

A CARDY FORMULA FOR OFF-DIAGONAL 3-PT  
COEFFICIENTS;

*or,*

HOW THE GEOMETRY BEHIND THE HORIZON GETS  
DISENTANGLLED

**AURELIO ROMERO BERMÚDEZ**

with Philippe Sabella Garnier & Koenraad Schalm

arXiv:[1804.08899](https://arxiv.org/abs/1804.08899)

*Gauge/Gravity Duality 2018, Würzburg*



Universiteit Leiden



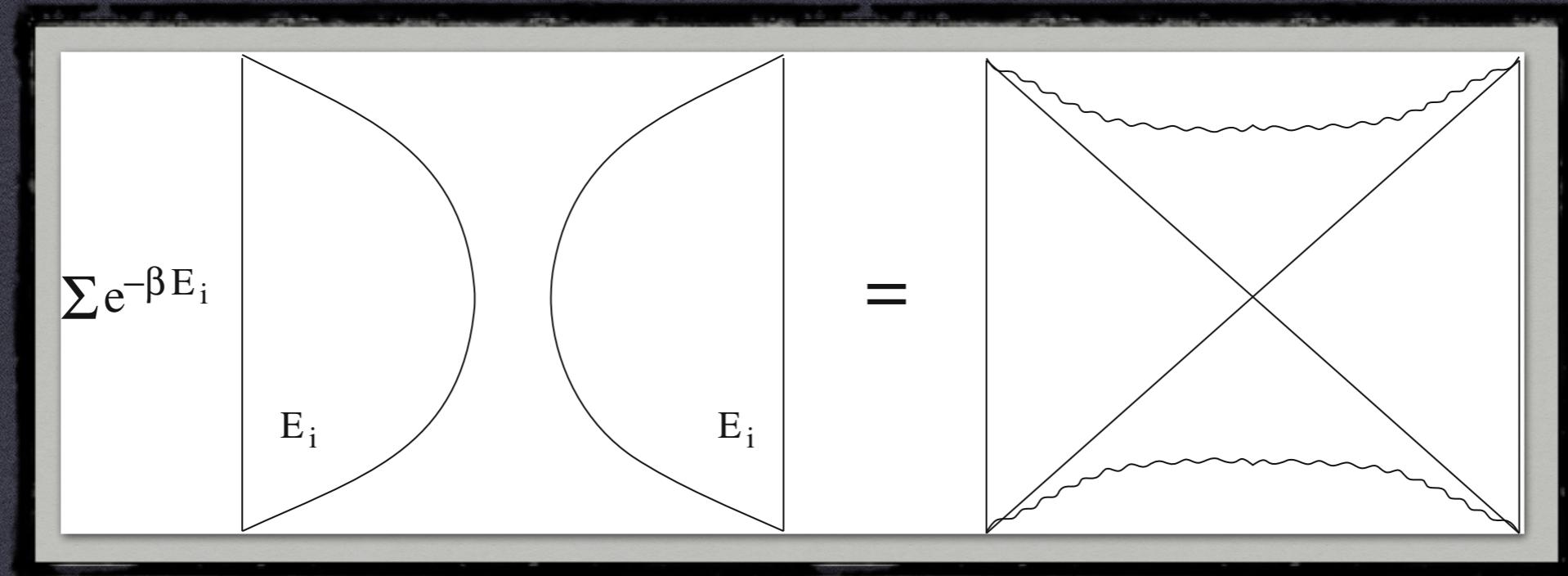
# INTRO I: TFD AND GEOMETRY

$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$$

$$|\Psi(\beta)\rangle = \sum_i e^{-\frac{\beta E_i}{2}} |E_i\rangle \otimes |E_i\rangle$$

Quant. superpos. of  
disconnected  
spacetimes

Eternal  
BH



$$\text{Tr}_2(|\Psi(\beta)\rangle\langle\Psi(\beta)|) = \rho_T(\beta)$$

Many authors to be cited

One-sided observer sees  
Schwarzschild BH

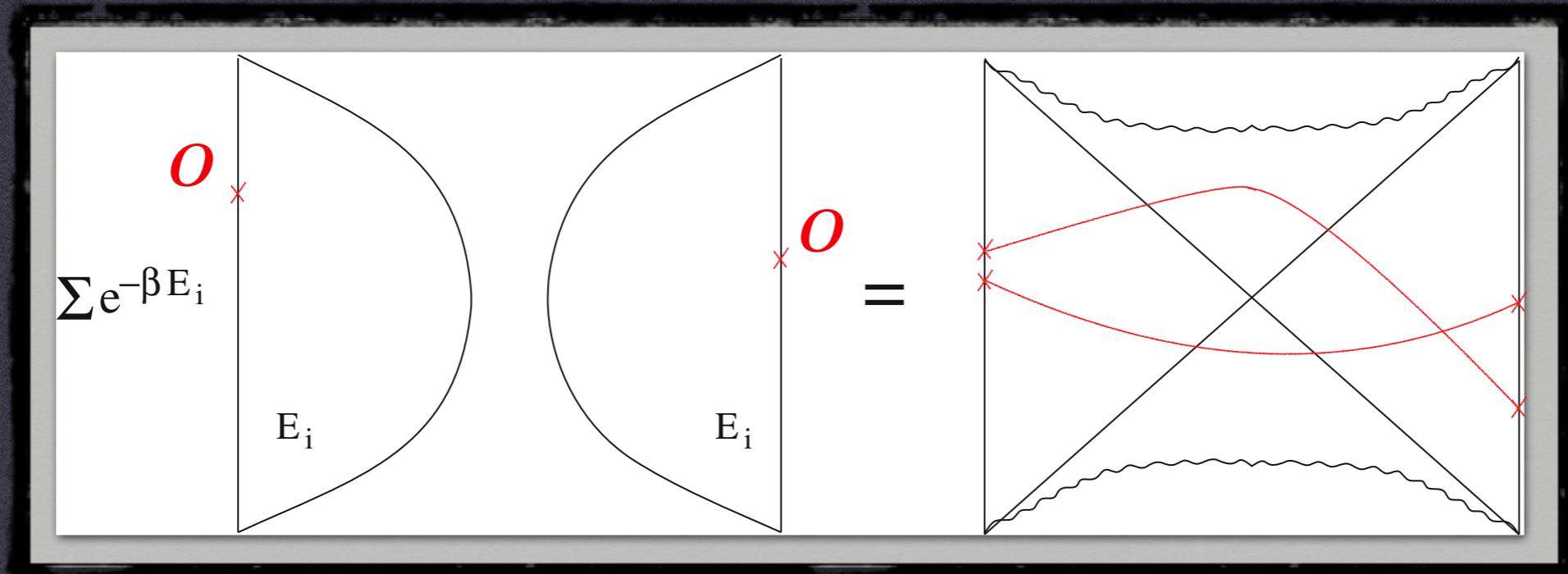
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Festuccia, Liu '09

$$G^L(t_{12}, \phi_{12}) = \frac{1}{Z(\beta)} \sum_{ab} e^{-\beta \frac{E_a + E_b}{2}} e^{-it_{12} E_{ab}} |\langle a | O | b \rangle|^2$$

# INTRO II: CONF. BLOCKS & THEIR DUAL INTERPRETATION

◆ Conformal blocks in CFTs  $\longleftrightarrow$

Exchange of primaries & gravitons

Witten diagrams

Fitzpatrick, Kaplan, Walters '14

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- ◆ Average (coarse-graining) of OPEs:

$$\overline{\langle E|O|E\rangle} \equiv \frac{\sum_i \langle i|O|i\rangle \delta(E - E_i)}{\rho(E)}$$

$$\overline{\langle E|O|E\rangle} \sim C_{O\chi\chi} \left( E - \frac{c}{12} \right)^{E_O/2} \exp \left[ -\frac{\pi c}{3} \left( 1 - \sqrt{1 - \frac{12E_\chi}{c}} \right) \sqrt{\frac{12E}{c} - 1} \right]$$

in the limit of  $\beta \rightarrow 0$ ,  $E/E_O \ll 1$ ,  $E \gg c$

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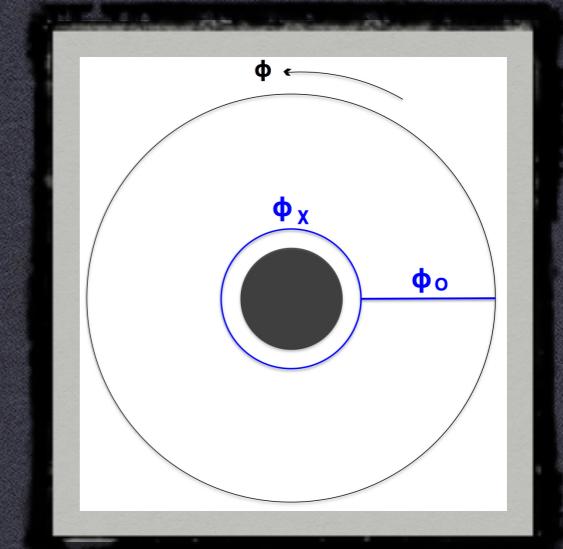
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## INTRO III: ETH

$$|\psi(0)\rangle = \sum_a c_a |\Psi_a\rangle \quad |\psi(t)\rangle = \sum_a c_a e^{-iE_a t} |\Psi_a\rangle$$

$$\langle O(t) \rangle = \langle \psi(t) | O | \psi(t) \rangle$$

$$\overline{\langle O(t) \rangle} \equiv \lim_{t \rightarrow \infty} \frac{1}{t} \int_0^t dt' \langle O(t') \rangle = \sum_a |c_a|^2 O_{aa}$$

