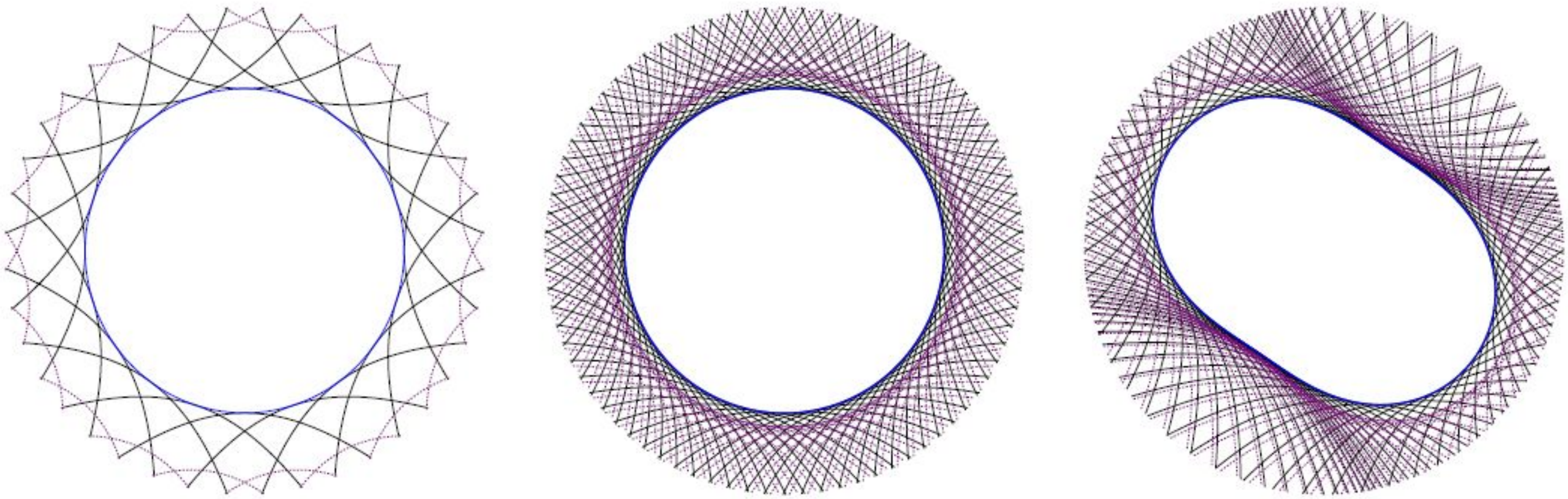
ASPECTS OF BULK RECONSTRUCTION FOR SUBREGIONS



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In this talk I will review recent works in the program of "HOLE-OGRAPHY",
which aims to RECONSTRUCT SPACETIME USING QUANTUM ENTANGLEMENT,
in the context of HOLOGRAPHY (or AdS/CFT)

Based on:

1) R. Espindola, A. Guijosa, A. Landetta, JP (arXiv:1708.02958)
2) R. Espindola, A. Guijosa, JP (arXiv:1804.05855)

BULK RECONSTRUCTION

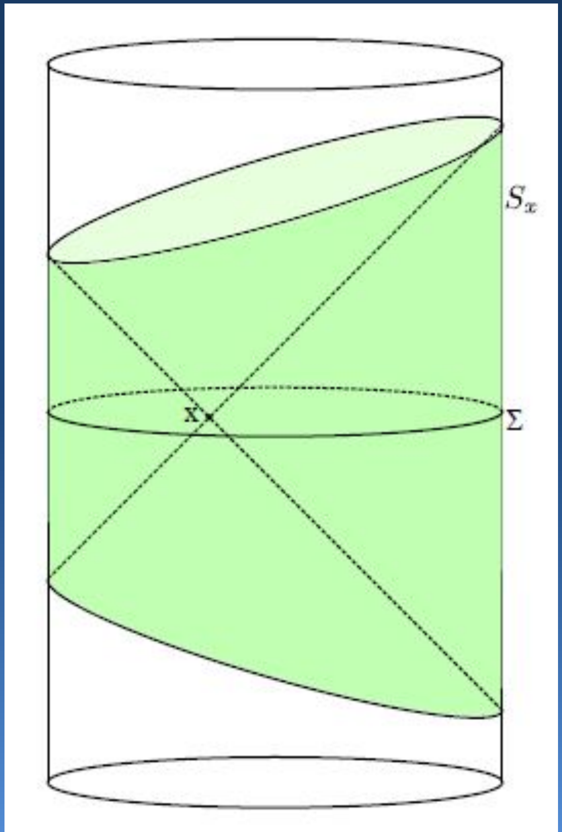
Two related but separate issues have been discussed:

1) Reconstruction of LOCAL BULK OPERATORS

HKLL prescription: smearing of CFT operator (UV/IR)

$$\varphi(x,z) = \int d^d x' K(x,z;x')O(x') + \ldots$$

[Banks,Douglas,Horowitz,Martinec; Bena; Hamilton,Kabat,Lifschytz,Lowe x3; Morrison; Heemskerk,Marolf,Polchinski,Sully; Kabat,Lifschytz; etc.]



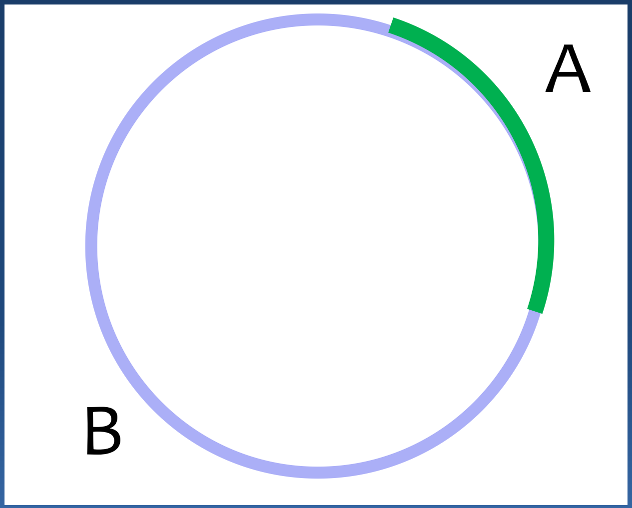
BULK RECONSTRUCTION

2) Reconstruction of the BULK GEOMETRY itself

Entangelement entropy (for Einstein gravity)

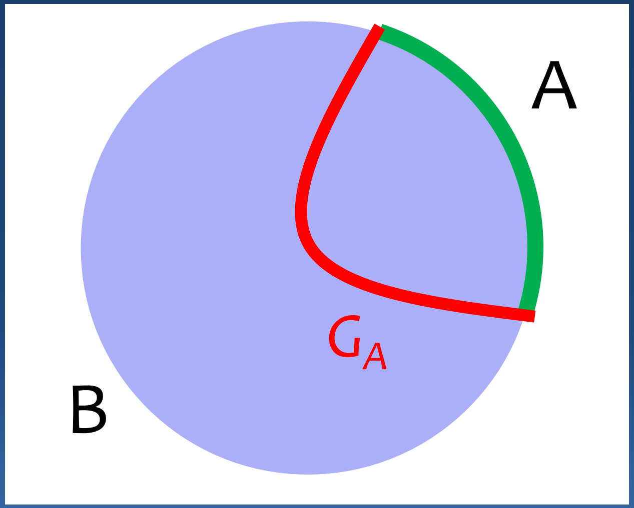
$$S_A = \frac{\text{Area of } \Gamma_A}{4G_N}$$

[Ryu,Takayanagi; Hubeny,Rangamani,Takayanagi; Lewkowycz,Maldacena; Dong,Lewkowycz,Rangamani]



CFT.on circle

=



GRAVITY THEORY on hyperbolic disk (AdS)

