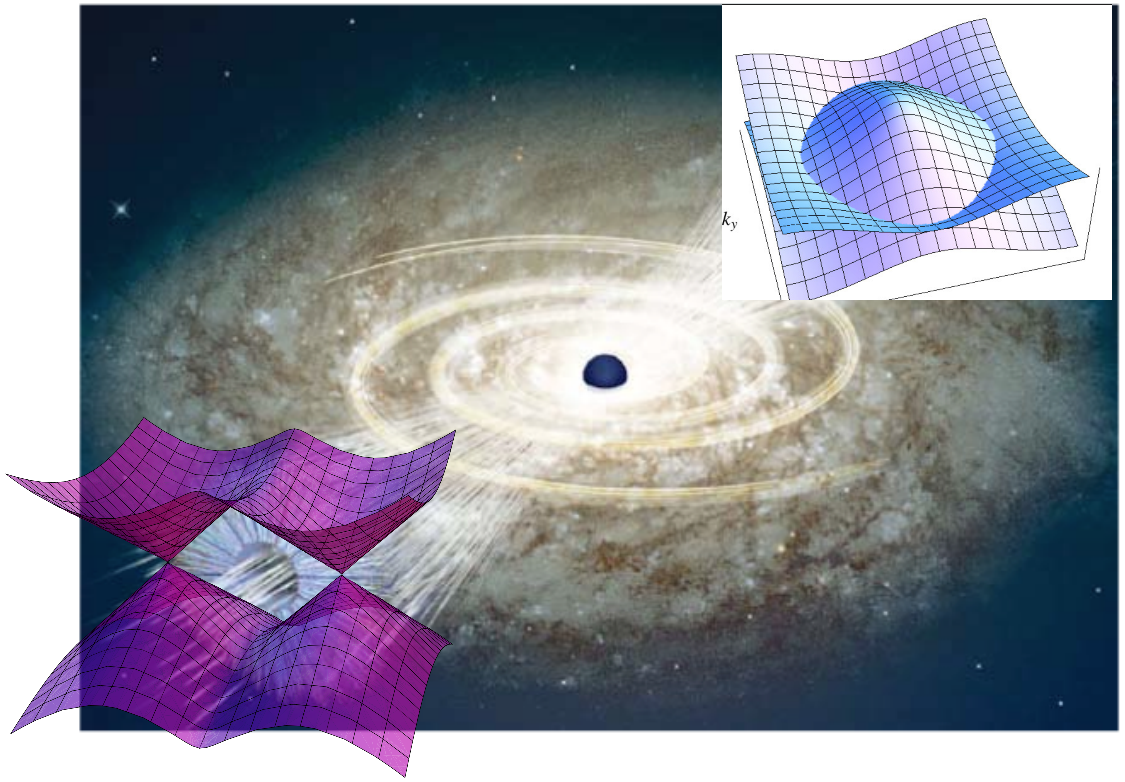
Holographic Topological Semimetals



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"Gauge/gravity duality 2018"

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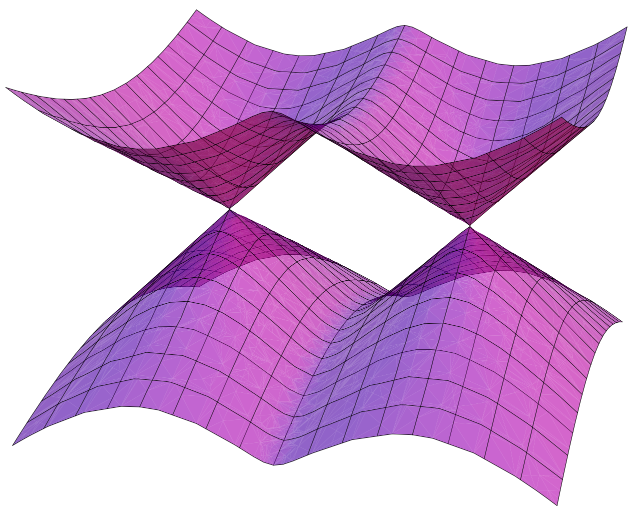
Based on collaborations with

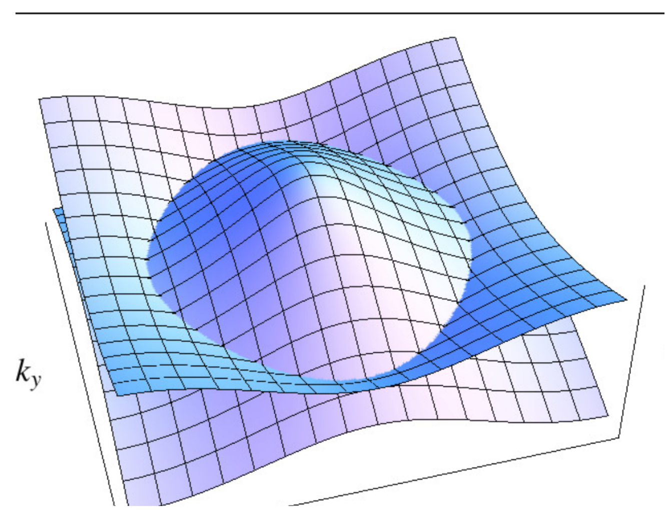
Karl Landsteiner, Ya-Wen Sun, Jun-Kun Zhao

arXiv: 1505.04772, 1511.05505, 1604.01346, 1801.09357, 1808.xxxxx and work in progress

Topological Semimetal (TSM)

- Weyl semimetal   Nodal line semimetal





- Beyond the Landau-Ginzburg paradigm
- Macroscopic effect of quantum anomaly (chiral anomaly, mixed gauge gravitational anomaly)

- Most known TSM: based on weak coupling

Motivation for Holographic TSM

- Topological semimetals (TSM) with strong interactions：
  How does TSM work in strongly coupled case?

    without quasiparticle
    no notion of band structure, Berry phase (Weyl points)

Motivation for Holographic TSM

- Topological semimetals (TSM) with strong interactions：How does TSM work in strongly coupled case?

  without quasiparticle
  no notion of band structure, Berry phase (Weyl points)

- Holography: strong-weak duality

  A holography model for TSM can teach us qualitative lessons!

  - New entry in the holographic dictionary: topological states of matter;
  - New predictions from holography for transport properties

Outline

- Holographic Weyl Semimetal
  [Landsteiner, YL, PLB, 2015; Landsteiner, YL, Y. W. Sun, PRL, 2016; YL, J.-K. Zhao, in progress] [Landsteiner's talk] [Fernandez-Pendas & Padhi's posters]

- Holographic Topological Nodal Line Semimetal
  [YL, Y. -W. Sun, 1801.09357; YL, Y. -W. Sun, to appear]

- Summary and open questions

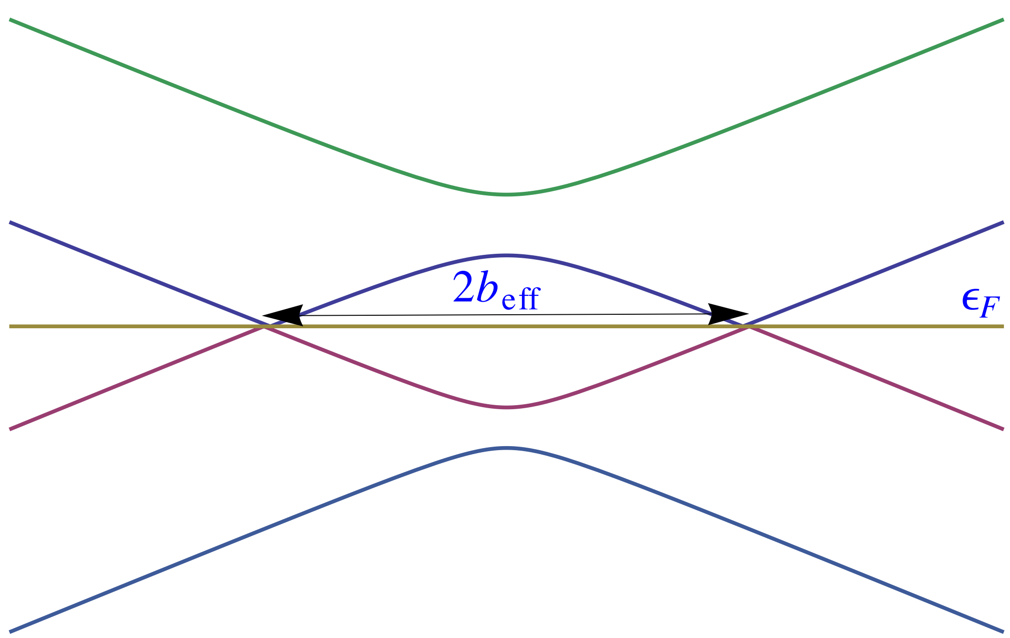
QFT of WSM

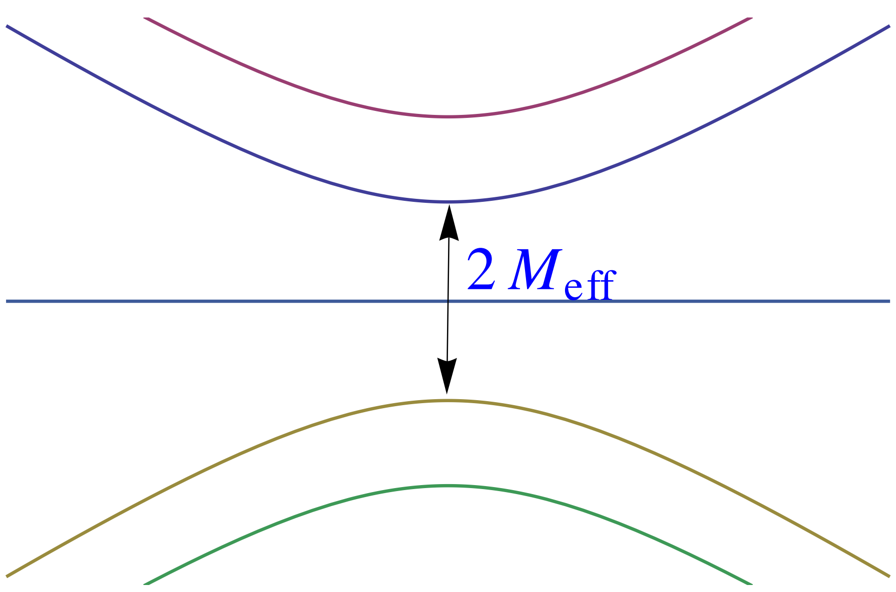
[Grushin; Jackiw; Burkov, Balents; Kostolecky et al.]

$$\mathcal{L} = \bar{\Psi} \left(i\gamma^\mu \partial_\mu + M - \gamma_5 \gamma_z b \right) \Psi \, .$$

Topological phase transition

$M < b : \quad b_{\text{eff}} = \sqrt{b^2 - M^2}$ $\qquad$ $M > b : \quad M_{\text{eff}} = \sqrt{M^2 - b^2}$





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$$M > b: \qquad M_{\text{eff}} = \sqrt{M^2 - b^2}$$

$$\mathcal{L}_{\text{eff}} = \bar{\psi}\left(i\gamma^\mu\partial_\mu - \gamma_5\gamma_z b_{\text{eff}}\right)\psi$$

$$\mathcal{L}_{\text{eff}} = \bar{\psi}\left(i\gamma^\mu\partial_\mu + M_{\text{eff}}\right)\psi$$

