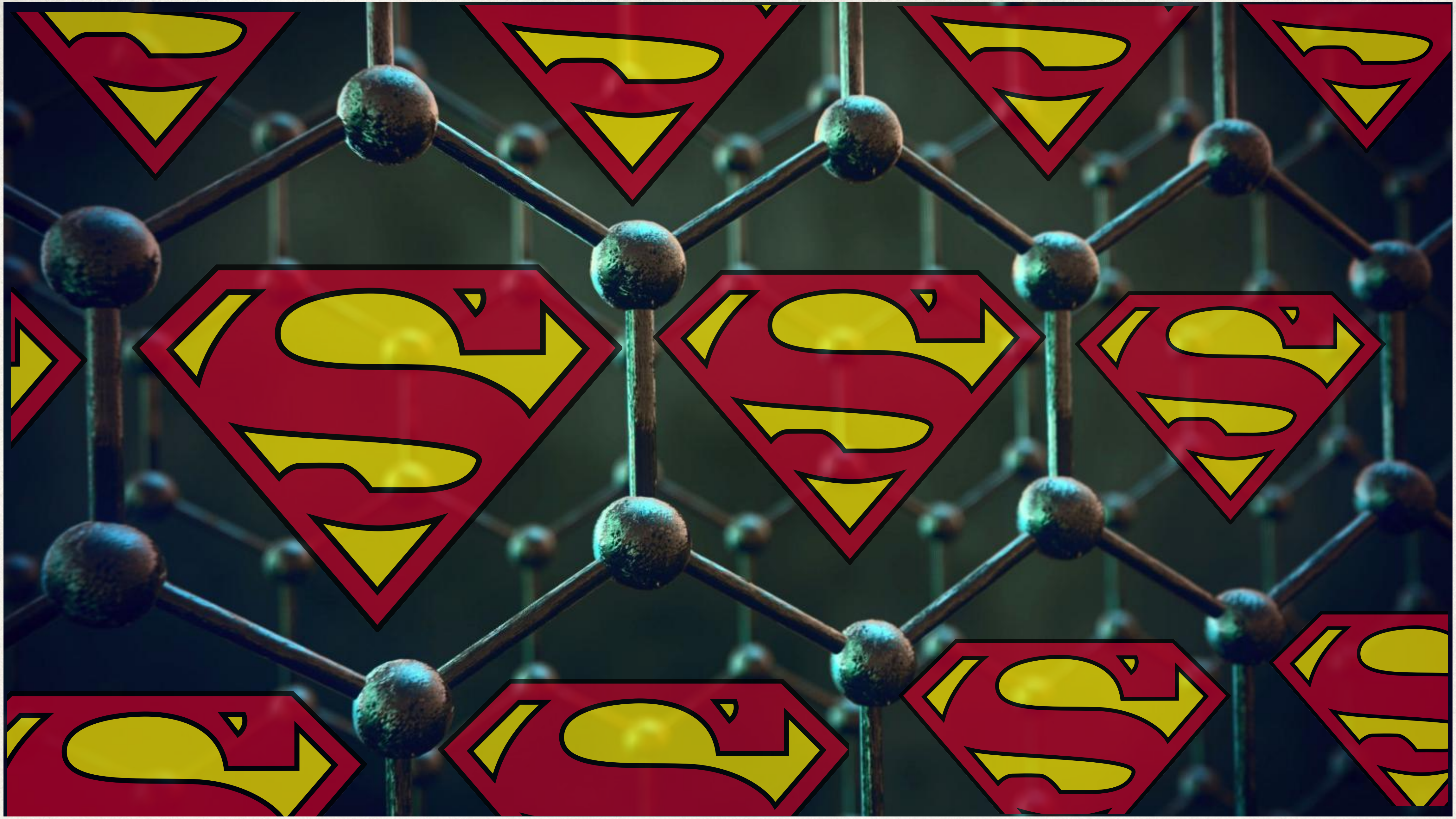

Graphene, Supersymmetry, and Boundary Conformal Field Theory

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(Huang, Jensen, Shamir, Virrueta)

I would like to convince you that studying

$$S = -\frac{1}{4}\int_{\mathcal{M}} \mathrm{d}^4x\, F^{\mu\nu}F_{\mu\nu} + \int_{\partial\mathcal{M}} \mathrm{d}^3x (i\bar{\psi}\not{D}\psi)$$

instead of the textbook

$$S = \int_{\mathcal{M}} \mathrm{d}^4x \left[-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + i\bar{\psi}\not{D}\psi\right]$$

is a very interesting thing to do.

