Analytic quasi normal modes in non-conformal plasmas

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[arXiv:1708.02252 & 1807.01718 with
P. Betzios, U. Gürsoy, and G. Policastro]

Introduction and motivation

- Idea: explore effects due to nonconformality in quark-gluon plasma via holography
- Several earlier studies using various approaches
  [Janik, Plewa, Soltanpanahi, Spalinski, Buchel, Heller, Myers, Ishii, Kiritsis, Rosen, Attems, Casalderrey-Solana, Mateos, Papadimitriou, Santos-Olivan, Sopuerta, Triana, Zilho, . . .]
- Our approach: A simple way to deviate from conformality – holographic dual of Einstein-Dilaton gravity with exponential potential (+modifications)
  - Use well-known black hole solutions [Chamblin, Reall]
  - Generalization of the AdS_5 solution which still can be controlled analytically

Analytic solutions in a critical limit

What can we do?

- Consider a critical limit of Chamblin-Reall (CR) geometries: dilaton potentials $V(\phi) = e^{\alpha\phi}$ with $\alpha = 4/3 - \epsilon$
- Analytic formulas for
  - all 2-point correlators of $T_{\mu\nu}$ at $q = 0$
  - transverse 2-point correlator of $T_{\perp\perp}$ at all q
- Consequently, information on non-hydrodynamic QNMs

Why is this interesting?

- The critical value $\alpha_c = 4/3$ singled out by improved holographic QCD in the IR
- Higher order Hawking-Page transitions obtained by perturbations around the critical potential [Gürsoy]
- Critical limit linked to the $D \to \infty$ limit of AdS_D through dimensional reduction [Gouteraux,Smolic[2],Skenderis,Taylor]
- At exactly critical value, the solution is the linear dilaton background [Witten; Dijkgraaf,Verlinde[2]; ...]

In terms of rescaled frequency and momentum

$$\varpi = \frac{\omega}{2\pi T}\,, \qquad q = \frac{k}{2\pi T}\,, \qquad \widetilde{S} = \sqrt{\varpi^2 - q^2 - 1}$$

and parametrizing the deviation from $\alpha_c = 4/3$ as

$$\xi = \frac{4 - \alpha^2/\alpha_c^2}{1 - \alpha^2/\alpha_c^2} \qquad\qquad (\xi \to +\infty \ \text{ as } \ \alpha \to \alpha_c \ \text{ from below})$$

we find

$$\langle T_{\perp\perp}(\varpi, q) T_{\perp\perp}(0)\rangle =$$

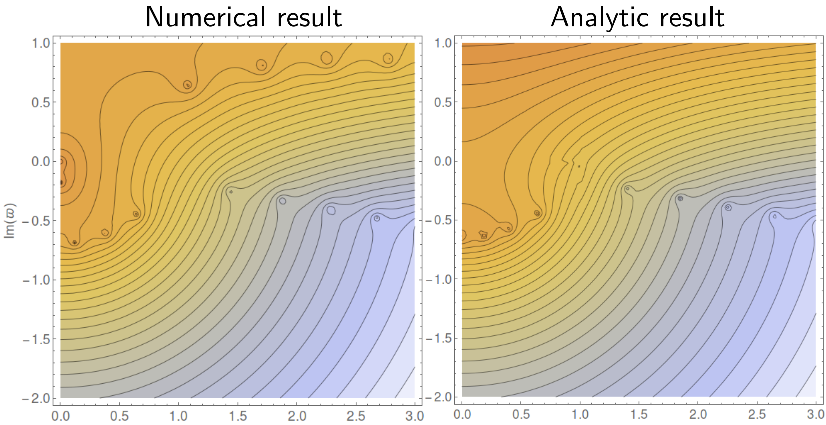
$$\frac{2\pi\,\xi^{\xi}\hat{r}_h^{-\xi}}{\Gamma\left(\frac{\xi}{2}\right)\Gamma\left(1+\frac{\xi}{2}\right)} \left(\frac{\left(\varpi^2 - q^2\right)}{16}\right)^{\frac{\xi}{2}} \left[i + \left(\frac{1+i\widetilde{S}}{1-i\widetilde{S}}\right)^{\frac{\xi}{2}} \frac{e^{-i\xi\widetilde{S}}}{\mathcal{R}}\right]^{-1} + \cdots$$

