Aspects of HISH: -
-Holography Inspired stringy hadron model

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There is a lore that the "physics" at large N and large $\lambda$ describes well that of $N=3$ and $\lambda \sim 1$.

For certain properties this is indeed the case and the passage involves $1/N$ or even $1/N^2$ corrections.

But there are circumstances where this is not the case. Here are some examples for large N:

- **Baryon** of $N=3$ is different from the very heavy object of large N number of quarks.
- **Nuclear matter** - At large N nuclear matter is a solid and only for $N \leq 8$ it behaves like a liquid.
Introduction

large $\lambda$ implies large tension of the flux tube

The behavior of hadrons at large tension and at order one is quite different.

It translates to the difference between the behavior of a string and of a particle

Lattice simulations show that properties like Wilson lines admit stringy behavior.

This obviously reflects on the nature of the holography needed to describe hadron physics.
Introduction

The **holographic duality** is after all an equivalence between certain bulk **string theories** and **boundary field theories**.

 Practically most of the applications of holography are based on relating **bulk fields** (not **strings**) and **operators** on the dual **boundary field theory**.

 This is based on the usual limit of $\alpha' \to 0$ with which we go for instance from a **closed string theory** to a **gravity theory**.

 However, to describe real hadrons one needs **strings** since after all in reality the string tension is not very large ($\lambda$ of order one).
Introduction

There is a wide range of hadronic physical observables which cannot be faithfully described by bulk fields but rather require dual stringy phenomena.

It is well known that this is the case for Wilson, ‘t Hooft and Polyakov lines.

We argue here that in fact also the spectra decays width and scattering amplitudes of mesons, baryons, exotics and glueballs can be recast only by holographic stringy hadrons.
The major argument against describing the hadron spectra in terms of fluctuations of fields like bulk fields or modes on probe branes is that they generically do not admit properly the Regge-like behavior of the spectra.

For $M^2$ as a function of $J$ we get from flavor branes only $J=0, J=1$ mesons and there will be a big gap of order $\lambda$ in comparison to higher $J$ mesons if we describe the latter in terms of strings.

The attempts to get the linearity between $M^2$ and $n$ basically face problems whereas for strings it is an obvious property.

We argue that also to account for the decay width and scattering amplitudes one needs strings and not particles.
Outline

Introduction:
(a) Large versus finite N and λ
(b) Hadron String/gauge holography versus gravity/gauge holography

Review of the HISH model

Summary of past results:
(a) Spectra of mesons and baryons
(b) Predictions about glueballs and exotics
(c) Decay rates of hadrons

New aspects:
(a) The quantization of a string with particles on its ends
(b) The propagator of such strings
A brief review of Holography
Inspired stringy hadron model
The construction of the HISH model is based on the following steps.

(i) Analyzing string configurations that correspond to hadrons in confining holographic backgrounds.

(ii) Devising a transition from the holographic regime of large $N_c$ and large $\lambda$ to the real world that bypasses formal expansions $1/N_c$ and $1/\lambda$.

(iii) Proposing a model of stringy hadrons in flat four dimensions with massive endpoint particles that is inspired by the corresponding holographic model.

(iv) Dressing the endpoint particles with charge, spin, baryonic vertex, etc.

(v) Confronting the outcome of the models with experimental data.
The structure of a **holographic meson** is a **rotating string** that starts and ends on flavor branes (the same or different). For instance, a heavy quarkonium is

- large mass
- flavour brane
- intermediate mass
- flavour brane
- infrared “wall”
In the generalized Sakai Sugimoto model the meson looks like
Example: The $B$ meson
The vertical segments of the holographic hadronic string can be mapped to massive particles at the ends of the string.
In holography a baryon is a baryonic vertex which is a wrapped Dp brane on a p cycle and is connected with Nc strings to flavor branes.

The preferable layout is the asymmetric one.
A possible dynamical baryon is with $N_c$ strings symmetrically connected to the flavor brane and to the BV which is also on the flavor brane.
Naturally the analog at $N_c=3$ of the symmetric configuration with a central baryonic vertex is the old Y shape baryon.