Mixed dimensional QED has something for everyone

$$S = -\frac{1}{4}\int_{\mathcal{M}} \mathrm{d}^4x\, F^{\mu\nu}F_{\mu\nu} + \int_{\partial\mathcal{M}} \mathrm{d}^3x (i\bar{\psi}\not{D}\psi)$$

where $D_\mu = \nabla_\mu - igA_\mu$ boundary conditions: $F_{nA} = g\bar{\psi}\gamma_A\psi$

- relation to graphene
- relation to large N_f QED$_3$ (Kotikov-Teber '13)
- behavior under electric-magnetic duality (Son '17)
- example of a bCFT with an exactly marginal coupling
- supersymmetric versions
- playground for computing trace anomalies

our work

Relation to Graphene

Son's model of graphene (cond-mat/0701501):

$$-\sum_{a=1}^{N}\int dt\, d^2x(\bar{\psi}_a\gamma^0\partial_0\psi_a + v\bar{\psi}_a\gamma^i\partial_i\psi_a + iA_0\bar{\psi}_a\gamma^0\psi_a) + \frac{1}{2g^2}\int dt\, d^3x(\partial_i A_0)^2$$

things to note

- only electric interactions
- electrons travel at speed $v \approx c/300$

beta function for the electron velocity

$$p\frac{\partial v(p)}{\partial p} = -\frac{4}{\pi^2 N}v(p)$$

once v gets sufficiently large, can restore magnetic interactions and flow to a relativistic fixed point

Relation to large N_f QED$_3$

(Kotikov-Teber '13)

propagator for mixed dimensional QED (don't FT the normal direction y)

$$-i\frac{e^{-py}}{p}\eta^{AB}$$ (Feynman gauge)

propagator for large N_f QED$_3$, resummed

$$-i\frac{\eta^{AB}}{p^2(1+\Pi(p))} \quad \text{where} \quad \Pi(p) = \frac{N_f e^2}{8|p|} + O(N_f^0)$$

Compensated by vertices, 3d e drops out of the amplitudes.
For scattering processes on the boundary (y=0),
the Feynman rules are the same in the IR with the identification $\frac{1}{N_f} \sim g^2$

Behavior under EM Duality

(Hsiao-Son '17)

Using recent progress in 2+1 dimensional non-SUSY dualities

$$\int d^3x \left[i\bar{\Psi}\gamma^A(\partial_A - ia_A)\Psi - \frac{1}{4\pi}\epsilon^{ABC}A_A\partial_B a_C \right] - \frac{1}{4g^2}\int d^4x F_{\mu\nu}^2$$

Integrating out a_B and A_μ yields same mixed QED theory but with a new

$$\tilde{g} = 8\pi/g$$

Can use the duality to calculate the current-current and stress tensor correlation function at the self-dual point and at infinite coupling — calculate transport coefficients.

(similar in spirit to H, Kovtun, Sachdev, Son '07)

Mixed QED is a bCFT

$$g_0 Z_{A_\mu}^{1/2} Z_\psi = g Z_g$$

The usual Ward identity for QED relates $Z_\psi = Z_g$

The superficial degree of divergence of the photon self energy is one (compared with two in four dimensional QED).

The gauge invariant prefactor $p_\mu p_\nu - \delta_{\mu\nu} p^2$ of $\Pi^{\mu\nu}(p)$ cuts down the degree of divergence to -1.

In other words, Z_γ is finite.

$\Longrightarrow$ coupling is not perturbatively renormalized.

Our work…

$\mathcal{N} = 1$ Super Graphene

$$S_{\text{bulk}} = \int_{\mathcal{M}} \mathrm{d}^4x \left(-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{g^2\theta}{16\pi^2} F^{\mu\nu} \widetilde{F}_{\mu\nu} + \frac{i}{2} \bar{\lambda} \gamma^\mu \partial_\mu \lambda + \frac{1}{2} D^2 \right)$$

$$S_{\text{bry}} = \int_{\partial\mathcal{M}} \mathrm{d}^3x \Big(i\widetilde{\psi}\Gamma^A D_A \psi - |\mathrm{D}_A \phi|^2 + |F|^2 + ig\big(\widetilde{\lambda}_+ \psi \phi^* - \widetilde{\psi} \lambda_+ \phi\big) - \frac{1}{4} \bar{\lambda} \gamma^5 e^{\eta\gamma^5} \lambda - \frac{g^2\theta}{8\pi^2} \widetilde{\lambda}_+ \lambda_+ \Big)$$

