Spontaneous symmetry breaking of translations in holography

Andrea Amoretti

Université Libre de Bruxelles

Based on works with
with D. Arean, R. Argurio, B. Goutéraux,
D. Musso and L. P. Zayas

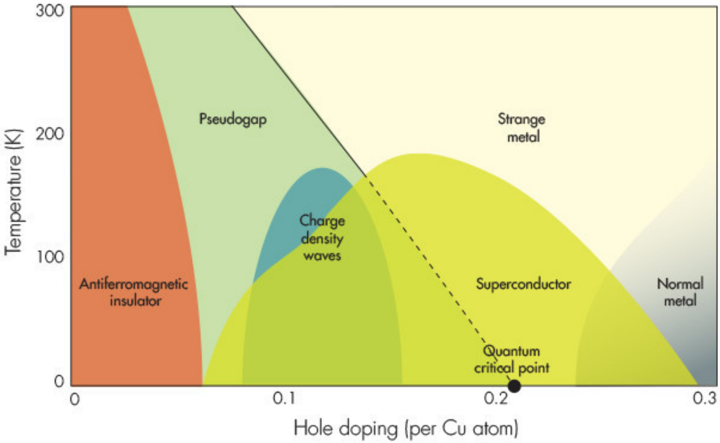
Strange metals phenomenology

Hussey et al. cond-mat/0404263

- Linear in T resistivities exceeding the Mott-Ioffe-Regel (MIR) bound (no long-lived quasi-particles)
- Optical conductivity: far IR peak moving off axis as T increases to room temperature.
- Planckian equilibration rate, given by dimensional analysis Sachdev, Zaanen

$$\tau_{eq} = \frac{\hbar}{k_B T}$$

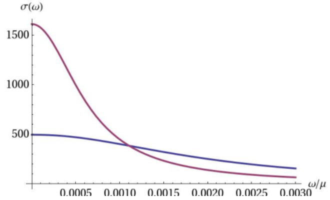
A lot of different intertwined orders



A possible mechanism...

Basov et al. PRL 81; Delacrétaz et al. 1612.04381

- No ever increasing scattering rate, no strong disorder. Rather, a mechanism for decoherence (absence of quasi particle is not sufficient to produce bad metallic behavior)
- Bad metallic behavior is associated largely with the absence of a zero frequency collective mode Hussey et al cond-mat/0404263
  Suppression of low-frequency spectral weight.



Pinned density wave → Gapped peak → Pseudo-Goldstone Bosons

...supported by hydro analysis

- Pseudo-GB mode due to weakly pinned, incommensurate density wave order
- Hydro provides a unified theoretical description of transport in terms of short range, quantum critical fluctuations of incommensurate density wave order Delacrtaz et al. 1702.05104
- Leads to a formula for the conductivity which encodes the gapped peak mechanism:

$$\sigma(\omega) = \sigma_{\text{inc}} + \frac{\rho^2}{\chi_{\pi\pi}} \frac{\Omega - i\omega}{(\Omega - i\omega)(\Gamma - i\omega) + \omega_o^2}$$

"Drude weight"

"Phase relaxation rate" (long-range order disrupted by phase-disordering mobile dislocations)

"incoherent conductivity", roughly the conductivity on the right of the peak

"Pinning", position of the gapped peak

"Momentum relaxation rate"

Technical issues

- Control on the thermodynamic and transport properties of a strongly correlated system (quantum critical fluctuations)
- Control on the low-energy dynamic (EFT) of a system featuring coexistence of spontaneous and explicit symmetry breaking.
- Typically a charge density wave is an insulating system, but we need a methallic one

Let us resort to holography

- As an alternative formulation of QFT we have all the standard techniques and machinery to control the kinetics of pseudo-symmetry breaking
- By means of explicit models and solutions thereof we have also the dynamical information

Breaking of translation in holography

- A lot of work has been done in the explicit case
  - Massive gravity arXiv:1301.0537
  - Linear axions arXiv:1311.5157
  - Q-lattice arXiv:1311.3292
- In the spontaneous case most of the known model are inhomogeneous and difficult to analyse arXiv:1401.5077
- A lot of work has been done in recent times along the homogeneous direction Alberte, Ammon, Andrade, Baggioli, Grozdanov, Jiménez-Alba, Krikun, Poovuttikul, Pujolas...

A homogeneous model to break translations spontaneously

$$S = \int d^{d+2}x \sqrt{-g} \left[R - \frac{1}{2}\partial\phi^2 - \frac{1}{4}\left(Z(\phi) + \lambda_1 Z_2(\phi) \sum_{i=1}^{d} \partial\psi_i^2 \right) F^2 \right.$$
$$\left. - V(\phi) - \frac{1}{2}\sum_{i=1}^{d} \left(Y(\phi)\partial\psi_i^2 + \lambda_2 Y_2(\phi)\partial\psi_i^4 \right) \right]$$

UV potential choice allows for spontaneous symmetry breaking:

$$\phi = \phi(r) \ , \qquad \psi_i = k x_i \ , \qquad Y_{1,2UV}(\phi) \sim \phi^2$$

Asymtpotically the model can be interpreted as perturbing the CFT with a complex scalar

$$\Phi = \phi(r) e^{i\psi}$$

Analogous to CDWs weak coupling effective models
arXiv:1407.4480

A homogeneous model to break translations spontaneously

$$V_{UV} = -d(d+1) + \frac{1}{2}m^2\phi^2 + \ldots, \;\; Z_{1UV} = 1 + Z_1\phi + \ldots,$$
$$Z_{2UV} = Z_2\phi^2 + \ldots \; Y_{1,2UV} = Y_2\phi^2 + \ldots$$