Anomalous Hall Effect (AHE) [Haldane, 1987]

$$\mathbf{J} = \frac{e^2}{2\pi^2}\mathbf{b}_{\text{eff}} \times \mathbf{E}$$

Holographic WSM

- Holographic model

$$\mathcal{L} = \frac{1}{2\kappa^2}\left(R + \frac{12}{L^2}\right) - \frac{1}{4}\mathcal{F}^2 - \frac{1}{4}F_5^2 + \frac{\alpha}{3}A_5 \wedge \left(F_5 \wedge F_5 + 3\mathcal{F} \wedge \mathcal{F}\right) + \quad \zeta A_5 \wedge R \wedge R + + |(\partial_\mu - iqA^5_\mu)\Phi|^2 - V(\Phi)$$

- Ward identity

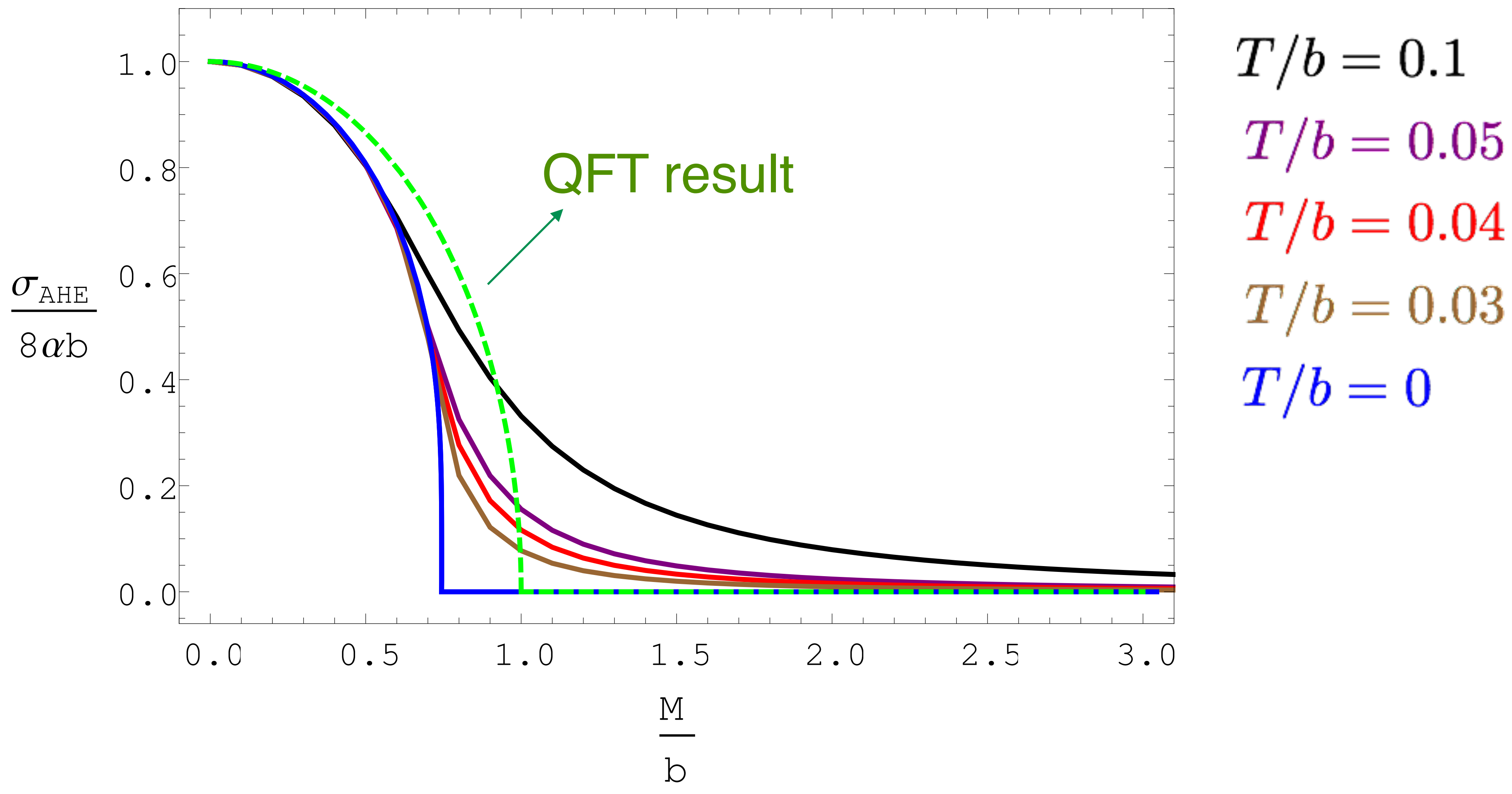
$$\partial_\mu J^\mu = 0$$

$$\partial_\mu J^\mu_5 = \left(\frac{\alpha}{3}\left[F_5 \wedge F_5 + 3\mathcal{F} \wedge \mathcal{F}\right] - iq\sqrt{-g}\left[\Phi(D_r\Phi)^* - \Phi^*(D_r\Phi)\right]\right)\Big|_{r\to\infty}$$

Holographic WSM

- Order parameter of topological WSM: AHE

$$\sigma_{\mathrm{AHE}} = 8\alpha A_z^5(0)$$



Holographic WSM

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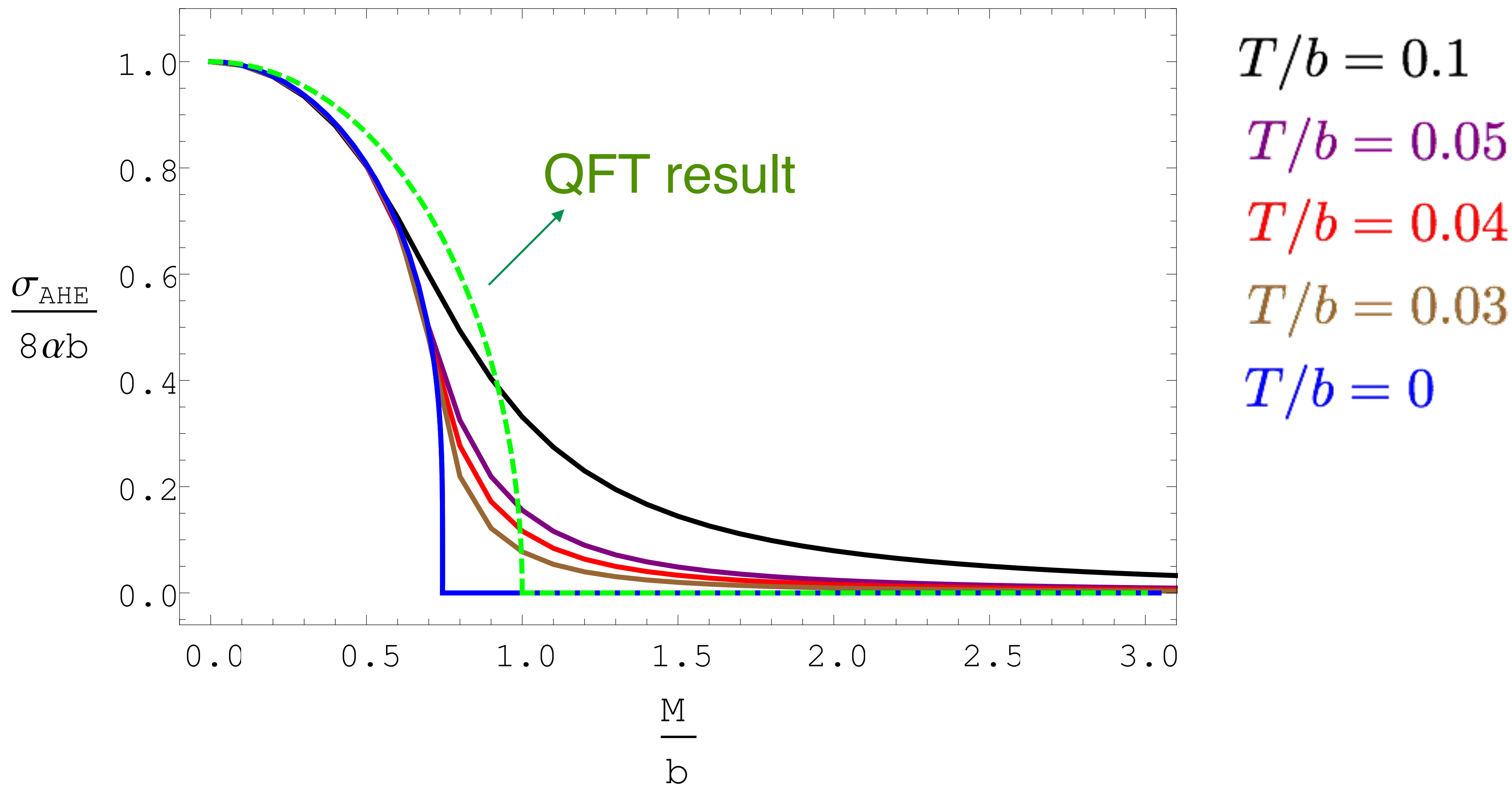
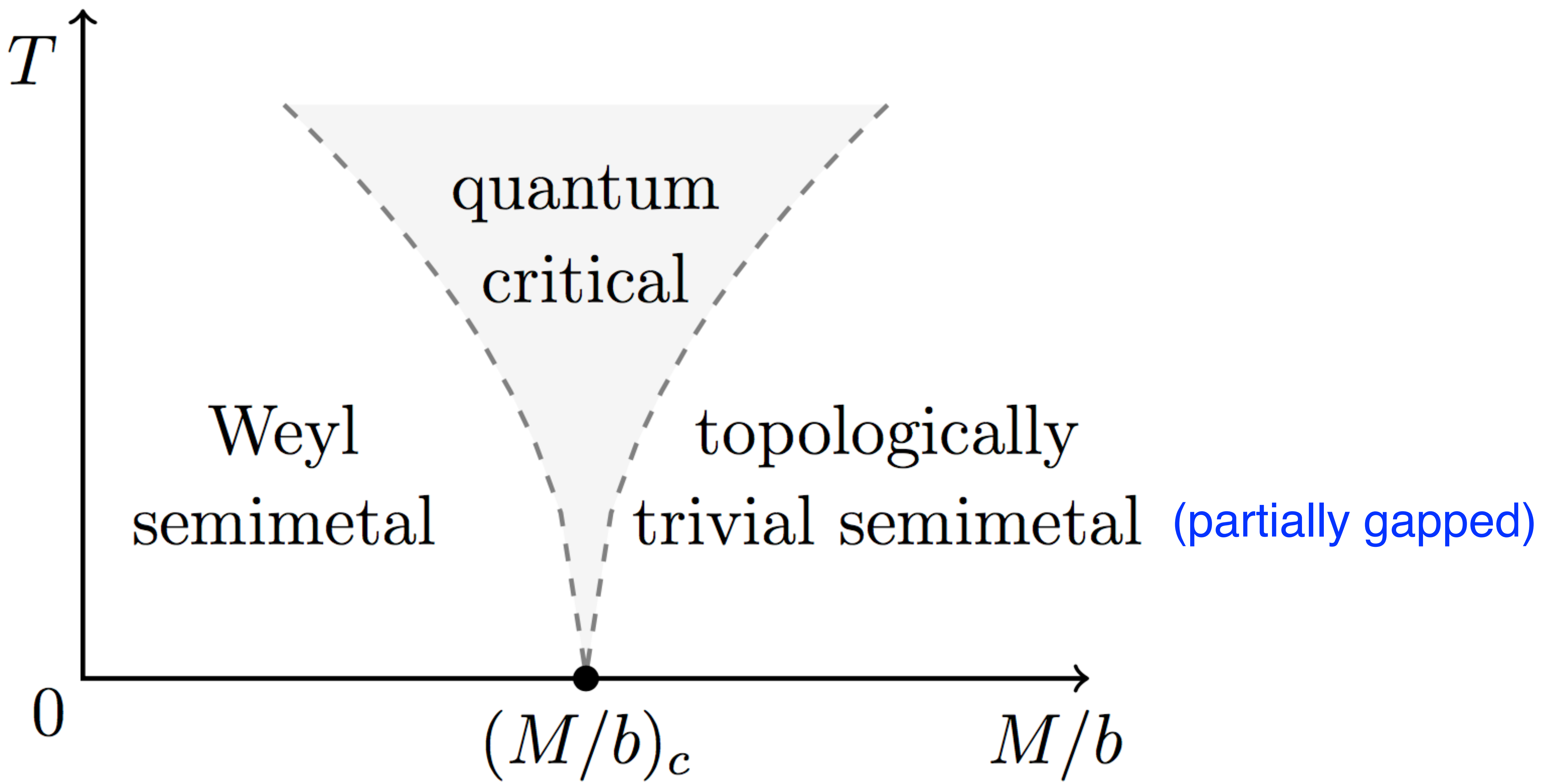


$$(\sigma_{\rm AHE}/b) \propto ((M/b)_c - M/b)^{\alpha} \qquad \alpha \approx 0.211$$

in contrast to the field theory result: 0.5

Holographic WSM



Transports in holographic WSM

- Conductivities, viscosities have peak/dip behaviour in the QC regime

- Temperature scaling behaviours of viscosities and conductivities in the QC regime: emergent Lifshitz-like symmetry in the IR at the transition point

