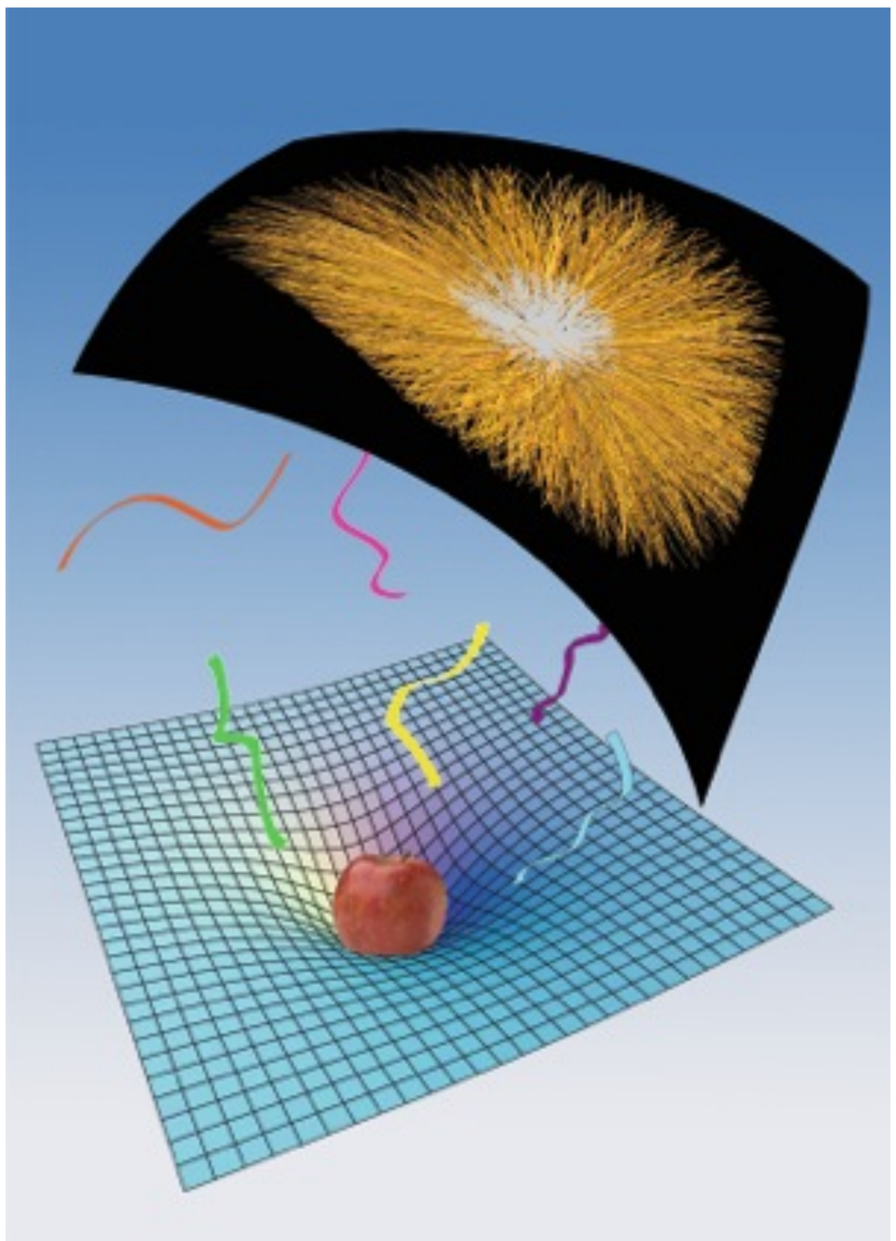
News about Heavy Quarks in Strongly Coupled Plasmas

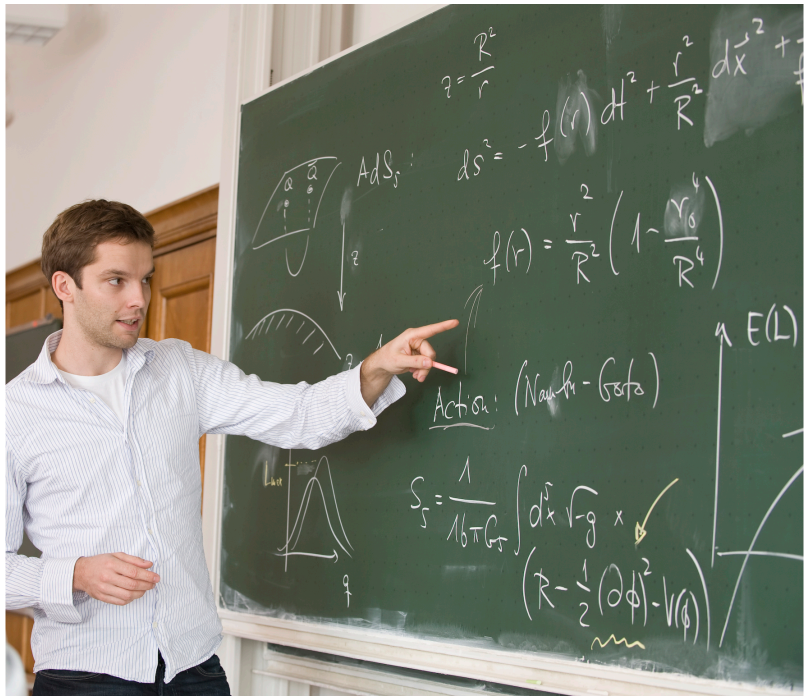


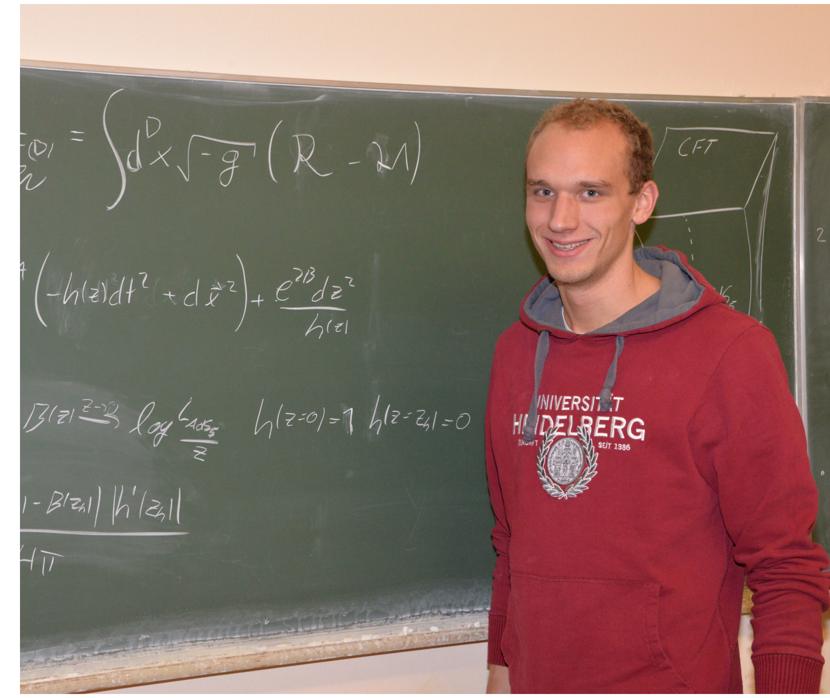
Carlo Ewerz

Heidelberg University
& EMMI/GSI
& FIAS

Gauge/Gravity Duality 2018
Würzburg
July 31, 2018

The Team





Konrad Schade

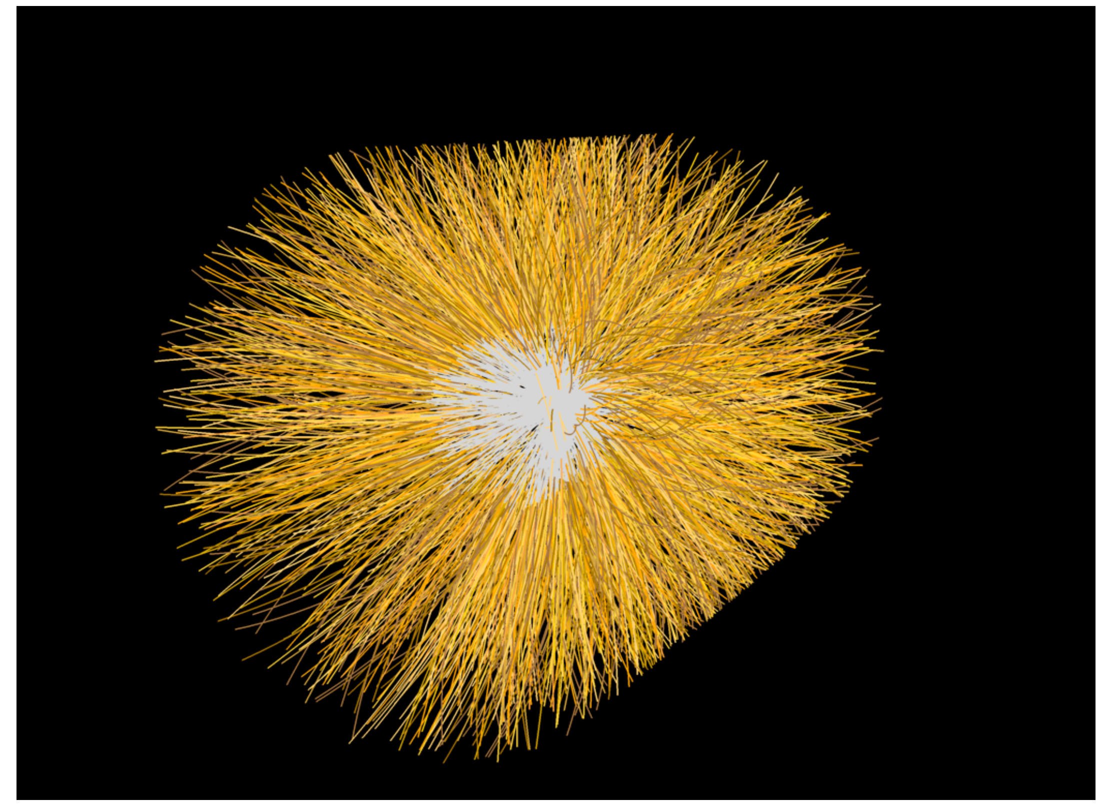
Paul Wittmer

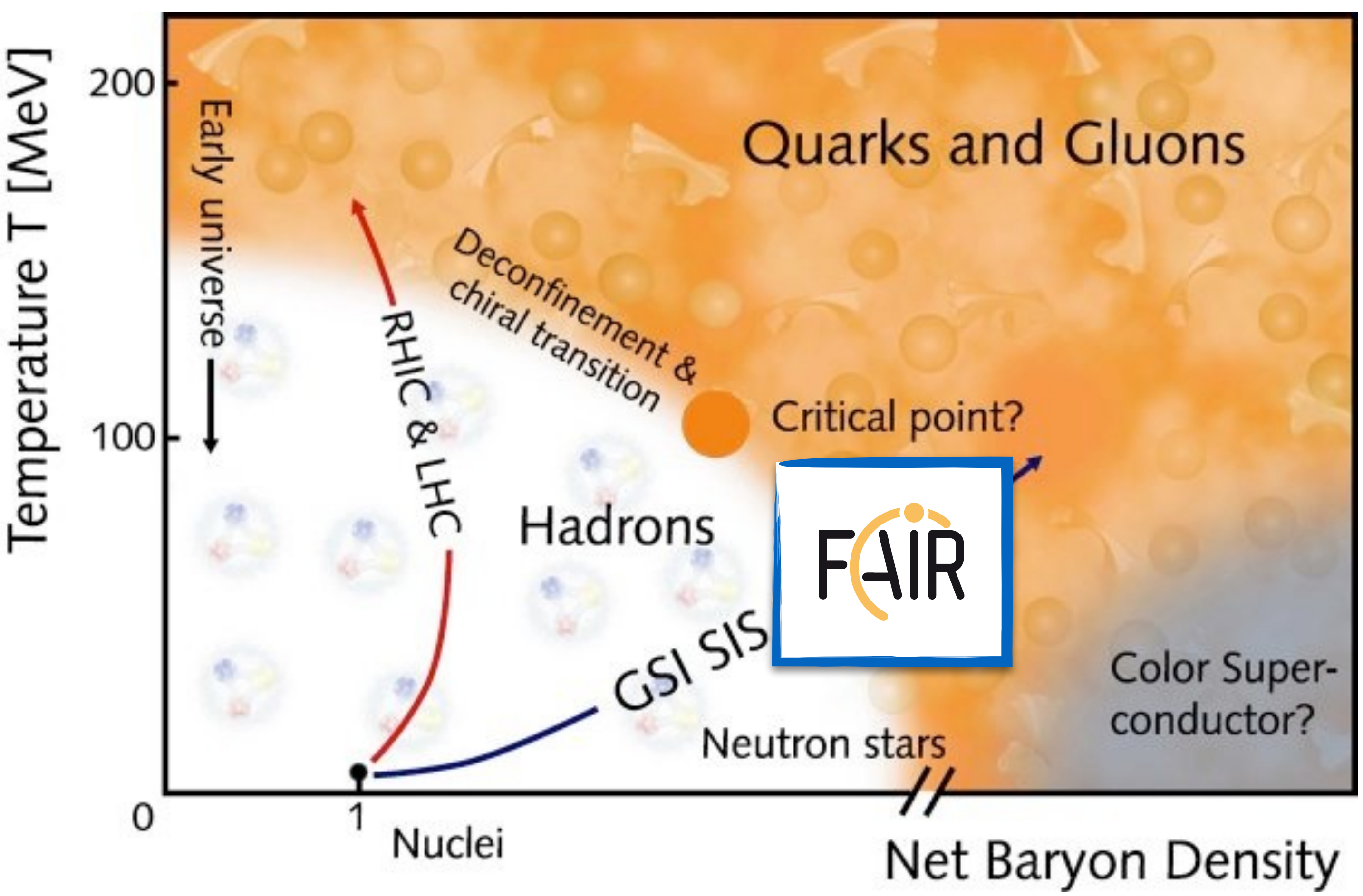
Andreas Samberg

and:
Olaf Kaczmarek

Motivation

quark-gluon plasma:
strongly coupled!
→ holography useful



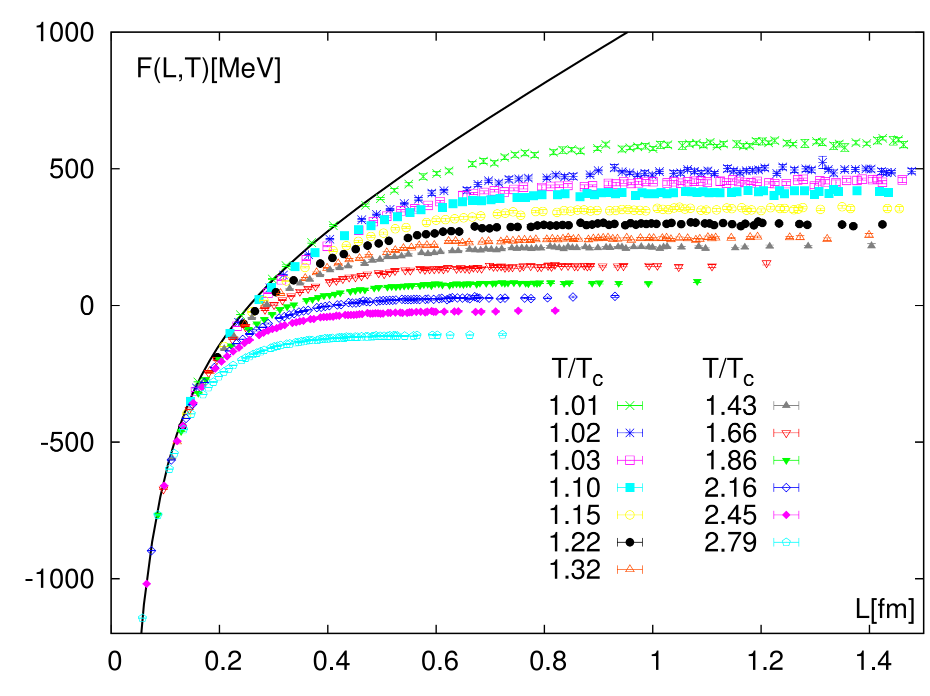


to approximate
real-world QCD
use non-conformal,
deformed AdS

Heavy Quarks in Quark-Gluon Plasma

Heavy quarks as probes of QGP:

- produced early in collision, witness full evolution of fireball to hadronization
- rich phenomenology in heavy-ion collisions
- heavy-quark bound states as sensitive probes of thermal medium
- well-studied in lattice QCD



Our Aim

look for universal or robust properties generically emerging in strongly coupled theories
→ classes of holographic models

