# Aspects of HISH:-
# -Holography Inspired stringy hadron model

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# Introduction

- There is a lore that the ``physics” at large N and large $\lambda$ describes well that of $N=3$ and $\lambda\sim1$.
- For certain properties this is indeed the case and the passage involves 1/N or even 1/N^2 corrections.
- But there are circumstances where this is not the case. Here are some examples for large N
- Baryon of N=3 is different from the very heavy object of large N number of quarks.
- Nuclear matter- At large N nuclear matter is a solid and only for N \leq 8 it behaves like a liquid

# Introduction

- large $\lambda$ implies large tension of the flux tube
- The behavior of hadrons at large tension and at order one is quite different.
- It translates to the difference between the behavior of a string and of a particle
- Lattice simulations show that properties like Wilson lines admit stringy behavior.
- This obviously reflects on the nature of the holography needed to describe hadron physics.

# Introduction

- The holographic duality is after all an equivalence between certain bulk string theories and boundary field theories.
- Practically most of the applications of holography are based on relating bulk fields ( not strings) and operators on the dual boundary field theory.
- This is based on the usual limit of $\alpha' \rightarrow 0$ with which we go for instance from a closed string theory to a gravity theory .
- However, to describe real hadrons one needs strings since after all in reality the string tension is not very large ($\lambda$ of order one)

# Introduction

- There is a wide range of hadronic physical observables which cannot be faithfully described by bulk fields but rather require dual stringy phenomena

- It is well known that this is the case for Wilson, 't Hooft and Polyakov lines

- We argue here that in fact also the **spectra decays width** and **scattering amplitudes** of

  mesons, baryons, exotics and glueballs

can be recast only by holographic stringy hadrons

# Introduction

- The major argument against describing the hadron spectra in terms of fluctuations of fields like bulk fields or modes on probe branes is that they generically do not admit properly the Regge-like behavior of the spectra.
- For $M^2$ as a function of J we get from flavor branes only J=0,J=1 mesons and there will be a big gap of order $\lambda$ in comparison to higher J mesons if we describe the latter in terms of strings.
- The attempts to get the linearity between $M^2$ and n basically face problems whereas for strings it is an obvious property.
- We argue that also to account for the decay width and scattering amplitudes one needs strings and not particles

# Outline

- Introduction :
- (a) Large versus finite N and $\lambda$
- (b) Hadron String/gauge holography versus gravity/gauge holography
- Review of the HISH model
- Summary of past results:

  (a) Spectra of mesons and baryons

  (b) Predictions about glueballs and exotics

  (c) Decay rates of hadrons
- New aspects:
- (a) The quantization of a string with particles on its ends
- (b) The propagator of such strings

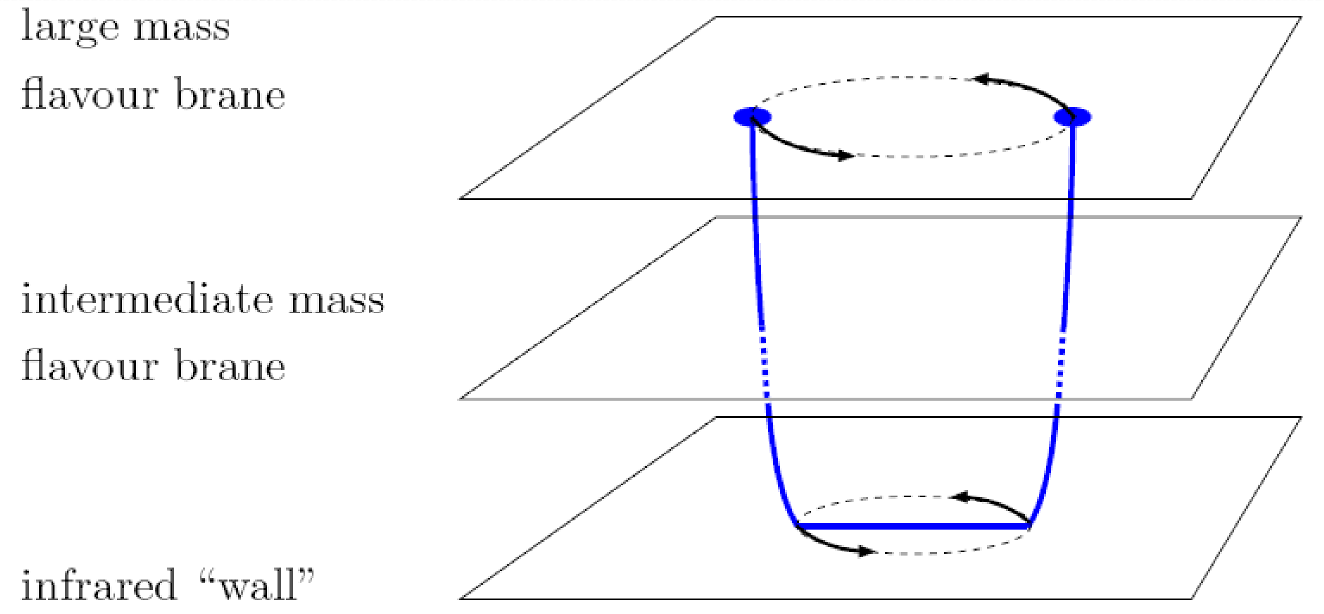
# A brief review of Holography Inspired stringy hadron model

# HISH- Holography Inspired Stringy Hadron

- The construction of the HISH model is based on the following steps.
- (i) Analyzing string configurations that correspond to hadrons in confining holographic backgrounds.
- (ii) Devising a transition from the holographic regime of large Nc and large λ to the real world that bypasses formal expansion $\frac{1}{N_c}$ and $\frac{1}{\lambda}$
- (iii) Proposing a model of stringy hadrons in **flat four dimensions with massive endpoint particles** that is **inspired** by the corresponding holographic model
- (iv)Dressing the endpoint particles with **charge, spin, baryonic vertex**, etc.
- (v) Confronting the outcome of the models with **experimental data** .

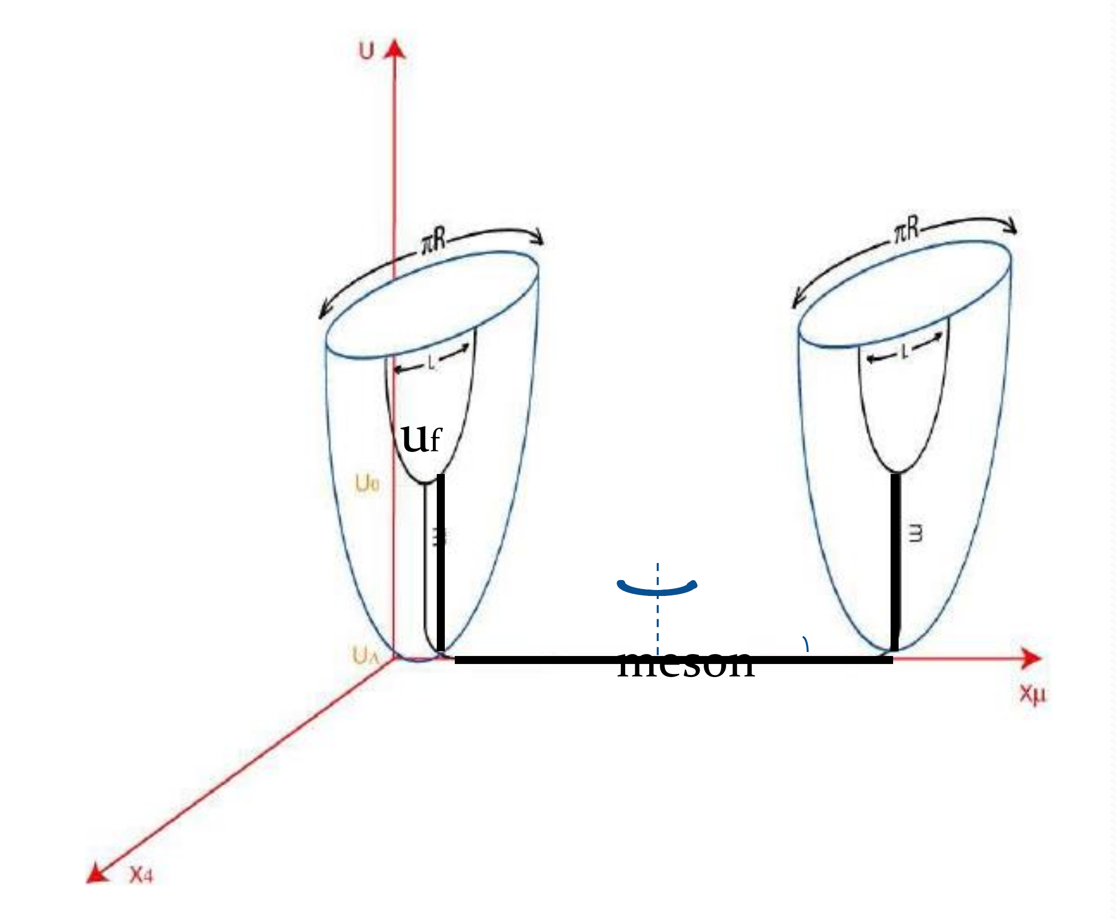
# (i) The structure of a holographic meson

- The structure of a **holographic meson** is a **rotating string** that starts and ends on flavor branes ( the same or different). For instance a heavy quarkqonium is

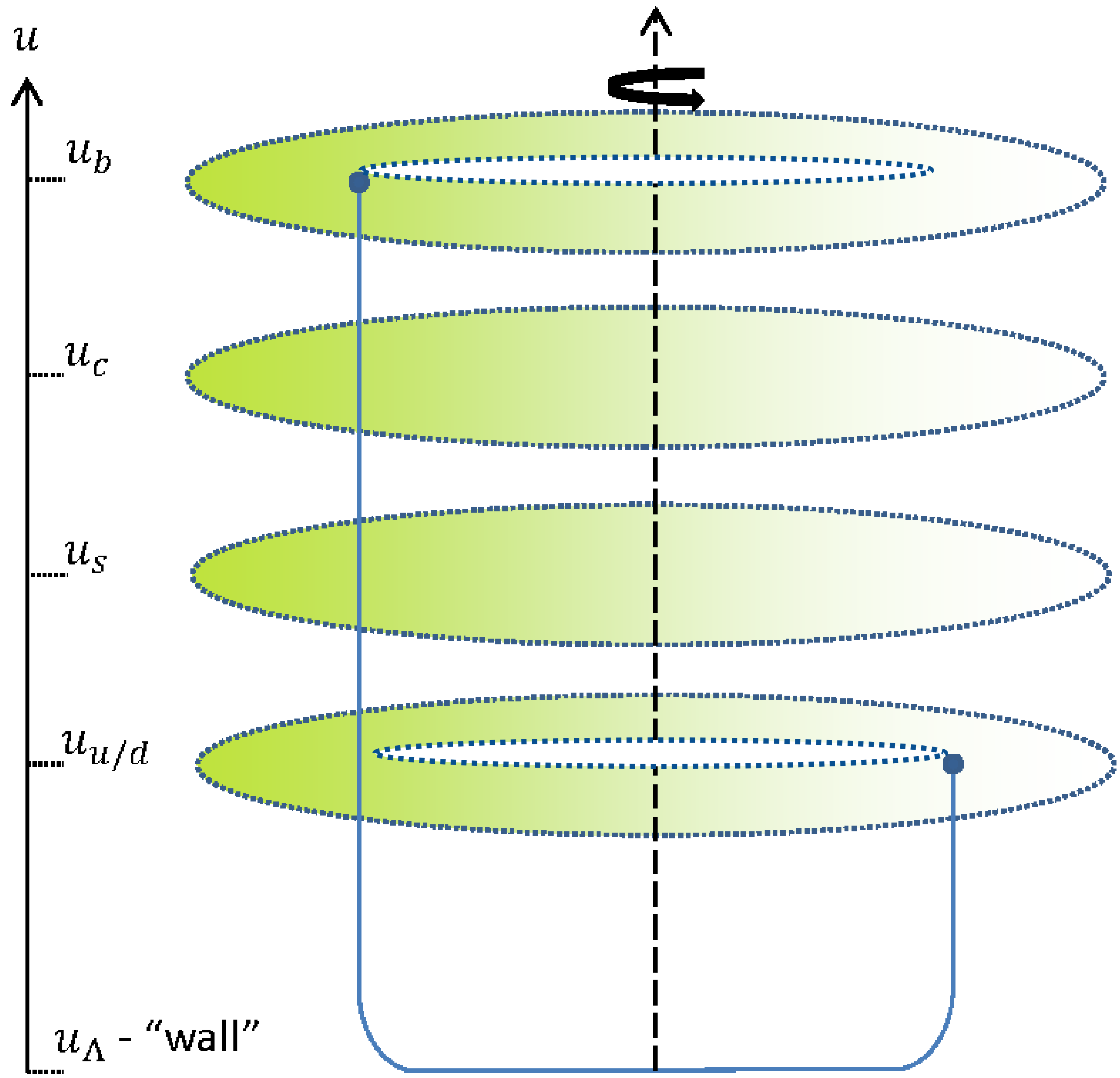


# Stringy meson in U shape flavor brane setup

- In the generalized Sakai Sugimoto model the meson looks like

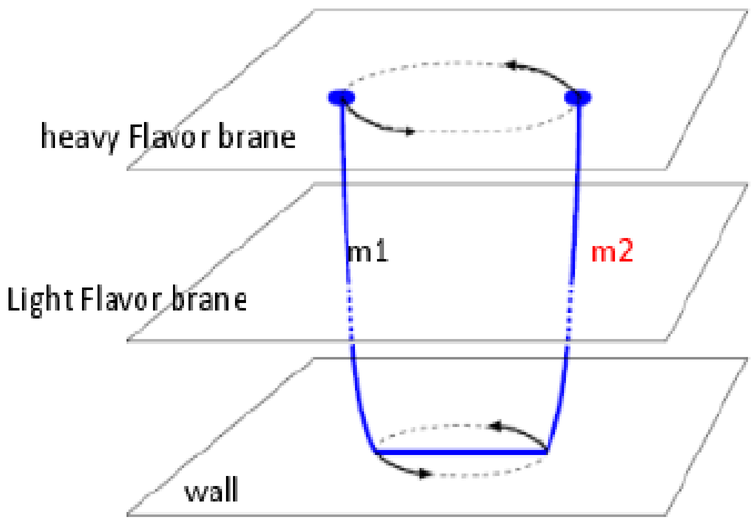


# Example: The B  meson

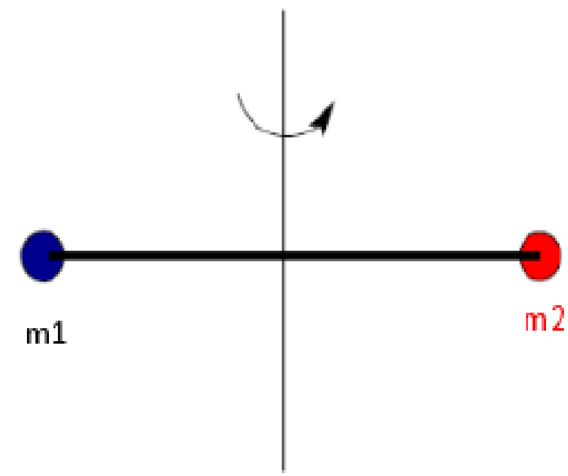


# HISH map to strings in flat space-time

- The vertical segments of the holographic hadronic string can me mapped to massive particles at the ends of the string