$\mathcal{N} = 1$ Super Graphene

$$S_{\text{bulk}} = \int_{\mathcal{M}} \mathrm{d}^4x \left(-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{g^2\theta}{16\pi^2} F^{\mu\nu} \widetilde{F}_{\mu\nu} + \frac{i}{2} \bar{\lambda} \gamma^\mu \partial_\mu \lambda + \frac{1}{2} D^2 \right)$$

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possibility of a
theta term (that we
could have added before)

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$$- \frac{1}{4}\bar{\lambda}\gamma^5 e^{\eta\gamma^5}\lambda - \frac{g^2\theta}{8\pi^2}\widetilde{\lambda}_+\lambda_+ \Big)$$

possibility of a
theta term (that we
could have added before)

SUSY gives us
a photino and auxiliary
field *D* as well

$\mathcal{N} = 1$ Super Graphene

$$S_{\text{bulk}} = \int_{\mathcal{M}} \mathrm{d}^4x \left(-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{g^2\theta}{16\pi^2} F^{\mu\nu} \widetilde{F}_{\mu\nu} + \frac{i}{2} \bar{\lambda} \gamma^\mu \partial_\mu \lambda + \frac{1}{2} D^2 \right)$$

$$S_{\text{bry}} = \int_{\partial\mathcal{M}} \mathrm{d}^3x \Big(i\widetilde{\psi}\Gamma^A D_A \psi - |\mathrm{D}_A \phi|^2 + |F|^2 + ig\big(\widetilde{\lambda}_+ \psi \phi^* - \widetilde{\psi} \lambda_+ \phi\big) - \frac{1}{4} \bar{\lambda} \gamma^5 e^{\eta\gamma^5} \lambda - \frac{g^2\theta}{8\pi^2} \widetilde{\lambda}_+ \lambda_+ \Big)$$

boundary breaks half the SUSY;
to preserve the remainder,
these terms are needed

$\mathcal{N} = 1$ Super Graphene

$$S_{\text{bulk}} = \int_{\mathcal{M}} \mathrm{d}^4x \left(-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{g^2\theta}{16\pi^2} F^{\mu\nu} \widetilde{F}_{\mu\nu} + \frac{i}{2} \bar{\lambda} \gamma^\mu \partial_\mu \lambda + \frac{1}{2} D^2 \right)$$

$$S_{\text{bry}} = \int_{\partial\mathcal{M}} \mathrm{d}^3x \Big(i\widetilde{\psi}\Gamma^A D_A \psi - |\mathrm{D}_A \phi|^2 + |F|^2 + ig\big(\widetilde{\lambda}_+ \psi \phi^* - \widetilde{\psi}\lambda_+ \phi\big) - \frac{1}{4} \bar{\lambda} \gamma^5 e^{\eta\gamma^5} \lambda - \frac{g^2\theta}{8\pi^2} \widetilde{\lambda}_+ \lambda_+ \Big)$$

the photon and photino couple
to the boundary electron and selectron

$\mathcal{N} = 1$ Super Graphene

$$S_{\text{bulk}} = \int_{\mathcal{M}} \mathrm{d}^4 x \left(-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{g^2 \theta}{16\pi^2} F^{\mu\nu} \widetilde{F}_{\mu\nu} + \frac{i}{2} \bar{\lambda} \gamma^\mu \partial_\mu \lambda + \frac{1}{2} D^2 \right)$$

$$S_{\text{bry}} = \int_{\partial\mathcal{M}} \mathrm{d}^3 x \Big(i\widetilde{\psi} \Gamma^A D_A \psi - |\mathrm{D}_A \phi|^2 + |F|^2 + ig\big(\widetilde{\lambda}_+ \psi \phi^* - \widetilde{\psi} \lambda_+ \phi\big) - \frac{1}{4} \bar{\lambda} \gamma^5 e^{\eta\gamma^5} \lambda - \frac{g^2 \theta}{8\pi^2} \widetilde{\lambda}_+ \lambda_+ \Big)$$

$\mathcal{N} = 2$ Super Graphene

- The photino is symplectic Majorana instead of just Majorana
- Two extra bulk scalars, X and Y with corresponding extra Yukawa terms.
- The boundary multiplet can be kept the same, but there is now a preserved U(1) R-symmetry