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ETH  

Initial energy  $E_0 = \langle \psi(0) | H | \psi(0) \rangle$  

Details of IC 

Rigol; Srednicki; ...  
late 2000s, 2010s

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ETH

$$= \frac{1}{N_{E_0, \Delta E}} \sum_a_{|E_a - E_0| < \Delta E} O_{aa}$$

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$O_{aa}$  do not fluctuate much between eigenstates close in energy:

$$O_{aa} - O_{a+1,a+1} \sim e^{-N} \text{ for an N-body Hamiltonian}$$

$$O_{ab} \sim e^{-N} \text{ for } a \neq b$$

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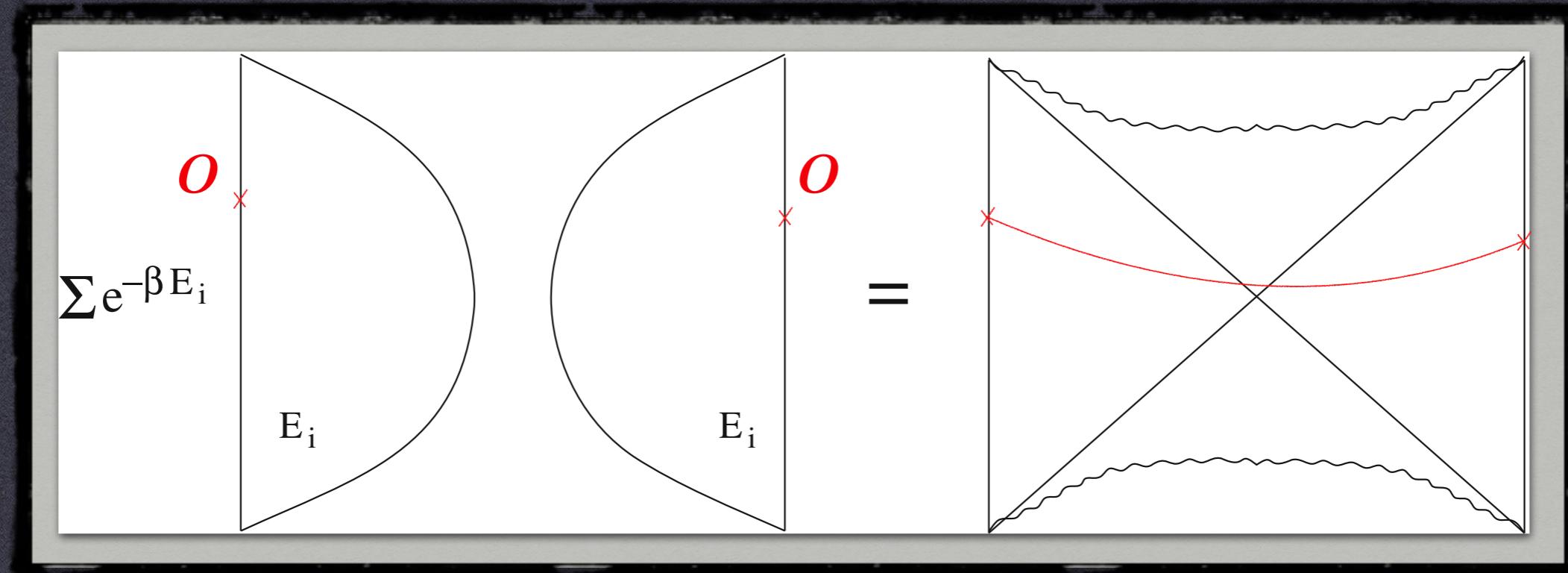
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Non-Integrability is  
important for ETH

# Goal for the day

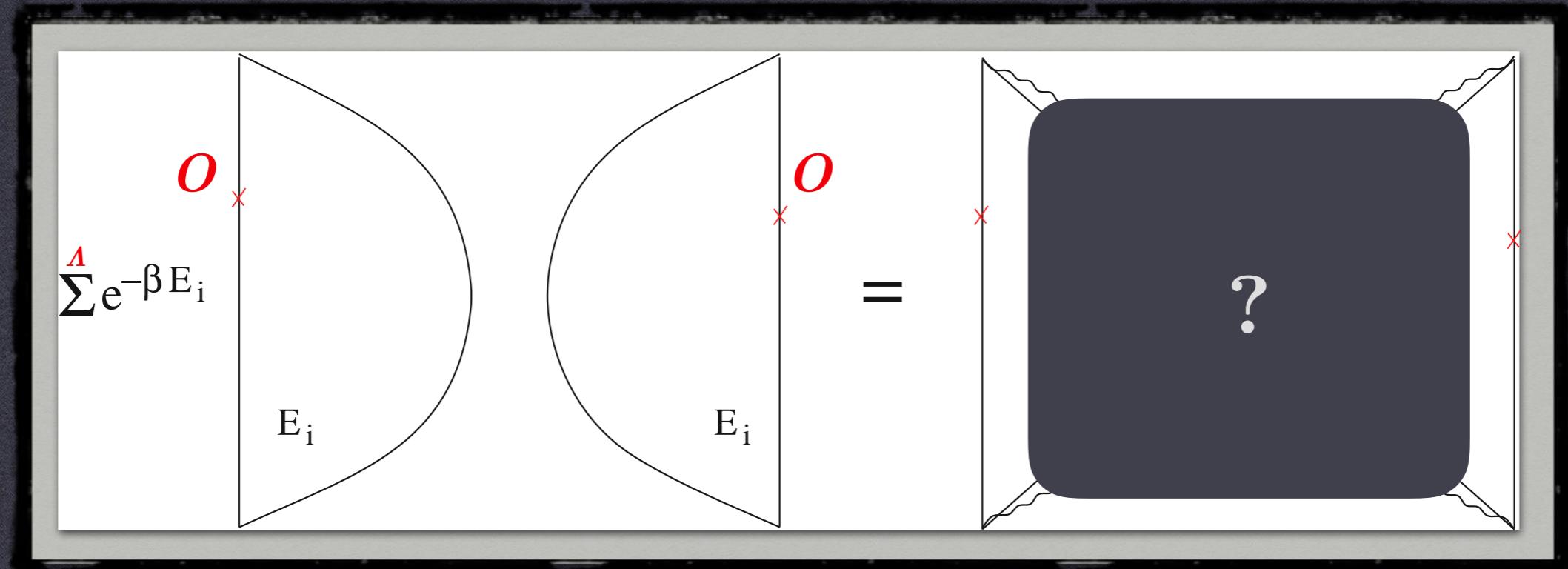
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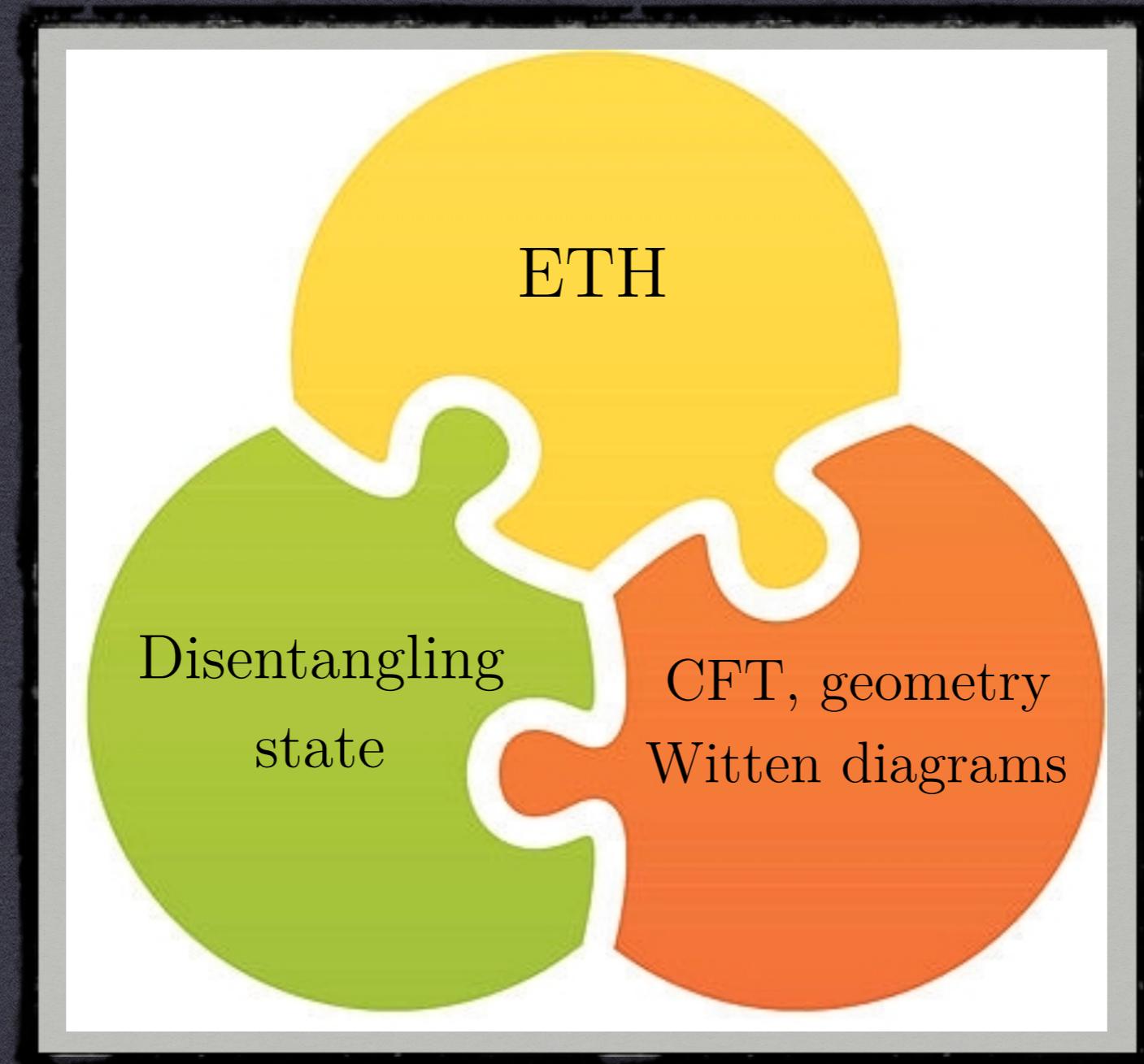
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$E_a, E_b < \Lambda$