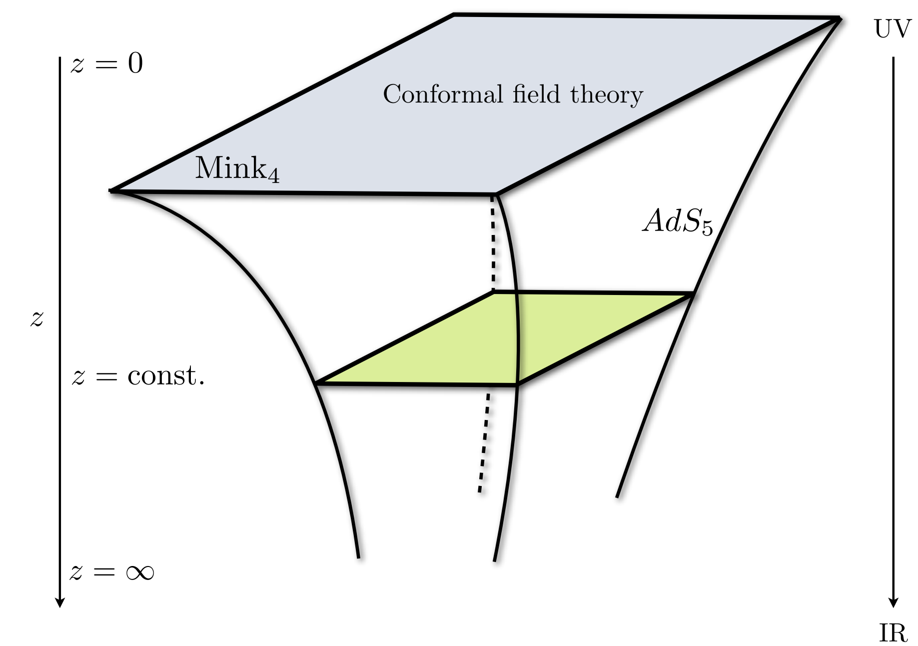
aim is not to find a precise model for QCD

AdS Models for the Plasma

N=4 SYM at Finite Temperature

zero temperature: AdS5

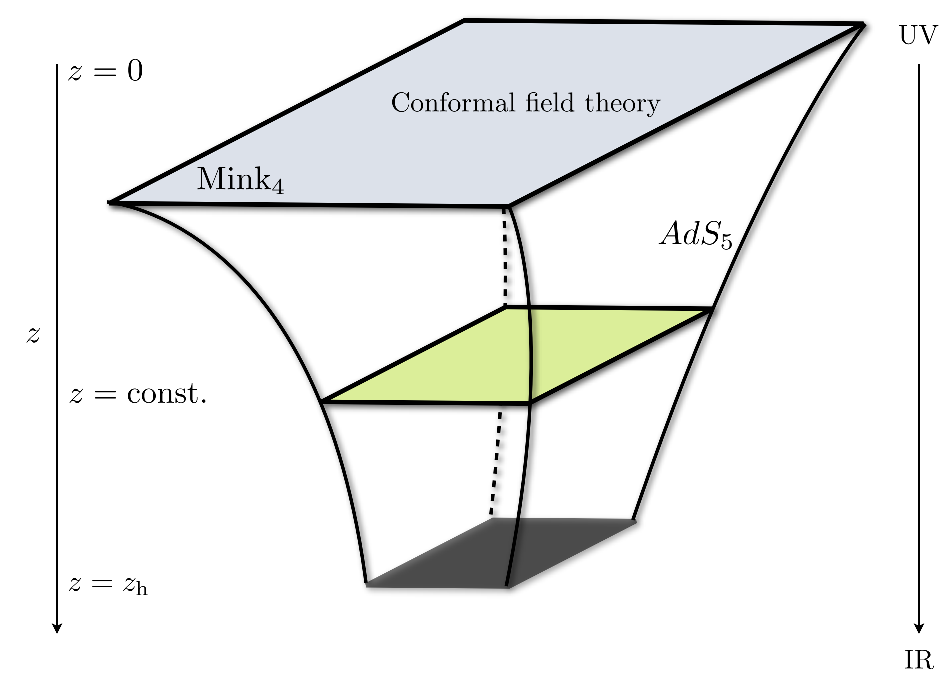
$$ds^2 = \frac{L^2}{z^2}\left(- \mathrm{d}t^2 + \mathrm{d}\vec{x}^{\,2} + \mathrm{d}z^2 \right)$$



finite temperature T:
AdS5 with black hole

$$ds^2 = \frac{L^2}{z^2}\left(- h\,\mathrm{d}t^2 + \mathrm{d}\vec{x}^{\,2} + \frac{\mathrm{d}z^2}{h} \right)$$

with $h = 1 - \frac{z^4}{z_h^4}$ and $T = \frac{1}{\pi\, z_h}$



Consistent Non-conformal Model

Start with five dimensional gravity action S_{EHs} :

$$S_{\mathrm{EHs}} = \frac{1}{16\pi\, G_{\mathrm{N}}^{(5)}} \int \mathrm{d}^5x \sqrt{-G} \left(\mathcal{R} - \frac{1}{2} \left(\partial\Phi\right)^2 - V(\Phi) \right)$$

with general ansatz

$$\mathrm{d}s^2 = \mathrm{e}^{2A(z)} \left(-\, h\, \mathrm{d}t^2 + \mathrm{d}\vec{x}^{\,2} \right) + \mathrm{e}^{2B(z)} \, \frac{\mathrm{d}z^2}{h}$$

$$T = \mathrm{e}^{A(z_{\mathrm{h}}) - B(z_{\mathrm{h}})} \, \frac{|h}{\;}$$

leads to 3 independent equations of motion but 5 unknown functions $V,\ \Phi,\ A,\ B,\ h$.

- 2-parameter model:
  with parameters ϕ, c

  DeWolfe, Rosen; Gubser; 2009

$$\mathrm{e}^{2A(z)} = \mathrm{e}^{c\, z^2} \, \frac{L^2}{z^2} \quad \text{and} \quad \Phi(z) = \sqrt{\frac{3}{2}}\, \phi\, z^2$$

$$\alpha = \frac{c}{\phi}$$

- 1-parameter model:
  with parameter κ

  Schade

$$\mathrm{e}^{2A(z)} = \frac{L^2}{z^2} \quad \text{and} \quad \Phi(z) = \sqrt{\frac{3}{2}} \kappa z^2$$

Consistent Non-conformal Model

Start with five dimensional gravity action S_{EHs} :

$$S_{\mathrm{EHs}} = \frac{1}{16\pi\, G_{\mathrm{N}}^{(5)}} \int \mathrm{d}^5x\sqrt{-G}\left(\mathcal{R} - \frac{1}{2}\left(\partial\Phi\right)^2 - V(\Phi)\right)$$

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$$\mathrm{d}s^2 = \mathrm{e}^{2A(z)}\left(-\,h\,\mathrm{d}t^2 + \mathrm{d}\vec{x}^{\,2}\right) + \mathrm{e}^{2B(z)}\,\frac{\mathrm{d}z^2}{h}$$

$$T = \mathrm{e}^{A(z_{\mathrm{h}})-B(z_{\mathrm{h}})}\,\frac{|h'(z_{\mathrm{h}})|}{4\pi}$$

scalar Φ :

can be dilaton ('string frame model')

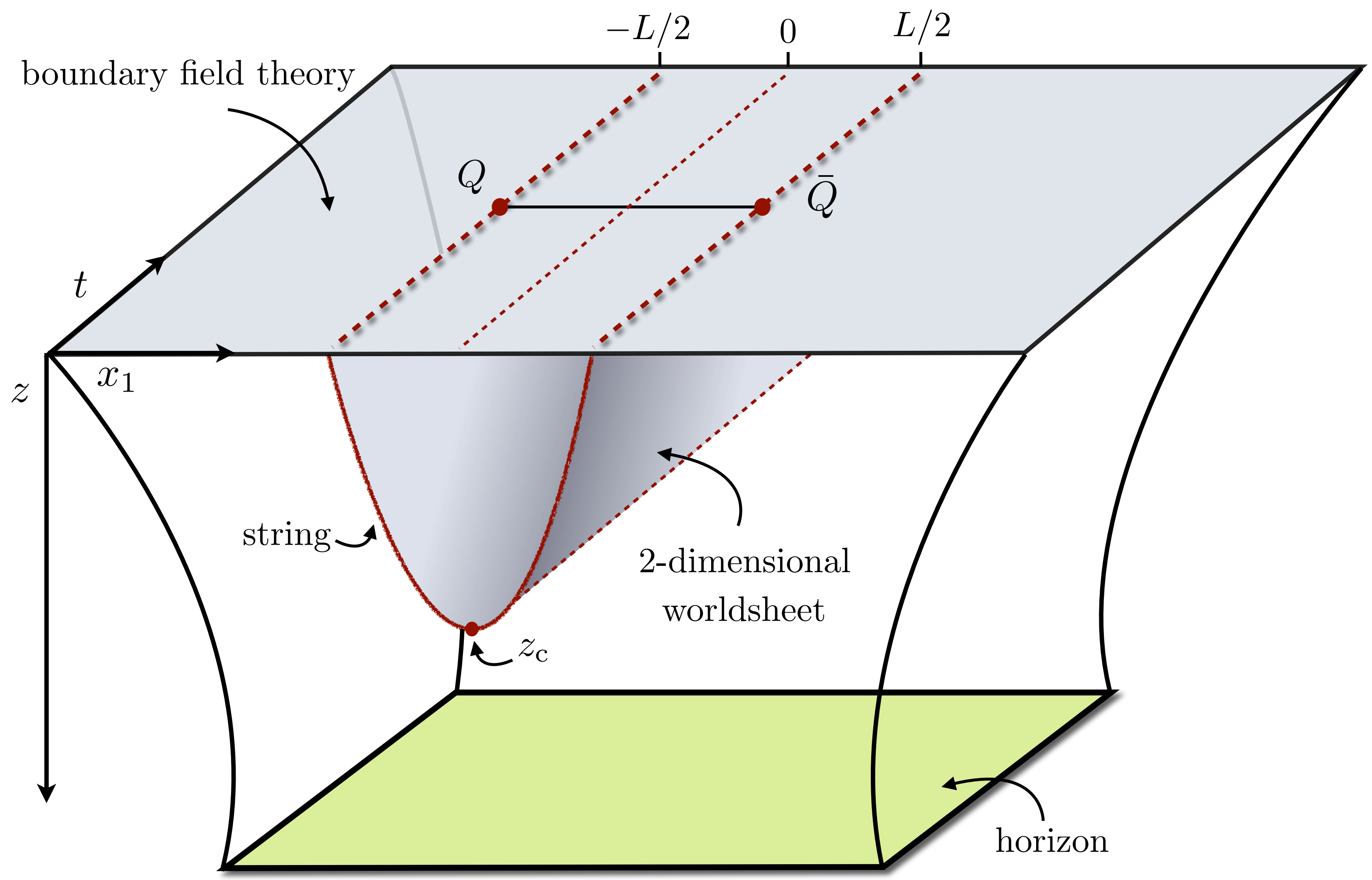
or not ('Einstein frame model')

We consider both possibilities as independent models.

Screening Distance

Konrad Schade, CE

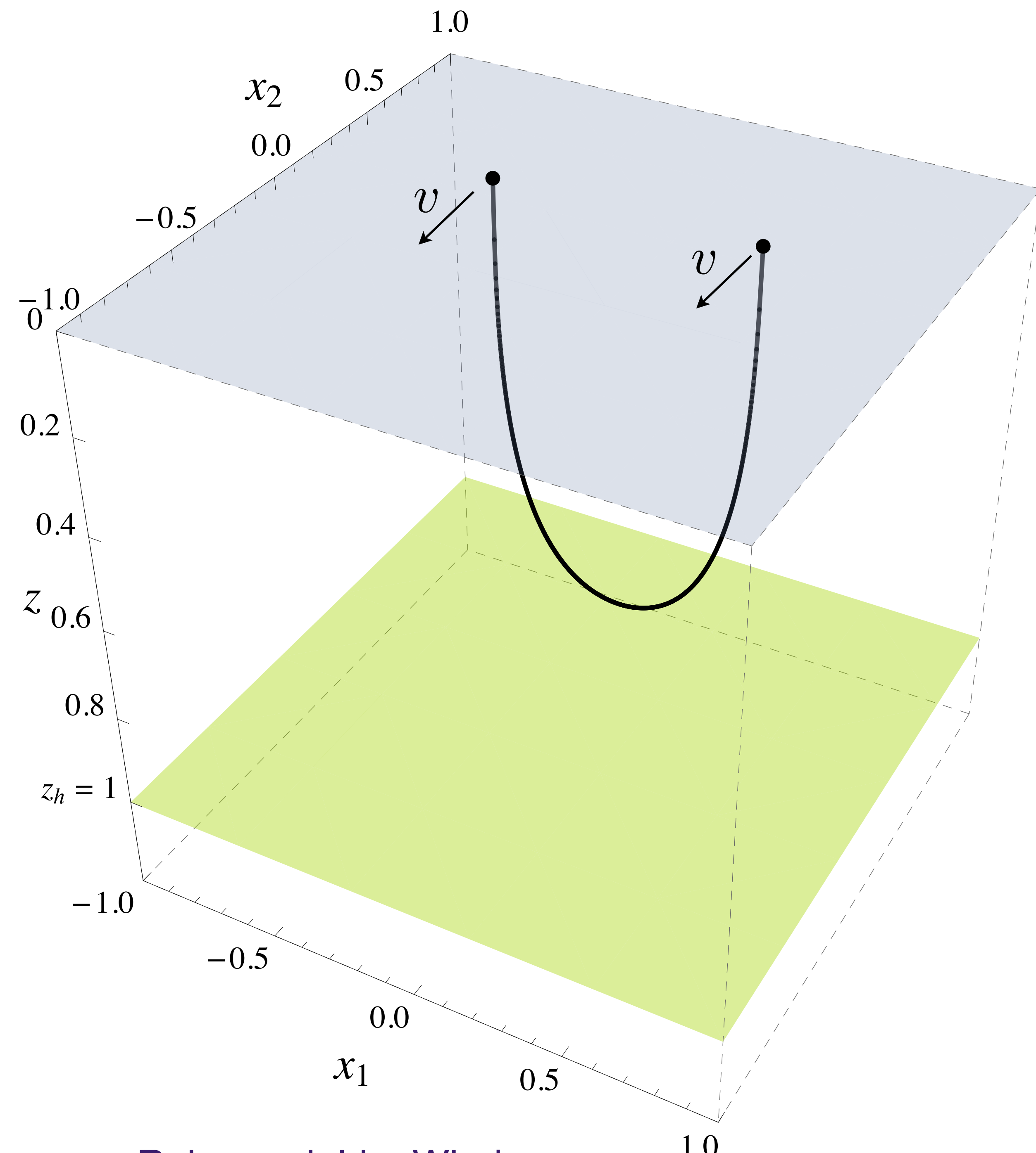
Consider static (heavy) quark-antiquark pair



Expectation value of temporal Wegner-Wilson loop in boundary field theory dual to macroscopic string hanging into the bulk

Screening Distance

- static $Q\bar{Q}$-pair in a hot plasma wind blowing in x_2-direction
- velocity is given by $v = \tanh \eta$
- orientation angle θ w.r.t. wind