$\mathcal{N} = 2$ Super Graphene

$$S_{\text{bulk}} = \int_{\mathcal{M}} \mathrm{d}^4x \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{i}{2} \bar{\lambda}_i \gamma^\mu \partial_\mu \lambda^i - \frac{1}{2} (\partial_\mu X)^2 - \frac{1}{2} (\partial_\mu Y)^2 + \frac{1}{2} \vec{D}^2 \right)$$

$$\begin{aligned} S_{\text{bry}} = \int_{\partial\mathcal{M}} \mathrm{d}^3x \Bigg(& -\frac{1}{4} \bar{\lambda}_i \, \vec{v} \cdot \vec{\tau}^{\,i}{}_j \gamma^5 e^{\eta\gamma^5} \lambda^j - X(\vec{v} \cdot \vec{D} + \partial_n X) \\ & + i\widetilde{\psi} \Gamma^A D_A \psi - |D_A \phi|^2 + |F|^2 + \sqrt{2} i g \big(\phi^* \, \widetilde{\lambda}_+ \psi - \phi \, \widetilde{\psi} \lambda_+ \big) \\ & + g \widetilde{\psi} Y \psi - g^2 |\phi|^2 Y^2 - g(\vec{v} \cdot \vec{D} + \partial_n X) |\phi|^2 \Bigg) \end{aligned}$$

…we could do something similar with $\mathcal{N} = 4$ super graphene

Claim: Mixed dimensional QED along with $\mathcal{N} = 1, 2,$ and 4 super graphene are all bCFTs where the gauge coupling is exactly marginal.

Put Graphene Like Theories to Work Computing Trace Anomalies…

Trace Anomaly with a Codimension One Boundary

K_{AB} extrinsic curvature
hat on K removes trace

2D $\langle T^\mu{}_\mu \rangle = \frac{c}{24\pi}(R + 2K\delta(x^n))$

Jensen-O'Bannon ('15) b-theorem

$a_{\rm UV} > a_{\rm IR}$

3D $\langle T^\mu{}_\mu \rangle = \frac{1}{4\pi}(-aR + b\,\mathrm{tr}\,\hat{K}^2)\delta(x^n)$

4D $\langle T^\mu{}_\mu \rangle = \frac{1}{16\pi^2}(cW^2 - aE.D. + (-b_1\,\mathrm{tr}\,\hat{K}^3 + b_2 K^{AB} W_{nAnB})\delta(x^n))$

Solodukhin-Fursaev ('16) conjecture

5D $\langle T^\mu{}_\mu \rangle \sim \delta(x_\perp)(b_1 W^2 + b_2 K^4 + \ldots)$

$b_2 = 8c$

6D $\langle T^\mu{}_\mu \rangle \sim a\,E.D. + c_1 W^3 + c_2 W^3 + c_3 W\Box W + \delta(x_\perp)(b_1 K W^2 + \ldots)$