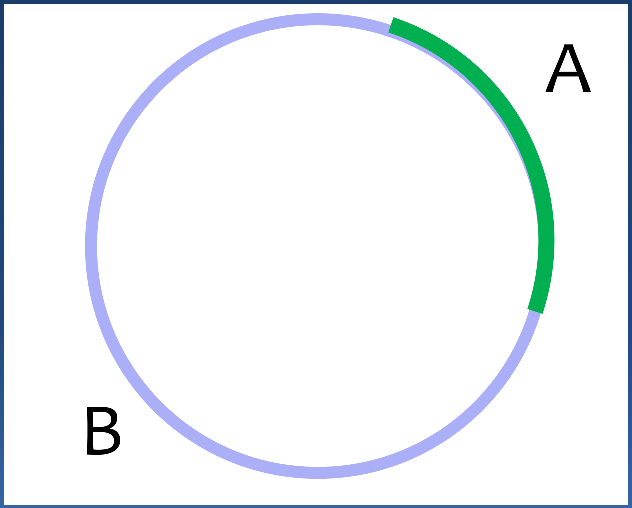
BULK RECONSTRUCTION

2) Reconstruction of the BULK GEOMETRY itself

For simplicity, we'll work with a CFT in d=2

The dual geometry is 2+1 dim AdS

"Extremal surfaces" are then. GEODESICS



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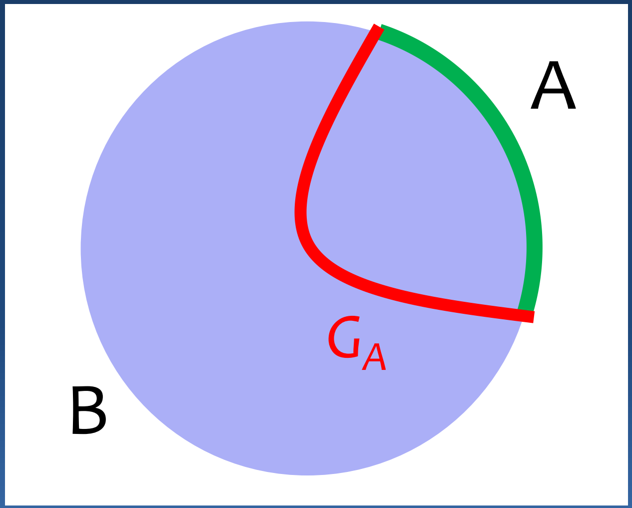
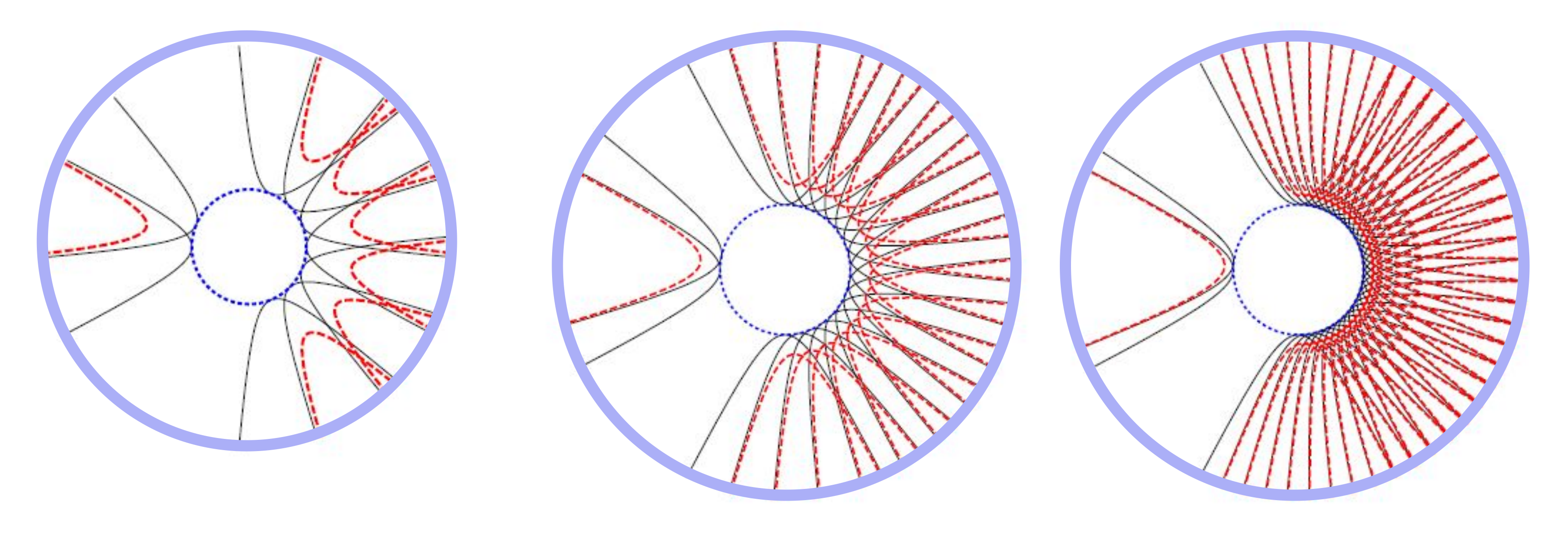


GRAVITY THEORY on hyperbolic disk (AdS)

BULK RECONSTRUCTION

2) Reconstruction of the BULK GEOMETRY itself



`HOLE-OGRAPHY' : to reconstruct ANY SPACELIKE CURVE, can add and subtract geodesics that are TANGENT to the curve.

Area encoded in `Differential Entropy': $E = \mathrm{A}$

[Balasubramanian,Chowdhury,Czech,de Boer,Heller Myers,Rao,Sugishita; Czech,Dong,Sully; Czech,Lamprou,McCandlish,Sully; Headrick,Myers,Wien]

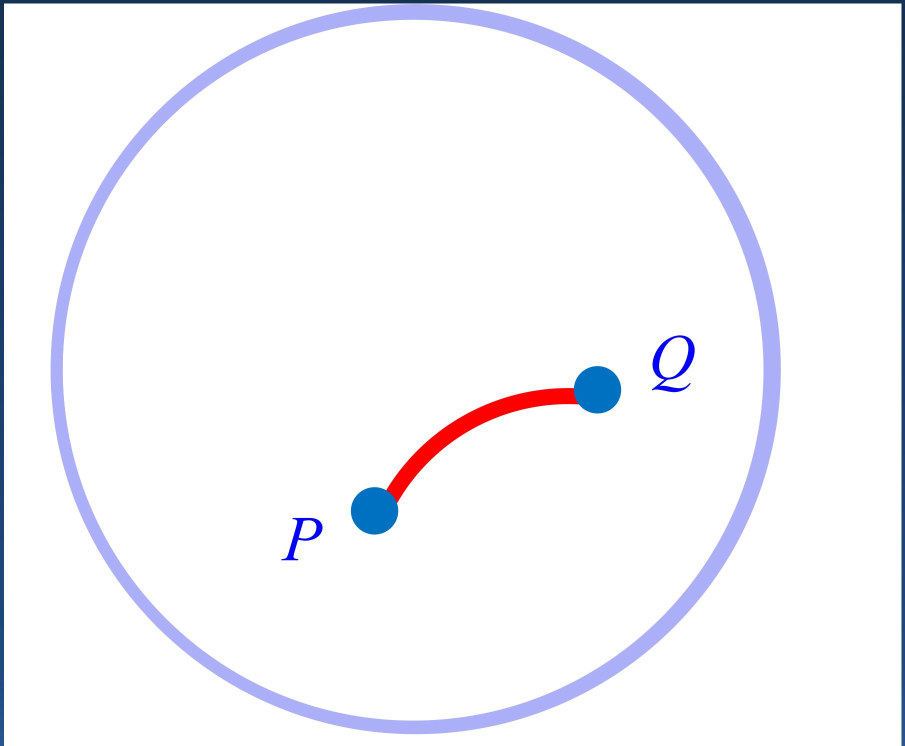
BULK RECONSTRUCTION

2) Reconstruction of the BULK GEOMETRY itself

Once we have a closed curve, we can shrink it down to a BULK POINT

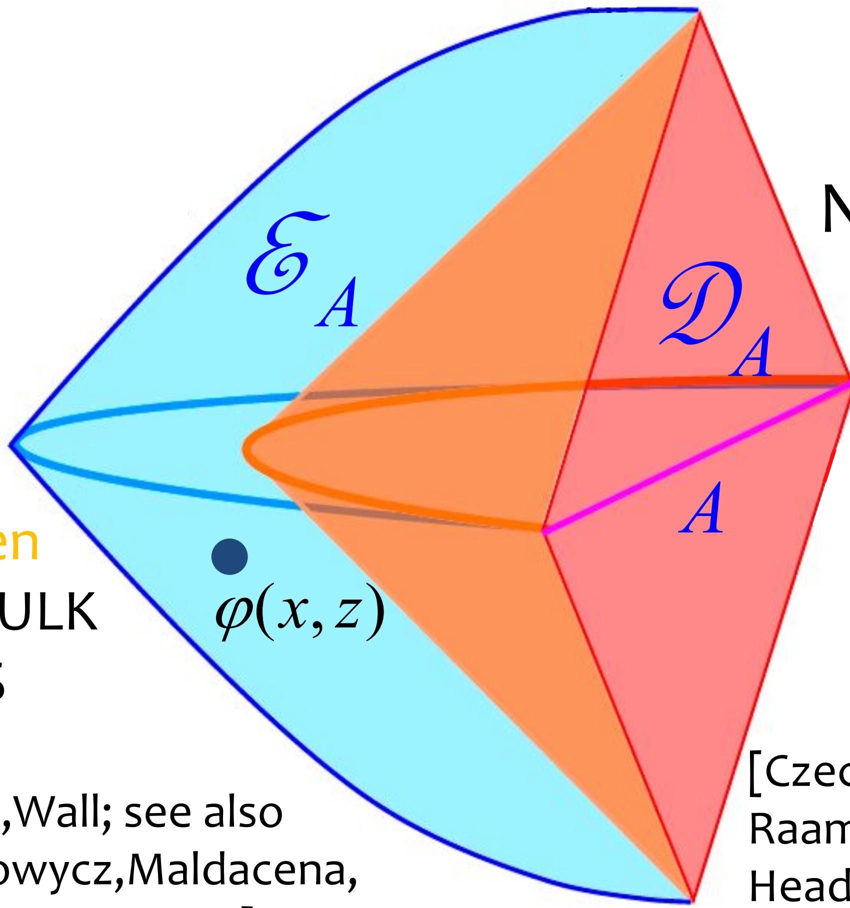
- It is possible to compute the DISTANCE between two bulk points *P* and *Q* in terms of the corresponding DIFFERENTIAL ENTROPIES

[Czech,Lamprou]



So the most basic ingredients of the geometry, POINTS and DISTANCES, can be recovered purely from the pattern of entanglement in the CFT state!!

