Holographic Second Laws of Black Hole Thermodynamics

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PERIMETER
INSTITUTE

Based on arXiv: 1803.03633 with A. Bernamonti, R. Myers and J. Oppenheim

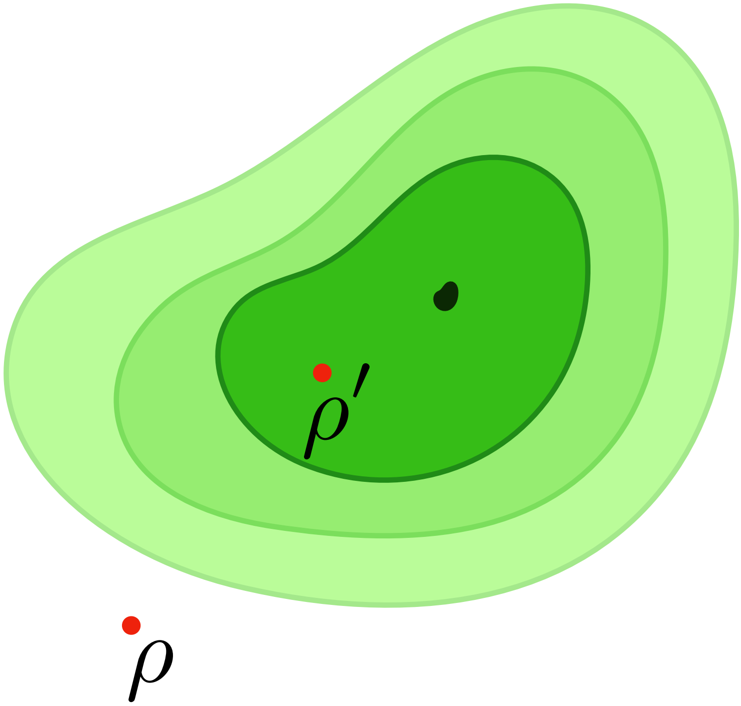
Second Law

- **Second law of thermodynamics:** the entropy of a closed system can never decrease

$$S(0) \leq S(t)$$

Necessary condition for any state transformation

$$\rho \xrightarrow{t} \rho'$$



- **Second law of black hole mechanics**

$$S_{BH} = \frac{A}{4G_N}$$

horizon area is a non-decreasing function of time in any classical process

[Bekenstein; Hawking]

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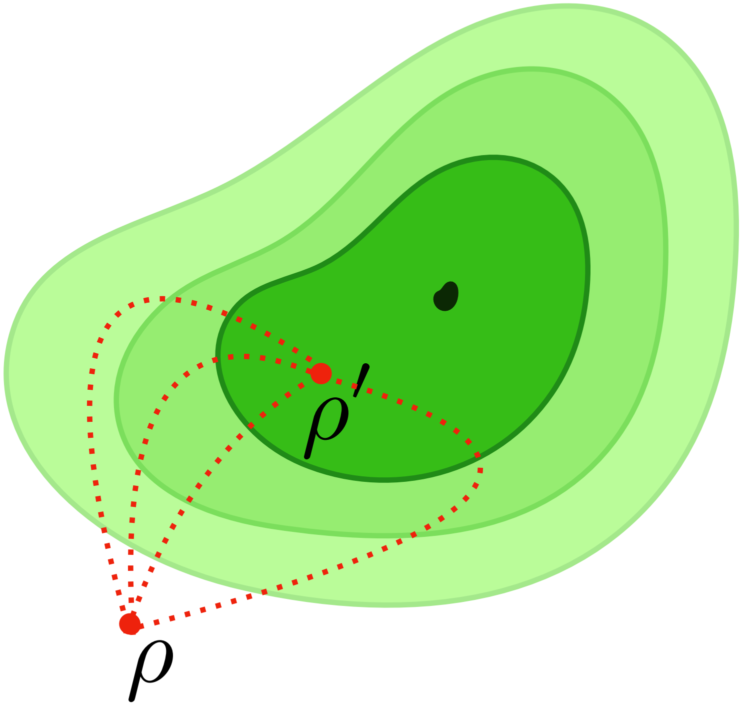

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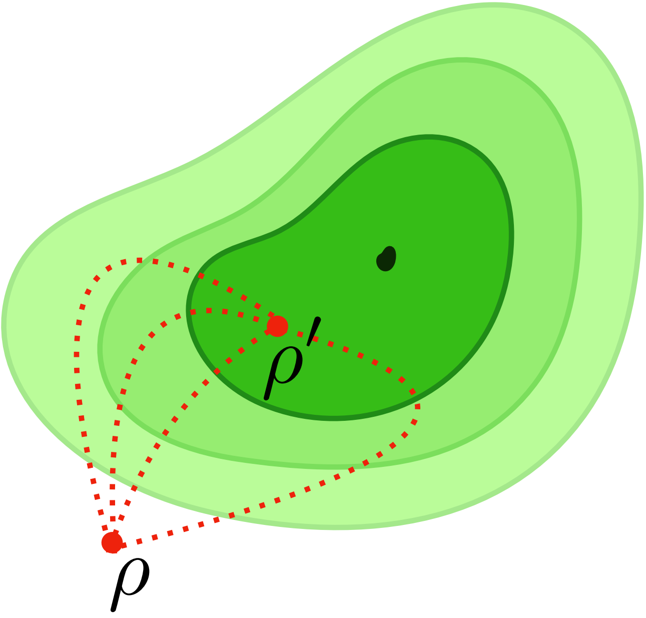
Q: is there more?

