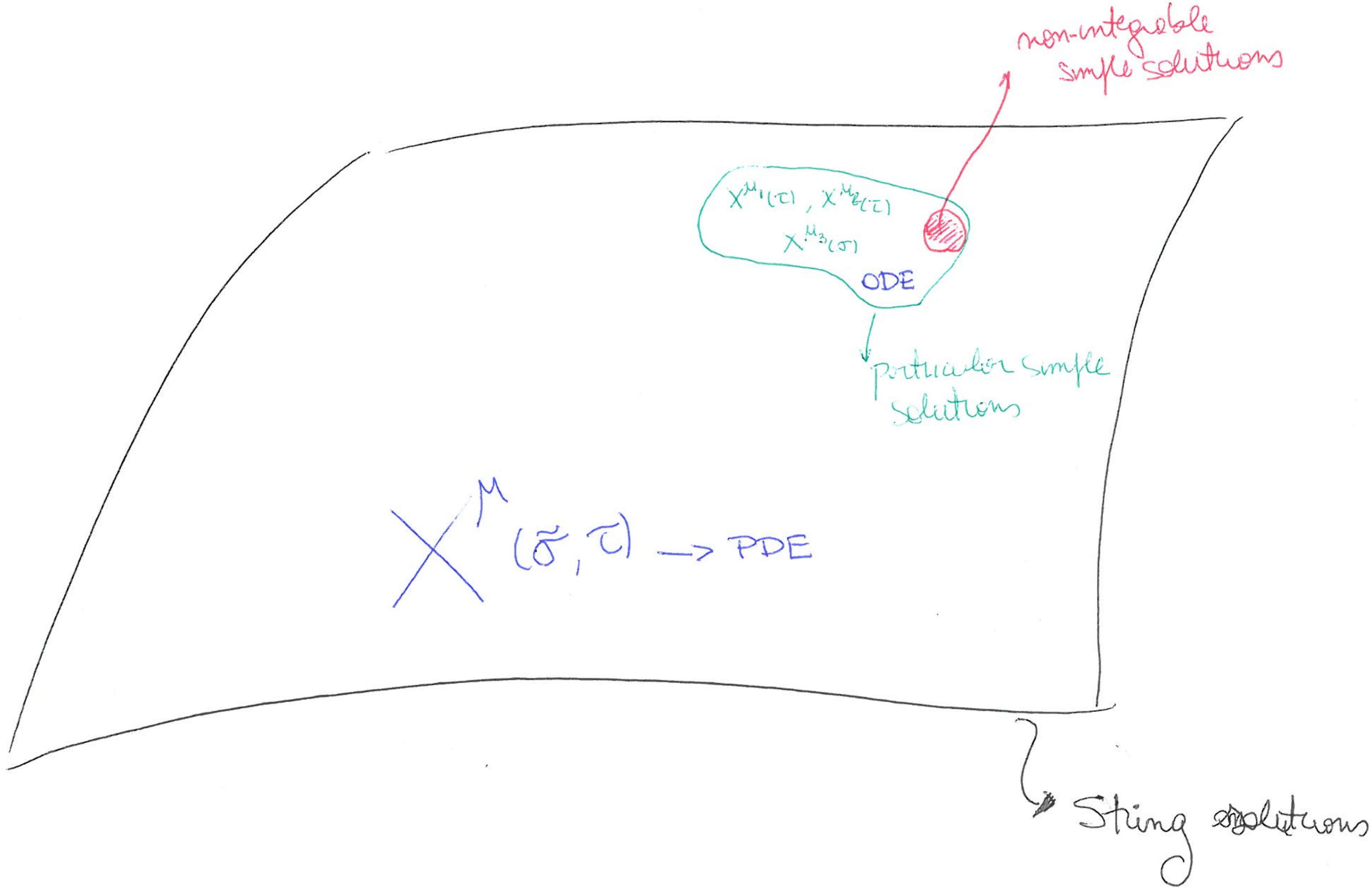
A similar solution can be found in Type IIB after dualities.