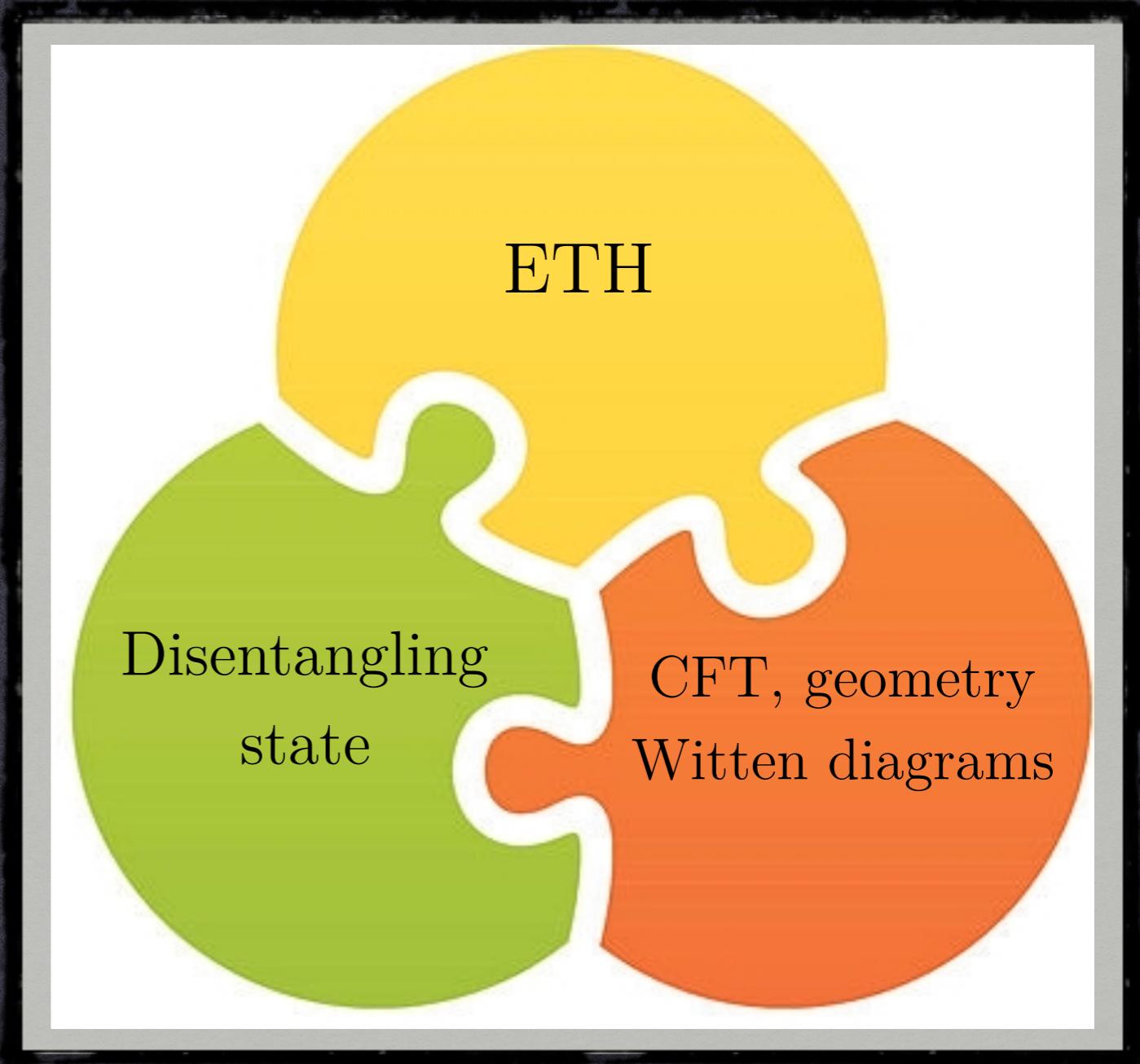
*‘Disentangling experiment’*

# PUZZLE & OUTLINE



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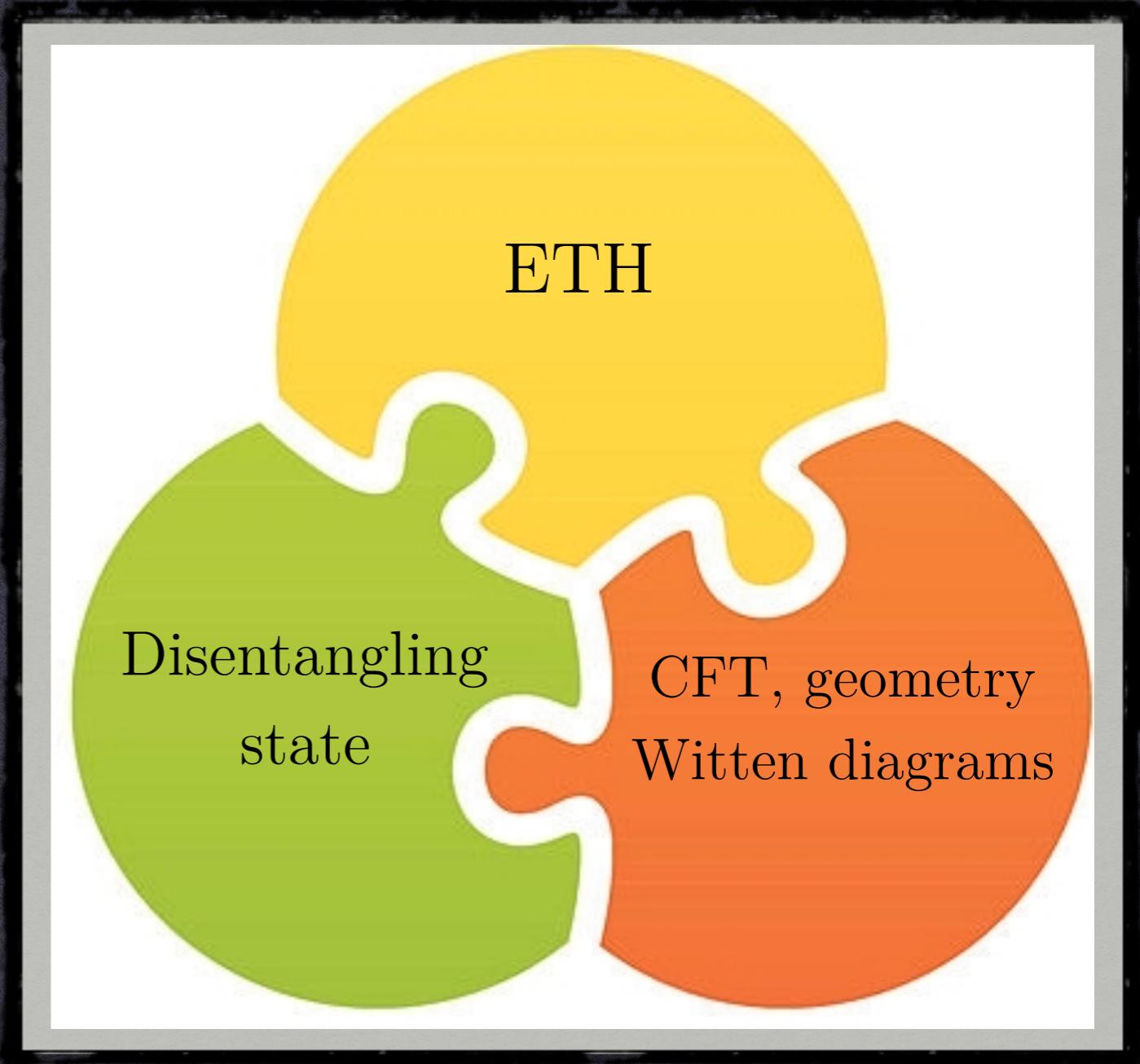
$CFT_2$  on the circle (line)

