Instituto de Física Teórica UAM-CSIC

FROGS
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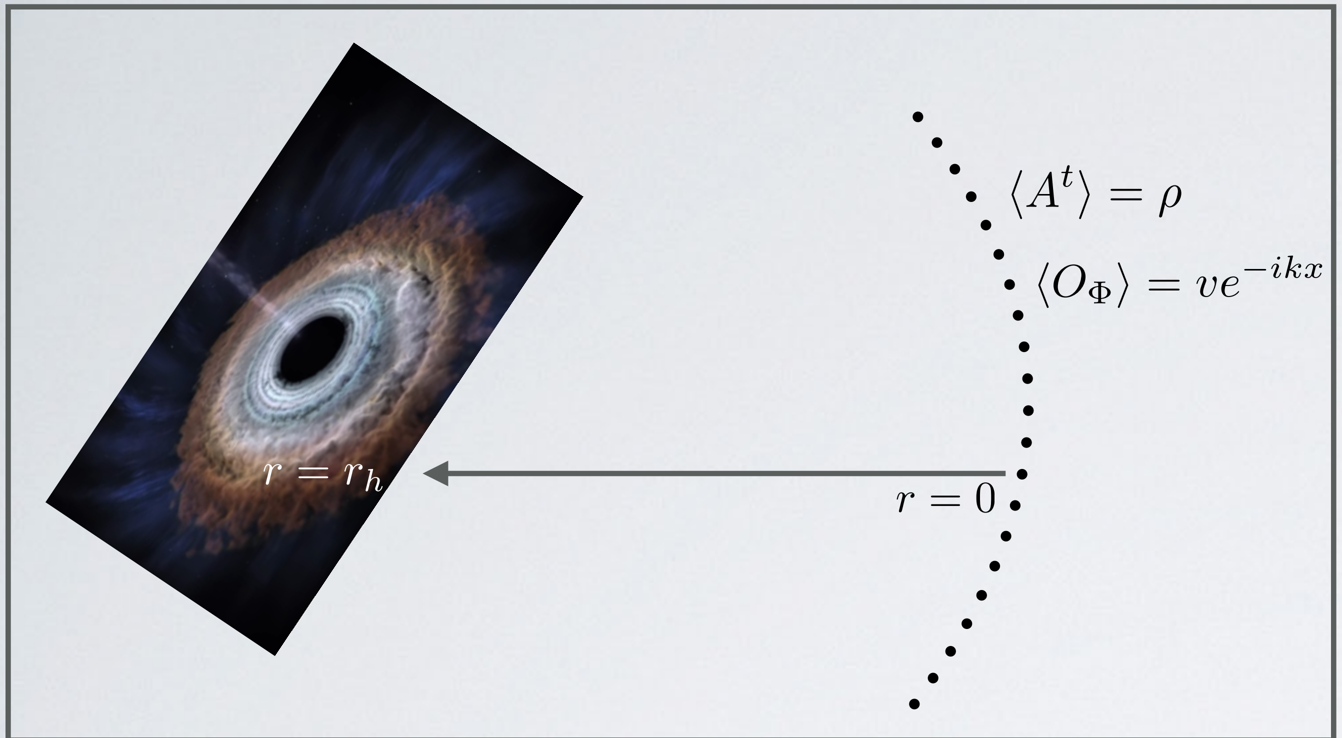
# From Q-lattices to Bad Metals

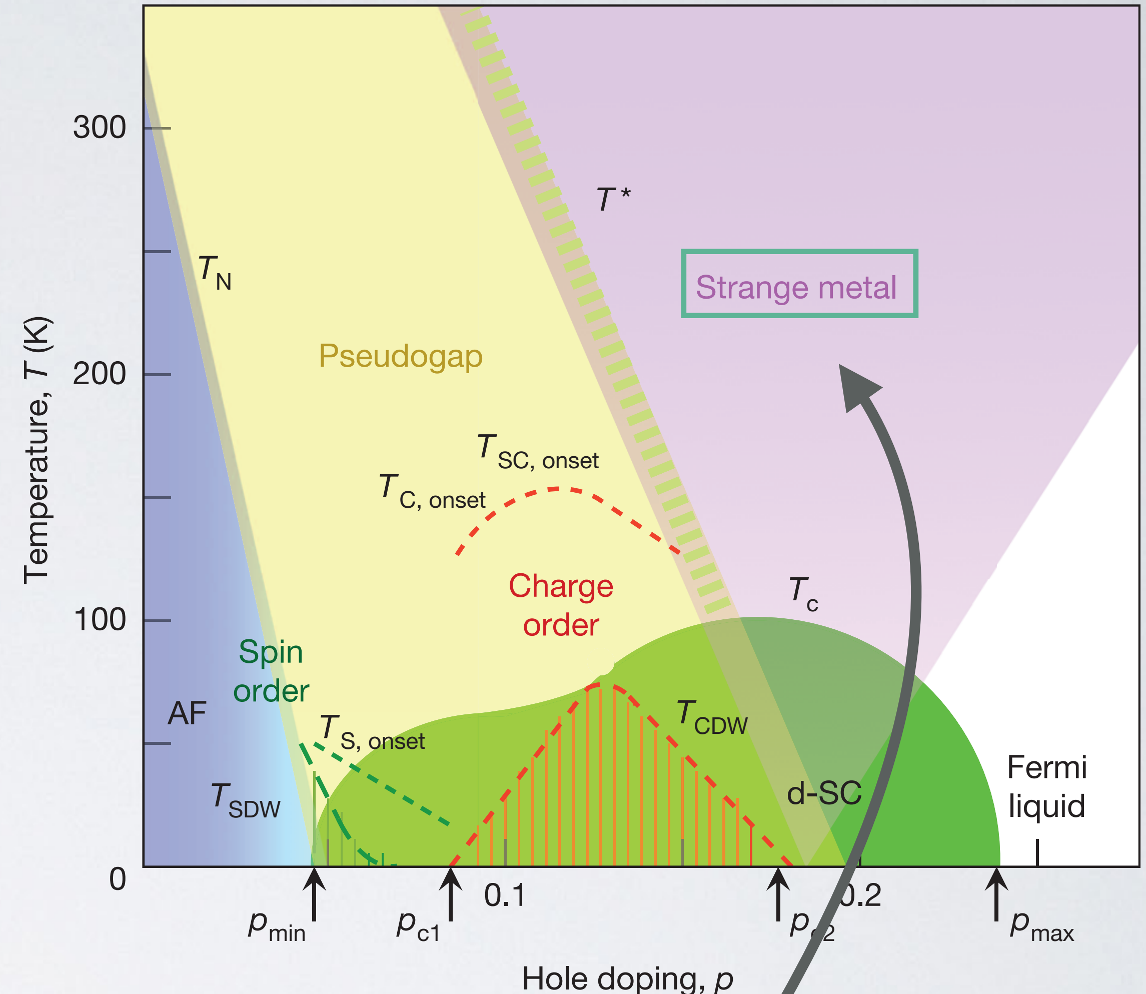

with A. Amoretti, B. Goutéraux and D. Musso

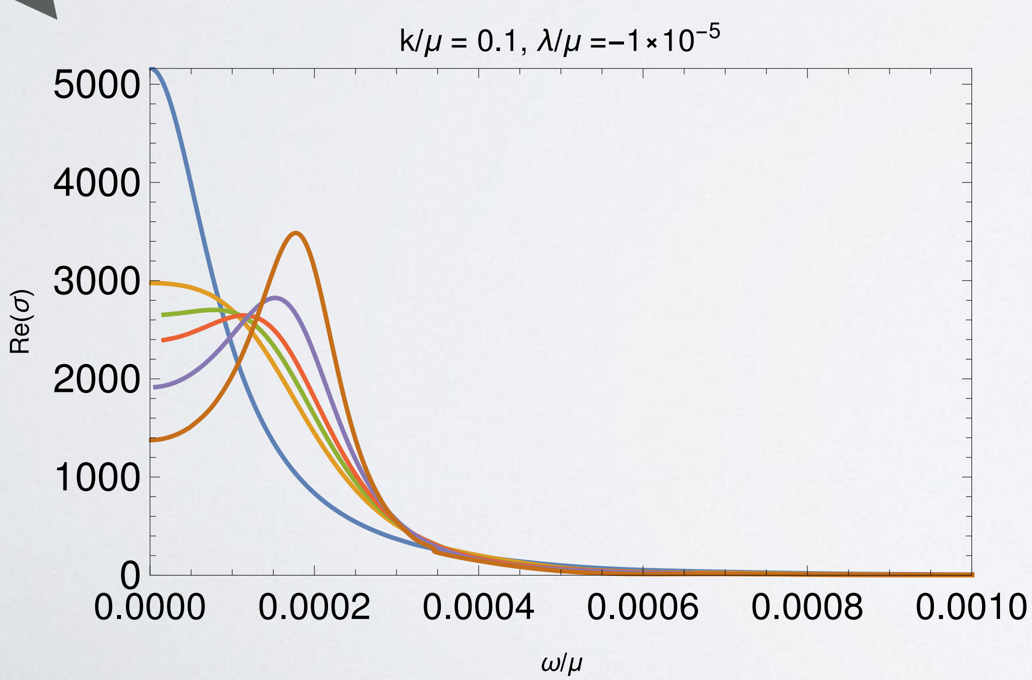

[see also 1711.06610 and 1712.07994]