Rotating holographic string

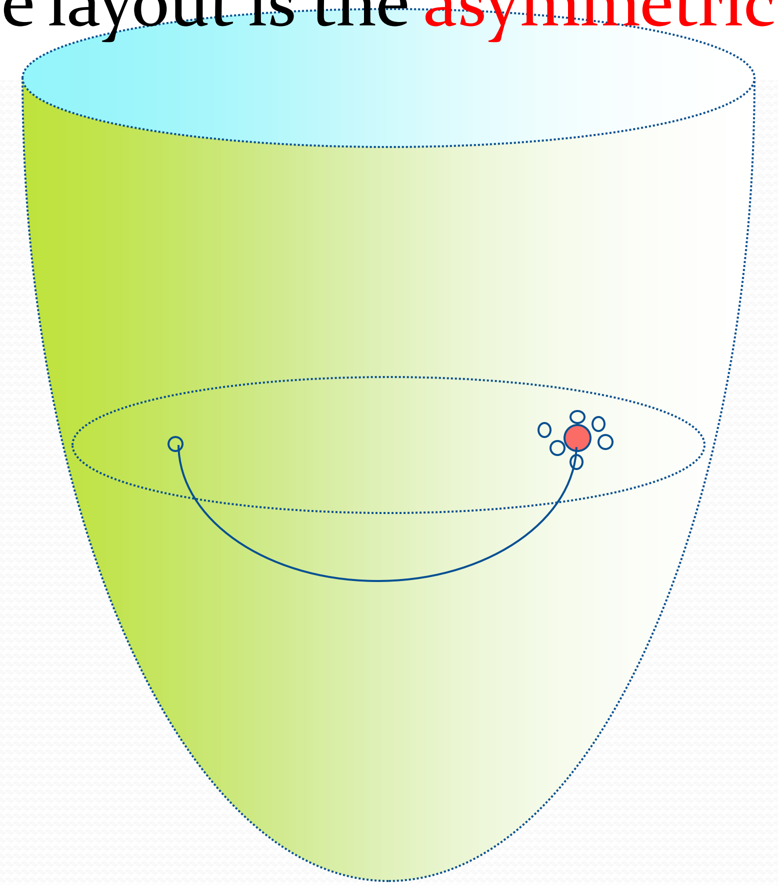



Rotating sting in flat space-time with massive endpoints

with mass $$m_{sep} = T \int_{u_0}^{u_f} g(u)du = T \int_{u_0}^{u_f} \sqrt{G_{00}G_{uu}}du$$

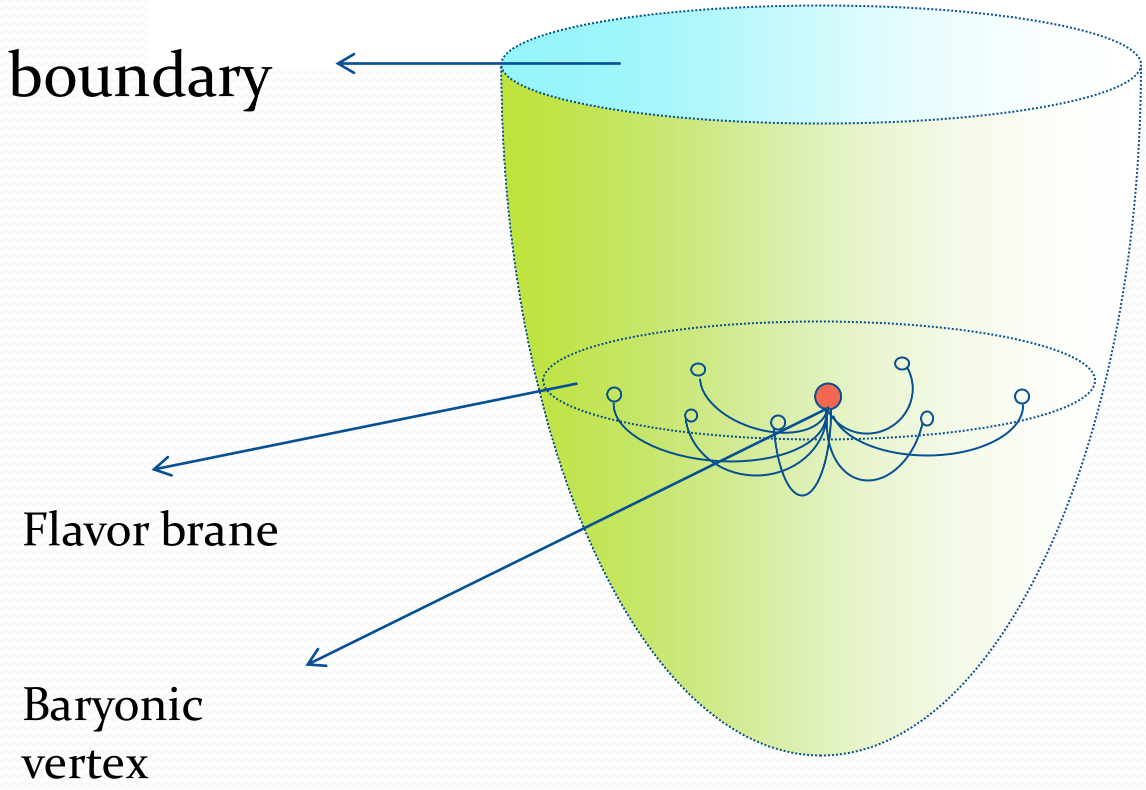
## (2) Holographic Baryon

- In holography a baryon is a baryonic vertex which is a wrapped Dp brane on a p cycle and is connected with Nc strings to flavor branes.
- The preferable layout is the asymmetric one.



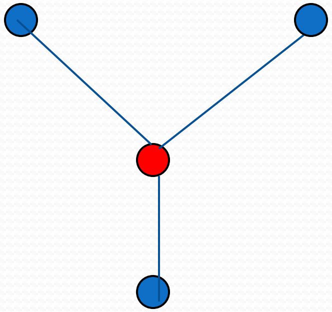
# A possible baryon layout

- A possible dynamical baryon is with Nc strings symmetrically connected to the flavor brane and to the BV which is also on the flavor brane.

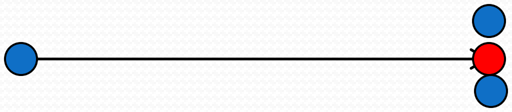


# From large Nc to three colors

- Naturally the analog at Nc=3 of the symmetric configuration with a central baryonic vertex is the old Y shape baryon

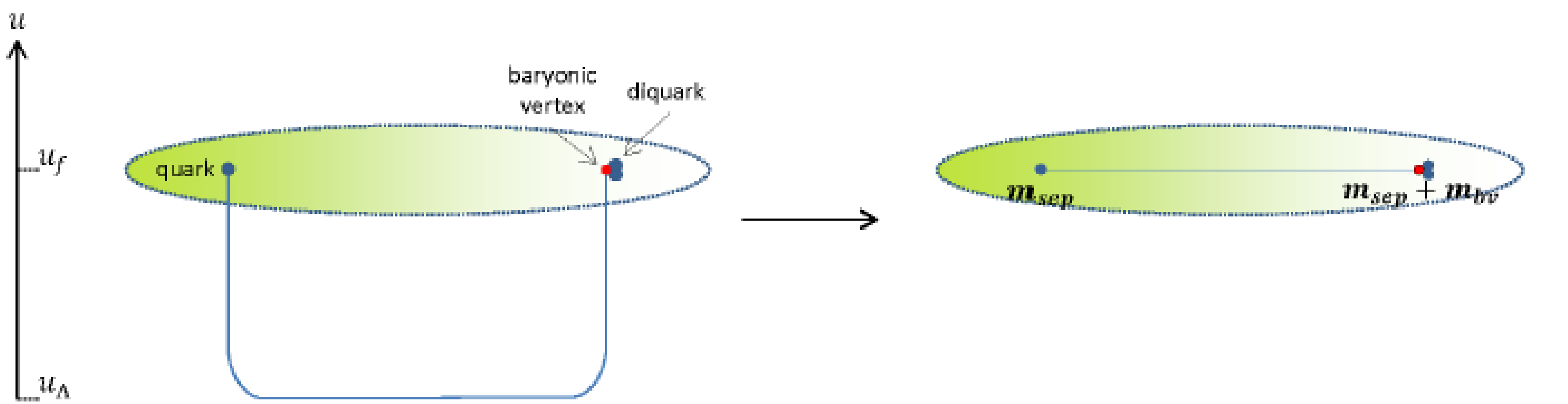


- The analog of the asymmetric setup with one quarks on one end and Nc-1 on the other is a straight string with quark and a di-quark on its ends.



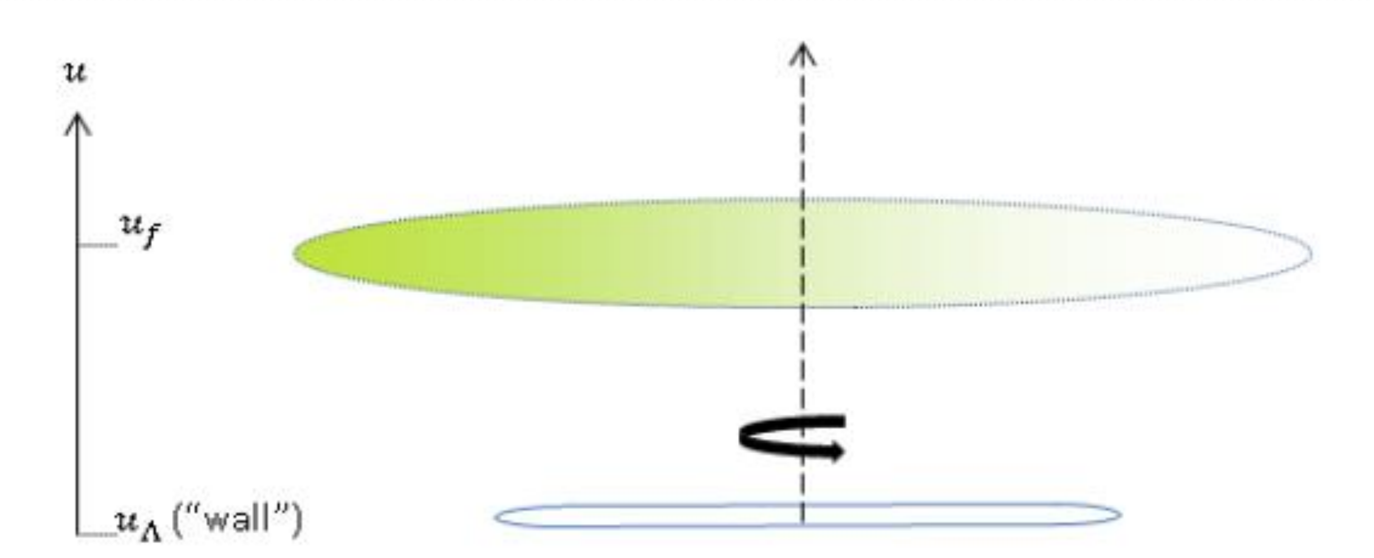
# HISH Baryon

- In HISH the holographic baryon is mapped into a single string that connects a quark on one side and a diquark on the other side



# (3) Holographic Glueballs-

- Mesons are open strings with a massive quark and an anti-quark on its ends.
- Baryons are open strings with a massive quarks connected to a baryonic vertex
- What are glueballs?
- Since they do not incorporate quarks it is natural to assume that they are **rotating folded closed strings**



# Closed strings versus open strings

- The spectrum of states of a closed string admits

$$M^2 = \frac{2}{\alpha'}\left(N + \tilde{N} + A + \tilde{A}\right)$$

- The spectrum of an open string

$$M^2_{open} = \frac{1}{\alpha'}(N + A)$$

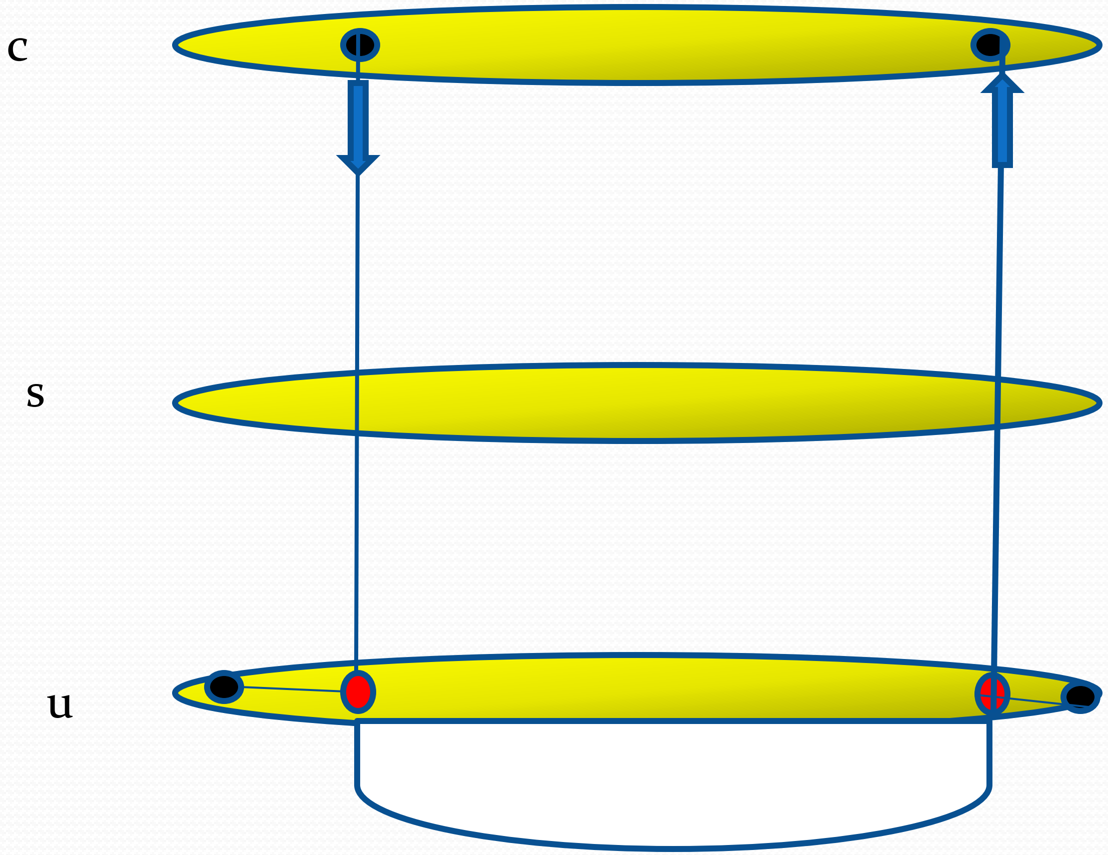
- **The slope of the closed string is ½ of the open**
- The closed string ground states has

