Gravitational and electromagnetic perturbations of (near-)$AdS_2 \times S^2$ in (near-)extreme Reissner-Nordstrom

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The connection problem for AdS/CFT

- Near the horizon of near-extreme black holes: AdS-like spacetimes
- With AdS boundary conditions: Paradigm-shifting AdS/CFT results
- In the real world gravity does not obey AdS boundary conditions

The connection problem:

Extending anti-de Sitter solutions away from the near-horizon region of (near-)extreme black holes and connecting them with solutions in the far asymptotically flat region.

- AdS_2 is most advantageous over its higher dimensional counterparts:
  1. $AdS_2 \times S^2$ is an exact solution of 4D pure Einstein-Maxwell theory
  2. $AdS_2 \times S^2$ is the near-horizon of extreme Reissner-Nordstrom

  [Bertotti, Robinson (1959)]

AdS_2 vs near–AdS_2

Consider a Reissner-Nordstrom black hole of mass M and charge Q and parameterize the deviation from extremality by

$$\kappa \equiv \sqrt{1 - \frac{Q^2}{M^2}}$$

- The extreme Reissner-Nordstrom (ERN) is given by: $\kappa = 0$
- The near-extreme Reissner-Nordstrom (NERN) is given by: $\kappa \ll 1$

Let r be a dimensionless radial coordinate s.t. the horizon is at $r = 0$

The near-horizon spacetimes are as follows

- In ERN for $r \ll 1$ the geometry is given by $AdS_2 \times S^2$:

$$\frac{1}{M^2} ds^2 = -r^2 dt^2 + \frac{dr^2}{r^2} + d\Omega^2 \,, \qquad A_t = Mr \,.$$

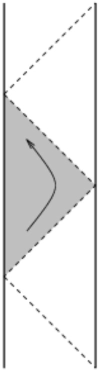
- In NERN for $r \sim \kappa \ll 1$ the geometry is given by near–$AdS_2 \times S^2$:

$$\frac{1}{M^2} ds^2 = -r(r + 2\kappa) dt^2 + \frac{dr^2}{r(r + 2\kappa)} + d\Omega^2 \,, \qquad A_t = M(r + \kappa) \,.$$

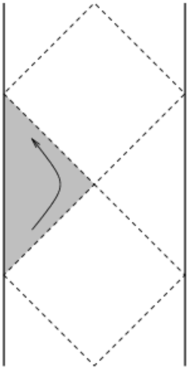
AdS_2 vs near–AdS_2

[Maldacena, Michelson, Strominger (1999)]
[Spradlin, Strominger (1999)]

- Locally, AdS_2 and near–AdS_2 are diffeomorphic
- On Penrose diagram of global AdS_2 they cover the following patches:



AdS_2: Poincare patch

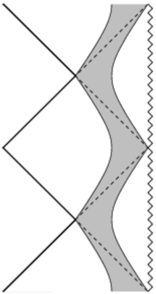


near–AdS_2: Rindler patch

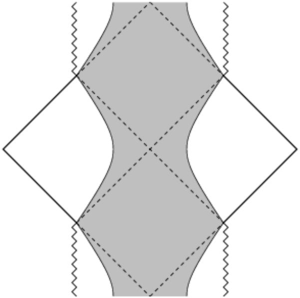
- Physically distinct connections to asymptotically flat region:
  - Attaching flat region to shaded triangle on left $\Rightarrow$ ERN physics
  - Attaching flat region to shaded triangle on right $\Rightarrow$ NERN physics

AdS_2 vs near–AdS_2

Near-horizon anti-de Sitter arises in ERN and NERN as follows:



ERN



NERN

In this context, solve the connection problem for coupled gravitational and electromagnetic perturbations of $AdS_2 \times S^2$ and near–$AdS_2 \times S^2$

Solving the connection problem: Outline

[APP (arXiv:1805.12409)]
[APP (arXiv:1806.07097)]

Step 1: Reduce the linearized Einstein-Maxwell eqs on the background of the (near-)extreme RN to a single 4^{th} order radial ODE

Step 2: Derive the anti-de Sitter eqs as an appropriate near-horizon approximation of the (near-)extreme RN ones, and solve them

Step 3: Find and solve intermediate and far region approximations such that, together with the near-horizon region, the full (near-)extreme RN spacetime is covered

Step 4: Use the method of matched asymptotic expansions to glue the solutions together in overlapping regions

All computations are entirely analytical!