Daniel Areán. Würzburg 2018

# > ⟨~~translations~~⟩ with pseudo-spontaneous Q-lattices



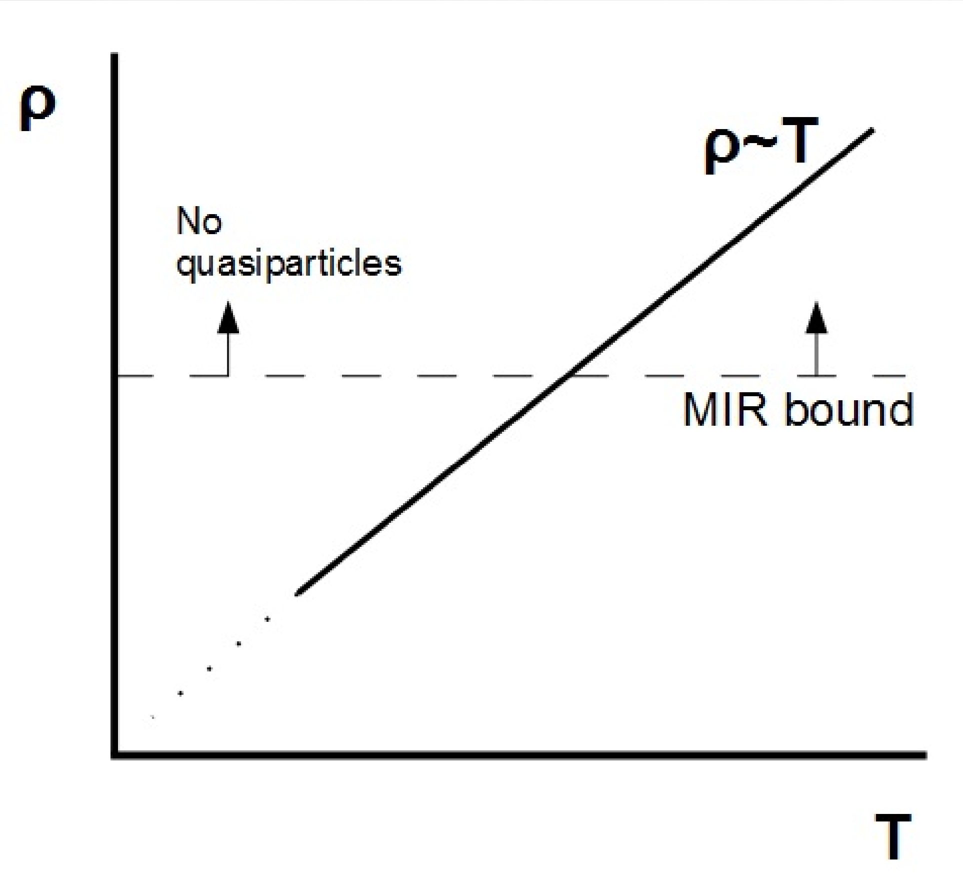



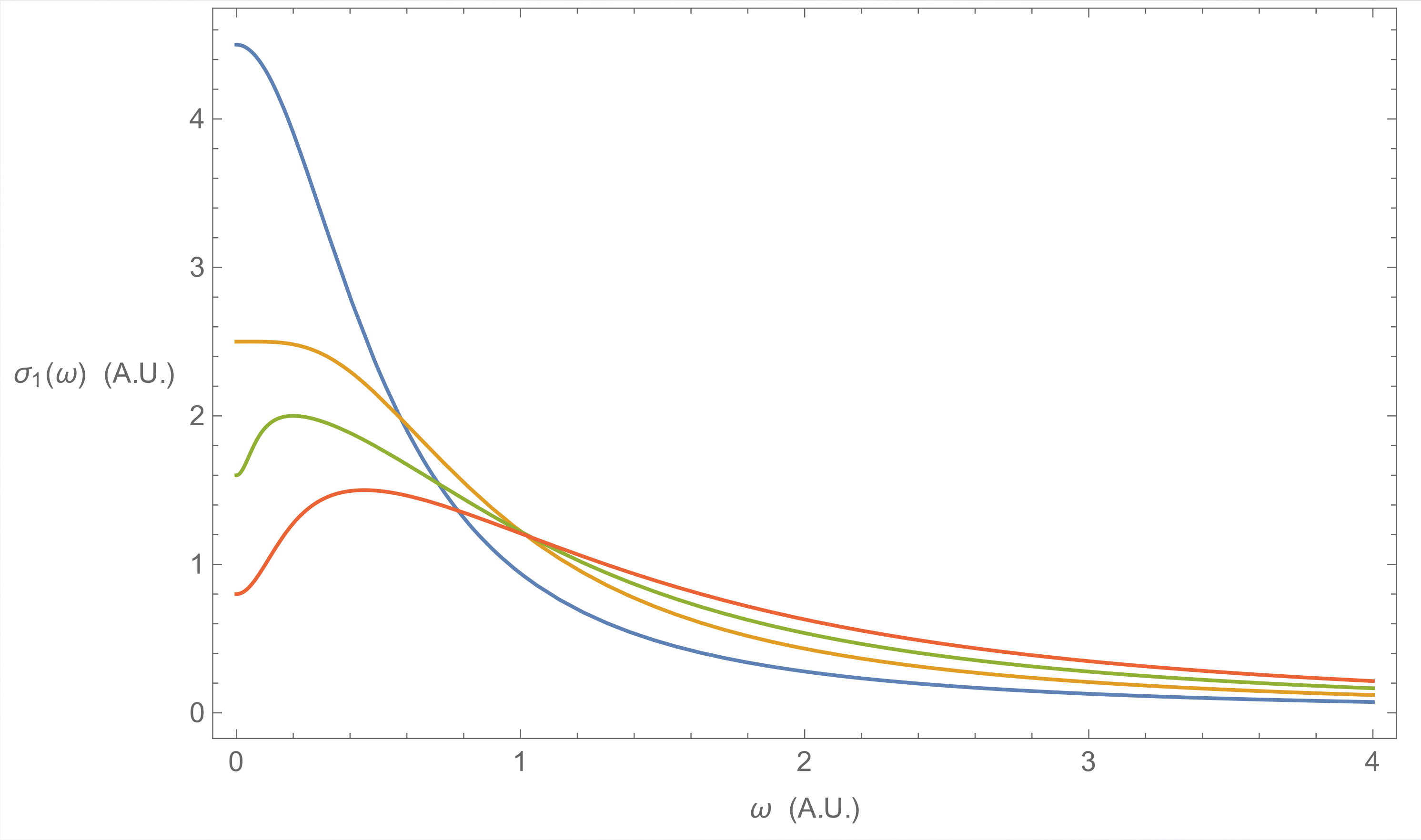

$$\sigma(\omega) = \sigma_{inc} + \frac{\rho^2}{\chi_{PP}} \frac{\Omega - i\omega}{(\Omega - i\omega)(\Gamma - i\omega) + \omega_0^2} \, .$$

*Linear resistivity and peak -> off-axis as T ↑*

# > Bad metals are challenging [see cond-mat/0404263]




[from 1405.3651]




[1702.05104]

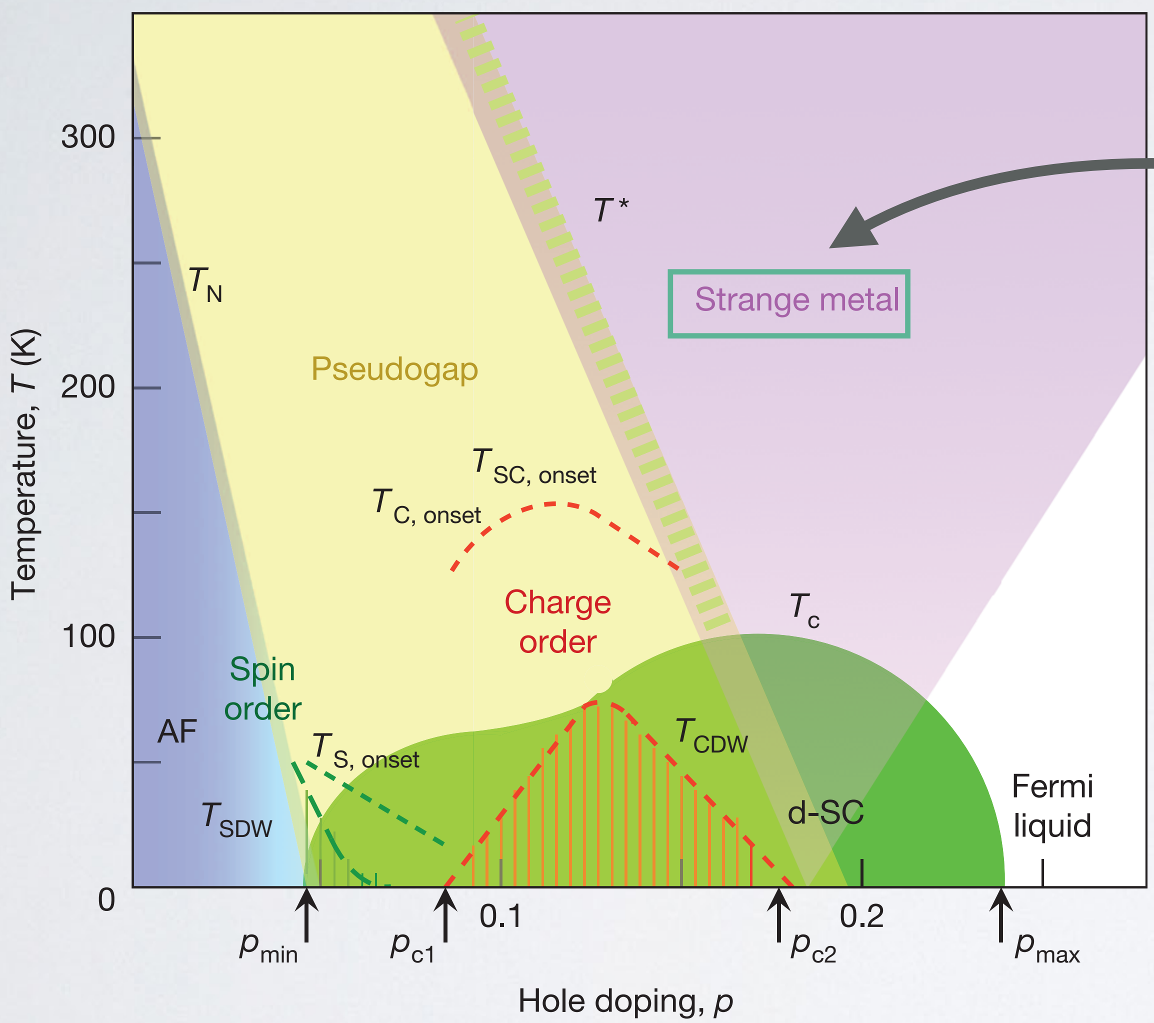
- Resistivity linear in T w/out long-lived quasiparticles (~~MIR bound~~)
- Far IR peak in $\sigma_{AC}$ moving off-axis as T is increased.

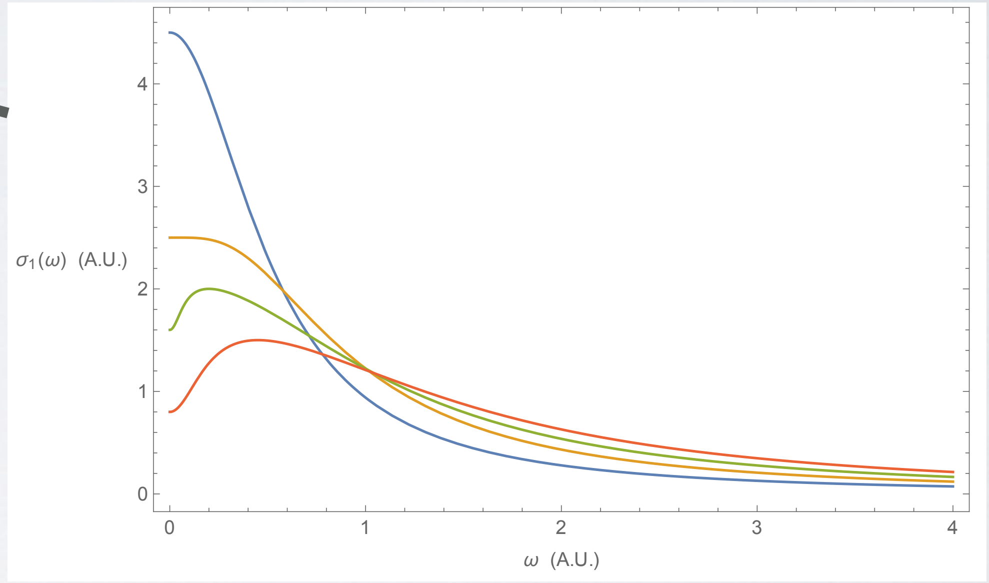
# > Bad metals and quantum criticality

- Hints of universality [1405.3651, 1612.04381]

$$\rho \sim T$$

$$\omega_{\text{peak}} \sim \frac{1}{\tau_{\text{p}}} \sim \frac{K_{\text{B}}}{\hbar} T$$