$$M^2 = \frac{2}{\alpha'}(A + \tilde{A}) = \frac{2-D}{6\alpha'}$$

- The intercept is **2 times** that of an open string

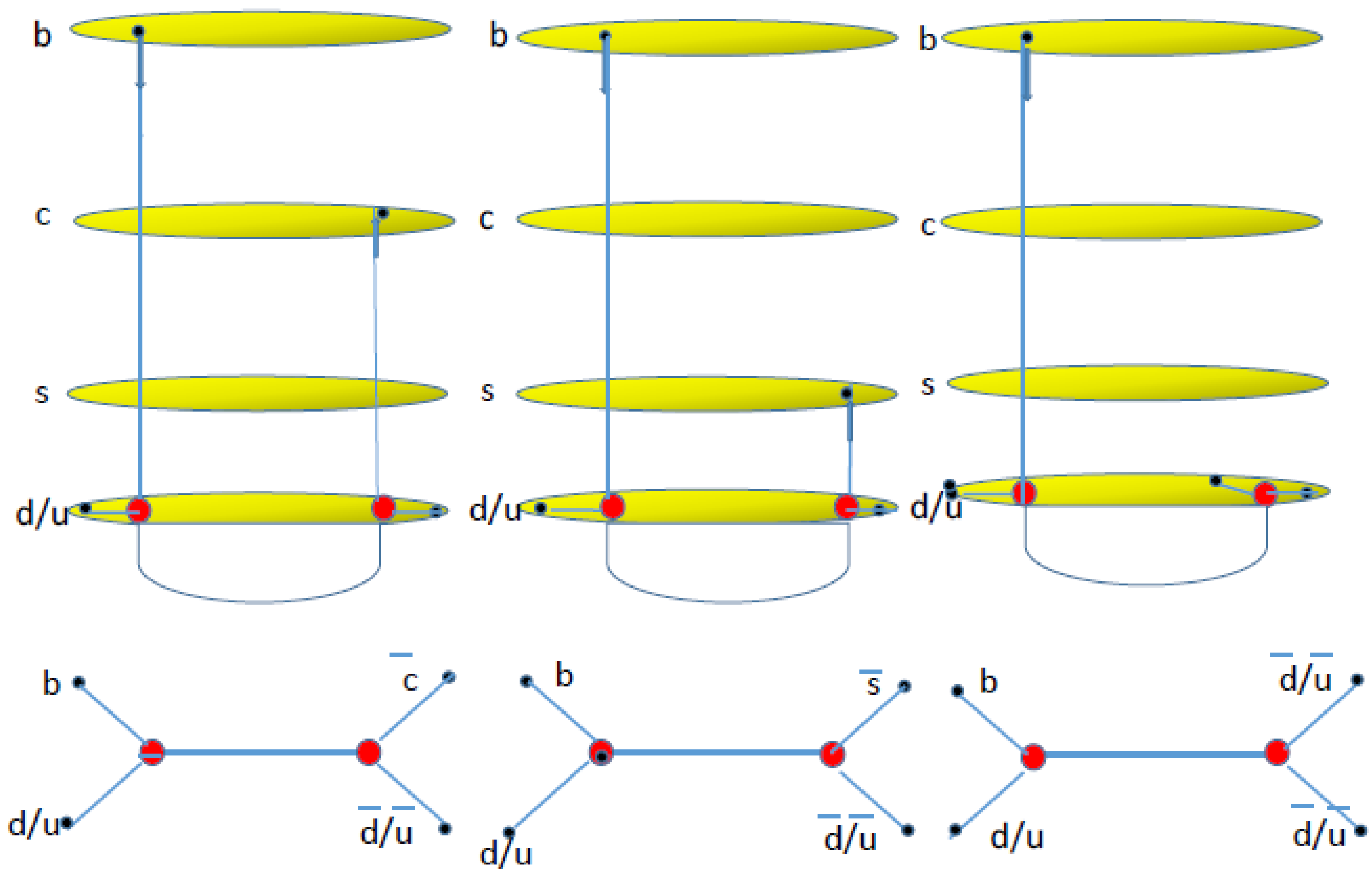
# (4) Holographic Exotics

- A tetra quark based on a baryonic vertex connected to a u c di-quark and connected to an anti-baryonic vertex which is connected to anti- u and anti- c



# HISH exotic hadrons

- In the same way that we map holographic mesons and baryons we can also map from holographic to HISH exotics

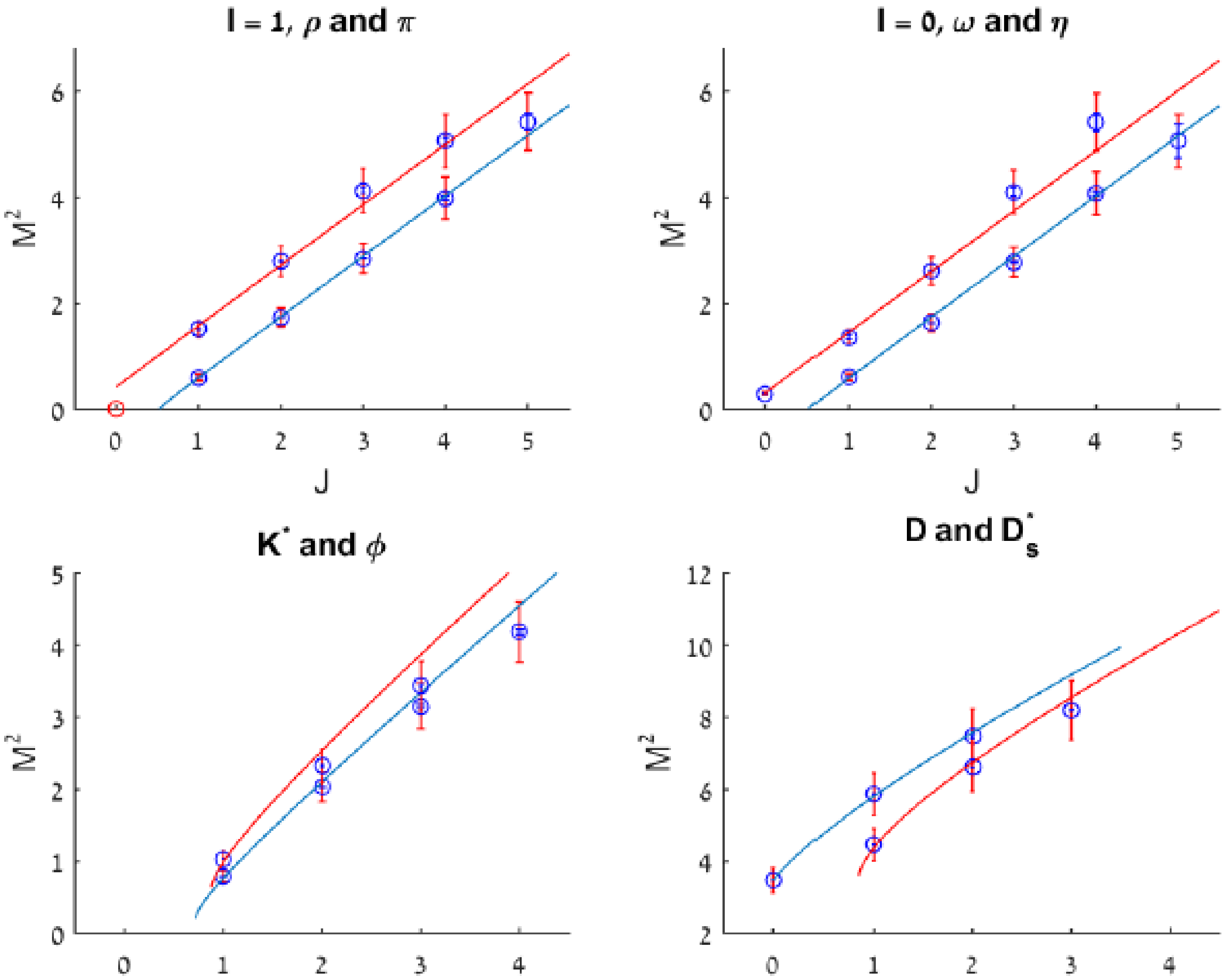


# Past results of the HISH model

# Spectrum of Mesons and Baryons

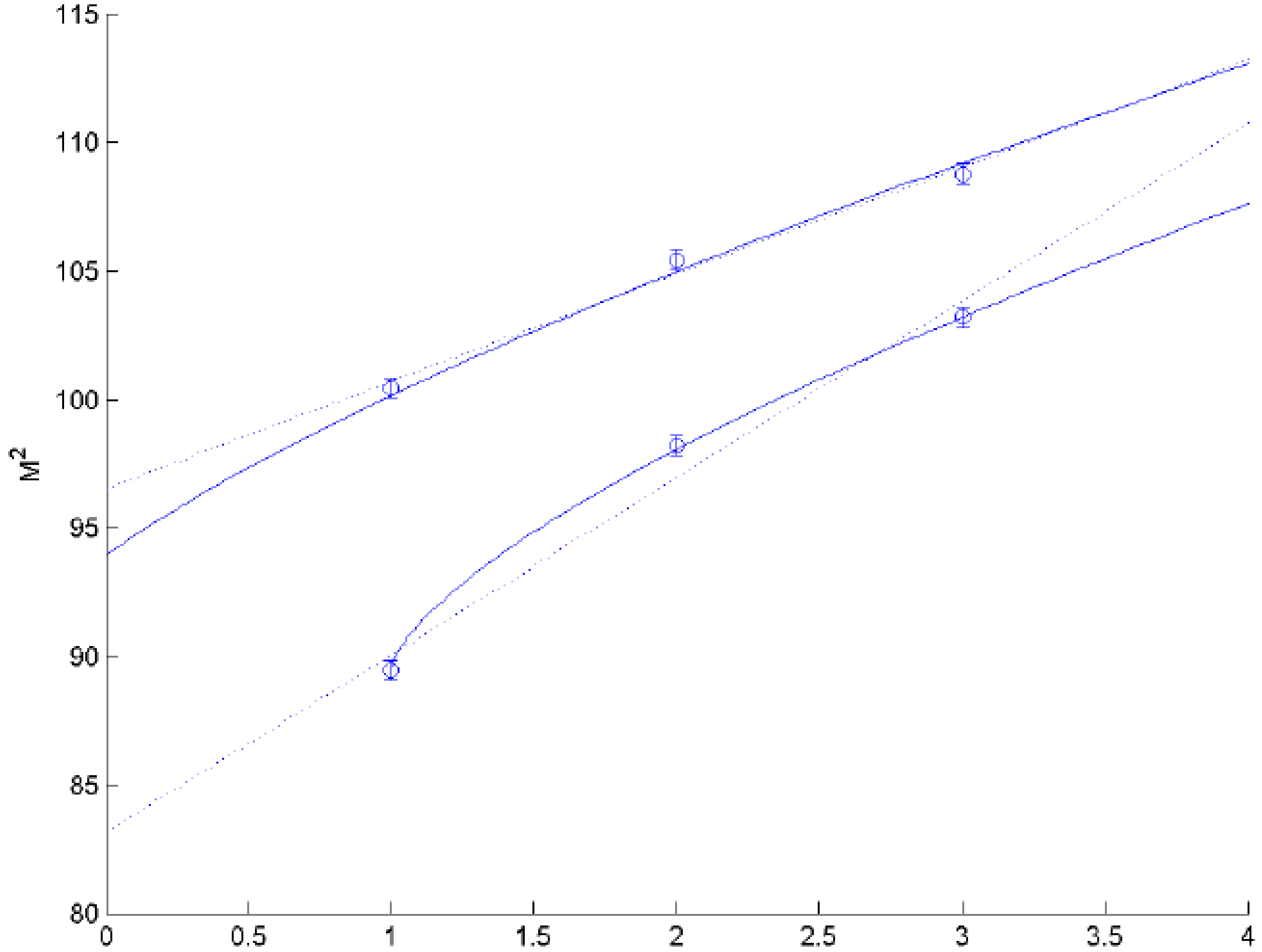
## The spectra fits

- The best fits of HISH to meson states



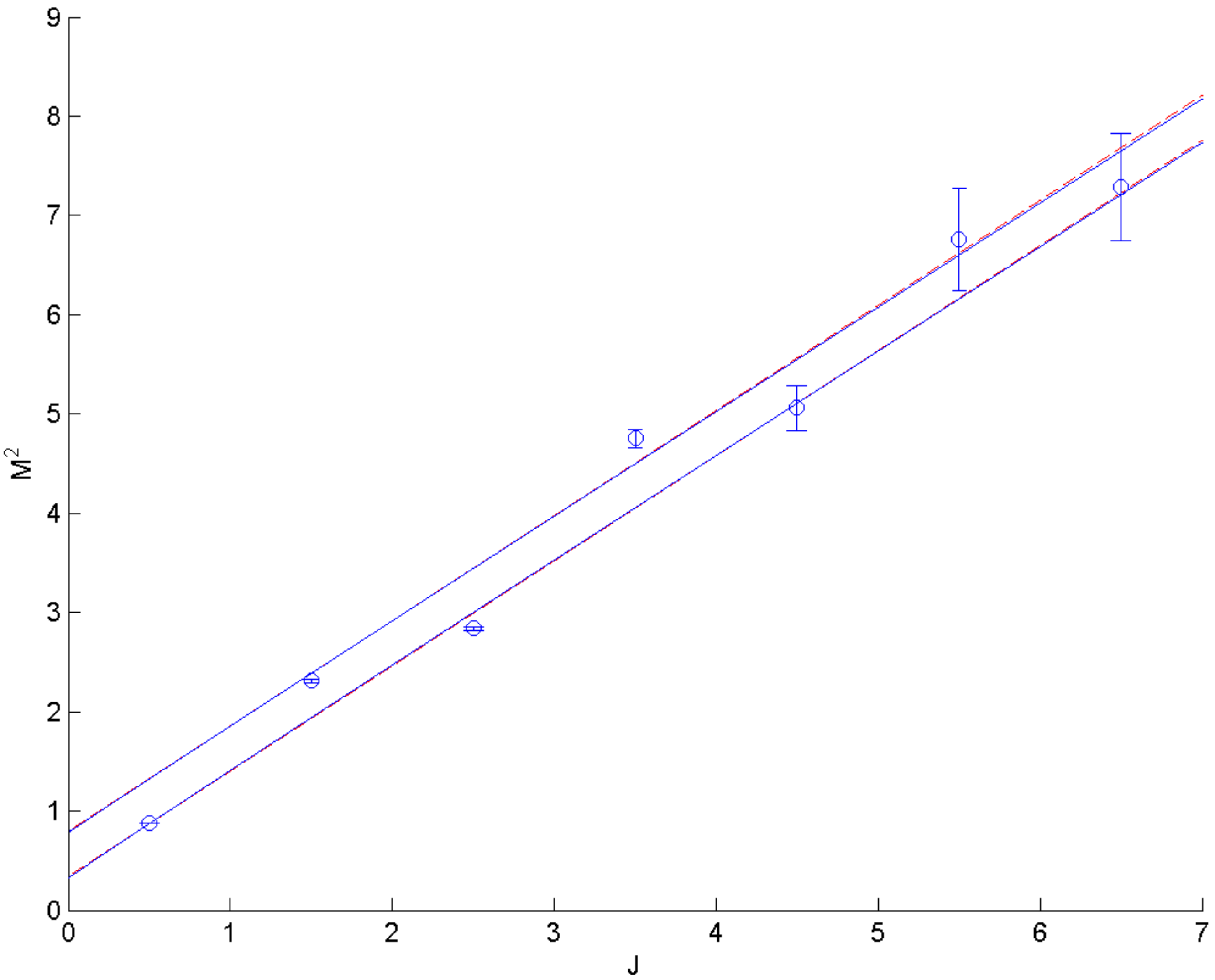
# The botomonium trajectories

- To emphasize the deviation from linearity due to the massive endpoints here are the botomonium