Rajagopal, Liu, Wiedemann

- Nambu-Goto action:

$$S = \frac{1}{2\pi\alpha'} \int d\sigma d\tau \sqrt{-\det g_{\alpha\beta}}$$

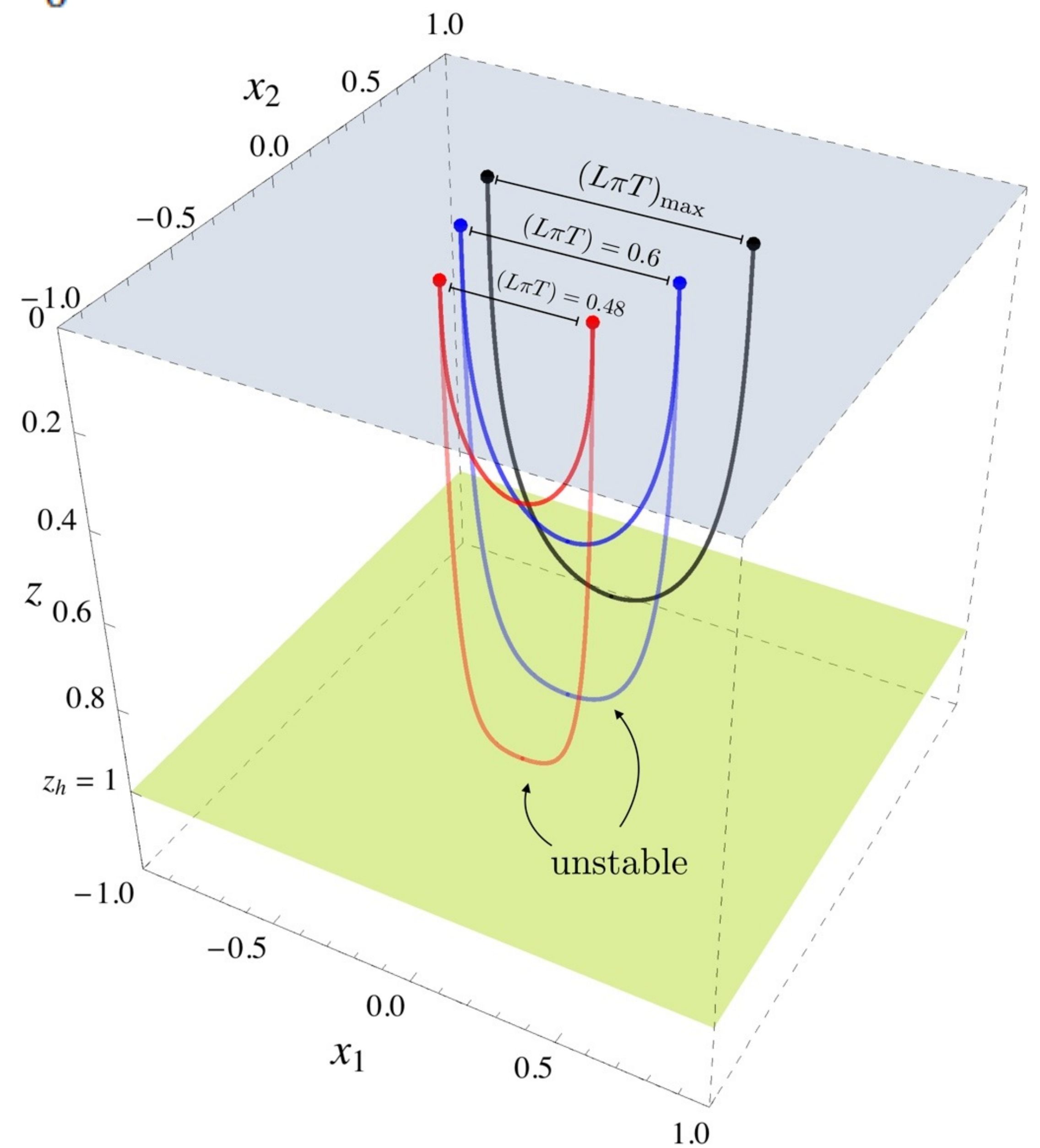
with $g_{\alpha\beta} = G_{\mu\nu} \partial_\alpha x^\mu \partial_\beta x^\nu$

Screening Distance

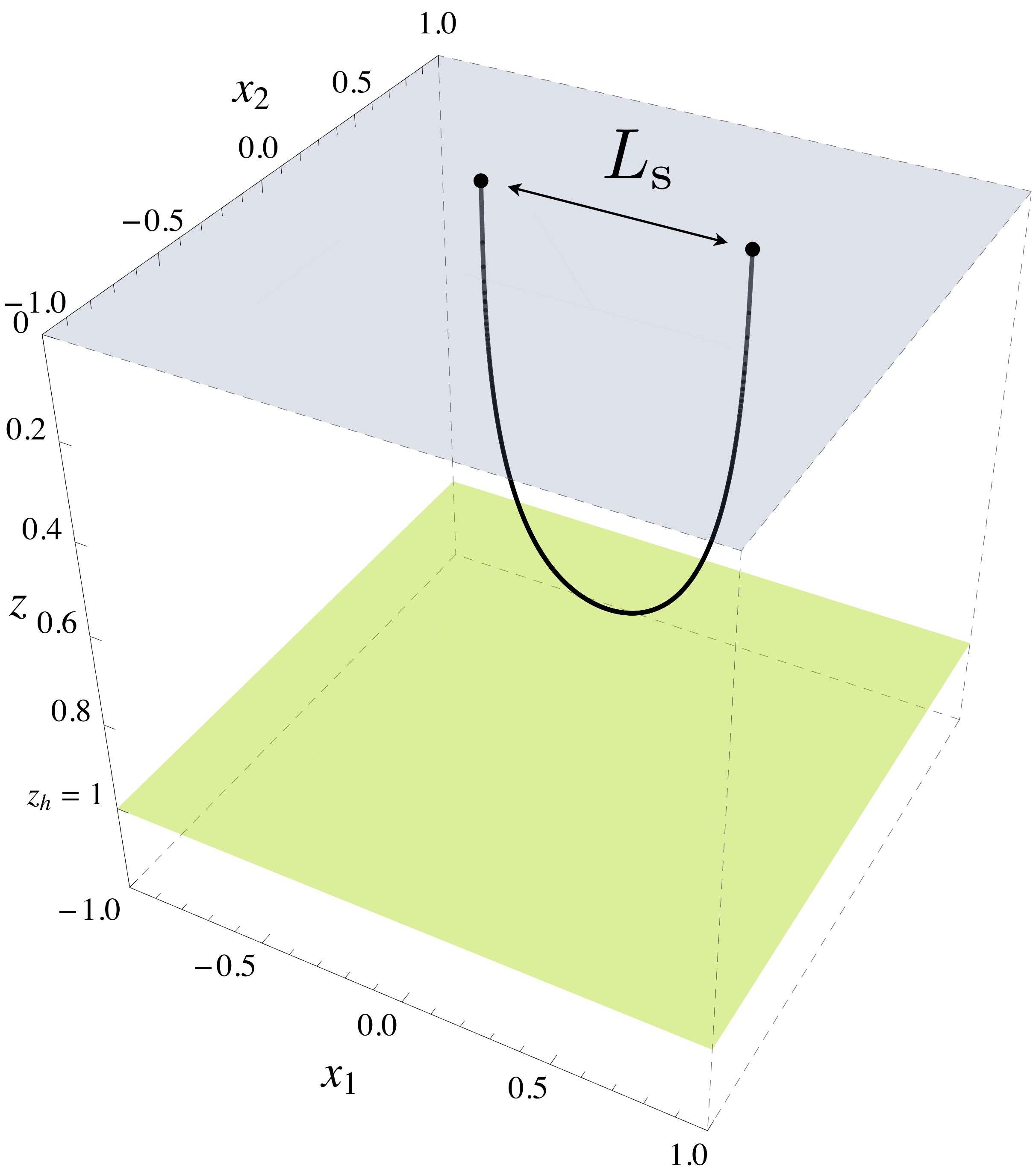
distance L:

$$L\pi T = 2\pi T \int_0^{L/2} \mathrm{d}\sigma = 2\pi T \int_0^{L/2} \frac{\mathrm{d}z}{z'} = 2\pi T \int_0^{z_c} \mathrm{d}z \sqrt{\frac{h(z_c)\, e^{2B(z)+4A(z_c)-2A(z)}}{h(z)^2\, e^{4A(z)} - h(z)\, h(z_c)\, e^{4A(z_c)}}}$$

lowest point z_c parametrizes different configurations



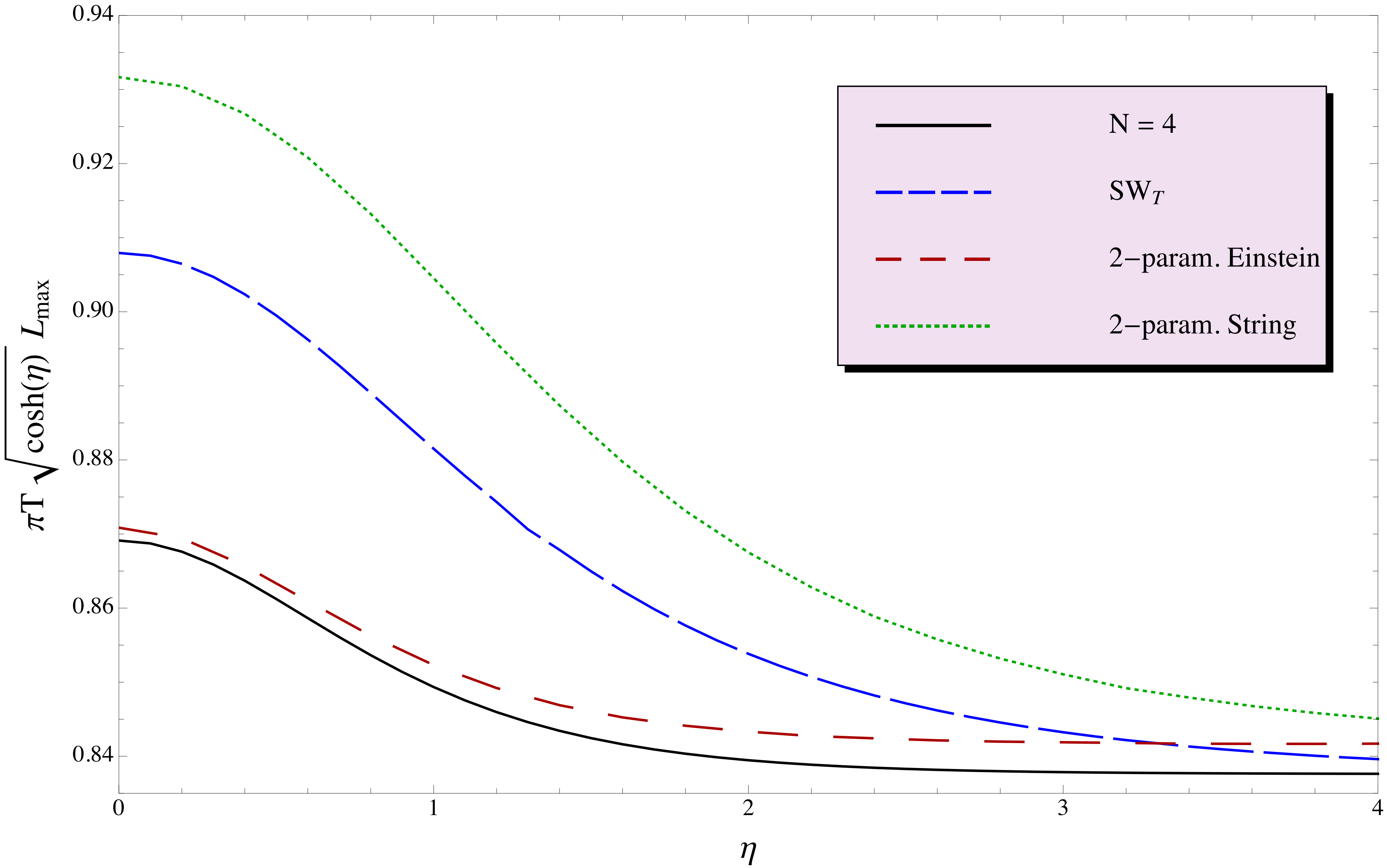
Screening Distance



maximal distance L_s
is screening distance

(different from Debye screening length)

Screening Distance



- **velocity lowers screening distance**

$$\propto 1/\sqrt{\gamma} \;\; \propto (\text{boosted energy density})^{-1/4}$$

Screening Distance Conjecture

Konrad Schade, CE

Observation:

At given T screening distance in N=4 SYM is smaller than in all consistently deformed models studied.

- holds for all kinematical parameters

Conjecture:

Screening distance in N=4 SYM is lower bound in a large class of (or maybe all?) consistent theories.

Energies

A. Samberg, O. Kaczmarek, CE

An old problem…

Free energy / potential of heavy quark-antiquark pair calculated in 1998:
Rey, Theisen, Yee;
Brandhuber, Itzhaki, Sonnenschein, Yankielowicz
and many times since then.
However, … actually not.

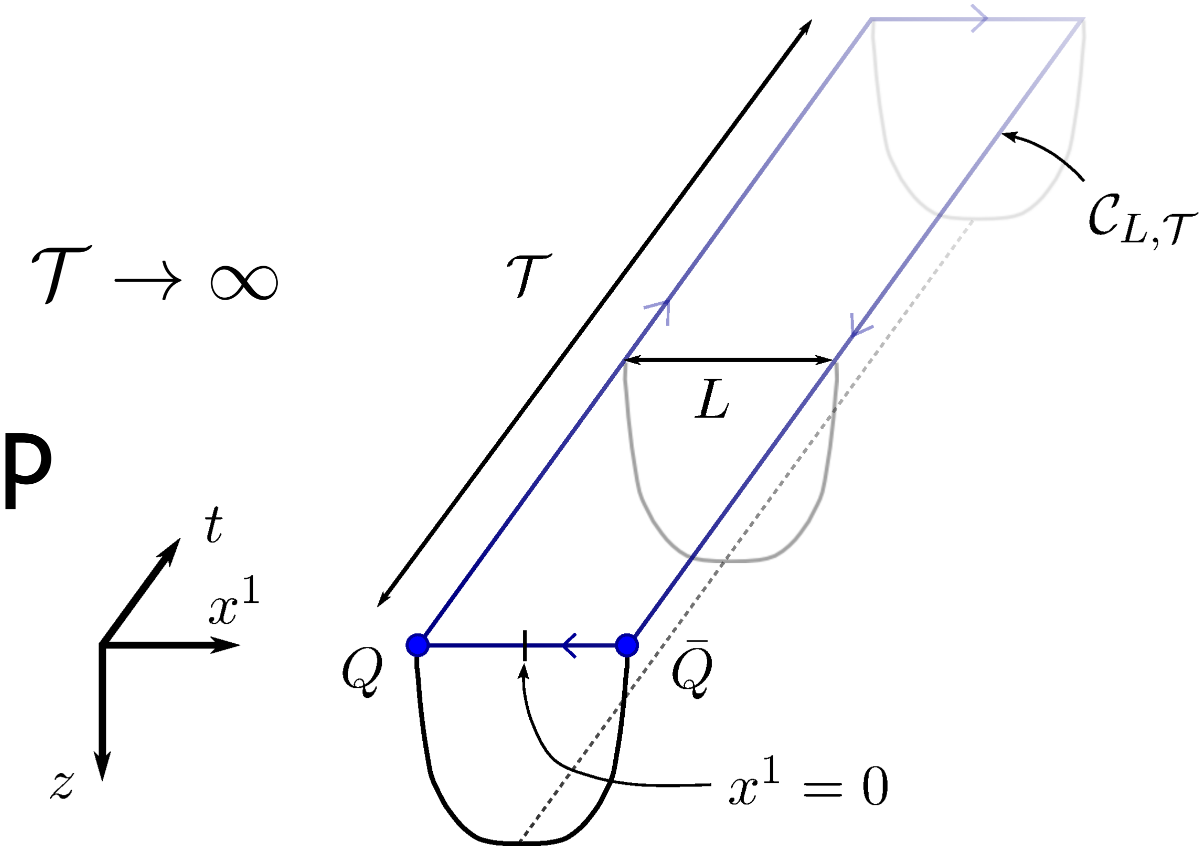
Physical expectation:
potential independent of T at small distances

Free Energy of Heavy QQbar pair

in field theory:

$$\langle W(\mathcal{C}_{L,\mathcal{T}})\rangle \sim \exp\left(-\mathrm{i}F_{Q\bar{Q}}(L)\mathcal{T}\right) \,, \qquad \mathcal{T}\to\infty$$

for temporal Wilson loop



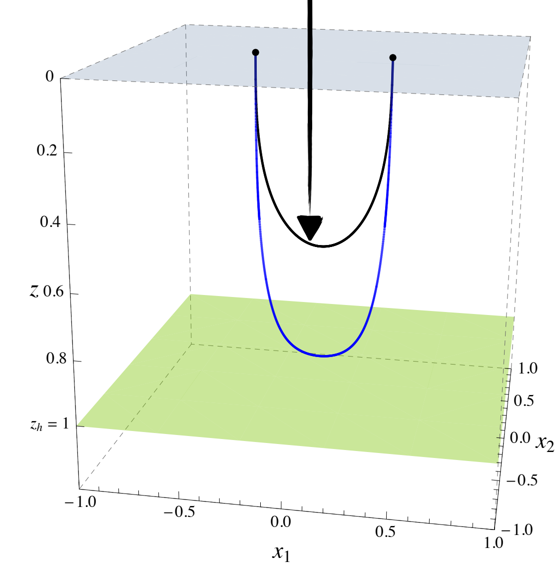
calculation in AdS/CFT:

$$\langle W(\mathcal{C})\rangle \sim \exp\left(\mathrm{i}S_{\mathrm{NG}}[\mathcal{C}]\right)$$