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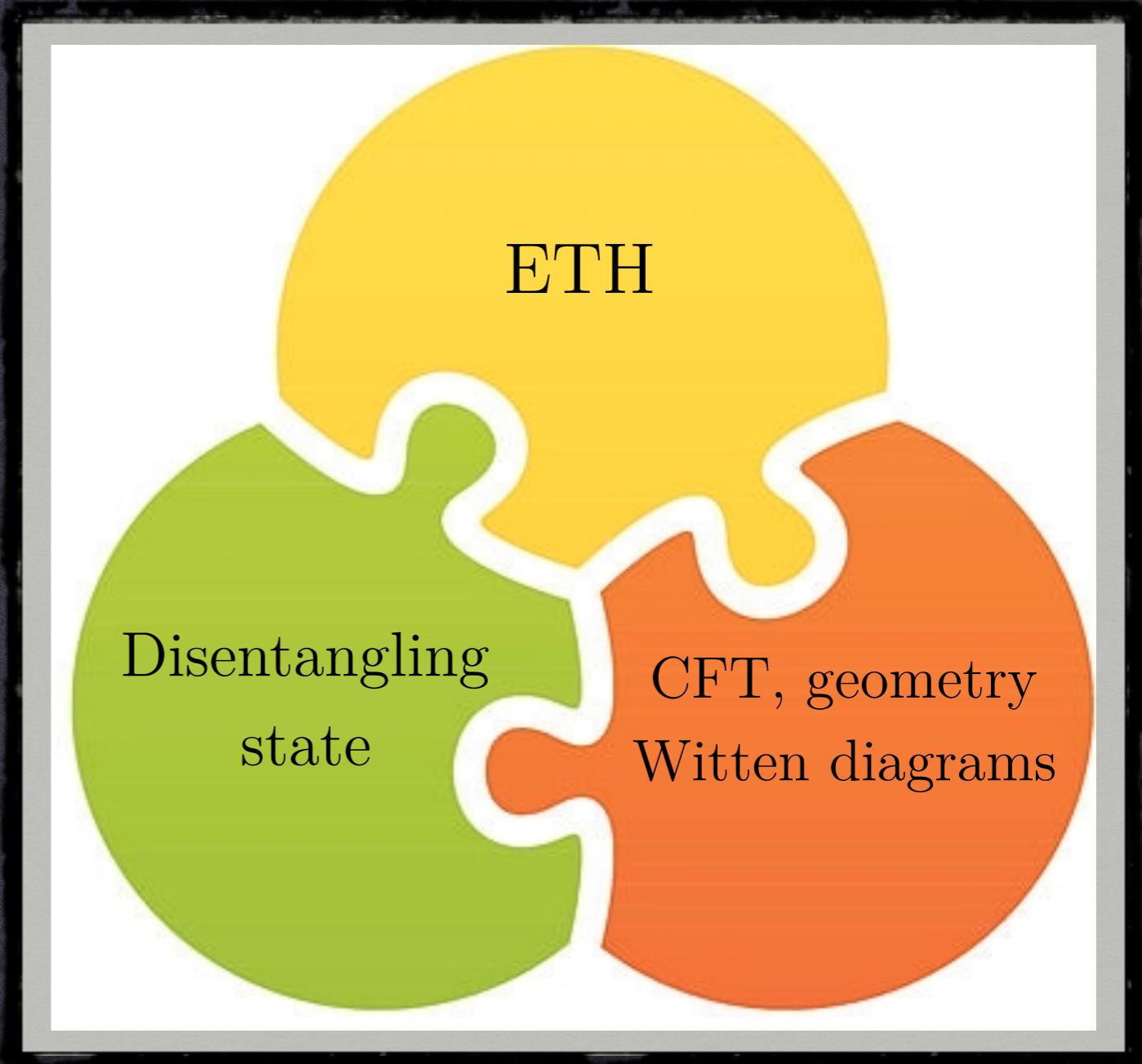
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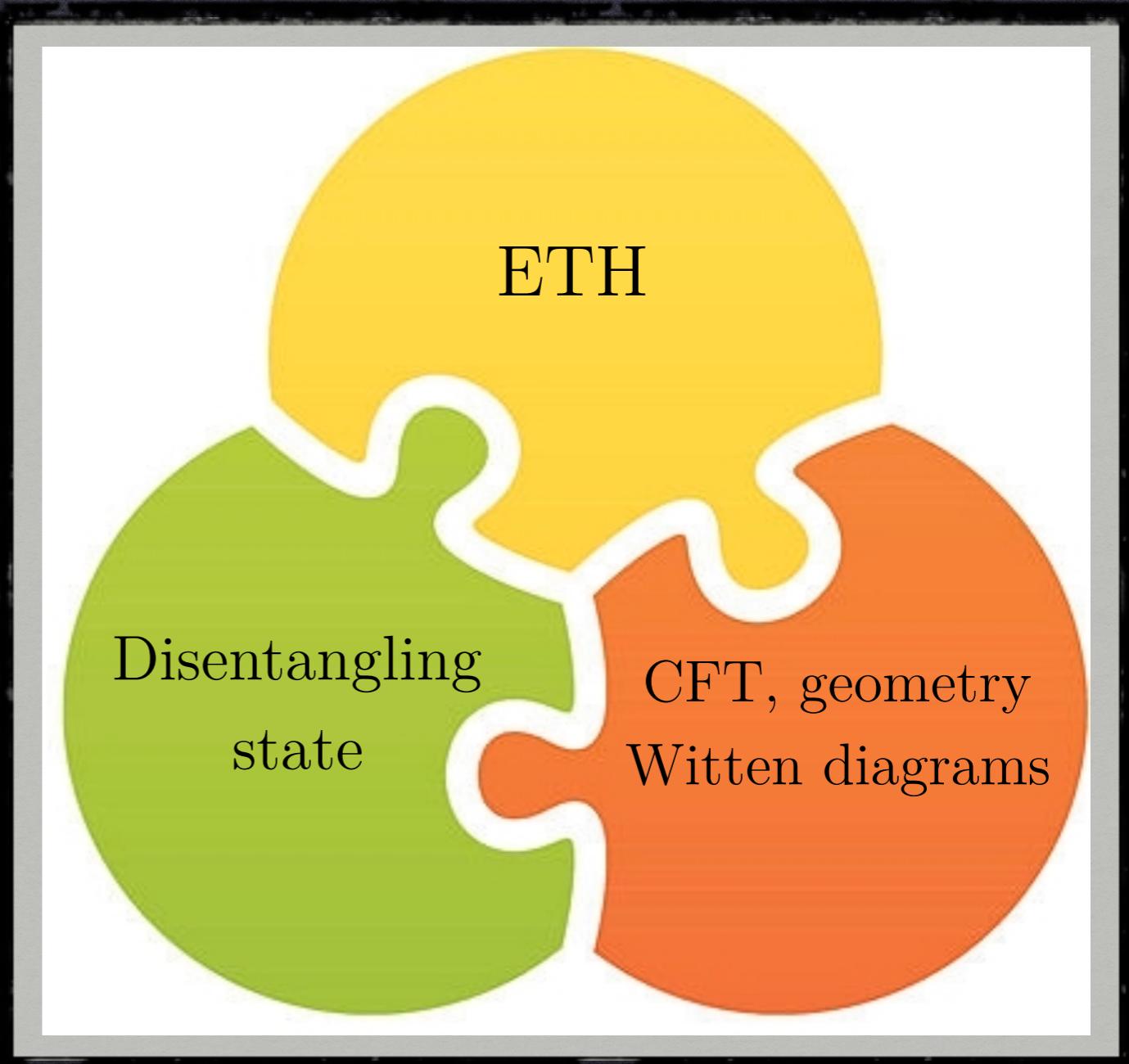
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4- Plug back into:

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$E_a, E_b < \Lambda$



$CFT_2$  on the circle (line)

# 1- Starting point and sketch of derivation

Known from CFT:

$$Z(\beta) = e^{\frac{\pi^2 c}{3\beta}} + O(c^0) \quad G^L(t_{12}, \phi_{12}) = \sum_{n \in \mathbb{Z}} \frac{(2\pi T)^{2\Delta}}{[\cosh(2\pi T(\phi_{12} + 2\pi n)) + \cosh(2\pi T t_{12}) + i\epsilon]^\Delta}$$

$$G^L(t_{12}, \phi_{12} = 0) = \frac{1}{Z(\beta)} \sum_{ab} e^{-\beta \frac{E_a + E_b}{2}} e^{-it_{12} E_{ab}} |\langle a | O | b \rangle|^2$$

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$$= \frac{1}{Z(\beta)} \sum_{E_a, E_b} e^{-\beta \frac{E_a + E_b}{2} - iE_{ab}t_{12}} \sum_{\substack{a | E(a) = E_a \\ b | E(b) = E_b}} |\langle a | \mathcal{O} | b \rangle|^2$$

$$= \frac{1}{Z(\beta)} \sum_{E_a, E_b} e^{-\frac{E_a + E_b}{2} - it_{12} E_{ab}} e^{S(E_a)} e^{S(E_b)} \mathcal{F}(E_a, E_b, \Delta)$$

$$\overline{|\langle a | O | b \rangle|^2}$$

## 2- Derivation: key assumption

$$G^L(t_{12}) = \frac{1}{Z(\beta)} \sum_{E_a, E_b} e^{-\frac{E_a + E_b}{2} - it_{12}E_{ab}} e^{S(E_a)} e^{S(E_b)} \mathcal{F}(E_a, E_b, \Delta)$$

$$\sum_{E_a, E_b} e^{S(E_a)} e^{S(E_b)} \rightarrow \int dE_a dE_b \rho(E_a) \rho(E_b)$$

