Hall viscosities in semi-Dirac Semimetals

Gauge/Gravity duality,
Wurzburg 2018

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arXiv:1805.09827

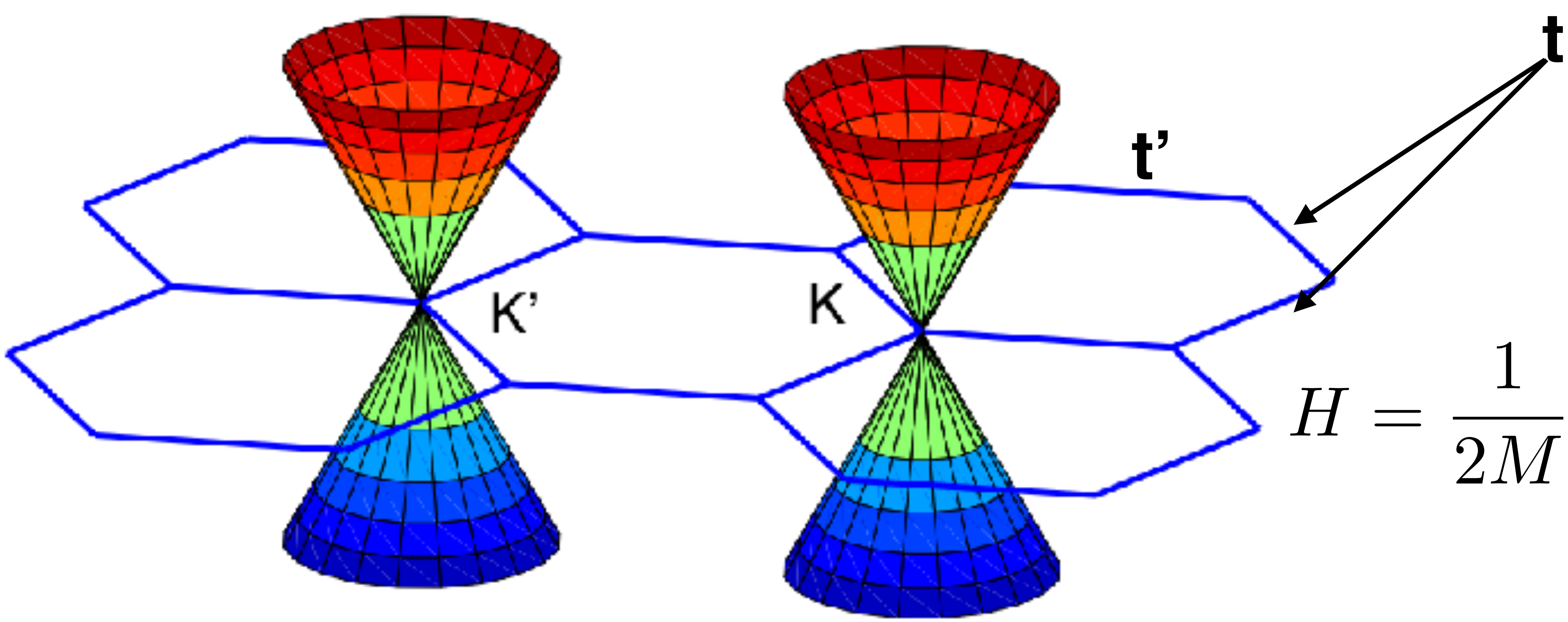
MPI for the Physics of Complex Systems

- Dirac materials are a new laboratory to study QFTs
- Topology plays an important role (some examples)
  - Quantum Hall states in 2d
  - Anomaly-induced transport in 3d
- In 2d system with rotational invariance and without T-inversions, the Viscosity tensor can have an odd antisymmetric contribution parametrised by a single quantity
- When rotational invariance is broken more odd viscosities appear