Quantum Thermodynamics applies perspective and tools of information theory to examine thermodynamics

Provides additional constraints on equilibration processes for quantum systems and for macroscopic systems with long range correlations



[Horodecki, Oppenheim '11]
[Brandao, Horodecki, Ng, Oppenheim, Wehner '13]
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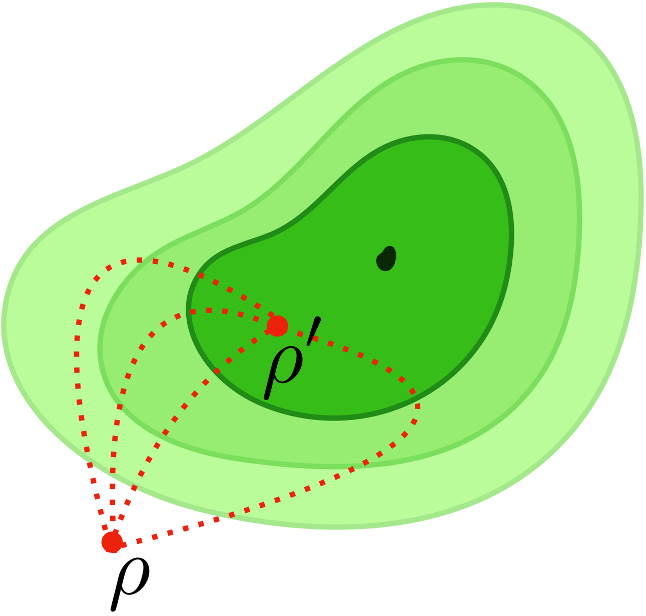

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➡ Relevant for quantum field theory and gravity!

Rényi Divergences

- Notion of distance between states

$$D_\alpha(\rho||\rho_\beta) \equiv \frac{\mathrm{sgn}(\alpha)}{\alpha - 1} \log \mathrm{tr}(\rho^\alpha \rho_\beta^{1-\alpha}) \qquad \forall \alpha \in \mathbb{R}$$

reference thermal state

[Petz '86]

Ex: Relative entropy distance from the thermal state

$$\lim_{\alpha \to 1} D_\alpha(\rho||\rho_\beta) = S(\rho||\rho_\beta) = \beta(F(\rho) - F(\rho_\beta))$$

second law in a closed system $F(\rho) \geq F(\rho')$ is equivalent to the statement that the relative entropy does not increase

$$D_1(\rho||\rho_\beta) \geq D_1(\rho'||\rho_\beta)$$

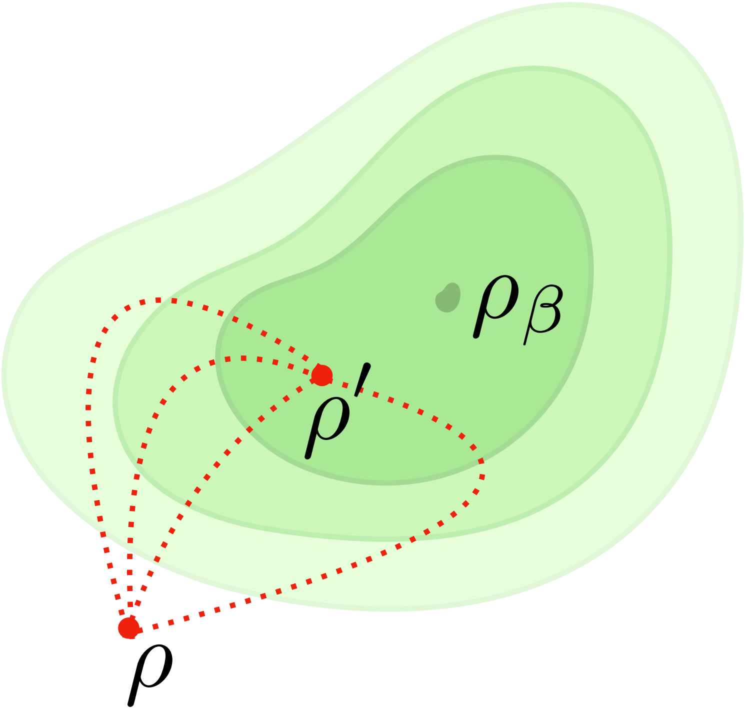
Additional Second Laws

- Monotonicity of Rényi Divergences generalizes the second law

$$D_\alpha(\rho||\rho_\beta) \geq D_\alpha(\rho'||\rho_\beta)$$

One-parameter family of necessary conditions for the equilibration dynamics

[Brandao, Horodecki, Ng, Oppenheim, Wehner '13]



