From Q-lattices to Bad Metals

with A. Amoretti, B. Goutéraux and D. Musso
[see also 1711.06610 and 1712.07994]

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\[ \langle A^t \rangle = \rho \]
\[ \langle O_\Phi \rangle = ve^{-ikx} \]

\[ \sigma(\omega) = \sigma_{inc} + \frac{\rho^2 \chi_{PP}}{\Omega - i\omega} \frac{\Omega - i\omega}{(\Gamma - i\omega) + \omega_0^2} \cdot \]

Linear resistivity and peak \( \rightarrow \) off-axis as \( T \uparrow \)
Bad metals are challenging

- Resistivity linear in $T$ w/out long-lived quasiparticles (MIR bound)
- Far IR peak in $\sigma_{AC}$ moving off-axis as $T$ is increased.
Bad metals and quantum criticality

- Hints of universality \[1405.3651, 1612.04381\]

\[
\rho \sim T
\]

\[
\omega_{\text{peak}} \sim \frac{1}{\tau_p} \sim \frac{K_B}{\hbar} T
\]

- Spatial Ordering \(\neq 0\) in Pseudo-Gap phase (PDW)
  Residual resistivity upturn below \(T_{\text{order}}\) \[PRL 88 (03 2002) 147003\]
- Off-axis peaks in \(\sigma_{\text{AC}}\) as Quantum Critical CDW \[1612.04381, 1702.05104\]
CALCULABLE (toy-) Model of \( \langle \text{translations} \rangle \) with spontaneous Q-lattices

\[
\langle A^t \rangle = \rho \\
\langle O_\Phi \rangle = ve^{-ikx}
\]

AdS Boundary

\[ r = 0 \]

Charged Black Hole

\[ r = r_h \]

[see Andrea’s talk for more details; 1711.06610 and 1712.07994]
\( \langle \text{translations} \rangle \) with \('\text{pseudo-}\text{Spontaneous Q-lattices}'\)

\[
S = \int d^{d+2}x \sqrt{-g} \left[ R - \frac{1}{2} \partial \phi^2 - \frac{1}{4} Z(\phi) F^2 - V(\phi) - \frac{1}{2} Y(\phi) \sum_{i=1}^{d} \partial \psi_i^2 \right].
\]

\( \Rightarrow \text{ANSATZ} \)

\( \sim \text{AdS charged black hole} \)

\[
ds^2 = -D(r) dt^2 + B(r) dr^2 + C(r) d\bar{x}^2
\]

\( A = A(r) dt \quad \longrightarrow \quad A \sim \mu \quad \text{Chemical potential} \)

\[
\phi = \phi(r) \quad \longrightarrow \quad \Phi_i = \phi e^{ikx^i}
\]

\( \psi_i = kx^i \)
\[ S = \int d^{d+2}x \sqrt{-g} \left[ R - \frac{1}{2} \partial \phi^2 - \frac{1}{4} Z(\phi) F^2 - V(\phi) - \frac{1}{2} Y(\phi) \sum_{i=1}^{d} \partial \psi_i^2 \right]. \]

> **pseudo-Spontaneous Q-lattices**

\[
\langle A^i \rangle = \rho \\
\langle O_\phi \rangle = v e^{-i k x}
\]

\[
\phi = \phi(r) \\
\psi_i = k x^i \\
\Phi_i = \phi e^{i k x^i}
\]

Asymptotically \( O_\phi \sim \phi e^{i k x} \) with source \( \ll v e v \)
\[ S = \int d^{d+2}x \sqrt{-g} \left[ R - \frac{1}{2} \partial \phi^2 - \frac{1}{4} Z(\phi) F^2 - V(\phi) - \frac{1}{2} Y(\phi) \sum_{i=1}^{d} \partial \psi_i^2 \right]. \]

\[ \phi = \phi(r) \]
\[ \psi_i = k x^i \]

\[ \Phi_i = \phi e^{ikx^i} \text{ with source } \ll \text{ vev} \]