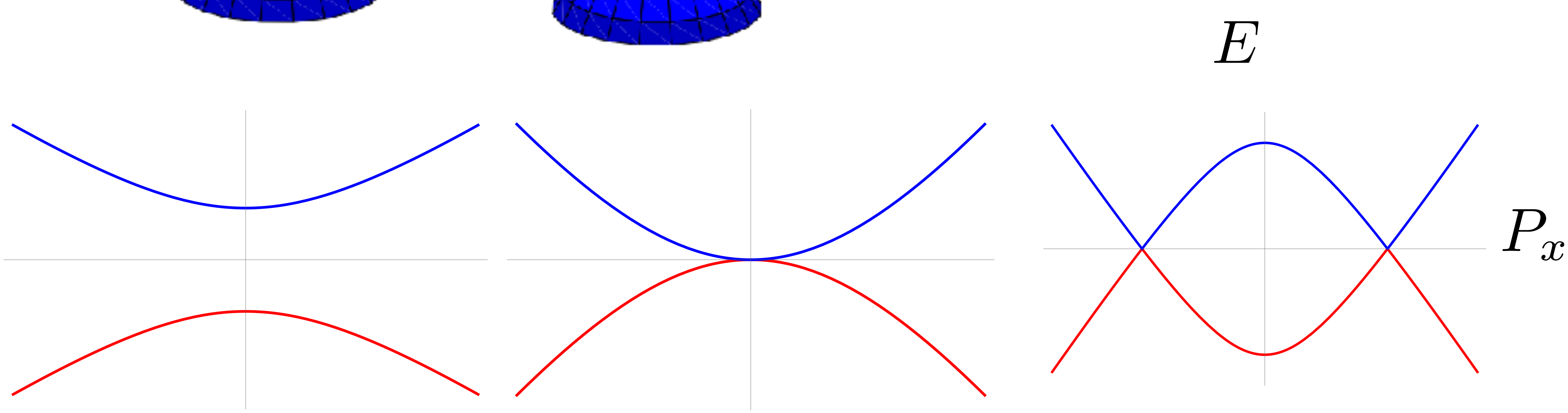
outline

- Semi-Dirac
- Hall viscosity tensor
- Summary
- Outlook

Semi-Dirac



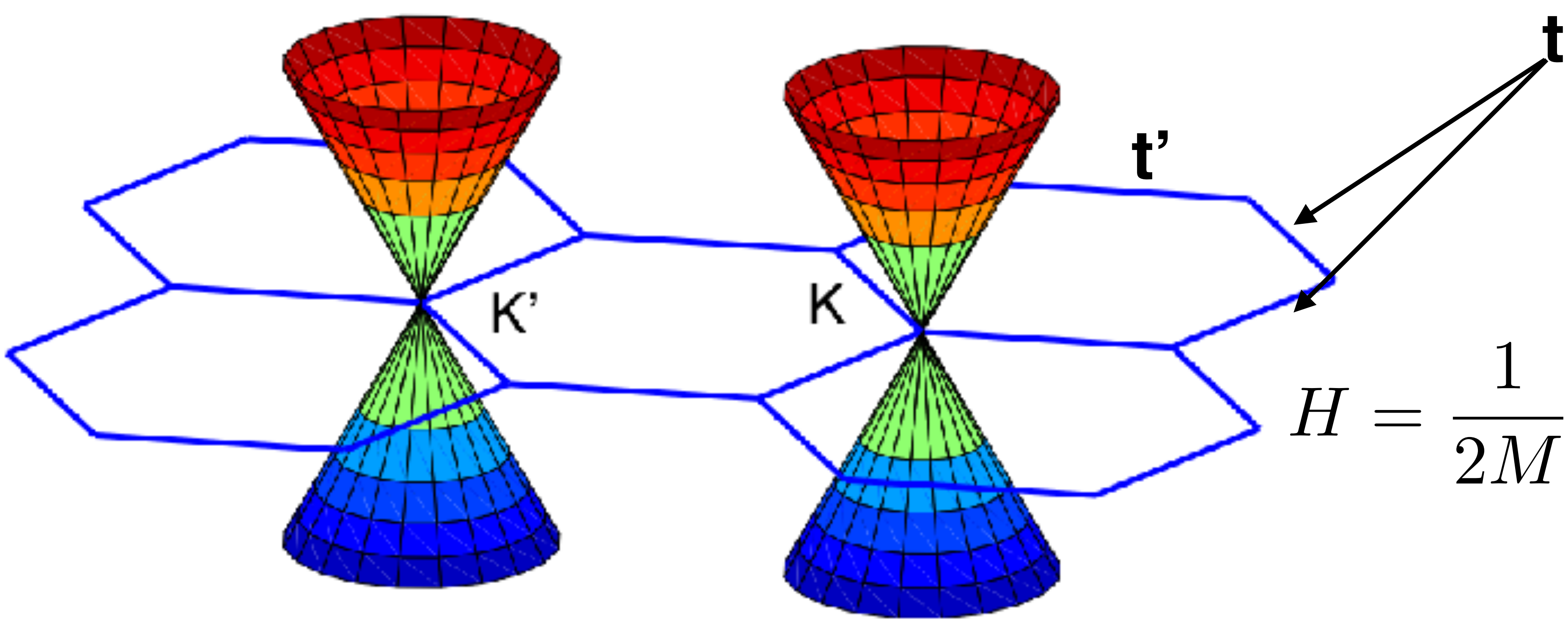
$$H = \frac{1}{2M}\left(P_x^2 - \delta^2\right)\sigma_x + P_y\sigma_y$$



$$t' = 2t \implies \delta = 0$$

[G. Montambaux, F. Piechon, J.-N. Fuchs, and M.O. Goerbig]

