Non-Relativistic Geometry in Limits of Gauge/Gravity Duality

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based on work:

1712.05784 (JHEP) (Hartong,Lei,NO,Oling),

1705.03535 (PRD) (Harmark,Hartong,NO),

1712.03980 (CQG) (Grosvenor,Hartong,Keeler,NO)

1807.04765 (Hansen,Hartong,NO)

to appear (Harmark,Hartong,Menculini,NO,Yan)

Introduction/Outline

this talk is about non-relativistic geometry and its appearance in gauge/gravity duality

- brief intro to non-relativistic geometry: Newton-Cartan geometry and avatars
- limit of AdS3/CFT2 (zooming in to near-BPS bound)
- non-relativistic string theory and limits of AdS5/CFT4
- outlook

punchline: non-relativistic (bulk) theories of gravity play a role in limits of gauge/gravity duality and string theory

more generally: non-relativistic (quantum) gravity as an alternate route towards relativistic quantum gravity

Motivation

gravity/holography

- interesting in own right to find new theories of gravity
  (w. other local symmetries and still diff inv.)
- applications in holography as new bulk theories
  (relevant for non-AdS holography, but also limits of AdS/CFT)
- cosmology
- condensed matter
- (toy) models of quantum gravity, new insights into quantum behavior (BH ?)

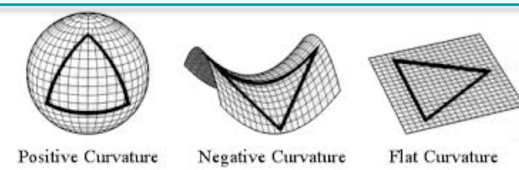
string theory

- tractable limits of string theory
- non-relativistic dispersion relations seen in limits of AdS/CFT
- new string theories ? (non-relativistic sigma models on a non-rel WS)

Space-Time symmetries and Geometry

local symmetries of space and time ←→ geometry of space and time

Einstein: Lorentz ←→ (pseudo-)Riemannian geometry



Einstein equivalence principle:
freely falling observers do not experience gravity
& laws of physics obey special relativity (local Lorentz sym.)

Cartan: Galilean ←→ Newton-Cartan geometry

[Eisenhart,Trautman,Dautcourt,Kuenzle,Duval,Burdet,Perrin,Gibbons,Horvathy,Nicolai,Julia…] ..

- can be used to geometrize Poisson equation of Newtonian gravity
  & more general non-rel gravity: freely falling observers see Galilean laws of physics

Non-Relativistic (NR) Geometry (general view)

very generally: take some symmetry algebra that includes space and time translations and spatial rotations (assume isotropic): "Aristotelian" symmetries
gauge the symmetry and turn space/time translations into local diffeomorphisms

well-known: Poincare -> pseudo-Riemannian geometry (relativistic)

(extra symmetry in this case: Lorentz boost)

→
there are various versions of NR geometry having in common that
they follow similarly from gauging space-time symmetry algebras that include:

$(H \quad , P_a) \quad \& \; G_a$ (Galilean boosts), …..

e.g. Bargmann algebra

Torsional Newton-Cartan geometry

$$\underbrace{\overbrace{H, P_a, J_{ab}, G_a}^{\text{Galilean}} \quad N}_{\text{Bargmann}}$$

(Galilean algebra is c to infinity limit of Poincare)

$$[H, G_a] = P_a \qquad [P_a, G_b] = 0$$

$$[P_a, G_b] = N\delta_{ab}$$

central element N is the mass generator

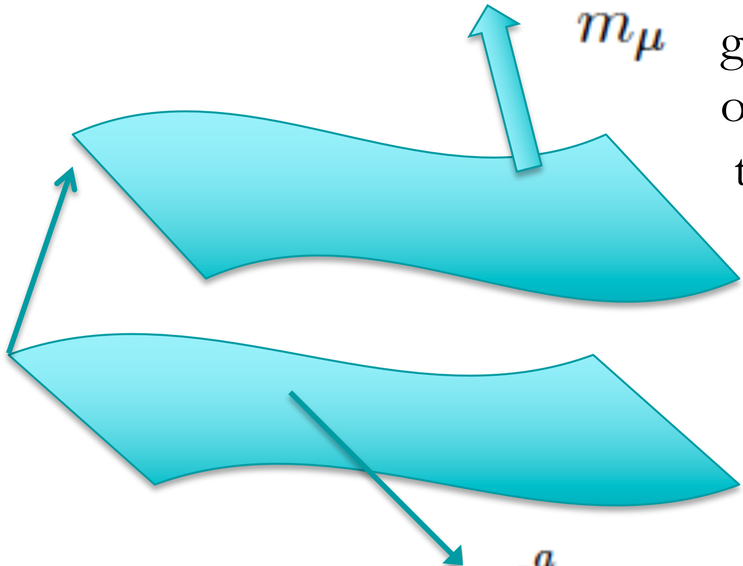
Newton-Cartan geometry: tangent space is Bargmann (central ext. Gal.) invariant