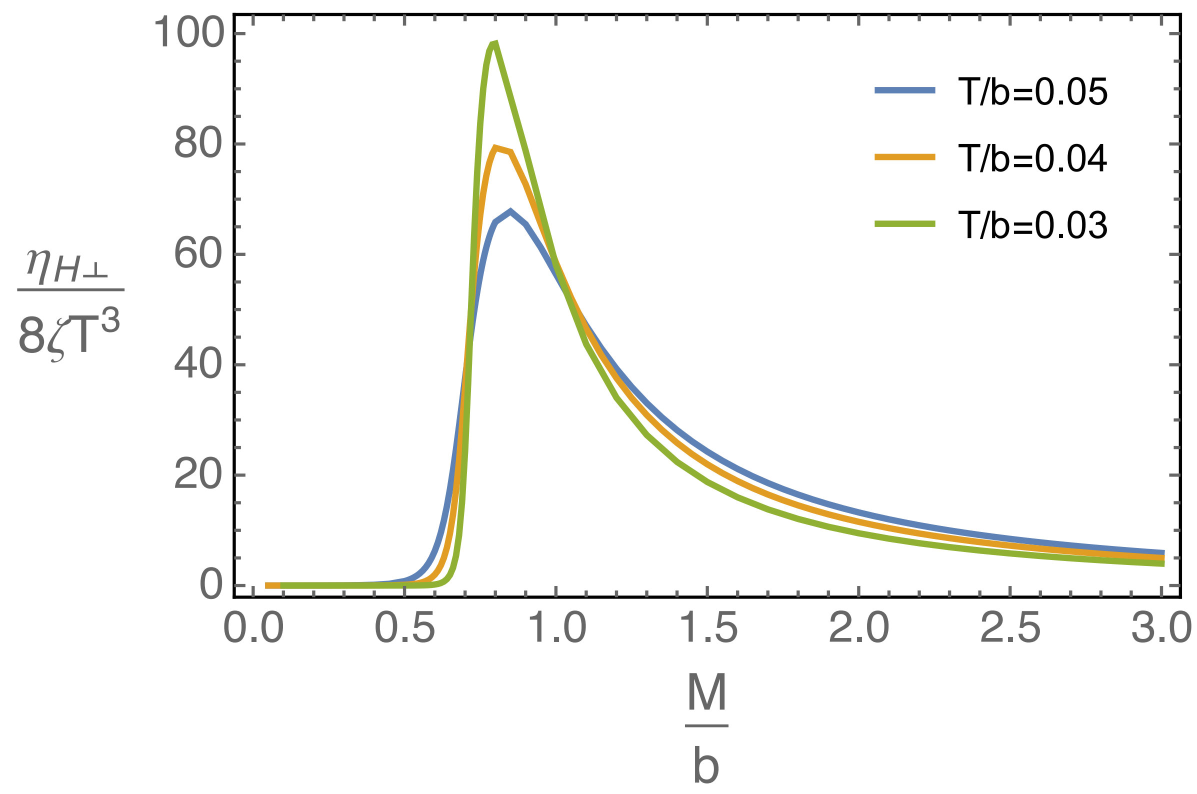
$$
\begin{aligned}
\eta_{\|}/s \propto T^{2-2\beta}\,, &\quad \eta_{H_{\|}} \propto T^{4-\beta}\,,\\
\eta_{H_{\perp}} \propto T^{2+\beta}\,, &\quad \sigma_{\|} \propto T^{2-\beta}\,,\\
\sigma_{\perp} \propto T^{\beta}\,, &\quad \sigma_{\mathrm{AHE}} \propto T^{\beta}\,,
\end{aligned}
$$

Transports in holographic WSM

- Odd viscosity is due to the presence of *the mixed gauge-gravitational anomaly*

$$\eta_{H_\perp} = 8\zeta q^2\phi^2 A_z g_{xx}\Big|_{r=r_0}$$

mixed axial-gravitational anomaly

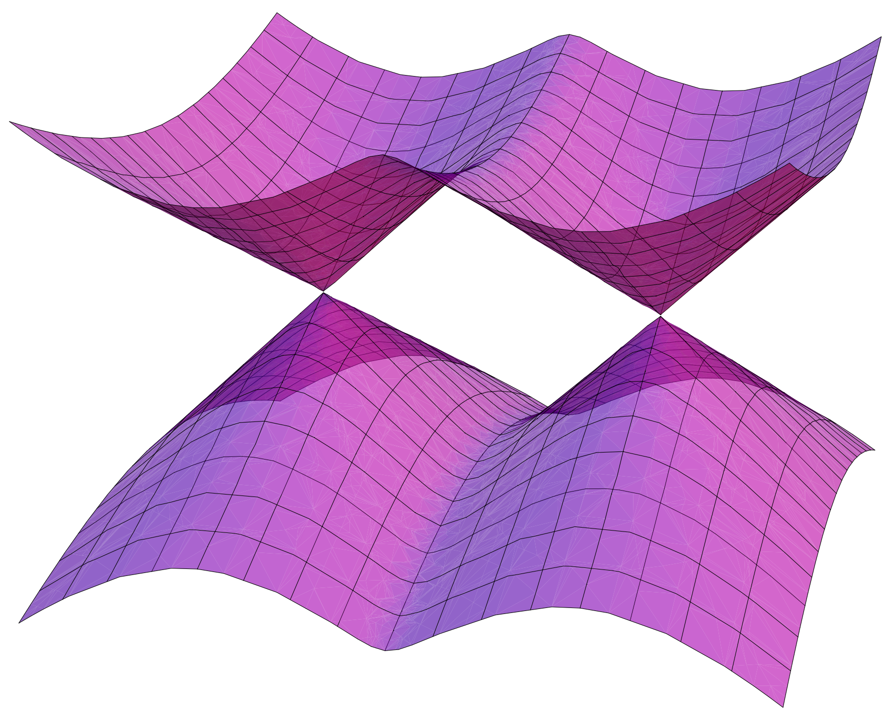


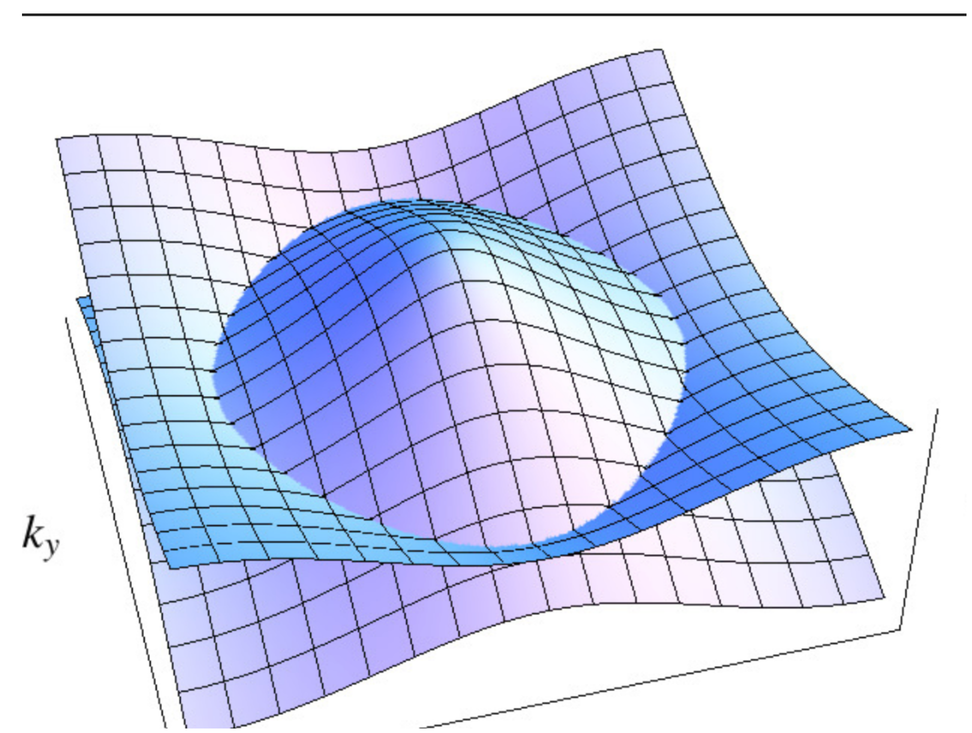
- analytic nontrivial relation

$$\frac{\eta_\parallel}{\eta_\perp} = \frac{2\eta_{H_\parallel}}{\eta_{H_\perp}} = \frac{\sigma_\parallel}{\sigma_\perp} = \frac{g_{xx}}{g_{zz}}\Big|_{r=r_0}$$

Holographic Nodal Line Semimetal

[YL, Y.-W. Sun, 1801.09357]