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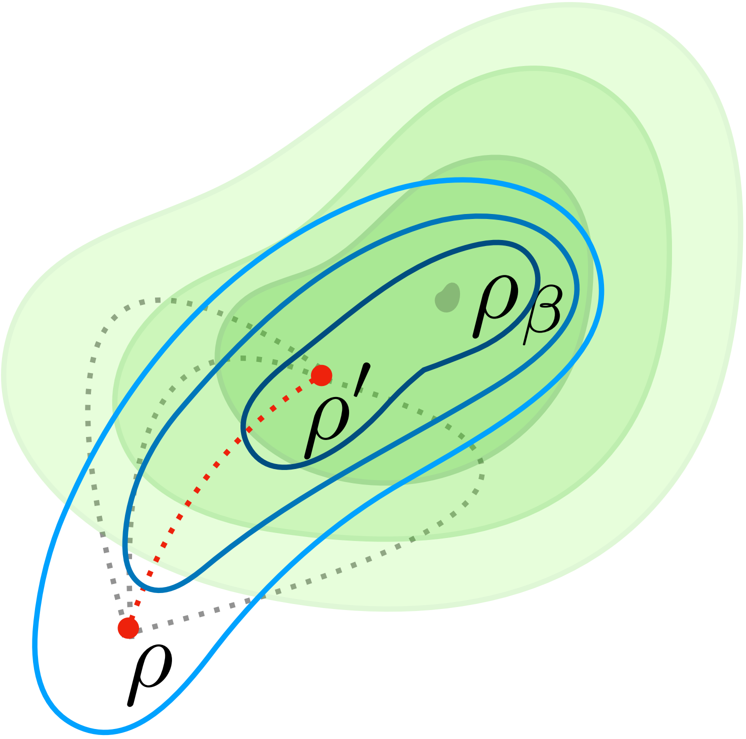


➡ They may provide additional constraints for allowed transitions!

Additional Second Laws

- Does monotonicity of Rényi divergences imply new constraints for quantum field theory and black holes, beyond the second law?

- AdS/CFT framework:

[Bernamonti, FG, Myers, Oppenheim ’18]

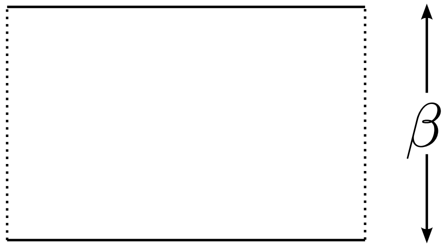
Class of CFT_2 excited states ⟷ AdS_3 black holes with scalar excitations

Path Integral Approach

- Euclidean path integral construction for $\mathrm{tr}(\rho^{\alpha}\rho_{\beta}^{1-\alpha})$ in 2d CFT

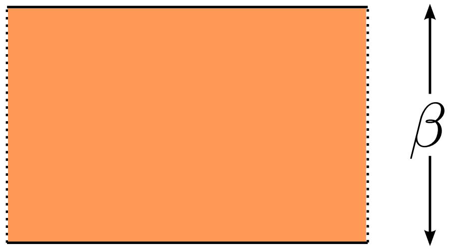
Thermal state

$$\rho_{\beta} = e^{-\beta H_{\mathrm{CFT}}} =$$



Excited state $\{\lambda, \Delta\}$: relevant deformation of the thermal state

$$\rho = e^{-\beta H_{\Delta}} =$$



excited state for the theory governed by H_{CFT}

$$H_{\Delta} = H_{\mathrm{CFT}} + \lambda \int dx \mathcal{O}_{\Delta}$$

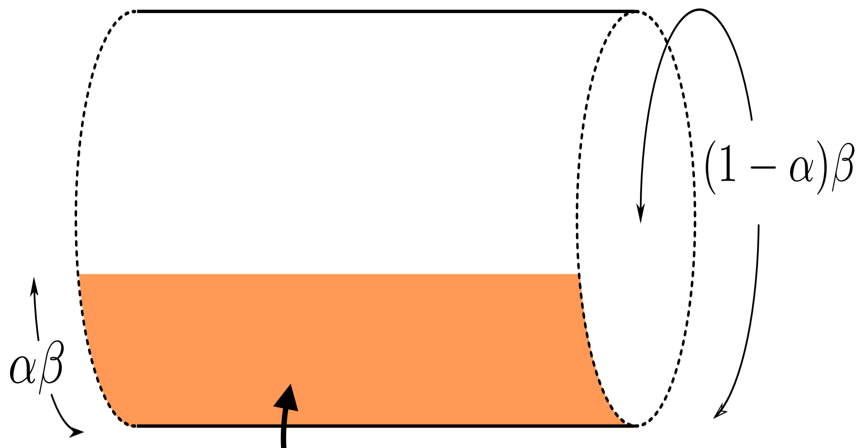
amplitude

relevant operator
$0 < \Delta < 2$

Path Integral Approach

- Rényi Divergences trace function

$$Z_{\mathrm{CFT}} = \mathrm{tr}(\rho^{\alpha}\rho_{\beta}^{1-\alpha}) =$$



relevant deformation $\mathcal{O}_{\Delta}$ turned on and off along the Euclidean time
“Euclidean quantum quench”