Andringa,Bergshoeff,Panda,de Roo
Bergshoeff,Hartong,Rosseel/Hartong,NO

Note: Bargmann algebra can be obtained from a c → infinity limit (Ionuni-Wigner contraction) of: Poincare x U(1):

$$P_0 = \frac{1}{2}H + \alpha^2 N \qquad Q = -\frac{1}{2}H + \alpha^2 N \qquad \alpha \to \infty$$

torsional Newton-Cartan geometry



gauge field of central extension
of Galilean algebra (Bargmann)
t-component: Newtonian potential

(inverse) spatial metric:

$$h^{\mu\nu} = e^\mu_a e^\nu_b \delta^{ab}$$

(degenerate: rank D-1)

NC geometry = no torsion $\longrightarrow$ $\tau_\mu = \partial_\mu t$ notion of absolute time

TTNC geometry = twistless torsion $\longrightarrow$ $\tau_\mu = \mathrm{HSO}$ preferred foliation in equal time slices

TNC geometry no condition on τ_μ

- in TTNC: torsion measured by $a_\mu = \mathcal{L}_{\hat{v}} \tau_\mu$
  geometry on spatial slices is Riemannian

Different perspectives on TNC geometry

recall: manifestations of Einstein's equivalence principle:

- gauging Poincare
- cosets (Minkowski = Poincare/Lorentz)
- Noether procedure in relativistic field theory

- Torsional Newton-Cartan geometry:
apply equivalence principle to any a Galilean boost structure

- gauging Bargmann

  Andringa,Bergshoeff,Panda,de Roo/Bergshoeff,Hartong,Rosseel/Hartong,NO
- cosets

  Grosvenor,Hartong,Keeler,NO
- Noether procedure in Galilean/Bargmann field theory

  Festuccia,Hansen,Hartong,NO

also:

- Null reductions Julia,Nicolai
- Limits (c → infinity) Bergshoeff,Rosseel,Zojer/van Bleeken/Hansen,Hartong,NO

Non-relativistic FTs couple to TNC geometry

appears as boundary geometry of class of non-AdS Lifshitz spacetimes known as Lifshitz spactimes

$$ds^2 = -\frac{dt^2}{r^{2z}} + \frac{1}{r^2}\left(dr^2 + d\vec{x}^2\right)$$

[Kachru,Liu,Mulligan][Taylor]

characterized by anisotropic (non-relativistic) scaling between time and space

$$t \to \lambda^z t\,, \qquad \vec{x} \to \lambda \vec{x}$$

near boundary: lightcones open up (for z>1) → TNC geometry

Christensen,Hartong,Rollier,NO (1311)
Hartong,Kiritsis,NO (1409)

- more general use in non-relativistic field theory:

- use background field methods for systems with non-relativistic symmetries (compute EM tensor, Ward identities, anomalies)
- symmetry principle for construction of effective theories (e.g. Son, 2013)
- covariant formulation of hydrodynamics

[Jensen(2014)] [Kiritsis,Matsuo(2015)], [Hartong,NO,Sanchioni(2016)]

[deBoer,Hartong,NO,Sybesma,Vandoren](2017) ….]