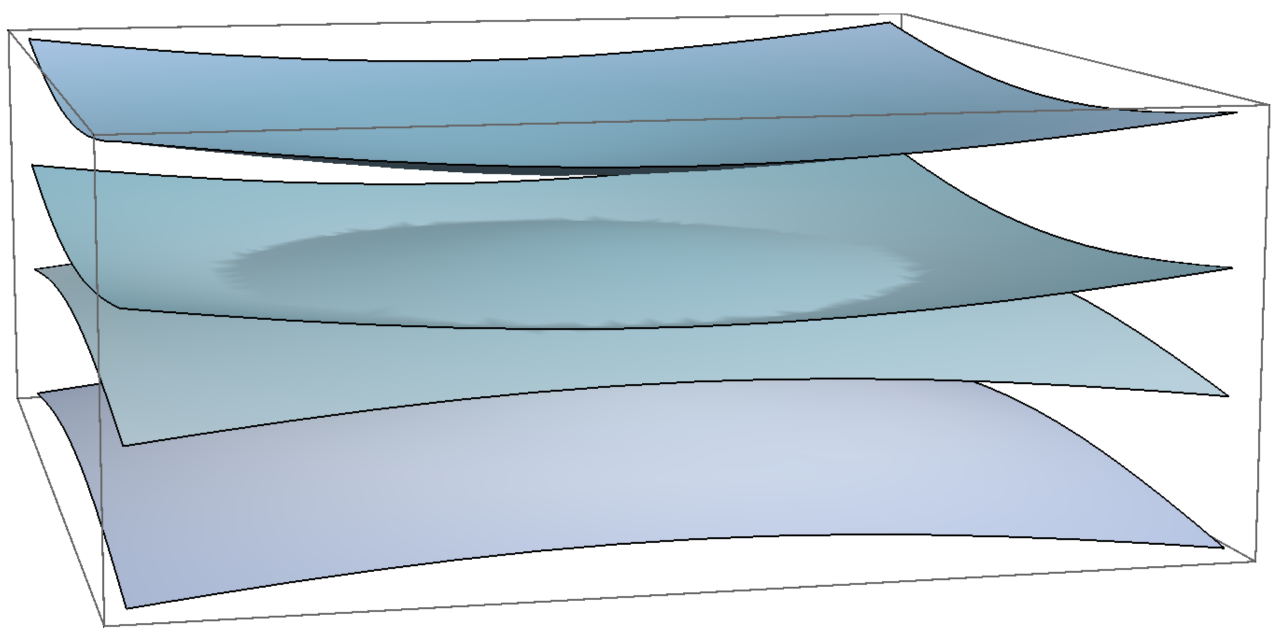




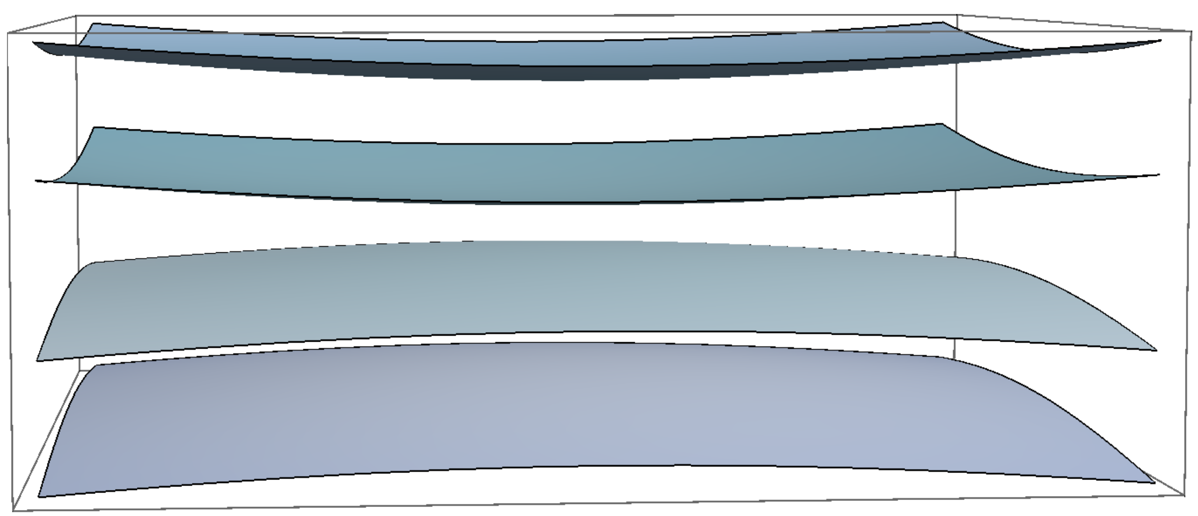
Holographic Nodal Line Semimetal

- Weakly coupled field theory model for NLSM

$$\mathcal{L} = i\bar{\psi}\left(\gamma^{\mu}\partial_{\mu} - m - \gamma^{\mu\nu}b_{\mu\nu}\right)\psi \qquad \gamma^{\mu\nu} = \frac{i}{2}[\gamma^{\mu}, \gamma^{\nu}]$$



$m^2 < 4b_{xy}^2$



$m^2 > 4b_{xy}^2$

Holographic Nodal Line Semimetal

- Conservation equation

$$\partial_\mu J^\mu = 0\,,$$
$$\partial_\mu J_5^\mu = im\bar{\psi}\gamma^5\psi + 2ib_{\mu\nu}\bar{\psi}\gamma^{\mu\nu}\gamma^5\psi\,,$$

Operator	Field
$\bar{\psi}\psi\,,\ \bar{\psi}\gamma^5\psi$	Axially charged Complex scalar filed
$\bar{\psi}\gamma^{\mu\nu}\psi\,,\ \bar{\psi}\gamma^{\mu\nu}\gamma^5\psi$	Axially charged complex two form field

Holographic Nodal Line Semimetal

- Holographic model

$$S = \int d^5x\sqrt{-g}\bigg[\frac{1}{2\kappa^2}\Big(R+\frac{12}{L^2}\Big)-\frac{1}{4}\mathcal{F}^2-\frac{1}{4}F^2+\frac{\alpha}{3}\epsilon^{abcde}A_a\Big(3\mathcal{F}_{bc}\mathcal{F}_{de}+F_{bc}F_{de}\Big)$$
$$-(D_a\Phi)^*(D^a\Phi)-V_1(\Phi)-\frac{1}{3\eta}\big(\mathcal{D}_{[a}B_{bc]}\big)^*\big(\mathcal{D}^{[a}B^{bc]}\big)-V_2(B_{ab})-\lambda|\Phi|^2B^*_{ab}B^{ab}\bigg]$$

- Two axially charged matter fields

- Ward identity

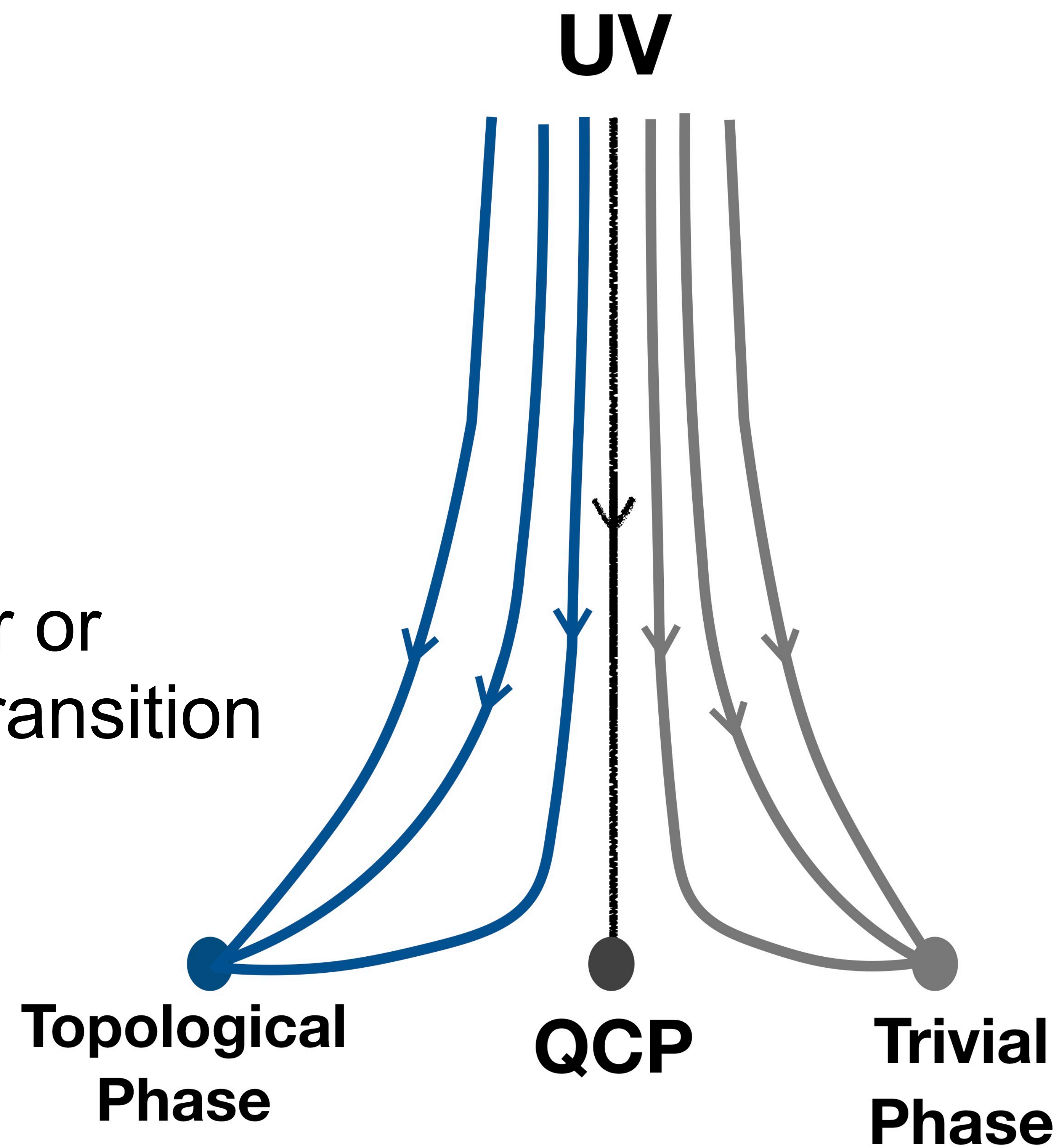
$$\partial_\mu J^\mu = 0\,,$$
$$\partial_\mu J^\mu_5 = \lim_{r\to\infty}\sqrt{-g}\bigg(-\frac{\alpha}{3}\epsilon^{r\alpha\beta\rho\sigma}(F_{\alpha\beta}F_{\rho\sigma}+\mathcal{F}_{\alpha\beta}\mathcal{F}_{\rho\sigma})+iq_1\Big[\Phi^*(D^r\Phi)-\Phi(D^r\Phi)^*\Big]+$$
$$+\frac{iq_2}{\eta}\big(B^*_{\mu\nu}\mathcal{D}^{[r}B^{\mu\nu]}-(\mathcal{D}^{[r}B^{\mu\nu]})^*B_{\mu\nu}\big)\bigg)+\text{c.t.}\,.$$

Holographic Nodal Line Semimetal

The phase transition is continuous

No local order parameter or transport for the phase transition

QCP is stable



Holographic Nodal Line Semimetal

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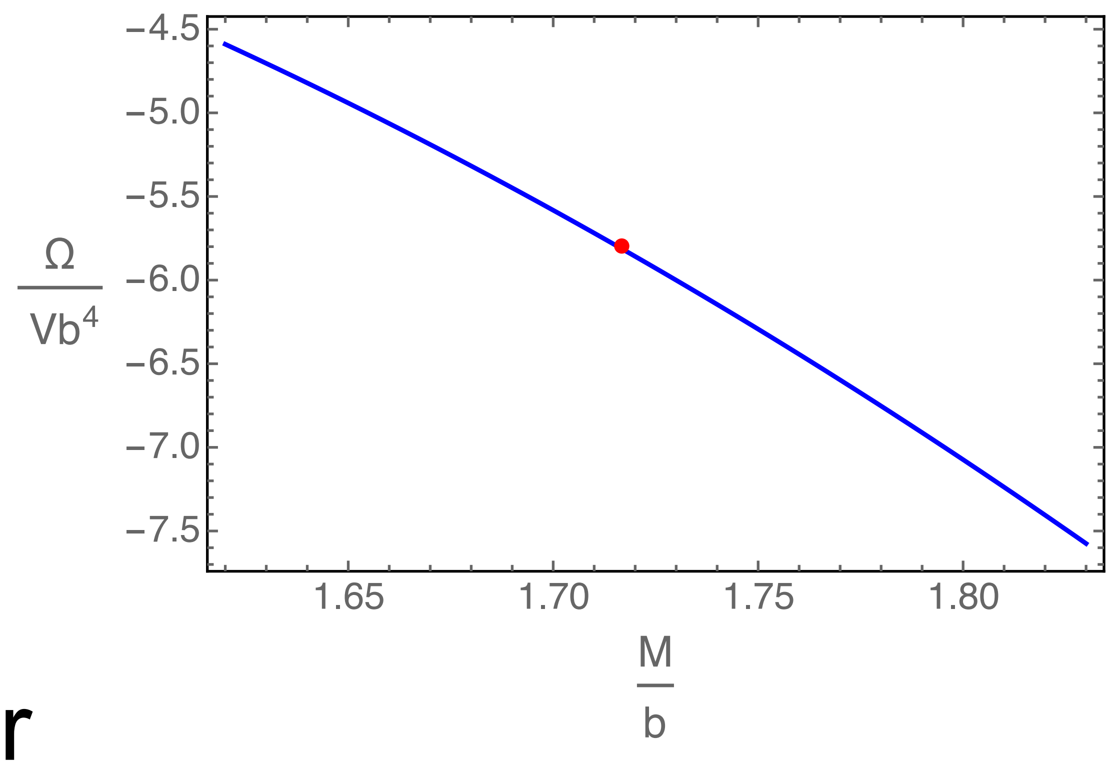