$\overline{\langle a | O | b \rangle^2}$  smooth function of  
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$$E = \frac{E_a + E_b}{2}, \quad \chi = E_a - E_b$$

$$G^L(t_{12}) = \frac{1}{Z(\beta)} \int_{-\infty}^{\infty} d\chi \int_{-c/12}^{\infty} dE \ e^{i\chi t_{12}} e^{-\beta E} \rho\left(E + \frac{\chi}{2}\right) \rho\left(E - \frac{\chi}{2}\right) \mathcal{F}(E, \chi, \Delta)$$

### 3- Derivation: inversion

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(3.1) Inversion of Fourier Transform

$$\int\limits_{-c/12}^{\infty} dE \ e^{-\beta E} \rho\left(E + \frac{\chi}{2}\right) \rho\left(E - \frac{\chi}{2}\right) \mathcal{F}(E, \chi, \Delta) = Z(\beta) \left(\frac{2\pi}{\beta}\right)^{2\Delta} \sum_{n \in \mathbb{Z}} F_n(\Delta, \chi, \beta)$$

(3.2) Inversion of Laplace Transform: Mellin inversion formula

# Result: $\overline{\langle a | O | b \rangle^2}$ and Wightman

◆  $\overline{\langle a | O | b \rangle^2} \equiv \mathcal{F}(E, \chi, \Delta)$

$$\mathcal{F}(E, \chi, \Delta) \rho\left(E + \frac{\chi}{2}\right) \rho\left(E - \frac{\chi}{2}\right) = \sum_{n \in \mathbb{N}} \int_0^E dx \, \rho(x, \chi, \Delta) \, \hat{F}_n(E - x, \chi, \Delta)$$

known

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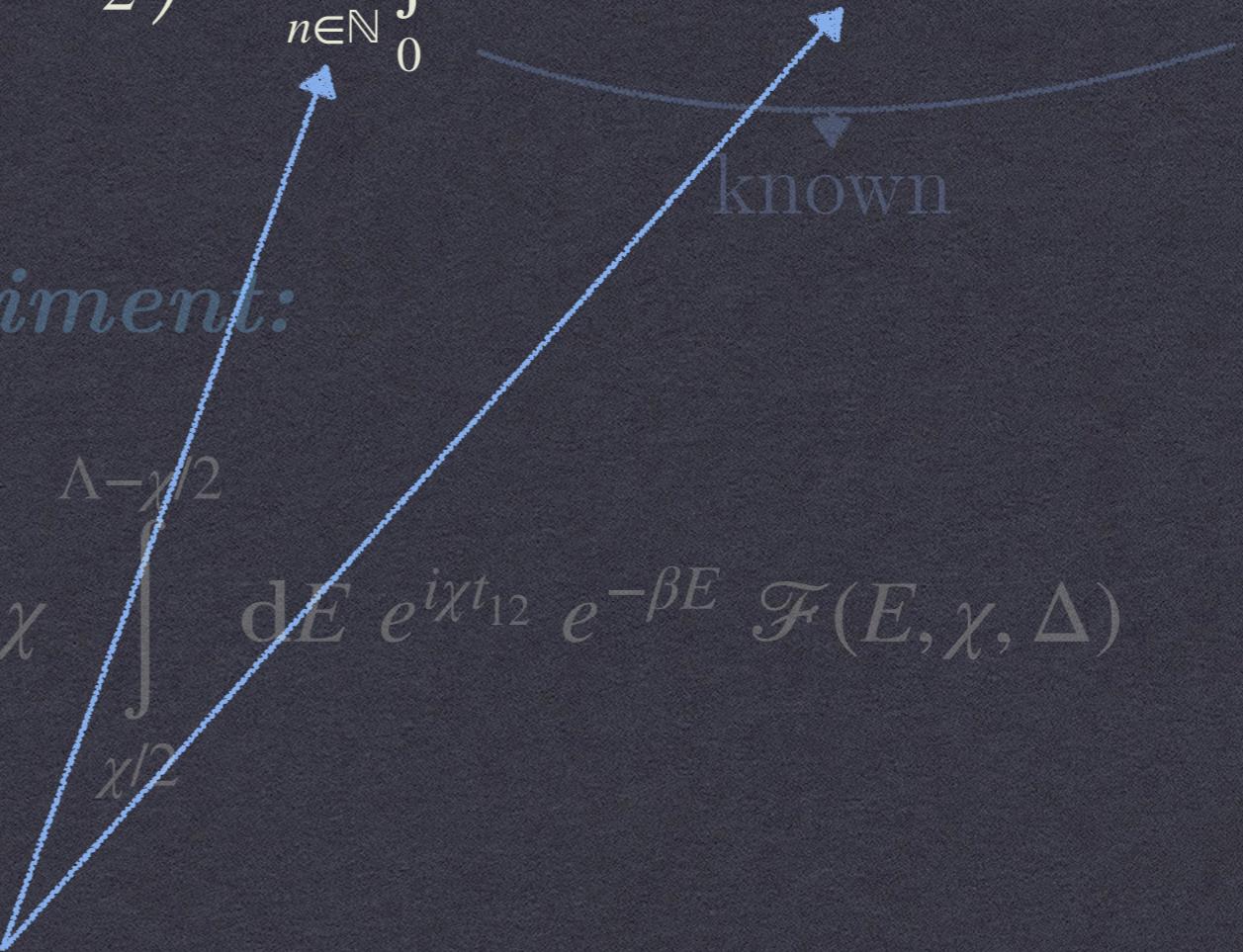
◆ *Disentangling experiment:*

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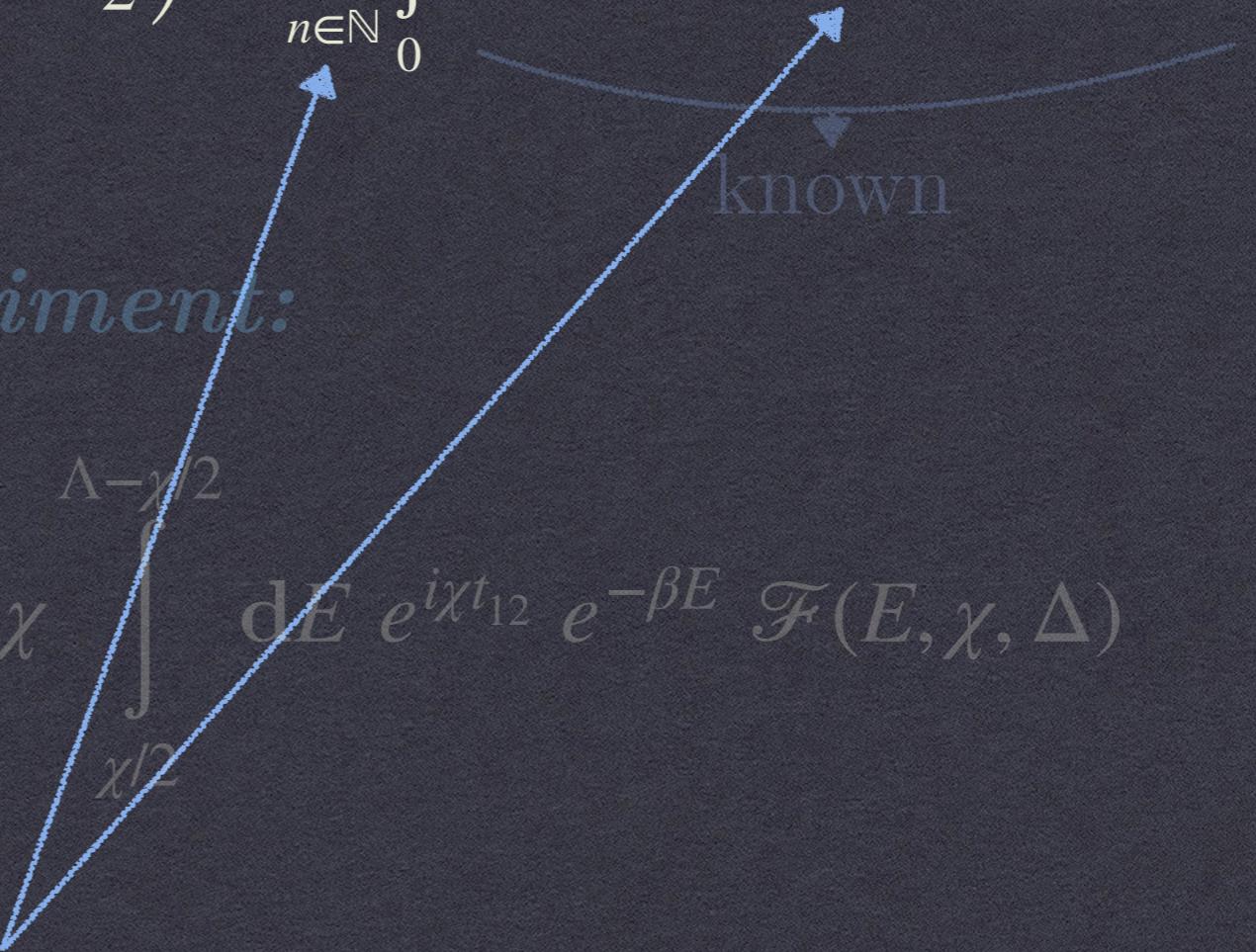
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- Two sums we cannot do