Non-relativistic gravity and AdS/CFT limits

I. Chern-Simons theories on non-relativistic algebras and near-BPS limits of AdS/CFT

II. Non-relativistic strings and limits of AdS/CFT

Chern-Simons theories on non-relativistic algebras

3D Einstein gravity = CS gauge theory
-> insights into classical and quantum properties of theory
holographic dualities with 2D CFTs
black holes

- can 3D non-relativistic gravity theories be reformulated as CS ?
- can we get this from a limiting procedure of AdS3 gravity ?

yes:

→ CS on (alternate real form of) Newton-Hooke: provides 3D CS theory that can be considered the non-relativistic analogue of AdS3/CFT2

on CFT side: zooming into near-BPS bound Hartong,Lei,NO,Olling(2017)

note: CS on extended Bargmann first considered by: Papageorgiou,Schroers (0907)
- relation to 3D Horava-Lifshitz gravity: Hartong,Lei,NO(2016)
- from non-relatvistic limit of SO(1,2) CS with two U(1) fields added (& SUSY extension): Bergshoeff,Rosseel (2016)

Chern-Simons on NR algebras need extension

CS Lagrangian $$\mathcal{L} = \mathrm{Tr}\left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A\right)$$

need invariant bilinear form -> non-trivial requirement for non. rel algebras (non-semi simple Lie algebras)

Galilean algebra $$[J\,, P_a] = \epsilon_{ab} P_b\,, \quad [J\,, G_a] = \epsilon_{ab} G_b\,, \quad [H\,, G_a] = P_a$$

Bargmann algebra
(central extension = mass generator) $$[P_a\,, G_b] = N\delta_{ab}$$

- 2+1 dim special:

can further extend with a central term $$[G_a, G_b] = S\epsilon_{ab},$$

number of generators: 8

Newton-Hooke (cosmo cst.): $$[G_a\,, G_b] = S\epsilon_{ab}\,, \quad [H\,, P_a] = -\Lambda_c G_a\,,$$ $$[P_a\,, P_b] = \Lambda_c S\epsilon_{ab}\,.$$

Zooming into near-BPS in AdS3/CFT2

any d-dimensional CFT with U(1) flavor symmetry, BPS bound and free coupling constant admits limit in which one zooms in to spectrum close to lightest charged state of theory on $\mathbb{R} \times S^d$

- turn on chemical potential for charge: $D - Q$ has non-negative spectrum
- zoom into the 1-loop corrections of the dilatation operator

→ symmetry algebra is Inonu-Wigner contraction of $so(2, d+1) \oplus u(1)$
leading to algebra with scale but no conformal generators

- for 2D CFT: rotation is abelian and useful to add 2nd U(1) and considering contraction of $so(2,2) \oplus u(1) \oplus u(1)$

i.e. two copies of $sl(2,\mathbb{R}) \oplus u(1)$.

after limit: two copies of P_2^c, two-dimensional centrally extended Poincare

admits infinite dimensional extension: left and right moving warped Virasoro algebra

Bulk and boundary interpretations of algebra

bulk

SO(2,2) AdS3 isometry x U(1)x U(1)

boundary

SL(2,R) x U(1) x SL(2,R) x U(1)

limit:

phase space maps !

3D Newton-Hooke
(alternative real form)

$$P_2^c \oplus P_2^c$$

→ gives rise to CS action that describes:

pseudo Newton-Cartan geometry

$$[\mathcal{P}_a, \mathcal{K}_b] = -2i\mathcal{N}\eta_{ab} - 2i\mathcal{S}\epsilon_{ab}\,,$$

$$[\mathcal{D}, \mathcal{P}_a] = i\mathcal{P}_a\,, \quad [\mathcal{D}, \mathcal{K}_a] = -i\mathcal{K}_a\,,$$

$$[\mathcal{M}, \mathcal{P}_a] = i\epsilon_a{}^b\mathcal{P}_b\,, \quad [\mathcal{M}, \mathcal{K}_a] = i\epsilon_a{}^b\mathcal{K}_b\,.$$