# The spectra fits of Nucleons

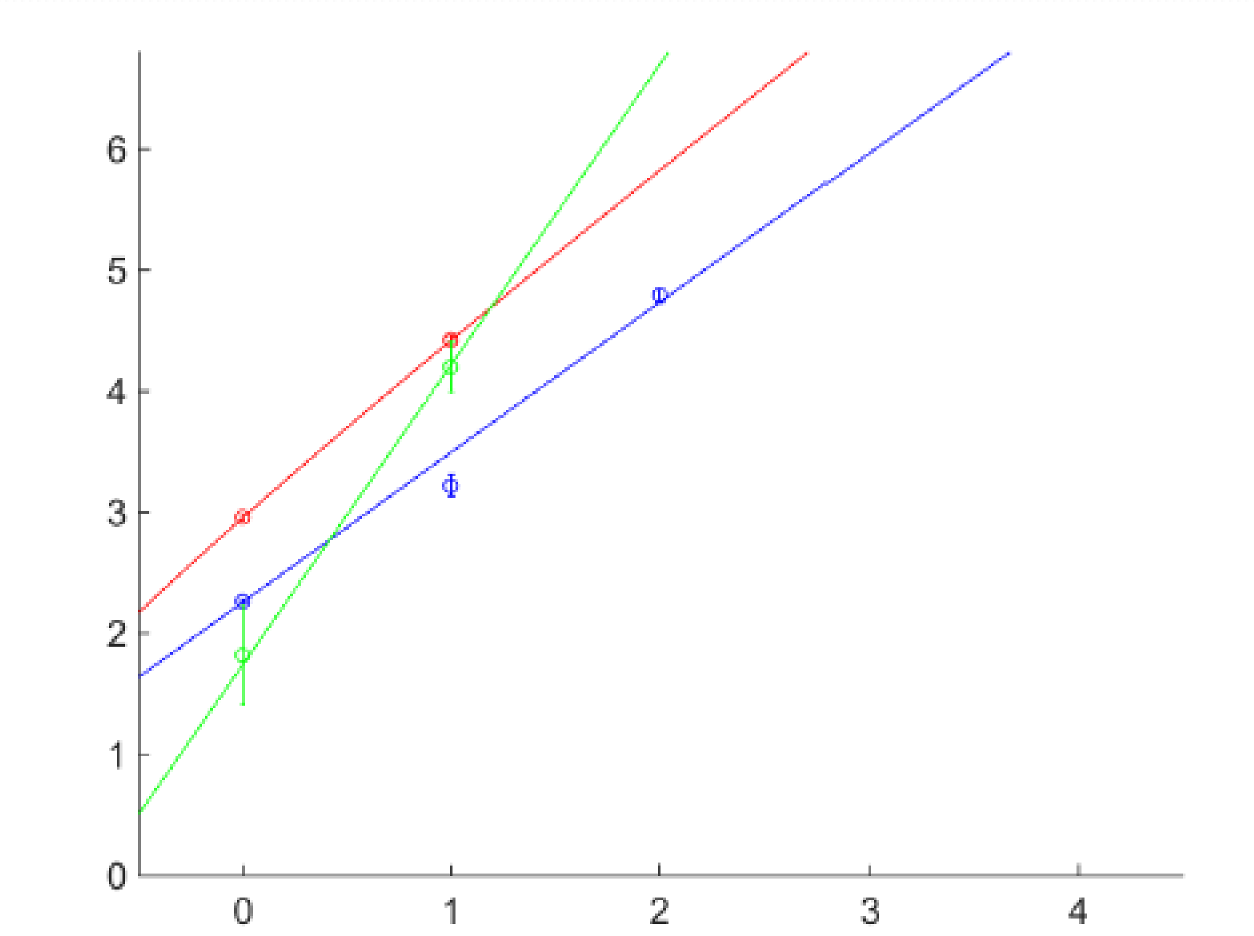
- Trajectories for even and odd J nucleons



# Predictions for the search of Glueballs and Exotic Hadrons

# Glueball 0++ fits of experimental data

- The meson and glueball trajectories based on $f_0(1380)$ as a glueball lowest state.



# Predictions of tetra quarks based on the Y(4630)

- Based on the Y(4630) that was observed to decay predominantly to $\Lambda_c^+ \Lambda_c^-$. If we assume that it is on a Regge-like trajectory and we borrow the slop and the endpoint masses from the $J/\Psi$ trajectory we get

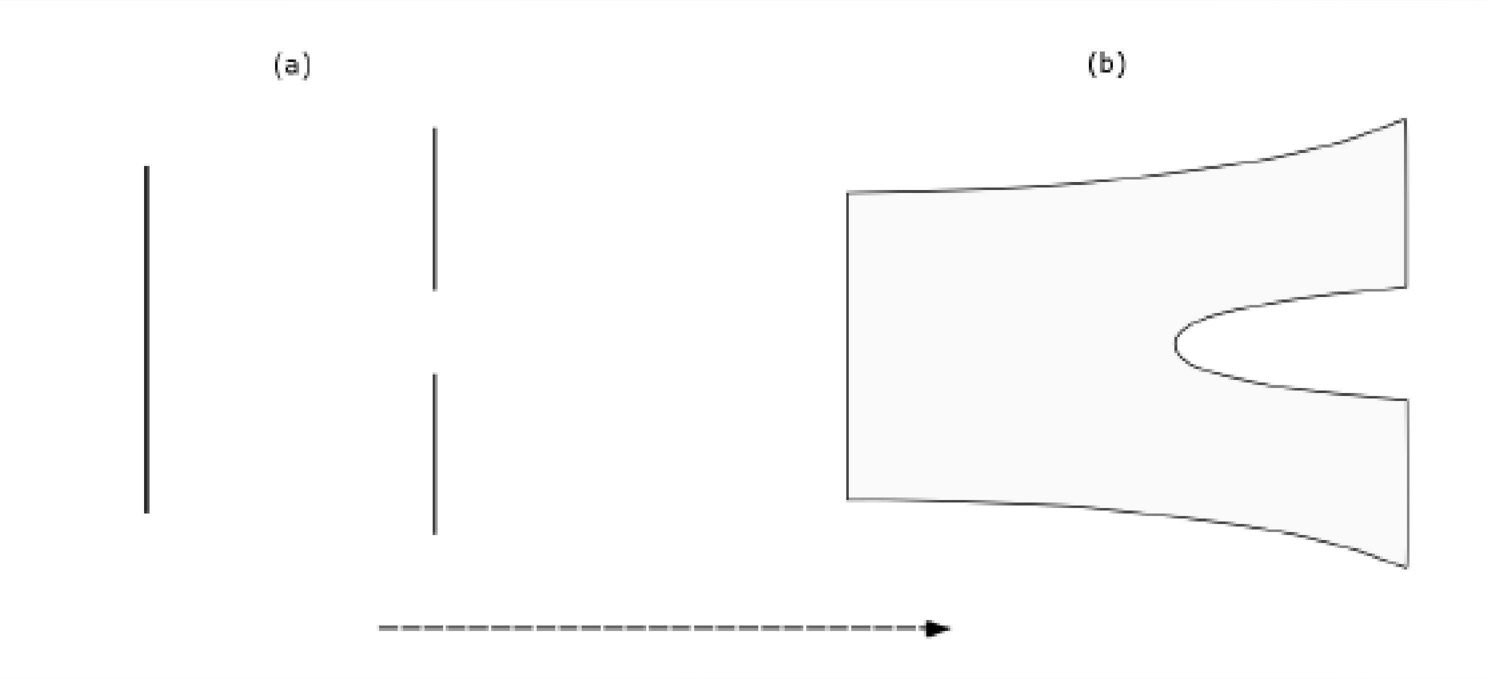
| $n$ | Mass | Width |
|---|---|---|
| 0 | $4634^{+9}_{-11}$ | $92^{+41}_{-32}$ |
| 1 | 4902±95 | 103±46 |
| 2 | 5148±99 | 114±51 |
| 3 | 5378±104 | 124±55 |
| 4 | 5594±109 | 134±60 |

| $J^{PC}$ | Mass | Width |
|---|---|---|
| $1^{--}$ | $4634^{+9}_{-11}$ | $92^{+41}_{-32}$ |
| $2^{++}$ | 4791±64 | 98±44 |
| $3^{--}$ | 4939±66 | 105±47 |
| $4^{++}$ | 5080±67 | 111±49 |
| $5^{--}$ | 5215±69 | 117±52 |

# The Decay Width of Hadrons

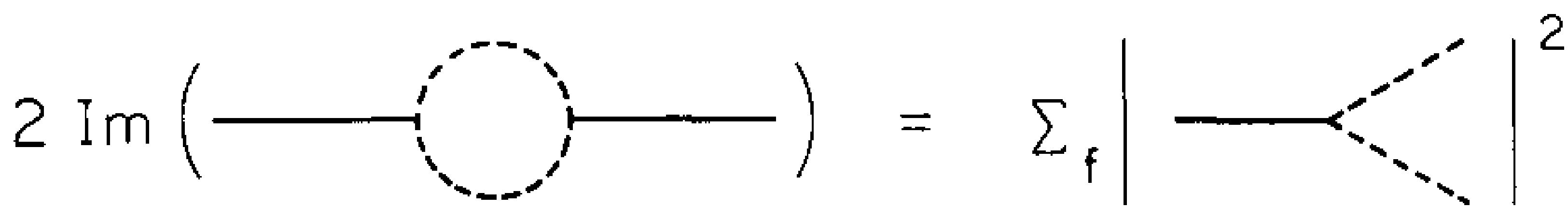
# The decay of a long string in critical flat space-time

- The decay of a hadron is in fact the **breaking of a string into two strings**
- Obviously a type I open string can undergo such a split



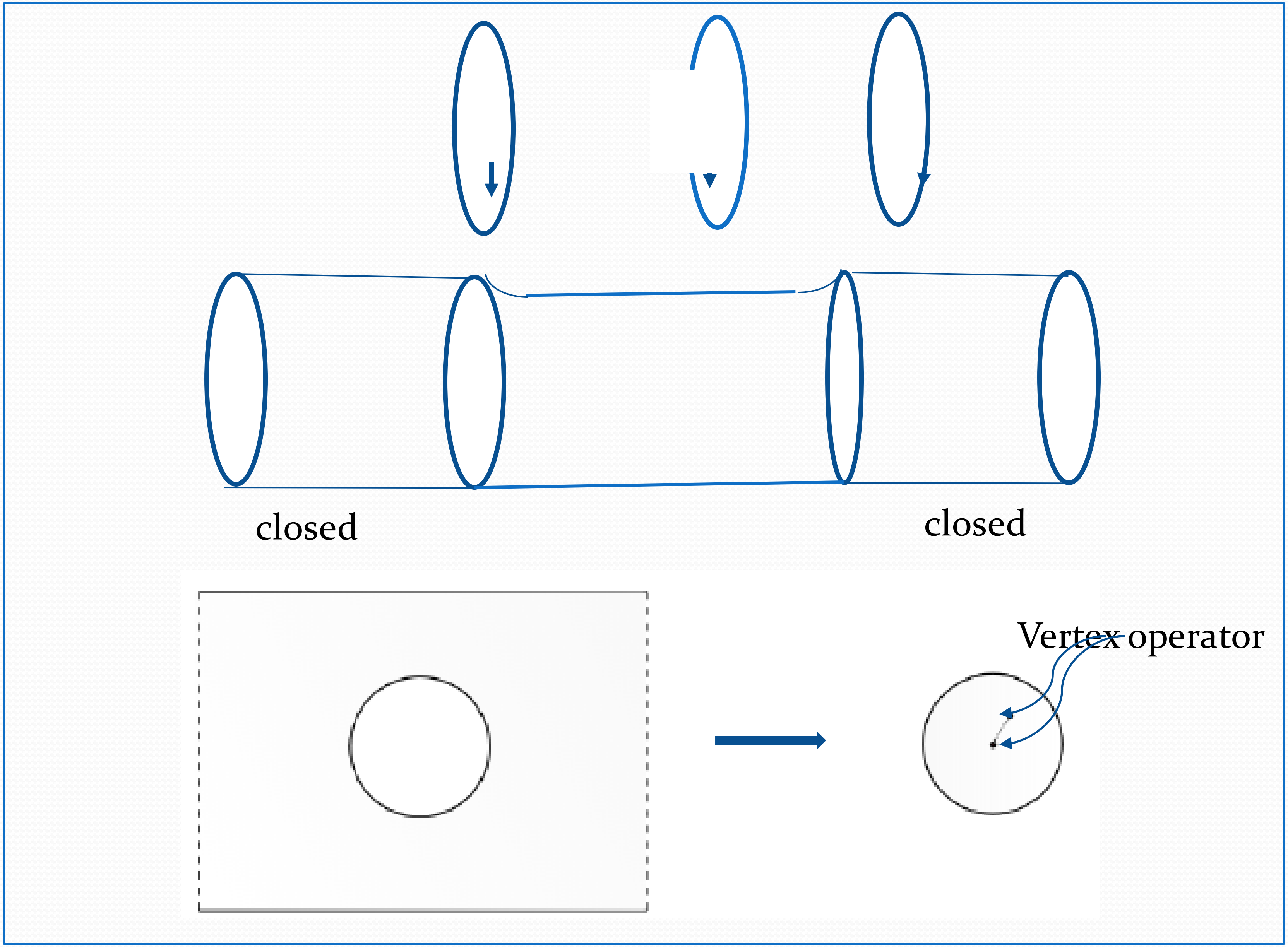
# The decay of a long string in critical flat space-time

- The total decay width is related by the optical theorem to the imaginary part of the self-energy diagram




- A trick that Polchinski et al used is to compactify one space coordinate and consider incoming and outgoing strings that wrap this coordinate so one can avoid an annulus open string diagram and instead compute a disk diagram with simple vertex operator of a closed string

# The string amplitude



# The decay of a long string in critical flat space-time

- We would like to determine the dependence of the string amplitude on the string length L

$$iA_2 = \frac{iTN}{g^2} L \left[\frac{\kappa}{2\pi\sqrt{L}}\right]^2 \int_{|z|<1} \mathrm{d}^2 z \, \langle : e^{ip\cdot X(0)} :: e^{-ip\cdot X(z)} : \rangle$$

open string coupling

Zero mode

Gravitational coupling

Normalization Of the vertex

Vertex operator

# The decay of a long string in flat space-time

- Substituting the propagators the amplitude reads

$$\boxed{i\mathcal{A}_2 = \frac{iTN\kappa^2}{2\pi g^2}\lim_{t\to 0}\frac{\Gamma(t-1)\Gamma(1-\tilde{J})}{\Gamma(t-\tilde{J})}}$$

$$\tilde{J} \equiv \frac{L^2T}{2\pi} - 2$$

$$\boxed{= \frac{iTN\kappa^2}{2\pi g^2}\left(\tilde{J}\partial_{\tilde{J}}\ln[\Gamma(-\tilde{J})] + \lim_{t\to 0}\frac{\tilde{J}}{t}\right)}$$

regulator

- The imaginary par $\sum_k \pi k\delta(J-k)$ for $k = 1, \ldots$

$$\boxed{\mathrm{Im}\mathcal{A}_2 = -\frac{iTN\kappa^2}{2g^2}\tilde{J}}$$

# The decay of a long string in flat space-time

- Since A2 is the mass square shift the total decay width

$$\boxed{\Gamma = -\mathrm{Im}\delta(m) = -\mathrm{Im}\frac{1}{2m}\delta(m^2) = \frac{TN\kappa^2}{4g^2}\frac{\tilde{J}}{E}}$$

