A complexity/fidelity susceptibility g-theorem for $AdS_3/BCFT_2$

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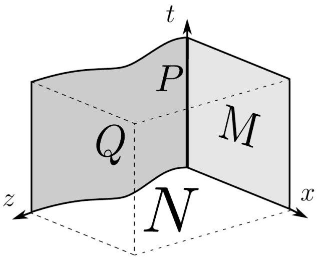
Overview

- Part I: AdS/BCFT
  - (holographic) boundary CFTs
  - A holographic Kondo model
- Part II: Complexity and fidelity susceptibility
  - Definitions
  - For the holographic Kondo model
  - For general $AdS_3/BCFT_2$

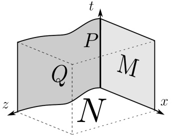
Part I: (holographic) boundary CFTs

- Boundary CFTs: CFTs that live on a space with a boundary, e.g. the half plane. Can be used to describe interaction of CFT with defects.
- Holographic models [Takayanagi 1105.5165]:



N: AdS bulk, *M*: asymptotic (AdS) boundary, *P*: boundary/defect of CFT, *Q*: *dynamic* boundary of spacetime *N*, "*brane*".

Part I: (holographic) boundary CFTs



$$\mathcal{S} = \frac{1}{2\kappa}\int_N d^{d+1}x\,\sqrt{-g}\,(R - 2\Lambda + \kappa\mathcal{L}_M) - \frac{1}{\kappa}\int_Q d^d x\,\sqrt{-\gamma}\,(K + \kappa\mathcal{L}_Q) + S_{c.t.}^{(M,P)}$$

Equation for geometry of Q (similar to *Israel junction conditions* [Israel, 1966]):

$$K_{ij} - \gamma_{ij}K = -\kappa\, S_{ij} \qquad (1)$$

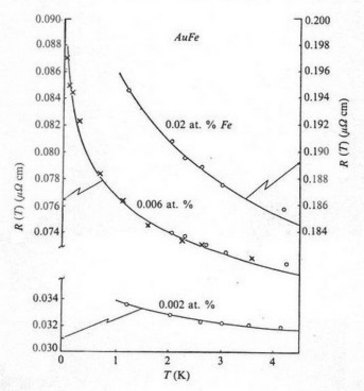
S_{ij}: energy momentum tensor on Q, γ_{ij}: induced metric,

K: extrinsic curvature depending on embedding.

⇒ Embedding (location of the brane Q) will be a dynamical function $x(z)$ with (1) its own equations of motion.

The Kondo model

Spin-spin interaction of electrons with a localised magnetic impurity impacts resistivity [Kondo 1964], at low temperatures electrons form a bound state around impurity, the *Kondo cloud*.



The holographic Kondo model

Spin-spin interaction of electrons with a localised magnetic impurity impacts resistivity [Kondo 1964], at low temperatures electrons form a bound state around impurity, the *Kondo cloud*.

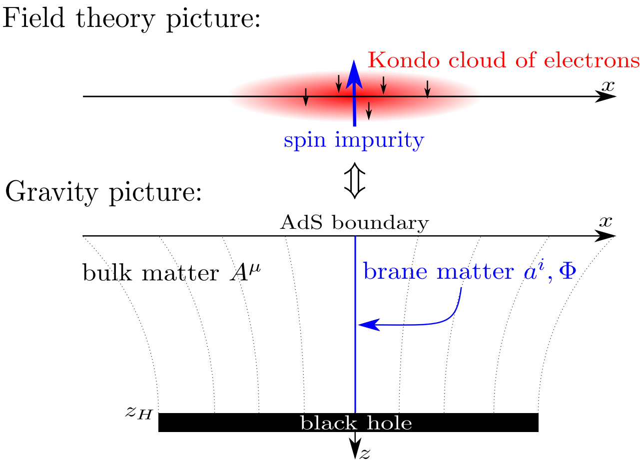
Holographic bottom-up Kondo model [Erdmenger et al. 1310.3271]:
Effectively, a holographic superconductor on Q in $\mathrm{AdS}_3/\mathrm{BCFT}_2$.