Semi-Dirac



$$H = \frac{1}{2M}\left(P_x^2 - \delta^2\right)\sigma_x + P_y\sigma_y$$

TiO_2/VO_2 Heterostructure

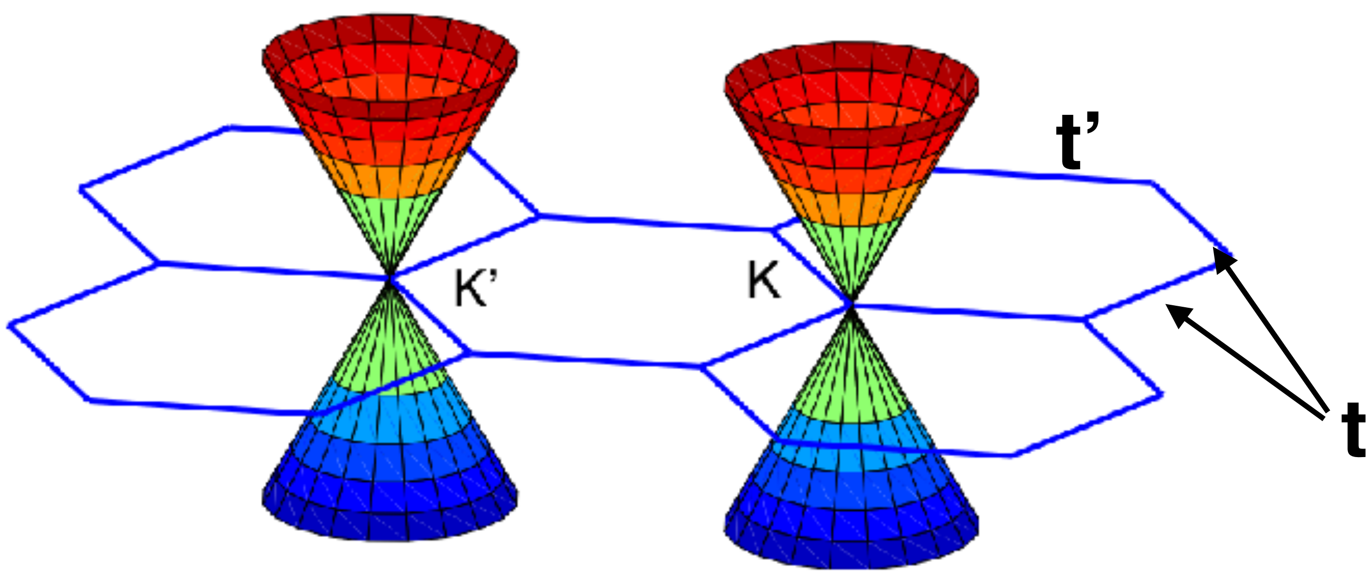
$(BEDT-TTF)_2I_3$ Organic salts

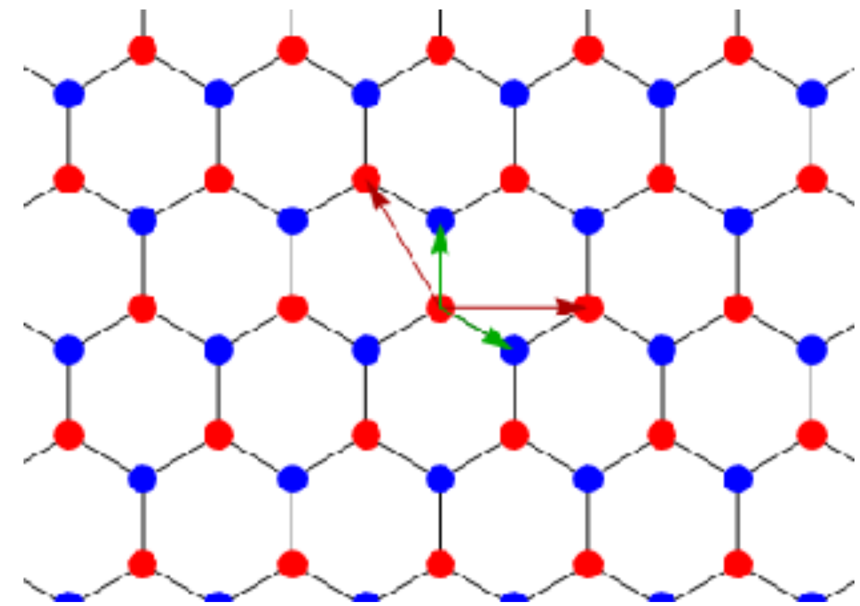
Photonic materials

The goal is to apply a magnetic field and then compute the Hall viscosity

[G. Montambaux, F. Piechon, J.-N. Fuchs, and M.O. Goerbig]

Semi-Dirac





$$H' = -\sigma_x (k_x + K') + \sigma_y k_y$$

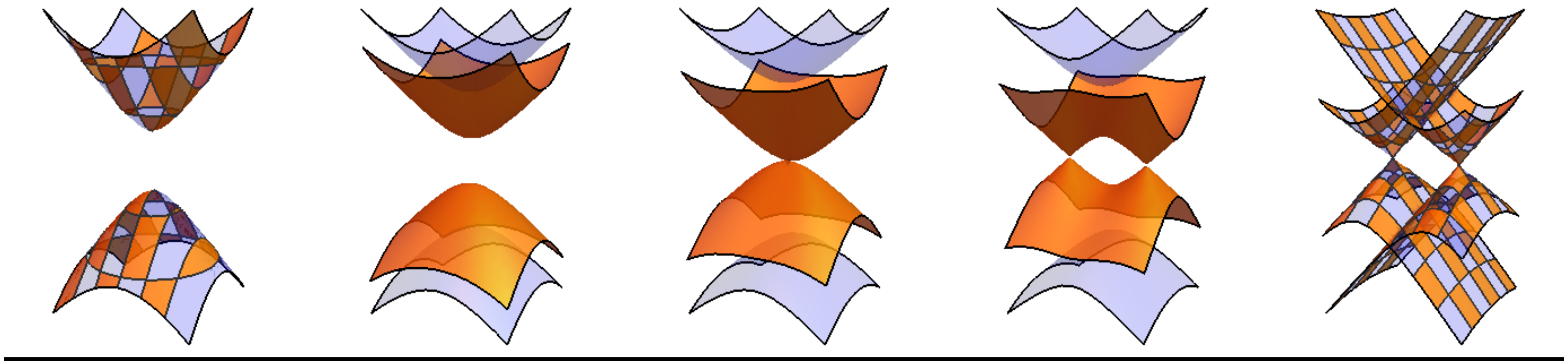
$$H = \sigma_x (k_x - K) + \sigma_y k_y$$