where the reflection amplitude is

$$\mathcal{R}(\varpi, q) = -\frac{\Gamma\left(1+i\widetilde{S}\right)\,\Gamma\left(\frac{1}{2}\left(1-i\varpi-i\widetilde{S}\right)\right)^2}{\Gamma\left(1-i\widetilde{S}\right)\,\Gamma\left(\frac{1}{2}\left(1-i\varpi+i\widetilde{S}\right)\right)^2}$$

Comparison to numerics

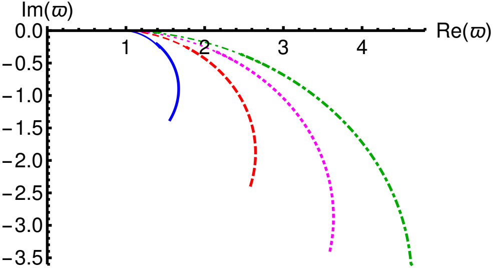
Check of correlators (log of absolute value) at $\xi \simeq 17$, $q = 0$



- Result only depends on T through $\varpi = \omega/2\pi T$
- Difference $\sim 1/\xi$ for generic ϖ

Quasi Normal Modes

Evolution of nonhydro modes from $\xi = 4$ (conformal) to $\xi = \infty$



- Nonhydro QNMs accumulate on the real axis, $\varpi > \sqrt{1 + q^2}$
- A branch cut on the real axis?
- Nonhydro modes dominate time dependence as $\xi \to \infty$ (for nonzero momentum q)
  - Early breakdown of hydro?
  - Infinitely many modes but gapped, $|\varpi| > 1$

Extensions and implications

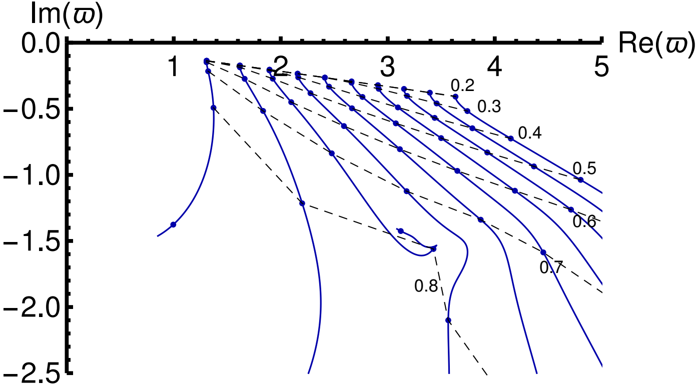
1. Our results describe the QNMs in the $T \to 0$ limit for any dilaton potential with the IR asymptotics

$$V(\phi) \sim e^{(4/3-\epsilon)\phi} \qquad \phi \to \infty$$

2. In particular, the method applies for RG flows from AdS_5 in the UV to near critical Chamblin-Reall geometry in the IR
3. Mimic such a flow by gluing AdS_5 to CR geometry directly
   - Analytic control remains even at finite T!
   - Nontrivial T dependence of QNMs
   - Critical limit $\xi \to \infty$ now regular

Results after gluing AdS+CR

Nonhydro QNM evolution with T extracted from the analytic correlator for $\xi \simeq 27$ and $q = 0$