Results for Boundary Charges

Can we say anything more about

3d

aR conjecture

$b\,\mathrm{tr}\,\hat{K}^2$ yes

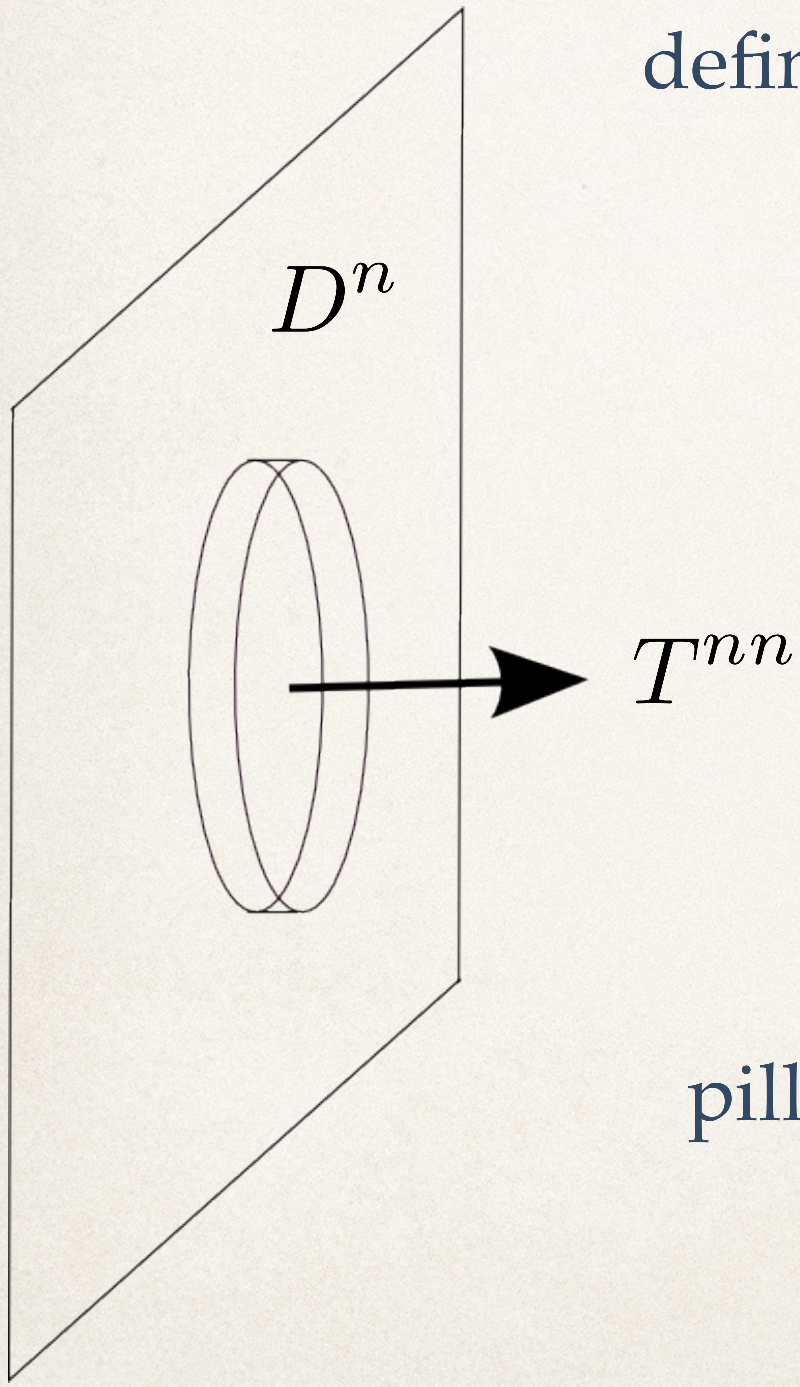
4d

$b_1\,\mathrm{tr}\,\hat{K}^3$ yes

$b_2 K^{AB} W_{AnBn}$ yes

Related to displacement operator two and three point functions

Displacement Operator



definition: operator sourced by small changes in the embedding

$$\frac{\delta I}{\delta x^n} \equiv D^n$$

diffeomorphism Ward identity:

$$\partial_\mu T^{\mu n} = D^n \delta(x^n)$$

$$\partial_\mu T^{\mu A} = 0$$

tangential components still conserved

pill box argument implies

$$T^{nn}(\vec{x}, x^n)|_{x^n=0} = D^n(\vec{x})$$

Results

$$\langle D^n(\vec{x}) D^n(0) \rangle = \frac{c_{nn}}{|\vec{x}|^{2d}}$$

$$\langle D^n(\vec{x}) D^n(\vec{x}') D^n(0) \rangle = \frac{c_{nnn}}{|\vec{x}|^d |\vec{x}'|^d |\vec{x} - \vec{x}'|^d}$$

3d

aR (?)

$b \operatorname{tr} \hat{K}^2$ $\quad b = \frac{\pi^2}{8} c_{nn}$

4d

$b_1 \operatorname{tr} \hat{K}^3$ $\quad b_1 = \frac{2\pi^3}{35} c_{nnn}$

$b_2 K^{AB} W_{AnBn}$ $\quad b_2 = \frac{2\pi^4}{15} c_{nn}$

Argument analogous to Osborn and Petkou's result for *c* in 4d. (Fails for *a* in 3d because R is topological.)

Next: Look at b_1 and b_2 for the graphene-like theories

Summary of Perturbative Results

$$b_1^{(\mathcal{N}=0)} = \frac{16}{35} - \frac{3g^2}{35} N_f + \mathcal{O}(g^4)$$

$$b_2^{(\mathcal{N}=0)} = \frac{4}{5} - \frac{g^2}{10} N_f + \mathcal{O}(g^4)$$

$$b_1^{(\mathcal{N}=1)} = \frac{3}{5} - \frac{9g^2 N_f}{40} + \mathcal{O}(g^4)$$

$$b_2^{(\mathcal{N}=1)} = 1 - \frac{g^2 N_f}{4} + \mathcal{O}(g^4)$$

$$b_1^{(\mathcal{N}=2)} = \frac{38}{45} - \frac{19g^2 N_f}{60} + \mathcal{O}(g^4)$$