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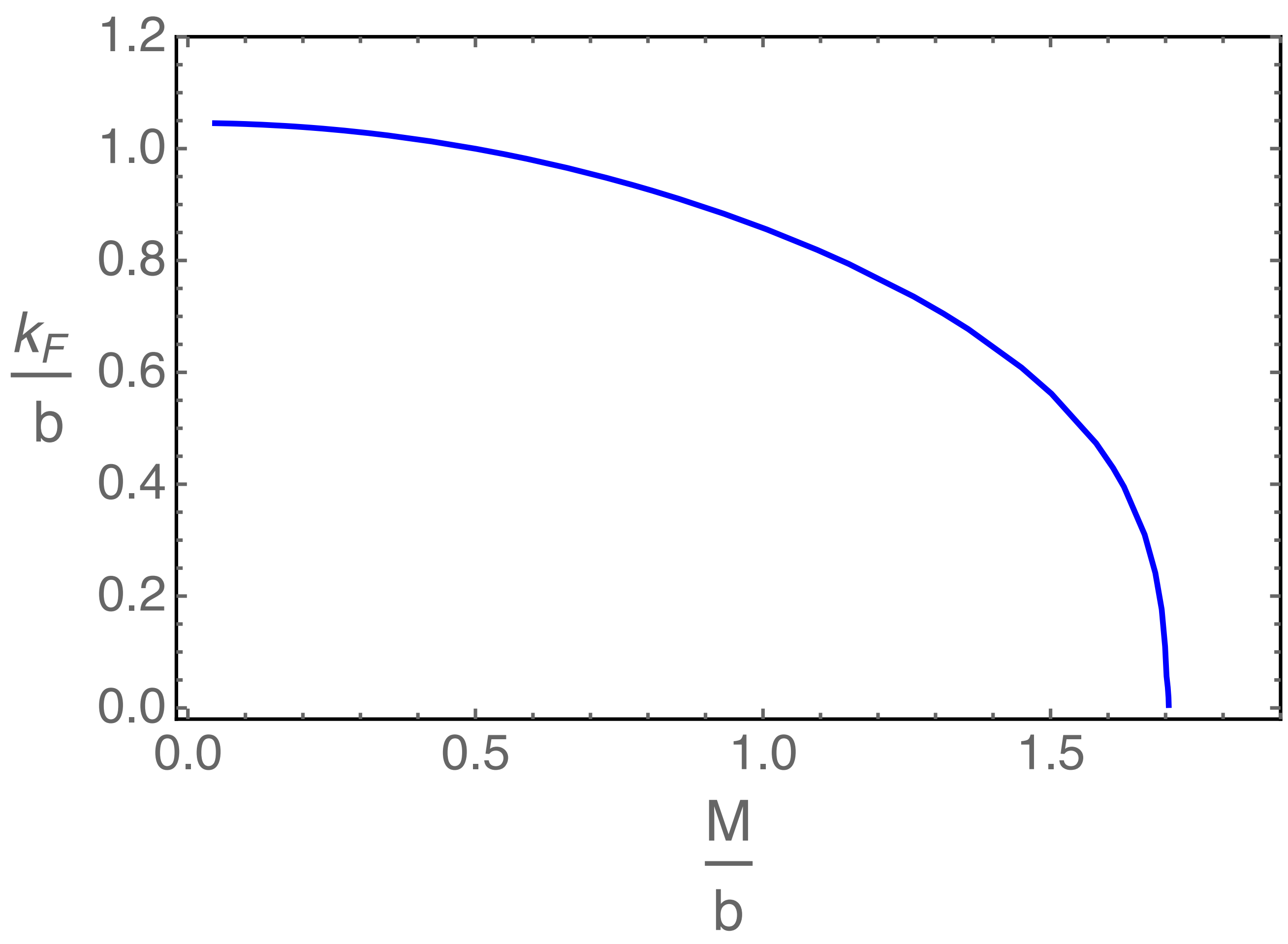
Holographic Nodal Line Semimetal

- **APRES:**

- (1) Multiple while discrete Fermi nodal lines
- (2) Fermi nodal lines forming circles in the nodal line semimetal phase with two bands crossing



Topological invariants

[YL, Y.-W. Sun, to appear]

- distinguish different topology of the quantum wave function in the momentum space

- **Topological invariants** for interacting systems: the *topological Hamiltonian method,* the zero frequency Green's function contains all topological information [Z. Wang and S.C.Zhang, 2012, 2013]

$$\mathcal{H}_t(\mathbf{k}) = -G^{-1}(0, \mathbf{k})$$

Topological invariants

[YL, Y.-W. Sun, to appear]

- For pure AdS, the topological invariants of dual Dirac nodes (@ $\omega = \mathbf{k} = 0$) are ± 1.
- For holographic WSM with very small M/b, the dual Weyl nodes are at $\omega = 0$ and $k_z = \pm A_z|_{\rm hor}$, the topological invariants are ± 1 respectively.
- For holographic NLSM, for one set of NL the Berry phase is Pi; while for another set of NL, Berry phase is undetermined
- Bulk topological structure: (1) different bulk IR solutions which are not adiabatic connected; (2) scalar field is to generate gap, another matter field is to generate FS, they behave differently in different solutions

Holographic WSM/insulator transition

[YL, J.-K. Zhao, in progress]

- Starting from the most general holographic model [Grignani, et al. 2017]

$$\mathcal{S} = \int d^5x\sqrt{-g}\bigg[\frac{1}{2\kappa^2}(R+12) - \frac{Y(\phi)}{4}\mathcal{F}^2 - \frac{Z(\phi)}{4}F^2 + \frac{\alpha}{3}\epsilon^{abcde}A_a\Big(F_{bc}F_{de} + 3\mathcal{F}_{bc}\mathcal{F}_{de}\Big) - \frac{1}{2}(\partial\phi)^2 - \frac{W(\phi)}{2}(A_a - \partial_a\theta)^2 - V(\phi)\bigg]$$

with

$$Z(\phi) = 1\,,$$

$$W(\phi) = -w_0\Big[1 - \cosh\Big[\sqrt{\frac{2}{3}}\phi\Big]\Big]\,,$$

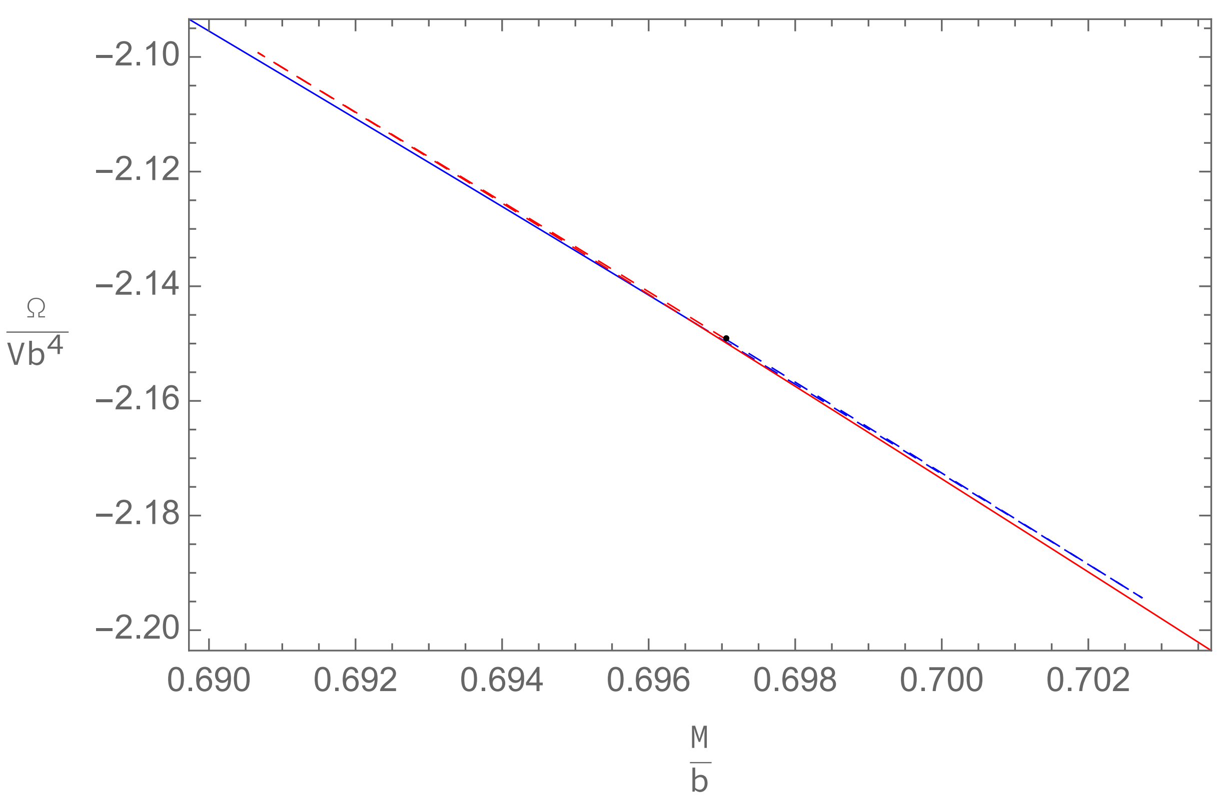
$$V(\phi) = \frac{9}{2}\Big[1 - \cosh\Big[\sqrt{\frac{2}{3}}\phi\Big]\Big]\,,$$

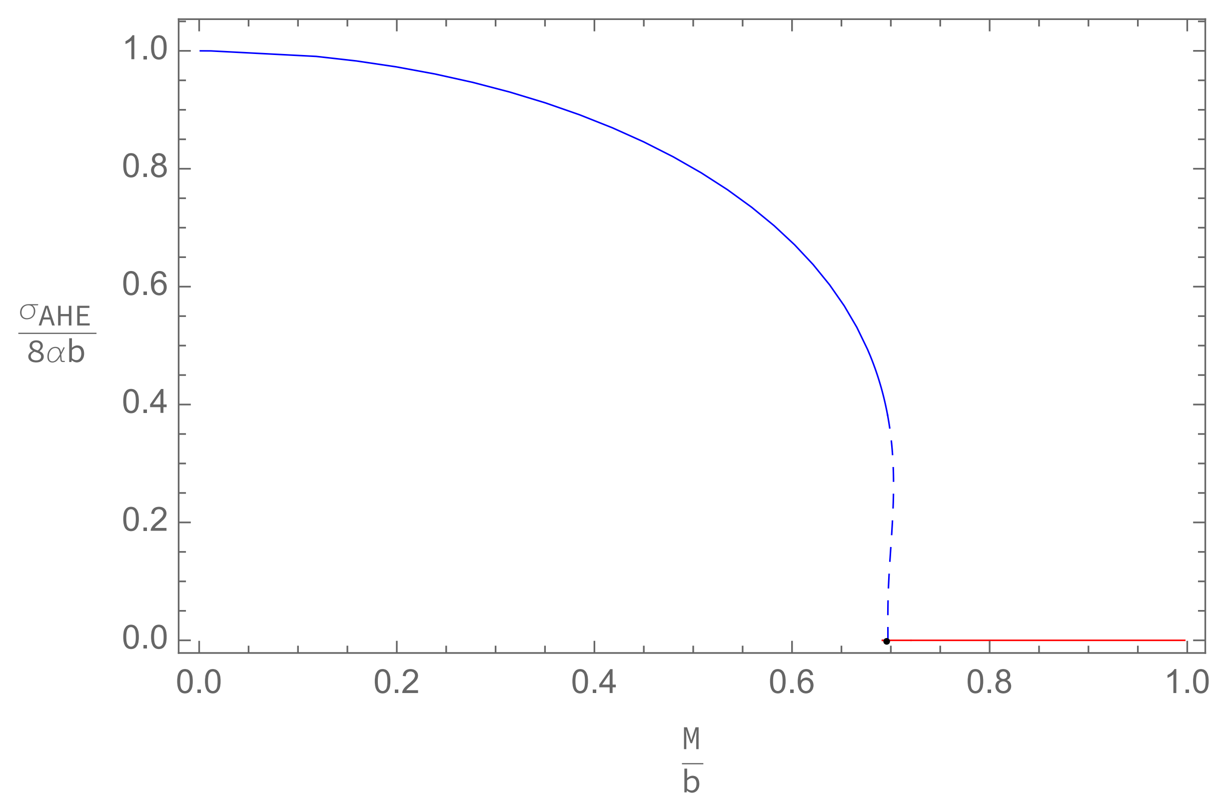
$$Y(\phi) = \cosh\Big[\sqrt{\frac{2}{3}}\phi\Big]\,.$$

