Analytic quasi normal modes in non-conformal plasmas

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Introduction and motivation

- Idea: explore effects due to nonconformality in quark-gluon plasma via holography

- Several earlier studies using various approaches
  [Janik, Plewa, Soltanpanahi, Spalinski, Buchel, Heller, Myers, Ishii, Kiritsis, Rosen, Attems, Casalderrey-Solana, Mateos, Papadimitriou, Santos-Olivan, Sopuerta, Triana, Zilho, ...]

- Our approach: A simple way to deviate from conformality – holographic dual of Einstein-Dilaton gravity with exponential potential (+modifications)
  - Use well-known black hole solutions [Chamblin, Reall]
  - Generalization of the AdS₅ solution which still can be controlled analytically
Analytic solutions in a critical limit

What can we do?

- Consider a critical limit of Chamblin-Reall (CR) geometries: dilaton potentials $V(\phi) = e^{\alpha \phi}$ with $\alpha = 4/3 - \epsilon$
- **Analytic formulas for**
  - all 2-point correlators of $T_{\mu\nu}$ at $q = 0$
  - transverse 2-point correlator of $T_{\perp\perp}$ at all $q$
- Consequently, information on **non-hydrodynamic QNMs**

Why is this interesting?

- The critical value $\alpha_c = 4/3$ singled out by improved holographic QCD in the IR
- Higher order Hawking-Page transitions obtained by perturbations around the critical potential
- Critical limit linked to the $D \to \infty$ limit of AdS$_D$ through dimensional reduction
- At exactly critical value, the solution is the linear dilaton background

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[\text{Gürsoy}]
[\text{Gouteraux}, \text{Smolic}^2, \text{Skenderis}, \text{Taylor}]
[\text{Witten}; \text{Dijkgraaf}, \text{Verlinde}^2; \ldots]
In terms of rescaled frequency and momentum

\[ \varpi = \frac{\omega}{2\pi T}, \quad q = \frac{k}{2\pi T}, \quad \tilde{S} = \sqrt{\varpi^2 - q^2 - 1} \]

and parametrizing the deviation from \( \alpha_c = 4/3 \) as

\[ \xi = \frac{4 - \alpha^2/\alpha_c^2}{1 - \alpha^2/\alpha_c^2} \quad (\xi \to +\infty \text{ as } \alpha \to \alpha_c \text{ from below}) \]

we find

\[ \langle T_{\perp\perp}(\varpi, q) T_{\perp\perp}(0) \rangle = \]

\[ \frac{2\pi \xi \xi \hat{r}_h^{-\xi}}{\Gamma \left( \frac{\xi}{2} \right) \Gamma \left( 1 + \frac{\xi}{2} \right)} \left( \frac{(\varpi^2 - q^2)}{16} \right)^{\xi/2} \left[ i + \left( \frac{1 + i\tilde{S}}{1 - i\tilde{S}} \right)^{\xi/2} \frac{e^{-i\xi\tilde{S}}}{\mathcal{R}} \right]^{-1} + \cdots \]

where the reflection amplitude is

\[ \mathcal{R}(\varpi, q) = -\frac{\Gamma \left( 1 + i\tilde{S} \right) \Gamma \left( \frac{1}{2} \left( 1 - i\varpi - i\tilde{S} \right) \right)^2}{\Gamma \left( 1 - i\tilde{S} \right) \Gamma \left( \frac{1}{2} \left( 1 - i\varpi + i\tilde{S} \right) \right)^2} \]
Comparison to numerics

Check of correlators (log of absolute value) at $\xi \simeq 17, \ q = 0$

- Numerical result
- Analytic result

- Result only depends on $T$ through $\varpi = \omega / 2\pi T$
- Difference $\sim 1/\xi$ for generic $\varpi$
Quasi Normal Modes

Evolution of nonhydro modes from $\xi = 4$ (conformal) to $\xi = \infty$

- Nonhydro QNMs accumulate on the real axis, $\omega > \sqrt{1 + q^2}$
- A branch cut on the real axis?
- Nonhydro modes dominate time dependence as $\xi \to \infty$
  (for nonzero momentum $q$)
  - Early breakdown of hydro?
  - Infinitely many modes but gapped, $|\omega| > 1$
Extensions and implications

1. Our results describe the QNMs in the $T \to 0$ limit for any dilaton potential with the IR asymptotics

$$V(\phi) \sim e^{(4/3 - \epsilon)\phi} \quad \phi \to \infty$$

2. In particular, the method applies for RG flows from AdS$_5$ in the UV to near critical Chamblin-Reall geometry in the IR

3. Mimic such a flow by gluing AdS$_5$ to CR geometry directly
   - Analytic control remains even at finite $T$!
   - Nontrivial $T$ dependence of QNMs
   - Critical limit $\xi \to \infty$ now regular
Results after gluing AdS+CR

Nonhydro QNM evolution with $T$ extracted from the analytic correlator for $\xi \simeq 27$ and $q = 0$

As $T$ decreases, QNMs move closer to real line (units: $T$ of linear dilaton bg)

Evolution stops at the locations determined by the CR geometry as $T \to 0$
Critical limit – branch cut

Limit of small black holes at $\xi = \infty$:

$$\langle T_{\perp\perp}(\varpi, q) T_{\perp\perp}(0) \rangle$$

at

$$\frac{r_h}{r_c} = 20$$

Discrete modes

$$r_h \rightarrow \infty$$

Branch cut
We studied the QNMs of a non-conformal plasma analytically

- “Large” deviation from CFT near a critical point
- Our results should be contrasted with other studies where broken scale dependence has milder effects on the QNMs [e.g. Janik et al; Mateos et al, . . . ]
- Infinitely many gapped long lived modes in the critical limit, forming a branch cut – relations to/applications in
  - Weak coupling physics, kinetic theory?
  - Continuous phase transition with divergent correlation length? Relevant in quark-gluon plasma? [Gürsoy]
- How does the (gapped) branch cut affect hydrodynamics?
- Extension to charged backgrounds in progress
Extra slides