The analog of the asymmetric setup with one quarks on one end and $N_c-1$ on the other is a straight string with quark and a di-quark on its ends.
In HISH the holographic baryon is mapped into a single string that connects a quark on one side and a diquark on the other side.
(3) Holographic Glueballs-

**Mesons** are open strings with a massive quark and an anti-quark on its ends.

**Baryons** are open strings with a massive quarks connected to a baryonic vertex

**What are glueballs?**

Since they do not incorporate quarks it is natural to assume that they are rotating folded closed strings
Closed strings versus open strings

- The spectrum of states of a **closed** string admits
  \[ M^2 = \frac{2}{\alpha'} \left( N + \tilde{N} + A + \tilde{A} \right) \]

- The spectrum of an **open** string
  \[ M^2_{\text{open}} = \frac{1}{\alpha'} (N + A) \]

- The slope of the closed string is $\frac{1}{2}$ of the open
- The closed string **ground states** has
  \[ M^2 = \frac{2}{\alpha'} (A + \tilde{A}) = \frac{2 - D}{6\alpha'} \]

- The **intercept** is 2 **times** that of an open string
Holographic Exotics

A tetra quark based on a baryonic vertex connected to a u c di-quark and connected to an anti-baryonic vertex which is connected to anti- u and anti- c
HISH exotic hadrons

In the same way that we map holographic mesons and baryons we can also map from holographic to HISH exotics
Past results of the HISH model
Spectrum of Mesons and Baryons
The spectra fits

- The **best fits** of HISH to meson states

![Graphs showing the fits of HISH to meson states](image)
To emphasize the deviation from linearity due to the massive endpoints here are the botomonium trajectories.
The spectra fits of Nucleons

Trajectories for even and odd J nucleons
Predictions for the search of Glueballs and Exotic Hadrons
Glueball $0^{++}$ fits of experimental data

The meson and glueball trajectories based on $f_0(1380)$ as a glueball lowest state.
Based on the $Y(4630)$ that was observed to decay predominantly to $\Lambda_c^+ \Lambda_c^-$. If we assume that it is on a Regge-like trajectory and we borrow the slope and the endpoint masses from the $J/\Psi$ trajectory we get

$$
\begin{array}{|c|c|c|}
\hline
n & \text{Mass} & \text{Width} \\
\hline
 0 & 4634^{+9}_{-11} & 92^{+41}_{-32} \\
 1 & 4902\pm95 & 103\pm46 \\
 2 & 5148\pm99 & 114\pm51 \\
 3 & 5378\pm104 & 124\pm55 \\
 4 & 5594\pm109 & 134\pm60 \\
\hline
\end{array}
$$

$$
\begin{array}{|c|c|c|}
\hline
J^{PC} & \text{Mass} & \text{Width} \\
\hline
1^{--} & 4634^{+9}_{-11} & 92^{+41}_{-32} \\
2^{++} & 4791\pm64 & 98\pm44 \\
3^{--} & 4939\pm66 & 105\pm47 \\
4^{++} & 5080\pm67 & 111\pm49 \\
5^{--} & 5215\pm69 & 117\pm52 \\
\hline
\end{array}
$$
The Decay Width of Hadrons
The decay of a long string in critical flat space-time

The decay of a hadron is in fact the breaking of a string into two strings.

Obviously a type I open string can undergo such a split.
The decay of a long string in critical flat space-time

The total decay width is related by the optical theorem to the imaginary part of the self-energy diagram

\[
2 \text{ Im } \left( \begin{array}{c}
\text{Diagram}
\end{array} \right) = \sum_f \left| \begin{array}{c}
\text{Diagram}
\end{array} \right|^2
\]

A trick that Polchinski et al. used is to compactify one space coordinate and consider incoming and outgoing strings that wrap this coordinate so one can avoid an annulus open string diagram and instead compute a disk diagram with simple vertex operator of a closed string
The string amplitude
The decay of a long string in critical flat space-time

We would like to determine the dependence of the string amplitude on the string length $L$

$$iA_2 = \frac{iTN}{g^2} L \left[ \frac{\kappa}{2\pi \sqrt{L}} \right]^2 \int_{|z|<1} d^2 z \langle : e^{ip \cdot X(0)} : e^{-ip \cdot X(z)} : \rangle$$

- open string coupling
- Gravitational coupling
- Zero mode
- Normalization of the vertex
- Vertex operator
The decay of a long string in flat space-time

Substituting the propagators, the amplitude reads

\[
iA_2 = \frac{iTN\kappa^2}{2\pi g^2} \lim_{t \to 0} \frac{\Gamma(t - 1)\Gamma(1 - \tilde{J})}{\Gamma(t - \tilde{J})}
\]

\[
= \frac{iTN\kappa^2}{2\pi g^2} \left( \tilde{J} \partial \tilde{J} \ln[\Gamma(-\tilde{J})] + \lim_{t \to 0} \frac{\tilde{J}}{t} \right)
\]