- The leading behavior for string in d=26 is

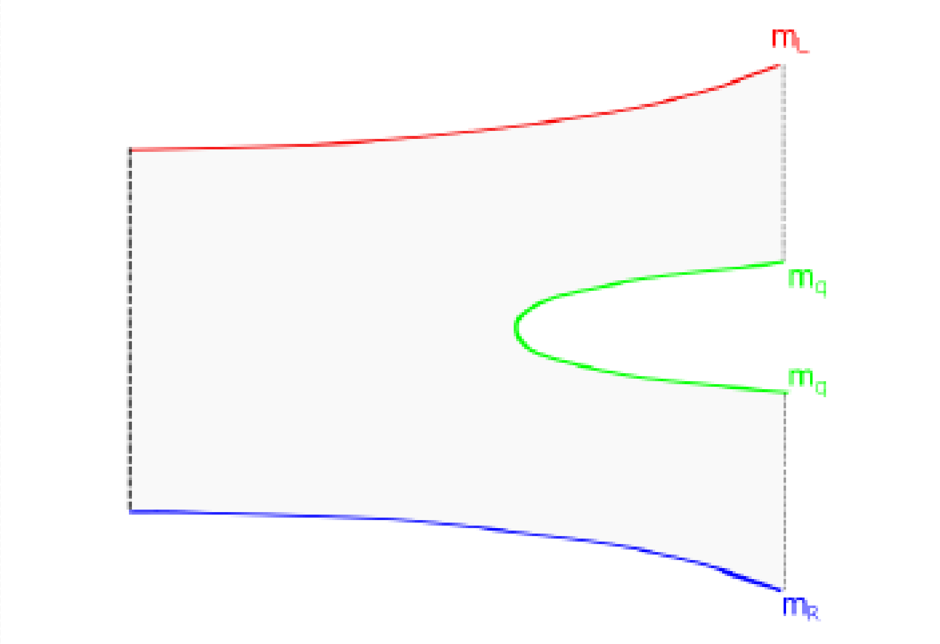
$$\boxed{\frac{\Gamma}{L} = \frac{g^2 T^{13} N}{4(4\pi)^{12}}}$$

$$\boxed{\Gamma = \frac{TN\kappa^2}{4g^2}\left[L_{tot} + \frac{4\pi}{T}\frac{1}{L_{tot}}\right]}$$

$$L_{tot} = \sqrt{L^2 - \frac{8\pi}{T}}$$

# The decay width of a string with massive endpoints

- The decay of a string with **massive particles on its ends**




- The dependence on the masses:

  (a) The length L(m1,m2)

  (b) The boundary conditions ( not anymore Neumman)

  ( c) The vertex operators should be modified

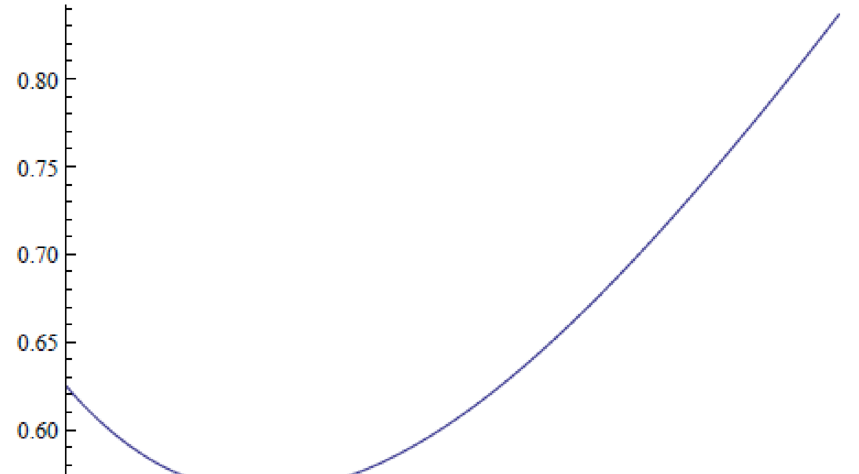
# The decay width of a string with massive endpoints

- The main result is the linearity with L with a universal constant A

$$\Gamma = \frac{\pi}{2} A T L(M, m_1, m_2, T)\,.$$

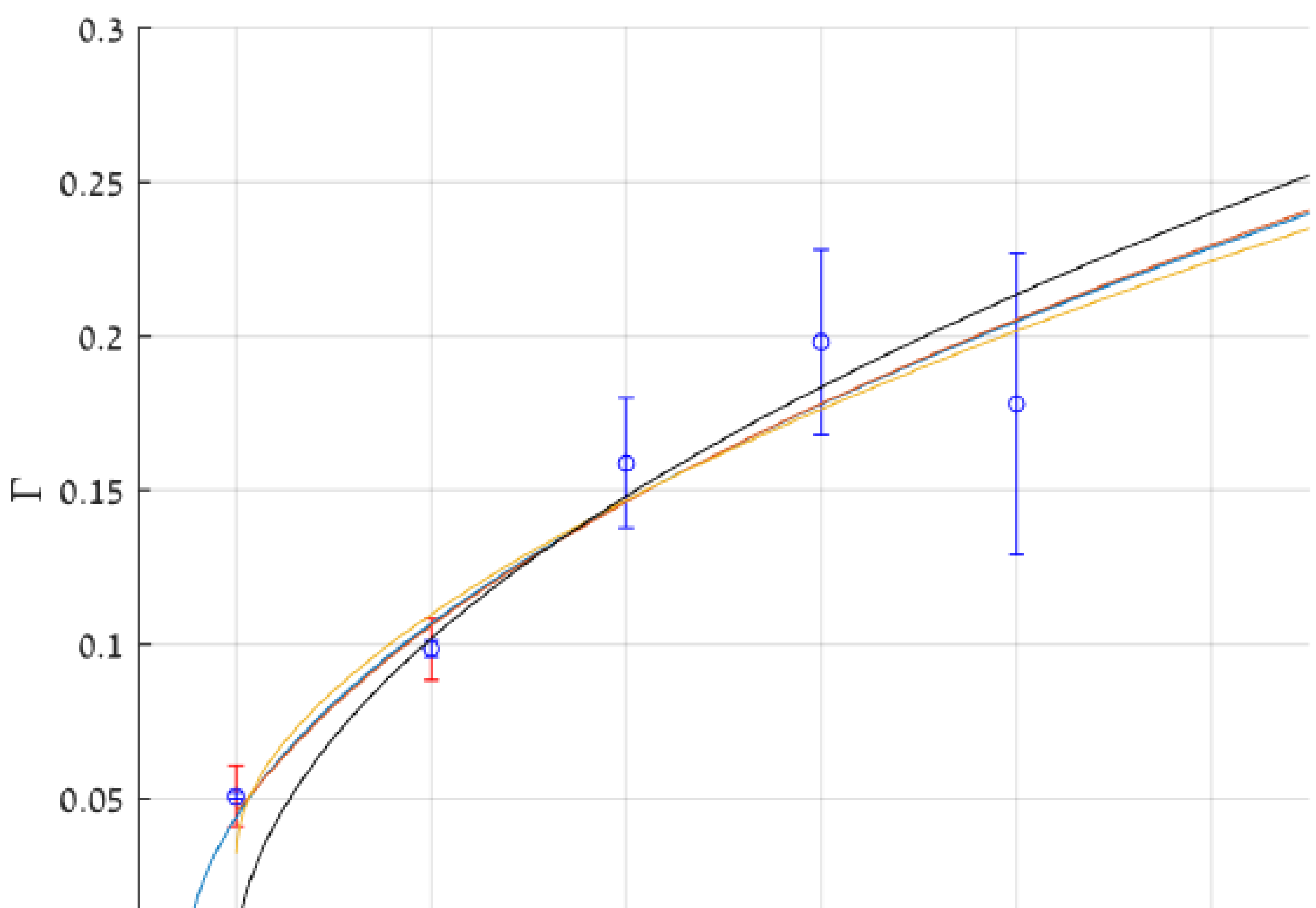
- For small endpoint masses we can expand *

$$\Gamma \propto \frac{\pi}{4} T L + \frac{\pi}{4} m - \frac{2\sqrt{2}}{3} m^{3/2} (TL)^{-1/2} + \mathcal{O}(L^{-3/2})$$



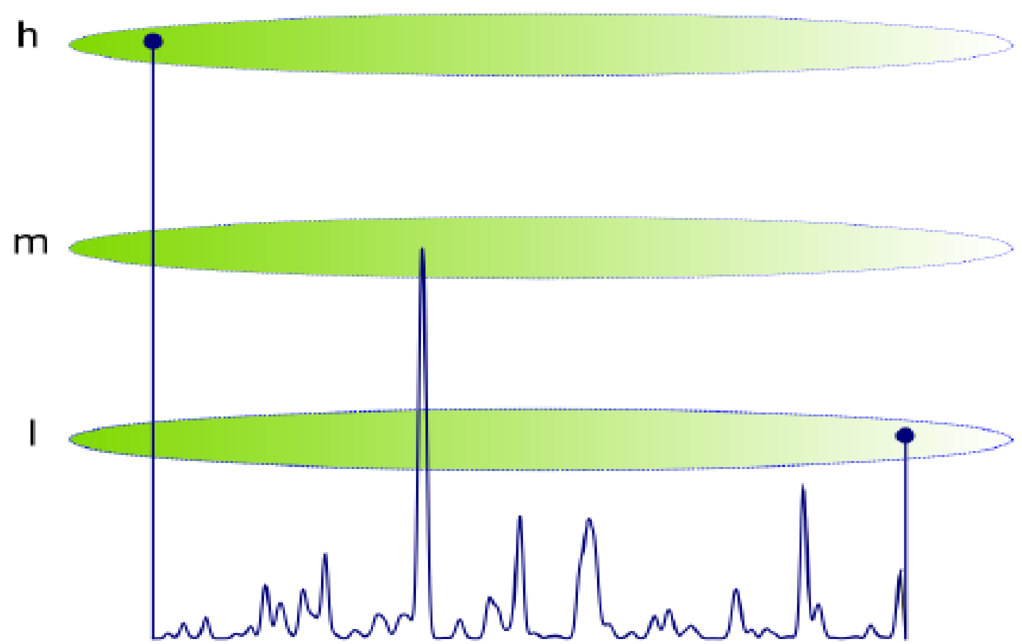
# Test case: the decay of the K* states

- Different methods of inserting the intercept
- blue $J \rightarrow J - a$, red $L^2 \rightarrow L^2 - 2a/T$, yellow force bc



# The suppression factor for stringy holographic hadrons

- The horizontal segment of the stringy hadron **fluctuates** and can reach flavor branes
- When this happens the string may **break up** , and the two new endpoints connect to a flavor brane



# Determination of the suppression factor

- Assuming first that the string stretches in flat space-time we found ( J.S, K. Peeters , M. Zamamklar) using both a string bit model and a continues one that

$$\Gamma = \text{const.} \cdot \exp\left(-1.0\,\frac{z_B^2}{\alpha'_{\text{eff}}}\right) \cdot T_{\text{eff}}\,\mathcal{P}_{\text{split}} \cdot L$$

$$\exp\left(-1.0\,\frac{z_B^2}{\alpha'_{\text{eff}}}\right) = \exp\left(-2\pi\,\frac{m_{sep}^2}{T_{\text{eff}}}\right)$$

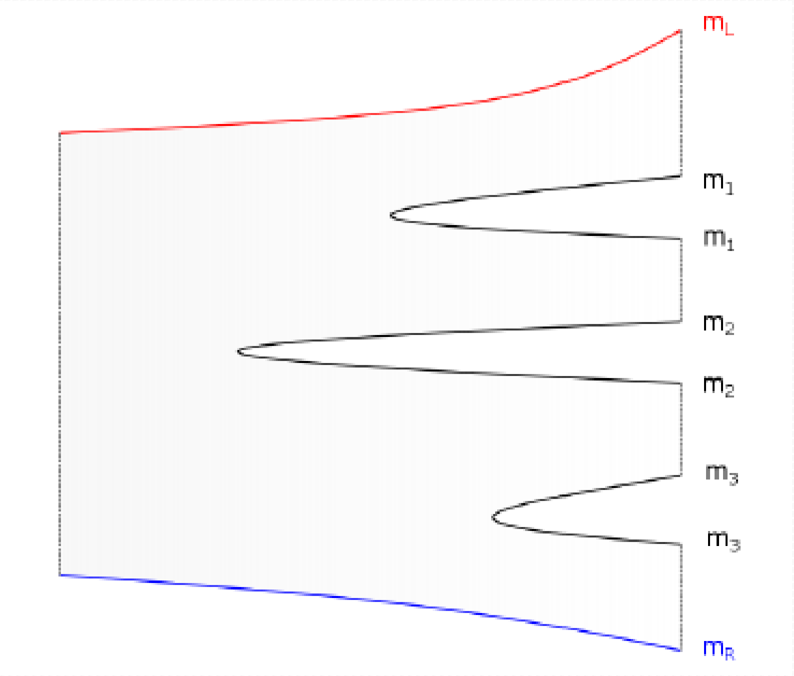
- There are further corrections due to the curvature and due to the massive endpoints. The final result is

- $$\Gamma = \exp\left(-2\pi C(T_{\text{eff}}, M, m_i)\frac{m_{sep}^2}{T_{\text{eff}}}\right)$$

$$C(T_{\text{eff}}, M, m_i) \approx 1 + c_c\frac{M^2}{T_{\text{eff}}} + \sum_{i=1}^{2} c_{m_i}\frac{m_i}{M}$$

# Multi string breaking and string fragmentation

- The basic process of a string splitting into two strings can of course repeat itself and thus eventually describe a decay of a single string into n strings



# Multi string breaking and string fragmentation

- The probability of a string to decay into n strings is

$$\boxed{\mathcal{P} = \frac{T_{\rm eff}^2}{\pi^3}\sum_{n=1}^{\infty}\frac{1}{n^2}\sum_{\{n_f\}}\prod_f \exp\left(-2\pi\tilde{C}\frac{m_{sep,f}^2}{T_{\rm eff}}n_f\right)}\,.$$

- We sum over all combinations of nf such that

$\sum_f n_f = n,$

- String fragmentation is supposed to be the generator of jets in hadron and $e^+e^-$ collisions.
- It is incorporated in the event generator Pythia
- In our picture the endpoint quark anti-quark do not carry momenta.

# New aspects of HISH

# (( The Quantization of the string with massive endpoints

# The quantization of the string with massive endpoints

- Four decades ago Chodos and Thorn suggested a model of a bosonic string with two massive particles on its ends.
- Their motivation probably was the physics of a flux tube and quark and an anti-quark on its ends.

- Through the years this model has been addressed but never fully quantized and clearly not in four dimension. This is what we have recently acheieved
- For us this model is an approximation of the stringy hadron in holographic model (HISH)
- On root to the quantization of the model we have further developed an alternative method of renormalization
- In certain respects the hadronic string is better behaved that the ordinary bosoinc string

# The classical string with massive endpoints

- We start with the action and equations of motion
- The action is the NG action plus two relativistic point-particle action terms

$$S = S_{st} + S_{pp}|_{\sigma=-\ell} + S_{pp}|_{\sigma=\ell}$$

$$\boxed{S_{st} = -T\int d\tau d\sigma\sqrt{-h} = -T\int d\tau d\sigma\sqrt{\dot{X}^2X'^2 - (\dot{X}\cdot X')^2}}$$

$$-\infty < \tau < \infty \quad -\ell \leq \sigma \leq \ell$$

- The world sheet coordinates

and the induced metric is

$$h_{\alpha\beta} = \eta_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu$$

The relativistic particle action

$$\boxed{S_{pp} = -m\int d\tau\sqrt{-\dot{X}^2}}$$

# The equations of motion

- The bulk equation of motion

$$\partial_\alpha(\sqrt{-h}h^{\alpha\beta}\partial_\beta X^\mu) = 0$$

- The boundary equation of motion

$$\boxed{T\sqrt{-h}\partial^\sigma X^\mu \pm m\partial_\tau\left(\frac{\dot{X}^\mu}{\sqrt{-\dot{X}^2}}\right) = 0}$$

- The + for $\sigma = \ell$ and – for $\sigma = -\ell$.

