Some recent progress on bulk reconstructions from CFT

Debajyoti Sarkar

ITP, Bern

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Würzburg University

Aim and outline

- To understand emergent spacetime and locality in the particular set-up of AdS/CFT.
- AdS/CFT is custom-made to study such questions. The problem boils down to finding local bulk physics in terms of the 'better-understood' boundary CFT.

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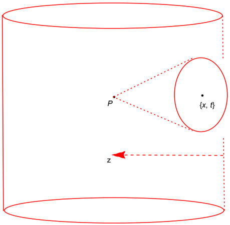
- To understand emergent spacetime and locality in the particular set-up of AdS/CFT.
- AdS/CFT is custom-made to study such questions. The problem boils down to finding local bulk physics in terms of the 'better-understood' boundary CFT.
- Briefly review the 'bulk' scalar field construction in a given background for a general CFT with large N, and large λ.
- Whether it is possible to recover the bulk metric purely from CFT techniques.
- Conclusions and future work.

Using normalizable mode prescription of Lorentzian AdS/CFT, a free, local massive scalar in AdS_{d+1} (using bulk dynamics at leading order in large N and large λ):

$$\phi^{(0)}(t,x,z) = c_{d,\Delta} \int_{t'^2+y'^2<z^2} dt' d^{d-1}y' (\sigma z')^{\Delta-d} \mathcal{O}_\Delta(t+t', x+iy'),$$

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2), \qquad \sigma(z,x|z',x') = \frac{z^2+z'^2+(x-x')^2}{2zz'}$$

Hamilton et al. 2006, 2007. Based on Balasubramanian et al.; Banks et al. 1998.; Bena 1999 ...



Lightning review

$$\phi^{(0)}(t,x,z) = c_{d,\Delta} \int_{t'^2+y'^2<z^2} dt' d^{d-1}y' (\sigma z')^{\Delta-d} \mathcal{O}(t+t', x+iy'),$$

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- Smearing function is fixed and has the correct behavior under AdS isometry. Holography is manifest.

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- The boundary integration is over finite boundary region (spacelike separated from the bulk field).
- Smearing function is fixed and has the correct behavior under AdS isometry. Holography is manifest.
- For interactions, just use microcausality: Bilocal operators or equivalently tower of higher dimensional operators to restore locality for scalar fields at the level of 3 point function.

$$\phi^{(1)}(z,x) = \int dx' K_\Delta(z,x|x')\mathcal{O}_\Delta(x') + \sum_l a_l \int dx' K_{\Delta_l}(z,x|x')\mathcal{O}_{\Delta_l}(x')$$

Kabat, Lifschytz, Lowe, 2011

- Clearly indicates that locality breaks down as we go to finite N. Not enough higher dimensional operators to recover locality.

Some extensions of HKLL (Cut for time)

- Locality for spin- 1, 2 and arbitrary integer spin s.

  Kabat, Lifschytz, Roy, DS 2012, DS and Xiao 2012

- Local operators in terms of fields on a cut-off surface instead of at the conformal boundary. Connection with similar construction in dS and applications towards holographic RG framework.

  DS 2014

- Incorporating perturbative and non-perturbative $1/N$ effects only using constraints of locality.

  Kabat, Lifschytz 2011, 2012, 2013, 2015, Roy, D.S 2015

- Operators in the black hole background; especially inside the horizon. Black hole information problem and discussions.

  . . . Roy, D.S 2015 . . .

- $1/\lambda$ corrections.

  Roy, D.S 2017

An alternative construction

- Although at sub-leading orders in $1/N$ or $1/\lambda$, one might recover the bulk fields purely from bulk micro-causality requirements without using any inputs from bulk dynamics, at the leading order one still requires to know the bulk action and in particular, the bulk metric.

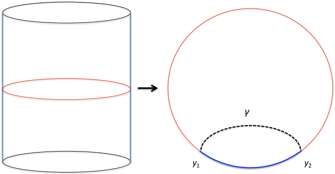
- Using lessons from entanglement, in the context of subregion duality, $\phi^{(0)}$ can be constructed without any reference to the bulk action.

- The boundary requirement is that the resulting bulk operators commute with 'extended modular Hamiltonian'.

  Kabat, Lifschytz 2017

- We will then extend this boundary technique to fully recover the bulk metric anywhere inside pure AdS.