The imaginary part \(\sum_k \pi k \delta(J - k)\) for \(k = 1, \ldots\)

\[
\text{Im}A_2 = -\frac{iTN\kappa^2}{2g^2} \tilde{J}
\]

\(\tilde{J} \equiv \frac{L^2T}{2\pi} - 2\)
The decay of a long string in flat space-time

Since $A^2$ is the mass square shift the total decay width

$$\Gamma = -\text{Im} \delta(m) = -\text{Im} \frac{1}{2m} \delta(m^2) = \frac{TN\kappa^2}{4g^2} \frac{J}{E}$$

The leading behavior for string in $d=26$ is

$$\frac{\Gamma}{L} = \frac{g^2T^{13}N}{4(4\pi)^{12}}$$

$$\Gamma = \frac{TN\kappa^2}{4g^2} \left[ L_{tot} + \frac{4\pi}{T} \frac{1}{L_{tot}} \right]$$

$$L_{tot} = \sqrt{L^2 - \frac{8\pi}{T}}$$
The decay width of a string with massive endpoints

The decay of a string with massive particles on its ends.

The dependence on the masses:
(a) The length \( L(m_1, m_2) \)
(b) The boundary conditions (not anymore Neumann)
(c) The vertex operators should be modified
The decay width of a string with massive endpoints

The main result is the linearity with $L$ with a universal constant $A$

$$\Gamma = \frac{\pi}{2} Atlantis(M, m_1, m_2, T).$$

For small endpoint masses we can expand *

$$\Gamma \propto \frac{\pi}{4} TL + \frac{\pi}{4} m - \frac{2\sqrt{2}}{3} m^{3/2} (TL)^{-1/2} + O(L^{-3/2}).$$
Test case: the decay of the $K^*$ states

- Different methods of inserting the intercept
- blue $J \rightarrow J - a$, red $L^2 \rightarrow L^2 - 2a/T$, yellow force bc
The suppression factor for stringy holographic hadrons

- The horizontal segment of the stringy hadron **fluctuates** and can reach flavor branes.
- When this happens the string may **break up** , and the two new endpoints connect to a flavor brane.
Determining the suppression factor

Assuming first that the string stretches in flat spacetime we found (J.S, K. Peeters, M. Zamamklar) using both a string bit model and a continues one that

\[ \Gamma = \text{const.} \cdot \exp \left( -1.0 \frac{z_B^2}{\alpha'_{\text{eff}}} \right) \cdot T_{\text{eff}} P_{\text{split}} \cdot L \]

There are further corrections due to the curvature and due to the massive endpoints. The final result is

\[ \Gamma = \exp \left( -2\pi C(T_{\text{eff}}, M, m_i) \frac{m_{\text{sep}}^2}{T_{\text{eff}}} \right) \]

\[ C(T_{\text{eff}}, M, m_i) \approx 1 + c_c \frac{M^2}{T_{\text{eff}}} + \sum_{i=1}^{2} c_m \frac{m_i}{M} \]
The basic process of a string splitting into two strings can of course repeat itself and thus eventually describe a decay of a single string into $n$ strings.
The probability of a string to decay into \( n \) strings is

\[
P = \frac{T_{\text{eff}}^2}{\pi^3} \sum_{n=1}^{\infty} \frac{1}{n^2} \sum_{\{n_f\}} \prod_f \exp \left( -2\pi \tilde{C} \frac{m_{\text{sep},f}^2}{T_{\text{eff}} n_f} \right).
\]

We sum over all combinations of \( n_f \) such that

\[
\sum_f n_f = n.
\]

String fragmentation is supposed to be the generator of jets in hadron and \( e^+e^- \) collisions.

It is incorporated in the event generator Pythia.

In our picture the endpoint quark anti-quark do not carry momenta.
New aspects of HISH
The Quantization of the string with massive endpoints
Four decades ago Chodos and Thorn suggested a model of a **bosonic string with two massive particles on its ends**.

Their motivation probably was the **physics of a flux tube and quark and an anti-quark on its ends**.

Through the years this model has been addressed but never fully quantized and clearly not in four dimension. This is what we have recently achieved.

For us this model is an approximation of the **stringy hadron in holographic model (HISH)**.

On root to the quantization of the model we have further developed an **alternative method of renormalization**.