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*Circle  $\rightarrow$  Line*

# $\overline{\langle a | O | b \rangle^2}$ on the Line

- ◆ Drop the sum over images,  $n=0$
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We can perform the sum corresponding to simple poles in

$$\overbrace{\rho\left(E + \frac{\chi}{2}\right)\rho\left(E - \frac{\chi}{2}\right)\mathcal{F}(E, \chi, \Delta)}^{\text{We can perform the sum corresponding to simple poles in}} = \frac{1}{2\pi} \int_0^E \rho(E') \hat{F}_0(E' - E, \chi, \Delta) dE'$$

$$\rho(E, \Delta) = \sqrt{\frac{\pi c}{6}} \frac{L}{E} I_1\left(2\sqrt{\frac{\pi c}{6} L E}\right)$$

$$\hat{F}_0(E, \chi, \Delta) \propto \chi^{2\Delta-2} \left\{ e^{-i2\pi\Delta\frac{E}{\chi}} {}_{2\Delta}F_{2\Delta-1} \left[ \begin{array}{c} a_1, \dots, a_{2\Delta} \\ b_1, \dots, b_{2\Delta-1} \end{array} \middle| -e^{i2\pi\frac{E}{\chi}} \right] + cc \right\}$$

# $\overline{\langle a | O | b \rangle^2}$ on the Line

Some analytical results:

- ◆ Saddle point approx.  $ELc \gg 1$

$$\mathcal{F}(E, \chi, \Delta) = \frac{C_{\mathcal{O}}}{2\pi} \frac{1}{\rho\left(E + \frac{\chi}{2}\right) \rho\left(E - \frac{\chi}{2}\right)} \frac{1}{2\pi i} \oint d\beta e^{\beta E} Z(\beta) \left( \frac{2\pi}{\beta} \right)^{2\Delta} F_0(\beta, \chi, \Delta)$$

↓  
peaked around  $\beta_0 = \sqrt{\frac{\pi c L}{6E}}$

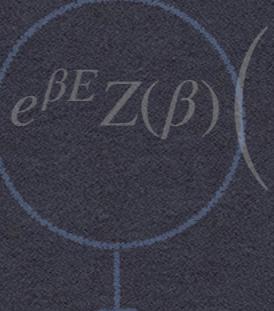
$$\mathcal{F}(E, \chi, \Delta) \simeq \frac{2^{3\Delta-2}}{\pi \Gamma(2\Delta)} \frac{\rho(E)}{\rho(E + \chi/2)\rho(E - \chi/2)} \left( \frac{6E\pi}{cL} \right)^{\Delta-1/2} \Gamma\left(\Delta - \frac{i\chi}{2\pi} \sqrt{\frac{\pi c L}{6E}}\right) \Gamma\left(\Delta + \frac{i\chi}{2\pi} \sqrt{\frac{\pi c L}{6E}}\right)$$

c.f. Brehm, Das, Datta, '18

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- Expand  $F_0$  in small  $\chi$ :

$$\begin{aligned} \mathcal{F}(E, \chi, \Delta) \simeq & \frac{C_{\mathcal{O}} (2\pi)^{2\Delta-2}}{\rho\left(E + \frac{\chi}{2}\right) \rho\left(E - \frac{\chi}{2}\right)} \frac{2^\Delta \Gamma(\Delta)^2}{\Gamma(2\Delta)} \left(\frac{6E}{\pi c L}\right)^{\Delta-1} \left[ I_{2\Delta-2} \left( 2\sqrt{\frac{EL\pi c}{6}} \right) \right. \\ & \left. - \chi^2 \left(\frac{6E}{\pi c L}\right)^{-1} \frac{\psi^{(1)}(\Delta)}{4\pi^2} I_{2\Delta-4} \left( 2\sqrt{\frac{EL\pi c}{6}} \right) + O(\chi^4 L^2/E^2) \right] \end{aligned}$$

# On the Line: a view to the spectrum

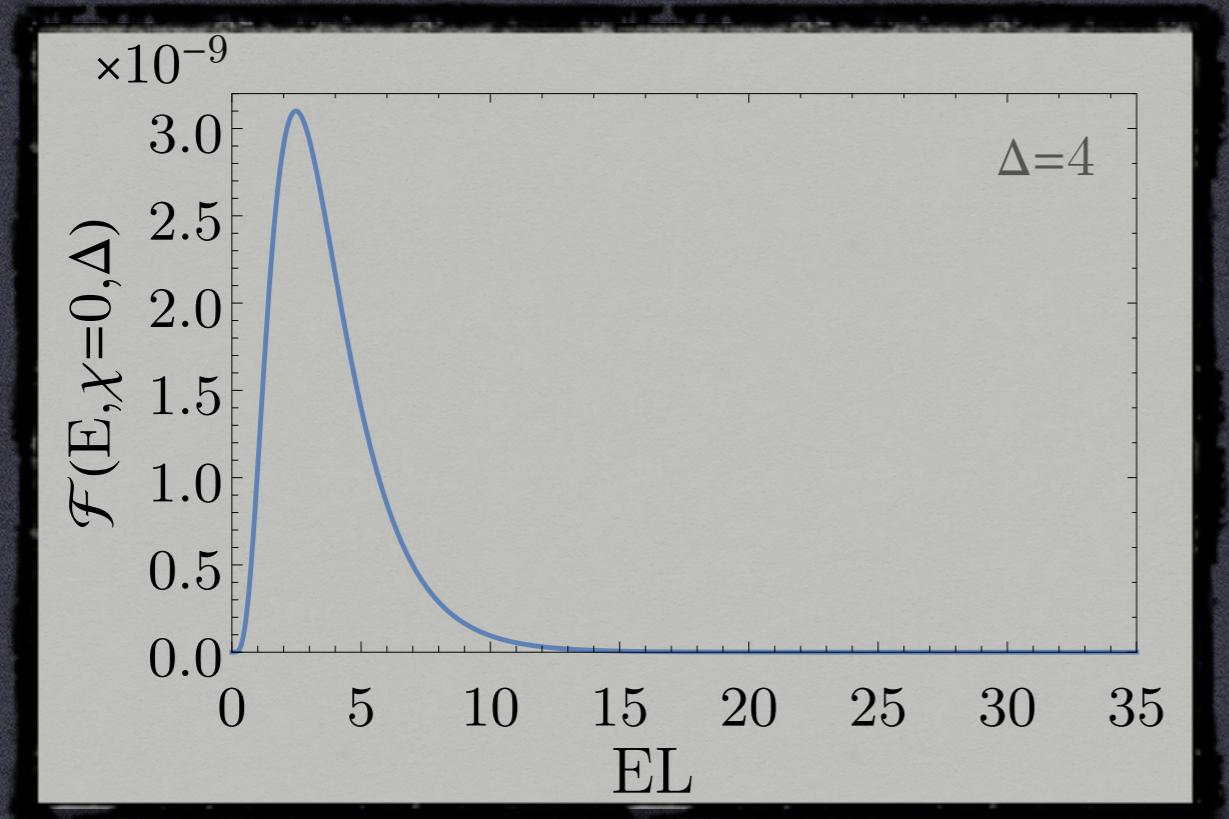
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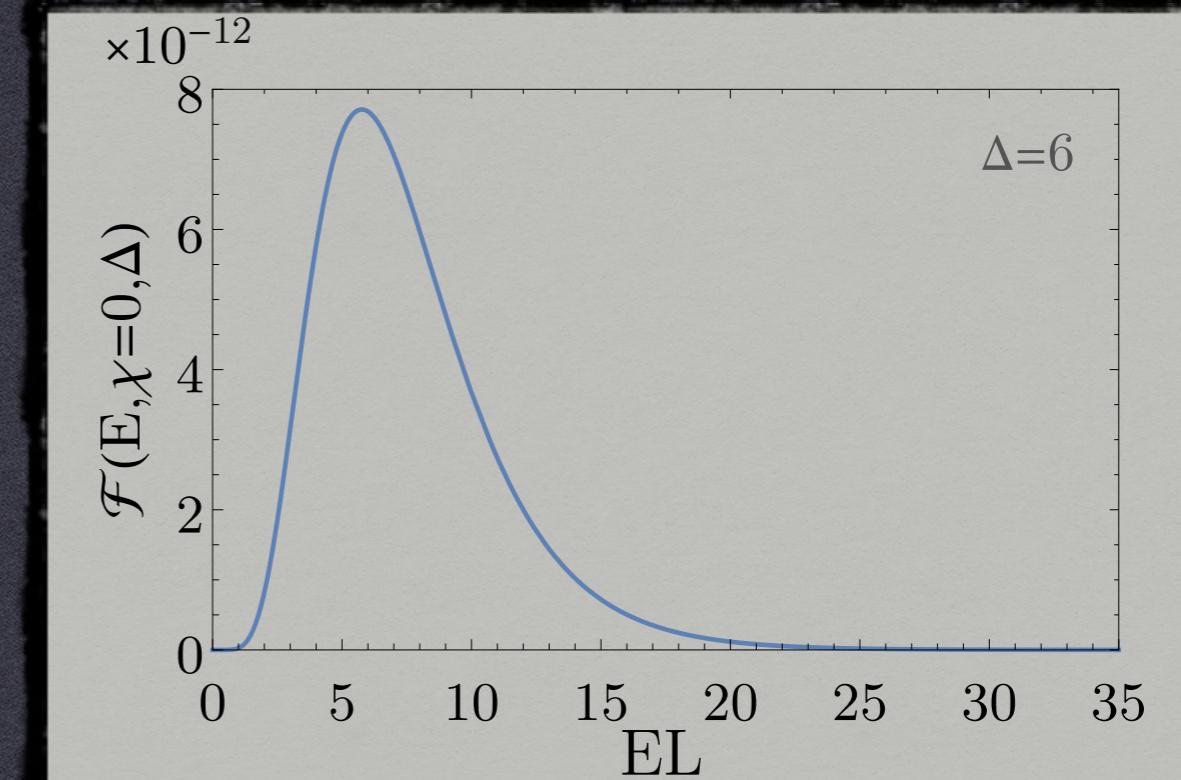
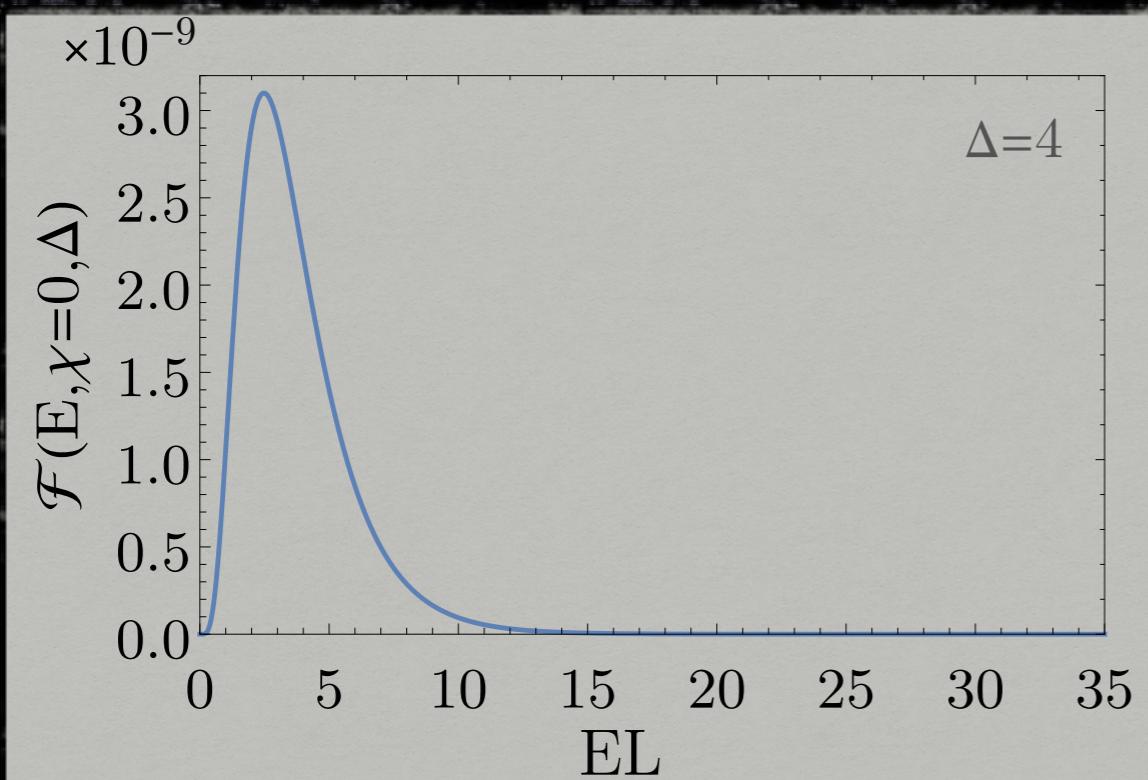
c=20