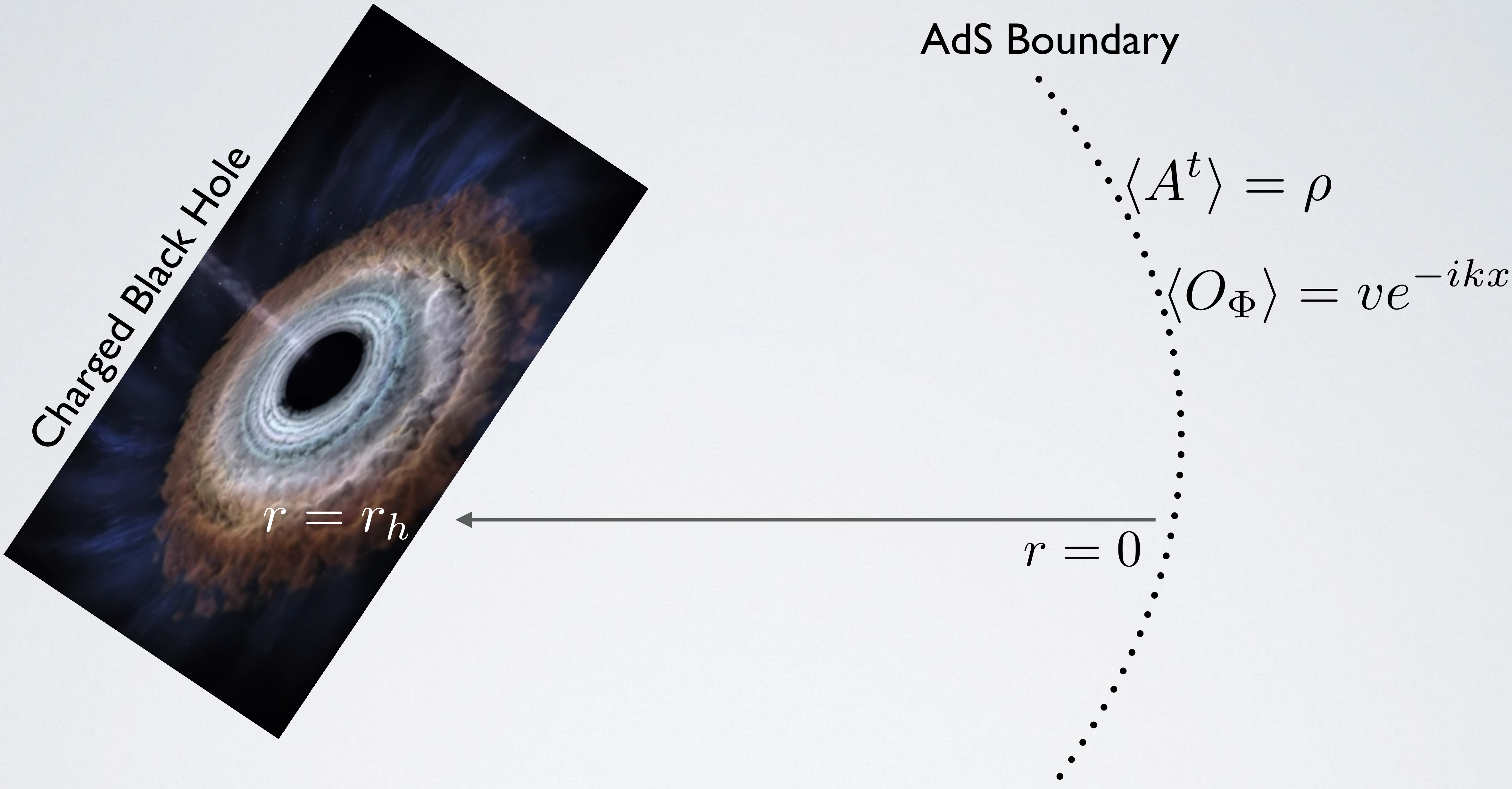



- ⟨Spatial Ordering⟩ ≠ 0 in Pseudo-Gap phase (PDW)
  Residual resistivity upturn below $T_{\text{order}}$ [PRL 88 (03 2002) 147003]
- Off-axis peaks in $\sigma_{AC}$ as Quantum Critical CDW [1612.04381, 1702.05104]

# > ⟨~~translations~~⟩ with spontaneous Q-lattices

[1311.3292, 1401.5077]




➥ **CALCULABLE (toy-) Model of ⟨~~translations~~⟩**

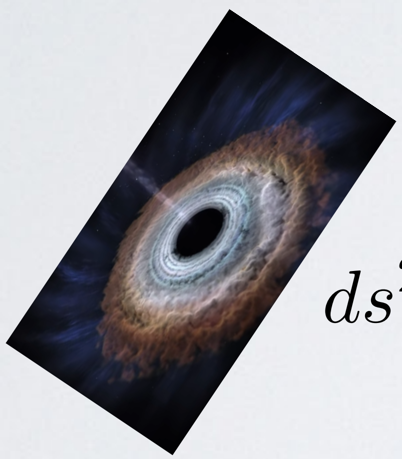
[see Andrea's talk for more details; 1711.06610 and 1712.07994]

## > ⟨~~translations~~⟩ with ‘pseudo-Spontaneous Q-lattices’

$$S = \int d^{d+2}x \sqrt{-g} \left[ R - \frac{1}{2}\partial\phi^2 - \frac{1}{4}Z(\phi)F^2 - V(\phi) - \frac{1}{2}Y(\phi)\sum_{i=1}^{d}\partial\psi_i^2 \right].$$

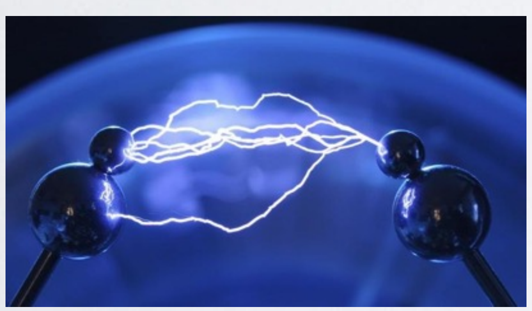
[1311.3292, 1401.5077]

## > ANSATZ



$\sim$AdS charged black hole

$$ds^2 = -D(r)dt^2 + B(r)dr^2 + C(r)d\vec{x}^2$$

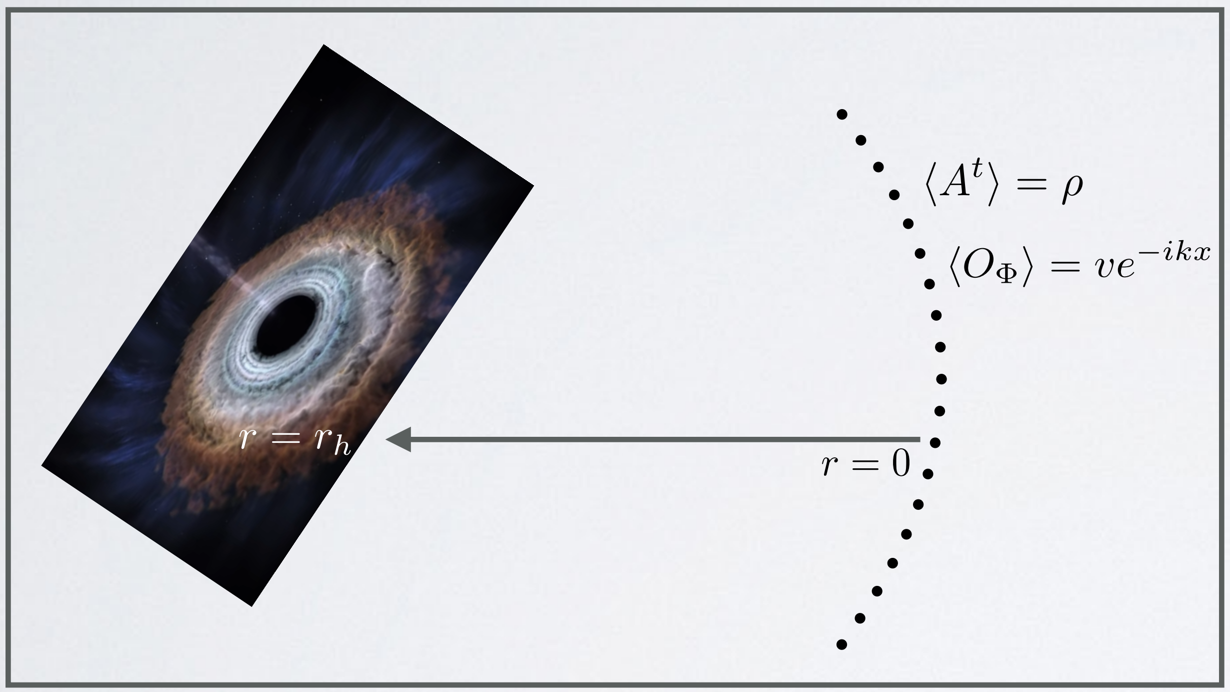


$$A = A(r)dt \longrightarrow A \sim \mu$$ Chemical potential

$$\begin{aligned} \phi &= \phi(r) \\ \psi_i &= kx^i \end{aligned} \longrightarrow \Phi_i = \phi\, e^{ikx^i}$$ Q-lattice(s)

$$S = \int d^{d+2}x\,\sqrt{-g}\left[R - \frac{1}{2}\partial\phi^2 - \frac{1}{4}Z(\phi)F^2 - V(\phi) - \frac{1}{2}Y(\phi)\sum_{i=1}^{d}\partial\psi_i^2\right].$$

> '**pseudo-**Spontaneous Q-lattices'




$$\begin{array}{l} \phi = \phi(r) \\ \psi_i = k x^i \end{array} \longrightarrow \Phi_i = \phi\, e^{ikx^i}$$

Assymptotically $O_\phi \sim \phi\, e^{ikx}$ with source << vev

## > ‘pseudo-Spontaneous Q-lattices’

$$S=\int d^{d+2}x\,\sqrt{-g}\left[R-\frac{1}{2}\partial\phi^2-\frac{1}{4}Z(\phi)F^2-V(\phi)-\frac{1}{2}Y(\phi)\sum_{i=1}^{d}\partial\psi_i^2\right].$$

$$\phi=\phi(r)$$
$$\psi_i=kx^i$$

$\longrightarrow$ $\boxed{\Phi_i=\phi\,e^{ikx^i}\ \text{with source << vev}}$

## > UV ASYMPTOTICS (~AdS)

$$V_{UV}=-d(d+1)+\frac{1}{2}m^2\phi^2+\dots,\quad Z_{UV}=1+Z_1\phi+\dots,\quad Y_{UV}=\boxed{Y_2\,\phi^2}+\dots$$

$$\phi(r\to 0)=\phi_{(0)}r^{d+1-\Delta}+\phi_{(1)}r^{\Delta}+\dots,\qquad m^2=\Delta(\Delta-d-1).$$

> with $\boxed{\phi_{(0)}\ll\phi_{(1)}}$

# > Quantum Critical 'Q-lattices'

Hyper-scaling IR solutions found & classified in 1401.5436

Numerically solve ODEs and find BH geometries:

AdS (UV) -> Scaling (IR):

$$ds^2 = r^\theta \left[ -f(r)\frac{dt^2}{r^{2z}} + \frac{L^2 dr^2}{r^2 f(r)} + \frac{d\vec{x}^2}{r^2} \right]$$

$$t \to \lambda^z t\,, \;\; \vec{x} \to \lambda \vec{x} \qquad s \sim T^{\frac{2-\theta}{z}}$$

We're interested in the case:

$$z \to \infty$$
$$\theta \to -\infty$$
$$\tilde{\theta} \equiv -\theta/z = 1$$

$$\boxed{s \sim T,\; 1/\sigma_{\mathrm{DC}} \sim T}$$

# > Quantum Critical 'Q-lattices'

*Hyper-scaling IR solutions found & classified in* 1401.5436

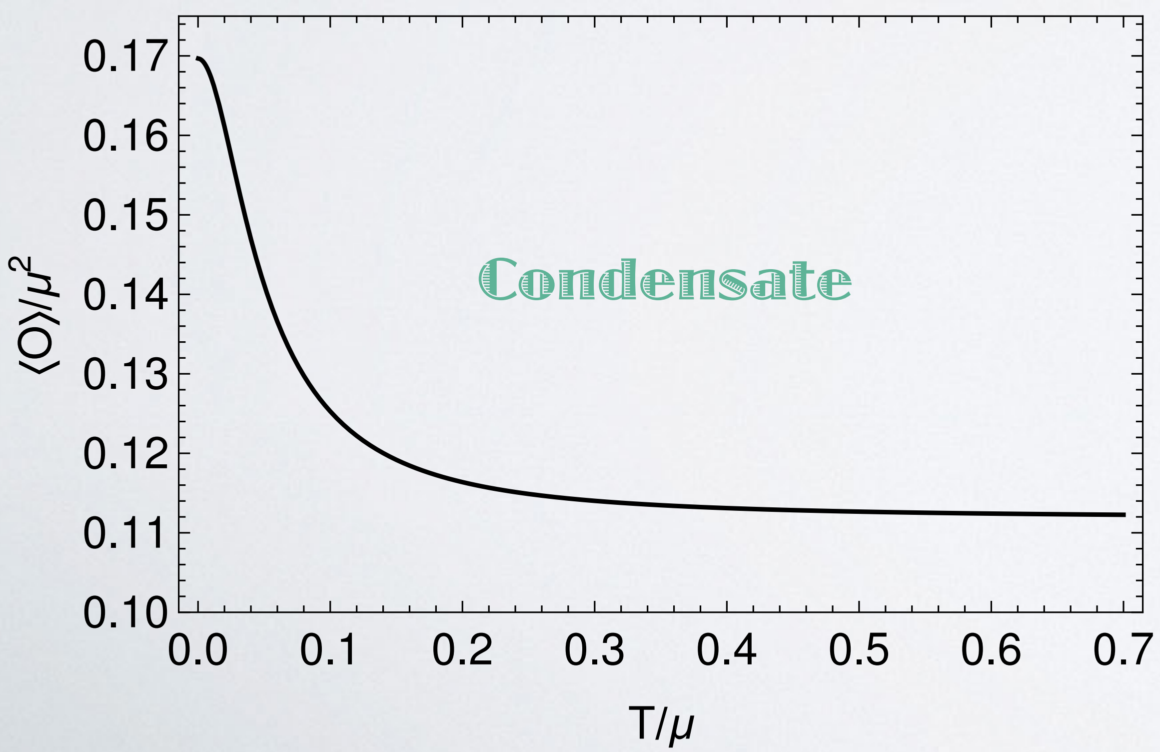
> **Scaling (IR) - Model**

$$S=\int d^{d+2}x\,\sqrt{-g}\left[R-\frac{1}{2}\partial\phi^2-\frac{1}{4}Z(\phi)F^2-V(\phi)-\frac{1}{2}Y(\phi)\sum_{i=1}^{d}\partial\psi_i^2\right].$$

