Holographic reconstruction of asymptotically flat spaces

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Main Question

To what extend can one extrapolate/mimic the AdS/CFT recipe to "flat space holography"?

Outline

1. AdS/CFT
   - Symmetries
   - Geometry on the boundary
2. Flat space

Symmetries

Global symmetry on the boundary $\Longleftrightarrow$ local symmetry in AdS

Asymptotic symmetries in the bulk $\Longrightarrow$
gauge fields on the boundary $\Longrightarrow$
conserved currents and symmetries in the boundary QFT.

Geometry and symmetries

Stress-energy tensor T^{ij} encodes spacetime symmetries!

$$J^i = T^i{}_j \xi^j_{\mathsf{CKV}} \implies \nabla_i J^i = 0.$$

Why all symmetries can be encoded in the single object: stress-energy tensor?
Because all the geometry is encoded in a single geometric entity: the metric.

AdS	✓				
Lifshitz	X	Higher spins	X	Flat space	X

The null boundary of flat space cannot be described using Riemannian geometry! $\implies$ No stress-energy tensor!

What is the right notion of geometry on the boundary of flat space? What is the analog of the stress-energy tensor?

Geometry and symmetries

Proposed approach

Use first order formalism, since it is well adapted to spacetimes with non-Riemannian geometry on the boundary.

Why?

- Makes geometry on the boundary explicit; provides a notion of covariance.
- Makes symmetry on the boundary explicit.
- Is easy to work with.

Asymptotically AdS space

Gauge fix:

$$E_r^{\hat{r}} = -1, \quad E_r^{\hat{a}} = 0, \quad \Omega_r^{\hat{A}\hat{B}} \sim 0.$$

General residual local transformation in the bulk is parametrized by

$$\xi^{\hat{a}}(x), \quad \lambda^{\hat{a}\hat{b}}(x), \quad \lambda_D(x), \quad \lambda_K^{\hat{a}}(x)$$

Fall-off behaviour of the fields is

$$\begin{aligned} E_i^{\hat{a}} &= e^r e_i^{\hat{a}} + e^{-r} f_i^{\hat{a}} + \dots, \\ E_i^{\hat{r}} &= b_i, \\ \Omega_i^{\hat{a}\hat{b}} &= \omega_i^{\hat{a}\hat{b}} + \dots . \end{aligned}$$

Asymptotically AdS space

Under general residual local transformation we get:

$$\delta e_i^{\hat{a}} = \partial_i \xi^{\hat{a}} + \xi_{\hat{b}} \omega_i^{\hat{a}\hat{b}} - \lambda^{\hat{a}\hat{b}} e_{i\hat{b}} + \xi^{\hat{a}} b_i - \lambda_D e_i^{\hat{a}},$$

$$\delta \omega_i^{\hat{a}\hat{b}} = \partial_i \lambda^{\hat{a}\hat{b}} + 2\omega_{i\hat{c}}{}^{[\hat{a}} \lambda^{\hat{b}]\hat{c}} - 4\lambda_K^{[\hat{a}} e_i^{\hat{b}]} + 4\xi^{[\hat{a}} f_i^{\hat{b}]},$$

$$\delta b_i = \partial_i \lambda_D + 2\lambda_K^{\hat{a}} e_{i\hat{a}} - 2\xi^{\hat{a}} f_{i\hat{a}},$$

$$\delta f_i^{\hat{a}} = \partial_i \lambda_K^{\hat{a}} - b_i \lambda_K^{\hat{a}} + \omega_i^{\hat{a}\hat{b}} \lambda_{K\hat{b}} - \lambda^{\hat{a}\hat{b}} f_{i\hat{b}} + \lambda_D f_i^{\hat{a}}.$$

Residual gauge transformations = gauged conformal algebra!

Generator	P_a	K_a	M_{ab}	D
Gauge field	$e_i^{\hat{a}}$	$f_i^{\hat{a}}$	$\omega_i^{\hat{a}\hat{b}}$	b_i
Gauge parameter	$\xi^{\hat{a}}$	$\lambda_K^{\hat{a}}$	$\lambda^{\hat{a}\hat{b}}$	λ_D

Asymptotically AdS space

Instead of

$$S_{\text{on-shell}}[g_{ij}]$$

we have

$$S_{\text{on-shell}}[e_i^{\hat{a}}, f_i^{\hat{a}}, \omega_i^{\hat{a}\hat{b}}, b_i],$$

with

$$P^i_{\hat{a}} = \frac{\delta S_{\text{on-shell}}}{\delta e_i^{\hat{a}}},$$
$$D^i = \frac{\delta S_{\text{on-shell}}}{\delta b_i}, \ldots$$

$\implies$ we have individual conformal currents and sources.