Let me focus on a particular aspect of these systems: Integrability. One can show that the string sigma models in a given background is classically integrable, if the equations of motion can be written in terms of a Lax pair. In general, it is very difficult to find such Lax pair.

It is much easier to 'disprove Integrability'. By proposing a semiclassical string soliton $X^{\mu}(\tilde{\sigma}, \eta)$ and studying the coupled non-linear partial differential equations of motion. This is still quite complicated to do in practice!

A more modest approach is to consider a simple string soliton, whose equations of motion admit a one-dimensional truncation and reduce to ordinary differential equations.

If this truncation is Liouville non-Integrable, the whole sigma model is also non-Integrable.


non-integrable simple solutions
$X^{\mu_1}(\tau)$, $X^{\mu_2}(\tau)$
$X^{\mu_3}(\sigma)$
ODE
particular simple solutions
$X^{\mu}(\sigma, \tau)$ —> PDE
String solutions

Consider the NS sector of the $\mathcal{N} = 2$ Gaiotto-Maldacena solutions

$$ds_{10}^2 = 4f_1 ds_{AdS_5}^2 + f_2(d\sigma^2 + d\eta^2) + f_3 d\Omega_2(\chi, \xi) + f_4 d\beta^2.$$
$$B_2 = f_5 d\Omega_2(\chi, \xi).$$

Propose a string solution of the form,

$$t = t(\tau), \quad \sigma = \sigma(\tau), \quad \eta = \eta(\tau), \quad \chi = \chi(\tau); \quad \xi = k\tilde{\sigma}, \quad \beta = \lambda\tilde{\sigma}.$$

Carefully studying the equations of motion and Virasoro constraint, one finds a set of non-linear and couple ordinary differential equations for $\ddot{\sigma}, \ddot{\eta}, \ddot{\chi}$, in terms of first derivatives and the potential function $V(\sigma, \tau)$.
One solution is $\sigma = 0$, $\eta = E\tau$, $\chi = 0$, $\dot{t} = E/f_1$.

One then follows developments by mathematicians.
Consider the previous simple solution and a variation of it

$$\eta(\tau) = \eta_s = E\tau, \quad \chi = 0 + z(\tau).$$

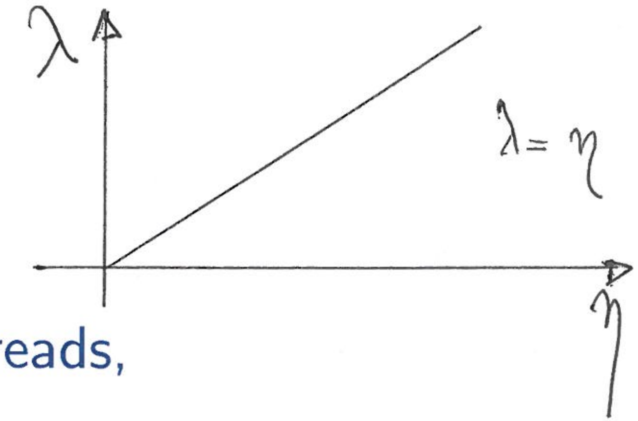
This leads us to a linear second order differential equation

$$\ddot{z}(\tau) + \mathcal{B}\dot{z}(\tau) + \mathcal{A}z(\tau) = 0,$$
$$\mathcal{A} = (k^2 - k\dot{\eta}\frac{\partial_\eta f_5}{f_3})|_{\eta=\eta_s}, \quad \mathcal{B} = (\dot{\eta}\partial_\eta \log f_3)|_{\eta=\eta_s}.$$

There are criteria due to Kovacic to decide the Liouvillian integrability (or not) of this differential equation.
When applied to the potentials written in terms of a Fourier-Bessel series, we find that all of them are non-integrable.
Except for one $V(\sigma, \eta)$!