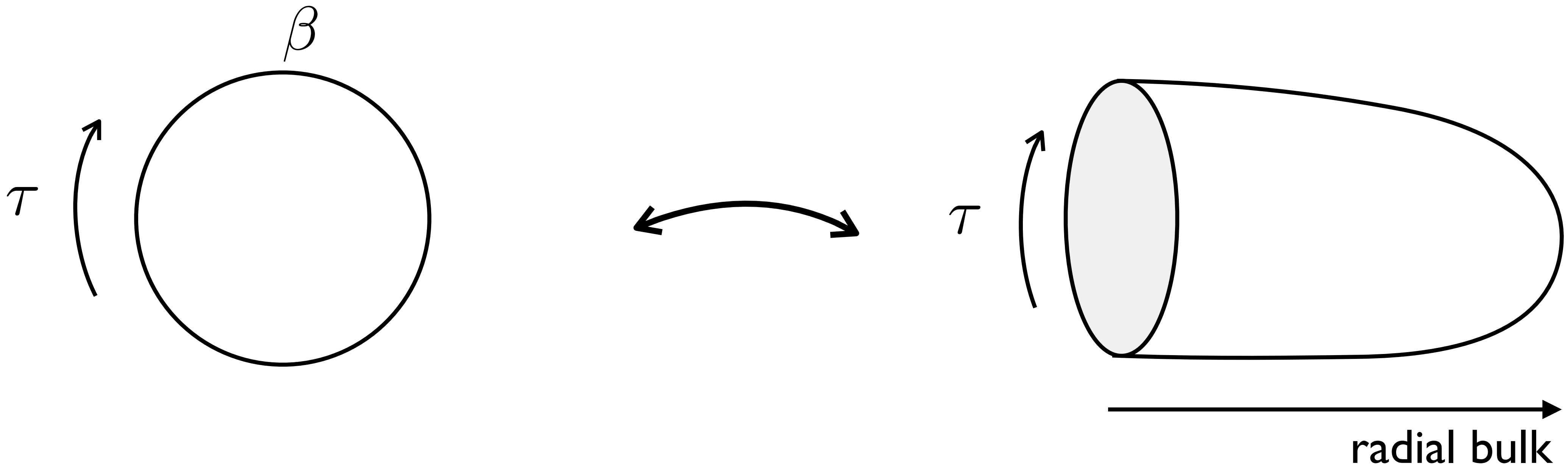
Notice : $0 \leq \alpha \leq 1$

Rényi Divergences in AdS/CFT

- Holographically:

CFT on thermal cylinder

AdS Euclidean BH



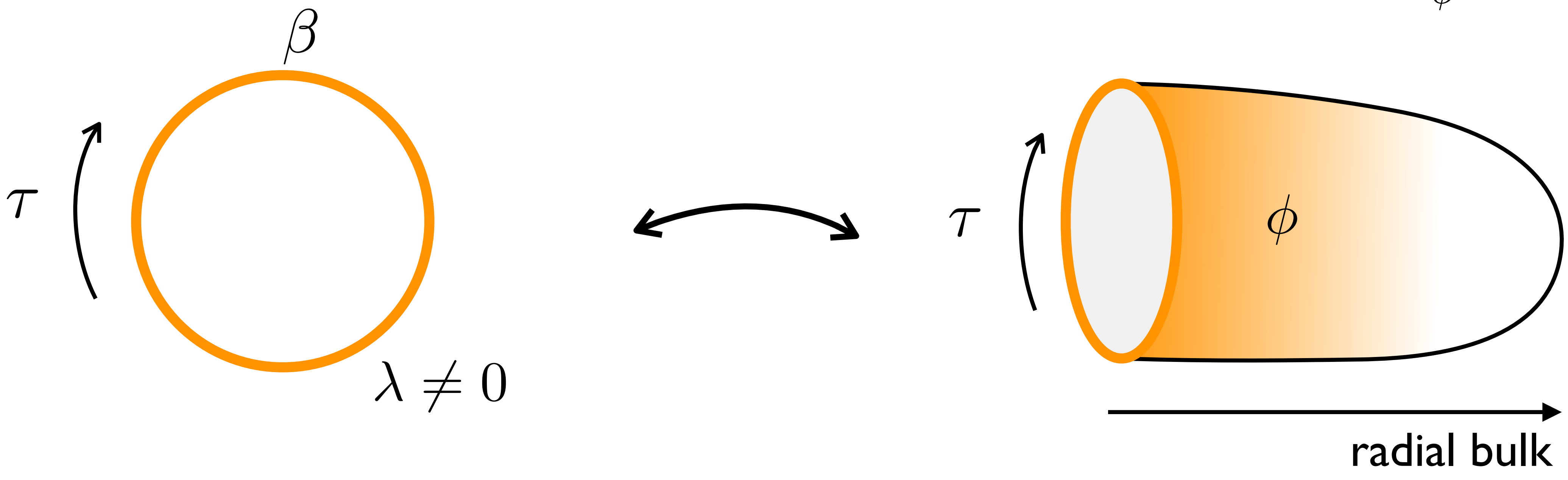
Rényi Divergences in AdS/CFT

- Holographically:

CFT on thermal cylinder + $\lambda \int dx \mathcal{O}_\Delta$

AdS Euclidean BH + bulk scalar

$$m_\phi^2 = \Delta(\Delta - 2)$$

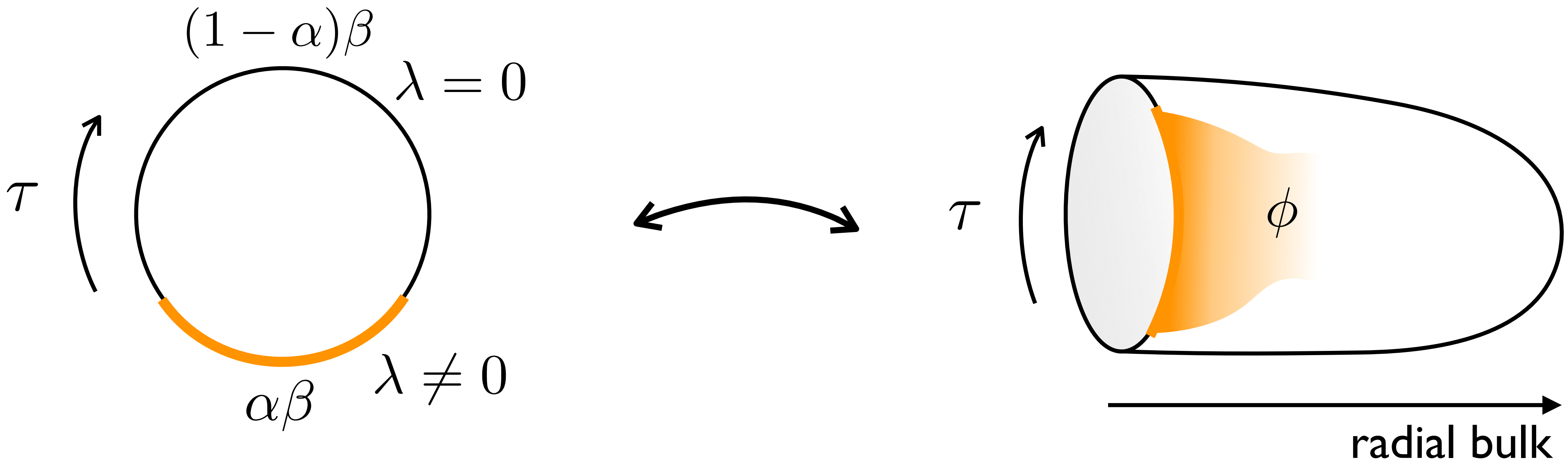


Rényi Divergences in AdS/CFT

- Holographically:

Euclidean quantum quench

AdS Euclidean BH + bulk scalar
time dependent b.dary conditions



$$Z_{\mathrm{CFT}} = \mathrm{tr}(\rho^{\alpha}\rho_{\beta}^{1-\alpha}) = Z_{\mathrm{Gravity}} = e^{-S_E(g,\phi)}$$

Rényi Divergences in AdS/CFT

- At leading order in scalar amplitude expansion $\lambda\beta^{2-\Delta}$

$$D_\alpha(\rho||\rho_\beta) = \frac{1}{\alpha-1}\log\frac{\mathrm{tr}(\rho^\alpha\rho_\beta^{1-\alpha})}{(\mathrm{tr}\rho)^\alpha(\mathrm{tr}\rho_\beta)^{1-\alpha}}$$

$$\approx \lambda^2\left(\frac{2\pi}{\beta}\right)^{2(\Delta-2)}\frac{c\,L}{6\pi\beta}\,\frac{(\Delta-1)^2}{2^{\Delta+3}}\,\frac{I(\alpha,\Delta)-\alpha I(1,\Delta)}{\alpha-1}$$

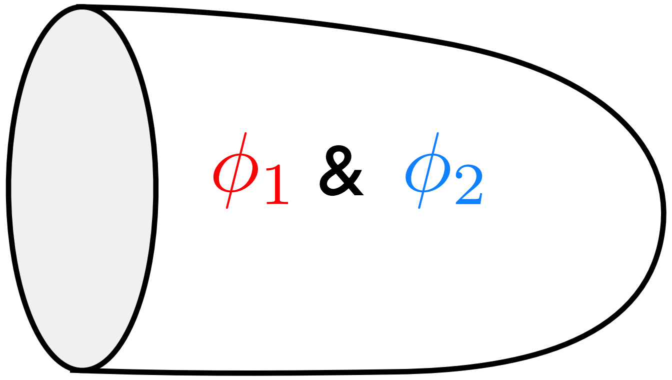
with

$$I(\alpha,\Delta) = \frac{2^{2-\Delta}\sqrt{\pi}\Gamma(\Delta)}{\Gamma\left(\Delta+\frac{1}{2}\right)}\int_0^{2\pi\alpha}dp\,(2\pi\alpha-p)\,F\left[\Delta,\Delta,\Delta+\frac{1}{2},\frac{1+\cos p}{2}\right]$$

$$I(1,\Delta) = \frac{2\pi^{3/2}\Gamma\left(\frac{1-\Delta}{2}\right)\Gamma\left(\frac{\Delta}{2}\right)^2}{\Gamma(\Delta)\Gamma\left(1-\frac{\Delta}{2}\right)}$$

New Holographic Constraints

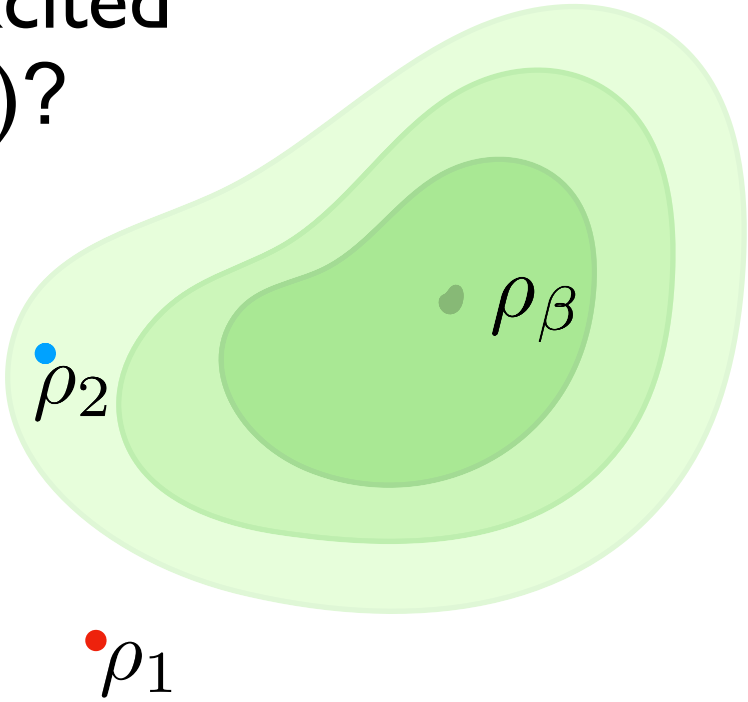
- Two scalar fields in AdS black hole



Bulk interaction

$$U(\phi_1, \phi_2) = g(\phi_1\phi_2^2 + \phi_2\phi_1^2)$$

Possible transition from a state ρ_1 with ϕ_1 excited to a state ρ_2 with ϕ_2 excited (at fixed energy)?