# Rotation solution of the equations of motion

- A rotating solution in the (1,2) plane

$$\boxed{X^0 = \tau, \qquad X^1 = R(\sigma)\cos(k\tau), \qquad X^2 = R(\sigma)\sin(k\tau)}$$

For any choice of $R(\sigma)$

Substituting it to the boundary equation of motion

$$T\frac{\sqrt{(1-k^2R^2)R'^2}}{R'} \mp m\frac{k^2R}{\sqrt{1-k^2R^2}} = 0$$

Which translates into the tension being balanced by the centrifugal force

$$\boxed{\frac{T}{\gamma} = \frac{2\gamma m\beta^2}{L}}$$

# The classical energy and angular momentum

- E and J for the rotating string are

$$E = \frac{2m}{\sqrt{1-\beta^2}} + TL\frac{\arcsin\beta}{\beta}$$

$$J = \frac{mL\beta}{\sqrt{1-\beta^2}} + \frac{1}{4}TL^2\frac{\arcsin\beta - \beta\sqrt{1-\beta^2}}{\beta^2}$$

- In the limit of small masses

$$J = \frac{1}{2\pi T}E^2\left(1 - \frac{8\sqrt{\pi}}{3}\left(\frac{m}{E}\right)^{3/2} + \frac{2\pi^{3/2}}{5}\left(\frac{m}{E}\right)^{5/2} + \dots\right)$$

- In the limit of large masses

$$J_4 = \frac{2m^{1/2}}{T3\sqrt{3}}(E-2m)^{3/2} + \frac{7}{\sqrt{1083}m^{1/2}T}(E-2m)^{5/2} - \frac{1003}{\sqrt{2332803}Tm^{3/2}}(E-2m)^{7/2}$$

# The quantum string with massive endpoints

# Fluctuations and gauge choice

- Consider the quantum fluctuations around the rotating solution

$$X^{\mu}=X^{\mu}_{cl}+\delta X^{\mu}=\left(t\,,\rho\,,\theta\,,z^{i}\right)=\left(\tau+\lambda\delta t,R(\sigma)+\lambda\delta\rho,k\tau+\lambda\delta\theta,\lambda\delta z^{i}\right)$$

- Where $\lambda$. is a formal expansion parameter later will be expressed in terms of $\frac{1}{mL}$
- We fix the reparameterization invariance using the orthogonal gauge

$$\boxed{\begin{aligned}&\frac{1}{2}(h_{\tau\tau}+h_{\sigma\sigma})=\frac{1}{2}(\dot{X}^{2}+X'^{2})=0\\&h_{\tau\sigma}=h_{\sigma\tau}=\dot{X}\cdot X'=0\end{aligned}}$$

- We further use the static gauge $\tau=X^{0} \quad \Rightarrow \quad \delta t=0$

$$R(\sigma)=\sigma \text{ or } R(\sigma)=\frac{1}{k}\sin(k\sigma$$

# Transverse fluctuations

- Consider the fluctuations transverse to the plane of rotation. We truncate the action to second order

$$S_{st,\delta z} = -T\lambda^2 \int d\tau d\sigma \left[\frac{1}{2}\left(\sqrt{R'^2}g\right)^{-1}\delta z'^2 - \frac{1}{2}\left(\sqrt{R'^2}g\right)\delta\dot{z}^2\right]$$

$$S_{pp,\delta z} = m\lambda^2 \int d\tau \frac{1}{2}\gamma\delta\dot{z}^2$$

- To properly normalize the kinetic term define

$$f_t \equiv (\sqrt{R'^2}g)^{1/2}\delta z$$

- For the case

$$R(\sigma) = \tfrac{1}{k}\sin(k\sigma)$$

$$S_{st,\delta z} = -T\lambda^2 \int d\tau d\sigma \left(\frac{1}{2}f_t'^2 - \frac{1}{2}\dot{f}_t^2\right)$$

$$S_{pp,\delta z} = m\lambda^2 \int d\tau \frac{1}{2}\gamma \dot{f}_t^2$$

# The transverse fluctuations

- The world sheet Hamiltonian

$$H = \frac{1}{2}T\lambda^2 \left( \int_{-\ell}^{\ell} d\sigma (\dot{f}_t^2 + f_t'^2) + \frac{\gamma m}{T} \dot{f}_t^2 |_{\pm\ell} \right)$$

- The Mode expansion and canonical quantization

$$f_t = f_0 + i\sqrt{\mathcal{N}} \sum_{n \neq 0} \frac{\alpha_n}{\omega_n} e^{-i\omega_n \tau} f_n(\sigma)$$

- The canonical quantization condition

$$[f_t(\sigma), \pi_t(\sigma')] = i\delta(\sigma - \sigma')$$

- Can be achieved upon choosing $\mathcal{N} = \frac{1}{2T\ell^2\lambda^2}$

# The transverse fluctuations

- The fluctuations have to obey the bulk EOM

$$f_n'' + \omega_n^2 f_n = 0$$

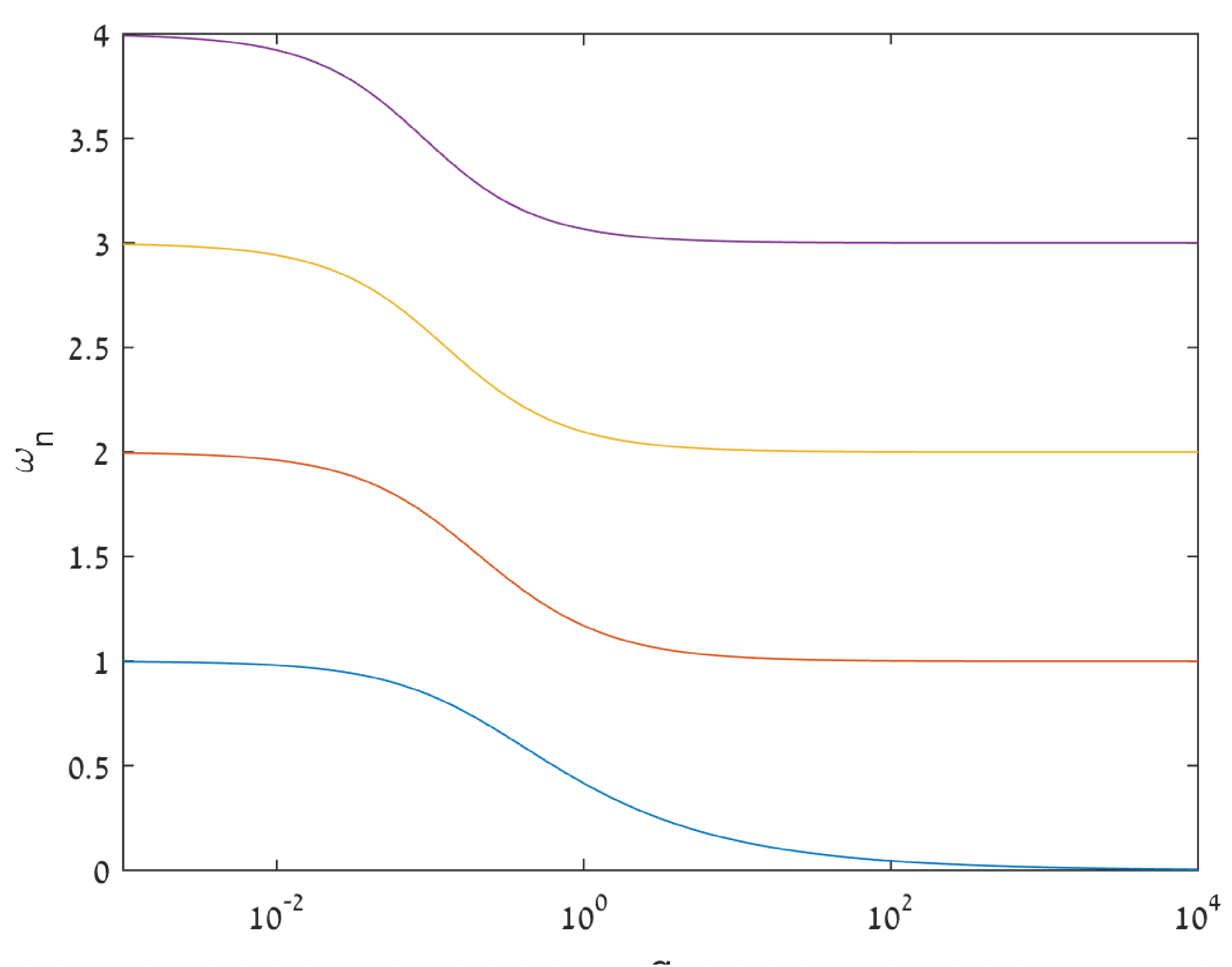
- The boundary equations are $T f_n' \mp \gamma m \omega_n^2 f_n = 0$

- The eigenfquencies  are subject to

$$\boxed{2\delta \cot(\delta) x \cos(2x) - (\delta^2 - \cot^2(\delta) x^2) \sin(2x) = 0}$$

- Where $x \equiv \omega_n \ell \qquad \delta \equiv k\ell = \arccos(\gamma^{-1})$

# The Eigenfrequencies



# Energy and angular momentum

- E and J associated with the transverse modes are

$$E_{st,f_t} = T\lambda^2 \int_{-\ell}^{\ell} d\sigma \frac{1}{2} \frac{1}{\cos^2(k\sigma)} (f_t'^2 + \dot{f}_t^2)$$

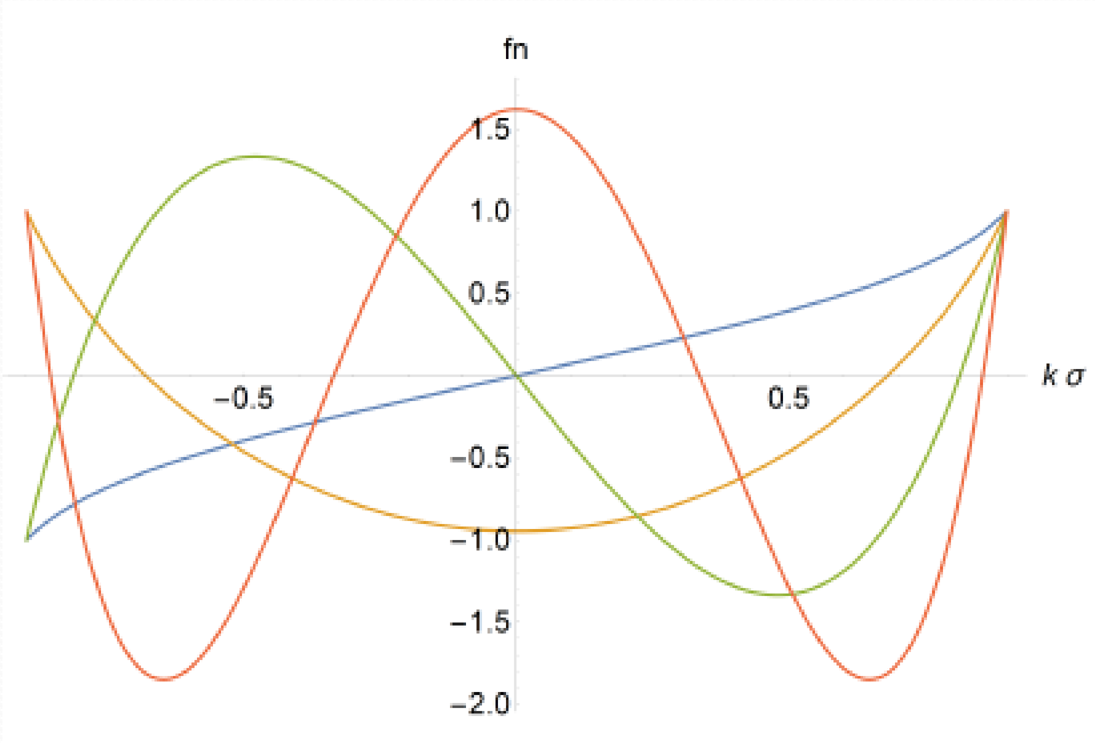
$$E_{pp,f_t} = \frac{1}{2} m\lambda^2 \gamma^3 \dot{f}_t^2$$

$$J_{st,f_t} = T\lambda^2 \int_{-\ell}^{\ell} d\sigma \frac{1}{2} \frac{\tan^2(k\sigma)}{k} (f_t'^2 + \dot{f}_t^2)$$

$$J_{pp,f_t} = \frac{1}{2} m\lambda^2 \gamma^3 \frac{\sin^2(k\ell)}{k} \dot{f}_t^2$$

# The transverse modes

- The first transverse modes



# The quantum intercept

# The intercept

- We would like to determine the impact of the fluctuations on E and J , namely the quantum correction of the trajectory
- We found before the classical E and J

$$E = E(m, T, \gamma) \qquad J = J(m, T, \gamma)$$

- The classical trajectory is defined by $J = J_{cl}(E)$
- We define the intercept as

$$\boxed{a = \langle \delta J - \frac{1}{k}\delta E \rangle} \qquad \frac{1}{k} = \frac{1}{2}\frac{L}{\beta}$$

- We proved that the intercept is given in terms of the expectation value of the world sheet Hamiltonian

$$\boxed{a = -\frac{1}{k}\langle H \rangle}$$

# The intercept

- Using the mode expansion and the orthogonality relations we get

$$H = -\frac{1}{2}\frac{T\lambda^2}{2}\frac{1}{T\ell\lambda^2}\sum_{n\neq 0}\alpha_{-n}\alpha_n = \frac{1}{2\ell}\sum_{n\neq 0}\alpha_{-n}\alpha_n$$

- Thus the contribution of the transverse modes is

$$\boxed{a_t = -\frac{1}{k}\langle H\rangle = -\frac{1}{2}\sum_{n>0}\frac{\omega_n\ell}{\delta}}$$

- For the massless case since $\delta = \frac{\pi}{2}$ and $\omega_n\ell = \frac{\pi}{2}n$

$$\boxed{a_t(m=0) = -\frac{1}{2}\sum_{n>0} n = \frac{1}{24}}$$

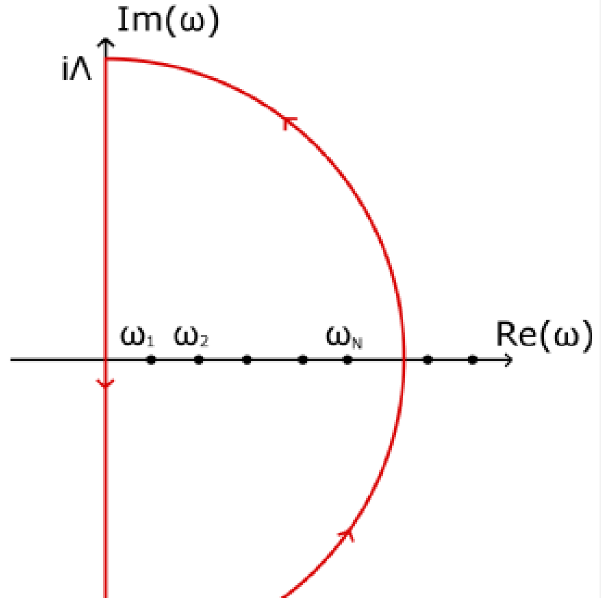
- Using zeta function regulatization

# The renormalization of the sum of the eigenfrequencies

- The zeta function cannot be used for the massive case. We convert the infinite sum into a contour integral using Cauchy integral formula

$$\boxed{\frac{1}{2\pi i}\oint dz z \frac{d}{dz}\log f(z) = \frac{1}{2\pi i}\oint dz z \frac{f'(z)}{f(z)} = \sum_j n_j z_j - \sum_k \tilde{n}_k \tilde{z}_k}$$