> **UV ASYMPTOTICS (~AdS)**

\[ V_{UV} = -d(d+1) + \frac{1}{2} m^2 \phi^2 + \ldots, \quad Z_{UV} = 1 + Z_1 \phi + \ldots, \quad Y_{UV} = Y_2 \phi^2 + \ldots \]

\[ \phi(r \to 0) = \phi_0 r^{d+1-\Delta} + \phi_1 r^\Delta + \ldots, \quad m^2 = \Delta(\Delta - d - 1). \]

> with \[ \phi_0 \ll \phi_1 \]
Quantum Critical ‘Q-lattices’

Hyper-scaling IR solutions found & classified in 1401.5436

Numerically solve ODEs and find BH geometries:

**AdS (UV) -> Scaling (IR):**

\[
\begin{align*}
    ds^2 &= r^\theta \left[-f(r)\frac{dt^2}{r^{2z}} + \frac{L^2 dr^2}{r^2 f(r)} + \frac{d\tilde{x}^2}{r^2}\right] \\
    t &\to \lambda^z t, \quad \bar{x} \to \lambda \bar{x} \quad s \sim T^{\frac{2-\theta}{z}}
\end{align*}
\]

We're interested in the case:

\[
\begin{align*}
    z &\to \infty \\
    \theta &\to -\infty \\
    \tilde{\theta} &\equiv -\theta/z = 1
\end{align*}
\]

\[
\begin{align*}
    s &\sim T, \quad 1/\sigma_{\text{DC}} \sim T
\end{align*}
\]
Quantum Critical ‘Q-lattices’

Hyper-scaling IR solutions found & classified in 1401.5436

Scaling (IR) - Model

\[ S = \int d^{d+2}x \sqrt{-g} \left[ R - \frac{1}{2} \partial^2 \phi^2 - \frac{1}{4} Z(\phi) F^2 - V(\phi) - \frac{1}{2} Y(\phi) \sum_{i=1}^{d} \partial^2 \phi_i^2 \right] \]

\[ V(\phi) = -6 \cosh \left( \frac{\phi}{\sqrt{3}} \right), \quad Z(\phi) = \exp \left(-\sqrt{3}\phi \right), \quad Y(\phi) = (1 - \exp \phi)^2, \]

We’ve constructed numerical (pseudo-)spontaneous solutions

\textit{AdS (UV) -\textgreater Scaling (IR)}
Quantum Critical regions w/ pseudo-\langle \text{translations} \rangle

\[ s \sim T \]

\[ \frac{1}{\sigma_{\text{DC}}} \sim T \]

\[ \langle A' \rangle = \rho \]
\[ \langle O_{\Phi} \rangle = v e^{-ikx} \]
AC conductivity

\[ \sigma(\omega) = \frac{i}{\omega} G_{JJ}^R(\omega, q = 0) \]

In the spontaneous case (see Andrea's talk) \( \sigma \sim \sigma_{inc} + \frac{i}{\omega} \)

with \( \sigma_{inc} \sim T \)

In the 'pseudo' case the Goldstone is gapped. Expect no \( \omega = 0 \) pole, and \( \sigma_{DC} \sim \sigma_{inc} \sim T \) (see hydro model 1702.05104)

Let's compute \( \sigma_{AC} \) (numerics needed) and see if it agrees with the hydro model of a pinned-CDW
AC conductivity

1) Breaking is pseudo-spontaneous: source $\sim 10^{-5} \ll vev \sim 0.1$
2) Resistivity linear in $T$ up to $T/\mu \sim 0.005$
3) $\sigma_{DC}$ finite and Drude peak moves off-axis as $T$ is increased!

[See also 1708.07837, 1708.08306 for holo-pinning]
Bad metals from **pseudo-spontaneous** ‘Q-lattices’?