The only potential, for which one finds an integrable soliton is very simple,

$$V_{ST} = \eta \log \sigma - \eta \frac{\sigma^2}{2} + \frac{\eta^3}{3}.$$



The background that is derived from this potential reads,

$$ds^2 = AdS_5 + \frac{d\sigma^2 + d\eta^2}{1-\sigma^2} + \frac{\eta^2(1-\sigma^2)}{4\eta^2 + (1-\sigma^2)^2} d\Omega_2 + \sigma^2 d\beta^2,$$

$$e^{-2\phi} = (1-\sigma^2)[4\eta^2 + (1-\sigma^2)^2], \quad B_2 = \frac{2\eta^3}{4\eta^2 + (1-\sigma^2)^2} d\Omega_2,$$

$$A_1 = 2(1-\sigma^2)^2 d\beta, \quad F_4 = B_2 \wedge F_2.$$

This background was written by Sfetsos and Thompson in 2011. They obtained it by applying non-Abelian T-duality on $AdS_5 \times S^5$. Further, for this background a Lax pair can be written. The system is classically integrable.

Let me draw some general lessons from this.

We started by considering $\mathcal{N} = 2$ linear quiver SCFTs. We worked with their dual string description.

All the dynamical information is encoded in a function $V(\sigma, \eta)$ solving a Laplace equation with boundary condition in terms of $\lambda(\eta)$.

Inside all of these different potentials V and functions λ, there is a particular one. It is the potential leading to the Sfetsos-Thompson background.

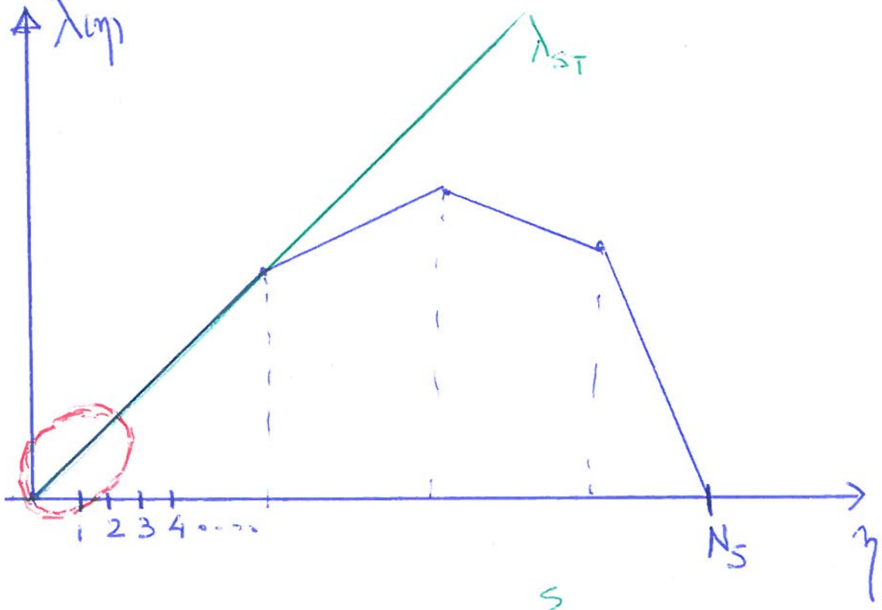
The field theoretical interpretation of such isolated background is quite dubious. Aside from this, the Sfetsos-Thompson background has the special property of having an integrable sigma model.