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Step 1: Reducing the linearized Einstein-Maxwell eqs

- Even parity RN perturbation ansatz in Regge-Wheeler-Zerilli gauge:
  [Regge, Wheeler (1957)] [Zerilli (1974)]

$$h_{\mu\nu} = \begin{pmatrix} -Y(r) & X(r) & 0 & 0 \\ & V(r) & 0 & 0 \\ & & K(r) & 0 \\ & & & K(r)\sin^2\theta \end{pmatrix} e^{i\omega t}\, Y_{l,0}(\theta,\phi)$$

$$a_\mu = (\chi(r) \quad \psi(r) \quad 0 \quad 0)\, e^{i\omega t}\, Y_{l,0}(\theta,\phi)$$

  With this ansatz, the linearized Einstein-Maxwell eqs may be reduced to
  [APP (arXiv:1805.12409)] [APP (arXiv:1806.07097)]

$$a_4(r)K'''' + a_3(r)K''' + a_2(r)K'' + a_1(r)K' + a_0(r)K = 0$$

  Y, V, X, χ, ψ obtained by algebraic expressions of K and its derivatives.

  Note: an alternative to the well-known reductions to two 2^{nd} order ODEs

- It is convenient to change variables according to

$$K(r) = \left(\frac{r}{1+r}\right)^2 H(r)$$

- Generic modes do not survive in the near-horizon limit. The modes which do survive and solve the $AdS_2 \times S^2$ and near–$AdS_2 \times S^2$ eqs are

$$\text{the low energy modes:} \qquad \omega \ll 1$$

Steps 2,3: Anti-de Sitter, Static, and Far solns

Divide the spacetime into three regions:

$$\begin{aligned} &\text{Near:} && r \ll 1 \\ &\text{Static:} && \max(\kappa, \omega) \ll r \ll 1/\omega \\ &\text{Far:} && 1 \ll r \end{aligned}$$

- In the intermediate Static region the soln is given by

$$H^s(r) = (1+r)^3 \left[C_1^s\, r^{l-1} + C_2^s\, r^{l-3}(l - r + 2lr) + C_3^s\, r^{-l-2} + C_4^s\, r^{-l-4}(l + 1 + 3r + 2lr) \right]$$

- In the Far region the soln is given by

$$\begin{aligned} H^f(r) = {} & C_1^f\, r\omega \left((l+1)(l+2) r\omega h_{l+2}^{(1)}(r\omega) - \left((l+1)(l+2)(2l+3) - 2r^2\omega^2 \right) h_{l+1}^{(1)}(r\omega) \right) \\ & + C_2^f\, r\omega \left((l+1)(l+2) r\omega h_{l+2}^{(2)}(r\omega) - \left((l+1)(l+2)(2l+3) - 2r^2\omega^2 \right) h_{l+1}^{(2)}(r\omega) \right) \\ & + C_3^f \left(r\omega \left(l(l+1)^3(l+2) - 2\left(l^2 + l + 2 \right) r^2\omega^2 \right) h_{l+2}^{(1)}(r\omega) \right. \\ & \qquad \left. -(l+2)\left(l(l+1)^3(2l+3) - (l+3)\left(l^2 + l + 2 \right) r^2\omega^2 \right) h_{l+1}^{(1)}(r\omega) \right) \\ & + C_4^f \left(r\omega \left(l(l+1)^3(l+2) - 2\left(l^2 + l + 2 \right) r^2\omega^2 \right) h_{l+2}^{(2)}(r\omega) \right. \\ & \qquad \left. -(l+2)\left(l(l+1)^3(2l+3) - (l+3)\left(l^2 + l + 2 \right) r^2\omega^2 \right) h_{l+1}^{(2)}(r\omega) \right) \end{aligned}$$

where $h_m^{(1)}, h_m^{(2)}$ are the spherical Hankel functions.