$$\psi_K = (\psi_{KA}, \psi_{KB})$$

$$\psi_{K'} = (\psi_{K'A}, \psi_{K'B})$$

$$\psi = (\psi_{KA}, \psi_{KB}, \psi_{K'B}, \psi_{K'A})$$

$$H = \gamma^0 \left(\gamma^x \left(k_x - \frac{\Delta k}{2} \gamma^5 \right) + \gamma^y k_y \right) \; {\color{red} +m_0 \gamma^0}$$



$$H = \gamma^0 \left(\gamma^x \left(k_x - \frac{\Delta k}{2} \gamma^5 \right) + \gamma^y k_y \right) \; +m_0 \gamma^0$$

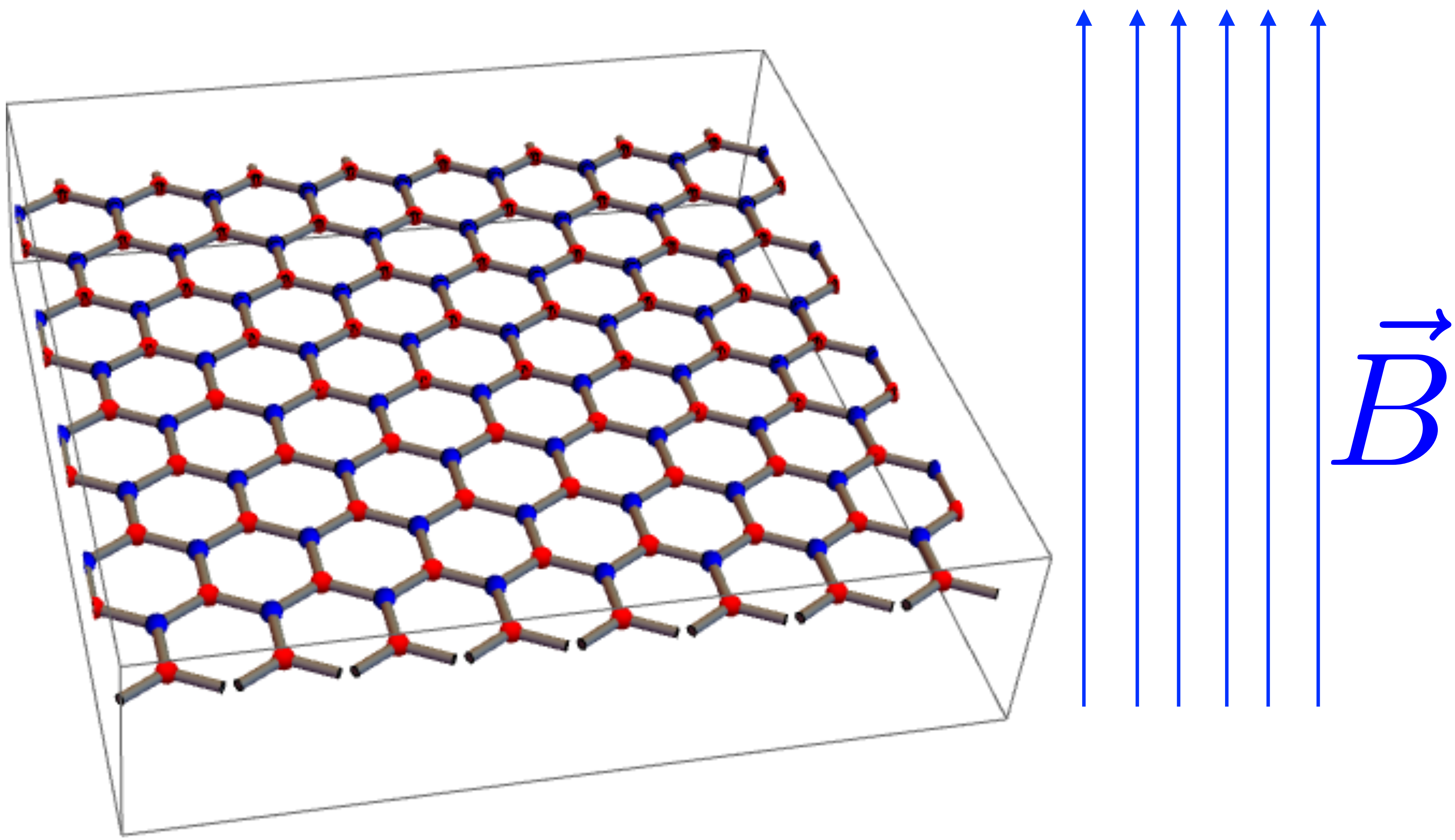
$$\Delta K_{eff} = \sqrt{(\Delta K)^2 - m_0^2} \qquad \Delta K_{eff}^2 = \delta^2 + m_0^2$$

$$\delta / m_0 \ll 1$$

$$E_\pm^2 = \frac{1}{4m_0^2} \left(k_x^2 - \delta^2 \right)^2 + k_y^2$$

$$H = \frac{1}{2M} \left(P_x^2 - \delta^2 \right) \sigma_x + P_y \sigma_y$$

Applying a magnetic field



T-inversion is broken, therefore Hall viscosity appears

[Avron, et al]