- We will use a function $f(\omega)$ with only simple zeros at $\omega = \omega_n$: which are on the positive real axis



# The renormalization of the sum of the eigenfrequencies

- The sum of the eigen-frequencies is the Casimir energy

$$E_C = \frac{1}{2}\sum_{n=1}^{\infty}\omega_n \quad = -\frac{2\beta a}{L}$$

- The semi-circle regularizes the Casimir energy

$$E_C^{(reg)} = \frac{1}{2}\sum_{n=1}^{N(\Lambda)}\omega_n$$

- We renormalize the result in the same way that we do for the Casimir effect. We subtract from the force for a string on lengy L the one of an infinite string

$$\boxed{E_C^{(ren)} = \lim_{\Lambda\to\infty}\left(E_C^{(reg)}(m,T,L) - E_C^{(reg)}(m,T,L\to\infty)\right)}$$

# The renormalization of the sum for the massless case

- For the ordinary string with no endpoint particles

$$f(\omega) = \sin(\pi\omega\ell) = 0$$

- Usually we use the zeta function renormlization

$$\sum_{n=1}^{\infty} n = \zeta(-1) = -\tfrac{1}{12}$$

- Using the contour integral method

$$\boxed{E_C(m=0) = \frac{1}{2}\sum_{n=1}^{\infty}\omega_n = \frac{1}{4\pi i}\oint \omega \frac{f'(\omega)}{f(\omega)}d\omega = \frac{1}{4i}\oint \omega\ell\cot(\pi\omega\ell)d\omega}$$

$$\frac{1}{4i}\oint \omega\ell\cot(\pi\omega\ell)d\omega = -\frac{1}{4}\int_{-\Lambda}^{\Lambda} y\ell\coth(\pi y\ell)dy + \frac{1}{4}\Lambda^2\ell\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{2i\theta}\cot(\pi\Lambda\ell e^{i\theta})d\theta$$

# The intercept for the massless case

- The regularized energy reads

$$E_C^{(reg)} = \left(-\frac{\Lambda^2 \ell}{4} - \frac{1}{24\ell}\right) + \frac{1}{2}\Lambda^2 \ell = \frac{\Lambda^2 \ell}{4} - \frac{1}{24\ell} = \frac{\Lambda^2 L}{8} - \frac{1}{12L}$$

- The corresponding force

$$F_C^{(reg)} = -\frac{d}{dL} E_C^{(reg)} = -\frac{\Lambda^2}{8} + \frac{1}{12L^2}$$

- The renormalized force

$$\boxed{F_C^{(ren)} = \lim_{\Lambda \to \infty} \left(F_C^{(reg)}(L) - F_C^{(reg)}(L \to \infty)\right) = \frac{1}{12L^2}}$$

- The renormalized energy

$$E_C^{(ren)} = -\frac{1}{12L} \qquad \Rightarrow \qquad a = \frac{1}{24}$$

# The renormatization for the rotating massive string

- The integral of the renormalized Casimir energy

$$E_C^{ren} = \frac{1}{\pi}\int_0^\infty \log\left( \frac{2y\beta^2\sqrt{1-\beta^2}\cosh\left(\frac{2y\arcsin\beta}{\beta}\right) + (\beta^4+(1-\beta^2)y^2)\sinh\left(\frac{2y\arcsin\beta}{\beta}\right)}{\frac{1}{2}\left((1-\beta^2)y^2+2\beta^2\sqrt{1-\beta^2}+\beta^4\right)\exp\left(\frac{2y\arcsin\beta}{\beta}\right)} \right)$$

- The corresponding result for the intercept reads

$$\boxed{a_t = -\frac{1}{2\pi\beta}\int_0^\infty \log\left[1-\exp\left(-\frac{4\arcsin\beta}{\beta}y\right)\left(\frac{y-\gamma\beta^2}{y+\gamma\beta^2}\right)^2\right]}$$

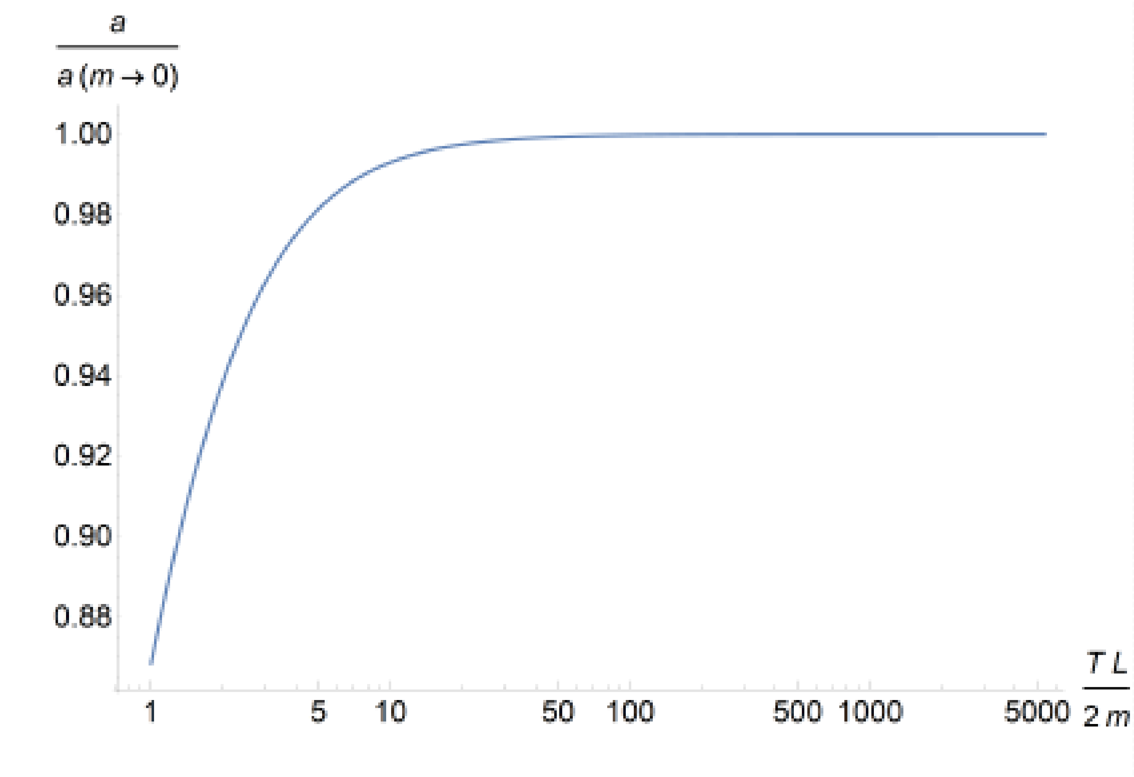
- This goes back to the standard massless result

$$a_t(\beta\to 1) = -\frac{1}{2\pi}\int_0^\infty \log\left(1-e^{-2\pi y}\right) = \frac{1}{24}$$

- For small masses

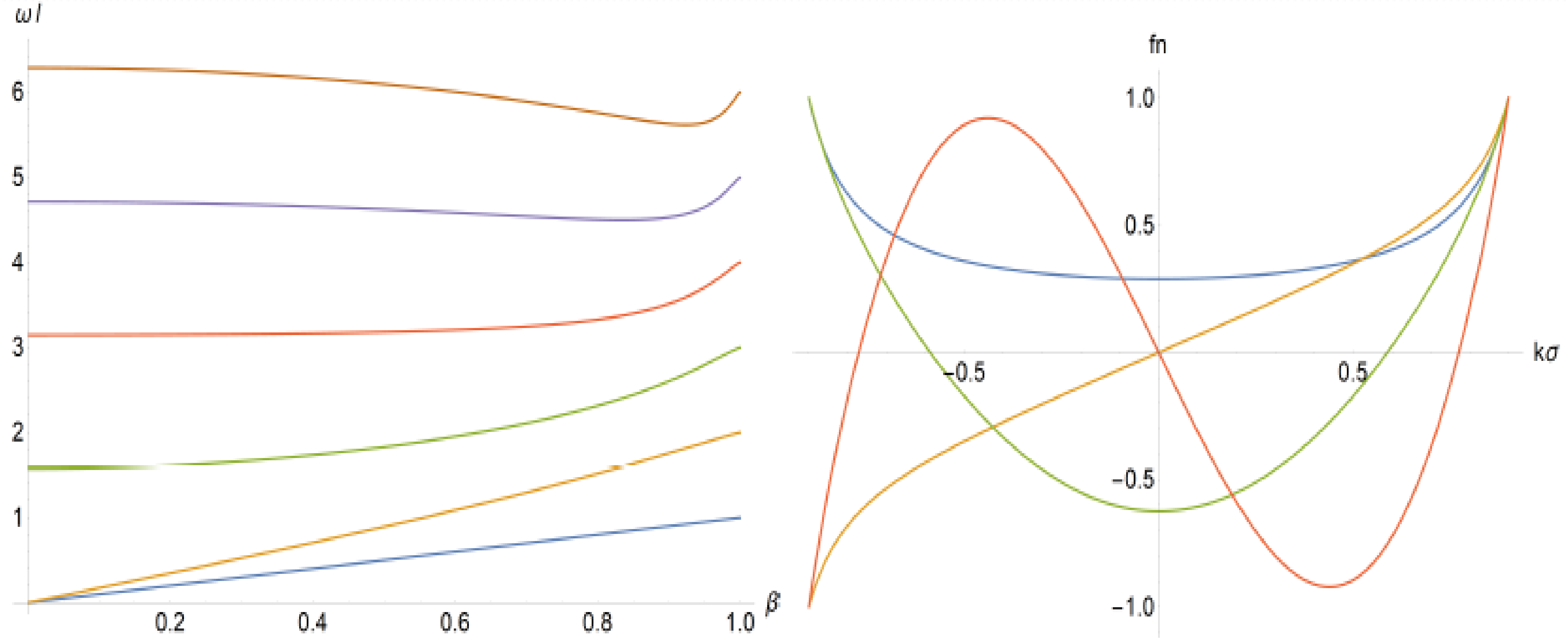
$$\boxed{a_t = \frac{1}{24} - \frac{11}{360\pi}\frac{1}{\gamma^3} + \ldots = \frac{1}{24} - \frac{11}{360\pi}\left(\frac{2m}{TL}\right)^{3/2}}$$

# The contribution of the a trasverse mode to the intercep



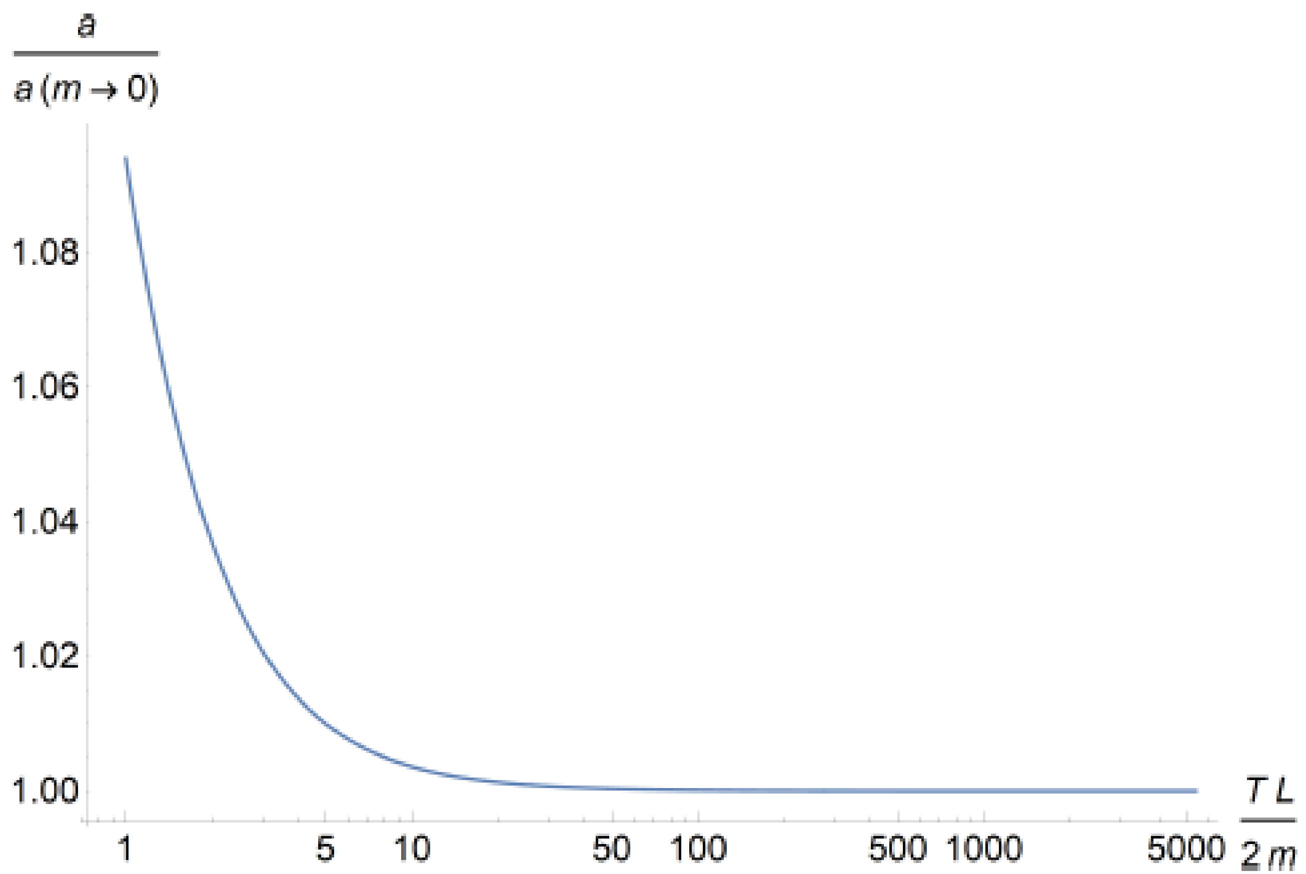
# The contribution to the intercept of the planar mode

- We have done the analysis also for the planar mode
- The eigenmodes and eigenfrequencies



# The planar mode

- The contribution of the planar mode to the intercept



# The Quantization of the non-critical string

# The Polchiski Strominger term

- It is well known that the quantization of the Polyakov string in non-critical dimensions is done by adding to the action the Liouville term
- Polchinsky and Strominger mapped this in the formulation of the NG string by taking a "composite Liouville" mode

$$\phi = -\tfrac{1}{2}\log\left(\partial_+ X \cdot \partial_- X\right)$$

- In the orthogonal gauge the PS term reads

$$\boxed{S_{PS} = \frac{B}{2\pi}\int d^2\sigma \partial_+\phi\partial_-\phi = \frac{B}{2\pi}\int d^2\sigma \frac{(\partial_+^2 X \cdot \partial_- X)(\partial_-^2 X \cdot \partial_+ X)}{(\partial_+ X \cdot \partial_- X)^2}}$$