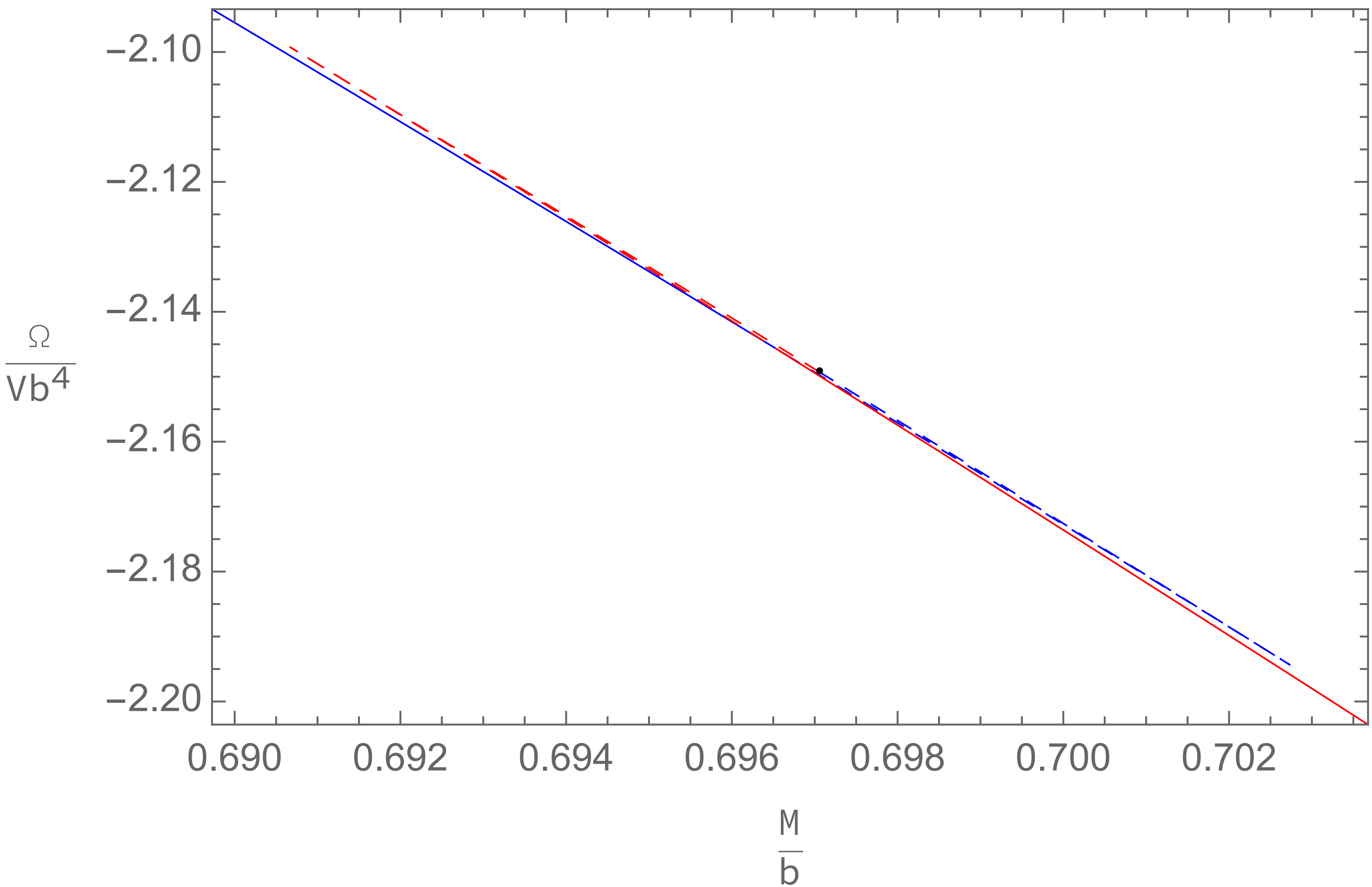
Holographic WSM/insulator transition

[YL, J.-K. Zhao, in progress]

- Three different phases, however, the QCP is unstable.
- The IR geometry in the insulating phase is a gapped geometry
- There is a first order quantum phase transition







−2.10
−2.12
−2.14
−2.16
−2.18
−2.20
Ω / Vb^4
0.690
0.692
0.694
0.696
0.698
0.700
0.702
M / b

Holographic WSM/insulator transition

[YL, J.-K. Zhao, in progress]

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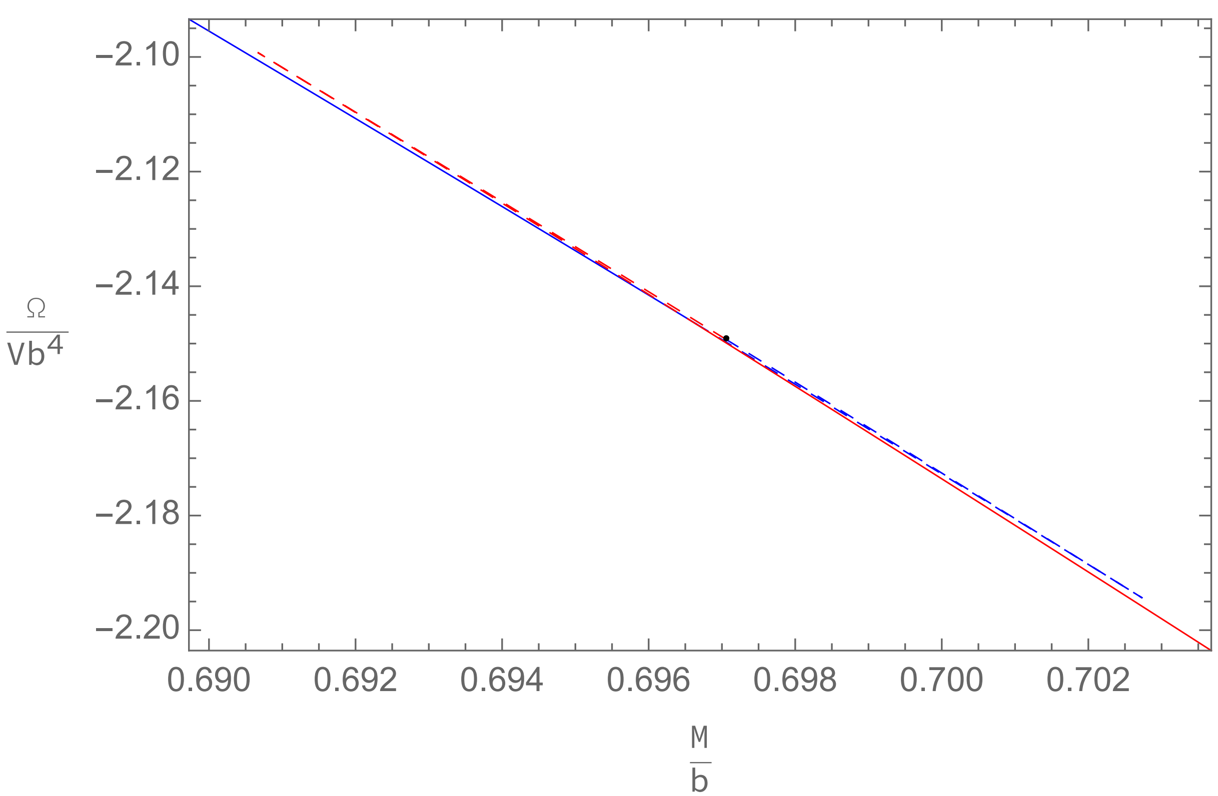

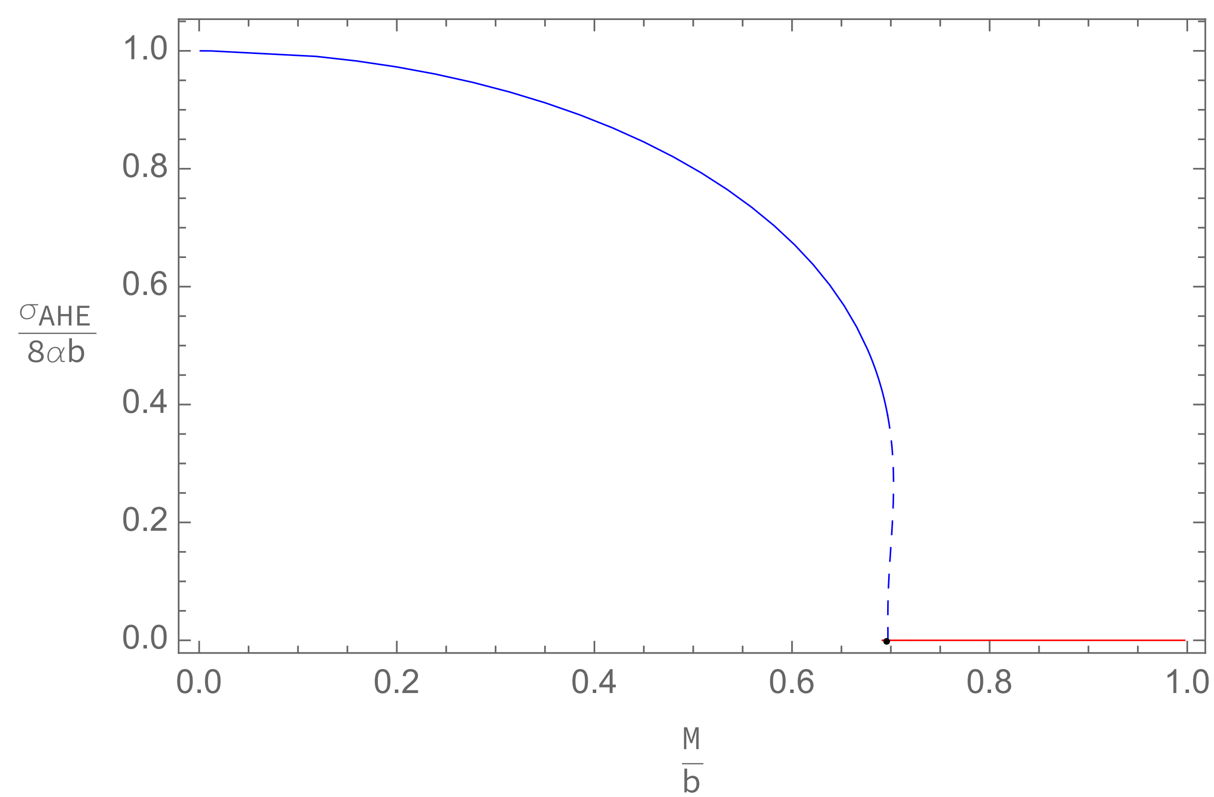

Summary （1）

- **Weakly coupled WSM vs. Holographic WSM**

	Weyl Semimetal	Holographic WSM
Symmetry	Time reversal or invesion symmetry breaking	Time reversal breaking
Features in transport	Anomalous Hall conductivity	AHE Quantum critical physics Mixed anomaly: Odd viscosity
Edge States	Gapless surface state Fermi arc	Surface current [Ammon et al, PRL, 2017]
Topological invariants	± 1	± 1
laboratory	TaAs, TaP, etc.	??