Subregion duality:



$$S_{EE} = -\mathrm{Tr}\left(\rho_R \log \rho_R\right) = \frac{\text{Area}(\gamma)}{4G_N}, \qquad \text{with} \qquad \rho_R = e^{-H_{mod}}$$

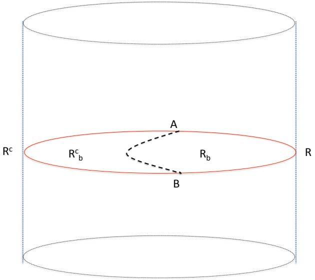
Ryu, Takayanagi 2006

Fixing notations/ conventions

- For example, the modular Hamiltonian H_{mod} for a spherical subregion in CFT_2 is explicitly known

$$H_{mod} = 2\pi \left[\int_{y_1}^{y_2} \frac{(w - y_1)(y_2 - w)}{y_2 - y_1} T_{ww}(w) + \int_{y_1}^{y_2} \frac{(\bar{w} - y_1)(y_2 - \bar{w})}{y_2 - y_1} T_{\bar{w}\bar{w}}(\bar{w}) \right]$$

Casini, Huerta, Myers 2011



- However, the presence of the RT surface γ also divides the bulk time slice into two subregions. We call the modular Hamiltonian associated to subregion R_b as H_{bulk}.

Some properties of H_{mod}

- Using AdS/CFT, one can explicitly show that the boundary modular Hamiltonian for spherical entangling subregion is equal to the corresponding bulk modular Hamiltonian up to terms localized on the RT surface.

Jafferis et al. 2015

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- However, the RT surface (much like Rindler horizon), works as a bifurcation surface and quantities localized on RT surface are invariant under H_{bulk} transformations.

$$[\phi^{(0)}_{HKLL}, H_{mod}] = 0, \qquad \text{for } \phi^{(0)} \text{ on } \gamma$$

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Bulk fields from CFT modular Hamiltonian

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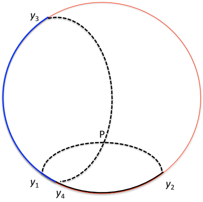

- If bulk fields localized at RT surface commute with $\tilde{H}_{mod} = H_{mod} - H^c_{mod}$, bulk fields localized at the intersection point must commute with that of both subregions (in AdS_3 e.g.).

- Thus the CFT requirement for a local bulk field is that

$$(y_2 - y_1)[\tilde{H}^{12}_{mod}, \phi(\xi, \bar{\xi})] = 0, \quad (y_4 - y_3)[\tilde{H}^{34}_{mod}, \phi(\xi, \bar{\xi})] = 0$$

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- Indeed, starting with an ansatz like

$$\phi(X) = \int dt' dy' g(p, q)\mathcal{O}(q, p),$$

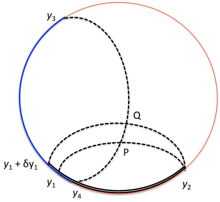
one indeed finds

$$\phi^{(0)}(Z, X_0) = c_\Delta \int_{Z^2 > y''^2 + t'^2} dt' dy'' \left(Z^2 - y''^2 - t'^2\right)^{\Delta - 2} \mathcal{O}(t', X_0 + iy'')$$

X_0 and the emergent direction Z coordinatizing the bulk point.

However, given this construction, the recovery of the bulk metric is imminent.

Roy and DS, 1801.07280.



The first step is to compute the bulk correlators between points P and Q. Guaranteed to reproduce the correct bulk to bulk correlator (dropping superscript (0))

$$\langle \phi(X_0, Z)\phi(\tilde{X}_0, \tilde{Z})\rangle_{bulk} = c_\Delta \frac{1}{\sqrt{\sigma^2 - 1}} \frac{1}{(\sigma + \sqrt{\sigma^2 - 1})^{\Delta - 1}}.$$

Bulk metric from CFT

- Consider WKB limit, where the conformal dimensions Δ of the operators are taken to be large.

- Parametrizing σ (in hindsight) in terms of a bulk length variable L along with a constant length parameter l_{bulk} as

$$\sigma = \cosh \frac{L}{l_{bulk}}.$$

immediately yields

$$\langle \phi(X_0, Z)\phi(\tilde{X}_0, \tilde{Z})\rangle_{bulk}\Big|_{\Delta \gg 1} \approx \frac{c_\Delta}{\sqrt{\sigma^2 - 1}} e^{-\frac{L\Delta}{l_{bulk}}} = c'_\Delta e^{-\frac{L\Delta}{l_{bulk}}}$$

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- However, this is precisely the leading order term in the free bulk to bulk propagator in the generally curved spacetimes, with L being identified with the associated geodesic length.