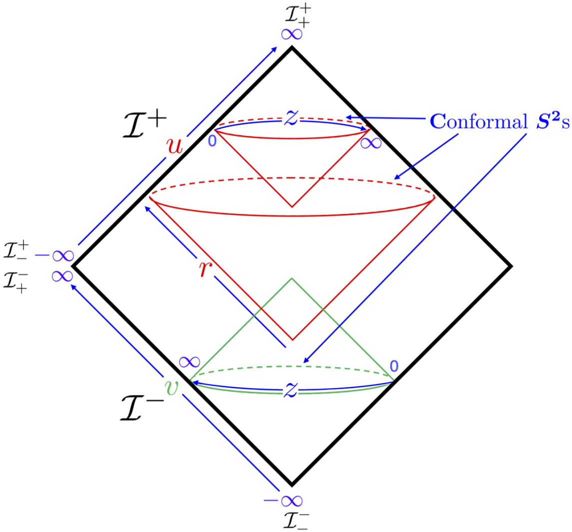
Relation to the usual formulation of CFT

Einstein's equation can be solved asymptotically. They are equivalent to conventional torsion/curvature constraints!

$$\omega_i^{\hat{a}\hat{b}} = \omega_i^{\hat{a}\hat{b}}[e_i^{\hat{a}}, b_i],$$
$$f_i^{\hat{a}} = -\frac{1}{d-2}\left(R_i^{\hat{a}} - \frac{1}{2(d-1)} R e_i^{\hat{a}}\right).$$

b_i is a pure gauge degree of freedom. Can be fixed to zero using $\lambda_K^{\hat{a}}$. This achieves the Fefferman-Graham gauge! We remain with $S_{\text{on-shell}}[e_i^{\hat{a}}]$ and diffeos $\times$ Weyl.

Asymptotically flat space



Asymptotically flat space

The procedure used in asymptotically AdS goes over smoothly:

- Split radial and boundary directions
- Gauge fix radial components
- Compute residual local transformation
- Obtain gauge algebra

Sources and spacetime symmetries

Generator	P_a	$-K_a$	M_{ab}	$-D$	P'	K'	B_a
Gauge field	$e^{\hat{a}}_{(0)i}$	$\omega^{\hat{a}}_{(0)i}$	$\omega^{\hat{a}\hat{b}}_i$	$h_{(0)i}$	b_i	$h_{(1)i}$	$e^{\hat{a}}_{(1)i}$
Gauge parameter	$\Xi^{\hat{a}}_{(0)}$	$\lambda^{\hat{a}}_{(0)}$	$\lambda^{\hat{a}\hat{b}}$	$\Xi^{\hat{u}}_{(0)}$	$\Xi^{\hat{r}}_{(0)}$	$\Xi^{\hat{u}}_{(1)}$	$\Xi^{\hat{a}}_{(1)}$

$10-$dimensional algebra for $3d-$boundary.
(D, M_{ab}, P_a, K_a) form a $(d-1)$-dimensional conformal algebra.
(P_a, P', B_a, M_{ab}) form a Carroll algebra, with B_a playing the role of the Carroll boosts.
This is the Poincaré algebra in anisotropic parametrisation.

Geometry on the boundary of asymptotically flat space

Einstein equations impose algebraic constraints among connections. After removing "pure gauge sources", we end up with

$$b_i(x), \qquad e_i^{\hat{a}}(x), \qquad e_{(1)i}^{\hat{a}},$$

as the minimal geometrical data needed to realize all of the symmetry on the boundary. This data is the flat space analog of the metric on the boundary of AdS.

Source - VEV relation in flat space

Traditionally assumed expansion is highly non-covariant:

$$\begin{aligned} ds^2 = &- du^2 - 2du dr + r^2 \gamma_{ab} d\Theta^a d\Theta^b \\ &+ \frac{2m}{r} du^2 + r C_{ab} d\Theta^a d\Theta^b + \dots \\ &+ \frac{1}{r}\left(\frac{4}{3} N_a + \dots\right) du d\Theta^a + \dots \end{aligned}$$

In our approach: m and N_a combine into a single representation of the asymptotic algebra!

$b_i(x)$ and $h_{(2)i} = (m, N_a)$ form a source-VEV pair!

$C_{ab} \sim e^{\hat{a}}_{(1)i}$ is the gauge field for supertranslations!

Summary

The main points are:

- First order formalism is well adapted to holographic description of asymptotically locally flat spaces.
- Spacetime symmetry on the boundary is made explicit and entirely geometric in the bulk.
- Novel notion of covariance at null infinity.
- Source-VEV relation
- Covariant counterterms