In certain respects the **hadronic string is better behaved that the ordinary bosonic string**.
The classical string with massive endpoints

- We start with the action and equations of motion.
- The action is the **NG action** plus two relativistic point-particle action terms.

\[ S = S_{st} + S_{pp}|_{\sigma=-\ell} + S_{pp}|_{\sigma=\ell} \]

\[ S_{st} = -T \int d\tau d\sigma \sqrt{-h} = -T \int d\tau d\sigma \sqrt{\dot{X}^2 X'^2 - (\dot{X} \cdot X')^2} \]

- The world sheet coordinates and the induced metric is

\[ h_{\alpha\beta} = \eta_{\mu\nu} \partial_{\alpha} X^\mu \partial_{\beta} X^\nu \]

- The relativistic particle action

\[ S_{pp} = -m \int d\tau \sqrt{-\dot{X}^2} \]
The equations of motion

The **bulk equation of motion**

\[ \partial_\alpha (\sqrt{-h} h^{\alpha \beta} \partial_\beta X^\mu) = 0 \]

The **boundary equation of motion**

\[ T \sqrt{-h} \partial^\sigma X^\mu \pm m \partial_\tau \left( \frac{\dot{X}^\mu}{\sqrt{-\dot{X}^2}} \right) = 0 \]

The + for \( \sigma = \ell \) and – for \( \sigma = -\ell \).
Rotation solution of the equations of motion

A rotating solution in the (1,2) plane

\[ X^0 = \tau, \quad X^1 = R(\sigma) \cos(k\tau), \quad X^2 = R(\sigma) \sin(k\tau) \]

For any choice of \( R(\sigma) \).

Substituting it to the boundary equation of motion of motion

\[ T \sqrt{\frac{(1 - k^2 R^2) R'^2}{R'}} \mp m \frac{k^2 R}{\sqrt{1 - k^2 R^2}} = 0 \]

Which translates into the tension being balanced by the centrifugal force

\[ \frac{T}{\gamma} = \frac{2\gamma m \beta^2}{L} \]
The classical energy and angular momentum

E and J for the rotating string are

\[
E = \frac{2m}{\sqrt{1 - \beta^2}} + TL \frac{\arcsin \beta}{\beta}
\]
\[
J = \frac{mL\beta}{\sqrt{1 - \beta^2}} + \frac{1}{4} TL^2 \frac{\arcsin \beta - \beta \sqrt{1 - \beta^2}}{\beta^2}
\]

In the limit of small masses

\[
J = \frac{1}{2\pi T} E^2 \left( 1 - \frac{8\sqrt{\pi}}{3} \left( \frac{m}{E} \right)^{3/2} + \frac{2\pi^{3/2}}{5} \left( \frac{m}{E} \right)^{5/2} + \ldots \right)
\]

In the limit of large masses

\[
J_4 = \frac{2m^{1/2}}{T3^{1/3}} (E - 2m)^{3/2} + \frac{7}{\sqrt{1083m^{1/2}T}} (E - 2m)^{5/2}
\]
\[
- \frac{1003}{\sqrt{2332803Tm^{3/2}}} (E - 2m)^{7/2}
\]
The quantum string with massive endpoints
Consider the **quantum fluctuations** around the rotating solution

\[ X^\mu = X_{\text{cl}}^\mu + \delta X^\mu = (t, \rho, \theta, z^i) = (\tau + \lambda \delta t, R(\sigma) + \lambda \delta \rho, \kappa \tau + \lambda \delta \theta, \lambda \delta z^i) \]

Where \( \lambda \) is a formal expansion parameter later will be expressed in terms of \( \frac{1}{mL} \).

We fix the reparameterization invariance using the orthogonal gauge

\[ \frac{1}{2} (h_{\tau \tau} + h_{\sigma \sigma}) = \frac{1}{2} (\dot{X}^2 + X''^2) = 0 \]

\[ h_{\tau \sigma} = h_{\sigma \tau} = \dot{X} \cdot X' = 0 \]

We further use the static gauge

\[ \tau = X^0 \quad \Rightarrow \quad \delta t = 0 \]

\[ R(\sigma) = \sigma \quad \text{or} \quad R(\sigma) = \frac{1}{k} \sin(k \sigma) \]
Transverse fluctuations