$$b_2^{(\mathcal{N}=2)} = \frac{4}{3} - \frac{g^2 N_f}{3} + \mathcal{O}(g^4)$$

$$b_1^{(\mathcal{N}=4)} = \frac{4}{3} - \frac{g^2 N_f}{2} + \mathcal{O}(g^4)$$

$$b_2^{(\mathcal{N}=4)} = 2 - \frac{g^2 N_f}{2} + \mathcal{O}(g^4)$$

in all theories, b_1 and b_2 depend on the coupling

a and c are unaffected by the coupling

(note however $b_1^{(\mathcal{N}=4)} - b_2^{(\mathcal{N}=4)}$ is coupling independent at one loop)

Marginal Directions

b_1 and b_2 depend on the exactly marginal coupling!

Unlike the situation for the bulk charges a and c in 4d.

Wess-Zumino consistency forces a
to be constant along marginal directions.

No such argument for c. However, SUSY fixes c
to be a constant, and it's unknown how to construct
4d CFTs with marginal directions but without SUSY
(and without boundaries).

In contrast, the charges b_1 and b_2 are not protected by supersymmetry.

Effect of the Theta Term

An integer part of θ can be traded for a boundary CS term, but the mixed dimensional nature means θ is neither integral nor periodic

$$\tan(\alpha) \equiv \frac{g^2\theta}{4\pi^2}$$

The perturbative results above hold with the replacement

$$g \to g\cos\alpha$$

The θ term "screens" the boundary charges by "rotating" the boundary condition.

$$b \,\mathrm{tr}\, \hat{K}^2$$

Summary of results

$$b_1 \,\mathrm{tr}\, \hat{K}^3$$

$$b_2 K^{AB} W_{AnBn}$$

- Presented graphene like theories both with and without SUSY that are bCFTs with an exactly marginal coupling.
- Related boundary central charges in three and four dimensional bCFTs to two and three point functions of the displacement operator.
- Discussed graphene like theories as examples where $b_2 \neq 8c$ and where both b_1 and b_2 depend on a marginal coupling.

Future Projects

- Verify our proposal for b_2 by computing it directly, for mixed QED in a curved space-time.
- Higher codimension defects. (Billo, Goncalves et al. '16)
- Find bounds on these boundary central charges. (Hofman-Maldacena '08)
- Computation of these central charges in AdS/CFT, for example in Janus solutions. (Takayanagi '11, Miao et al., Astaneh et al. '17)
- Models with only boundary interactions, like mixed QED.
- Boundary bootstrap. (Liendo et al. '12)

Larger Vision: Structure of QFT

- Constrain QFT by constraining CFTs
- Provide a more local view of QFT by figuring out how to deal with boundaries.

Thanks to my collaborators

- Kuo-Wei Huang (postdoc, Boston U)
- Kristan Jensen (faculty, San Francisco State)
- Itamar Shamir (postdoc, KCL)
- Julio Virrueta (PhD student, Stony Brook)

Extra Slides

$b\,\mathrm{tr}\,\hat{K}^2$ in 3d

(similar arguments for b_1 and b_2)

The term in the trace anomaly can be produced from an effective anomaly action in limit ε goes to zero where μ is a UV regulator:

$$I^{(b)} = \frac{b}{4\pi}\frac{\mu^\epsilon}{\epsilon}\int_{\partial M}\mathrm{tr}\,\hat{K}^2$$

For small deviations from planarity $K_{AB} \approx \partial_A \partial_B x^n$

$\Longrightarrow$ scale dependence of displacement 2-pt function

$$\mu\partial_\mu \langle D^n(\vec{x}) D^n(0)\rangle = \frac{b}{4\pi}\Box^2\delta(\vec{x})$$

The short distance behavior of $\langle D^n(\vec{x}) D^n(0)\rangle = \dfrac{c_{nn}}{|\vec{x}|^6}$

can be regulated by writing instead $\langle D^n(\vec{x}) D^n(0)\rangle = \dfrac{c_{nn}}{512}\Box^3(\log\mu^2\vec{x}^2)^2$

(Freedman, Johnson, Latorre '92)

Checks for Free Fields

Using heat kernel methods, a number of these charges were computed for free fields in the late 80s and early 90s (Melmed, Moss, Dowker, Schofield) and later revisited in the last couple of years (Solodukhin, Fursaev, Jensen, Huang, CPH).