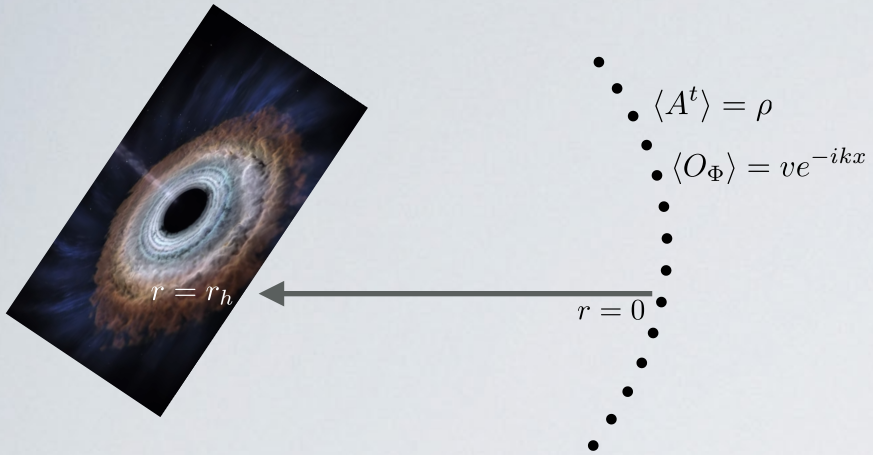
$$V(\phi)=-6\cosh\left(\frac{\phi}{\sqrt{3}}\right)\,,\quad Z(\phi)=\exp\left(-\sqrt{3}\phi\right)\,,\quad Y(\phi)=(1-\exp\phi)^2\,,$$