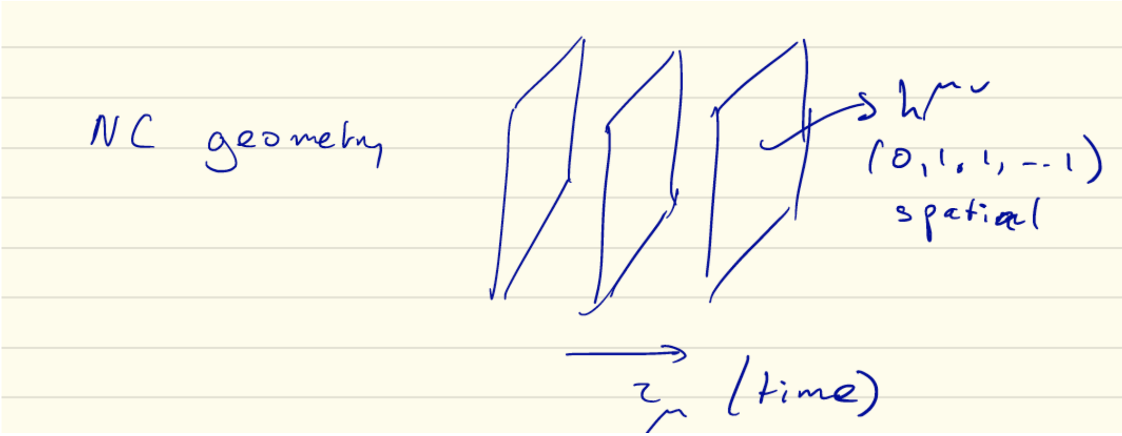
new algebra: scaling non-conformal alg.

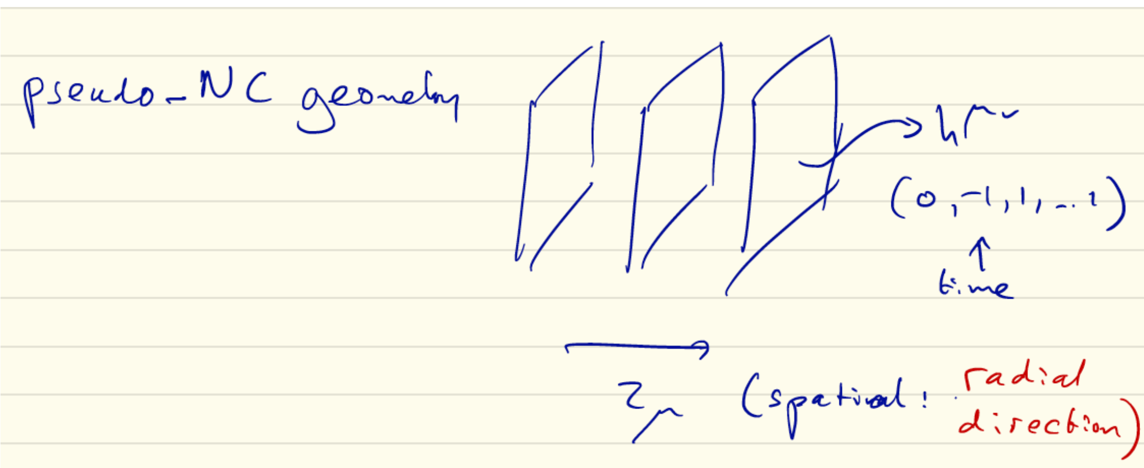
enhances to warped Virasoro:

$$[\mathcal{L}_m, \mathcal{L}_n] = (m-n)\mathcal{L}_{m+n} + 2\pi\gamma_1 m(m^2-1)\delta_{m+n},$$

$$[\mathcal{L}_m, \mathcal{N}_n] = -n\mathcal{N}_{m+n} - 4\pi i\gamma_2 m(m+1)\delta_{m+n}.$$

pseudo NC geometry as holographic spacetime





simple emergence of holographic direction (tau is radial direction)

- holography with non-relativistic bulk gravity dual and new type of CFT on bdry

Other forms of the algebra

- two copies of P_2^c

$$\mathcal{L}_{-1} = \frac{1}{2}(\mathcal{P}_1 + \mathcal{P}_0)\,, \quad \mathcal{L}_0 = \frac{1}{2}(\mathcal{D} + \mathcal{M})\,, \quad \mathcal{N}_0 = \mathcal{N} + \mathcal{S}\,, \quad \mathcal{N}_1 = \frac{1}{2}(\mathcal{K}_1 - \mathcal{K}_0)\,,$$

$$\bar{\mathcal{L}}_{-1} = \frac{1}{2}(\mathcal{P}_1 - \mathcal{P}_0)\,, \quad \bar{\mathcal{L}}_0 = \frac{1}{2}(\mathcal{D} - \mathcal{M})\,, \quad \bar{\mathcal{N}}_0 = \mathcal{N} - \mathcal{S}\,, \quad \bar{\mathcal{N}}_1 = \frac{1}{2}(\mathcal{K}_1 + \mathcal{K}_0)\,.$$

$$\boxed{[\mathcal{L}_{-1}\,,\mathcal{L}_0] = -i\mathcal{L}_{-1}\,, \qquad [\mathcal{L}_{-1}\,,\mathcal{N}_1] = -i\mathcal{N}_0\,, \qquad [\mathcal{L}_0\,,\mathcal{N}_1] = -i\mathcal{N}_1\,.}$$