Described by a pinned CDW?

\[
\sigma(\omega) = \sigma_{inc} + \frac{\rho^2}{\chi_{PP}} \frac{\Omega - i\omega}{(\Omega - i\omega)(\Gamma - i\omega) + \omega_0^2}.
\]

Let’s compute the QNMs, determine \(\Omega, \Gamma, \omega_0\) and see how it goes...
Bad metals from pseudo-spontaneous ‘Q-lattices’?

QNM spectrum \((\delta A_x, \delta g_{tx}, \delta \psi_x) \sim e^{-i\omega t}\)
Bad metals from pseudo-spontaneous ‘Q-lattices’?

QNM spectrum \((\delta A_x, \delta g_{tx}, \delta \psi_x) \sim e^{-i\omega t}\)

Let’s focus on the ‘bad metallic’ region

\[
\rho \sim T
\]

\[
(\Omega, \Gamma, \omega_0)
\]

\[
\sigma(\omega) = \sigma_{inc} + \frac{\rho^2}{\chi_{PP}} \frac{\Omega - i\omega}{(\Omega - i\omega)(\Gamma - i\omega) + \omega_0^2}
\]
Bad metals from **pseudo-spontaneous** ‘Q-lattices’?

\[ k = 0.1, \lambda = -10^{-5} \]

\[ \text{Im}(\omega/\mu) \]

\[ \text{Re}(\omega/\mu) \]

\[ k/\mu = 0.1, \lambda/\mu = -1 \times 10^{-5} \]

\[ \sigma(\omega) = \sigma_{inc} + \frac{\rho^2}{\chi_{PP}} \frac{\Omega - i\omega}{(\Omega - i\omega)(\Gamma - i\omega) + \omega_0^2}. \]
Our holographic setup is an effective model of a metallic pinned-CDW!

\[
\sigma(\omega) = \sigma_{inc} + \frac{\rho^2}{\chi_{PP}} \frac{\Omega - i\omega}{(\Omega - i\omega)(\Gamma - i\omega) + \omega_0^2}.
\]

\(\Gamma\): momentum relaxation rate. \(\sim (k/\mu)^2, \sim (\lambda/\mu)\)

[At high and low T agrees w/ memory-matrix] \(\Gamma \sim k^2 \text{ lim } \omega \to 0 \frac{G_R^{\psi \psi}}{\omega}\)

\(\omega_0\): 'mass of the phonon'. \(\sim \sqrt{(\lambda/\mu)} [GMOR^2], \sim (k/\mu)\)

\(\Omega\): \(\leftrightarrow k=0\) dissipative pole. \(\sim (k/\mu)^0, \sim (\lambda/\mu), \sim 1/T (at low T)\)
More on $\Omega$: it doesn't vanish as $k \to 0$, where $\psi$ fluctuations decouple... Let's study $\delta \psi \sim e^{-i\omega qx}$ at $k=0$

\[
\langle \psi \psi \rangle \sim \frac{1}{\omega + i\tilde{\Omega} + iDq^2}
\]

$\tilde{\Omega} \sim 1/T$, $\tilde{\Omega} \sim \lambda/\mu$

Remember! $\Omega \sim 1/T$
At finite $k$, the $\Omega$-mode' collides w/ 'Drude' as $T \uparrow$
At finite $k$ and very low $T$, the $k=0$ pole 'decouples'.

$k/\mu = 0.1$

$k = 0.1, \lambda = -10^{-5}$

$k/\mu = 0$

$k = 0, \lambda = -10^{-5}$

the low $T$ collision barely affected by $k$ (breaking translations)
Outlook

We've constructed a holo model breaking translations pseudo-spontaneously

\[ k/\mu = 0.1, \lambda/\mu = -1 \times 10^{-5} \]

Effective theory of pinned-CDW(\(\Omega\)): metallic, Drude-\(\rightarrow\)off-axis

Understand better \(\Omega\) (global \(U(1)\)?)

Play w/ parameters to get a larger \(T\)-linear region

Go for inhomogeneous, more realistic, models?