*We've constructed numerical (pseudo-)spontaneous solutions*

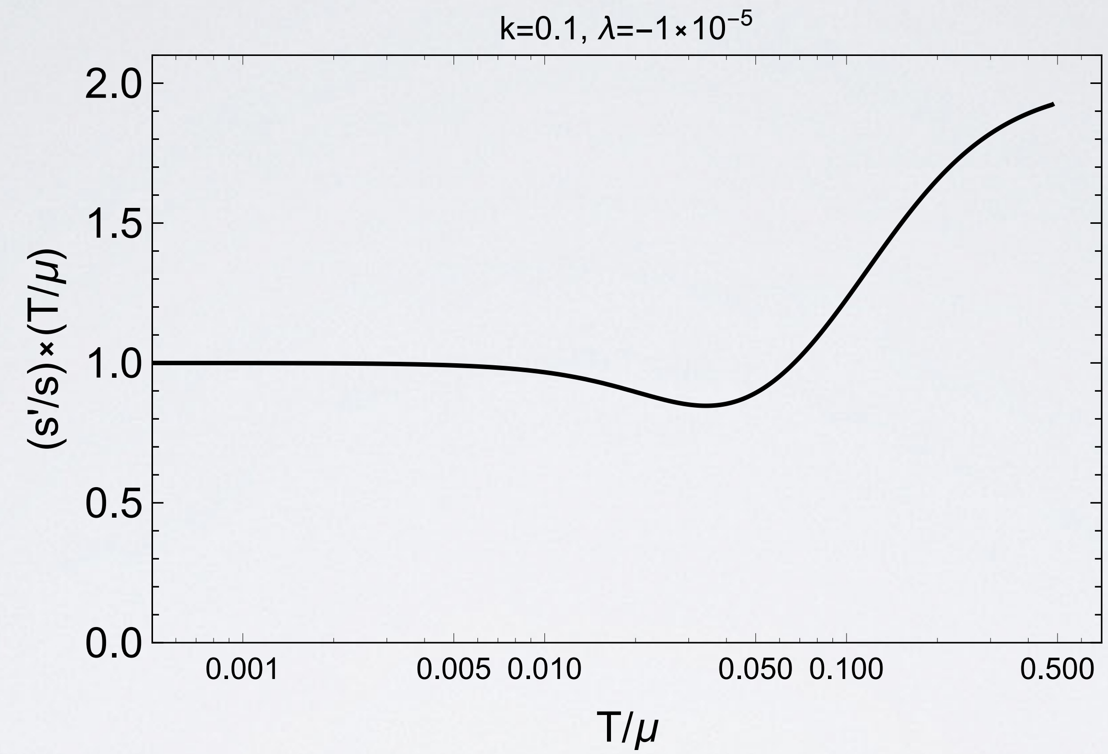
*AdS (UV) -> Scaling (IR)*




**[No Tc. AdS-RN isn't a sol.]**

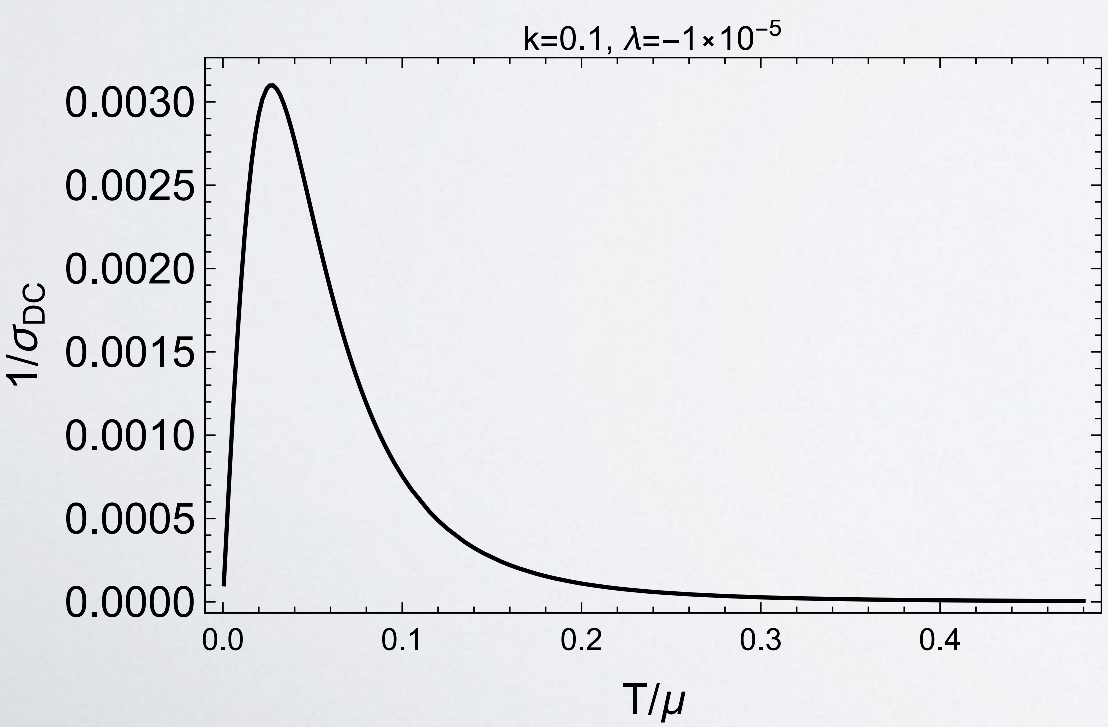


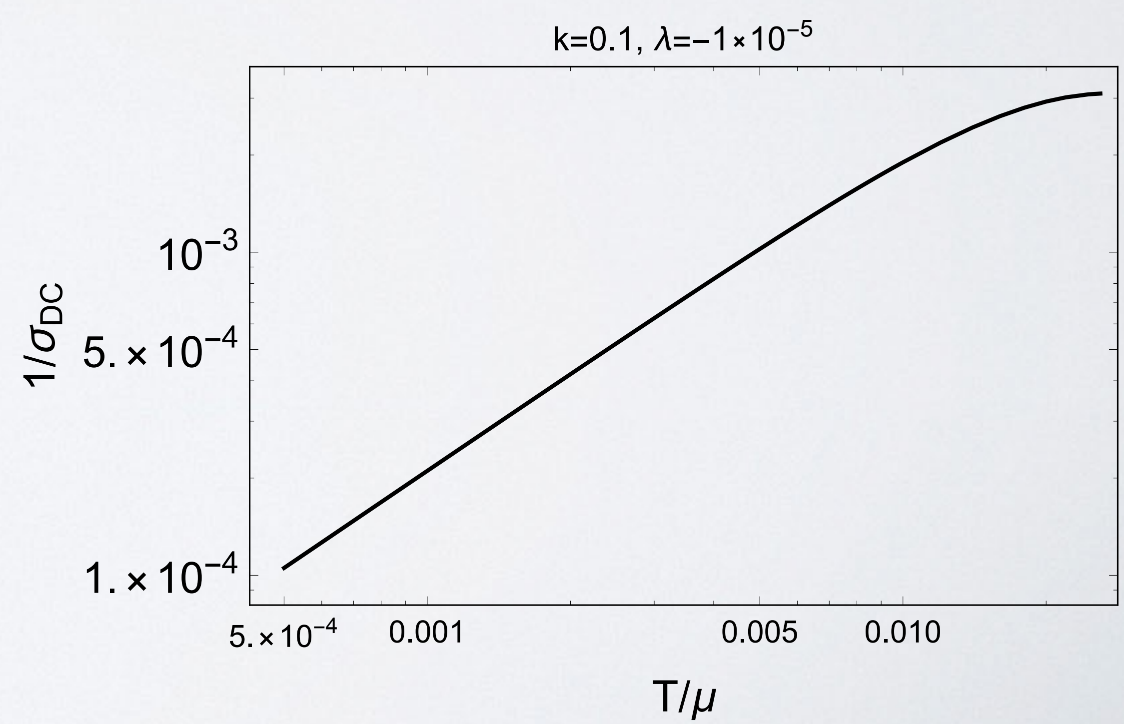

# Quantum Critical regions w/ pseudo-$\langle$~~translations~~$\rangle$

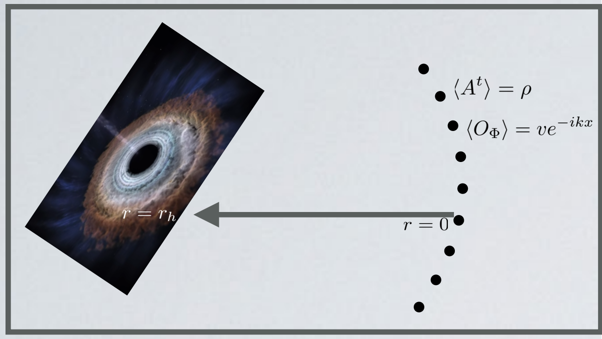



$$\boxed{s \sim T}$$

$$\boxed{1/\sigma_{\mathrm{DC}} \sim T}$$





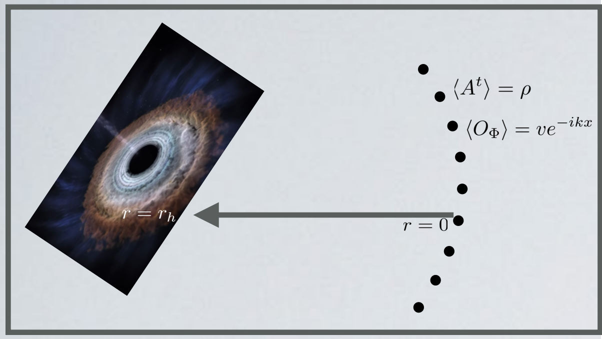
## > AC conductivity

$$\boxed{\sigma(\omega) = \frac{i}{\omega} G^R_{JJ}(\omega, q=0)}$$

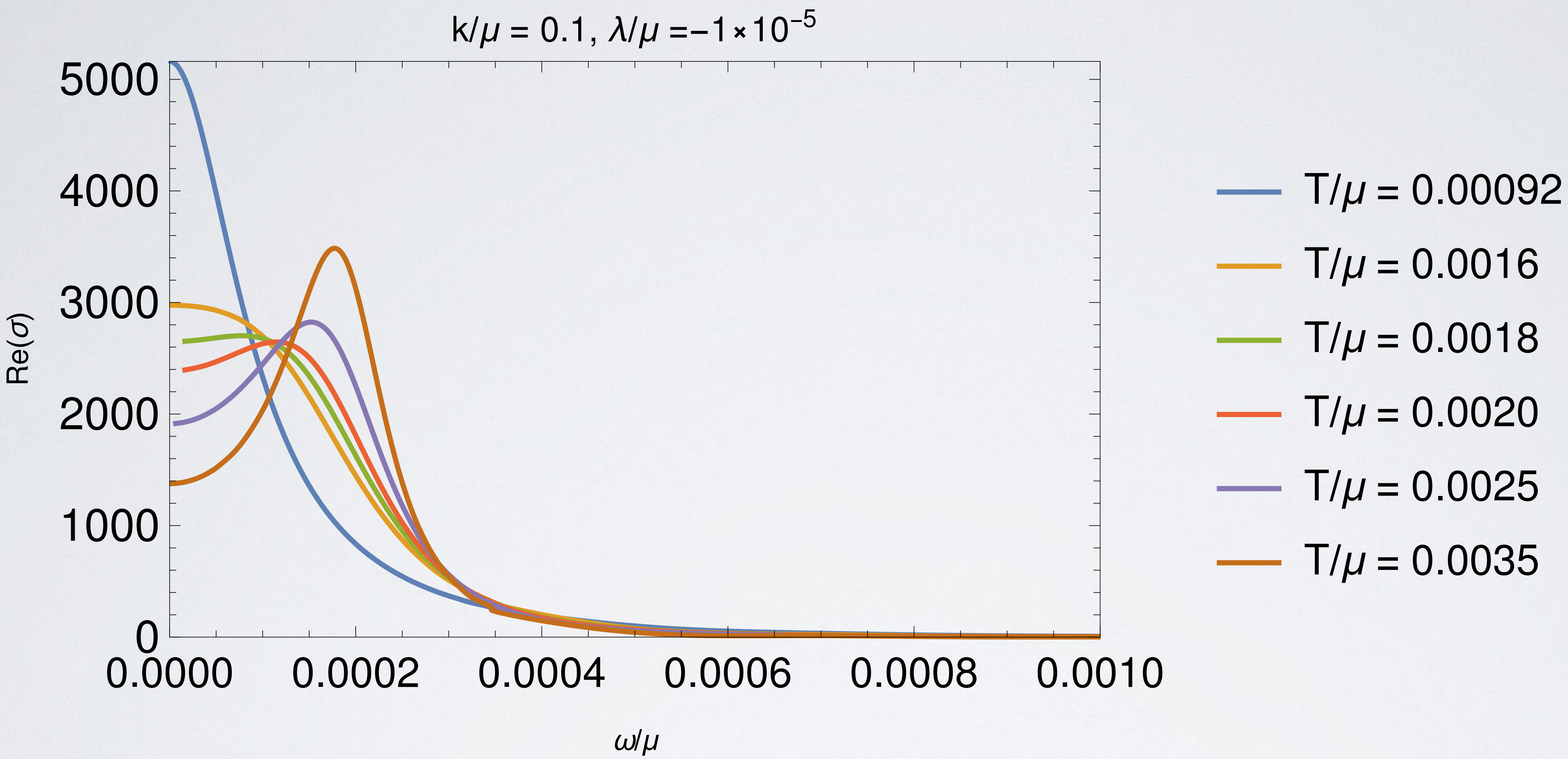
In the spontaneous case (see Andrea's talk) $\sigma \sim \sigma_{\rm inc} + \dfrac{i}{\omega}$
with $\sigma_{\rm inc} \sim T$

In the 'pseudo' case the Goldstone is gapped. Expect no $\omega=0$ pole, and $\sigma_{DC} \sim \sigma_{\rm inc} \sim T$ (see hydro model 1702.05104)

Let's compute $\sigma_{AC}$ (numerics needed) and see if it agrees with the hydro model of a pinned-CDW




# > AC conductivity



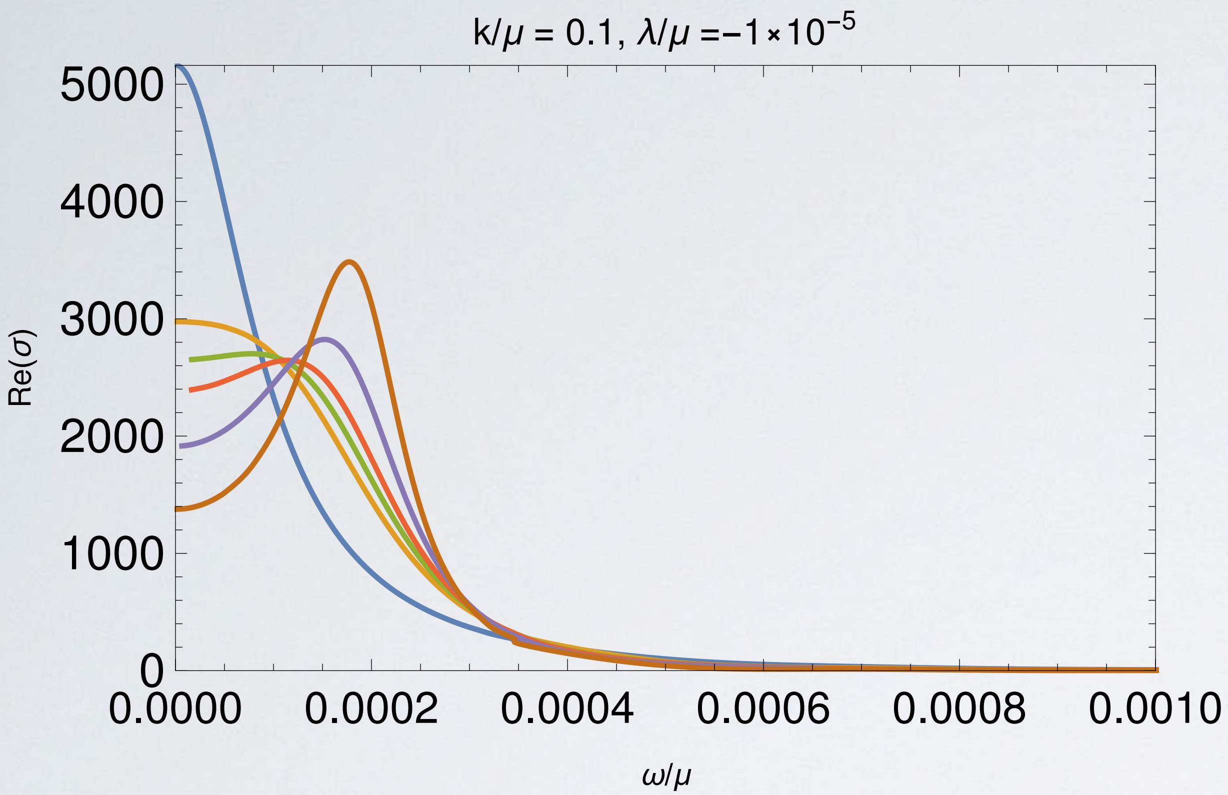

1) Breaking is pseudo-spontaneous: source ~ $10^{-5}$ << vev ~ 0.1

2) Resistivity linear in T up to T/μ ~0.005

3) $\sigma_{DC}$ finite and Drude peak moves off-axis as T is increased!

[See also 1708.07837, 1708.08306 for holo-pinning]

# > Bad metals from pseudo-spontaneous 'Q-lattices'?



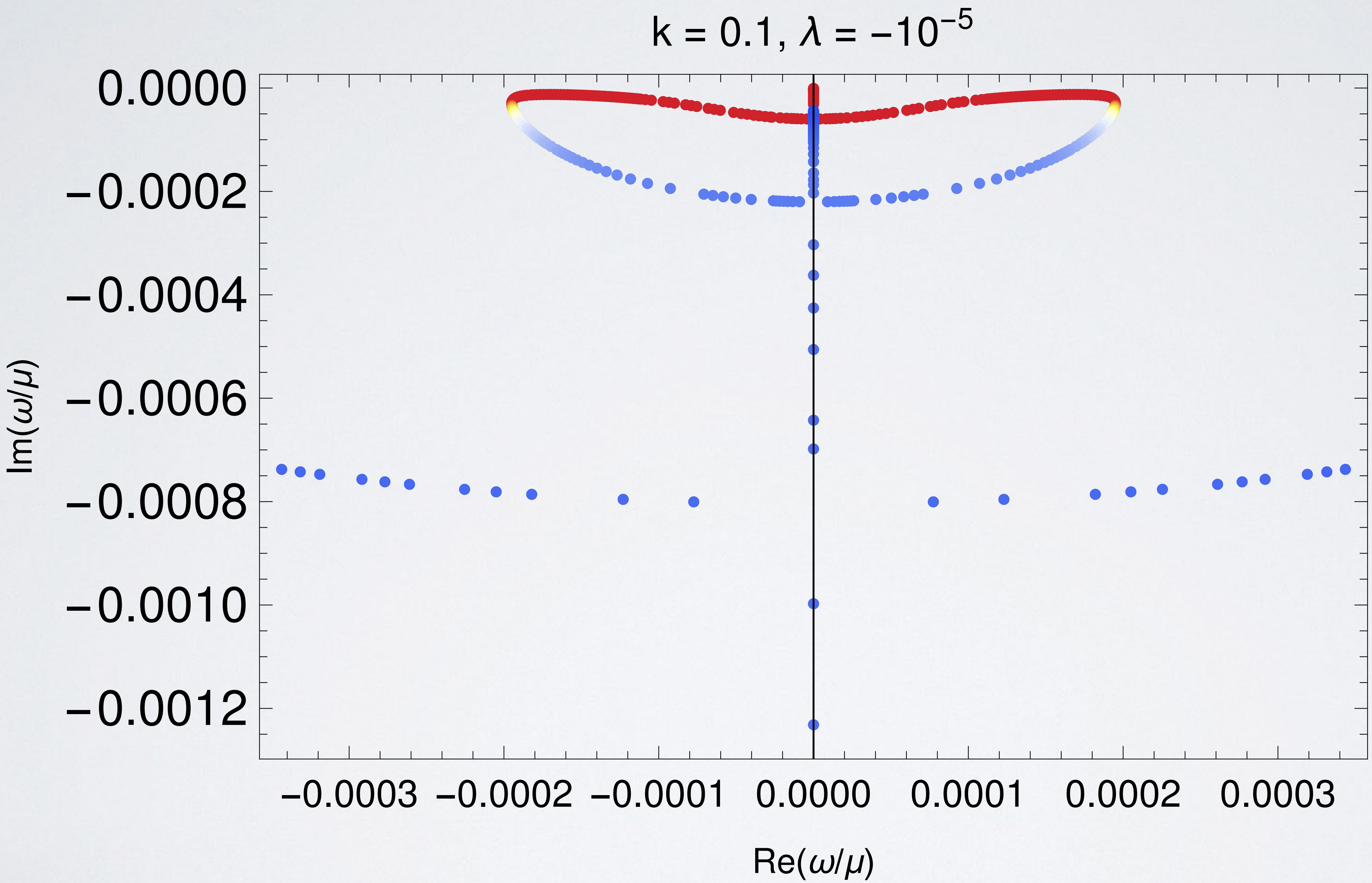

*Described by a pinned CDW?*

$$\sigma(\omega) = \sigma_{inc} + \frac{\rho^2}{\chi_{PP}} \frac{\Omega - i\omega}{(\Omega - i\omega)(\Gamma - i\omega) + \omega_0^2} \,.$$

*Let's compute the QNMs, determine $\Omega$, $\Gamma$, $\omega_0$ and see how it goes...*