SUBREGION DUALITY



Natural conjecture:

$\mathcal{E}_A \longleftrightarrow \mathcal{D}_A$
dual to

Has been proven for LOCAL BULK OPERATORS

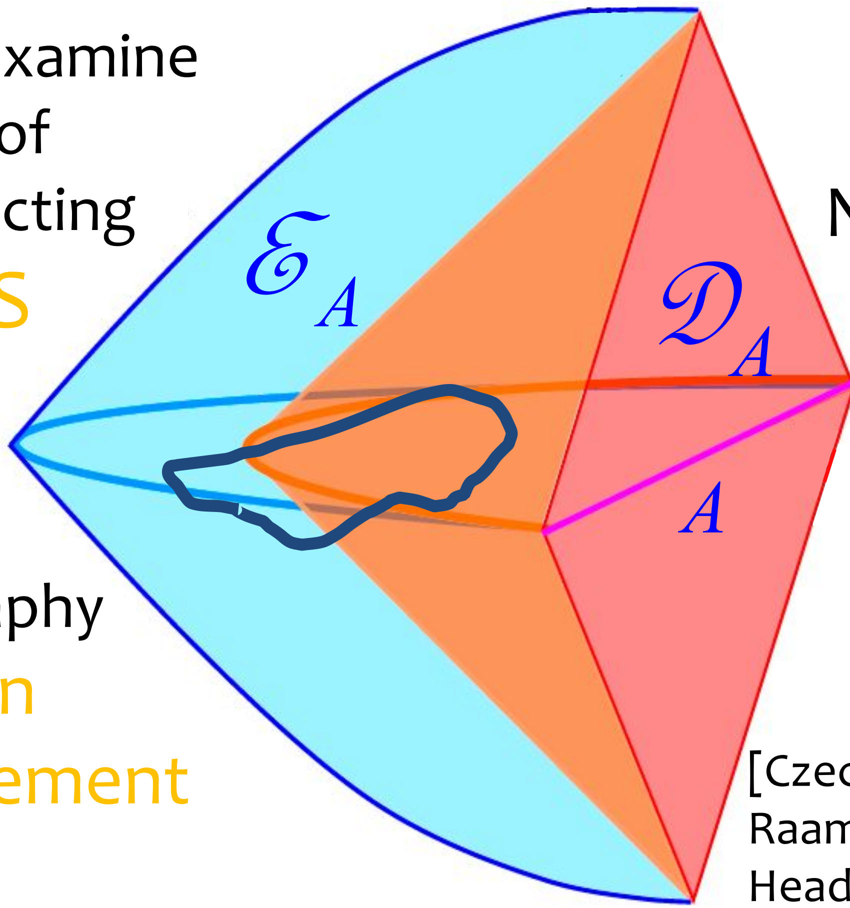
[Dong,Harlow,Wall; see also Jafferis,Lewkowycz,Maldacena, Suh; Faulkner,Lewkowycz]

[Czech,Karczmarek,Nogueira, Van Raamsdonk; Wall; Headrick,Hubeny,Lawrence, Rangamani]

SUBREGION DUALITY

Here, we'll examine the issue of reconstructing CURVES

I.e., we'll do hole-ography within an entanglement wedge



Natural conjecture:

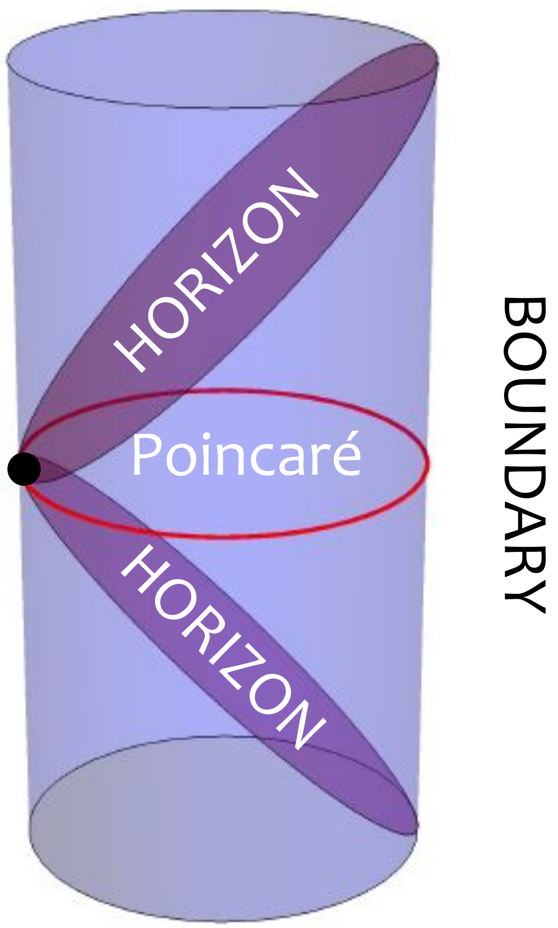
$\mathcal{E}_A \leftrightarrow \mathcal{D}_A$

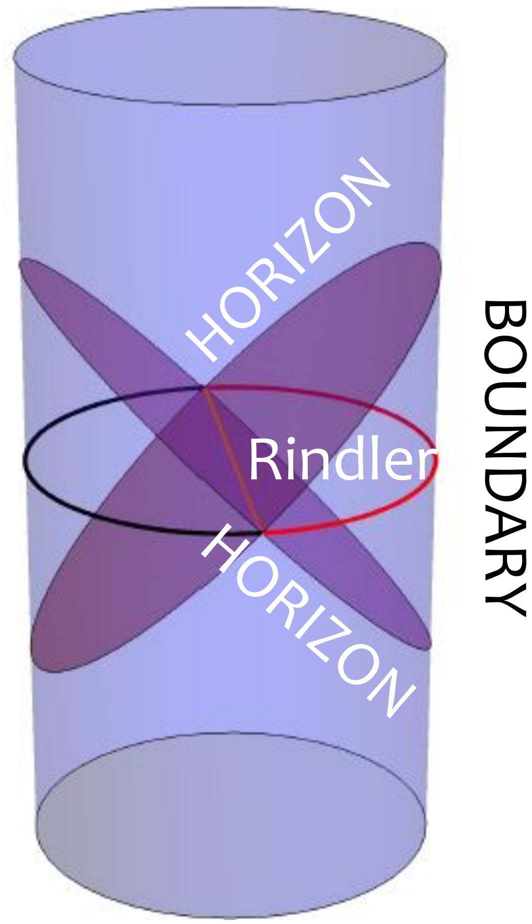
dual to

[Czech, Karczmarek, Nogueira, Van Raamsdonk; Wall; Headrick, Hubeny, Lawrence, Rangamani]

If A omits a single point, the entanglement wedge is the Poincaré wedge

For smaller A (mixed state), entanglement wedge is smaller: RINDLER WEDGE

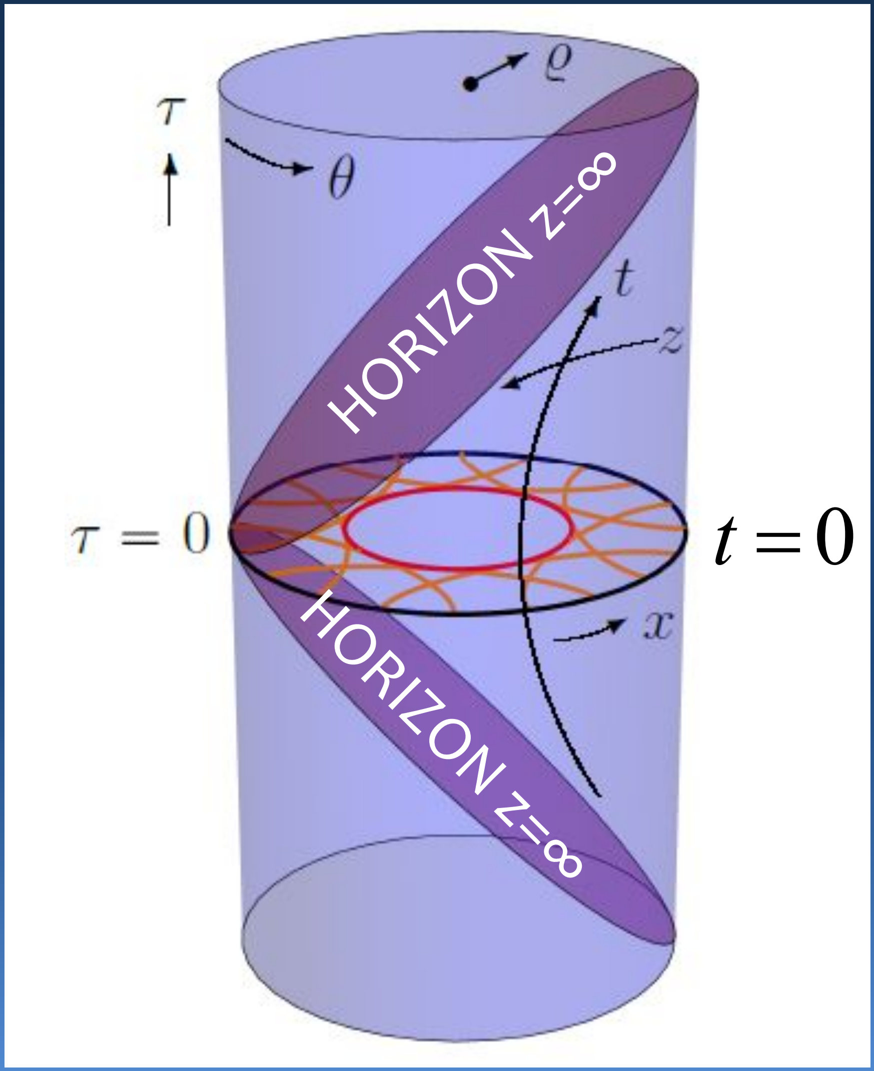




Hole-ography in Poincaré-AdS

Hole-ography in Poincaré

From known relation beween Poincaré and global AdS:



Slice of constant Poincaré time $t = 0$ coincides with slice of constant global time $\tau = 0$