- different real form of the (complexified) Newton-Hooke algebra

$$\mathcal{T}_a = \frac{1}{2}(\mathcal{P}_a + \mathcal{K}_a)\,, \qquad \mathcal{R}_a = \frac{1}{2}(\mathcal{P}_a - \mathcal{K}_a)\,.$$

$$[\mathcal{T}_a, \mathcal{R}_b] = i\mathcal{N}\eta_{ab}\,, \qquad [\mathcal{M}, \mathcal{T}_a] = i\epsilon_a{}^b\mathcal{T}_b\,, \qquad [\mathcal{M}, \mathcal{R}_a] = i\epsilon_a{}^b\mathcal{R}_b\,,$$

$$[\mathcal{D}, \mathcal{R}_a] = i\mathcal{T}_a\,, \qquad [\mathcal{D}, \mathcal{T}_a] = \frac{i}{l^2}\mathcal{R}_a\,,$$

$$[\mathcal{T}_a, \mathcal{T}_b] = -\frac{i}{l^2}\mathcal{S}\epsilon_{ab}\,, \qquad [\mathcal{R}_a, \mathcal{R}_b] = i\mathcal{S}\epsilon_{ab}\,.$$

Main features

- describes near-BPS sector of CFT (sym algebra is warped Virasoro)
- phase space of limiting theory continuously connected to parent theory
- vacuum is the homogeneous (non-rel) coset $(\mathbf{P}_2^c \times \mathbf{P}_2^c)/(\mathbf{P}_2^c \times U(1))$

→ pseudo Newton-Cartan geometry in some sense mimimal setup of holography (only need to reconstruct foliation structure)

bulk action in 2nd order form: $\mathcal{L} = e\left(h^{\mu\rho}h^{\nu\sigma}K_{\mu\nu}K_{\rho\sigma} - (h^{\mu\nu}K_{\mu\nu})^2 - \tilde{\Phi}\mathcal{R}\right)$ + cosmo constant

new: non-relativistic bulk gravity dual to a relativistic FT

- relation to spin chains: $E = \alpha^{-2}D = \mathcal{N} + \frac{g}{2}\mathcal{D}$, $J = -\alpha^{-2}Q_1 = \mathcal{N} - \frac{g}{2}\mathcal{D}$ $g = \alpha^{-2}$

$\mathcal{N}$ = length of spin chain

$\mathcal{D}$ = one-loop dilation operator

limit $g \to 0$, $(E-J)/g$ fixed

Non-relativistic Strings on Newton-Cartan geometry

- null-reduction of relativistic point particle gives action of (massive) non-relativistic point particle coupling to TNC geometry

→ generalize to: null reducing Polyakov action [Harmark,Hartong,NO](1705)

$$\boxed{\tilde{S} = \int d^2\sigma \left(-P_u^\alpha m_\alpha + \frac{1}{2} P_u^\gamma \tau_\gamma \left(v^\alpha v^\beta - e^\alpha e^\beta \right) h_{\alpha\beta} \right)}$$

- has all local TNC symmetries for P_u = constant
- for $P_u^\sigma = 0,\ P_u^\tau = P = \text{cst}$ and use static gauge $\tau = t$,

- action on flat NC background becomes standard non-rel string which has 1+1 dimensional world volume Poincare sym.

$$S = T \int dt dx \left(\frac{1}{2} \left(\partial_t \vec{Y} \right)^2 - \frac{1}{2} \left(\partial_x \vec{Y} \right)^2 \right)$$

- can be related to non-rel closed strings (Ooguri,Gomis) and string NC geometry

[Andringa,Bergshoeff,Gomis,de Roo (1206)], Bergshoeff,Gomis,Yan(1706) Harmark et al (to appear)