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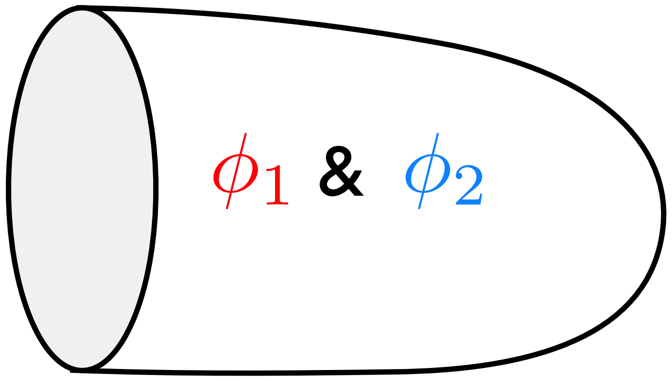

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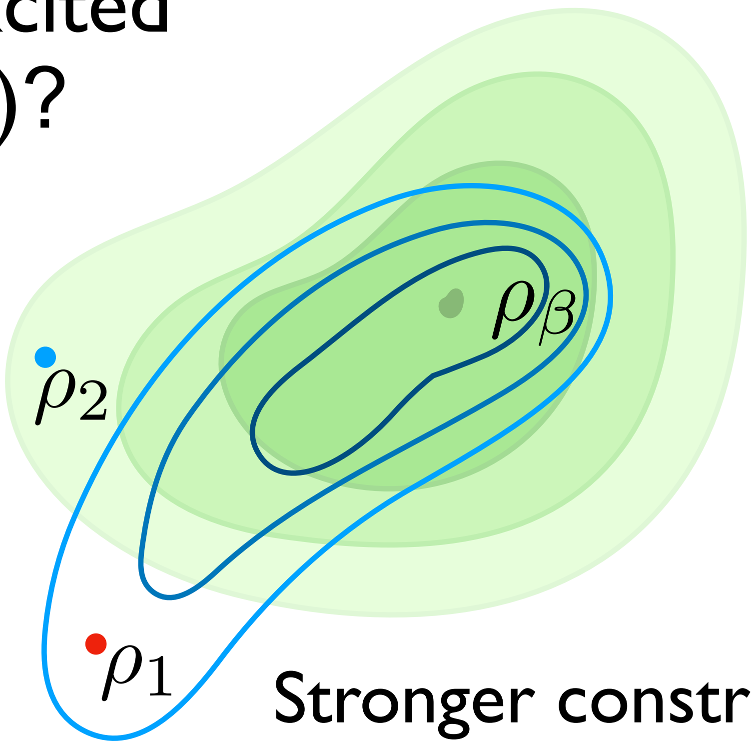
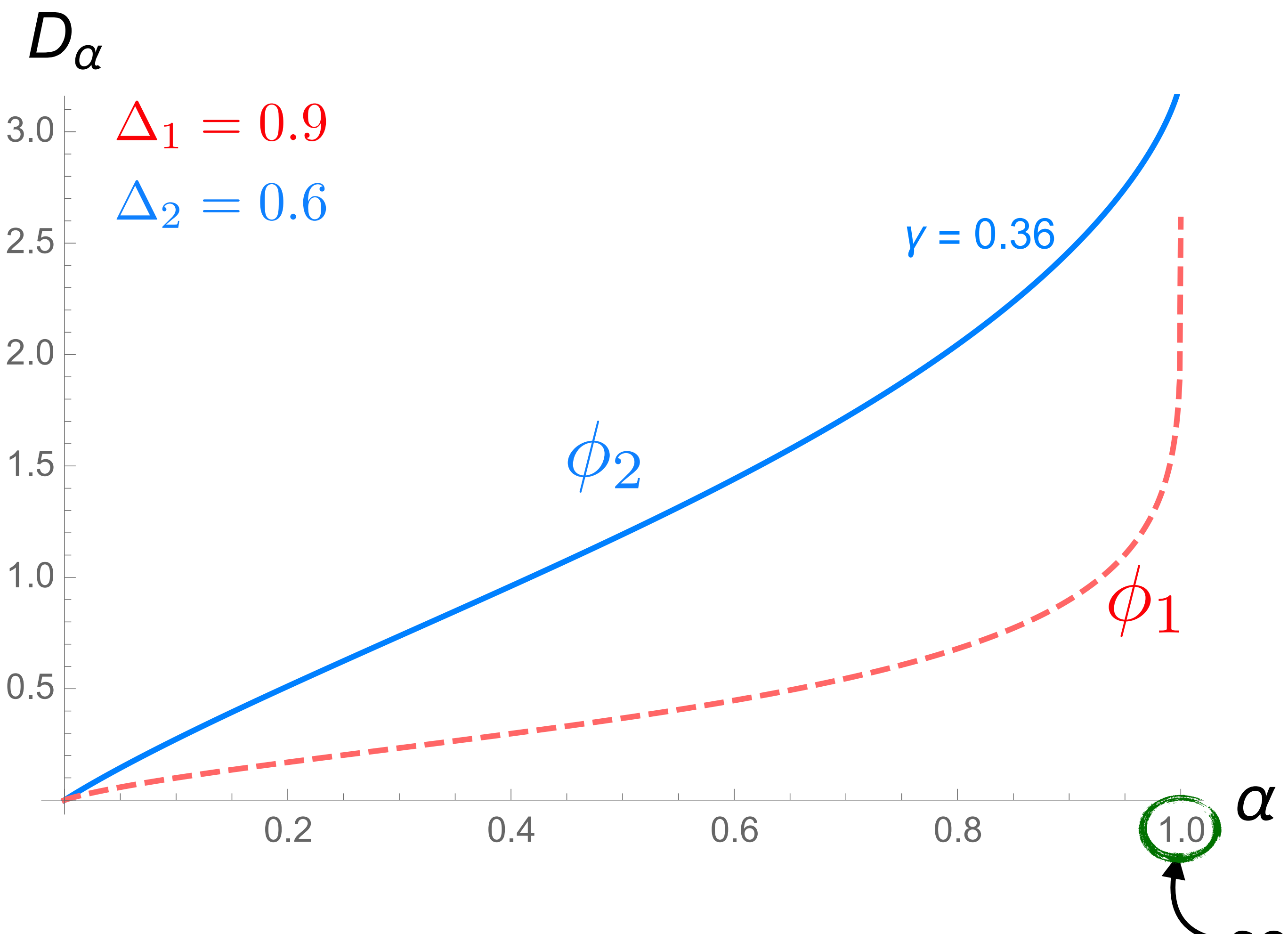