So at $t = 0$ we have all the geodesics we need for reconstruction of curves

And by *t*-independence of

$$ds^2 = \frac{L^2}{z^2}\left(-dt^2 + dx^2 + dz^2\right)$$

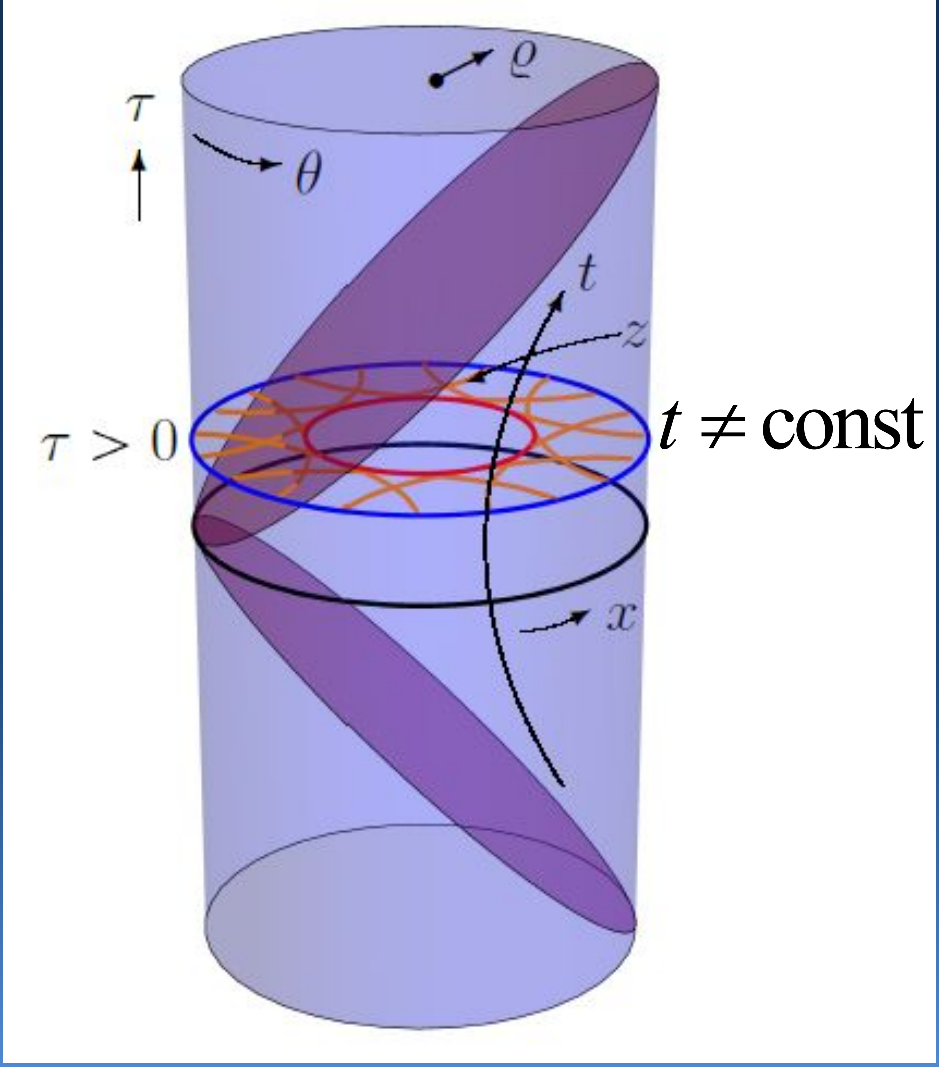
won't be missing geodesics on ANY fixed-*t* slice, either

Hole-ography in Poincaré

From known relation beween Poincaré and global AdS:

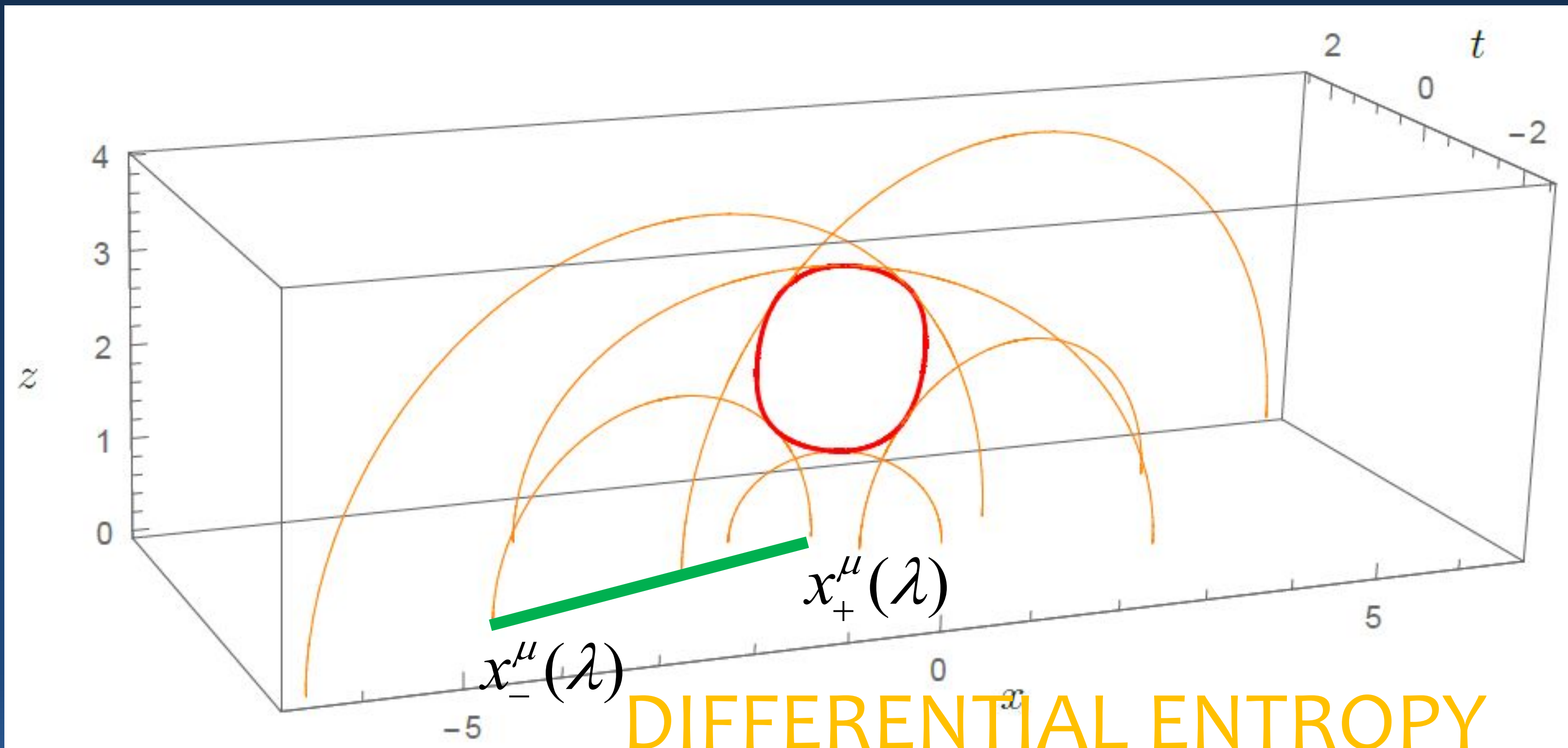
On the other hand, slice of constant global time $\tau \neq 0$ (corresponding to VARYING Poincaré time) is partly OUTSIDE of the Poincaré wedge,
so on this slice we have curves that are
NOT FULLY RECONSTRUCTIBLE
within Poincaré AdS!!

This is a CHALLENGE to hole-ography



Hole-ography in Poincaré

In more detail, given a curve, can find TANGENT geodesics



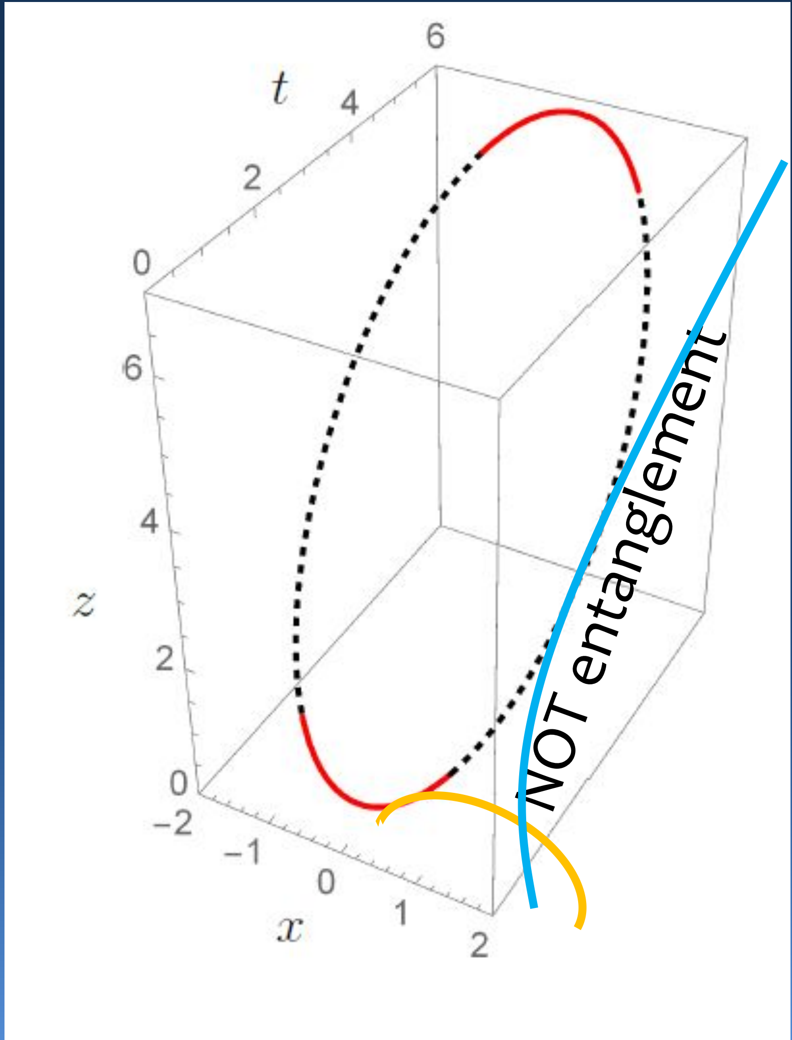
DIFFERENTIAL ENTROPY

$$E \equiv \oint d\lambda \left(\frac{\partial S(x_-(\lambda), x_+(\bar{\lambda}))}{\partial \bar{\lambda}} \right) \Bigg|_{\bar{\lambda}=\lambda}$$

[Balasubramanian et al.; Myers et al.; Headrick et al.]