# On the Line: a view to the spectrum

$$E = \frac{E_a + E_b}{2}$$
$$\chi = E_a - E_b$$

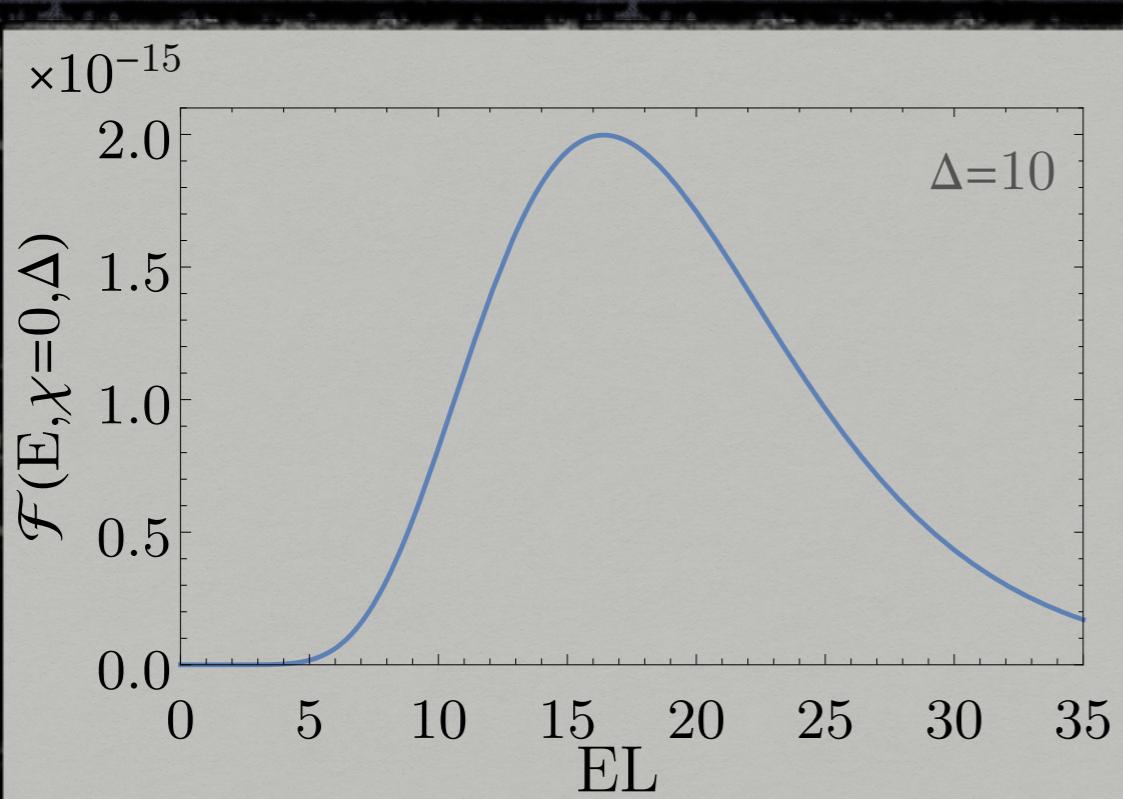
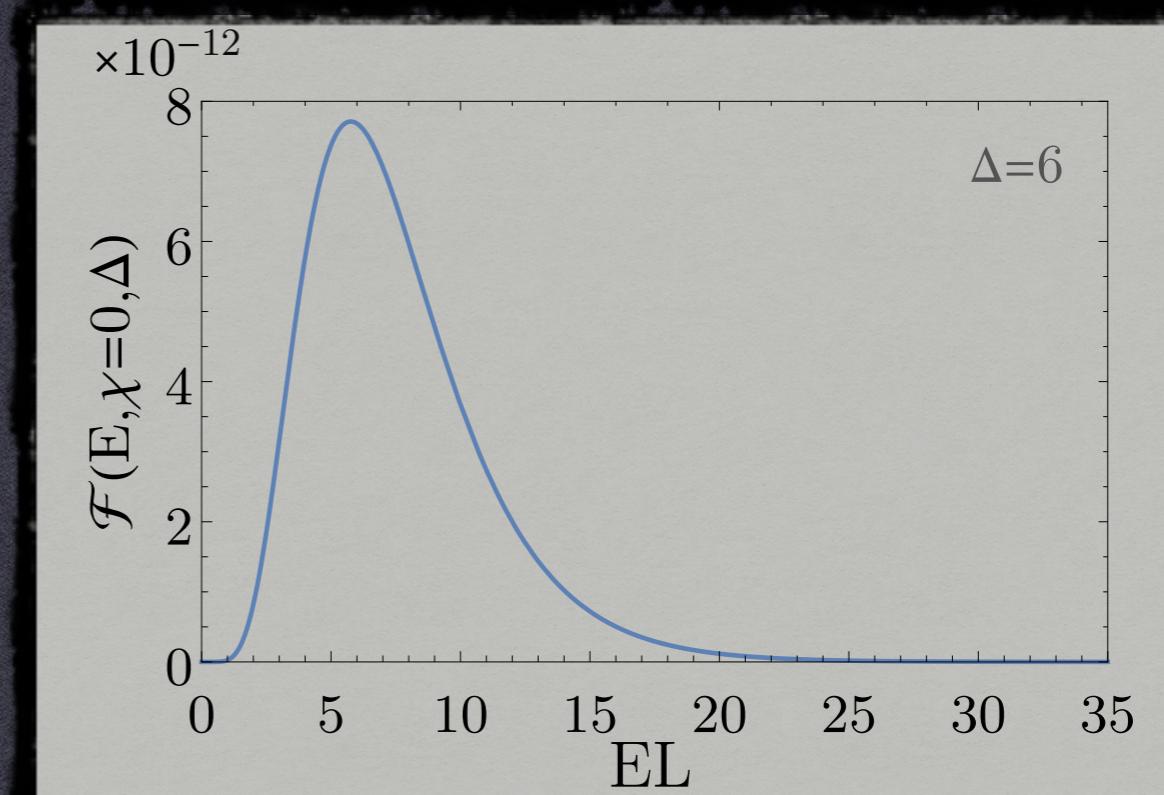
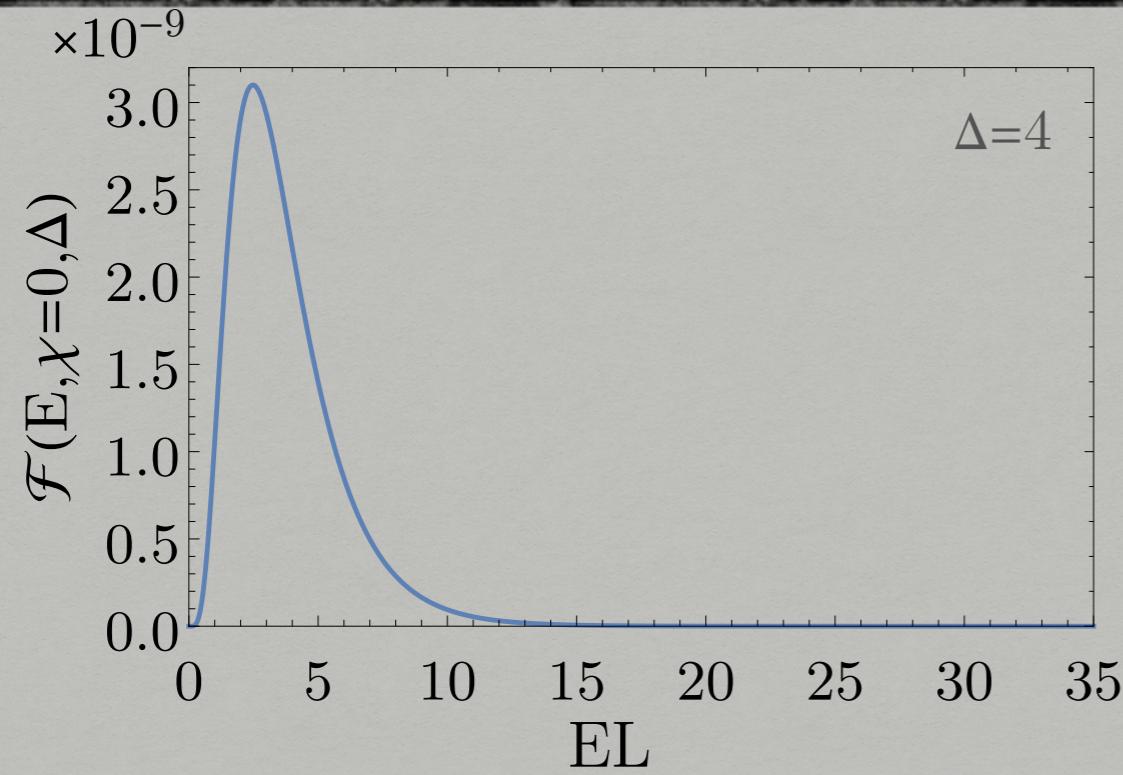
c=20