$$b^{s=0} = \frac{1}{64}\,(\mathrm{D\ or\ R})\ , \quad b^{s=\frac{1}{2}} = \frac{1}{32}\ ,$$

$$b_1^{s=0} = \frac{2}{35}(\mathrm{D})\ , \quad b_1^{s=0} = \frac{2}{45}(\mathrm{R})\ , \quad b_1^{s=\frac{1}{2}} = \frac{2}{7}\ , \quad b_1^{s=1} = \frac{16}{35}\ ,$$

$$b_2 = 8c$$

The displacement operator correlation functions yield the same results!

Why is $b_2 = 8c$ for free fields?

conformal symmetry means the stress tensor 2-pt functions depends on a single function $\alpha(v)$ of a cross ratio

$$v = \frac{(x-x')^2}{(x-x')^2 + 4x^n x'^n}$$

$v \to 1:$ boundary limit

$v \to 0:$ coincident limit

For free theories $\alpha(v) \sim 1 + v^{2d}$

theory without boundary

effect of image points on other side of the boundary

$$\Longrightarrow \; 2\alpha(0) = \alpha(1)$$

$\alpha(0) \sim c$ by the old Osborn-Petkou ('93) argument

What about interactions?

Wilson-Fisher fixed point for ϕ^4 scalar field theory, starting in 4d

McAvity and Osborn ('93, '95) showed, both in the ε expansion and in a large N expansion that

$$2\alpha(0) \neq \alpha(1)$$

Downside: We need to be in exactly 4d to connect to b_2, and in exactly 4d ϕ^4 scalar field theory is free

We need some more examples …. enter our graphene like theories.

Perturbative corrections to α(1)

In standard Feynman gauge, for Maxwell theory with a boundary

$$\langle A_A(x)A_B(x')\rangle = \delta_{AB}\left(\frac{1}{(x-x')^2}+\frac{1}{(\vec{x}-\vec{x}')^2+(y+y')^2}\right)$$

(Neumann boundary condition for tangential components.)

The coupling to the boundary produces a small change

$$\langle A_A(x)A_B(x')\rangle = \delta_{AB}\left(\frac{1}{(x-x')^2}+\frac{1-O(g^2)}{(\vec{x}-\vec{x}')^2+(y+y')^2}\right)$$

For the stress tensor two point function

$$\implies \alpha(1) = \alpha(0)(2-O(g^2))$$

Conjecture for a in 3d

The coefficients can be written in terms of the stress tensor two point function

$$a_{(3d)} = \frac{\pi^2}{9}\left(\epsilon(1) - \frac{3}{4}\alpha(1) + 3C\right)$$

$$\langle T_{\mu\nu}(\mathbf{x}, y) T_{\rho\sigma}(\mathbf{0}, y')\rangle = A_{\mu\nu,\rho\sigma}(\mathbf{x}, y, y')\frac{1}{|\mathbf{x}|^{2d}} \ ,$$

$$A_{AB,CD}(\mathbf{x}, y, y') = \alpha(v)\frac{d}{d-1} I^{(d)}_{AB,CD} + \left(2\epsilon(v) - \frac{d}{d-1}\alpha(v)\right) I^{(d-1)}_{AB,CD}$$

gives the right answer for free fields

leads to a possible bound $\quad \dfrac{a_{(3d)}}{b} \geq -\dfrac{2}{3}$

(see also recent work by Prochazka)