Steps 2,3: Anti-de Sitter, Static, and Far solns

- In the Near region we have:
  - For ERN the Near soln is given by

$$H^n(r) = C_1^n\, r^{-3} h^{(1)}_{-l-2}(\omega/r) + C_2^n\, r^{-3} h^{(2)}_{-l-2}(\omega/r) + C_3^n\, r^{-3} h^{(1)}_{-l}(\omega/r) + C_4^n\, r^{-3} h^{(2)}_{-l}(\omega/r)$$

This is an *exact* soln w.r.t. the $AdS_2 \times S^2$ background.

  - For NERN the Near soln is given by

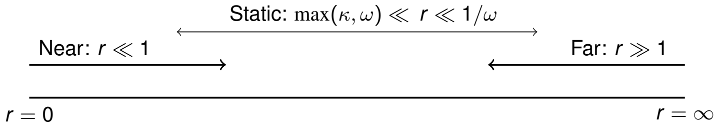
$$\begin{aligned} H^n(r) = {} & C_1^n\, r^{-2-\frac{i\omega}{2\kappa}} \left(\frac{r}{2\kappa}+1\right)^{\frac{i\omega}{2\kappa}} {}_2F_1\left(-1-l, l+2; 1-\frac{i\omega}{\kappa}; -\frac{r}{2\kappa}\right) \\ & + C_2^n\, r^{-2+\frac{i\omega}{2\kappa}} \left(\frac{r}{2\kappa}+1\right)^{-\frac{i\omega}{2\kappa}} {}_2F_1\left(-1-l, l+2; 1+\frac{i\omega}{\kappa}; -\frac{r}{2\kappa}\right) \\ & + C_3^n\, r^{-2-\frac{i\omega}{2\kappa}} \left(\frac{r}{2\kappa}+1\right)^{\frac{i\omega}{2\kappa}} {}_2F_1\left(1-l, l; 1-\frac{i\omega}{\kappa}; -\frac{r}{2\kappa}\right) \\ & + C_4^n\, r^{-2+\frac{i\omega}{2\kappa}} \left(\frac{r}{2\kappa}+1\right)^{-\frac{i\omega}{2\kappa}} {}_2F_1\left(1-l, l; 1+\frac{i\omega}{\kappa}; -\frac{r}{2\kappa}\right) \end{aligned}$$

This is an *exact* soln w.r.t. the near–$AdS_2 \times S^2$ background.

- Notice that the Static and Far solns are identical for ERN and NERN

Step 4: Matched Asymptotic Expansions

Since $\kappa\,,\omega \ll 1$ the Static region overlaps with both Near and Far regions:



- In the Near-Static overlap region $\max(\kappa, \omega) \ll r \ll 1$ the solution is

$$H^{ns}(r) = C_1^{ns} r^{l-1} + C_2^{ns} r^{l-3} + C_3^{ns} r^{-l-2} + C_4^{ns} r^{-l-4}$$

  Using this one may obtain linear relations between the C_i^n's and C_i^s's

- In the Far-Static overlap region $1 \ll r \ll 1/\omega$ the solution is

$$H^{fs}(r) = C_1^{fs} r^{l+2} + C_2^{fs} r^{l+1} + C_3^{fs} r^{-l+1} + C_4^{fs} r^{-l}$$

  Using this one may obtain linear relations between the C_i^f's and C_i^s's

Eliminating the C_i^s's one finds the linear relation between the C_i^f's and C_i^n's. This is the solution to the connection problem for (near-)$AdS_2 \times S^2$ in (N)ERN.