RG flow from UV to IR $\Leftrightarrow$ decreasing temperature from $T = T_c$ to $T = 0$
(OR increasing chemical potential $\mu \geq \mu_c$).

g-theorem for *boundary entropy* $\ln g$ [Affleck, Ludwig 1991, Friedan, Konechny 2004]:

$$T\partial_T \ln g(T) \geq 0$$

A holographic Kondo model



$$S = S_{CS}[A] - \int d^3x \delta(x) \sqrt{-g} \left(\frac{1}{4} f^{mn} f_{mn} + \gamma^{mn} (\mathcal{D}_m \Phi)^\dagger \mathcal{D}_n \Phi + V(\Phi^\dagger \Phi) \right)$$

[Erdmenger et al. 1310.3271]

Part II: Complexity and fidelity susceptibility

- A *quantum computer* is to compute an output state $|\psi\rangle$ from a simple input state $|0\rangle$ by implementing an operation $\mathcal{U}$ on the input:

$$|\psi\rangle = \mathcal{U}\,|0\rangle$$

- In practice, this will be accomplished by successively acting on the input with a series of specific *quantum gates* that are selected from a set of allowed operations $\{\mu_i\}$ ("Program" of the computer):

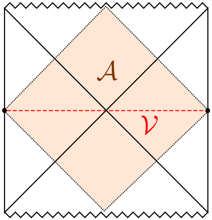
$$|\psi\rangle = \mathcal{U}\,|0\rangle = \mu_1\mu_2\mu_3...\,|0\rangle$$

- The *complexity* of the state $|\psi\rangle$ is the number of quantum gates μ_i that have to be applied in its computation. [Nielsen et al. Science 311, 5764, (2006)]

New entries to the holographic dictionary: Complexity

There are two proposals how to compute complexity $\mathcal{C}$ holographically:

- *Volume proposal:* $\mathcal{C} \propto \frac{\mathcal{V}}{LG_N}$ with Newton's constant G_N, the AdS scale L and the extremal surface volume $\mathcal{V}$. [Susskind Fortsch.Phys. 64 (2016) 24-43]
- *Action Proposal:* $\mathcal{C} = \frac{\mathcal{A}}{\pi\hbar}$ where $\mathcal{A}$ is the action of the bulk gravity integrated over the *Wheeler de-Witt patch*. [Brown et al. PRL 116, 191301 (2016)]



Conformal diagram of a black hole in AdS space. The vertical sides of the square are the conformal boundaries where the dual field theory state is understood to live. The dashed line is an extremal spacelike surface, the shaded region is the Wheeler de-Witt patch.

New entries to the holographic dictionary: Fidelity susceptibility

- *Fidelity susceptibility* $G_{\lambda\lambda}$ measures how much a state (labeled by a parameter λ) changes under variations $\delta\lambda$:

$$\underbrace{|\langle\psi(\lambda)|\psi(\lambda+\delta\lambda)\rangle|}_{\text{"Fidelity"}} = 1 - G_{\lambda\lambda}\delta\lambda^2 + \mathcal{O}(\delta\lambda^3)$$

[Braunstein, Caves, PRL 72, 3439 (1994)]

- If $|\psi(\lambda)\rangle$ is the ground state of a CFT perturbed by a marginal operator $\delta\lambda\mathcal{O}$, then holographically:

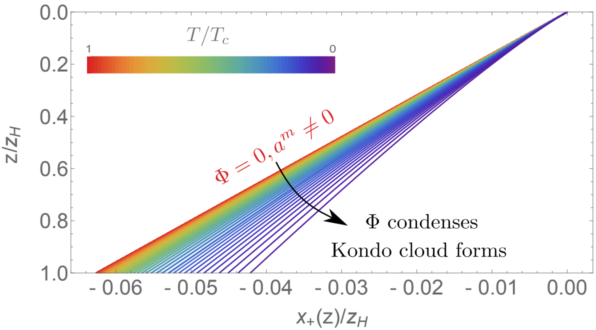
$$G_{\lambda\lambda} \sim \frac{\mathcal{V}}{L^d}$$

with the AdS scale L and $\mathcal{V}$ the volume of an extremal co-dimension one spacelike bulk surface. [Miyaji et al. PRL 115, 261602 (2015)]