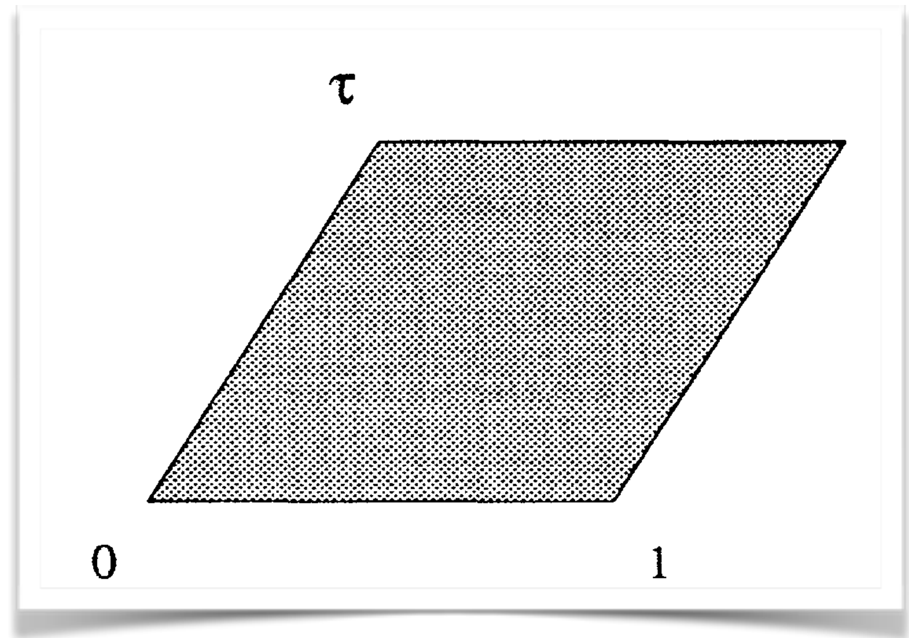
Adiabatic theorem and Berry curvature

$$\left\langle \frac{\partial H}{\partial x_j} \right\rangle = \frac{\partial E}{\partial x_j} - \Omega_{ij}\dot{x}_j .$$

$$\Omega_{ij} = i\left[\frac{\partial}{\partial x_i}\left\langle \psi \middle| \frac{\partial \psi}{\partial x_j}\right\rangle - \frac{\partial}{\partial x_j}\left\langle \frac{\partial \psi}{\partial x_i} \middle| \psi \right\rangle\right]$$

We put the theory on a torus

$$ds^2 = \frac{V}{\tau_2}\left(\mathrm{d}x^2 + 2\tau_1 \mathrm{d}x\mathrm{d}y + |\tau|^2 \mathrm{d}y^2\right)$$



Relation of the Hall viscosity
with the Berry curvature

$$\eta_{xxxy} = \frac{B}{2\pi N}(\tau_2^2 \Omega_{\tau_1\tau_2}) - \tau_2 \Omega_{\tau_1 V}$$

$$\eta_{xyyy} = \frac{B}{2\pi N}(\tau_2^2 \Omega_{\tau_1\tau_2}) + \tau_2 \Omega_{\tau_1 V}$$

$$\eta_{xxyy} = -\tau_2 \Omega_{\tau_2 V}$$

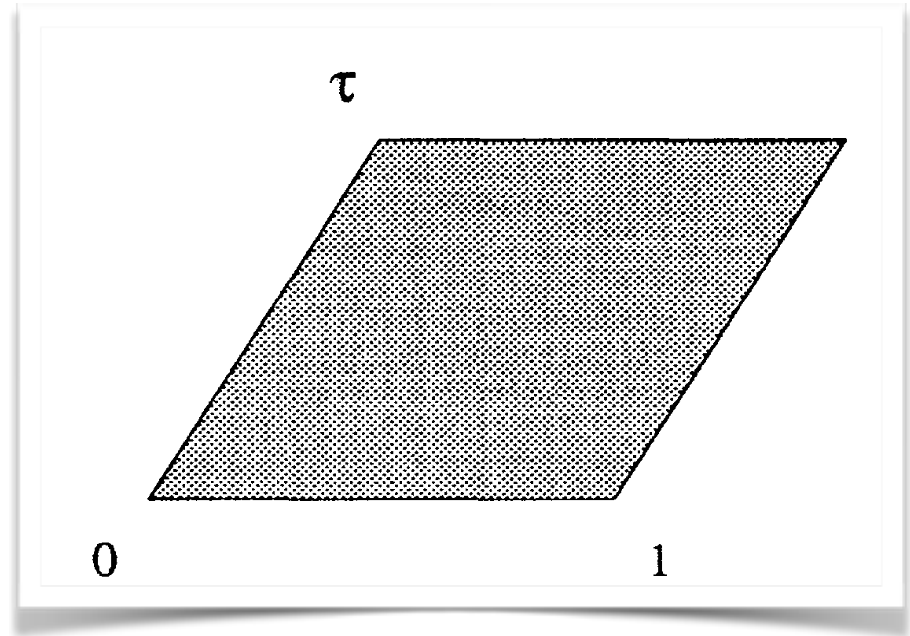
Isotropic systems

$$\eta_H = \bar{s}\rho/2$$

Isotropic graphene

$$\eta_H = \left(n(n+1) + \frac{1}{2}\right)\frac{N}{L^2}$$

$$ds^2 = \frac{V}{\tau_2}\left(\mathrm{d}x^2 + 2\tau_1 \mathrm{d}x\mathrm{d}y + |\tau|^2\mathrm{d}y^2\right)$$



As usual for a Landau system the Hamiltonian can be written in term of ladder operators.