Stronger constraint than the second law?

New Holographic Constraints

- Fixed Δ_1 & Δ_2 varying $\gamma = \left(\frac{2\pi}{\beta}\right)^{\Delta_2-\Delta_1} \frac{\lambda_2}{\lambda_1}$



second law

- $\rho_1 \cancel{\rightarrow} \rho_2$
- D_α **redundant** with second law

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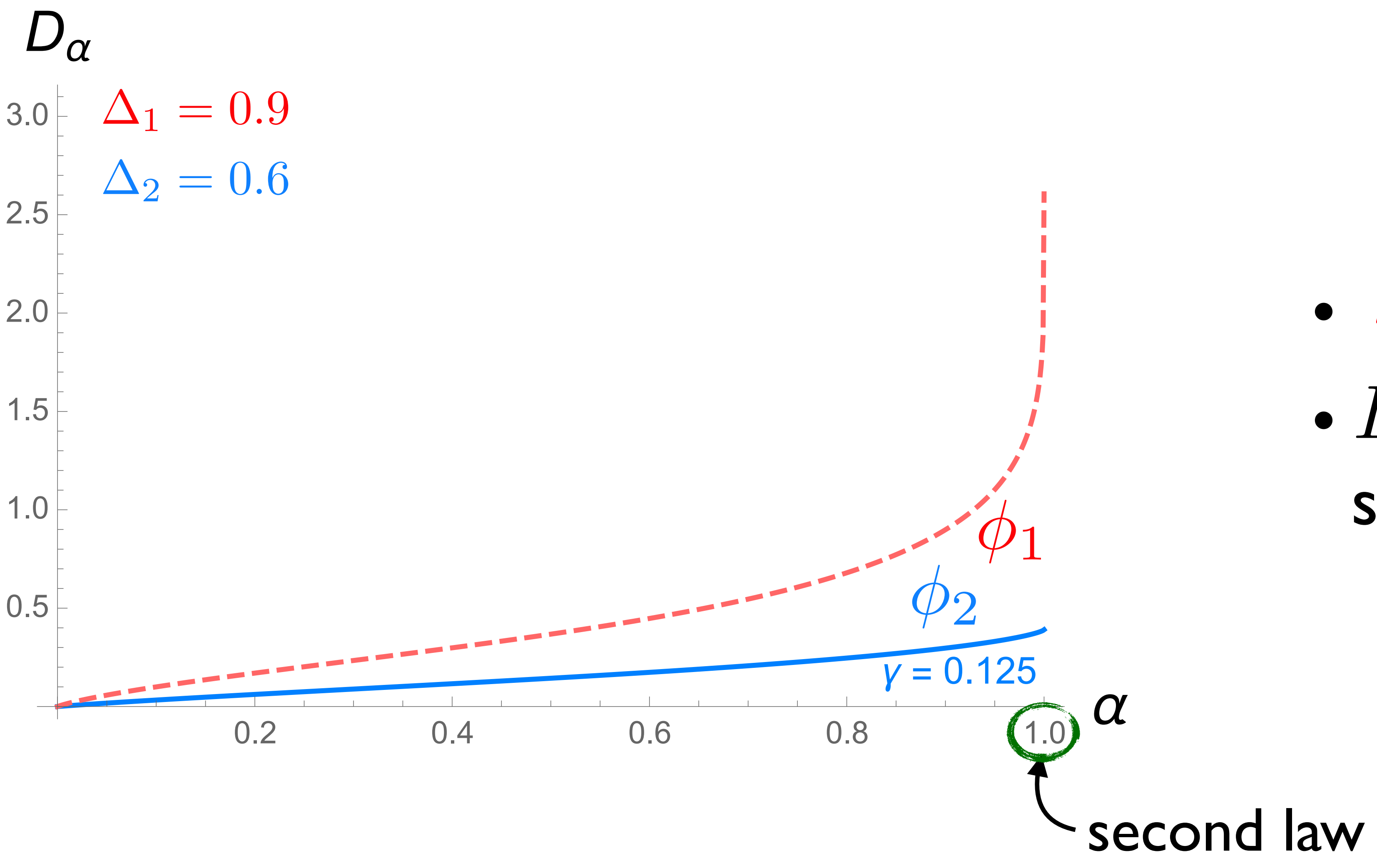