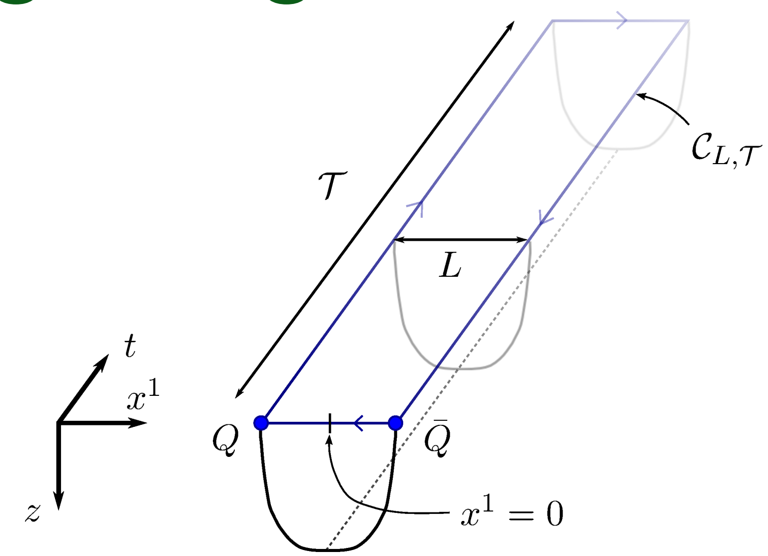
$$S_{\mathrm{NG}} = -\frac{1}{2\pi\alpha'}\int \mathrm{d}^2\sigma\sqrt{-\det g_{ab}}\,.$$

$$F_{Q\bar{Q}}(L) \sim -\frac{S_{\mathrm{NG}}[\mathcal{C}_{L,\mathcal{T}}]}{\mathcal{T}}\,, \qquad \mathcal{T}\to\infty$$

energetically favored string



Nambu-Goto action for hanging string



in general metric

$$\mathrm{d}s^2 = \mathrm{e}^{2A(z)} \left(-h(z)\,\mathrm{d}t^2 + \mathrm{d}\vec{x}^2\right) + \frac{\mathrm{e}^{2B(z)}}{h(z)}\,\mathrm{d}z^2$$

we have

$$S_{\mathrm{NG}}[\mathcal{C}_{L,\mathcal{T}}] = -\frac{\mathcal{T}}{\pi\alpha'}\int_0^{z_\mathrm{t}} \mathrm{d}z\,\mathrm{e}^{A+B}\sqrt{\frac{\mathrm{e}^{4A}h}{\mathrm{e}^{4A}h - \mathrm{e}^{4A_\mathrm{t}}h_\mathrm{t}}}$$

z_t: turning point

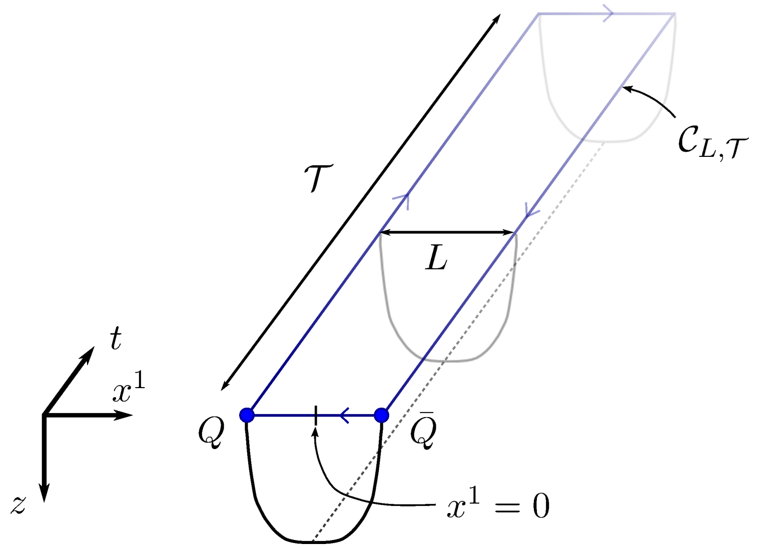
UV divergent:

$$S_{\mathrm{NG}}^{(\mathrm{reg})}[\mathcal{C}_{L,\mathcal{T}}] = -\frac{\mathcal{T}}{\pi\alpha'}\int_\varepsilon^{z_\mathrm{t}} \mathrm{d}z\,\mathrm{e}^{A+B}\sqrt{\frac{\mathrm{e}^{4A}h}{\mathrm{e}^{4A}h - \mathrm{e}^{4A_\mathrm{t}}h_\mathrm{t}}} \sim -\frac{\mathcal{T}L_{\mathrm{AdS}}^2}{\pi\alpha'}\left(\frac{1}{\varepsilon} + \ldots\right)$$

Subtraction for Nambu-Goto action

subtraction required:

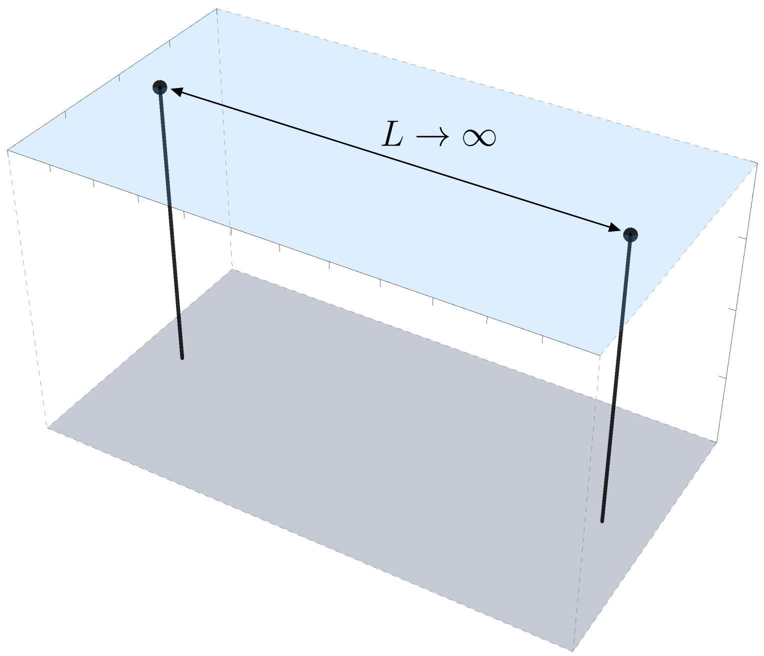
$$F^{(\mathrm{ren})}_{Q\bar{Q}}(L) = \lim_{\mathcal{T}\to\infty}\left(-\frac{S^{(\mathrm{reg})}_{\mathrm{NG}}[\mathcal{C}_{L,\mathcal{T}}] - \Delta S}{\mathcal{T}}\right)$$



subtractions in the literature:

- non-interacting string hanging down into black hole (2x)

$$S^{(\mathrm{reg})}_{\mathrm{NG}}[\text{straight string}] = -\frac{\mathcal{T}}{2\pi\alpha'}\int_{\varepsilon}^{z_{\mathrm{h}}} \mathrm{d}z\, \mathrm{e}^{A+B} \sim -\frac{\mathcal{T} L^2_{\mathrm{AdS}}}{2\pi\alpha'}\left(\frac{1}{\varepsilon} + \ldots\right)$$



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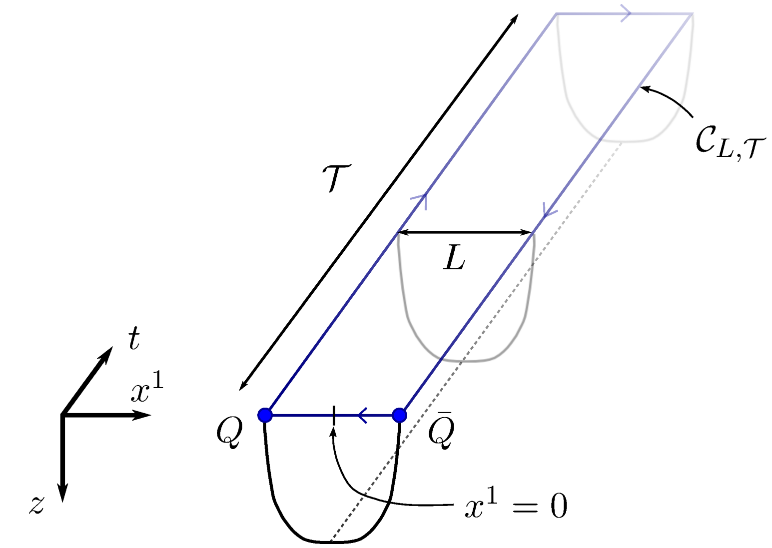

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- real part of action at L=∞

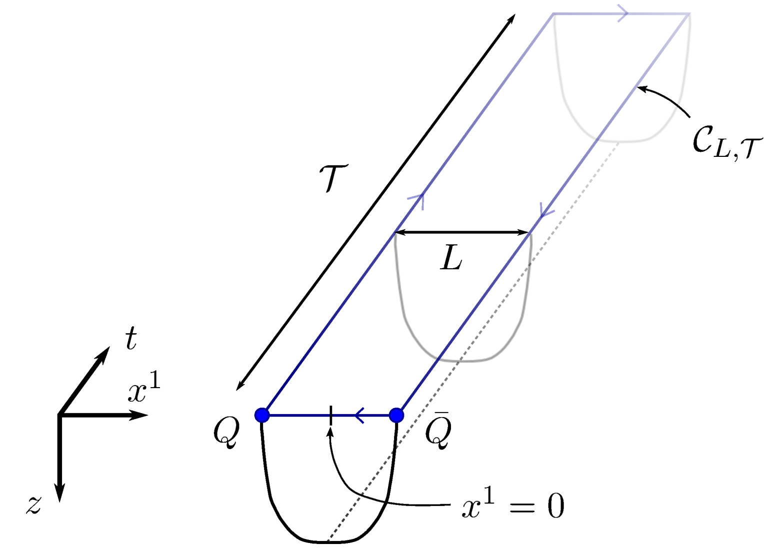
Albacete, Kovchegov, Taliotis

but: then F is T-dependent for small L - unphysical!

Subtraction for Nambu-Goto action

subtraction required:

$$F^{(\mathrm{ren})}_{Q\bar{Q}}(L) = \lim_{\mathcal{T}\to\infty}\left(-\frac{S^{(\mathrm{reg})}_{\mathrm{NG}}[\mathcal{C}_{L,\mathcal{T}}]-\Delta S}{\mathcal{T}}\right)$$



correct subtraction: only UV singularity

$$\Delta S_{\mathrm{min}} \equiv -\frac{\mathcal{T}L^2_{\mathrm{AdS}}}{\pi\alpha'}\int_{\varepsilon}^{\infty}\frac{\mathrm{d}z}{z^2} = -\frac{\mathcal{T}L^2_{\mathrm{AdS}}}{\pi\alpha'}\,\frac{1}{\varepsilon}$$

then no unphysical T-dependence!

Similar problem with regularization occurs in calculation of entanglement entropy.

Binding Energy of Heavy QQbar pair

quantity with hanging-string subtraction is binding energy

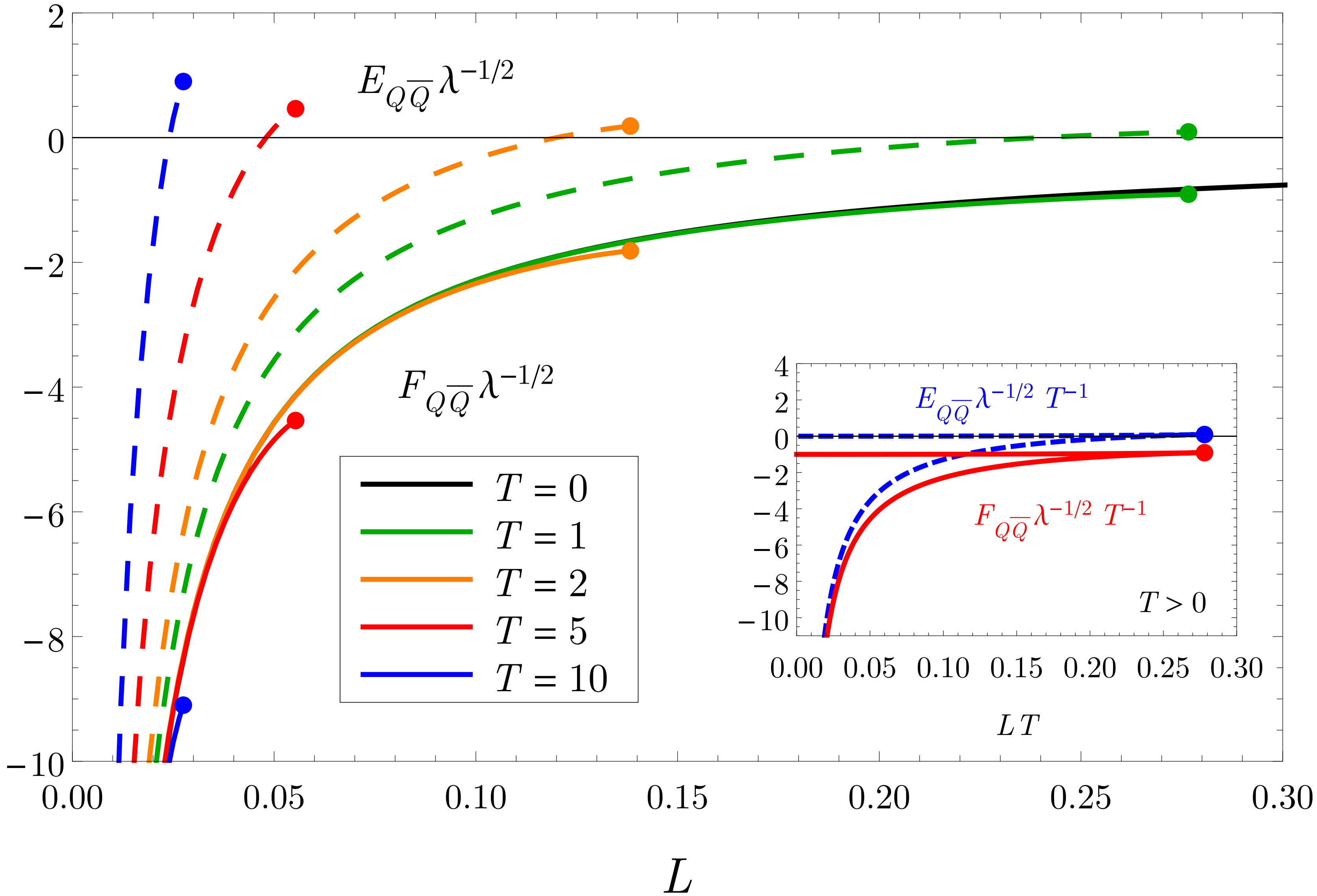
$$E_{Q\bar{Q}}(L) = \lim_{\mathcal{T}\to\infty}\left(-\frac{S_{\rm NG}[\mathcal{C}_{L,\mathcal{T}}] - 2S_{\rm NG}[\text{straight string}]}{\mathcal{T}}\right)$$

in fact difference of free energies:

$$\begin{aligned} E_{Q\bar{Q}}(L) &= \lim_{\mathcal{T}\to\infty}\left[-\frac{\big(S_{\rm NG}[\mathcal{C}_{L,\mathcal{T}}] - \Delta S_{\rm min}\big) - \big(2S_{\rm NG}[\text{straight string}] - \Delta S_{\rm min}\big)}{\mathcal{T}}\right] \\ &= F_{Q\bar{Q}} - F_{Q\,;\,\bar{Q}}\,, \end{aligned}$$

(note: defines single-quark free energy)