# > Bad metals from pseudo-spontaneous 'Q-lattices'?

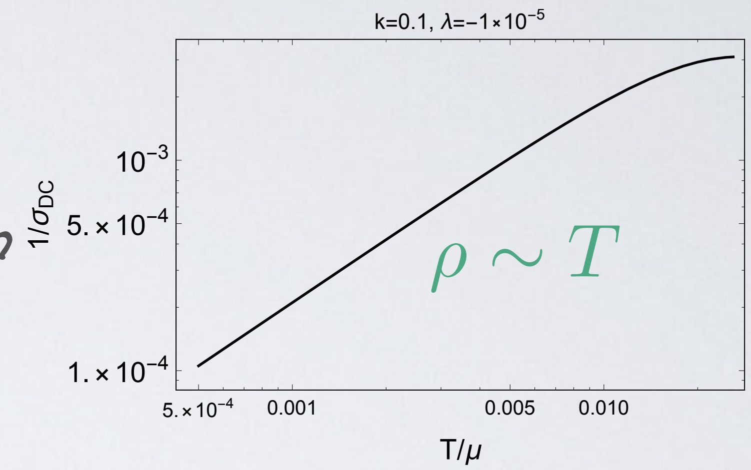
## > QNM spectrum $(\delta A_x,\, \delta g_{tx},\, \delta\psi_x) \sim e^{-i\omega\, t}$

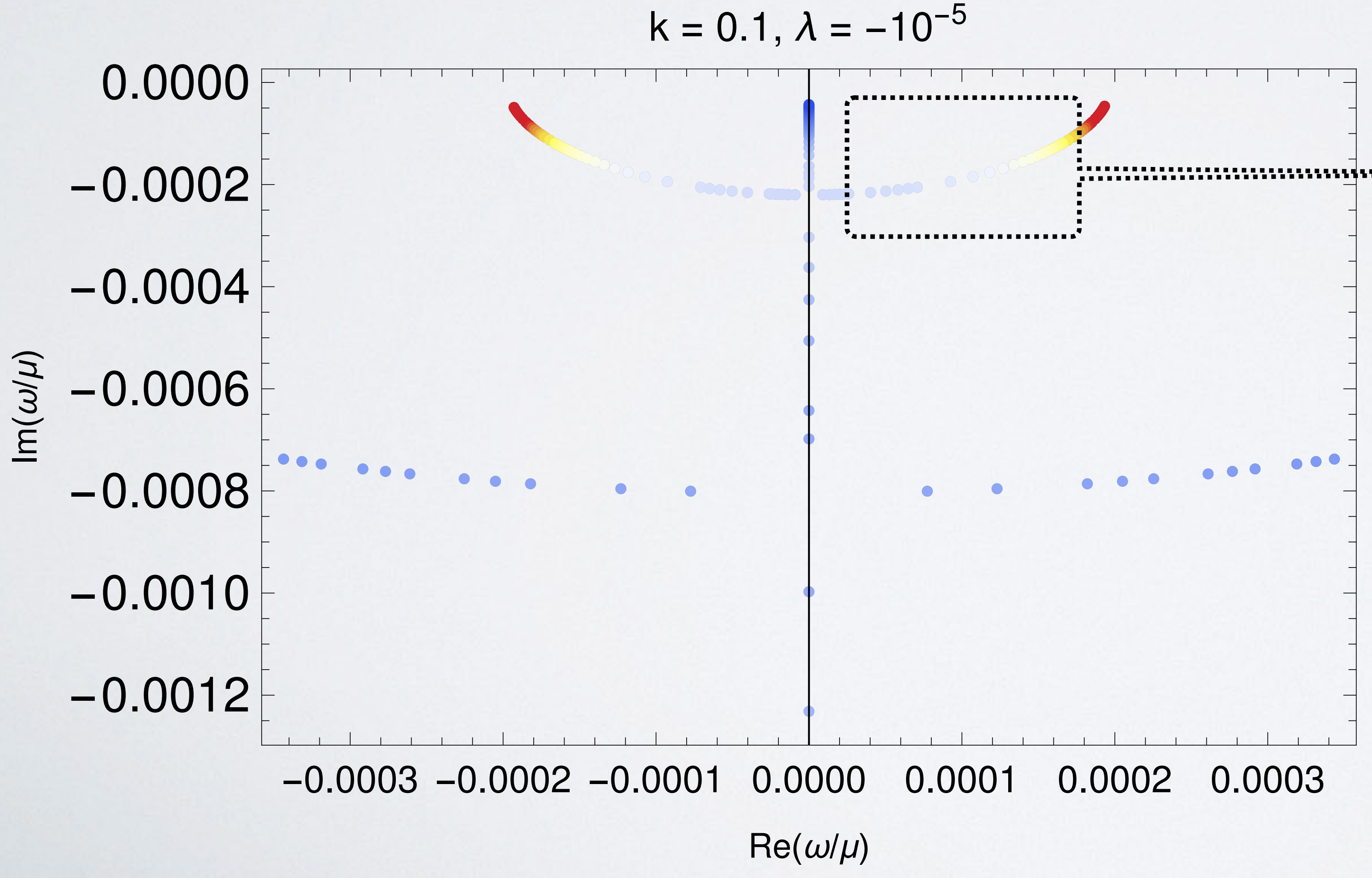


# > Bad metals from pseudo-spontaneous 'Q-lattices'?

## > QNM spectrum $(\delta A_x,\, \delta g_{tx},\, \delta\psi_x) \sim e^{-i\omega\, t}$

*Let's focus on the 'bad metallic' region*

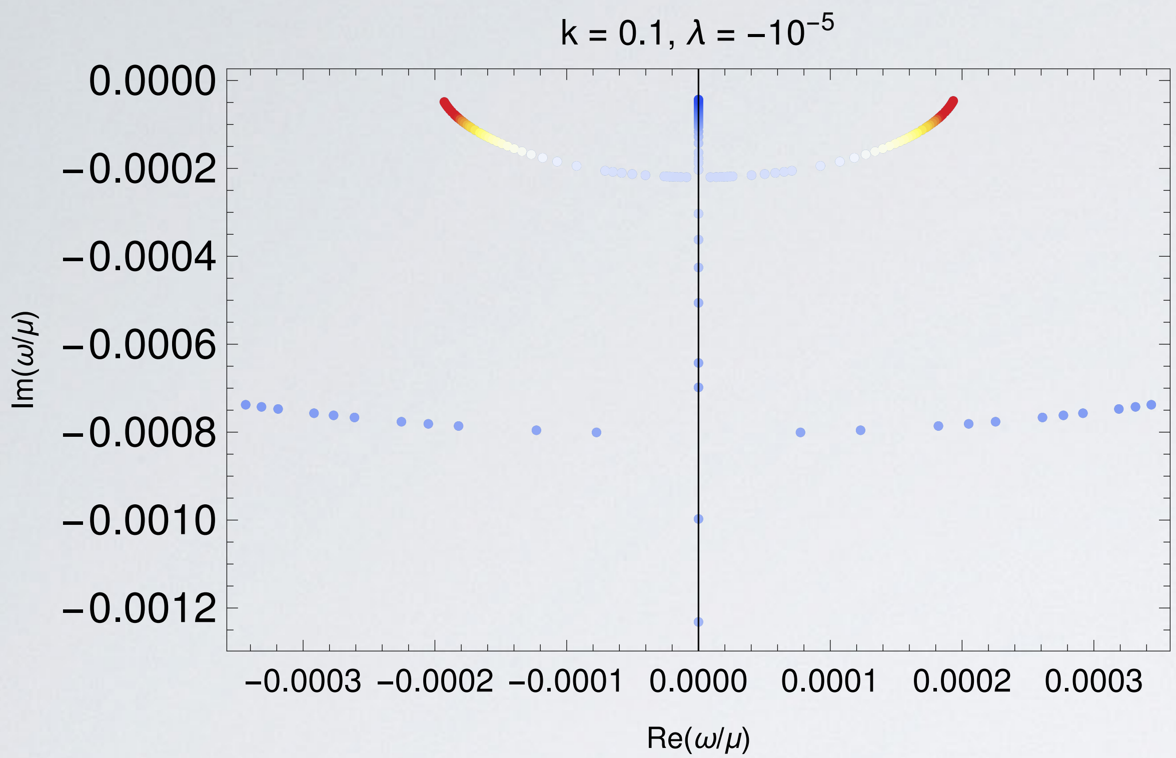



$(\Omega,\, \Gamma,\, \omega_0)$

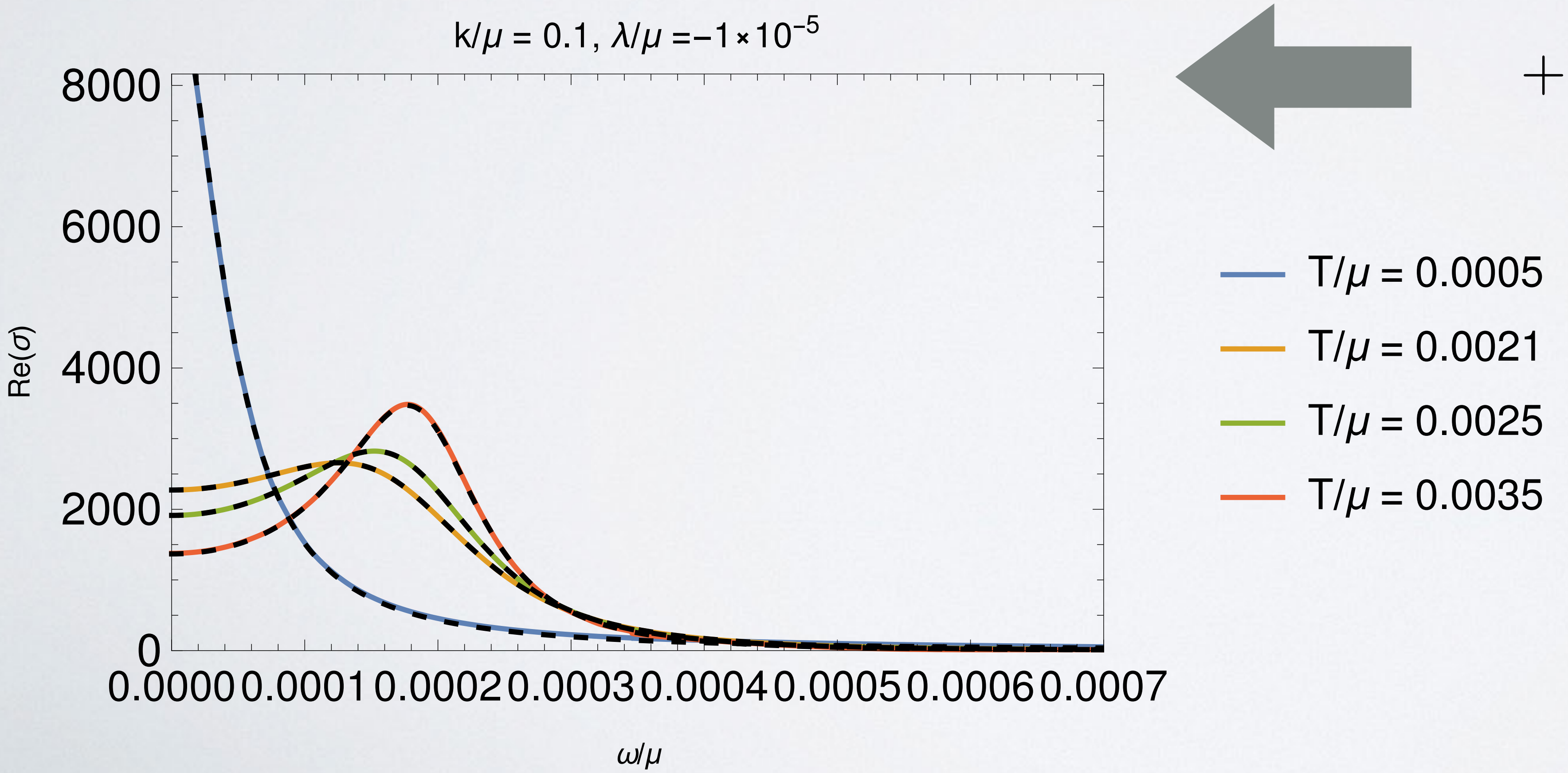
$$\sigma(\omega) = \sigma_{inc} + \frac{\rho^2}{\chi_{PP}} \frac{\Omega - i\omega}{(\Omega - i\omega)(\Gamma - i\omega) + \omega_0^2}$$