# The renormalization of the PS term

- The contribution of the PS to the intercept is achieved by substituting the classical solution into the PS action

$$\boxed{E_{PS} = \langle H_{PS} \rangle = -\int d\sigma \mathcal{L}_{PS} = \frac{B}{2\pi}\int_{-\ell}^{\ell} d\sigma k^2 \tan^2(k\sigma) = \frac{B}{\pi} k(\tan\delta - \delta)}$$

where . $\delta = k\ell$ The term $k\tan\delta$ diverges in the massless case since $\delta = \pi/2$

- For small masses it is finite but un-physically large
- Hellerman et all renormalized the PS term for the massless case by

$$S_{PS} \to S_{PS}^{(reg)} = \frac{B}{2\pi}\int d^2\sigma \frac{(\partial_+^2 X \cdot \partial_- X)(\partial_-^2 X \cdot \partial_+ X)}{(\partial_+ X \cdot \partial_- X)^2 + \alpha' \epsilon^4 (\partial_+^2 X \cdot \partial_-^2 X)}$$

- And adding a counterterm

$$S_{ct} \propto \frac{1}{\epsilon}\int d\tau (\ddot{X} \cdot \ddot{X})^{1/4}$$

# The renormalization of the PS term

- The result of the renormalization of the PS term in the massless case is that

$$a_{PS}(m=0) = \frac{26-D}{24}$$

- As a result the total intercept is in any D dimensions

$$\boxed{a(m=0) = \frac{D-2}{24} + \frac{26-D}{24} = 1}$$

- The massive endpoints serve as a regulator. Never the less we have to perform a subtraction since we subtract anyhow for the other divergenseis and also since we want to connect smoothly to the subtraction at m=o

# The renormalization of the PS term

- We can re-write the PS term as

$$\boxed{E_{PS} = \frac{B}{\pi}\left(\frac{T}{\tilde{m}} - \frac{2\delta^2}{\tilde{L}}\right)}$$

- Using the length and mass measured in the Lab frame

$$\tilde{m} = \gamma m \qquad \tilde{L} = L\frac{\arcsin\beta}{\beta} = \frac{2}{k}\sin\delta \times \frac{\delta}{\sin\delta} = \frac{2\delta}{k}$$

- Based on the boundary equation of motion

$$k\tan\delta = \frac{T\cos\delta}{m} = \frac{T}{\gamma m}$$

- Again we renormalize by subtracting from the force for the string of length L the force of the string wit $L \to \infty$

# The contribution of the PS term to the intercept

- Thus the renormalized contribution of the

$$\boxed{a_{PS} = -\frac{1}{k}E_{PS}^{(ren)} = \frac{26-D}{12\pi}\delta = \frac{26-D}{12\pi}\arcsin\beta}$$

- In the limit of $\beta \to 1$ $a_{PS} = \frac{26-D}{24}$

- As a function of $2m/TL$ it reads

$$a_{PS} = \frac{26-D}{12\pi}\arccos\left(\sqrt{\frac{2m}{2m+TL}}\right) = \frac{26-D}{24}\left[1 - \frac{2}{\pi}\left(\frac{2m}{TL}\right)^{1/2} + \frac{2}{3\pi}\left(\frac{2m}{TL}\right)^{3/2} + \dots\right]$$

- Thus the total intercept is

$$\boxed{a = (D-3)a_t + a_p + a_{PS} \approx 1 - \frac{26-D}{12\pi}(\frac{2m}{TL})^{1/2} + \frac{199-14D}{240\pi}(\frac{2m}{TL})^{3/2}}$$

# The puzzle of the realistic intercept

- In nature the intercept associated with all the hadrons whether mesons or baryons is **negative** when it is defined in the relation of the orbital and not the total angular momentum.
- For instance $\rho$ has a 0.5 and S=1 s so for L=J-S we get a= -0.5
- The attempt to explain this universal feature is probably the most important problem of the hadronic spectra.
- To account for it we study strings with different masses, electric charges and spins at their ends.
- We can get negative intercept but not yet in a fully satisfactory manner

# Asymmetric string

- So far we have assumed that the two endpoint particles carry the same mass. We generalize it now to the case of two different masses

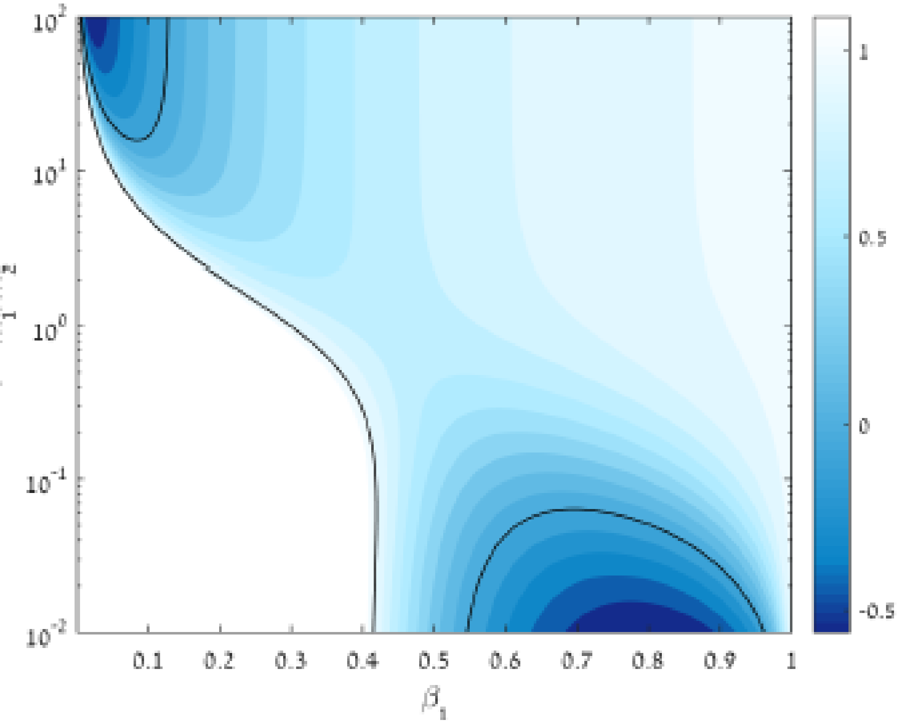
- The extreme case of one massless end and one infinitely heavy end is a Dirichlet-Neumann case which can analyzed as the usual case with $\omega_n = n - \frac{1}{2}$
- The intercept for such a case is

$$\boxed{a_{ND} = -\frac{1}{2}\sum_{n=1}^{\infty}(n - \frac{1}{2}) = -\frac{1}{48}}$$

- Instead of 1/24 for the DD or NN cases.
- The contributions of the transverse, planar and the PS mode to the intercept as a function of β1 and β2

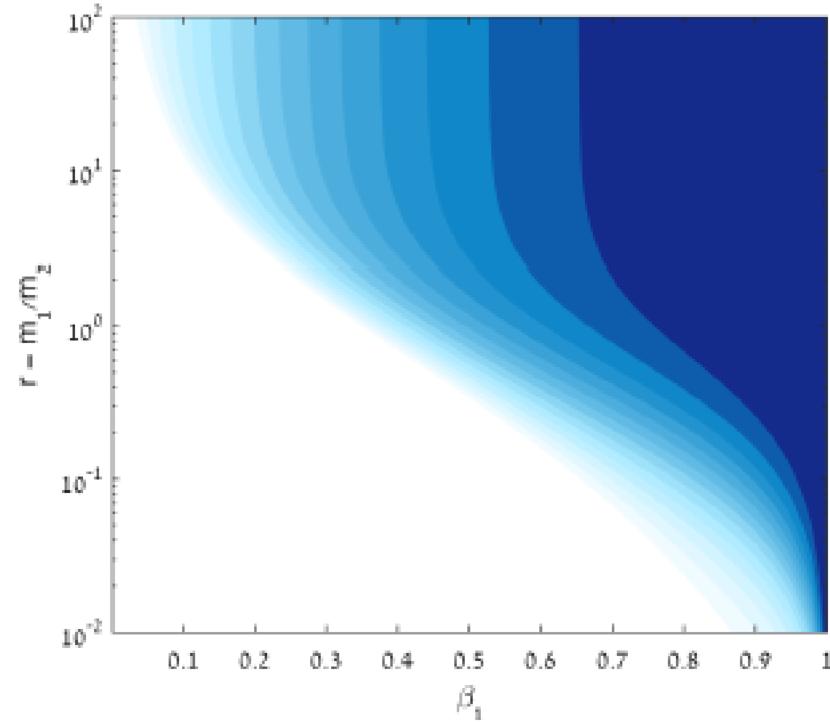
# The asymmetric intercept

- The intercept as a function of $r = m_1/m_2$. and $\beta_1$




Transverse

(2)

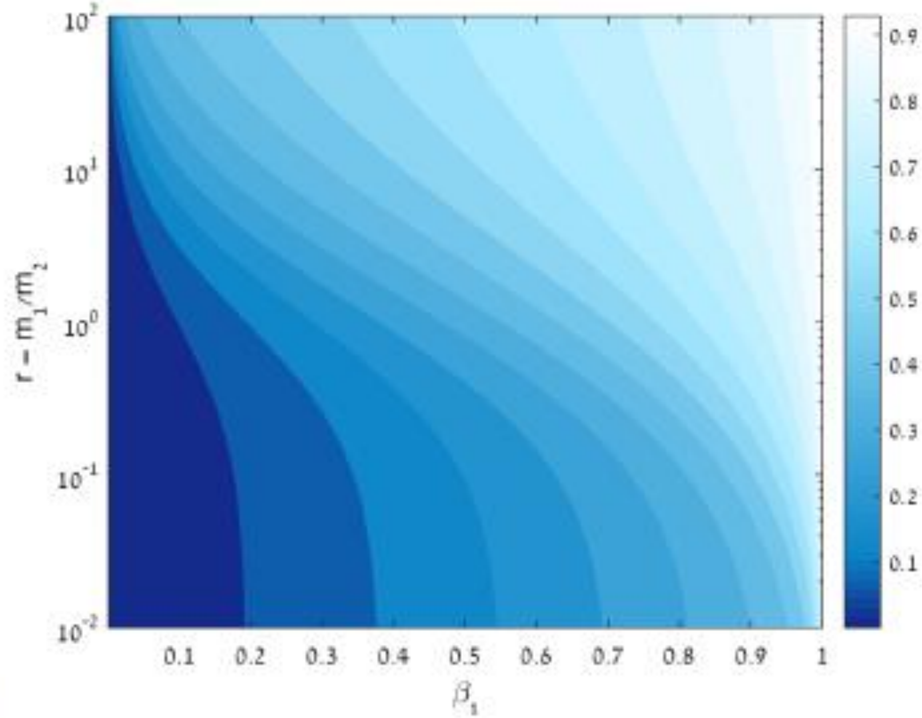



planar

# The asymmetric intercept

- The intercept

(3)

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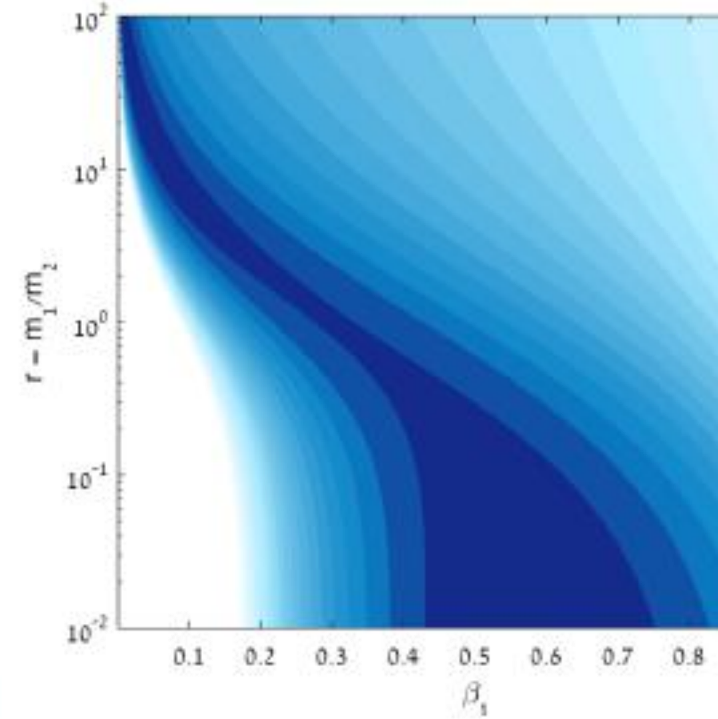


contribution of PS

full intercept

# The Propagator of the string with massive endpoints

# The propagator

- The propagator an ordinary bosonic string is

$$\boxed{X^\mu(z,\bar z)X^\nu(w,\bar w) \quad = -\frac{\alpha'}{2}\eta^{\mu\nu}\left(\log|z-w|^2+\log|z-\bar w|^2\right)}$$

- For the massive one we start with left and right moving modes

$$X_R^\mu = \frac{1}{2}(x^\mu - c^\mu) - \frac{1}{2}N_0 p^\mu(\tau-\sigma) + i\sqrt{N}\sum_{n\neq 0}\frac{\alpha_n^\mu}{\omega_n}e^{-i\frac{\pi}{\ell}\omega_n(\tau-\sigma)}$$

$$X_L^\mu = \frac{1}{2}(x^\mu + c^\mu) - \frac{1}{2}N_0 p^\mu(\tau+\sigma) + i\sqrt{N}\sum_{n\neq 0}\frac{e^{i\phi_n}\alpha_n^\mu}{\omega_n}e^{-i\frac{\pi}{\ell}\omega_n(\tau+\sigma)}$$

- We impose the boundary conditions and find for the right modes

$$\boxed{X_R^\mu(z)X_R^\nu(w) = -\frac{\alpha'}{2}\eta^{\mu\nu}\left(\log z - \sum_{n=1}^{\infty}\frac{1}{\omega_n}\left(\frac{w}{z}\right)^{\omega_n}\right)}$$

## The propagator

- Approximating the eigenfrequencis as

$$\omega_n = n + \delta_n \qquad \delta_n \approx -\frac{2}{q} n$$

Where $q \equiv T\ell/\gamma m$

- After a lengthy calculation we get that the HISH propagator in the limit of large q reads

$$X^\mu(z,\bar{z}) X^\nu(w,\bar{w}) = -\alpha' \eta^{\mu\nu} \left[ \frac{2\pi i}{q} \left( \frac{\bar{z}}{\bar{z} - w} - \frac{z}{z - \bar{w}} \right) + \left(1 + \frac{2}{q}\right) \left( \log|z - w| + \log|z - \bar{w}| \right) \right]$$

# Summary and open questions

- The HISH model is successful in deriving hadronic results that are difficult to account using QCD
- The trajectories that follows from the model fit very nicely the hadronic spectra
- The decay width of hadron were also computed and their fit to the data is also good
- We are currently computing the analog of the **Veneziano amplitude** for scattering of massive strings.
- Adding charges and spins are next to do facing the challenge of getting **negative intercept**
- Many more hadronic data to explain like **Jet physics**