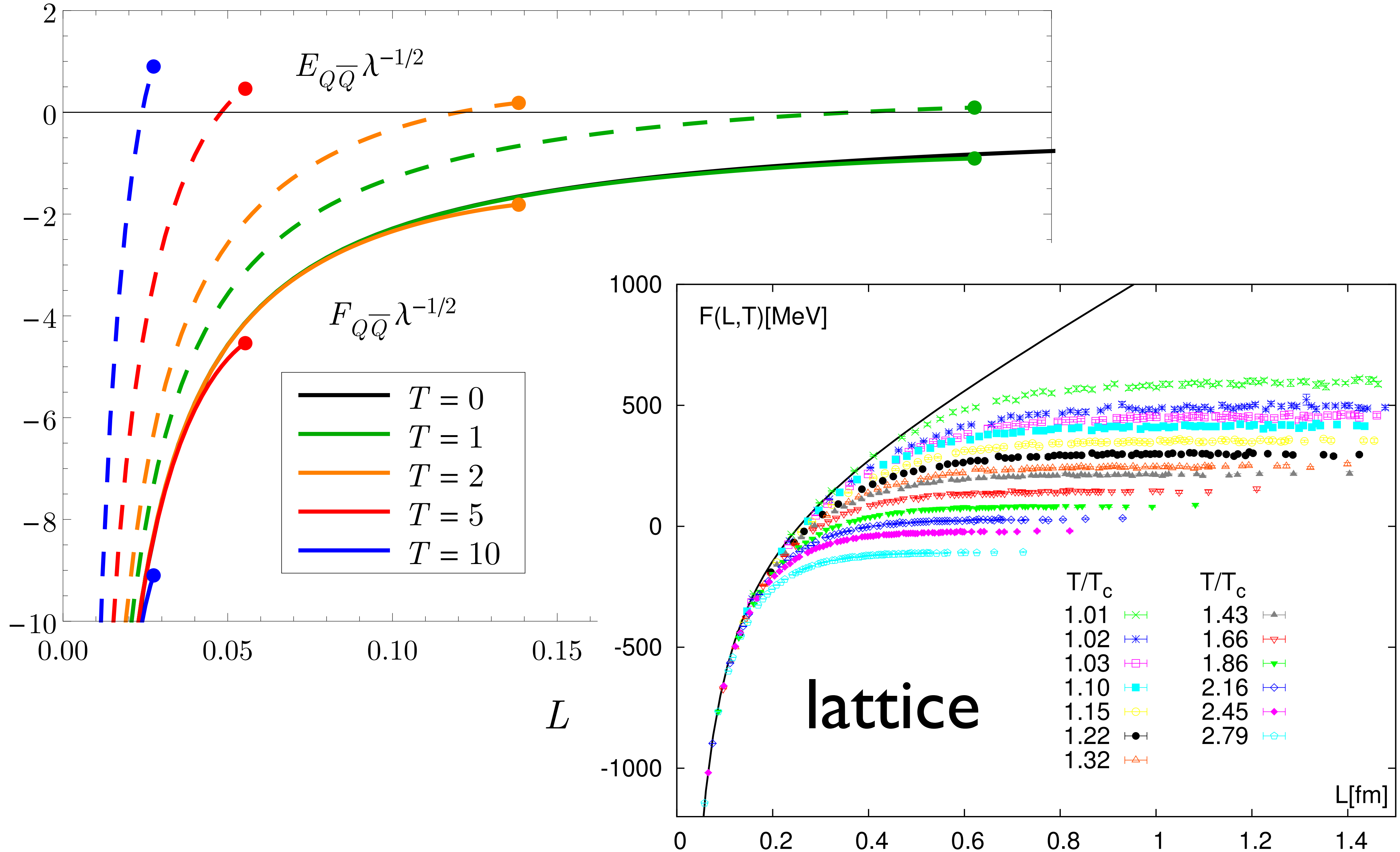
Free vs Binding Energy in N=4



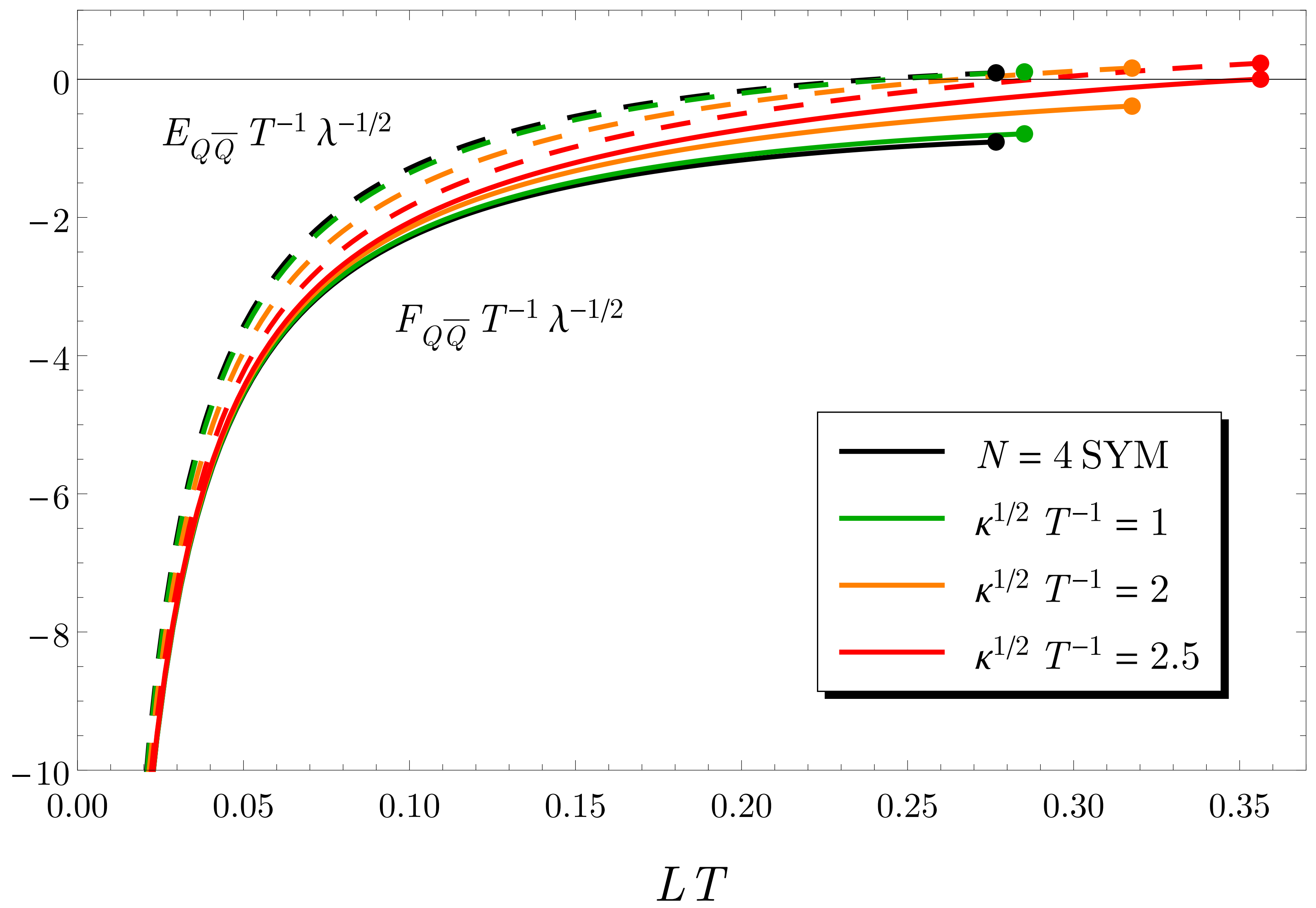
black: T=0 potential

$$V_{Q\bar{Q}}(L) = -\frac{4\pi^2\sqrt{\lambda}}{\Gamma^4\left(\frac{1}{4}\right)L}$$

Free vs Binding Energy in N=4



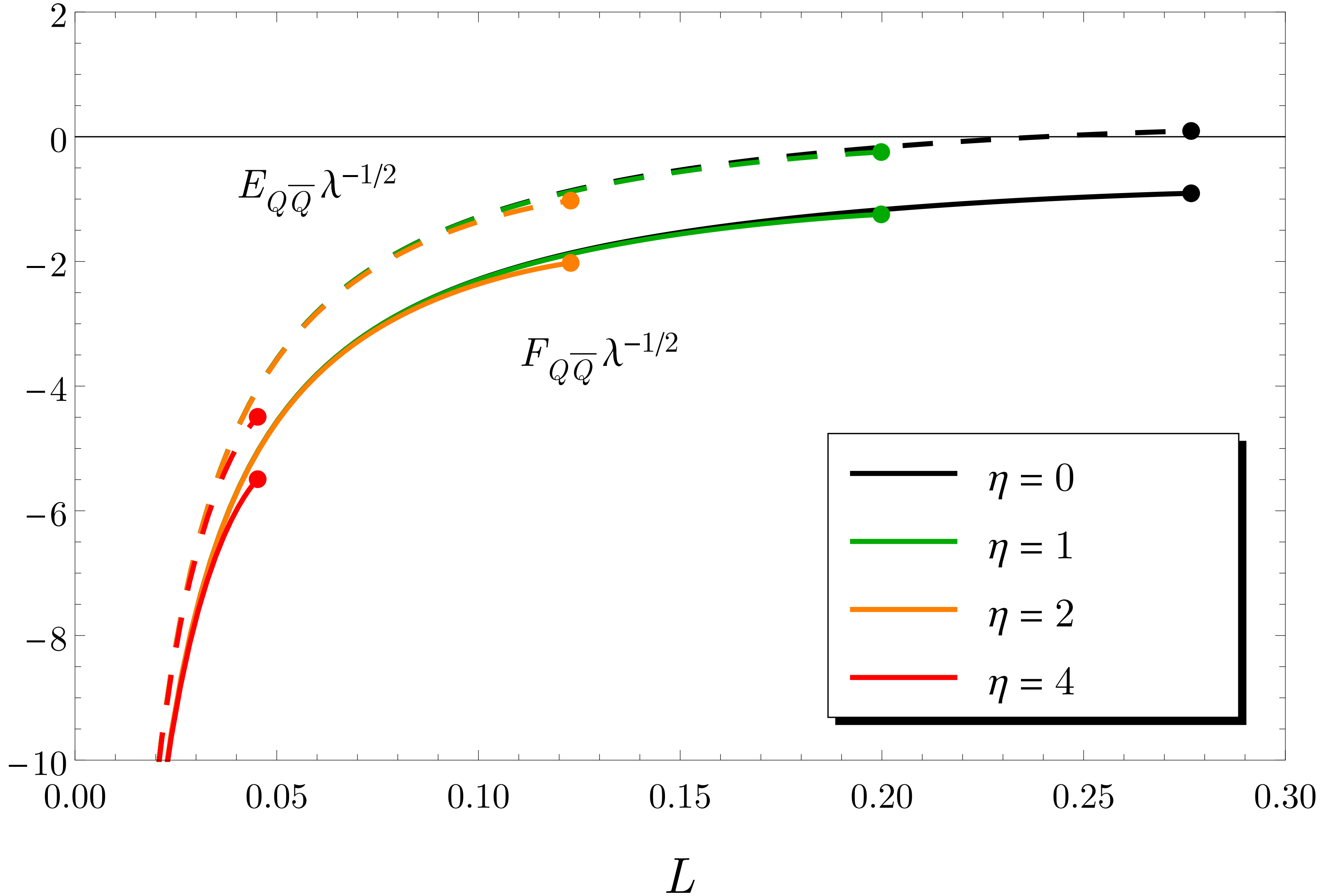
Free and Binding Energy - different non-conformalities



1-parameter string frame

Free and Binding Energy - quarks in motion

P. Wittmer, CE



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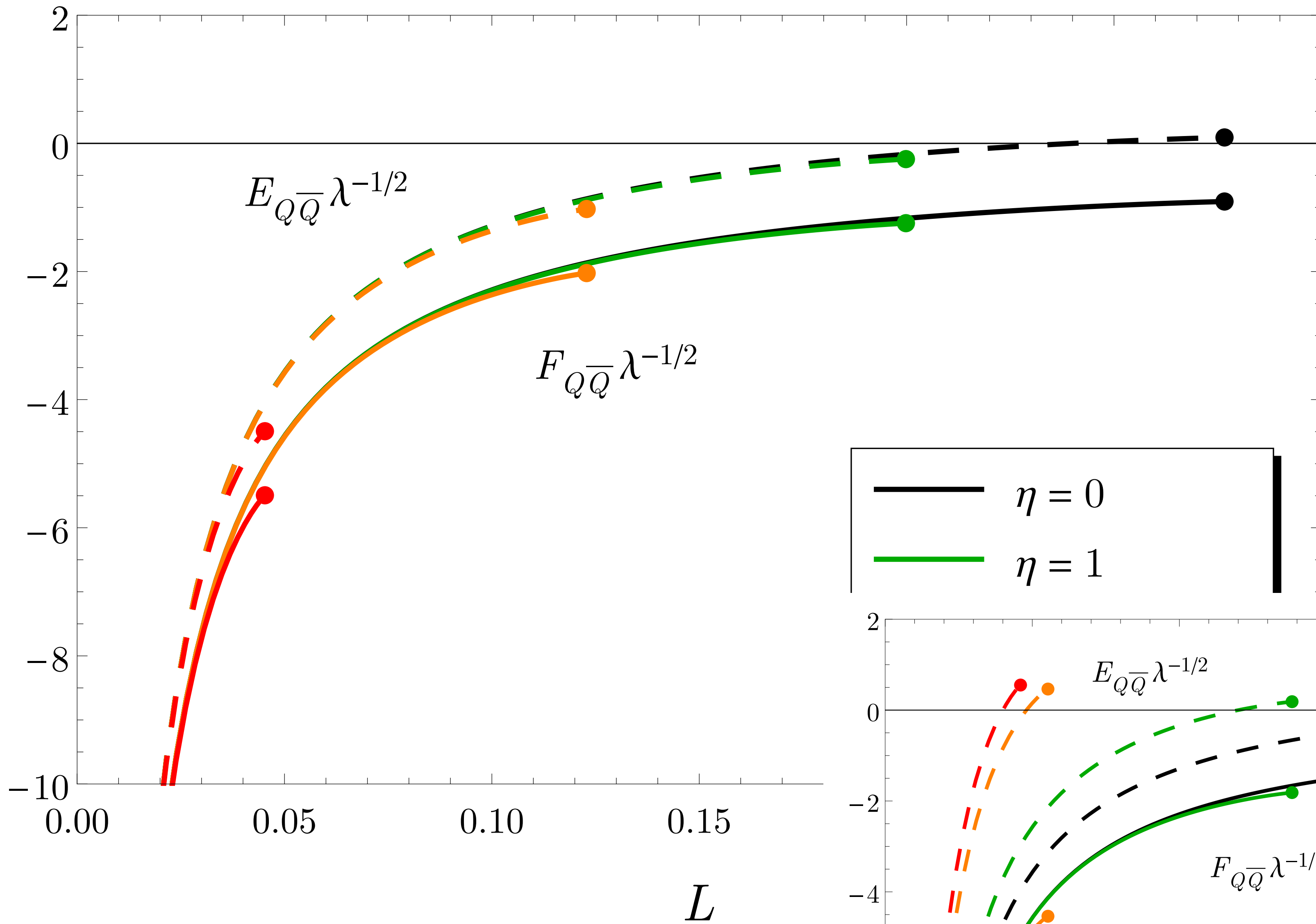
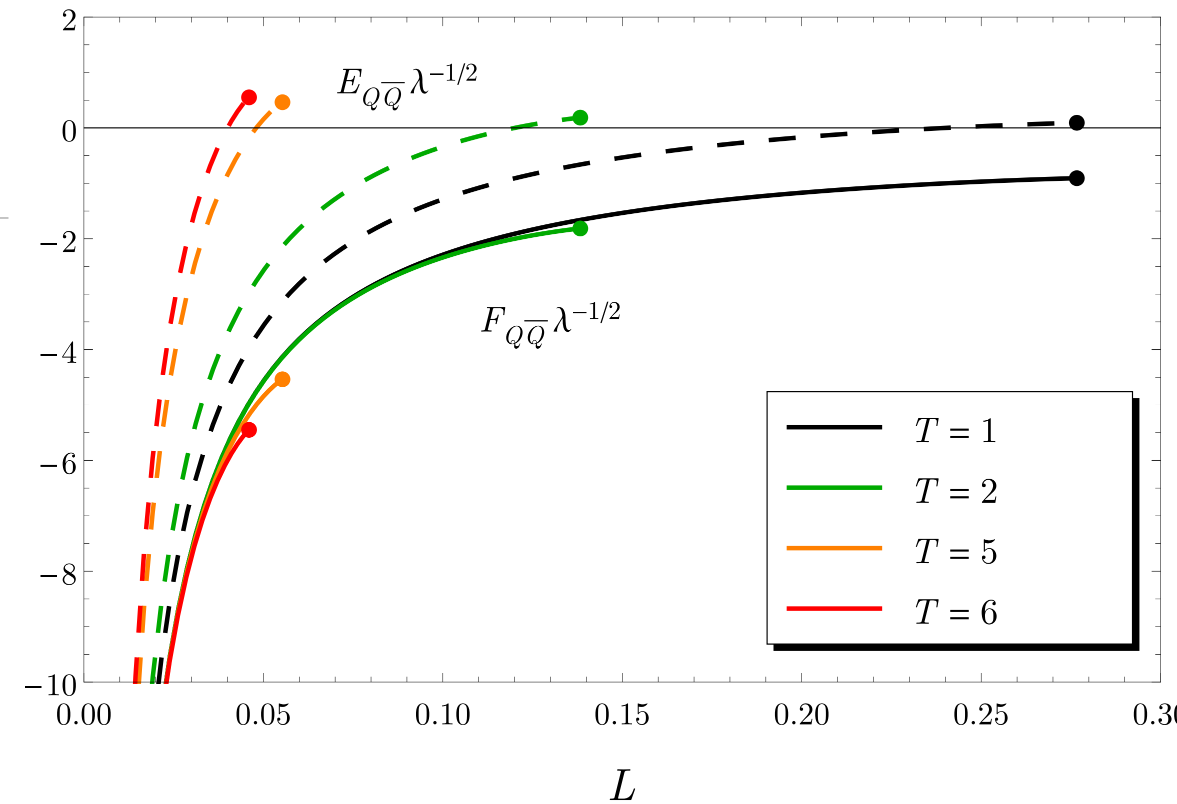