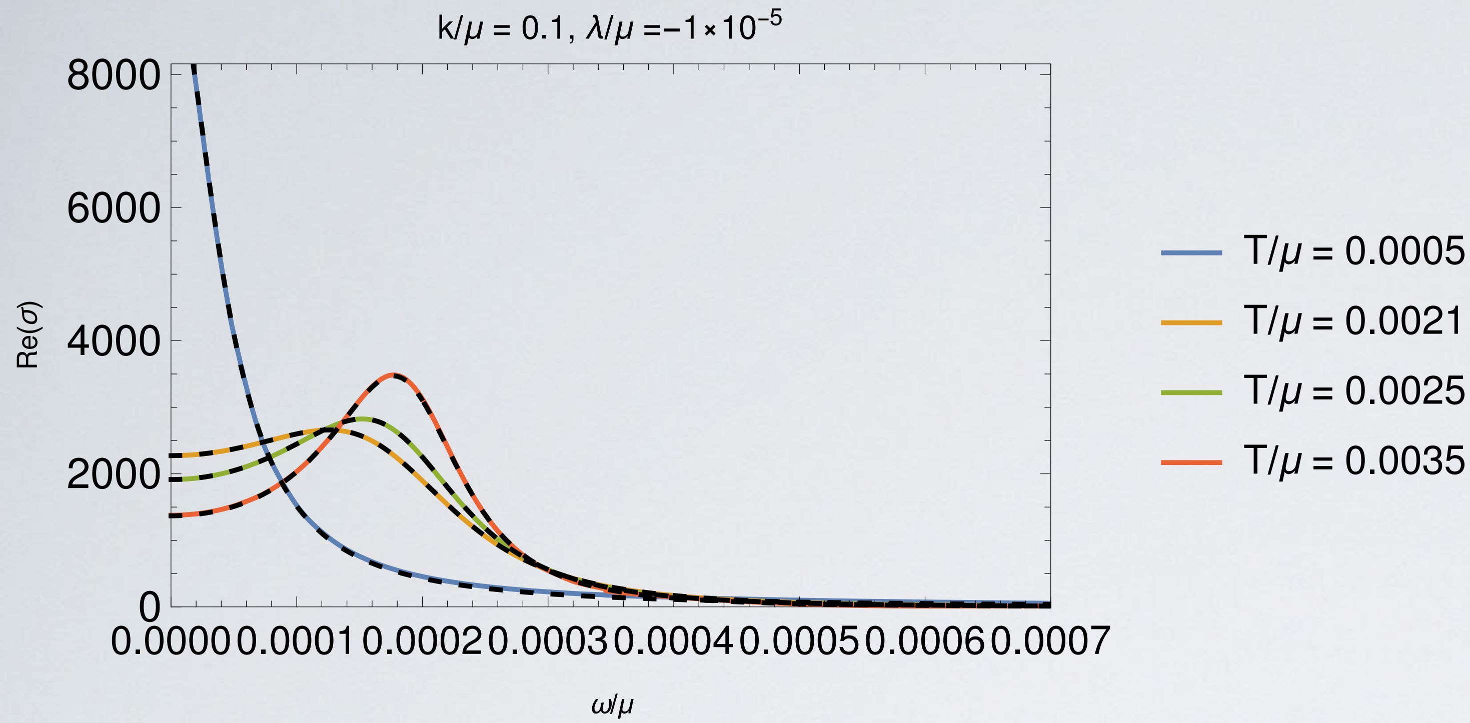
# > Bad metals from pseudo-spontaneous ‘Q-lattices’?




$$(\Omega,\ \Gamma,\ \omega_0)$$

$$\sigma(\omega) = \sigma_{inc} + \frac{\rho^2}{\chi_{PP}} \frac{\Omega - i\omega}{(\Omega - i\omega)(\Gamma - i\omega) + \omega_0^2} \,.$$






Our holographic setup is an effective model of a metallic pinned-CDW !

$$\sigma(\omega) = \sigma_{inc} + \frac{\rho^2}{\chi_{PP}} \frac{\Omega - i\omega}{(\Omega - i\omega)(\Gamma - i\omega) + \omega_0^2} \ .$$

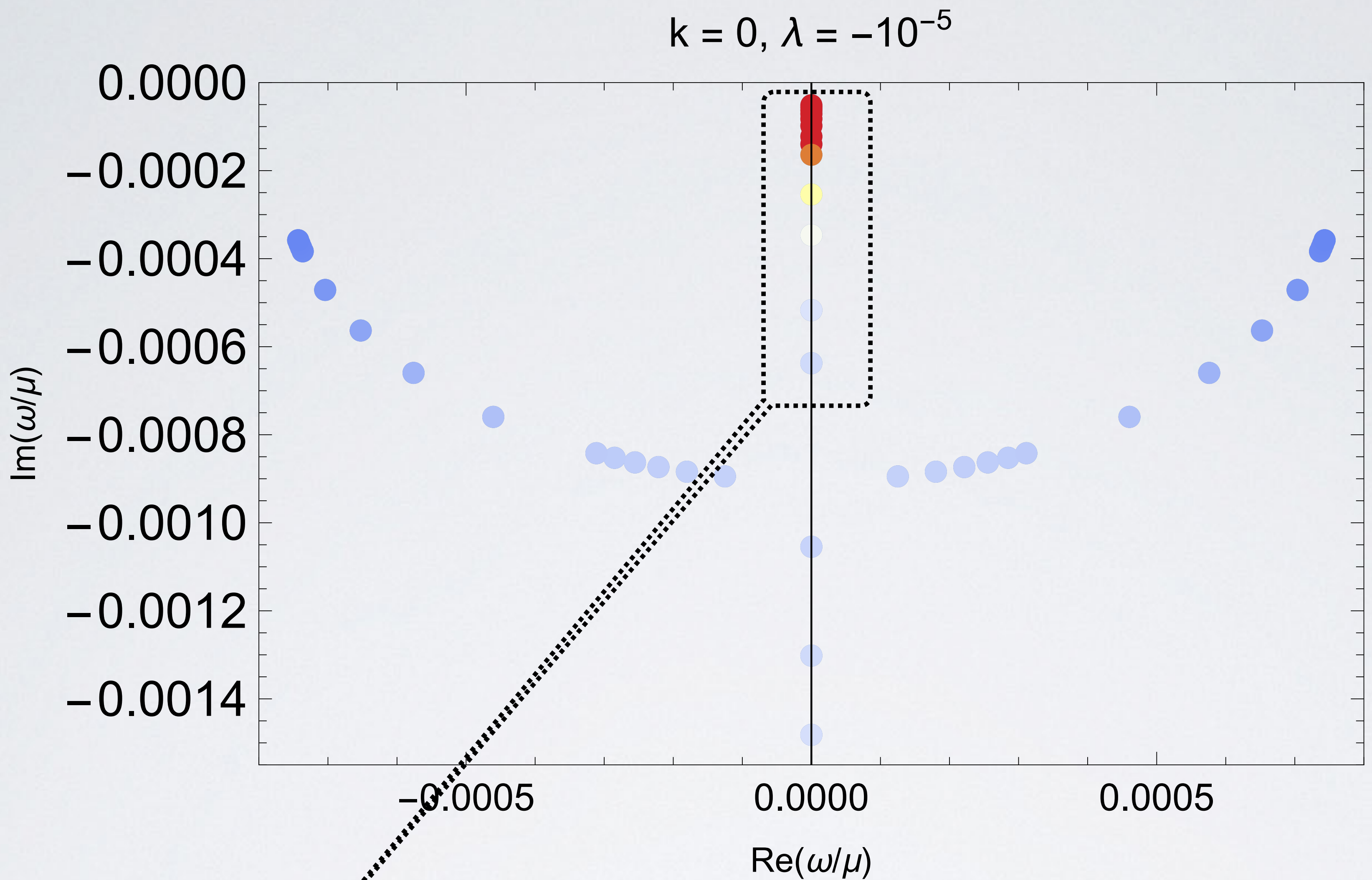
$\Gamma$ : momentum relaxation rate. $\sim (k/\mu)^2$, $\sim (\lambda/\mu)$

[at high and low T agrees w/ memory-matrix $\Gamma \sim k^2 \lim_{\omega\to 0} \mathrm{Im} \frac{G^{\mathrm{R}}_{\psi\,\psi}}{\omega}$ ]

$\omega_0$: 'mass of the phonon'. $\sim \sqrt{(\lambda/\mu)}$ [GMOR], $\sim (k/\mu)$

$\Omega$: $\Longleftrightarrow$ $k=0$ dissipative pole. $\sim (k/\mu)^0$ , $\sim (\lambda/\mu)$, $\sim 1/T$ (at low T)

More on $\Omega$: it doesn't vanish as k->0, where $\psi$ fluctuations decouple... Let's study $\delta\psi \sim e^{-i\omega + q\,x}$ at k=0



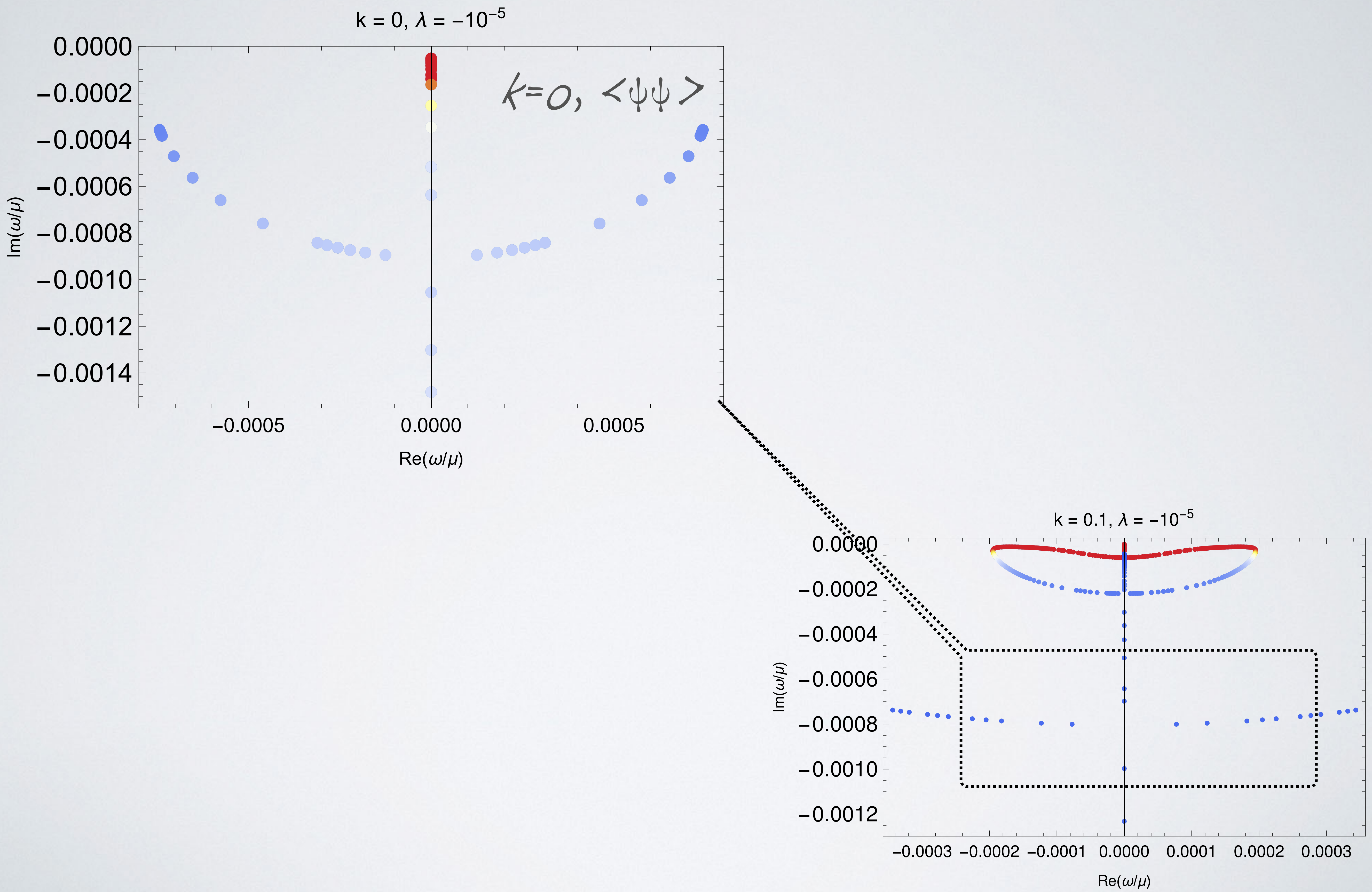

Pseudo-diffusive pole:

$$\langle \psi\,\psi \rangle \sim \frac{1}{\omega + i\tilde{\Omega} + iD\,q^2}$$

$$\tilde{\Omega} \sim 1/T\,,\ \tilde{\Omega} \sim \lambda/\mu$$

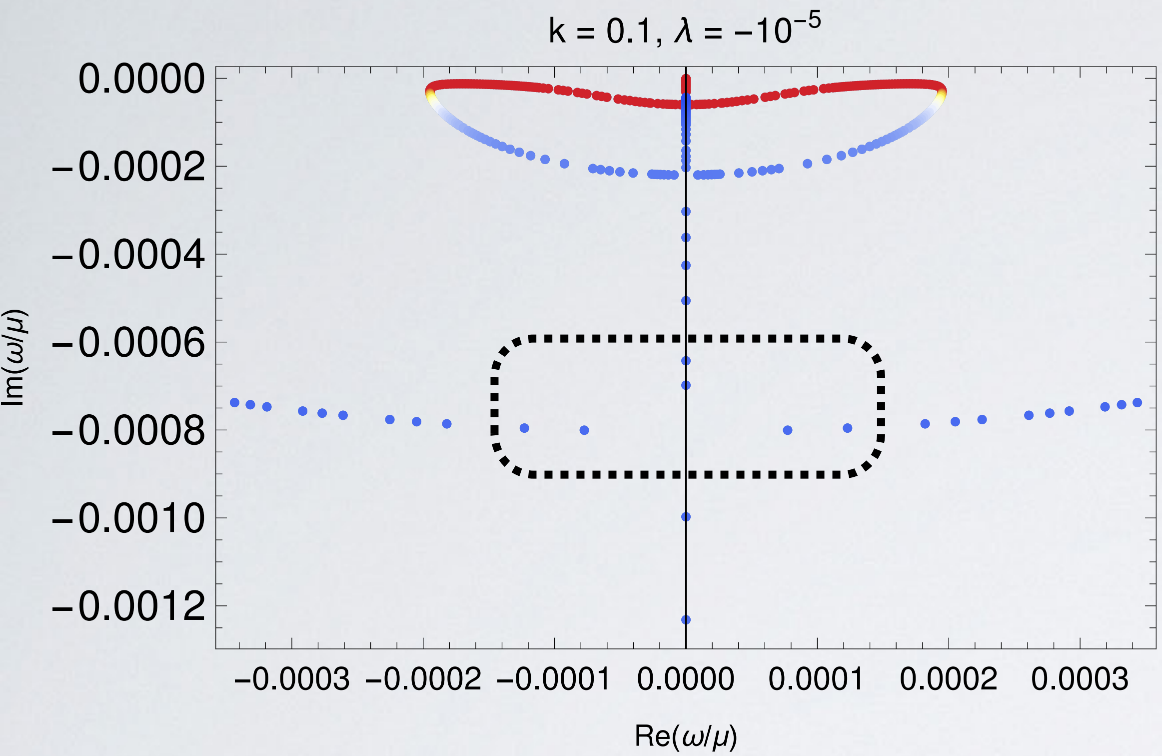
Remember! $\Omega \sim 1/T$

At finite k, the 'Ω-mode' collides w/ 'Drude' as T ↑

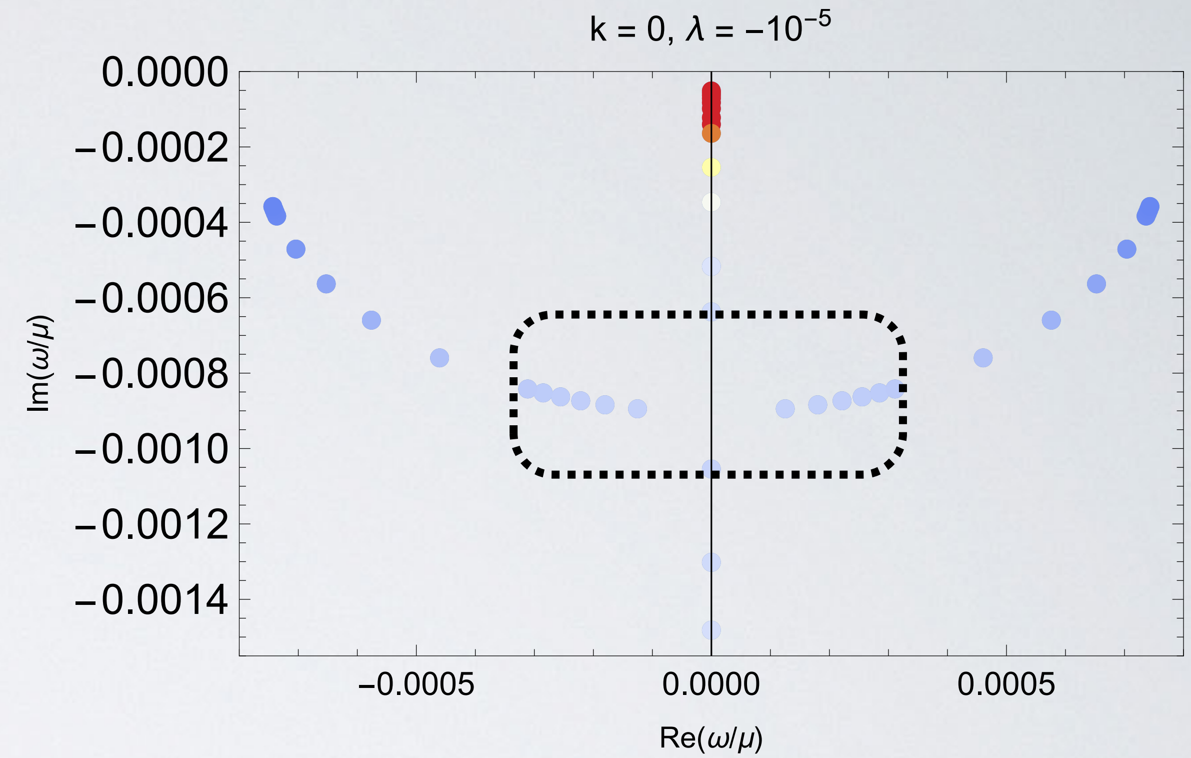


# At finite k and very low T, the k=0 pole 'decouples'

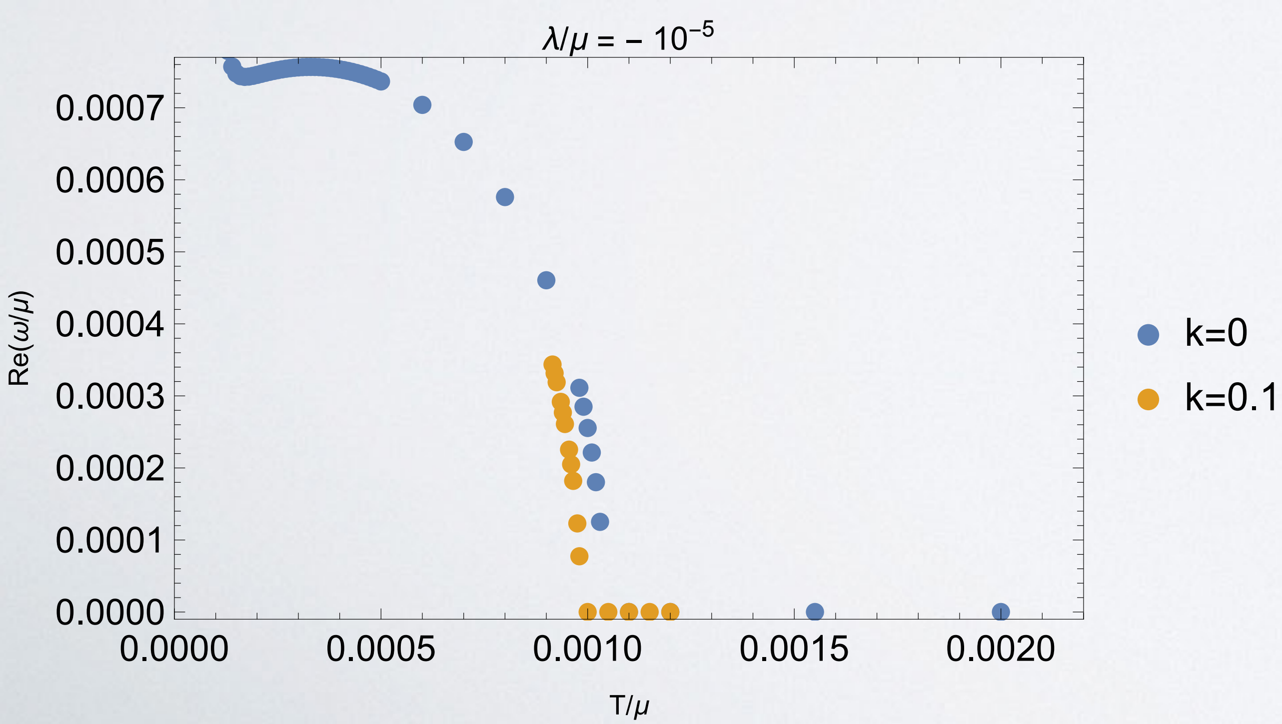
$k/\mu = 0.1$

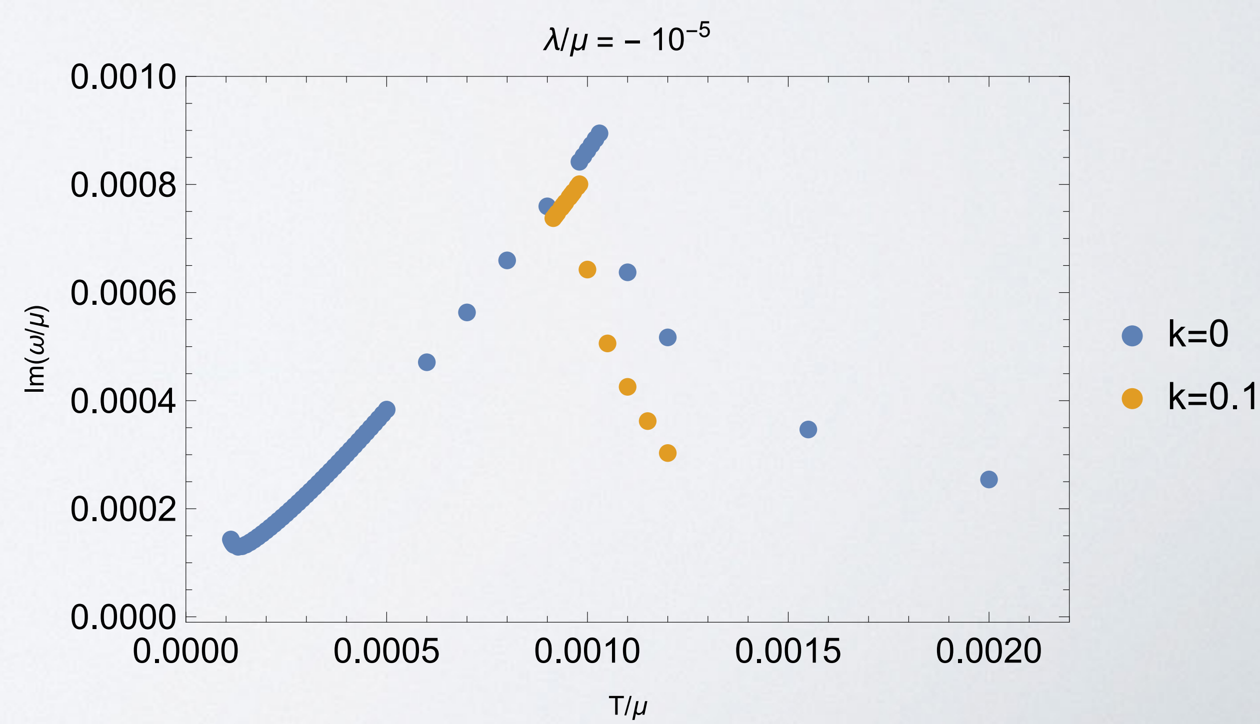



$k/\mu = 0$




## the low T collision barely affected by k (breaking translations)



# > Outlook

We've constructed a holo model breaking translations pseudo-spontaneously

Effective theory of pinned-CDW($\Omega$): metallic, Drude->off-axis

Understand better $\Omega$ (~~global U(1)?~~)

Play w/ parameters to get a larger T-linear region

Go for inhomogeneous, more realistic, models?