Consider the fluctuations transverse to the plane of rotation. We truncate the action to second order

\[ S_{st,\delta z} = -T \lambda^2 \int d\tau d\sigma \left[ \frac{1}{2} \left( \sqrt{R'^2 g} \right)^{-1} \delta z'^2 - \frac{1}{2} \left( \sqrt{R'^2 g} \right) \delta \dot{z}^2 \right] \]

\[ S_{pp,\delta z} = m\lambda^2 \int d\tau \frac{1}{2} \gamma \delta \ddot{z}^2 \]

To properly normalize the kinetic term define

\[ f_t \equiv (\sqrt{R'^2 g})^{1/2} \delta z \]

For the case

\[ R(\sigma) = \frac{1}{k} \sin(k\sigma) \]

\[ S_{st,\delta z} = -T \lambda^2 \int d\tau d\sigma \left( \frac{1}{2} f_t'^2 - \frac{1}{2} f_t^2 \right) \]

\[ S_{pp,\delta z} = m\lambda^2 \int d\tau \frac{1}{2} \gamma f_t^2 \]
The transverse fluctuations

- The world sheet Hamiltonian

\[ H = \frac{1}{2} T \lambda^2 \left( \int_{-\ell}^{\ell} d\sigma \left( \dot{f}_t^2 + f_t^2 \right) + \frac{\gamma m}{T} f_t^2 |_{\pm \ell} \right) \]

- The Mode expansion and canonical quantization

\[ f_t = f_0 + i \sqrt{N} \sum_{n \neq 0} \frac{a_n}{\omega_n} e^{-i\omega_n \tau} f_n(\sigma) \]

- The canonical quantization condition

\[ [f_t(\sigma), \pi_t(\sigma')] = i \delta(\sigma - \sigma') \]

- Can be achieved upon choosing

\[ N = \frac{1}{2T\ell^2\lambda^2} \]
The transverse fluctuations

The fluctuations have to obey the bulk EOM

\[ f''_n + \omega_n^2 f_n = 0 \]

The boundary equations are

\[ T f'_n + \gamma m \omega_n^2 f_n = 0 \]

The eigenfrequencies are subject to

\[ 2\delta \cot(\delta) x \cos(2x) - (\delta^2 - \cot^2(\delta) x^2) \sin(2x) = 0 \]

Where

\[ x \equiv \omega_n \ell \quad \delta \equiv k \ell = \arccos(\gamma^{-1}) \]
The Eigenfrequencies
Energy and angular momentum

E and J associated with the transverse modes are

\[
E_{st,ft} = T\chi^2 \int_{-\ell}^{\ell} d\sigma \frac{1}{2} \frac{1}{\cos^2(k\sigma)} (f_t'^2 + f_t^2)
\]

\[
E_{pp,ft} = \frac{1}{2} m \chi^2 \gamma^3 f_t^2
\]

\[
J_{st,ft} = T\chi^2 \int_{-\ell}^{\ell} d\sigma \frac{1}{2} \tan^2(k\sigma) \frac{k}{k} (f_t'^2 + f_t^2)
\]

\[
J_{pp,ft} = \frac{1}{2} m \chi^2 \gamma^3 \frac{\sin^2(k\ell)}{k} f_t^2
\]
The transverse modes

The first transverse modes
The quantum intercept
The intercept

We would like to determine the impact of the fluctuations on $E$ and $J$, namely the quantum correction of the trajectory.

We found before the classical $E$ and $J$

$$E = E(m, T, \gamma) \quad J = J(m, T, \gamma)$$

The classical trajectory is defined by

We define the intercept as

$$a = \langle \delta J - \frac{1}{k} \delta E \rangle$$

We proved that the intercept is given in terms of the expectation value of the world sheet Hamiltonian

$$a = -\frac{1}{k} \langle H \rangle$$
The intercept

- Using the **mode expansion** and the **orthogonality relations** we get

\[ H = -\frac{1}{2} \frac{T \chi^2}{2} \frac{1}{T \ell \chi^2} \sum_{n \neq 0} \alpha_{-n} \alpha_n = \frac{1}{2\ell} \sum_{n \neq 0} \alpha_{-n} \alpha_n \]

- Thus the contribution of the **transverse modes** is

\[ a_t = -\frac{1}{k} \langle H \rangle = -\frac{1}{2} \sum_{n > 0} \frac{\omega_n \ell}{\delta} \]

- For the **massless case** since \( \delta = \frac{\pi}{2} \) and \( \omega_n \ell = \frac{\pi}{2} n \)

\[ a_t(m = 0) = -\frac{1}{2} \sum_{n > 0} n = \frac{1}{24} \]