Failure of Reconstruction

But generically, curves contain segments whose tangent geodesics DO NOT reach boundary of Poincaré wedge



Failure of Reconstruction

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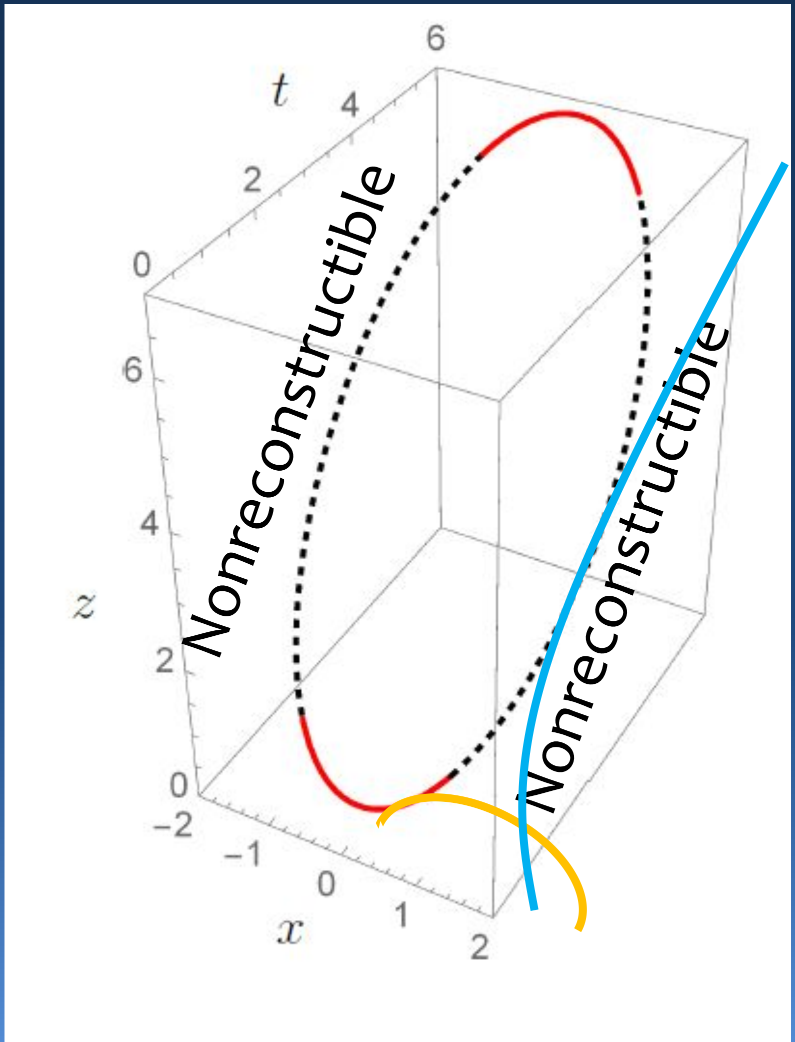


The CONDITION FOR RECONSTRUCTIBLITY is

$$-\left(u^t\right)^2 + \left(u^x\right)^2 > 0$$

where

$$u \equiv \left(t'(\lambda), x'(\lambda), z'(\lambda)\right)$$

is tangent vector to curve

Segments that violate this condition CANNOT BE RECONSTRUCTED using standard hole-ography

`Null Alignment'

Given a curve with TANGENT VECTOR $u \equiv \left(t'(\lambda), x'(\lambda), z'(\lambda)\right)$

$$E = \oint d\lambda \sqrt{\frac{L^2}{z^2}\left(-t'^2 + x'^2 + z'^2\right)}$$

$$= \oint d\lambda \sqrt{g_{mn} u^m u^n} = \mathrm{A}$$

Agreement E=A is maintained if we SHIFT TANGENT BY AN ORTHOGONAL NULL VECTOR,

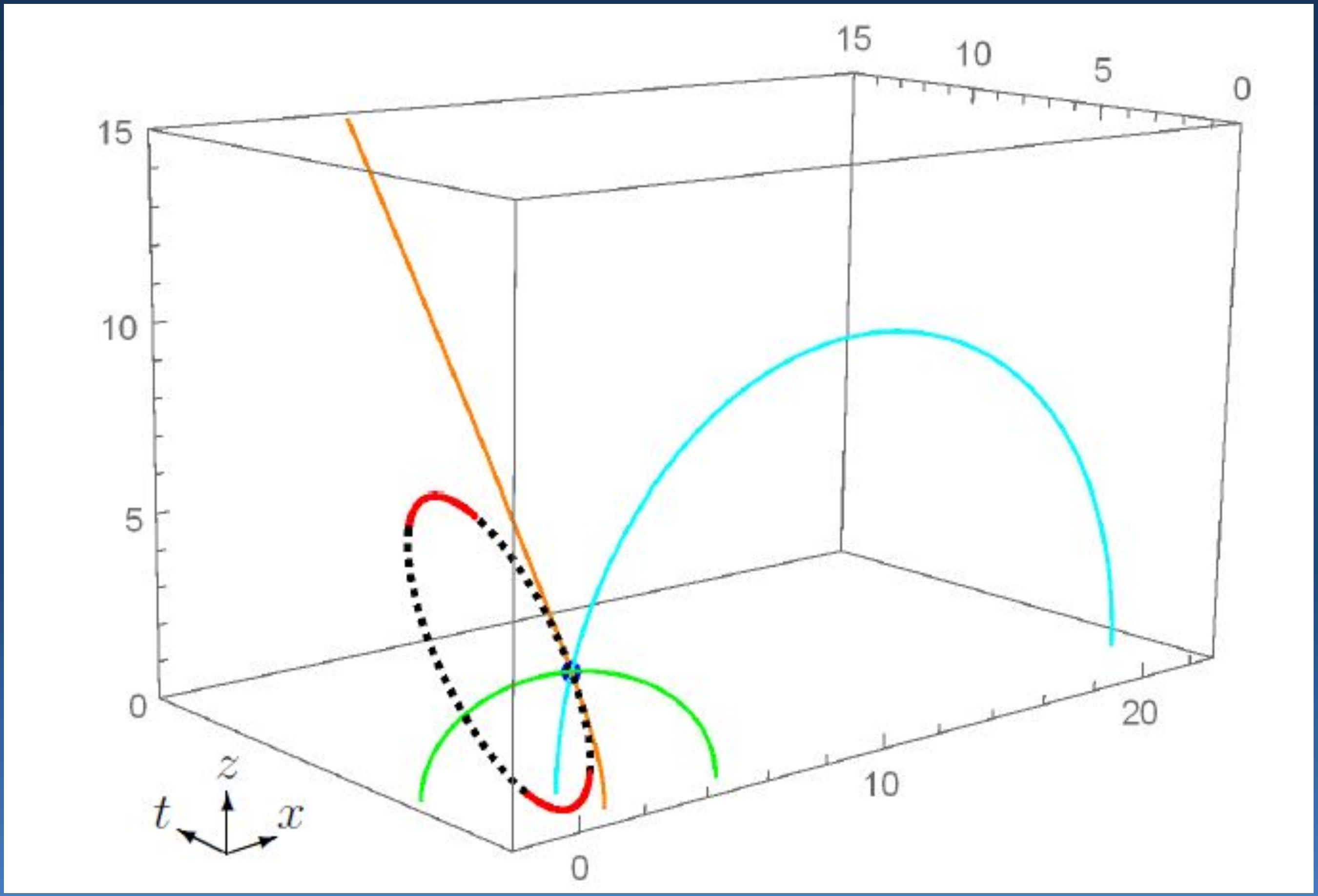
$$u \to U \equiv u + n, \quad n \cdot n = 0, \quad n \cdot u = 0$$

$$\Rightarrow U \cdot U = u \cdot u$$

[Headrick,Myers,Wien]

Reconstruction Achieved

We find that `null alignment' always allows us to SHOOT GEODESICS THAT DO REACH THE POINCARÉ BOUNDARY:

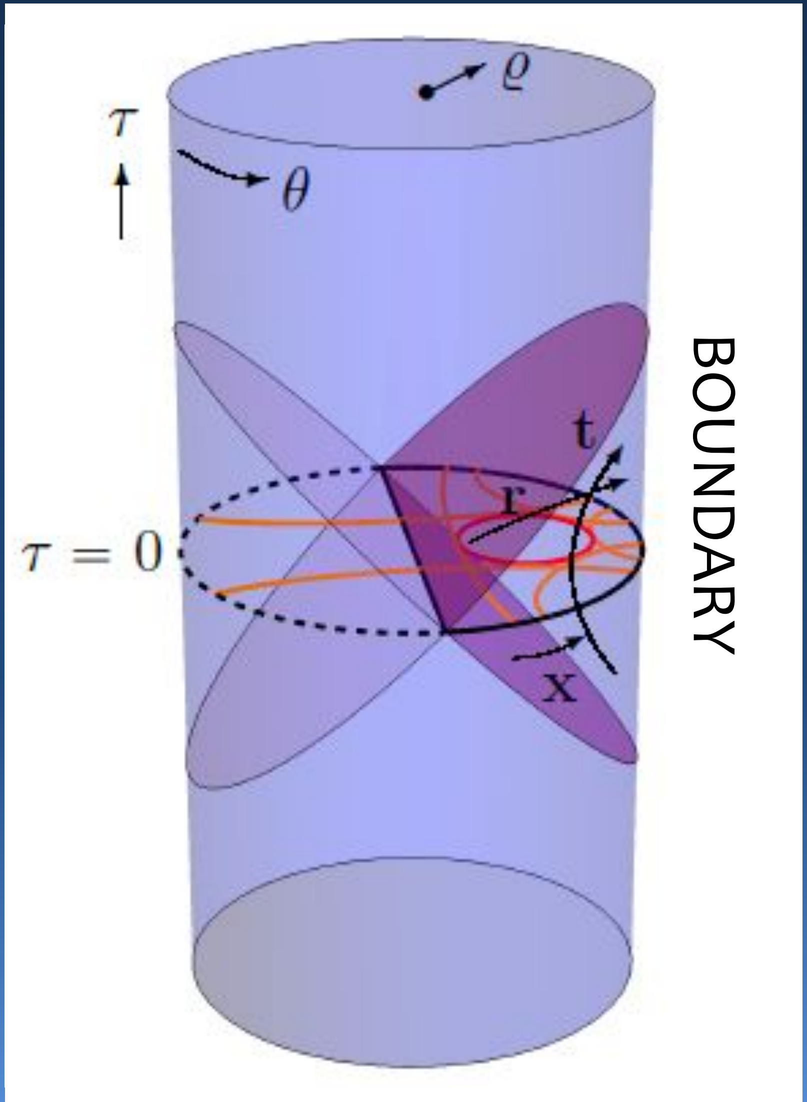


We thus conclude that ANY CURVE WITHIN THE POINCARÉ WEDGE CAN BE FULLY RECONS-TRUCTED