Notice the dependence on the deformation parameters is hidden in the definitions of **a, d, M**

$$H = \begin{pmatrix} 0 & (a+d)^\dagger & M & 0 \\ a+d & 0 & 0 & M \\ M & 0 & 0 & (d-a)^\dagger \\ 0 & M & d-a & 0 \end{pmatrix}$$

$$\mathfrak{a}_d^\dagger \mathfrak{a}_d |n, \alpha, d, \tau\rangle = n|n, \alpha, d, \tau\rangle$$

$$|\psi\rangle = \sum_n \mathbf{c}_n |n, d, \tau\rangle$$

$$H_{nm}\mathbf{c}_m = \epsilon_n \mathbf{c}_n \;, \quad \text{where} \quad H_{nm} = \langle n, d, \tau | H \;\; | m, d, \tau\rangle$$

Given the form of the states, the Berry curvature can be written as

$$\boxed{\Omega = i\mathrm{d}\left(\mathbf{c}_n^\dagger \cdot \mathrm{d}\mathbf{c}_n\right) + \mathrm{d}\left(\mathbf{c}_n^\dagger \cdot \mathbf{c}_m\right) \wedge \mathcal{A}^{nm} + \mathbf{c}_n^\dagger \cdot \mathbf{c}_m \mathcal{F}^{nm}}$$

$$\mathcal{A}_{\alpha\beta}^{nm} = i\langle n, \alpha, d, \tau|\mathrm{d}|m, \beta, d, \tau\rangle \qquad \mathcal{F}_{\alpha\beta}^{nm} = \mathrm{d}\mathcal{A}_{\alpha\beta}^{nm}$$

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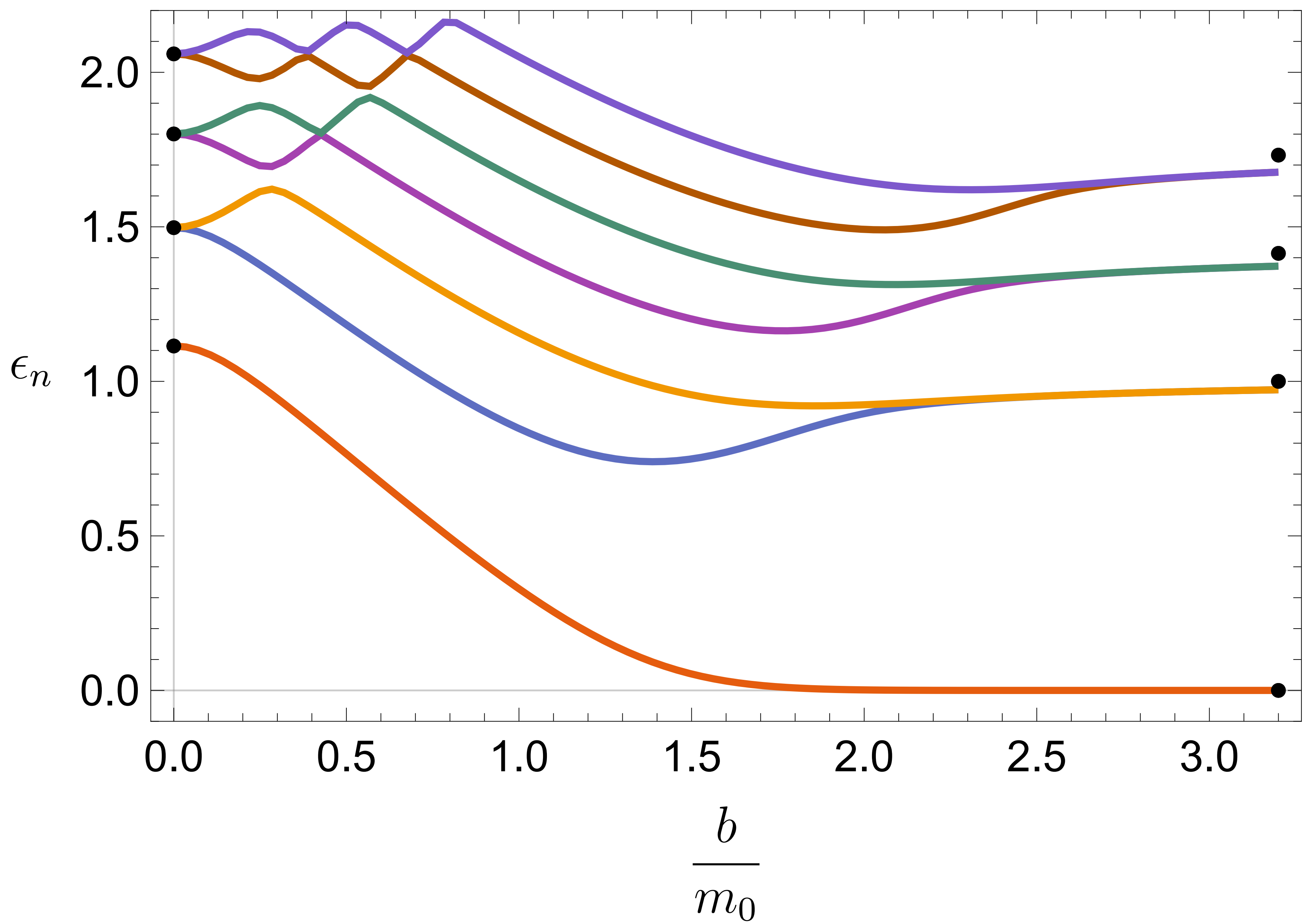
The deformations of the torus are is generated by acting with GL(2,R) transformations

$$[J_1, J_2] = -iJ_3 \quad , \quad [J_2, J_3] = iJ_1 \quad , \quad [J_3, J_1] = iJ_2 .$$