The solutions for the connection problems

- For $AdS_2 \times S^2$ and ERN we have: [APP (arXiv:1805.12409)]

$$C_{12}^{n+} = \frac{(-1)^{l+1}}{2^{2l+2}\pi} \frac{(l+1)(l+2)}{2l-1} \Gamma\left(-l-\tfrac{1}{2}\right)^2 \omega^{2l+3} \left((25l-8)\omega\, C_{12}^{f+} - 3l(l+1)^2(3l-1)\, C_{34}^{f+}\right)$$

$$C_{12}^{n-} = \frac{(-1)^l 2^{2l}}{\pi}(l-1)l(l+1)(2l+1)\Gamma\left(l+\tfrac{1}{2}\right)^2 \omega^{-2l-1} \left(3\omega\, C_{12}^{f-} + l^2(l+1)\, C_{34}^{f-}\right)$$

$$C_{34}^{n+} = \frac{(-1)^{l+1}}{2^{2l+2}\pi} l(l+1)(l+2)(2l+1)\Gamma\left(-l-\tfrac{1}{2}\right)^2 \omega^{2l+1} \left(3\omega\, C_{12}^{f+} - l(l+1)^2\, C_{34}^{f+}\right)$$

$$C_{34}^{n-} = \frac{(-1)^l 2^{2l}}{\pi} \frac{(l-1)l}{2l+3} \Gamma\left(l+\tfrac{1}{2}\right)^2 \omega^{-2l+1} \left((25l+33)\omega\, C_{12}^{f-} + 3l^2(l+1)(3l+4)\, C_{34}^{f-}\right)$$

$$C_{12}^{n\pm} = C_1^n \pm C_2^n\,, \quad C_{34}^{n\pm} = C_3^n \pm C_4^n\,,$$
$$C_{12}^{f\pm} = C_1^f \pm C_2^f\,, \quad C_{34}^{f\pm} = C_3^f \pm C_4^f\,.$$

The solutions for the connection problems

- For near–$AdS_2 \times S^2$ and NERN we have: [APP (arXiv:1806.07097)]

$$C_{12}^{n+} = \frac{i}{\pi}\frac{(l+1)(l+2)}{2l-1}2^{-2l-2}\,\Gamma\left(-l-\tfrac{1}{2}\right)^2\left((25l-8)\omega\, C_{12}^{f+} - 3l(l+1)^2(3l-1)\, C_{34}^{f+}\right)\times$$
$$\times\, (2\kappa)^{i\omega/2\kappa}\kappa^{l+2}\omega^{l}(-i\omega/\kappa)_{l+2}$$

$$C_{12}^{n-} = -\frac{i}{\pi}(l-1)l(l+1)(2l+1)2^{2l}\,\Gamma\left(l+\tfrac{1}{2}\right)^2\left(3\omega\, C_{12}^{f-} + l^2(l+1)\, C_{34}^{f-}\right)\times$$
$$\times\, \frac{(2\kappa)^{i\omega/2\kappa}}{\kappa^{l+2}\omega^{l}(i\omega/\kappa)_{l+2}}$$

$$C_{34}^{n+} = \frac{i}{\pi}l(l+1)(l+2)(2l+1)2^{-2l-2}\,\Gamma\left(-l-\tfrac{1}{2}\right)^2\left(3\omega\, C_{12}^{f+} - l(l+1)^2\, C_{34}^{f+}\right)\times$$
$$\times\, (2\kappa)^{i\omega/2\kappa}\kappa^{l}\omega^{l}(-i\omega/\kappa)_{l}$$

$$C_{34}^{n-} = -\frac{i}{\pi}\frac{(l-1)l}{2l+3}2^{2l}\,\Gamma\left(l+\tfrac{1}{2}\right)^2\left((25l+33)\omega\, C_{12}^{f-} + 3l^2(l+1)(3l+4)\, C_{34}^{f-}\right)\times$$
$$\times\, \frac{(2\kappa)^{i\omega/2\kappa}}{\kappa^{l}\omega^{l}(i\omega/\kappa)_{l}}$$

$$C_{12}^{n\pm} = C_1^n \pm \alpha\, C_2^n\,, \qquad C_{34}^{n\pm} = C_3^n \pm \beta\, C_4^n\,,$$
$$C_{12}^{f\pm} = C_1^f \pm C_2^f\,, \qquad C_{34}^{f\pm} = C_3^f \pm C_4^f\,,$$

$$\alpha = -(2\kappa)^{i\omega/\kappa}(-i\omega/\kappa)_{l+2}/(i\omega/\kappa)_{l+2}\,,\ \beta = -(2\kappa)^{i\omega/\kappa}(-i\omega/\kappa)_{l}/(i\omega/\kappa)_{l}$$

Thank you