✓

Hole-ography in Rindler-AdS

For smaller *A* (mixed state), entanglement wedge is smaller: RINDLER WEDGE



Now have GEODESICS EXITING THE WEDGE even at constant Rindler time

Does `null alignment' again SAVE THE DAY?

$$u \rightarrow U \equiv u + n,$$

$$n \cdot n = 0, \quad n \cdot u = 0$$

Failure of Reconstruction

Now one finds TWO CONDITIONS FOR RECONSTRUCTIBILITY:

$$(U^x)^2 - (U^t)^2 > 0$$

$$r^2(1+r^2)(U^x + U^t)^2 - (U^r)^2 > 0$$

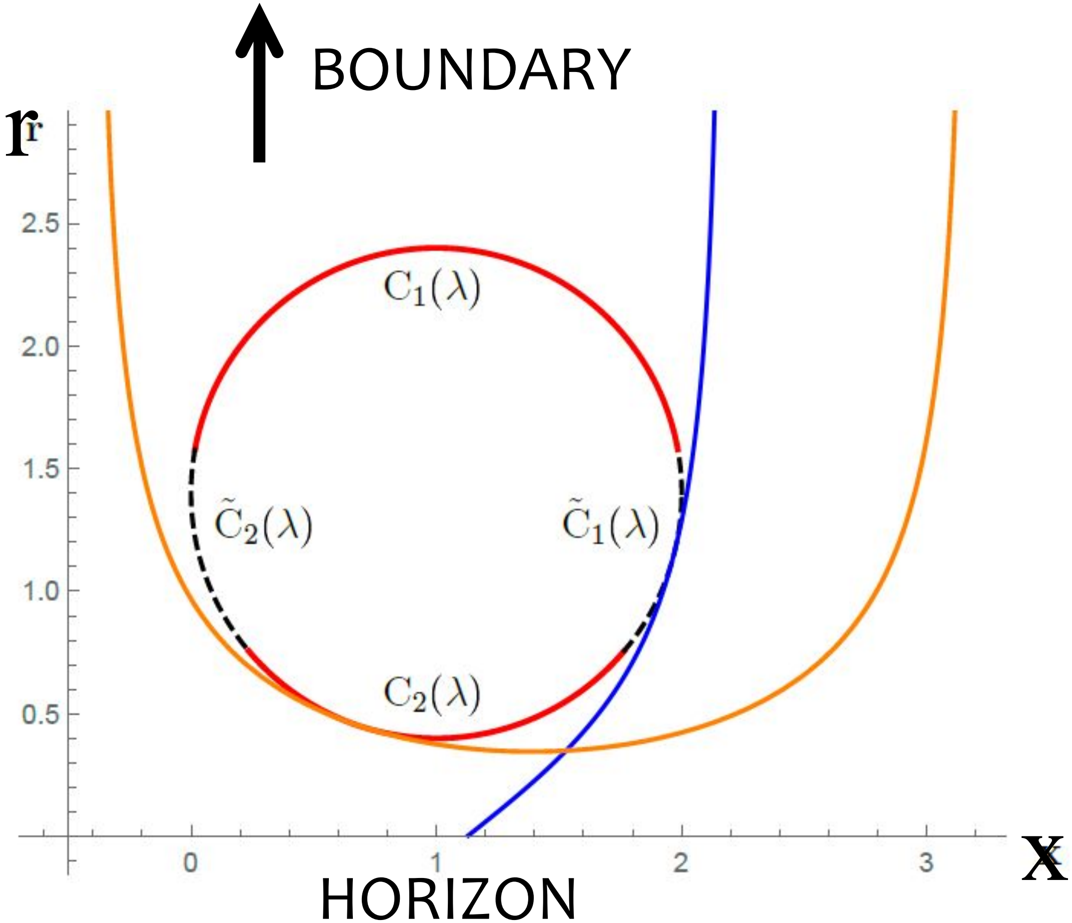
$$r^2(1+r^2)(U^x - U^t)^2 - (U^r)^2 > 0$$

(second condition implies third, or viceversa, depending on relative sign of U^x vs. U^t)

Problem: null alignment generally allows us to satisfy ONE of these, but not both at the same time

Failure of Reconstruction

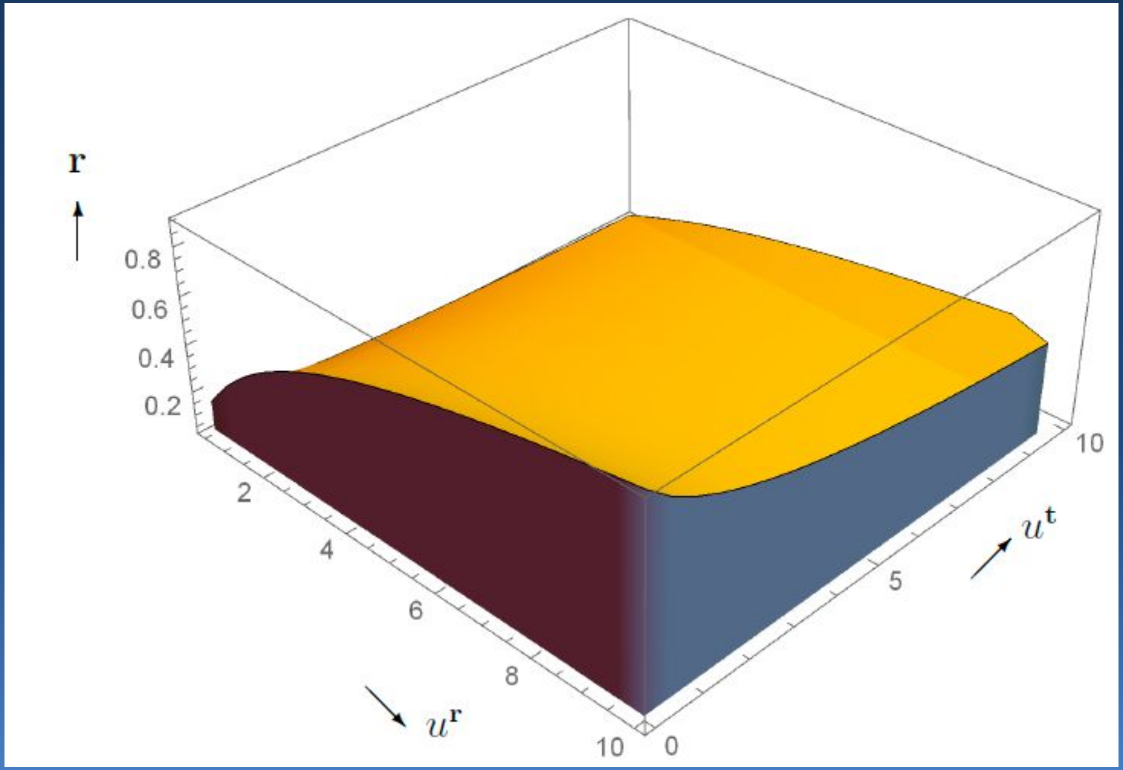
E.g., EVEN AFTER NULL ALIGNMENT, circle at constant Rindler time CANNOT BE RECONSTRUCTED on the sides



The generic problem is that **at any given radial depth r, sufficiently steep tangent vectors** *u* are such that no allowed *U=u+n* corresponds to a boundary-anchored geodesic

`Entanglement Shade'

This is analogous to the well-known phenomenon of "entanglement shadows": bulk regions not reached by geodesics [Hubeny,Maxfield,Rangamani,Tonni; Engelhardt,Wall; Balasubramanian,Chowdhury,Czech,de Boer; Freivogel,Jefferson,Kabir,Mosk,Yang; Engelhardt,Fischetti]