The UV behaviour for Y is crucual to have spontaneous solutions. The scalar ϕ behaves close to the boundary as

$$\phi(r \to 0) = \phi_{(s)} r^{d+1-\Delta} + \phi_{(v)} r^{\Delta} + \ldots\,, \qquad m^2 = \Delta(\Delta - d - 1)\,.$$

If we switch off the source $\phi_{(s)} = 0$, the scalar ψ behaves as

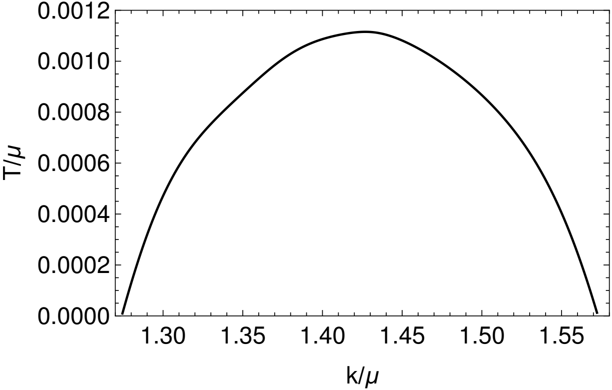
$$\psi_i(r \to 0) = r^{-\Delta}\left(\psi_{i,(s)} r^{d+1-\Delta} + \psi_{i,(v)} r^{\Delta}\right)$$

This shows that the scalars ψ_i have the same scaling dimension as ϕ, with $\psi_{(s)}$ the source and $\psi_{(v)}$ the vev.

Switching off the source for ϕ $\Rightarrow$ spontaneous breaking!

A homogeneous model to break translations spontaneously

The higher derivatives terms allow to construct an instability towards a finite k solution $\Rightarrow$ A genuine spontaneously breaking solution



A homogeneous model to break translations spontaneously

The dual renormalised stress energy tensor reads

$$\langle T^{tt} \rangle = \epsilon = -2\langle T^{xx} \rangle = -2p$$

It is compatible with the one of a solid

$$\langle T^{ij} \rangle = [p - (K - G)\partial \cdot \langle \Psi \rangle]\, \delta^{ij} + 2G\partial^{(i} \langle \Psi^{j)} \rangle$$

provided that there is no phase gradient at equilibrium $\partial \cdot \langle \Psi \rangle = 0$.
(G and K are the bulk and shear moduli)

The Ward Identities are indeed compatible with a spontaneous breaking of translations

$$\langle T^{\mu}_{\mu} \rangle = 0\ , \qquad \partial_\mu \langle T^{\mu\nu} \rangle = 0\ , \qquad \partial_\mu \langle J^{\mu} \rangle = 0$$

A homogeneous model to break translations spontaneously

The electric conductivity in the spontaneous case has a pole at $\omega = 0$ (as it should):

$$\sigma(\omega) = \sigma_{inc} + \frac{\rho^2}{\epsilon + p} \frac{i}{\omega}$$

where σ_0 is the incoherent conductivity

$$\sigma_{inc} = \lim_{\omega \to 0} \frac{G^R_{J_{inc} J_{inc}}(\omega, \vec{k} = 0)}{i\omega} \ , \qquad \langle J_{inc} P \rangle = 0$$

σ_{inc} is thus insensitive to momentum physics and can be expected to reflect universal properties of the QCP.

In our model we found:

$$\sigma_o = \left(\frac{sT}{sT + \mu\rho} \right)^2 \left(Z_{1,h} + 8\pi\lambda_1 k^2 \frac{Z_{2,h}}{s} \right),$$

Scaling IR solutions

It is possible to construct well behaved IR scaling solutions:

$$V = V_0 e^{-\delta\phi}, \quad Z_i = Z_{i,0} e^{\gamma_i \phi}, \quad Y_i = Y_{i,0} e^{\nu_i \phi},$$

$$ds^2 = \xi^\theta \left[\frac{L^2 d\xi^2}{\xi^2 f(\xi)} - f(\xi) \frac{dt^2}{\xi^{2z}} + \frac{d\vec{x}^2}{\xi^2} \right],$$

$$f(\xi) = \left(1 - \frac{\xi^{2+z-\theta}}{\xi_h^{2+z-\theta}} \right), \quad A = a\,\xi^{\zeta - z} dt\,,$$

The solution is scale covariant

$$t \to \lambda^z t\,, \qquad \xi \to \lambda \xi\,, \qquad \vec{x} \to \lambda \vec{x}$$

The entropy density scales accordingly:

$$S \sim T^{\frac{d-\theta}{z}}$$

Scaling IR solutions and metallic behaviour

Imposing the proper constraints on the coefficients σ_{inc} scales as:

$$\sigma_{inc} \sim T^{2+\frac{\zeta+2-2\theta}{z}}$$

- Within the allowed parameter space, the zero temperature resistivity can diverge or vanish: these states can be insulating or conducting.
- This is a non-trivial feature of relaxing Galilean symmetry: in Galilean systems, $\sigma_{inc} = 0$ by symmetry and the CDW is always a dc insulator.
- σ_{inc} controls the conductivity of a CDW also in the presence of a weak explicit momentum dissipation mechanism $\sigma = \sigma_{inc} + \mathcal{O}(\Gamma)$. The story is different in the presence of dislocations, see Daniel's talk

Conclusions

- We have constructed an effective strongly coupled holographic model to mimic the physics of a CDW
  - The model is homogeneous and easy to analyse
  - The Stress-energy tensor is compatible with the one of a solid
  - The transport is in accordance with the hydro picture
  - The DC conductivity is indipendnet on external scattering mechanisms
  - The system can be either metallic or insulating

Thank you