Summary (2)

- **Weakly coupled NLSM vs. Holographic NLSM**

	Nodal line semimetal	Holographic NLSM
Symmetry	Symmetry protected by mirror reflection symmetry, inversion symmetry	Symmetry protected by inversion symmetry
Features in transport	ARPES	ARPES (multiple NL) Other transport (??)
Edge States	No	??
Topological invariants	π	(1) π ; (2) undetermined
laboratory	$PbTaSe_2$, ZrTe etc.	??

Open questions

- Other approach (Kinetic theory, EFT) to odd viscosity?
- Disorder effects (or translational invariance breaking effects) on holographic TSM?
- Interesting transport physics in holographic NLSM?
- Classification of strongly coupled topological semimetals (Organisation principle)

Thank You!

Thank You!

Holographic Nodal Line Semimetal

- **Ansatz for T=0**

$$ds^2 = u(-dt^2 + dz^2) + \frac{dr^2}{u} + f(dx^2 + dy^2)$$

$$\Phi = \phi(r)\,,$$

$$B_{xy} = B(r)\,.$$

- Near UV

Metric: $ds^2|_{r\to\infty} = \frac{dr^2}{r^2} + r^2(-dt^2 + d\vec{x}^2)$

scalar field: $r\Phi|_{r\to\infty} = M$

two form field: $B_{xy}|_{r\to\infty} = br$

Holographic WSM

At zero temperature: 3 distinct classes of solutions (near horizon @ IR)

$$u = r^2\,,\quad h = r^2\,,\quad A_z = a_1 + \frac{\pi a_1^2\phi_1^2}{16r}e^{-\frac{2a_1 q}{r}}\,,\quad \phi = \sqrt{\pi}\phi_1\left(\frac{a_1 q}{2r}\right)^{3/2}e^{-\frac{a_1 q}{r}}\,;$$

$$u = u_0 r^2\left(1+\delta u\, r^\alpha\right),\quad h = h_0 r^\beta\left(1+\delta h\, r^\alpha\right),\quad A_z = r^\beta\left(1+\delta a\, r^\alpha\right),\quad \phi = \phi_0\left(1+\delta\phi\, r^\alpha\right)$$

$$u = \left(1+\frac{3}{8\lambda}\right)r^2\,,\quad h = r^2\,,\quad A_z = a_1 r^{\beta_1}\,,\quad \phi = \sqrt{\frac{3}{\lambda}} + \phi_1 r^{\beta_2}\,,$$

M/b<0.744 (Topological phase)

M/b=0.744 (Critical point)

M/b>0.744 (Trivial phase)

Holographic WSM

At zero temperature: 3 distinct classes of solutions (near horizon @ IR)

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M/b<0.744
(Topological phase)

$$u = u_0 r^2 \left(1 + \delta u\, r^\alpha\right),\quad h = h_0 r^\beta \left(1 + \delta h\, r^\alpha\right),\quad A_z = r^\beta \left(1 + \delta a\, r^\alpha\right),\quad \phi = \phi_0 \left(1 + \delta\phi\, r^\alpha\right)$$

M/b=0.744
(Critical point)

$$u = (1 + \frac{3}{8\lambda}) r^2\,,\quad h = r^2\,,\quad A_z = a_1 r^{\beta_1}\,,\quad \phi = \sqrt{\frac{3}{\lambda}} + \phi_1 r^{\beta_2}\,,$$

M/b>0.744
(Trivial phase)

From free energy, we find a continuous and smooth behaviour at the critical point.

Holographic Nodal Line Semimetal

- Three different IR solutions: Near horizon = leading + subleading

$$u=\frac{1}{8}(11+3\sqrt{13})r^2\Big(1+\delta u r^{\alpha_1}\Big)\,,$$
$$f=\sqrt{\frac{2\sqrt{13}}{3}-2\,b_0 r^{\alpha}\Big(1+\delta f r^{\alpha_1}\Big)}\,,$$
$$\phi=\phi_0 r^{\beta}\,,$$
$$B=b_0 r^{\alpha}\Big(1+\delta b r^{\alpha_1}\Big)\,,$$

$$u=u_0 r^2(1+\delta u r^{\beta})\,,$$
$$f=f_0 r^{\alpha}(1+\delta f r^{\beta})\,,$$
$$\phi=\phi_0(1+\delta\phi r^{\beta})\,,$$
$$B=b_0 r^{\alpha}(1+\delta b r^{\beta})\,,$$

$$u=(1+\frac{3}{8\lambda_1})r^2\,,$$
$$f=r^2\,,$$
$$\phi=\sqrt{\frac{3}{\lambda_1}}+\phi_1 r^{\frac{2\sqrt{160\lambda_1^2+84\lambda_1+9}}{3+8\lambda_1}-2}\,,$$
$$B=b_1 r^{2\sqrt{2}\sqrt{\frac{3\lambda+\lambda_1}{3+8\lambda_1}}}\,.$$

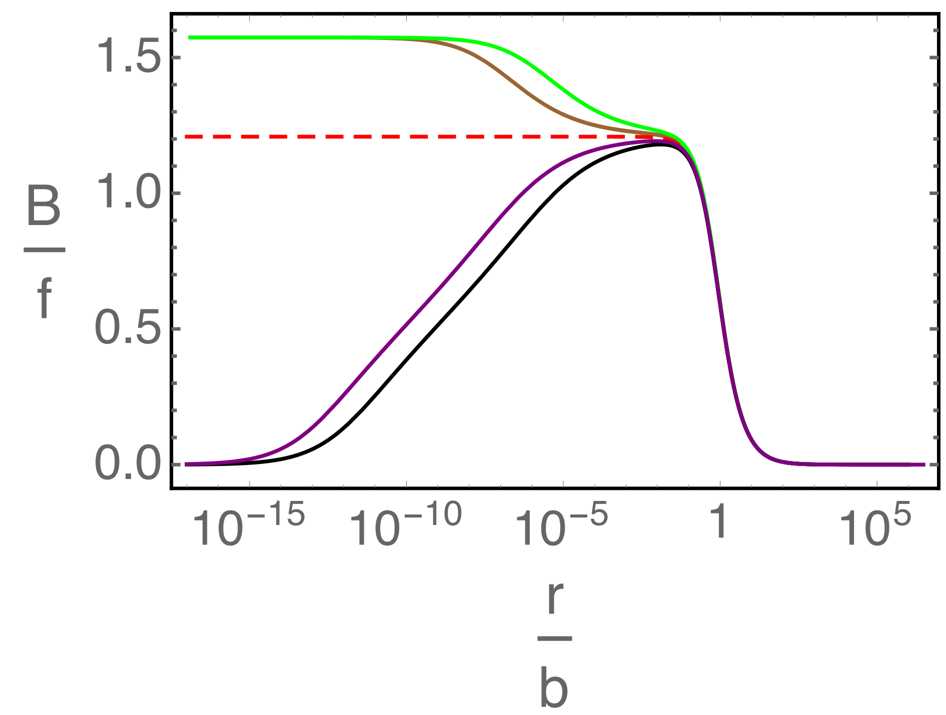
M/b<critical value (Topological NLSM phase)

M/b=critical value (Critical phase)

M/b>critical value (Trivial phase)