- $\rho_1 \longrightarrow \rho_2$ ✓
- D_α redundant with second law

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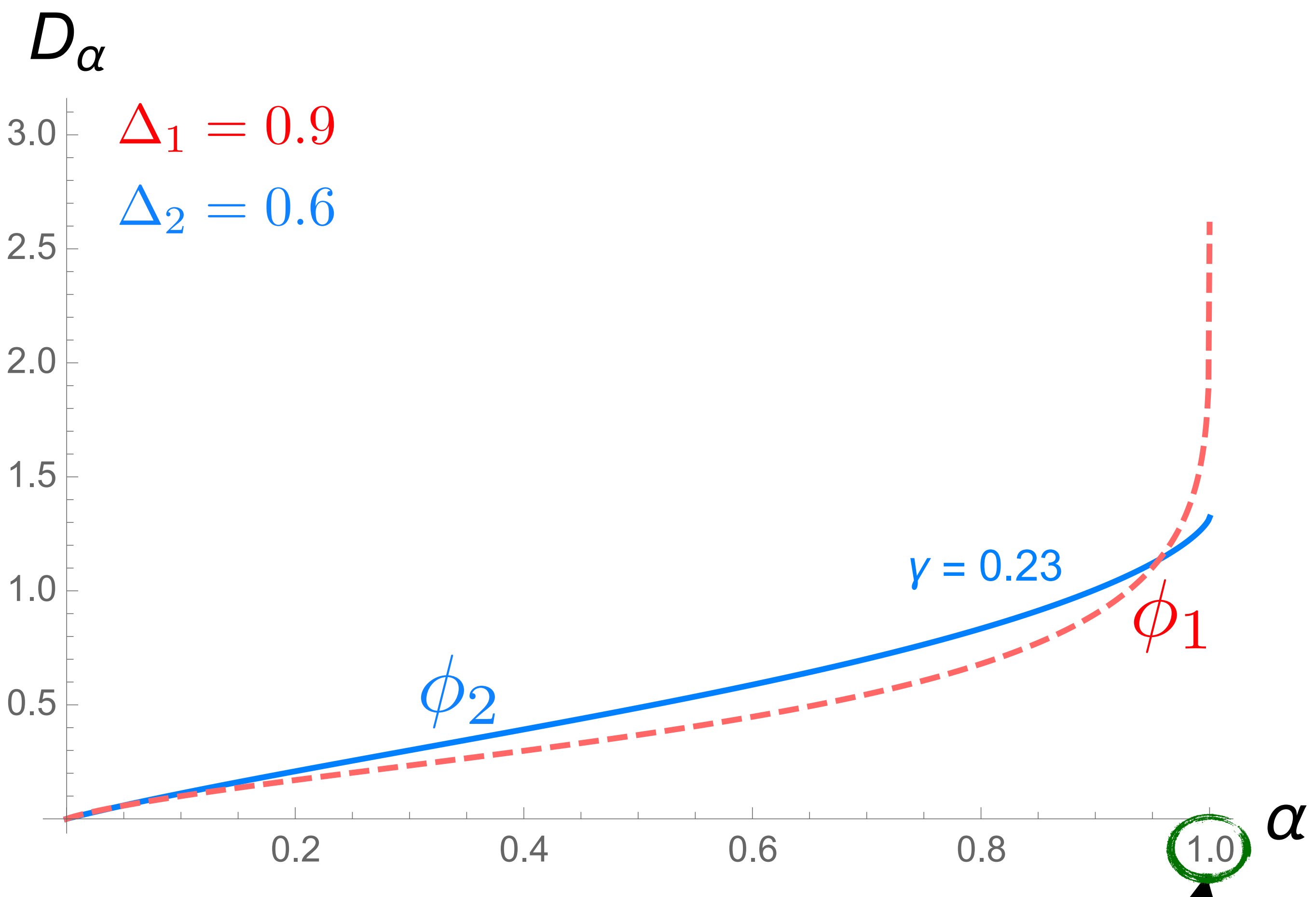

second law

- Transition allowed by the second law but forbidden for some values of α

$$\Longrightarrow \quad \rho_1 \cancel{\rightarrow} \rho_2$$

➡ Full range of D_α gives new constraints!

Conclusions & Outlook

- Quantum thermodynamics provides new constraints for field theory and gravitational dynamics
- Additional constraints come from varying the reference state

[Bernamonti, FG, Myers, Oppenheim '18]

$$D_\alpha(\rho||\sigma)$$

any equilibrium state

- More general states and larger range of α
- Compute D_α directly in gravity ?
  Additional laws for asymptotically flat black holes ? [in progress]
- Geometric interpretation ?

Thank you!