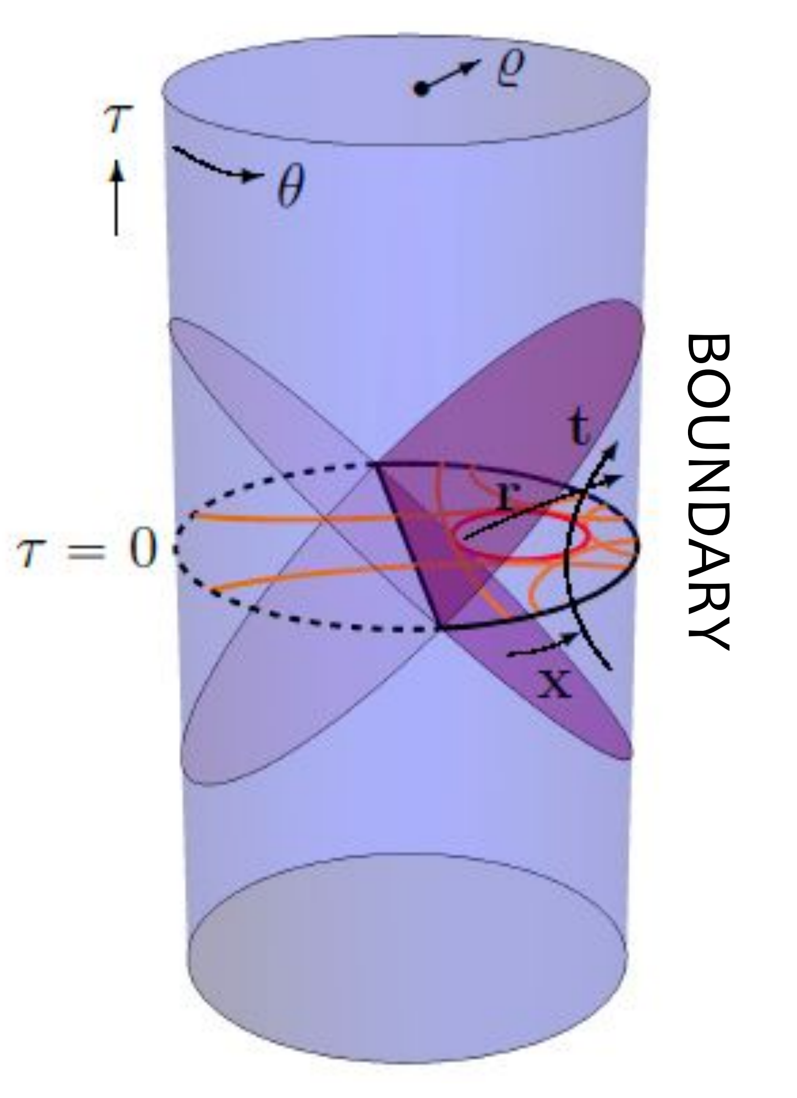


Except that the shadow here is well-delineated NOT on spacetime, but on the spacetime tangent bundle: "ENTANGLEMENT SHADE"

See also: [Freivogel,Jefferson, Kabir,Mosk,Yang]

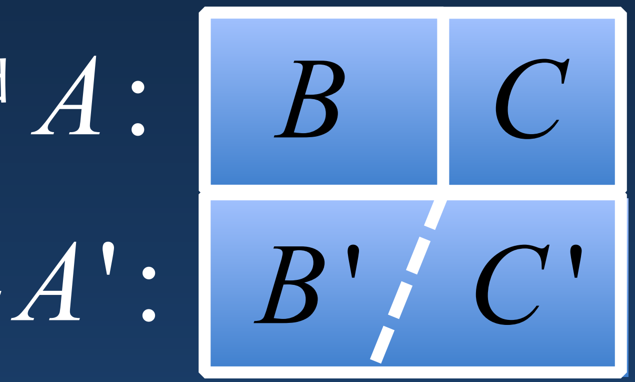
To reconstruct arbitrary curves within Rindler-AdS, then, we MUST resort to geodesics that are NOT boundary-anchored (they have at least one endpoint on the horizon instead of on the boundary)

These ARE NOT associated with entanglement entropies.
Do they have some interpretation in the CFT?



Entanglement of Purification

Consider some system *A*=*BC*, described by a density matrix r_A



If state is mixed, entanglement entropy

$$S_B \equiv -\mathrm{Tr}\left(\rho_B \ln \rho_B\right) \neq S_C \qquad \rho_B \equiv \mathrm{Tr}_C(\rho_A)$$

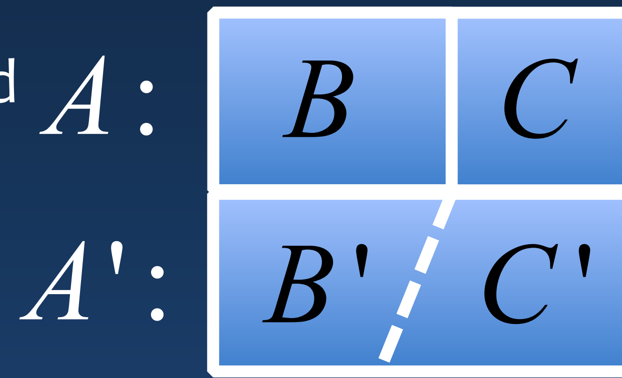
quantifies both quantum AND classical correlations

Can PURIFY the system, finding auxiliary degrees of freedom *A'*=*B'C'* and overall PURE state $|\psi\rangle$ for *AA'* such that

$$\rho_A = \mathrm{Tr}_{A'}(|\psi\rangle\langle\psi|)$$

Entanglement of Purification

Consider some system *A=BC*, described by a density matrix r_A



Purification is highly non-unique!
but we can select a special one by DOUBLE OPTIMIZATION

$$P(B:C) \equiv \min_{(|\psi\rangle, A'), B'} S_{BB'}$$

ENTANGLEMENT OF PURIFICATION

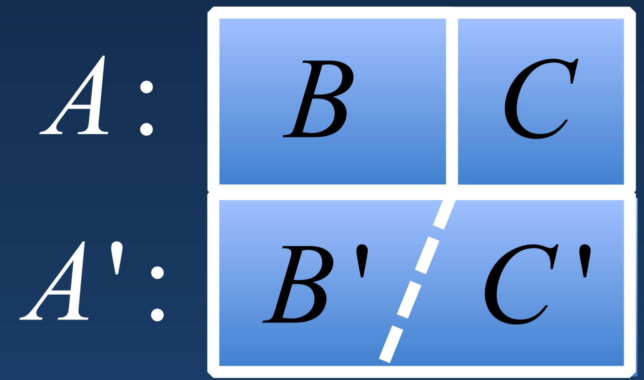
[Terhal,Horodecki, Leung,Di Vincenzo]

Reexpresses S_B in terms of purely quantum correlations

Nearly impossible to compute explicitly!

Entanglement of Purification

ENTANGLEMENT OF PURIFICATION is known to satisfy



$$P(B:C) \le \min\left(S_B, S_C\right)$$

$$P(B:CD) \ge P(B:C)$$

$$P(B:C) \ge \tfrac{1}{2} I(B:C)$$

$$P(B:CD) \ge \tfrac{1}{2} I(B:C) + \tfrac{1}{2} I(B:D)$$

$$P(B:CD) \le P(B:C) + P(B:D)$$

$$P(B:C) = S_{BC} \quad \text{if } \rho_A \text{ is pure}$$

$$S_{BC} = \left|S_B - S_C\right| \quad \Rightarrow \quad P(B:C) = \min\left(S_B, S_C\right)$$

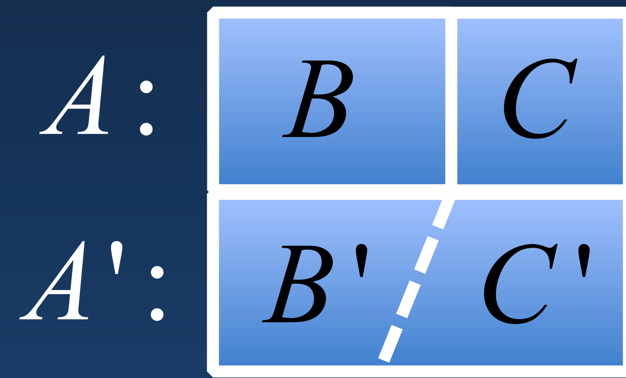
[Terhal,Horodecki, Leung,Di Vincenzo]

Entanglement of Purification

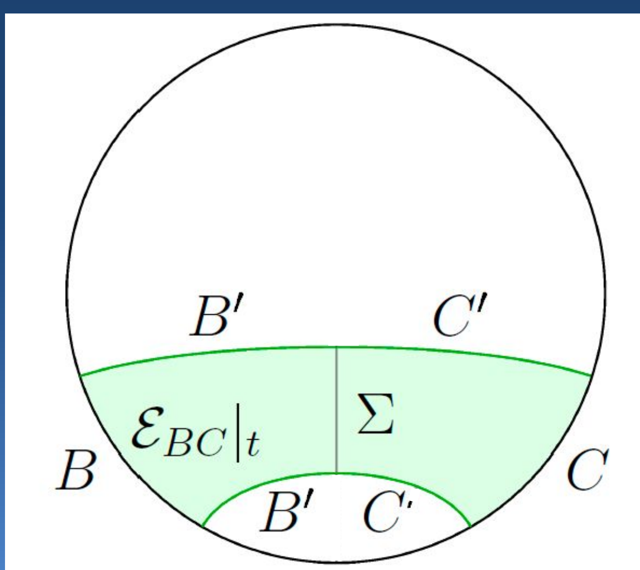
Recently conjectured holographic dual:

ENTANGLEMENT WEDGE CROSS SECTION

$$\mathrm{P}(B:C) \equiv \min \frac{\text{Area of } \Sigma}{4G_N}$$



[Takayanagi,Umemoto; Nguyen,Devakul,Halbasch,Zaletel,Swingle]



$\mathrm{P}(B:C)$ satifies all the SAME inequalities as $P(B:C)$!