Louko, Marolf, Ross 2000

- Once L is identified as the geodesic length, we can drop the WKB limit, and instead consider a limit where the bulk fields are infinitesimally separated.

- Using the resulting expression of σ
$$\sigma = \frac{Z^2 + \tilde{Z}^2 + (X_0 - \tilde{X}_0)^2}{2Z\tilde{Z}}.$$
in terms of 'bulk coordinates' and the relation between σ and $L = \sqrt{g^{bulk}_{\mu\nu} dx^\mu dx^\nu}$, one can recover the metric explicitly in any coordinate patch.

- Generalizable to exterior bulk regions in Rindler/ BTZ.

A subtlety

- A bulk point can be generated by an infinite number of intersecting modular Hamiltonian. Moreover there is a freedom of a multiplicative field redefinition of the bulk field $\phi(x) \to f(x)\phi(x)$.

- For translationally invariant states, c_Δ can only be a constant factor. For general states

$$\langle \phi(x)\phi(\tilde{x}) \rangle = e^{F-\Delta L}/\sqrt{\sigma^2 - 1}, \qquad \text{where} \qquad F = \log f_1(x) f_2(\tilde{x}).$$

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- Without any loss of generality, we can choose

$$F = \sum_{n\geq 0} c_n(x)(\Delta L)^n, \sum_{l\geq 1} c_l(x) \log^l(\Delta L), \sum_{m<0} c_m(x)(\Delta L)^m.$$

- However, for any such choice of F, the 'conformal bulk metric' is still recoverable. In line with the AdS/CFT ideas. The AdS vacuum states are recovered up to factors of R_{AdS}.

- Try to solve the modular constraints with a general ansatz such as

$$\phi^{(1)}(X) = \int dt' dy' g(p,q)\mathcal{O}_{\Delta}(q,p) + \sum_n a_n \int dt' dy' g_n(p,q)\mathcal{O}_{\Delta_n}(q,p)\,,$$

- The resulting constraint thus takes the form

$$(y_2 - y_1)\left[\tilde{H}^{12}_{mod}, \int dt' dy' g(p,q)\mathcal{O}_{\Delta}(q,p)\right] + \sum_n a_n (y_2 - y_1)\left[\tilde{H}^{12}_{mod}, \int dt' dy' g_n(p,q)\mathcal{O}_{\Delta_n}(q,p)\right] = 0\,,$$

and similarly for $\tilde{H}^{34}_{mod}$.

- Fortunately, the derivation is independent of the choice of Δ and only relies on the fact that the boundary operator $\mathcal{O}_{\Delta}$ is a conformal primary.

Interactions from H_{mod}

- Hence there is at least one set of solutions which gives the same functions $g_n(p,q)$'s as $g(p,q)$ with the only difference being the dependence on Δ.

$$g_n(p,q) = d_{\Delta_n}\left[Z^2 - (p-X_0)(q-X_0)\right]^{\Delta_n - 2}.$$

resulting in

$$\phi^{(1)}(t,\vec{x},z) = c_\Delta \int dt' d^{d-1}\vec{y}' K(t,\vec{x},z|t',\vec{y}')\mathcal{O}_\Delta(t+t',\vec{x}+i\vec{y}') + \sum_n a_n d_{\Delta_n}\int dt' d^{d-1}\vec{y}' K_{\Delta_n}(t,\vec{x},z|t',\vec{y}')\mathcal{O}_{\Delta_n}(t+t',\vec{x}+i\vec{y}')$$

- To evaluate suitable $a_n d_{\Delta_n}$'s one needs bulk lightcone singularity structures.

- Fortunately $\Phi^{(0)}$ is *sufficient* to determine the bulk metric up to an overall conformal factor, which in turn, *completely specifies* the bulk microcausality.

Summary

- Local bulk operator prescription for free (and for some, sub-leading in $1/N$ or $1/\lambda$) spin-0 and all integer HS gauge fields.

  Kabat, Lifschytz, Roy, DS 2012, DS and Xiao 2012, DS 2014, Roy and DS 2017

- Sub-horizon operator construction for hyperbolic BHs, TFD states and BHs formed by collapse.

  ... Roy, D.S 2015 ...

- Metric reconstruction in AdS_3 (and a clear prescription for higher dimensions) starting from purely boundary data. Connection with kinematic space, OPE blocks etc.

- Gauge field reconstruction in modular Hamiltonian approach?

- Fields (and background) reconstruction for entanglement wedge (and sub-horizon) in this approach. No perturbative construction seems possible.

THANKS FOR LISTENING