- Using **zeta function** regularization
The renormalization of the sum of the eigenfrequencies

The zeta function cannot be used for the massive case. We convert the infinite sum into a contour integral using Cauchy integral formula

\[ \frac{1}{2\pi i} \int dz \frac{dz}{z} \log f(z) = \frac{1}{2\pi i} \int dz \frac{f'(z)}{f(z)} = \sum_j n_j z_j - \sum_k \tilde{n}_k \tilde{z}_k \]

We will use a function \( f(\omega) \) with only simple zeros at \( \omega = \omega_n \), which are on the positive real axis
The renormalization of the sum of the eigenfrequencies

The sum of the eigen-frequencies is the Casimir energy

\[ E_C = \frac{1}{2} \sum_{n=1}^{\infty} \omega_n = -\frac{2\beta a}{L} \]

The semi-circle regularizes the Casimir energy

We renormalize the result in the same way that we do for the Casimir effect. We subtract from the force for a string on length \( L \) the one of an infinite string

\[ E_C^{(\text{ren})} = \lim_{\Lambda \to \infty} \left( E_C^{(\text{reg})}(m, T, L) - E_C^{(\text{reg})}(m, T, L \to \infty) \right) \]
The renormalization of the sum for the massless case

For the ordinary string with no endpoint particles

\[ f(\omega) = \sin(\pi \omega \ell) = 0 \]

Usually we use the zeta function renormalization

\[ \sum_{n=1}^{\infty} n = \zeta(-1) = -\frac{1}{12} \]

Using the contour integral method

\[
E_C(m = 0) = \frac{1}{2} \sum_{n=1}^{\infty} \omega_n = \frac{1}{4\pi i} \oint \omega \frac{f'(\omega)}{f(\omega)} d\omega = \frac{1}{4i} \int \omega \ell \cot(\pi \omega \ell) d\omega
\]

\[
\frac{1}{4i} \int \omega \ell \cot(\pi \omega \ell) d\omega = -\frac{1}{4} \int_{-\Lambda}^{\Lambda} y \ell \coth(\pi y \ell) dy + \frac{1}{4} \Lambda^2 \ell \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{2i\theta} \cot(\pi \Lambda e^{i\theta}) d\theta
\]
The intercept for the massless case

The regularized energy reads

\[
E_C^{(\text{reg})} = \left( -\frac{\Lambda^2 \ell}{4} - \frac{1}{24\ell} \right) + \frac{1}{2} \Lambda^2 \ell = \frac{\Lambda^2 \ell}{4} - \frac{1}{24\ell} = \frac{\Lambda^2 L}{8} - \frac{1}{12L}
\]

The corresponding force

\[
F_C^{(\text{reg})} = -\frac{d}{dL} E_C^{(\text{reg})} = -\frac{\Lambda^2}{8} + \frac{1}{12L^2}
\]

The renormalized force

\[
F_C^{(\text{ren})} = \lim_{\Lambda \to \infty} \left( F_C^{(\text{reg})}(L) - F_C^{(\text{reg})}(L \to \infty) \right) = \frac{1}{12L^2}
\]

The renormalized energy

\[
E_C^{(\text{ren})} = -\frac{1}{12L} \quad \Rightarrow \quad a = \frac{1}{24}
\]
The renormalization for the rotating massive string

- The integral of the renormalized Casimir energy

\[ E_C^{\text{ren}} = \frac{1}{\pi} \int_0^\infty \log \left( \frac{2y\beta^2 \sqrt{1 - \beta^2} \cosh \left( \frac{2y \arcsin \beta}{\beta} \right) + (\beta^4 + (1 - \beta^2)y^2) \sinh \left( \frac{2y \arcsin \beta}{\beta} \right)}{\frac{1}{2} \left( (1 - \beta^2)y^2 + 2\beta^2 \sqrt{1 - \beta^2} + \beta^4 \right) \exp \left( \frac{2y \arcsin \beta}{\beta} \right)} \right) \]

- The corresponding result for the intercept reads

\[ a_t = -\frac{1}{2\pi \beta} \int_0^\infty \log \left[ 1 - \exp \left( -\frac{4 \arcsin \beta}{\beta} \frac{y - \gamma \beta^2}{y + \gamma \beta^2} \right)^2 \right] \]