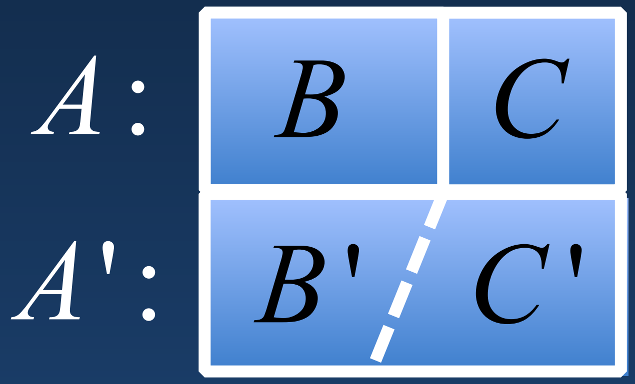
Conjecture: $\mathrm{P}(B:C) = P(B:C)$

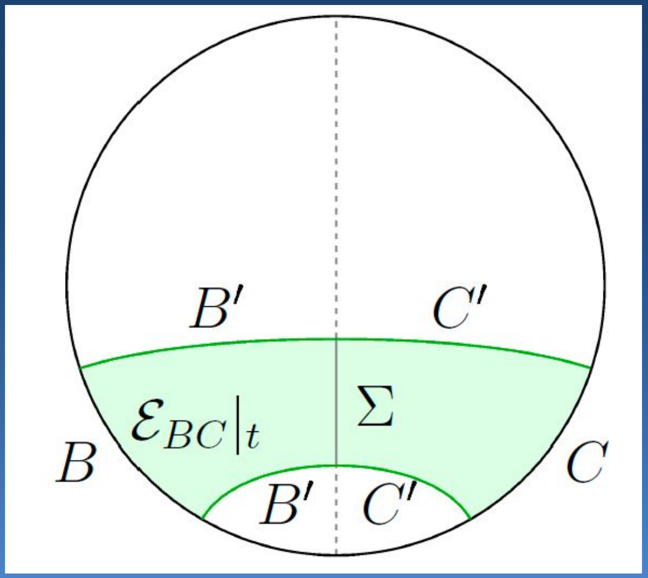
Entanglement of Purification

Recently conjectured holographic dual:

ENTANGLEMENT WEDGE CROSS SECTION

$$P(B:C) \equiv \min \frac{\text{Area of } \Sigma}{4G_N}$$



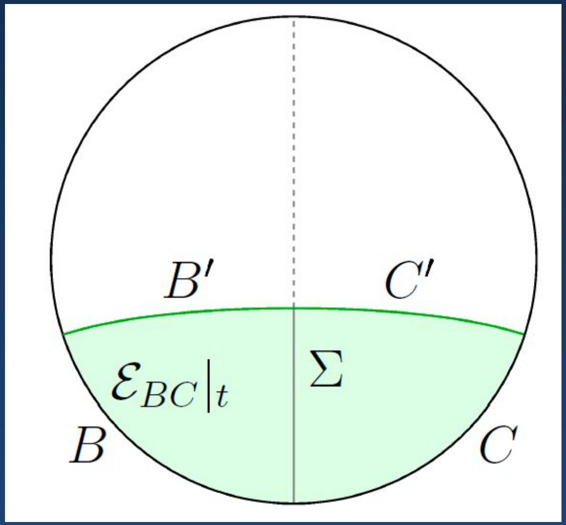


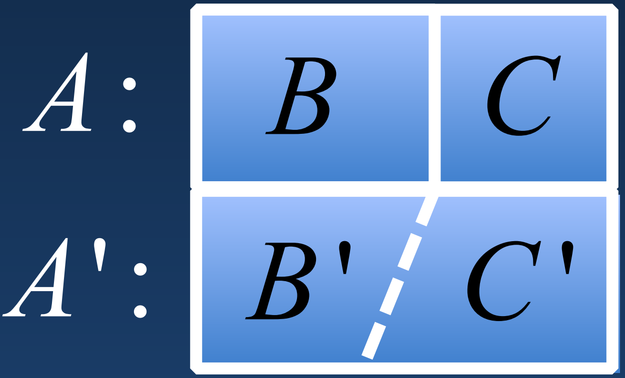
Notice: recipe uses truncated geodesic, so optimal purification $\left(|\psi\rangle, A'\right)$ is NOT dual to entire original geometry: the purifying d.o.f. ***A'*** "live on" horizon of entanglement wedge!

[Bhattacharya,Takayanagi,Umemoto; Hirai,Tamaoka,Yokoya]

Relation to Wedge Reconstruction

Consider contiguous *B* and *C*:

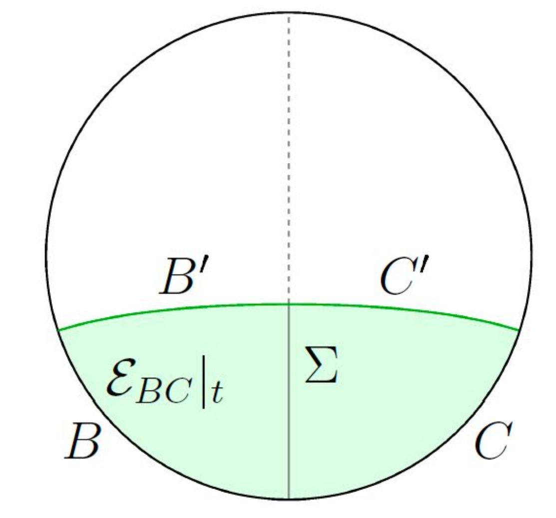


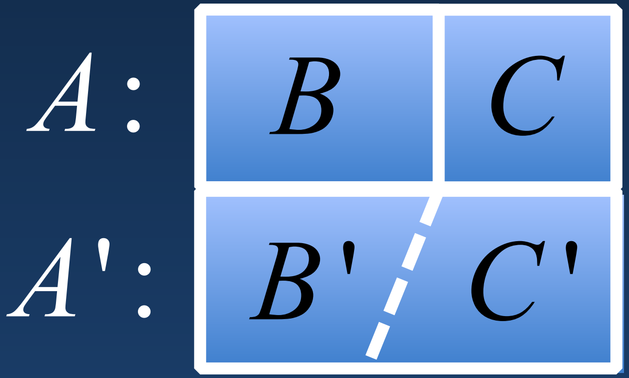


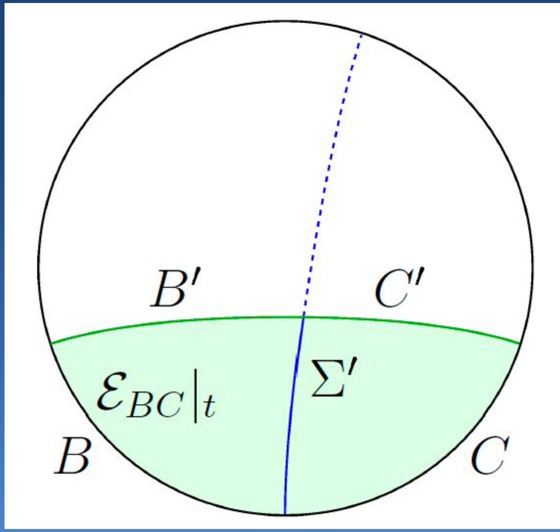
Σ is a geodesic exiting the wedge through the Rindler horizon

Relation to Wedge Reconstruction

Consider contiguous *B* and *C*:







Need MORE GENERIC geodesics Σ' of this type: slight generalization

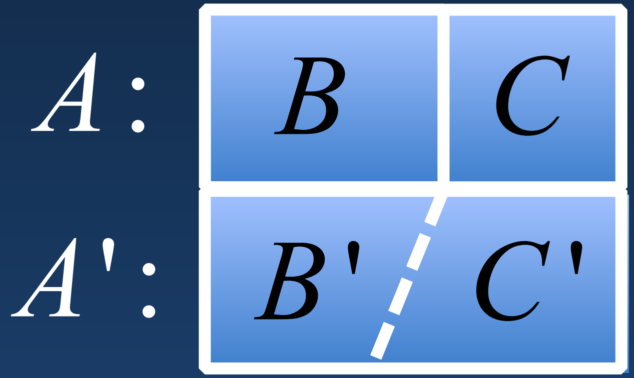
$$P'(B:C\,|\,B') \equiv \frac{\text{Area of } \Sigma'}{4G_N} = S_{BB'}\Big|_{(|\psi\rangle, A')}$$

Conditional Entanglement of Purification

[Espíndola,Guijosa,Pedraza]

Differential Purification

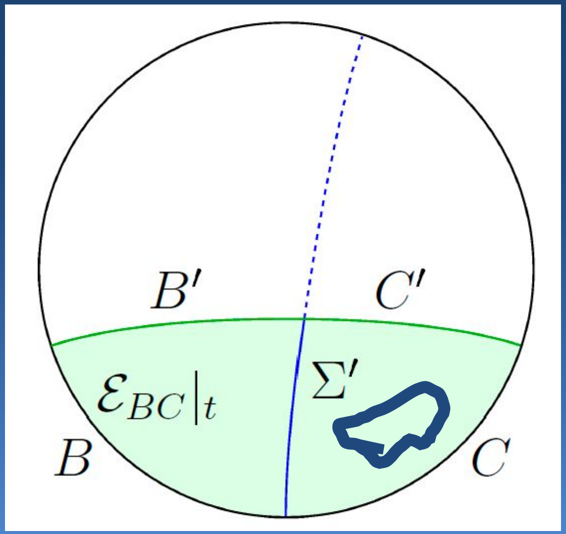
Using this entanglement of purification, can define



$$D \equiv \oint d\lambda \left(\frac{\partial P'(x_\infty(\lambda), x_h(\bar{\lambda}))}{\partial \bar{\lambda}} \right)\Bigg|_{\bar{\lambda}=\lambda}$$

DIFFERENTIAL PURIFICATION

[Espíndola,Guijosa,Pedraza]



and show that $D = A$ ✓

Extends to time dependent curves using notion of 'modular flow' ✓

CONCLUDING REMARKS

- Have seen that full reconstruction of bulk curves can be achieved for ARBITRARY SPACELIKE CURVES in Poincaré & Rindler AdS, using entanglement entropy +ENTANGLEMENT OF PURIFICATION (and possibly null alignment) ✓

- Can show that same recipe allows reconstruction within an ARBITRARY ENTANGLEMENT WEDGE in an ARBITRARY 3-DIM (aAdS) BULK GEOMETRY ✓