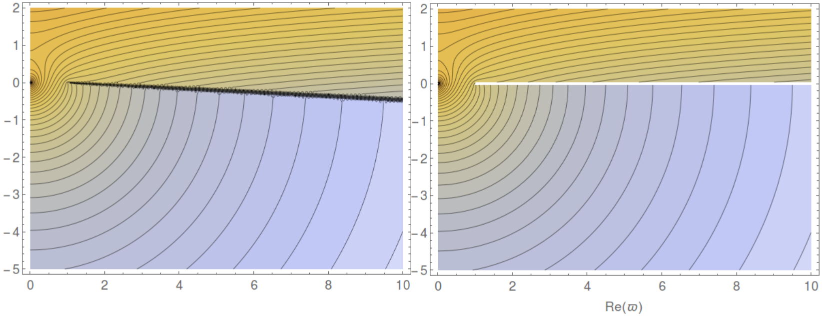
- As T decreases, QNMs move closer to real line (units: T of linear dilaton bg)
- Evolution stops at the locations determined by the CR geometry as $T \to 0$

Critical limit – branch cut

Limit of small black holes at $\xi = \infty$:

$\langle T_{\perp\perp}(\varpi, q) T_{\perp\perp}(0) \rangle$ at

$r_h/r_c = 20$	$r_h \to \infty$
Discrete modes	Branch cut

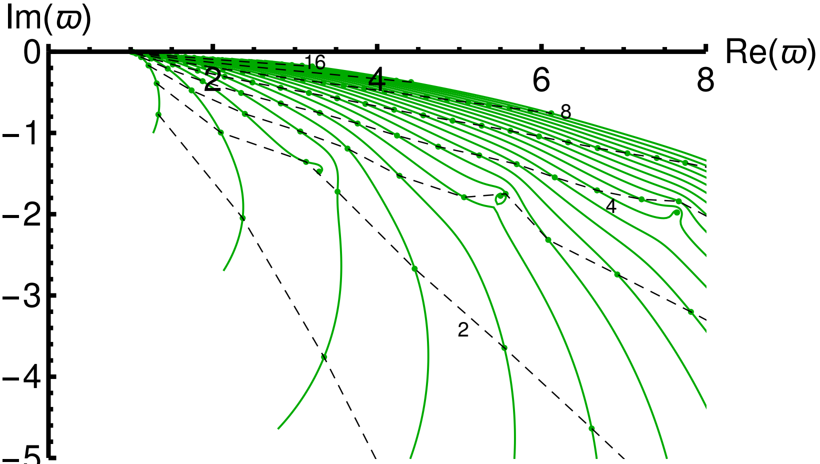


Conclusions & Discussion

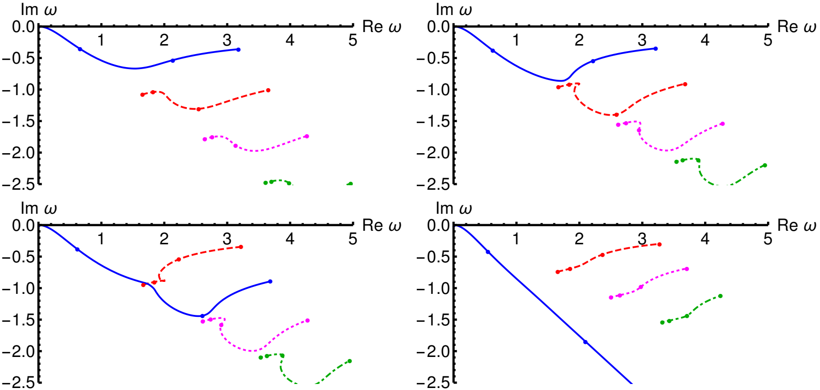
We studied the QNMs of a non-conformal plasma analytically

- "Large" deviation from CFT near a critical point
- Our results should be contrasted with other studies where broken scale dependence has milder effects on the QNMs
  [e.g. Janik et al; Mateos et al,...]
- Infinitely many gapped long lived modes in the critical limit, forming a branch cut – relations to/applications in
  - Weak coupling physics, kinetic theory?
  - Continuous phase transition with divergent correlation length? Relevant in quark-gluon plasma?
    [Gürsoy]
- How does the (gapped) branch cut affect hydrodynamics?
- Extension to charged backgrounds in progress

Extra slides



Im(ϖ)
Re(ϖ)
0
2
4
6
8
−1
−2
−3
−4
−5
16
8
4
2


Im ω
Re ω
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
1
2
3
4
5