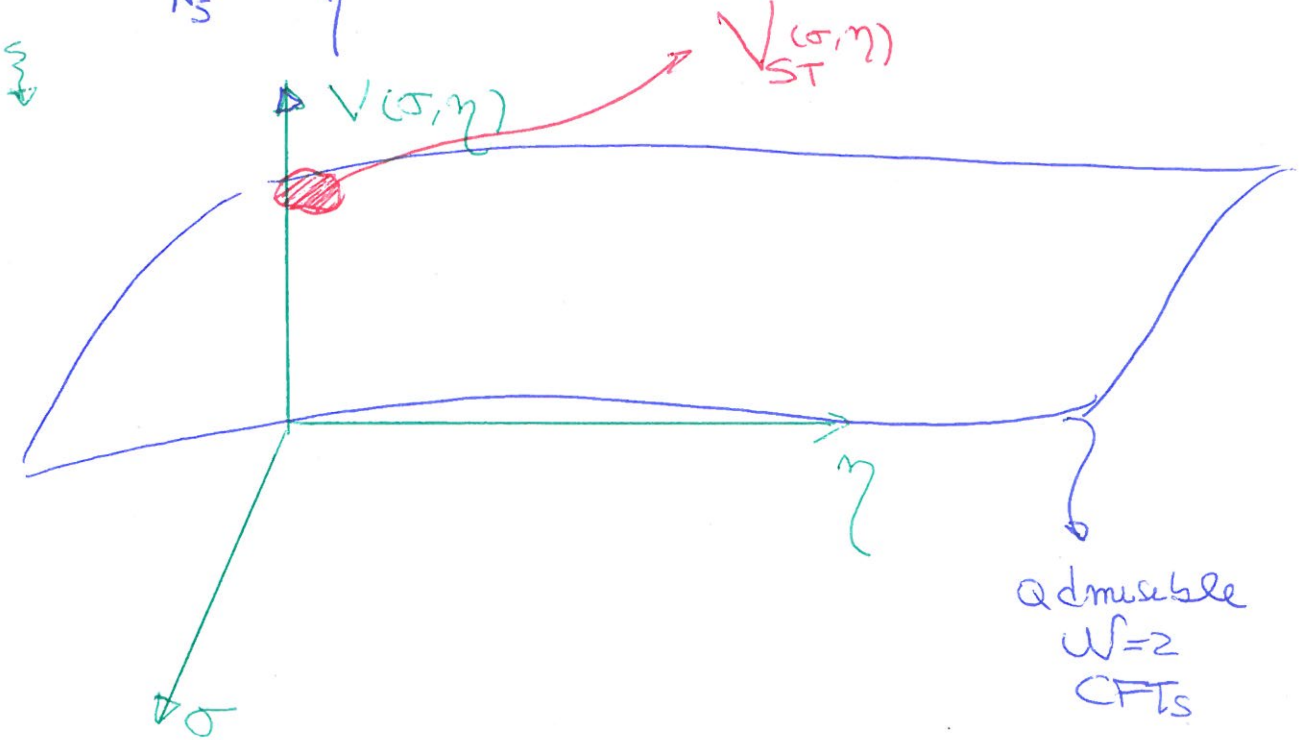
Probably, the Sfetsos-Thompson solution should be understood as 'needing a completion' that the Gaiotto-Maldacena backgrounds provide.

A graphic may clarify this!

$$V_{ST} = \eta \log \sigma + \eta \frac{\sigma^2}{2} + \frac{\eta^3}{3} \; ; \quad \lambda_{ST}(\eta) = \eta \rightsquigarrow$$

(NOT a good CFT in 4d!)





Some conclusions

The features discussed here for 4d $\mathcal{N}=2$ SCFTs: dual description in terms of a single function V and its derivatives. Writing of observable quantities in terms of V. Linearity of the PDE to determine V, etc.

Should repeat in the cases of SUSY CFTs in 2d, 3d, 5d, 6d. There should be 'core' solution, obtained by the application of non-Abelian T-duality on a given background. This core solution is integrable in some cases.

The systems can be thought in terms of D_p-D_{p+2}-NS_5 branes. There is always an AdS and an S^2, realising the $SO(2,p)$ and $SU(2)$ global symmetries.

It may be useful to study the taxonomy of these backgrounds and their deformations. There may be relations with integrable deformations.