Backreaction in the Kondo model

$$S_{brane}[a^m, \Phi] = -\int dV_{brane} \left(\frac{1}{4} f^{mn} f_{mn} + \gamma^{mn} (\mathcal{D}_m \Phi)^\dagger \mathcal{D}_n \Phi + V(\Phi^\dagger \Phi)\right)$$

- Brane starts at boundary and falls into black hole. As T is lowered (resp. μ is raised), it sweeps over the background like a curtain.



[Erdmenger et al.: 1511.03666]

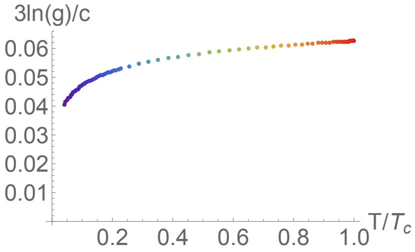
Entanglement entropy in the Kondo model

By calculating the *impurity entropy*

$$\ln g(T) = S_{imp}(T) \equiv S(T)|_{\text{impurity present}} - S(T)|_{\text{impurity absent}},$$

we can verify the g-theorem

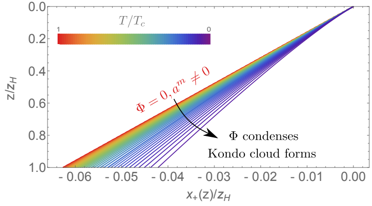
$$T\partial_T \ln g(T) \geq 0.$$

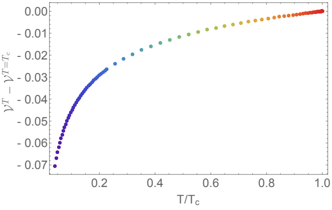


[Erdmenger et al.: 1511.03666]

Complexity and fid. susc. in the Kondo model

Along the RG-flow, *volume* of bulk spacetimes decreases:





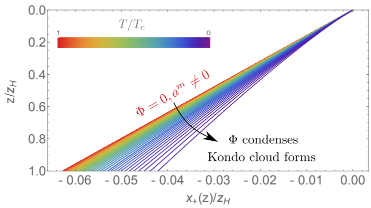
Similar to impurity entropy $S_{imp} \equiv S_{EE}|_{\text{impurity present}} - S_{EE}|_{\text{impurity absent}}$, define *relative complexity*

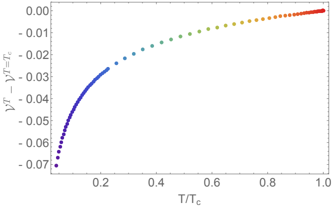
$$\Delta\mathcal{C} \propto \mathcal{V}^T - \mathcal{V}^{T=T_c}$$

with $\mathcal{V}^T$: bulk volume of condensed phase, $\mathcal{V}^{T=T_c}$: bulk volume of uncondensed phase.

[Erdmenger et al.: 1511.03666; MF 1702.06386]

Complexity and fid. susc. in the Kondo model



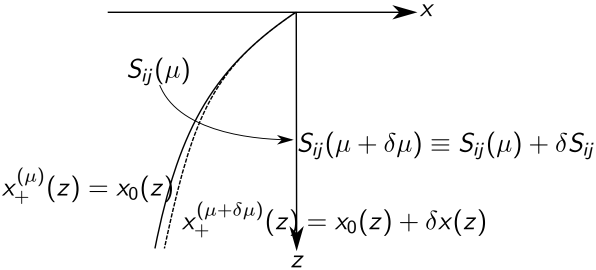


$\Delta\mathcal{C} \propto \mathcal{V}^T - \mathcal{V}^{T=T_c}$, i.e. the holographic Kondo model satisfies a *complexity/fidelity susceptibility analogue* of the g-theorem.