- This goes back to the standard massless result

\[ a_t(\beta \to 1) = -\frac{1}{2\pi} \int_0^\infty \log (1 - e^{-2\pi y}) = \frac{1}{24} \]

- For small masses

\[ a_t = \frac{1}{24} - \frac{11}{360\pi} \gamma^3 + \ldots = \frac{1}{24} - \frac{11}{360\pi} \left( \frac{2m}{TL} \right)^{3/2} \]
The contribution of the a trasverse mode to the intercept
The contribution to the intercept of the planar mode

We have done the analysis also for the planar mode

The eigenmodes and eigenfrequencies
The planar mode

The contribution of the **planar mode** to the intercept
The Quantization of the non-critical string
It is well known that the quantization of the Polyakov string in non-critical dimensions is done by adding to the action the Liouville term.

Polchinsky and Strominger mapped this in the formulation of the NG string by taking a “composite Liouville” mode:

\[ \phi = -\frac{1}{2} \log (\partial_+ X \cdot \partial_- X) \]

In the orthogonal gauge, the PS term reads:

\[ S_{PS} = \frac{B}{2\pi} \int d^2\sigma \partial_+ \phi \partial_- \phi = \frac{B}{2\pi} \int d^2\sigma \frac{\partial_+^2 X \cdot \partial_- X (\partial_-^2 X \cdot \partial_+ X)}{(\partial_+ X \cdot \partial_- X)^2} \]
The renormalization of the PS term

- The contribution of the PS to the intercept is achieved by substituting the classical solution into the PS action

\[ E_{PS} = \langle H_{PS} \rangle = - \int d\sigma \mathcal{L}_{PS} = \frac{B}{2\pi} \int_{-\ell}^{\ell} d\sigma k^2 \tan^2(k\sigma) = \frac{B}{\pi} k(\tan \delta - \delta) \]

where \( \delta = k\ell \) The term \( k\tan \delta \) diverges in the massless case since \( \delta = \pi/2 \)
- For small masses it is finite but un-physically large
- Hellerman et al renormalized the PS term for the massless case by

\[ S_{PS} \rightarrow S^{(reg)}_{PS} = \frac{B}{2\pi} \int d^2\sigma \frac{(\partial_+^2 X \cdot \partial_- X)(\partial_-^2 X \cdot \partial_+ X)}{(\partial_+ X \cdot \partial_- X)^2 + \alpha' \epsilon^4 (\partial_-^2 X \cdot \partial_+^2 X)} \]

- And adding a counterterm

\[ S_{ct} \propto \frac{1}{\epsilon} \int d\tau (\dot{X} \cdot \ddot{X})^{1/4} \]
The renormalization of the PS term

The result of the renormalization of the PS term in the massless case is that

\[ a_{PS}(m = 0) = \frac{26 - D}{24} \]

As a result the total intercept is in any D dimensions

\[ a(m = 0) = \frac{D - 2}{24} + \frac{26 - D}{24} = 1 \]

The massive endpoints serve as a regulator. Never the less we have to perform a subtraction since we subtract anyhow for the other divergences and also since we want to connect smoothly to the subtraction at m=0.
The renormalization of the PS term

We can re-write the PS term as

\[ E_{PS} = \frac{B}{\pi} \left( \frac{T}{\tilde{m}} - \frac{2\delta^2}{L} \right) \]

Using the length and mass measured in the Lab frame

\[ \tilde{m} = \gamma m \quad \tilde{L} = L \frac{\arcsin \beta}{\beta} = \frac{2}{k} \sin \delta \times \frac{\delta}{\sin \delta} = \frac{2\delta}{k} \]

Based on the boundary equation of motion

\[ k \tan \delta = \frac{T \cos \delta}{m} = \frac{T}{\gamma m} \]

Again we renormalize by subtracting from the force for the string of length \( L \) the force of the string with \( L \rightarrow \infty \).
The contribution of the PS term to the intercept

Thus the renormalized contribution of the

\[ a_{PS} = -\frac{1}{k} E_{PS}^{(ren)} = \frac{26 - D}{12\pi} \delta = \frac{26 - D}{12\pi} \arcsin \beta \]

In the limit of \( \beta \to 1 \)

\[ a_{PS} = \frac{26 - D}{24} \]

As a function of \( \frac{2m}{TL} \) it reads

\[ p_s = \frac{26 - D}{12\pi} \arccos \left( \sqrt{\frac{2m}{2m + TL}} \right) = \frac{26 - D}{24} \left[ 1 - \frac{2}{\pi} \left( \frac{2m}{TL} \right)^{1/2} + \frac{2}{3\pi} \left( \frac{2m}{TL} \right)^{3/2} + \ldots \right] \]