velocity not fully equivalent
to temperature change

Entropy and Internal Energy of QQbar pair

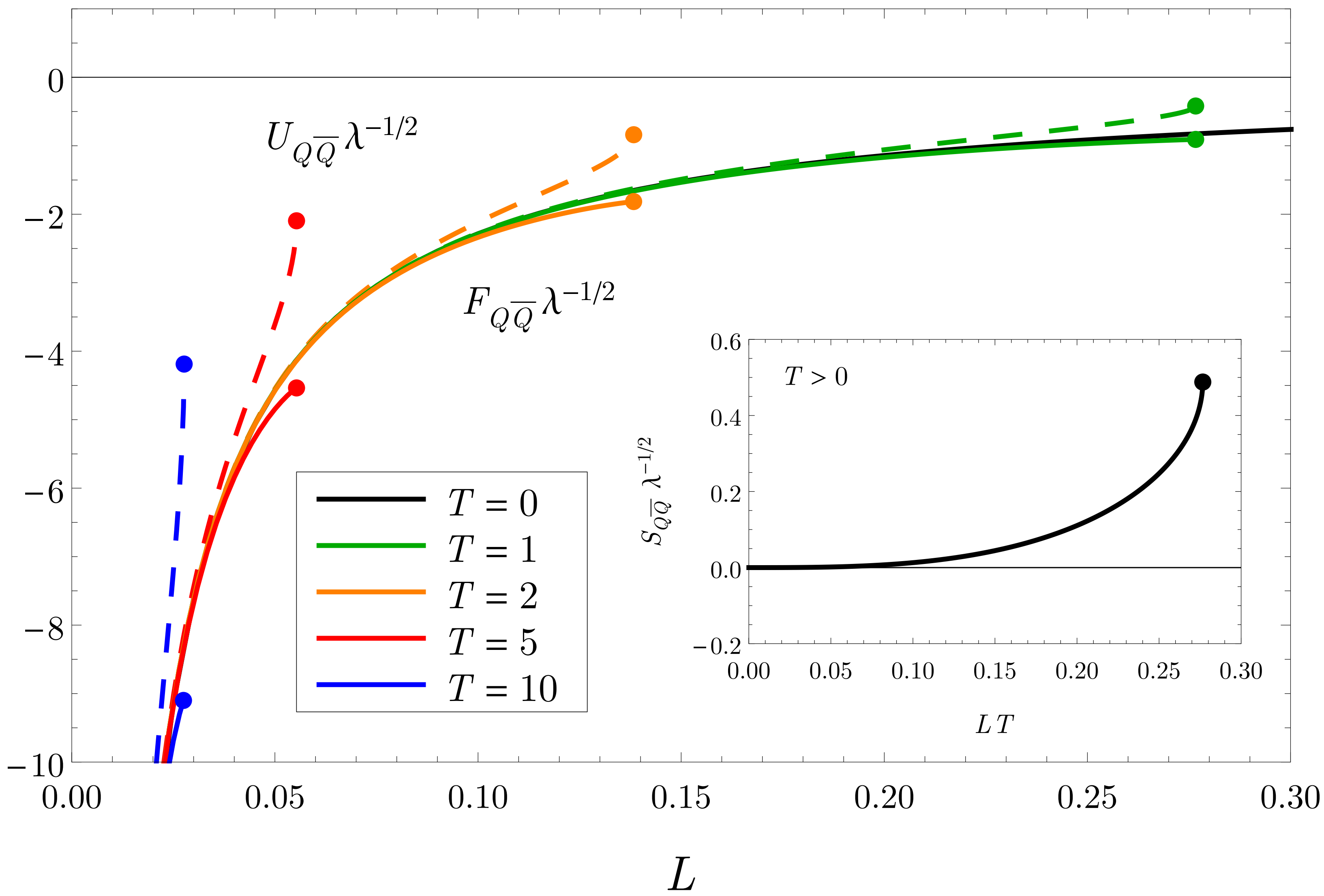
with (correct!) free energy obtain entropy

$$S_{Q\bar{Q}}(L,T) = -\frac{\partial F_{Q\bar{Q}}(L,T)}{\partial T}$$

and internal energy

$$U_{Q\bar{Q}}(L,T) = F_{Q\bar{Q}}(L,T) + TS_{Q\bar{Q}}(L,T)$$

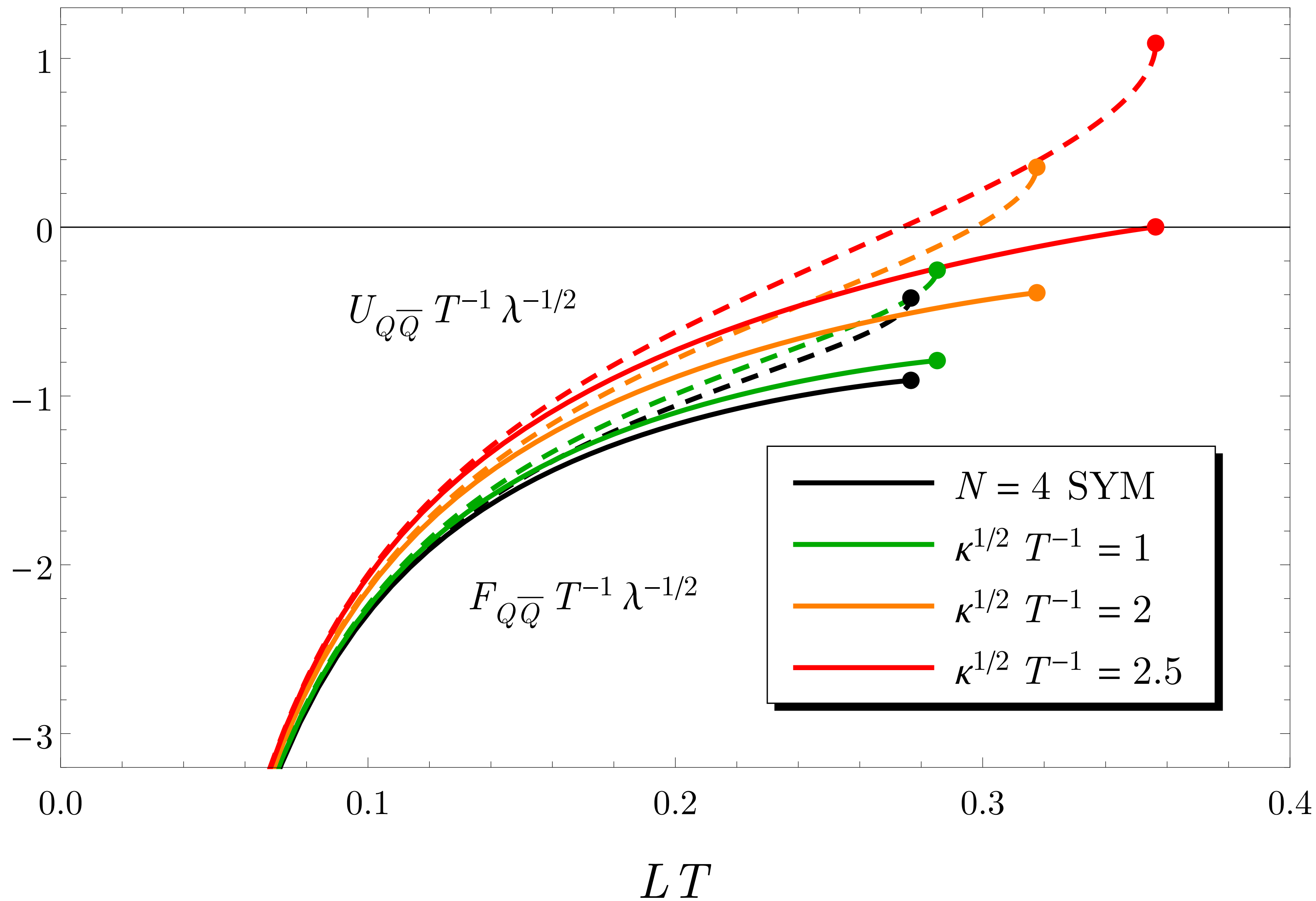
Entropy and Internal Energy in N=4



black: T=0 potential

$$V_{Q\bar{Q}}(L) = -\frac{4\pi^2\sqrt{\lambda}}{\Gamma^4\left(\frac{1}{4}\right) L}$$

Internal Energy - different non-conformalities



1-parameter string frame

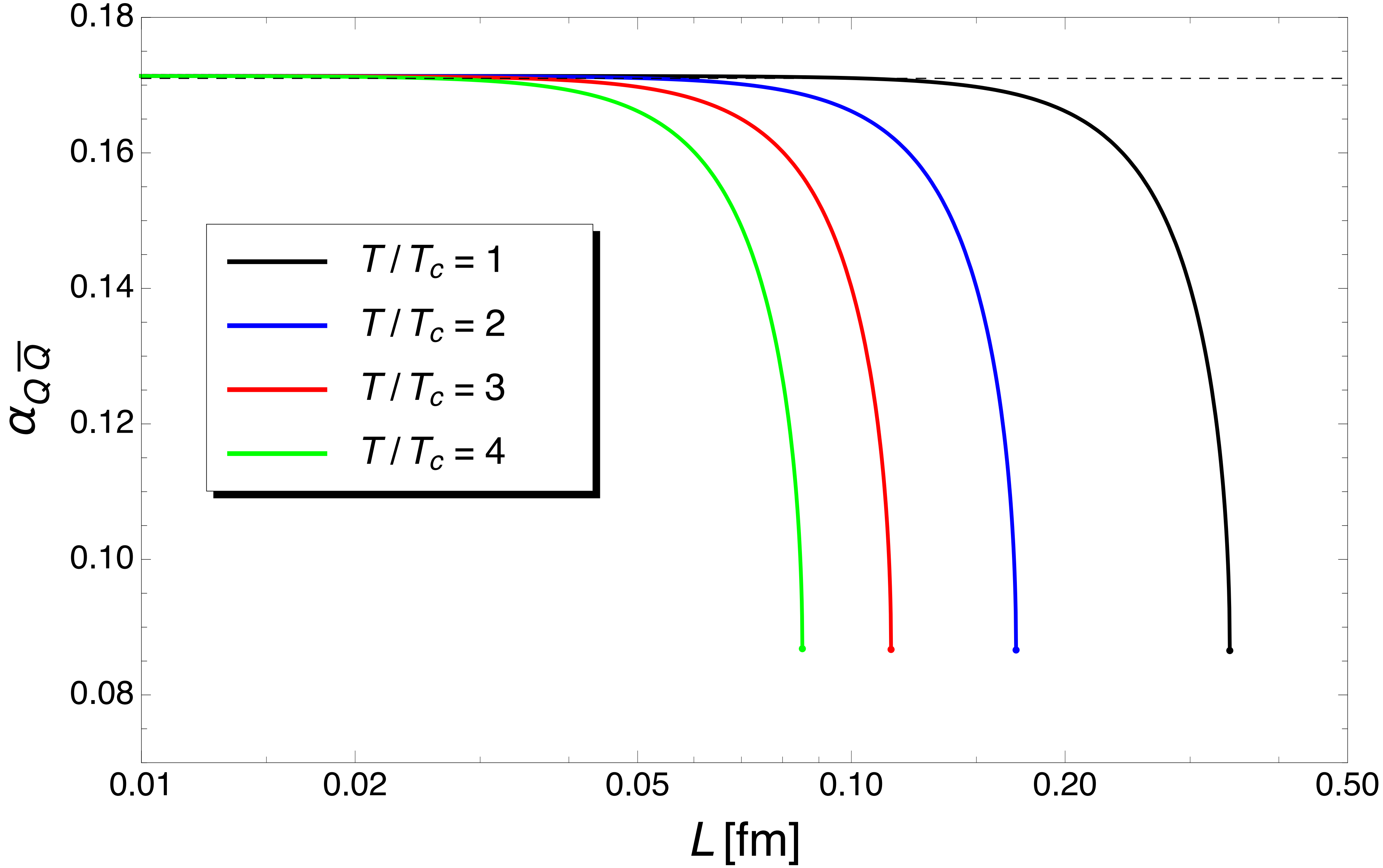
Running Coupling

Running Coupling α_{qq}

$$\alpha_{Q\bar{Q}} = \frac{3L^2}{4}\frac{dF(L,T)}{dL}$$

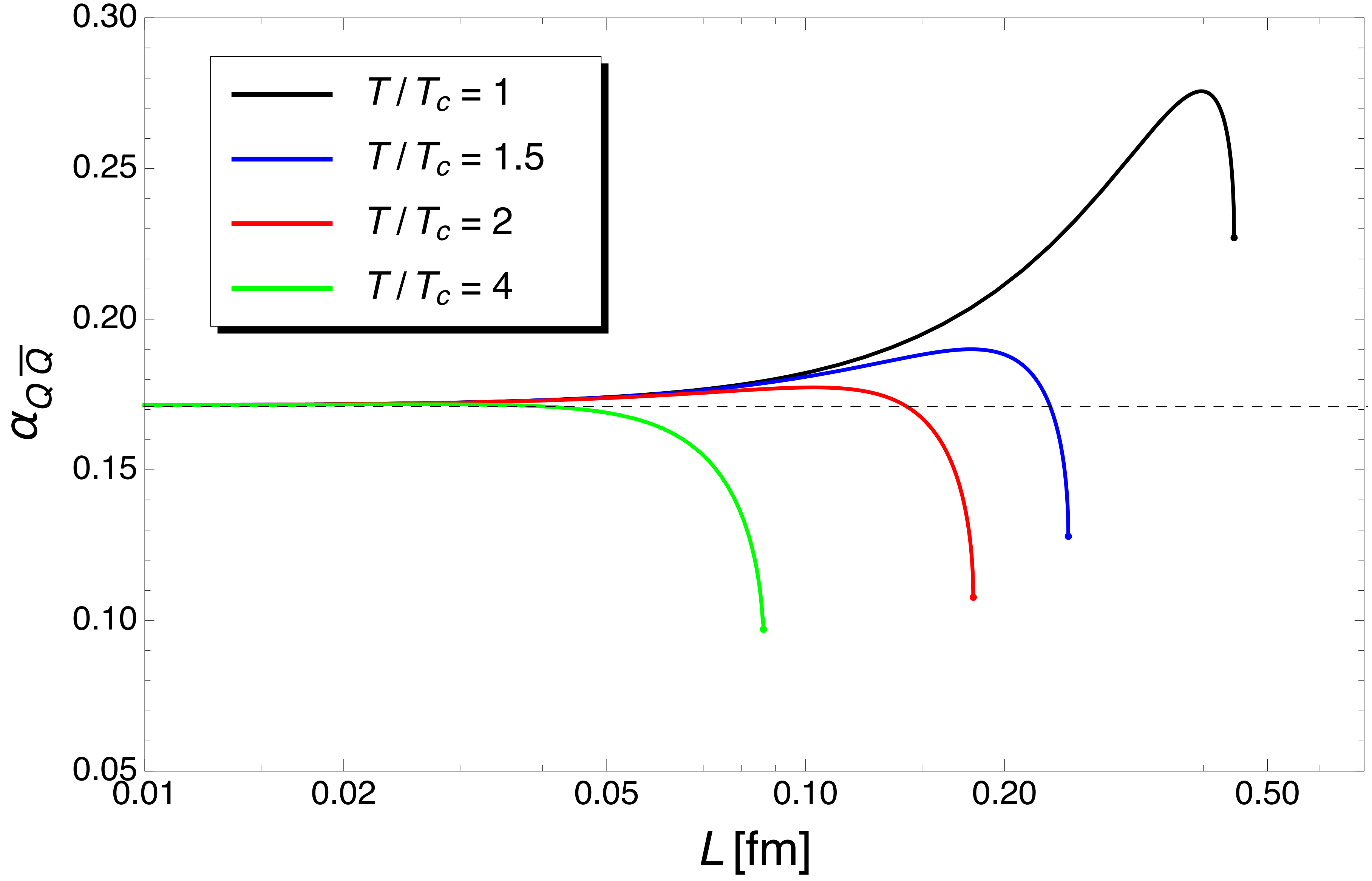
Konrad Schade, CE,
Paul Wittmer

N=4:



Running Coupling α_{qq}

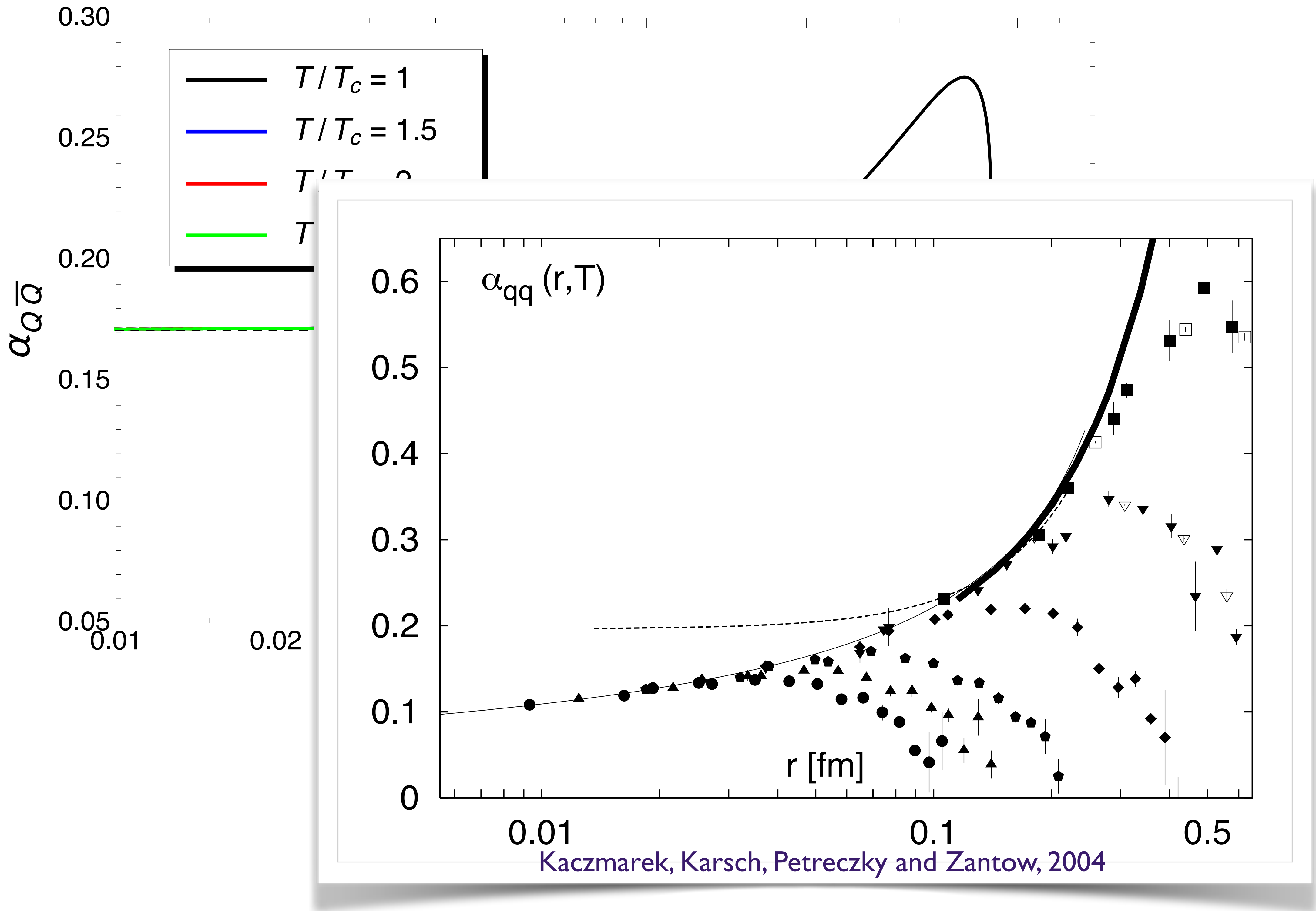
with non-conformal deformation:



- universal rise above conformal value

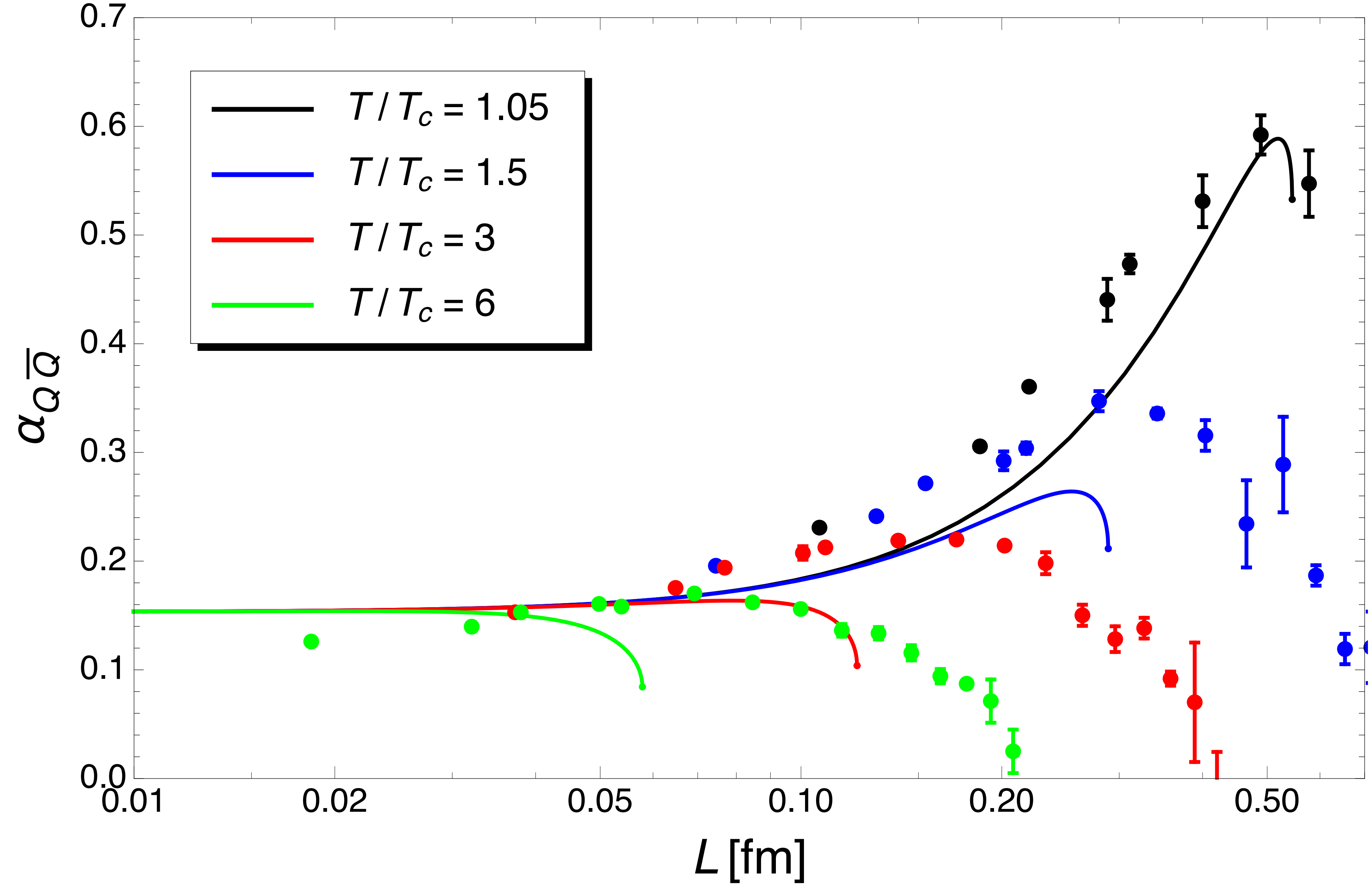
Running Coupling α_{qq}

with non-conformal deformation:



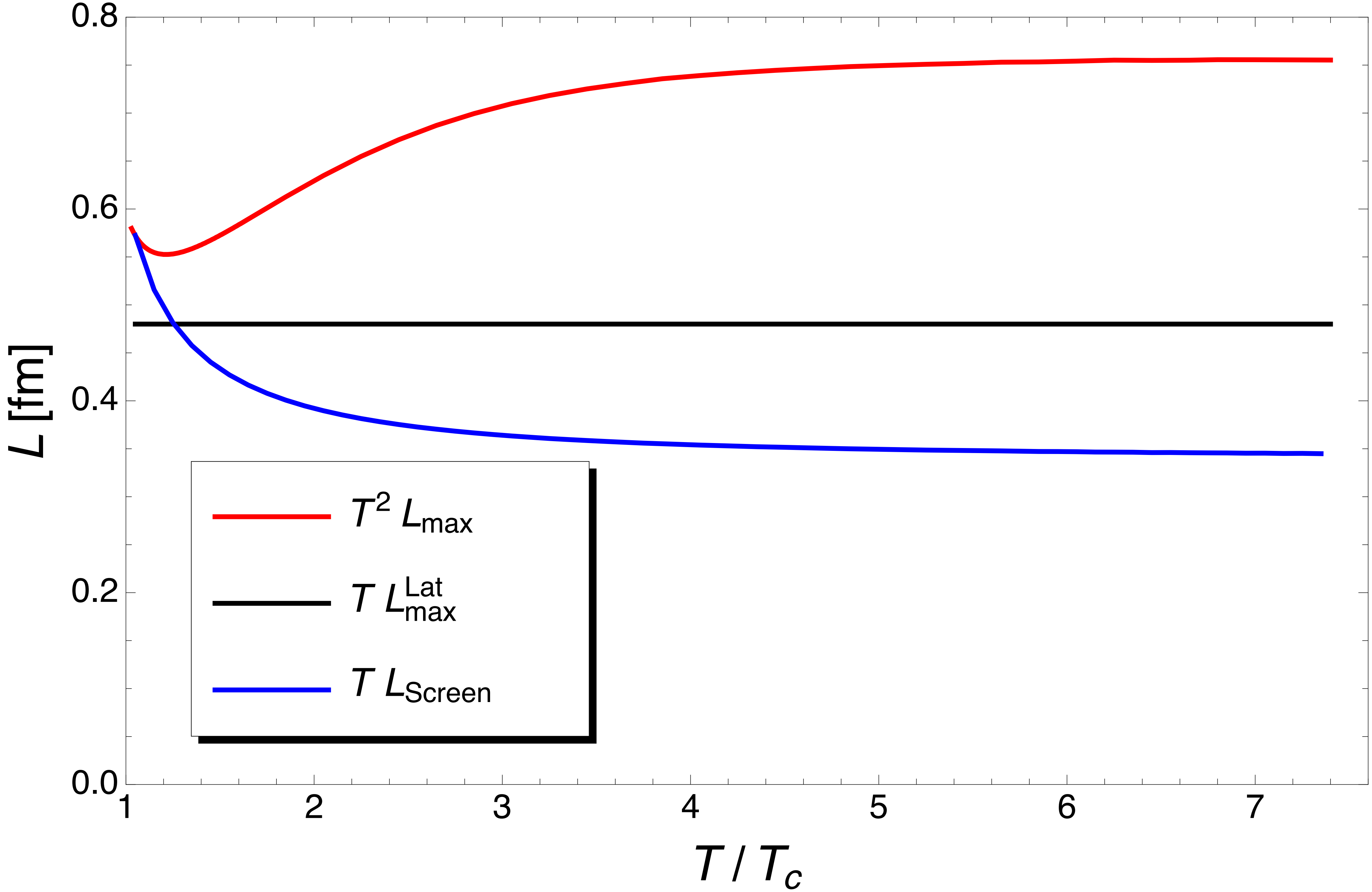
Kaczmarek, Karsch, Petreczky and Zantow, 2004

Running Coupling α_{qq}



- Coming close to QCD data if free parameters properly adjusted
- Parameters fixed from thermodynamics

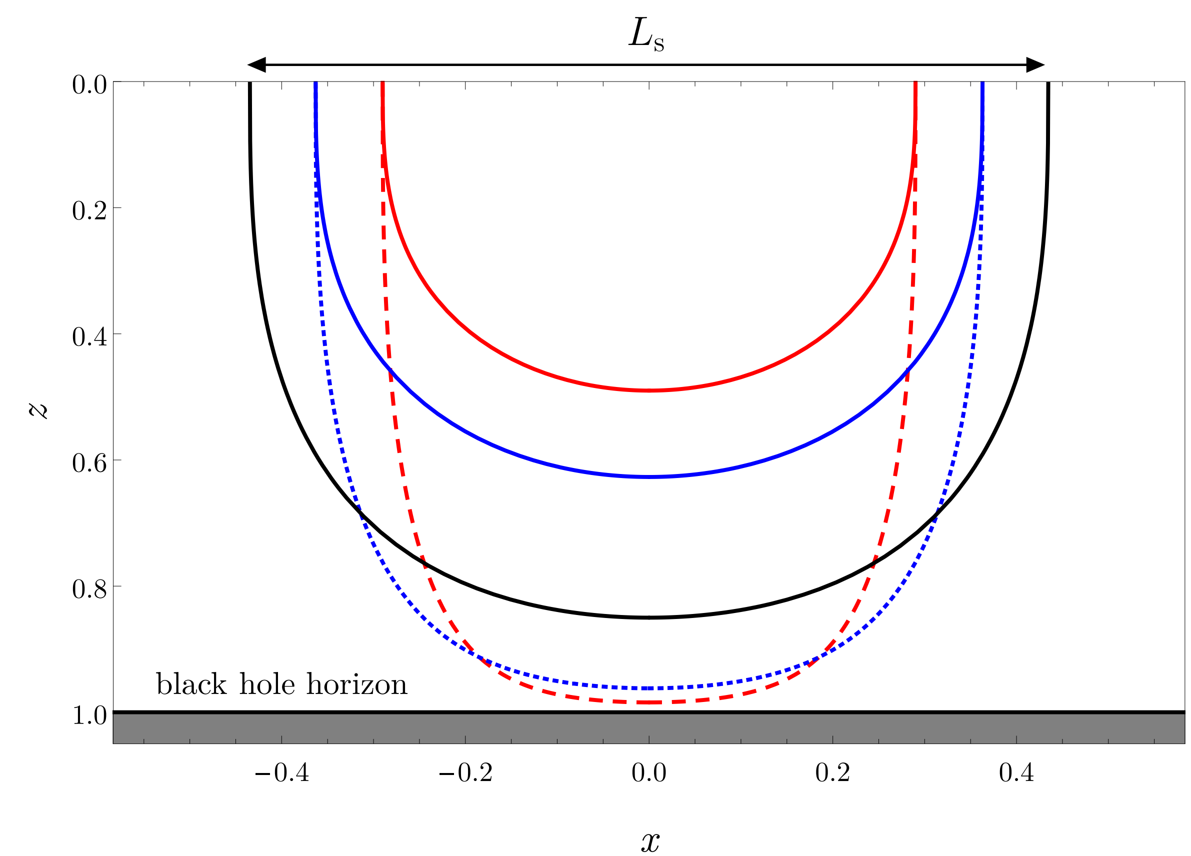
Running Coupling α_{qq}: Length Scales



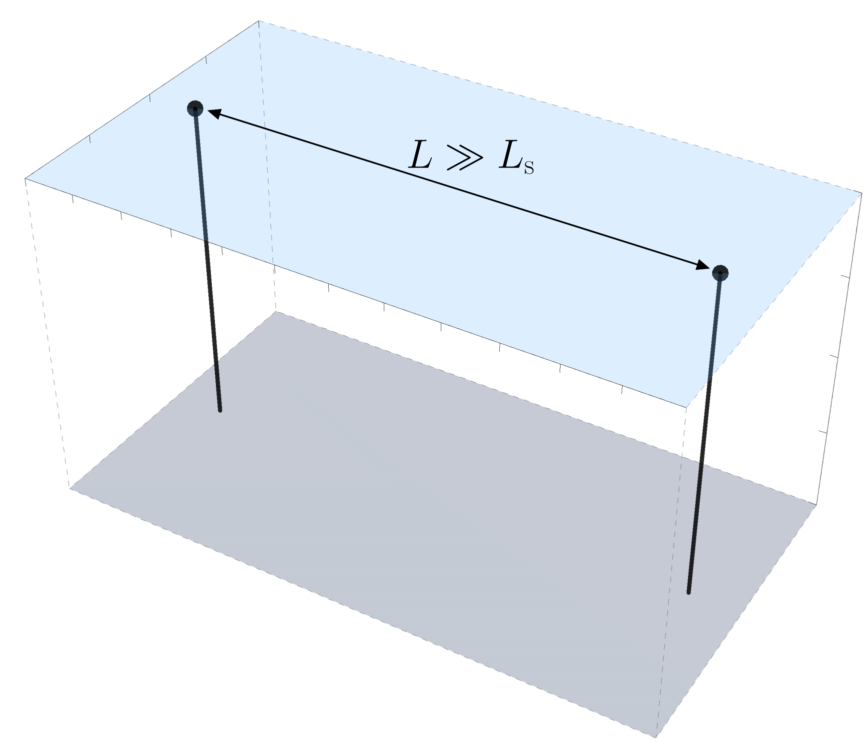
Complex Static Potential

QQbar Potential at larger distances?