Non-relativistic sigma models from WS scaling limit

take further limit:
tension to zero and compensate by rescaling coupling to time-like vielbein

$$T = \tilde{T}/c, \qquad \tilde{S} = -\frac{P}{2\pi}\int d^2\sigma \left(m_\mu \partial_0 X^\mu + \frac{1}{2} h_{\mu\nu} \partial_1 X^\mu \partial_1 X^\nu \right)$$

non-relativistic WS theory with only 1st order time derivatives

-> target space = U(1)-Galilean geometry (new version of NR geometry)

- specific non-rel. WS limit connects to previously studied limits of AdS/CFT
  Kruczenski (0311) Harmark,Orselli(1409)
- Landau-Lifshitz model (from Heisenberg XXX spin chain)
  is non-rel. ST with NC-like target space geometry

- non-relativistic nature already noticed before:
general magnon dispersion relation [Beisert(0511)] $E = \sqrt{1 + \lambda \sin^2(p)}$

limit $\lambda \to 0$ with $H = (E-1)/\lambda$ fixed gives $H = \frac{1}{2}\sin^2(p)$

Outlook (selected)

zooming into near-BPS of AdS/CFT and non-relativistic gravity

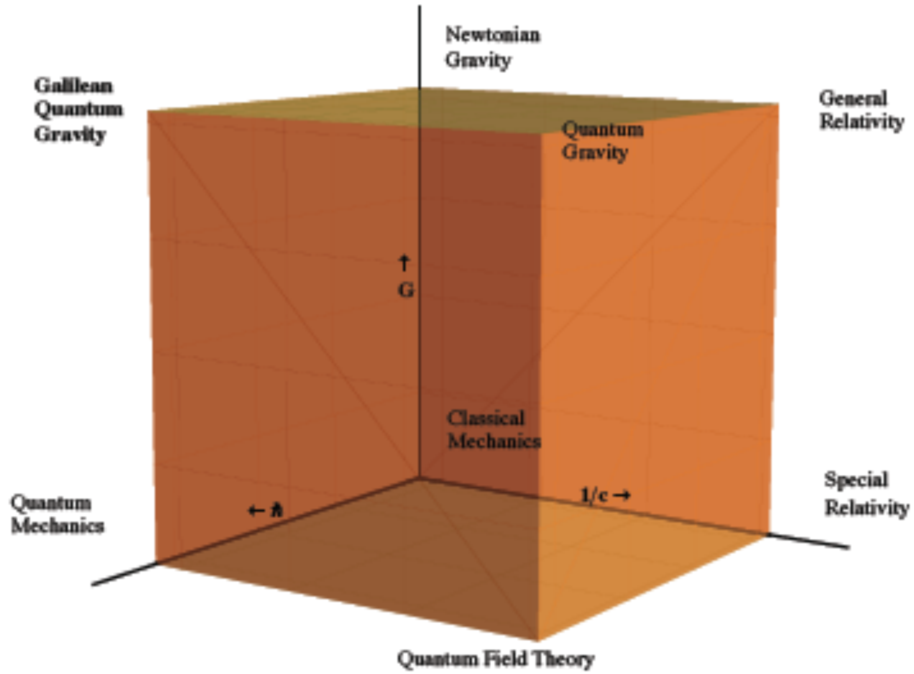
- well-defined limit: can study many well-studied aspects of AdS/CFT
- unitary representations
- WXW model on P2c/Wakimoto rep.
- D1-D5 brane system & N=(2,2) 3D supergravity
- BTZ black hole
- entanglement, higher spin, TMG
- connection with spin chains/spin Matrix theory

non-rel string theory:

- “beta functions” (target space dimension?), dynamics of the target space geometry
- higher-derivative corrections to the sigma model
- adding NS B-field/comparison to other non-rel limits (e.g. [Ooguri,Gomis])
- inclusion of WS fermions
- SUSY on WS/target space
- non-rel limits of DBI

The 7$^{\text{th}}$ corner of the cube of physical theories

$(\hbar, G_N, 1/c)$



so far: no coherent approach to describe the corner ”non-relativistic quantum gravity”

already interesting to find out what classical theory is

- new insights in the quantum theories of gravity
- approach relativistic quantum gravity by including 1/c corrections

Action principle for NR gravity (incl. Newtonian gravity): Hansen,Hartong,NO (1807)