Thus the total intercept is

\[ a = (D - 3)a_t + a_p + a_{PS} \approx 1 - \frac{26 - D}{12\pi} \left( \frac{2m}{TL} \right)^{1/2} + \frac{199 - 14D}{240\pi} \left( \frac{2m}{TL} \right)^{3/2} \]
The puzzle of the realistic intercept

In nature the intercept associated with all the hadrons whether mesons or baryons is **negative** when it is defined in the relation of the **orbital** and not the **total angular momentum**.

For instance $\rho$ has a $0.5$ and $S=1$ so for $L=J-S$ we get $a=-0.5$

The attempt to explain this universal feature is probably the most important problem of the hadronic spectra.

To account for it we study strings with different masses, electric charges and spins at their ends.

We can get **negative** intercept but not yet in a fully satisfactory manner.
Asymmetric string

So far we have assumed that the two endpoint particles carry the same mass. We generalize it now to the case of two different masses.

The extreme case of one massless end and one infinitely heavy end is a Dirichlet-Neumann case which can be analyzed as the usual case with \( \omega_n = n - \frac{1}{2} \).

The intercept for such a case is

\[
a_{ND} = -\frac{1}{2} \sum_{n=1}^{\infty} (n - \frac{1}{2}) = -\frac{1}{48}
\]

Instead of \( 1/24 \) for the DD or NN cases.

The contributions of the transverse, planar and the PS mode to the intercept as a function of \( \beta_1 \) and \( \beta_2 \)...
The asymmetric intercept

The intercept as a function of $r = m_1/m_2$ and $\beta_1$

Transverse planar
The asymmetric intercept

- The intercept

contribution of PS

full intercept
The Propagator of the string with massive endpoints
The propagator

The propagator an ordinary bosonic string is

\[ X^\mu(z, \bar{z}) X^\nu(w, \bar{w}) = -\frac{\alpha'}{2} \eta_{\mu\nu} \left( \log |z - w|^2 + \log |z - \bar{w}|^2 \right) \]

For the massive one we start with left and right moving modes

\[
X^\mu_R = \frac{1}{2} (x^\mu - c^\mu) - \frac{1}{2} N_0 p^\mu (\tau - \sigma) + i \sqrt{N} \sum_{n \neq 0} \frac{\alpha_n^\mu}{\omega_n} e^{-i \frac{\pi}{\alpha} \omega_n (\tau - \sigma)}
\]

\[
X^\mu_L = \frac{1}{2} (x^\mu + c^\mu) - \frac{1}{2} N_0 p^\mu (\tau + \sigma) + i \sqrt{N} \sum_{n \neq 0} \frac{e^{i \phi_n} \alpha_n^\mu}{\omega_n} e^{-i \frac{\pi}{\alpha} \omega_n (\tau + \sigma)}
\]

We impose the boundary conditions and find for the right modes

\[
X^\mu_R(z) X^\nu_R(w) = -\frac{\alpha'}{2} \eta_{\mu\nu} \left( \log z - \sum_{n=1}^{\infty} \frac{1}{\omega_n} \left( \frac{w}{z} \right) \omega_n \right)
\]
The propagator

Approximating the eigenfrequencies as

$$\omega_n = n + \delta_n \quad \delta_n \approx -\frac{2}{q} n$$

Where

$$q \equiv T\ell/\gamma m$$

After a lengthy calculation we get that the HISH propagator in the limit of large $q$ reads

$$X^\mu(z, \bar{z})X^\nu(w, \bar{w}) = -\alpha' \eta^{\mu\nu} \left[ \frac{2\pi i}{q} \left( \frac{\bar{z}}{\bar{z} - w} - \frac{z}{z - \bar{w}} \right) \right] + \left( 1 + \frac{2}{q} \right) \left( \log |z - w| + \log |z - \bar{w}| \right)$$
Summary and open questions

The HISH model is successful in deriving hadronic results that are difficult to account using QCD. The trajectories that follow from the model fit very nicely the hadronic spectra. The decay width of hadron were also computed and their fit to the data is also good. We are currently computing the analog of the Veneziano amplitude for scattering of massive strings.

Adding charges and spins are next to do facing the challenge of getting negative intercept. Many more hadronic data to explain like Jet physics.