- Volume difference is *finite*, divergencies near AdS boundary cancel!
- Prescription for calculating complexity: Volume vs. action?
- Relation to energy conditions/RG-flow/g-theorem for general AdS/BCFT?

[MF 1702.06386]

Complexity and fid. susc. in general $AdS_3/BCFT_2$ models



- Assume the RG-flow from UV to IR to be described by some parameter μ (T/T_c in Kondo model) which changes the embedding of the brane Q into BTZ ambient spacetime N.
- Assume $S_{ij}(\mu)$ satisfies "NEC" and violates "SEC". [Erdmenger et al. 1410.7811]
- Assume $\delta S_{ij}(\mu)$ satisfies NEC and SEC ("δNEC" and "δSEC").

[MF 1702.06386]

Complexity and fid. susc. in general $AdS_3/BCFT_2$ models

$$\delta SEC \Rightarrow \delta\mathcal{C}(\mu) \propto \mathcal{V}^{\mu+\delta\mu} - \mathcal{V}^{\mu} \leq 0$$

for every μ, hence

$$\delta SEC \Rightarrow \Delta\mathcal{C}(\mu) \propto \mathcal{V}^{\mu} - \mathcal{V}^{\mu_{UV}} \leq 0$$

This suggests the *complexity/fidelity susceptibility analogue* of the g-theorem for $AdS_3/BCFT_2$ models:

When going from the UV to the IR, the complexity/fidelity susceptibility of the BCFT state decreases. States get "simpler".

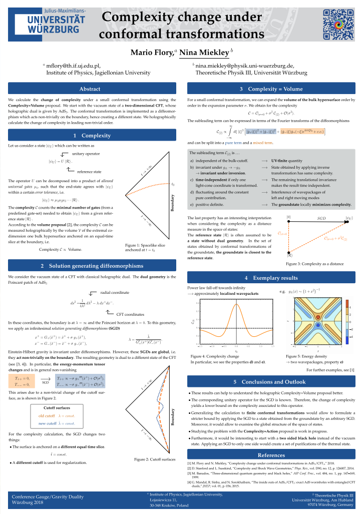
[MF 1702.06386]

Summary

- We studied an $AdS_3/BCFT_2$ model inspired by the Kondo effect and generic $AdS_3/BCFT_2$ models. [Takayanagi 1105.5165]
- We obtained general results constraining possible geometries of the brane by energy conditions. [Erdmenger et al. 1410.7811]
- The specific Kondo model was solved numerically, the Affleck-Ludwig g-theorem was verified. [Erdmenger et al. 1511.03666]
- A *complexity/fidelity susceptibility analogue* of the g-theorem for similar $AdS_3/BCFT_2$ models was proven assuming certain energy conditions. [MF 1702.06386]

Thank you very much for your attention

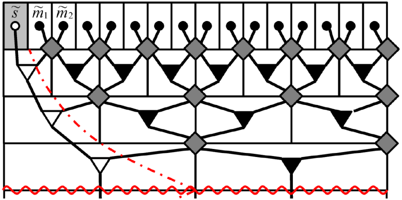
Please also visit the poster of Nina Miekley for our recent joint work

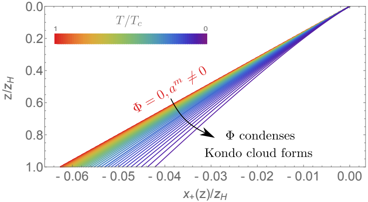


Back up slides...

Comparison to MERA model?

Coincidentally, there exists a *MERA model* of the Kondo effect [Matsueda 1208.2872]:





Curly red line: event horizon, dashed red line: "artificial horizon" $\Leftrightarrow$ our brane?