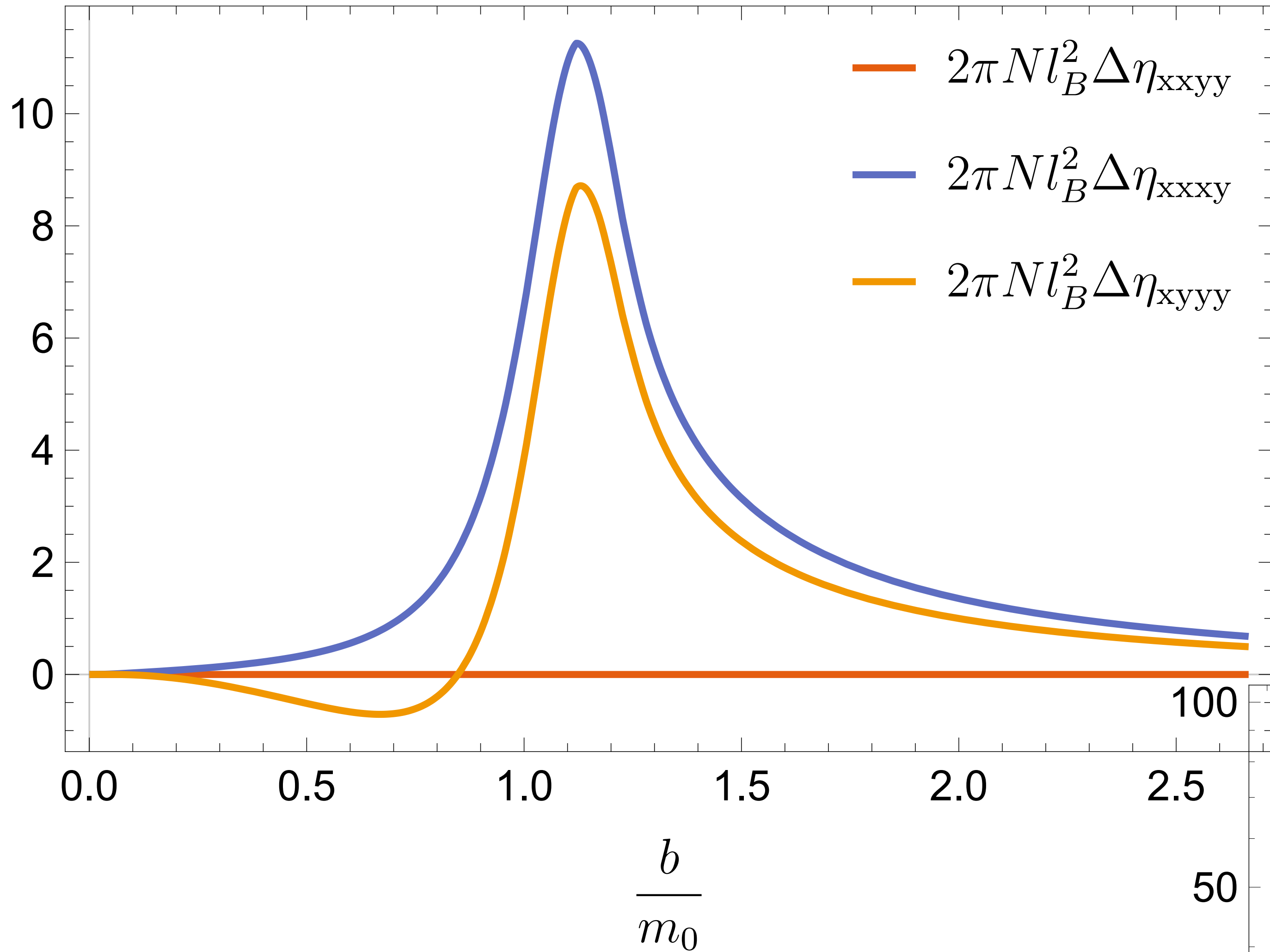
$$|n, \alpha, d, \tau\rangle = \mathcal{U}|n, \alpha, d, i\rangle \qquad \mathcal{A}_{\alpha\beta}^{nm} = i\langle n, \alpha, d, i|\mathcal{U}^{-1}\mathrm{d}\mathcal{U}|m, \alpha, d, i\rangle$$