$b_1 \operatorname{tr} \hat{K}^3$ in 4d

Analogous to b in 3d. Now we need to match scale dependent contributions of

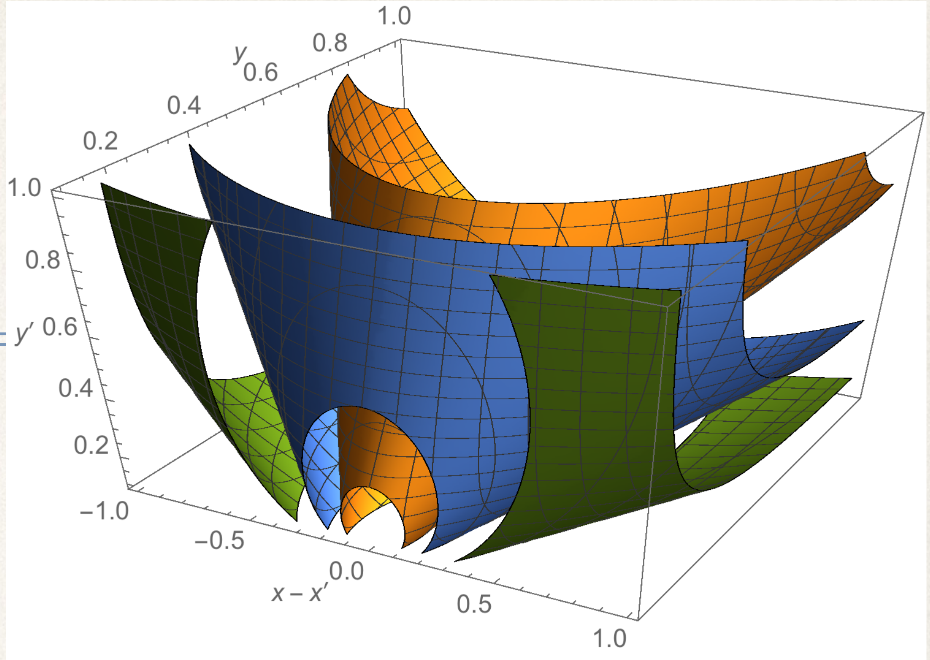
$$I^{(b_1)} = \frac{b_1}{16\pi^2} \frac{\mu^\epsilon}{\epsilon} \int_{\partial M} \operatorname{tr} \hat{K}^3$$

and

$$\langle D^n(\vec{x}) D^n(\vec{x}') D^n(0) \rangle = \frac{c_{nnn}}{|\vec{x}|^d |\vec{x}'|^d |\vec{x} - \vec{x}'|^d}$$

scale dependence of 3-pt function relies on recent work by Bzowski, Skenderis, and McFadden '13

$b_2 K^{AB} W_{nAnB}$ in 4d



Could extract from $\langle D^n \rangle_{W \neq 0}$

We'll do something more roundabout.

$$I^{(b_2)} = \frac{b_2}{16\pi^2} \frac{\mu^\epsilon}{\epsilon} \int_{\partial M} KW$$

We will cancel $\dfrac{\delta^2 I^{(b_2)}}{\delta g_{\mu\nu} \delta g_{\lambda\rho}}$

$$I^{(c)} = \frac{c}{16\pi^2} \frac{\mu^\epsilon}{\epsilon} \int_{M} W^2$$

against a boundary term in $\dfrac{\delta^2 I^{(c)}}{\delta g_{\mu\nu} \delta g_{\lambda\rho}}$

An order of limits issue spoils the naive relation between b_2 and c.

Aside on Stress Tensor 2-point function with boundary

Depends on a function $\alpha(v)$ of a cross ratio $\quad v = \dfrac{(x-x')^2}{(x-x')^2 + 4x^n x'^n}$

$v \to 1$: boundary limit

$v \to 0$: coincident limit

$$\langle T_{\mu\nu}(x) T_{\rho\sigma}(x') \rangle = \frac{1}{(x-x')^8} \left(\frac{4\alpha}{3} I_{\mu\nu,\rho\sigma} + (v^2 \partial_v^2 \alpha) \hat{I}_{\mu\nu,\rho\sigma} - \frac{1}{6} (v \partial_v \alpha)(\hat{\beta}_{\mu\nu,\rho\sigma} + 7 \hat{I}_{\mu\nu,\rho\sigma}) \right)$$

same tensor structure that appears in absence of a boundary

precise form of $\hat{\beta}_{\mu\nu,\rho\sigma}$ and $\hat{I}_{\mu\nu,\rho\sigma}$ not so important

essential point is that they can't cancel the b_2 boundary term

$b_2 K^{AB} W_{nAnB}$ in 4d (part 2)

$$I^{(c)} = \frac{c}{16\pi^2}\frac{\mu^\epsilon}{\epsilon}\int_M W^2$$ is valid in the coincident limit.

To reproduce the scale dependence of the 2-pt function in the boundary limit, $v \to 1$, we should use this effective action with a different value of c, in order to match to $\alpha(1)$.

Canceling $\dfrac{\delta^2 I^{(b_2)}}{\delta g_{\mu\nu}\delta g_{\lambda\rho}}$ against $\dfrac{\delta^2 I^{(c)}}{\delta g_{\mu\nu}\delta g_{\lambda\rho}}$

then leads to a relation between $\alpha(1)$, i.e. c_{nn}, and b_2.