- How much of the intuition and results gained from the holographic model can be applied to MERA model? (geometry of boundary/brane, g-theorem, coth-formula, volume decrease,...)
- MERA-geometry: AdS or kinematic space?

The top-down Kondo model

Holographic top-down model, brane setup:

	0	1	2	3	4	5	6	7	8	9
N D3	x	x	x	x						
N_7 D7	x	x			x	x	x	x	x	x
N_5 D5	x				x	x	x	x	x	

- $D3/D7$ strings: chiral fermions in 1+1 d $\rightarrow$ electrons ψ_L.
- $D3/D5$ strings: *slave fermions* in 0+1 d $\rightarrow$ impurity spin $\vec{S} = \chi^\dagger \vec{T} \chi$.
- $D5/D7$ strings: tachyonic scalar $\rightarrow$ Formation of Kondo cloud:

$$\langle \mathcal{O} \rangle \equiv \left\langle \psi_L^\dagger \chi \right\rangle \neq 0$$

[Erdmenger et al. 1310.3271]

The top-down Kondo model

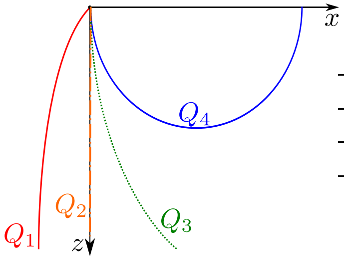
Holographic top-down model, near horizon limit: $D3 \Rightarrow AdS_5 \times S^5$, hence probe $D5 \Rightarrow AdS_2 \times S^4$, probe $D7 \Rightarrow AdS_3 \times S^5$.

Boundary	Bulk
k channels of chiral fermions	$U(k)$ Chern-Simons field
ψ_L	A_μ in AdS_3
slave fermions	Yang-Mills field
$q = \chi^\dagger \chi$	a_m in AdS_2
Operator	charged scalar
$\mathcal{O} = \psi_L^\dagger \chi$	Φ in AdS_2

[Erdmenger et al. 1310.3271]

Energy conditions

Utilising the *barrier theorem* [Engelhardt, Wall: 1312.3699], we can constrain the possible geometries allowed by different energy conditions.



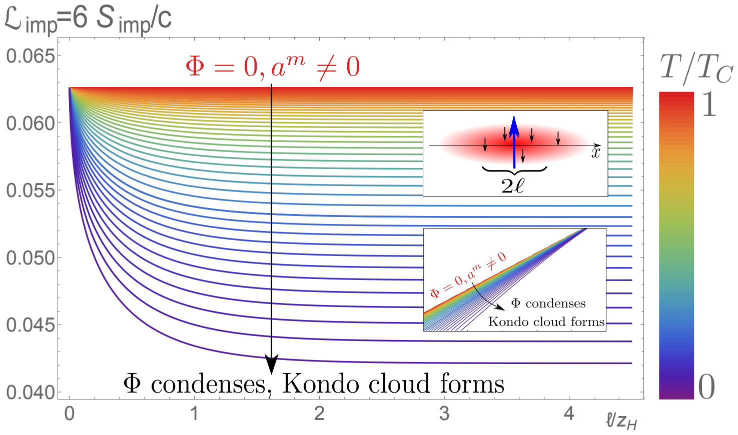
	NEC	WEC	SEC	comment
Q_1	yes	yes	no	
Q_2	yes	yes	yes	$S_{ij} = 0$
Q_3	yes	no	yes	
Q_4	yes	yes	yes	$\cup$ shaped

Whether or not a brane Q bends back to the boundary or goes deep into the bulk depends on whether S_{ij} satisfies or violates WEC and SEC.

[Erdmenger et al. 1410.7811]

Entanglement entropy in the Kondo model

Numerical results on *impurity entropy* $S_{imp} \equiv S_{EE}|_{\text{impurity present}} - S_{EE}|_{\text{impurity absent}}$.



[Erdmenger et al. 1511.03666]