Holographic Nodal Line Semimetal

Bulk profile for two-form field



$M/b = 1.682$

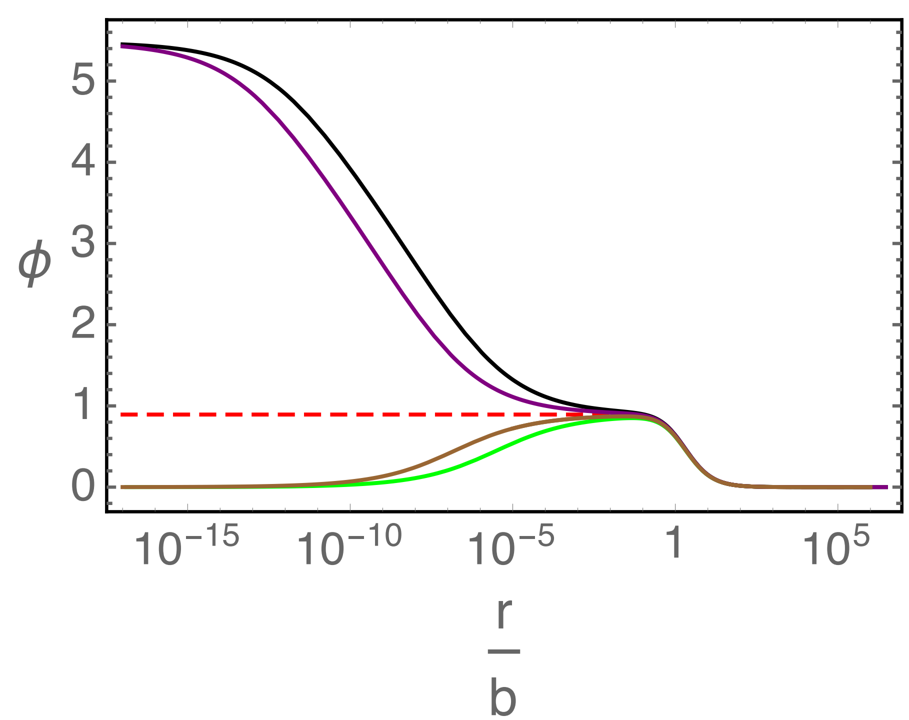
$M/b = 1.702$

$M/b = 1.717$

$M/b = 1.733$

$M/b = 1.750$

Bulk profile for scalar field



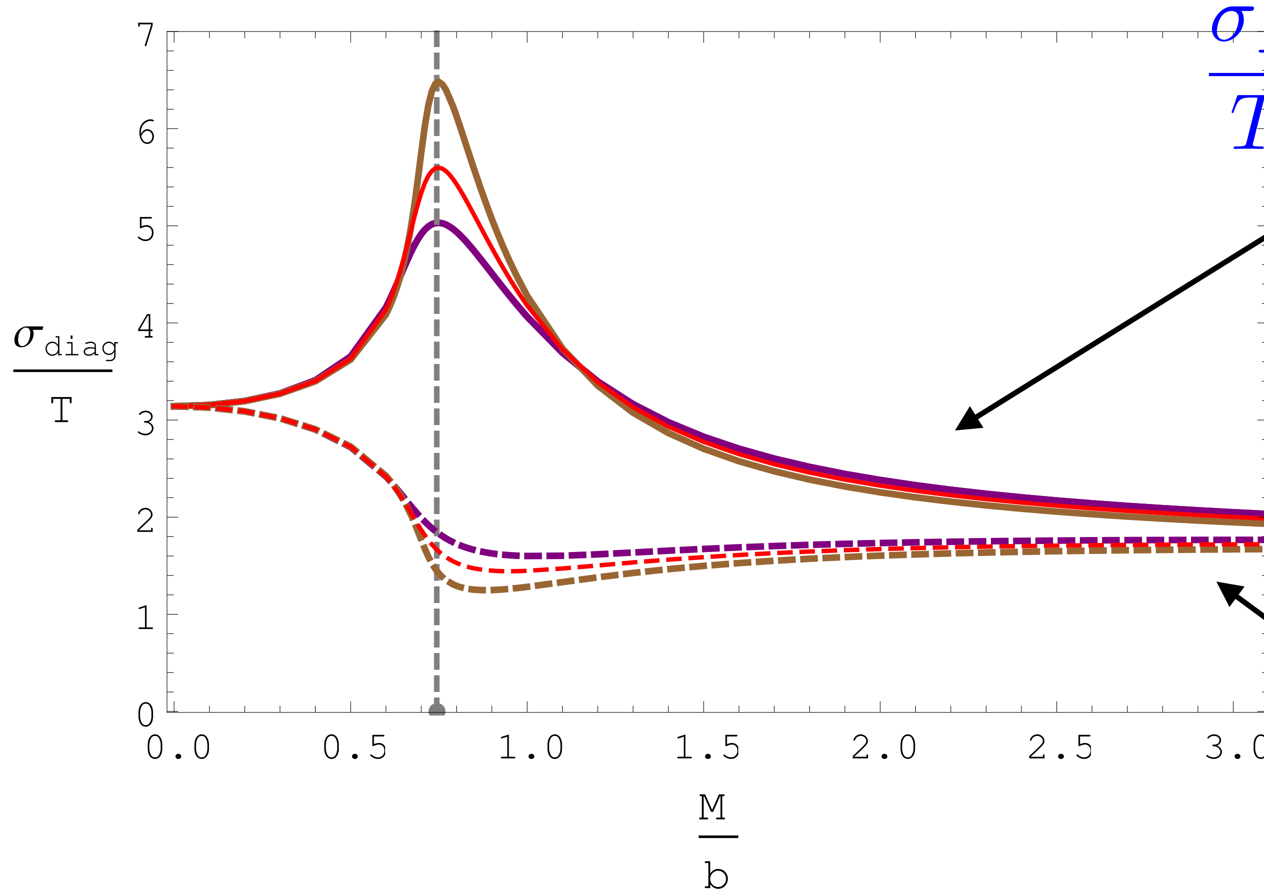
Predictions of Holographic WSM: **conductivity**

- Diagonal conductivities at T=0: $\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = 0$
- Diagonal conductivities at T>0:

$T/b = 0.03$

$T/b = 0.04$

$T/b = 0.05$



$$\frac{\sigma_\perp}{T} = \frac{\sigma_{xx}}{T} = \frac{\sigma_{yy}}{T}$$

$$\frac{\sigma_\parallel}{T} = \frac{\sigma_{zz}}{T}$$

Predictions of Holographic WSM: **viscosity**

- Axisymmetric system [Landau&Lifshitz, Vol. 10]

$$\tau_{xy} = \eta_\perp V_{xy} - \eta_\perp^H (V_{xx} - V_{yy})$$

$$\tau_{xz} = \eta_{||} V_{xz} + \eta_{||}^H V_{yz}$$

$$\tau_{yz} = \eta_{||} V_{yz} - \eta_{||}^H V_{xz}$$

$$V_{ij} = \frac{1}{2}(\partial_i v_j + \partial_j v_i)$$

odd viscosities

- In total: 3 shear, 2 bulk and 2 odd viscosities

- Viscosity: slightly deform the metric of spacetime

$$ds^2 = -dt^2 + (\delta_{ij} + h_{ij})dx^i dx^j$$

$$\partial_i v_j = \Gamma^0_{ij} u^0 = \partial_t h_{ij}$$

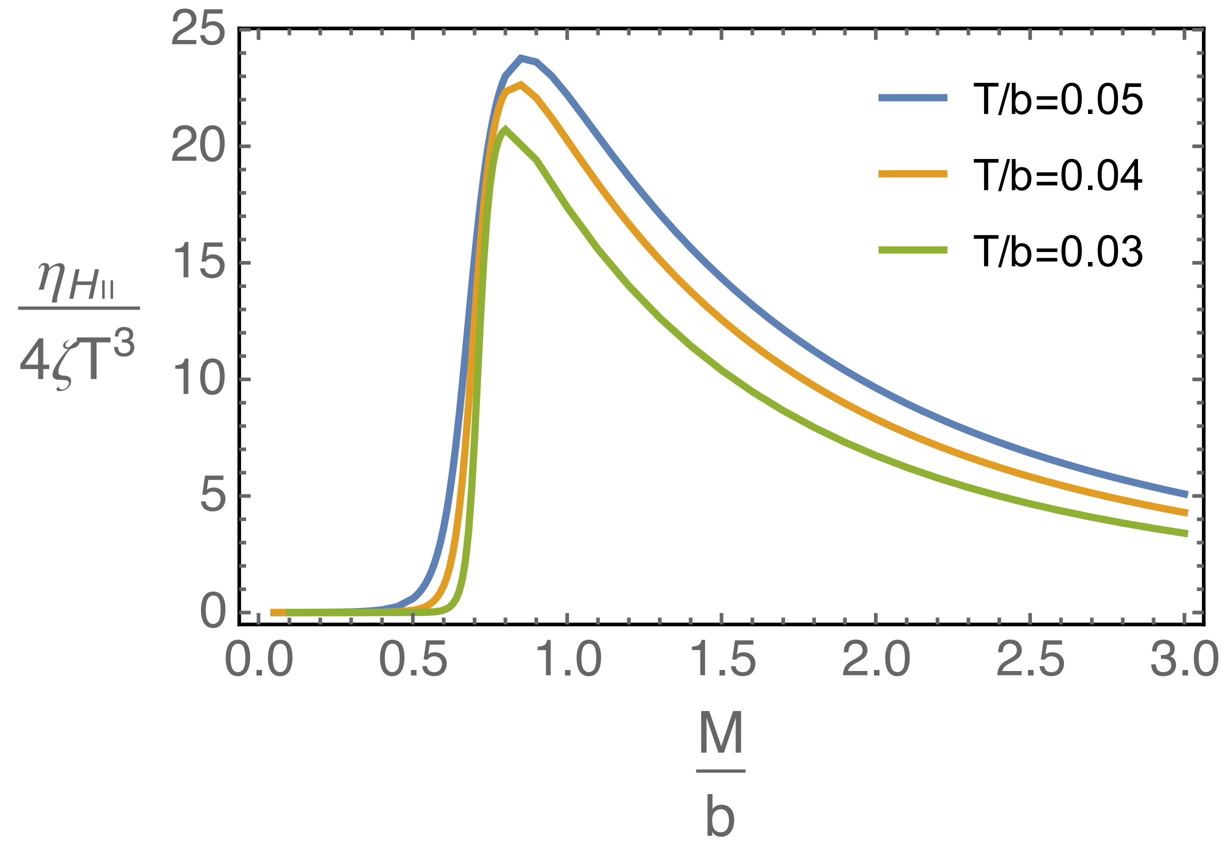
[Son, Saremi]

Predictions of Holographic WSM: **odd viscosity**

- at T=0, from field theory arguments, no substantial odd viscosity expected
- Odd viscosity determined by IR properties:

$$\eta_{H_{\parallel}} = 4\zeta \left. \frac{q^2 A_z \phi^2 g_{xx}^2}{g_{zz}} \right|_{r=r_0}$$

mixed axial-gravitational anomaly

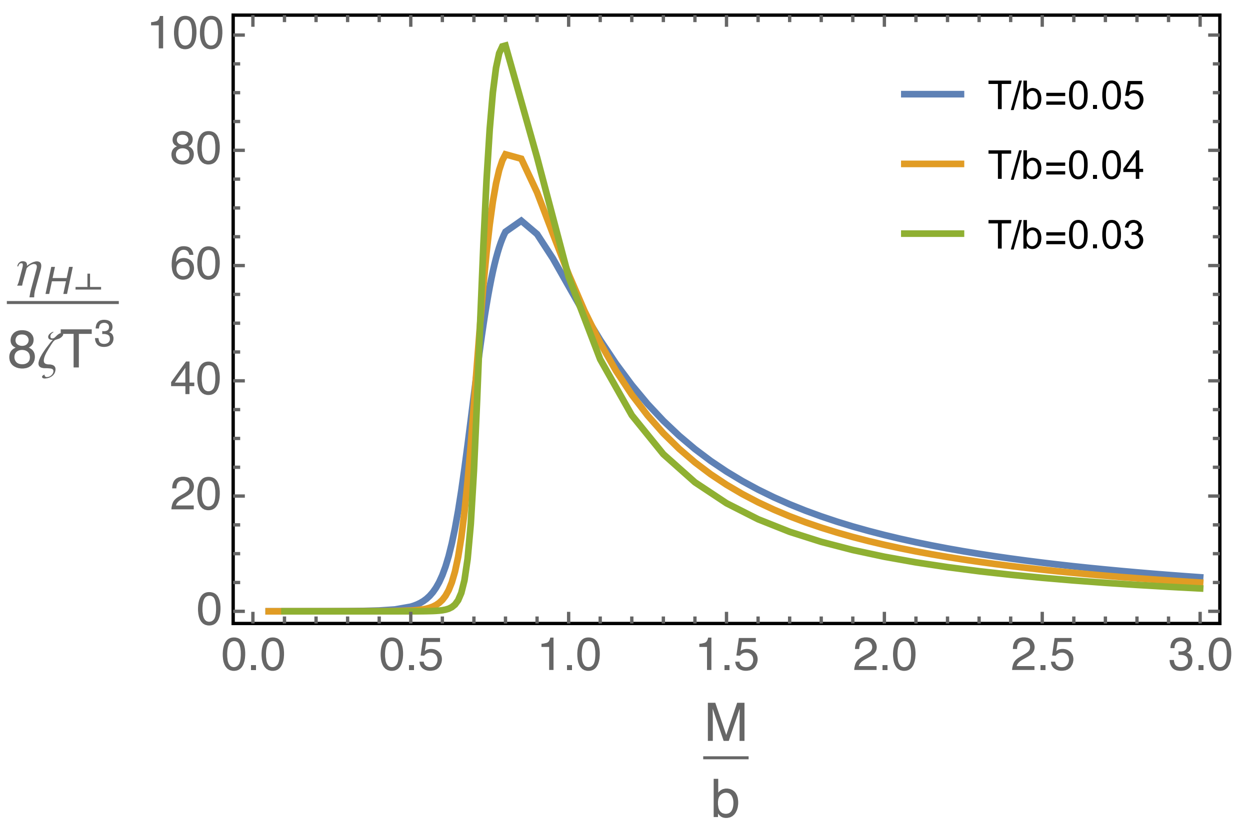


- *Highly suppressed in the WSM phase*
- *Rises steeply entering the QC region*
- *peaks at the critical point and drops slowly as M/b increases, finally reaching zero*

Predictions of Holographic WSM: **odd viscosity**

$$\eta_{H_\perp} = 8\zeta q^2 \phi^2 A_z g_{xx}\Big|_{r=r_0}$$

mixed axial-gravitational anomaly



Qualitatively the same as the other one

- *Highly suppressed in the WSM phase*
- *Rises steeply entering the QC region*
- *peaks at the critical point and drops slowly as M/b increases, finally reaching zero*

Predictions of Holographic WSM: **shear viscosity**

- Transverse shear viscosity

$$\eta_\perp = \eta_{xy,xy} = \eta_{(xx-yy),xx-yy} = g_{xx}\sqrt{g_{zz}}\Big|_{r=r_0} = \frac{s}{4\pi}$$

KSS bound

- Longitudinal shear viscosity

$$\eta_\parallel = \eta_{xz,xz} = \eta_{yz,yz} = \frac{g_{xx}^2}{\sqrt{g_{zz}}}\Bigg|_{r=r_0}$$