- small distances ($L < L_s$): simple string configuration



- very large distances: Debye screening, due to supergravity mode exchange between hanging strings (Bak, Karch, Yaffe)

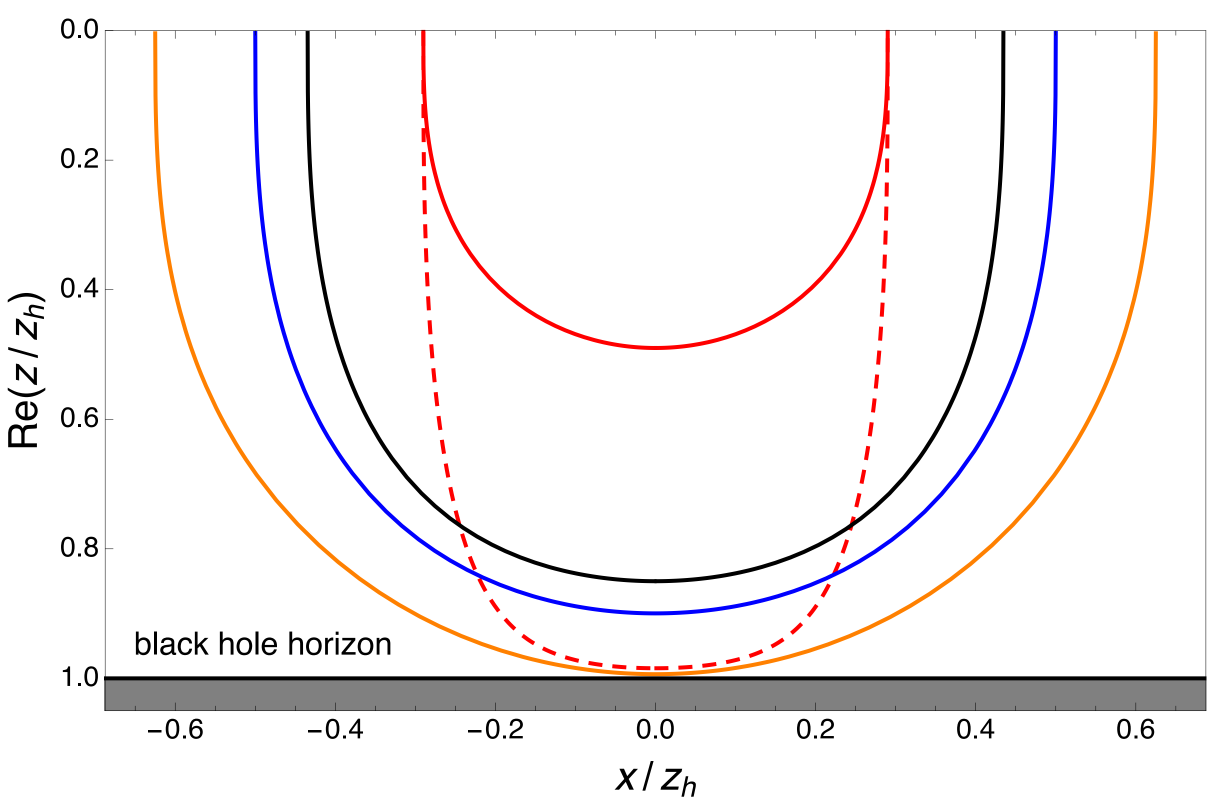


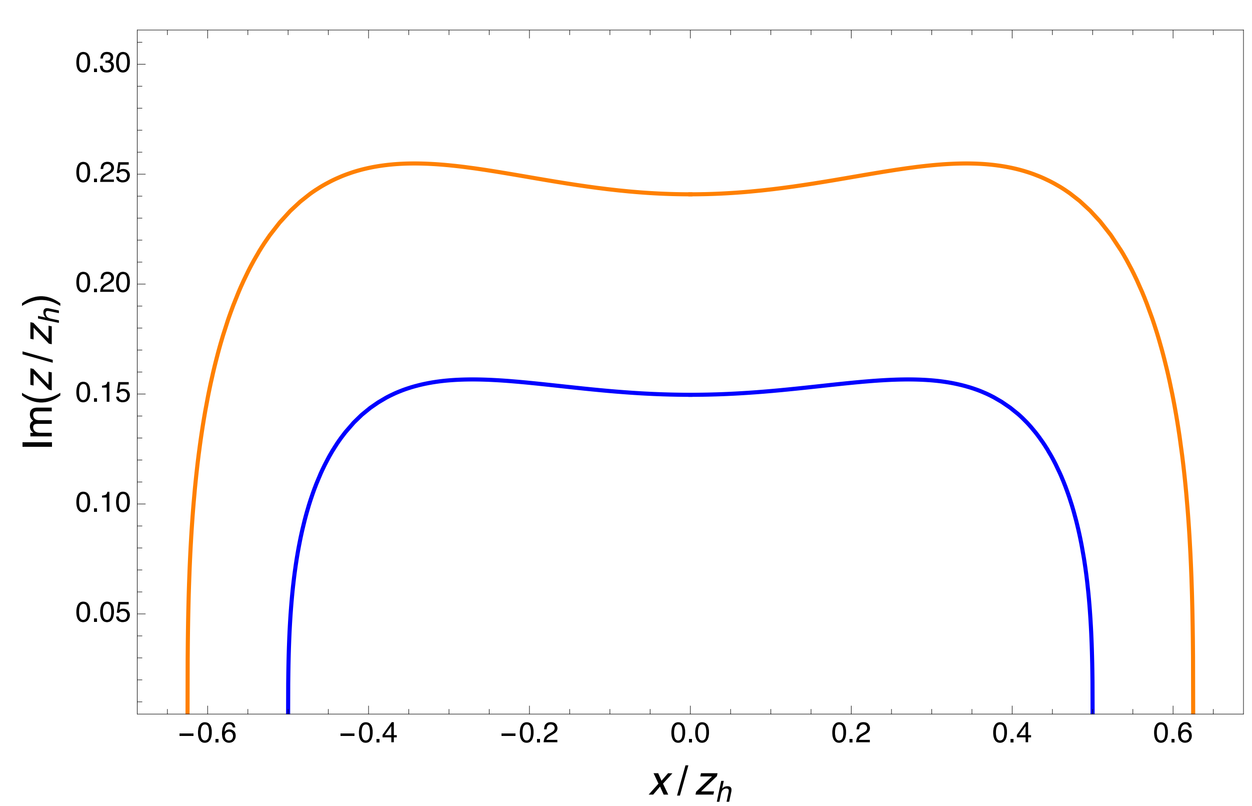
Complex QQbar Potential

Try to analytically continue simple string configuration beyond L_s

Kovchegov et al

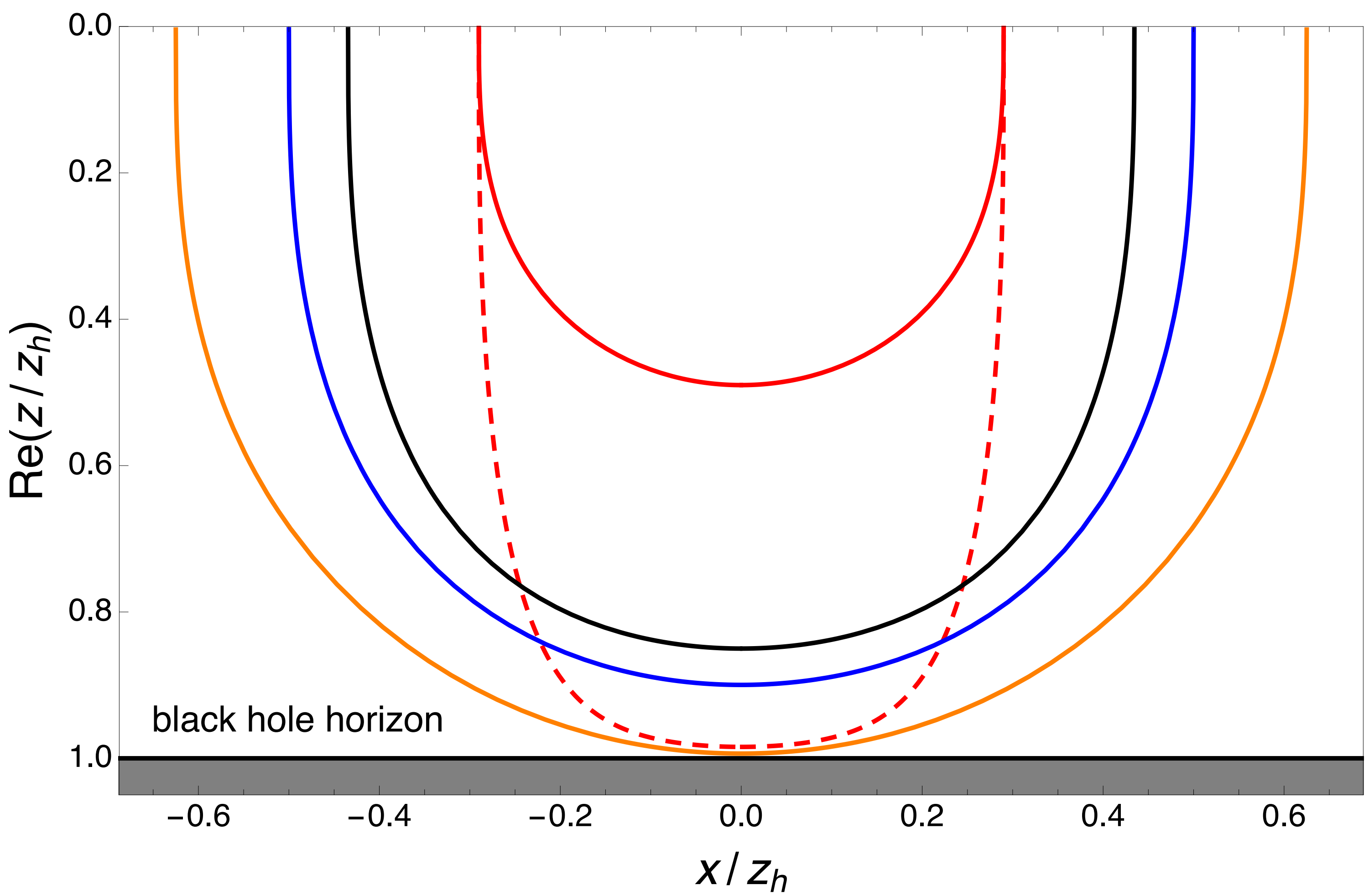
String coordinates & potential become complex





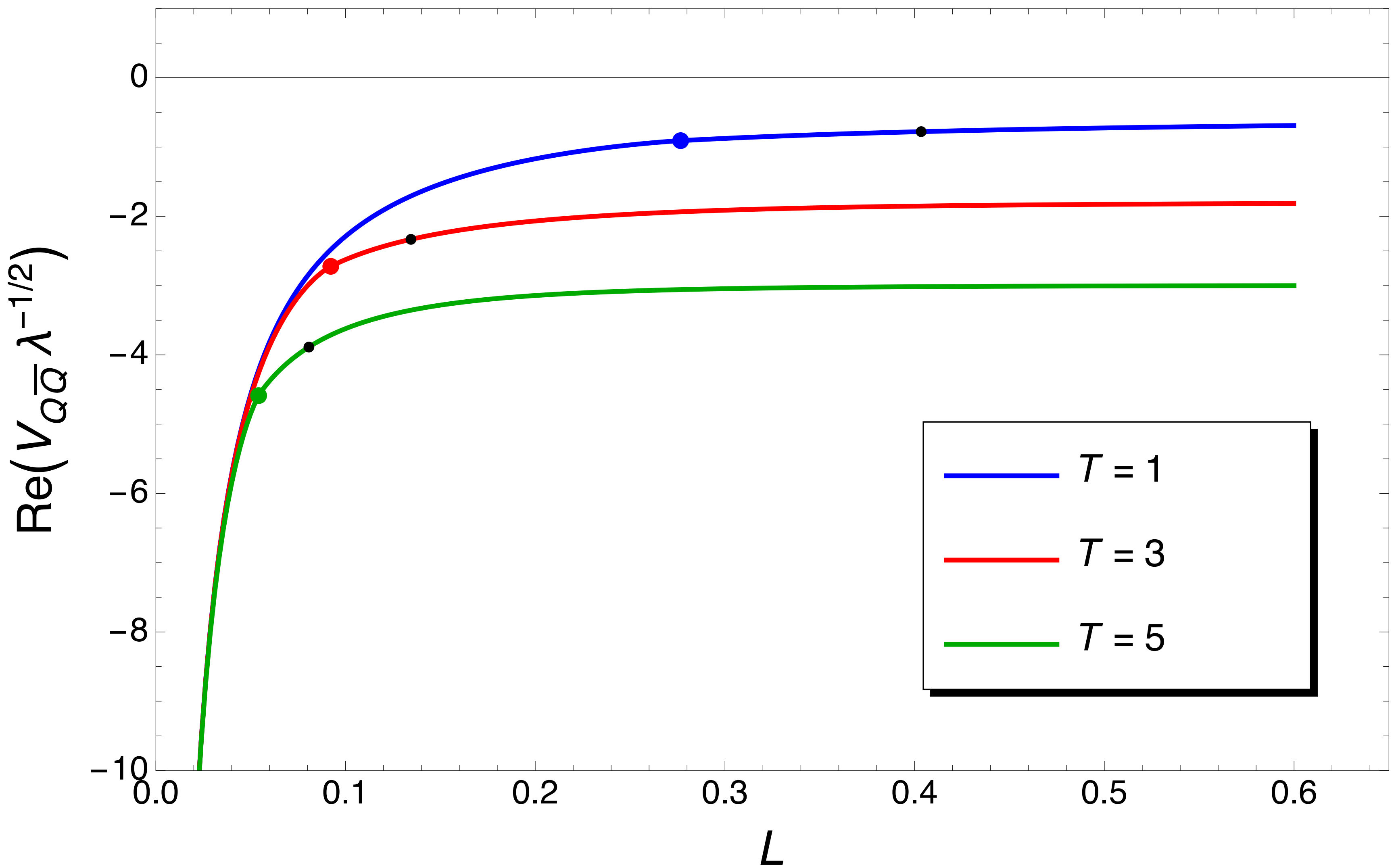
Complex QQbar Potential

At some distance L_h string hangs into black hole
→ quarks no longer causally connected,
better stop there?!



Complex QQbar Potential

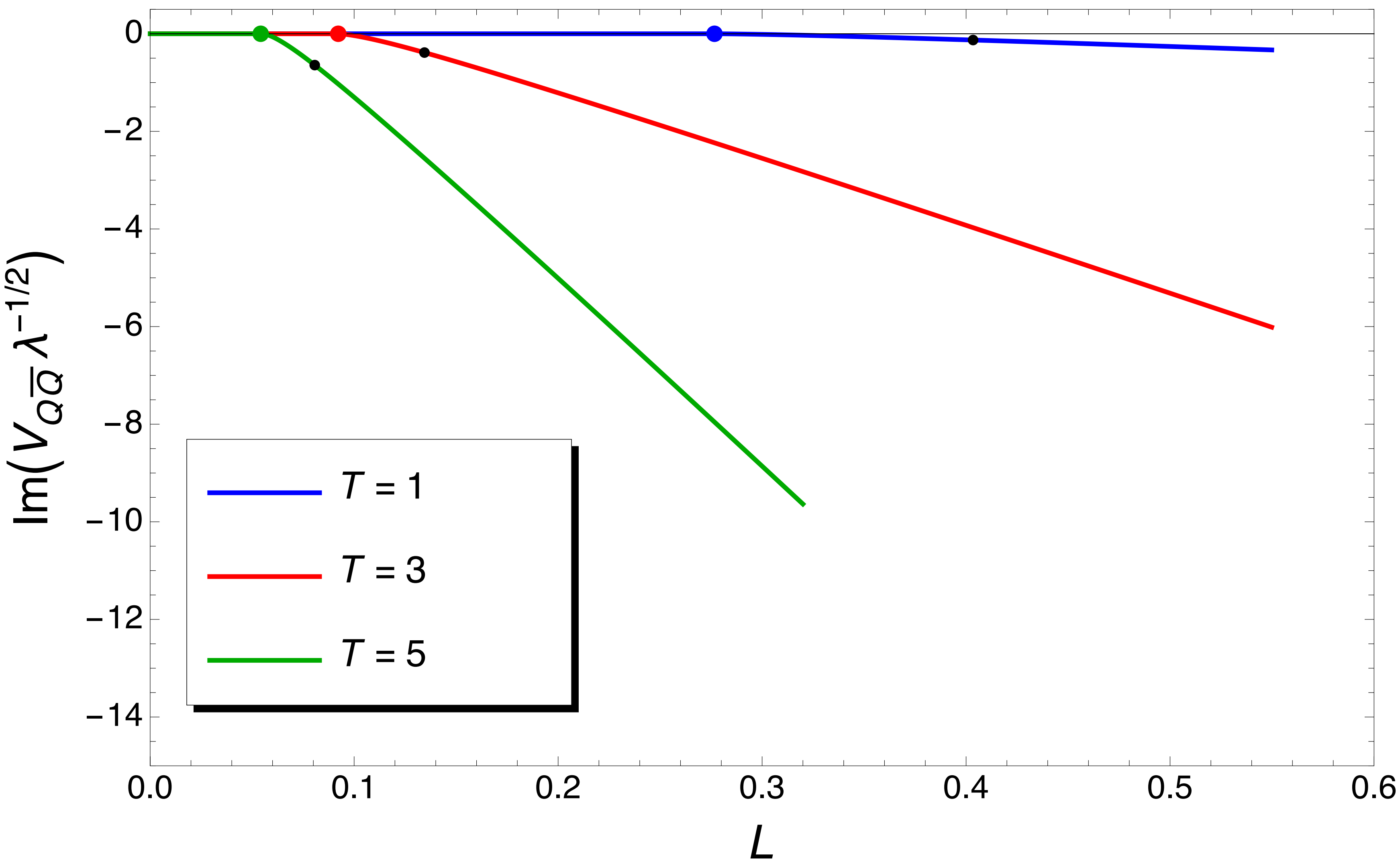
Real part of potential (with correct renormalization)



black points: L_h; colored points: L_s

Complex QQbar Potential

Imaginary part of potential



black points: L_h; colored points: L_s

Complex QQbar Potential

Questions:

How does imaginary part connect to real-valued Debye-screened potential at asymptotically large L?

Spectral function of heavy mesons from complex potential?

Summary

- Holography applied to heavy quark bound states / static potential in strongly coupled plasma
- Screening distance conjecture:
  L_s is bounded from below by its value in N=4 SYM
- First systematic calculation of free and internal energy of QQbar pair in holography
  (correct UV renormalization)
- Running coupling, complex static potential
- Universal strong-coupling behavior in these observables

Thanks for your interest!