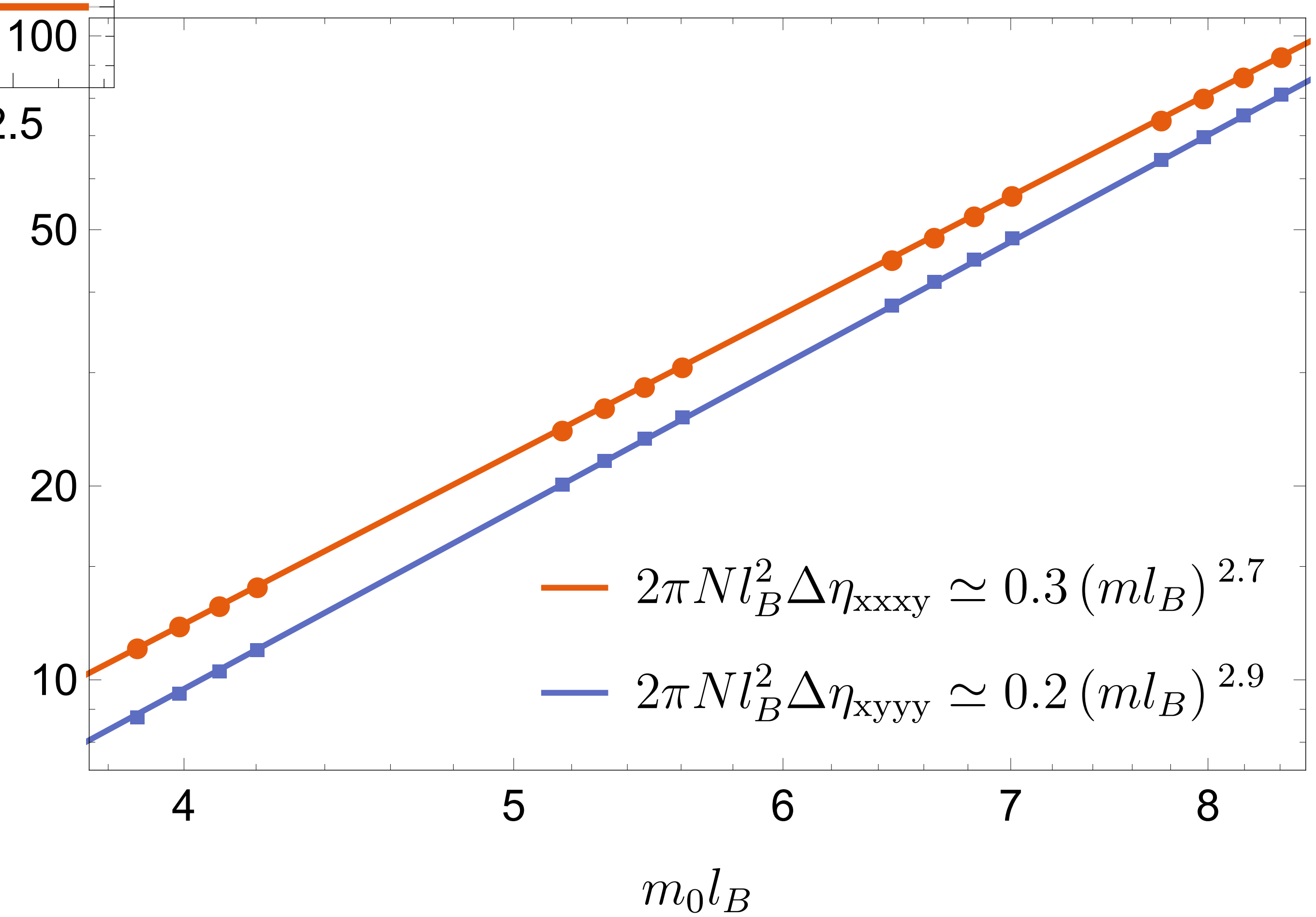
$$\mathcal{A}_{\alpha\beta}^{nm} = \omega_1\langle n, \alpha, d, i|J_1|m, \alpha, d, i\rangle + \omega_2\langle n, \alpha, d, i|J_2|m, \alpha, d, i\rangle + \omega_3\langle n, \alpha, d, i|J_3|m, \alpha, d, i\rangle$$

Landau Levels



Hall viscosities





$$\eta_{xxxy} = \frac{B}{2\pi N}(\tau_2^2 \Omega_{\tau_1\tau_2}) - \tau_2 \Omega_{\tau_1 V}$$

$$\eta_{xyyy} = \frac{B}{2\pi N}(\tau_2^2 \Omega_{\tau_1\tau_2}) + \tau_2 \Omega_{\tau_1 V}$$

$$\eta_{xxyy} = -\tau_2 \Omega_{\tau_2 V}$$

Summary

- Hall viscosity tensor can be computed even on systems with low symmetries
- We have written a general formula to obtain the Berry curvature depending on the isotropic states
- It seems the symmetries of the system are not broken enough because one of the components of the viscosity tensor remains zero

Outlook

- Can holography help to understand systems without rotational invariance?
    - In 3+1d yes! Karl Landsteiner showed yesterday how to do it
    - In 2+1d we suspect the answer is yes…
- How the relation $\eta_H = \bar{s}\rho/2$ gets modified?
    - When rotational invariance is broken by a tensor B. Bradlyn et al found $$\eta^H_{AB} = \frac{\bar{\rho}}{2}\left[s\delta_{AB} + \varsigma v_{AB} + \xi\left(v_{AC}v_{CB} - \delta_{AB}\right)\right]$$
    - When the breaking parameter is a vector we don't know the answer, and effective field theory analysis needs to be done.