# On the Line: a view to the spectrum

$$E = \frac{E_a + E_b}{2}$$

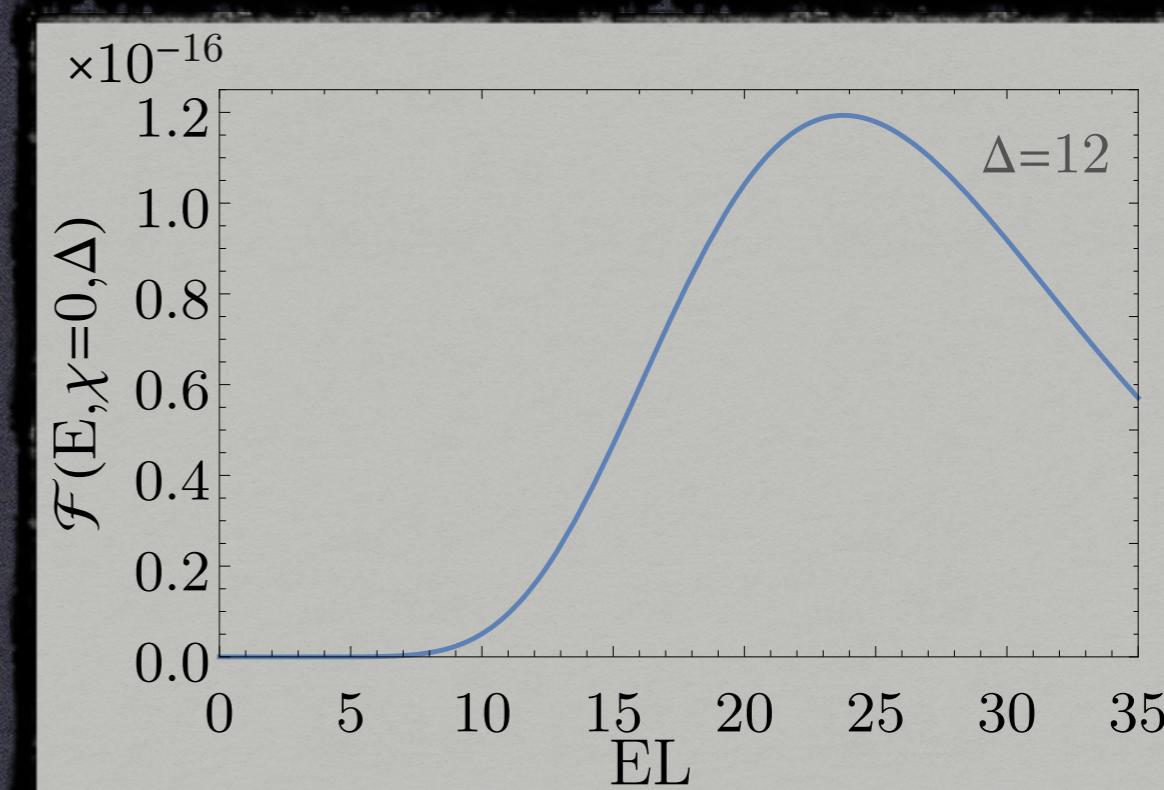
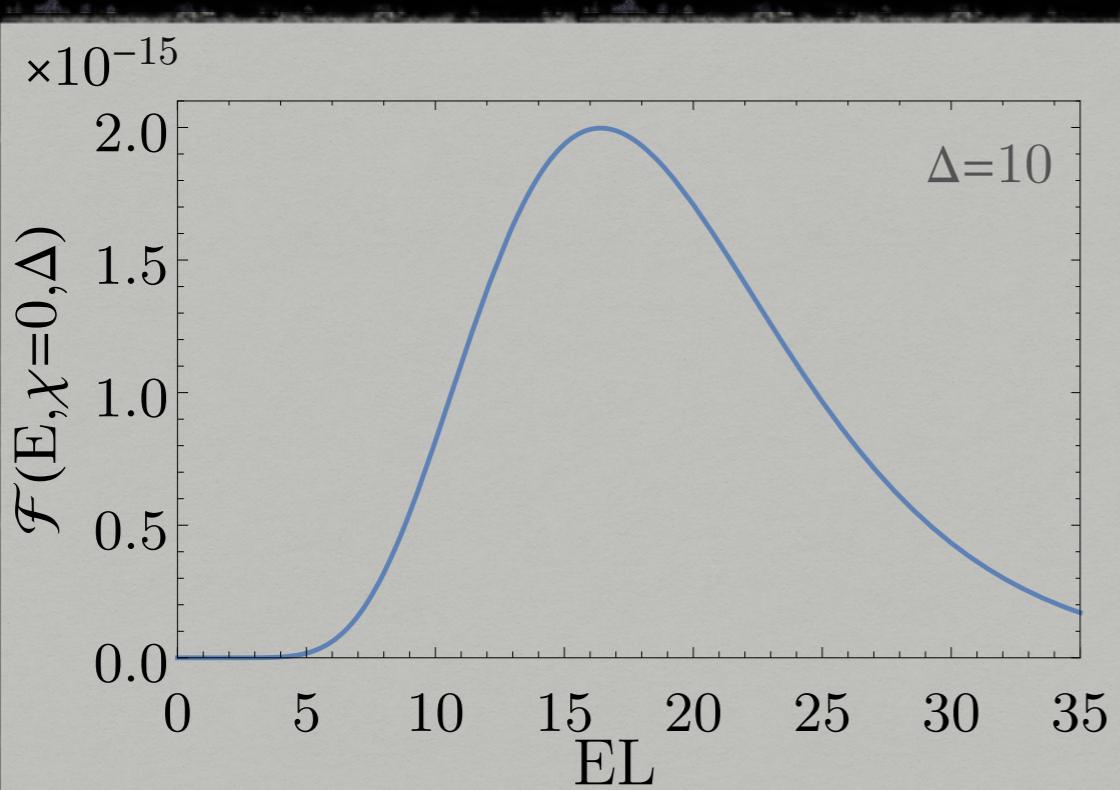
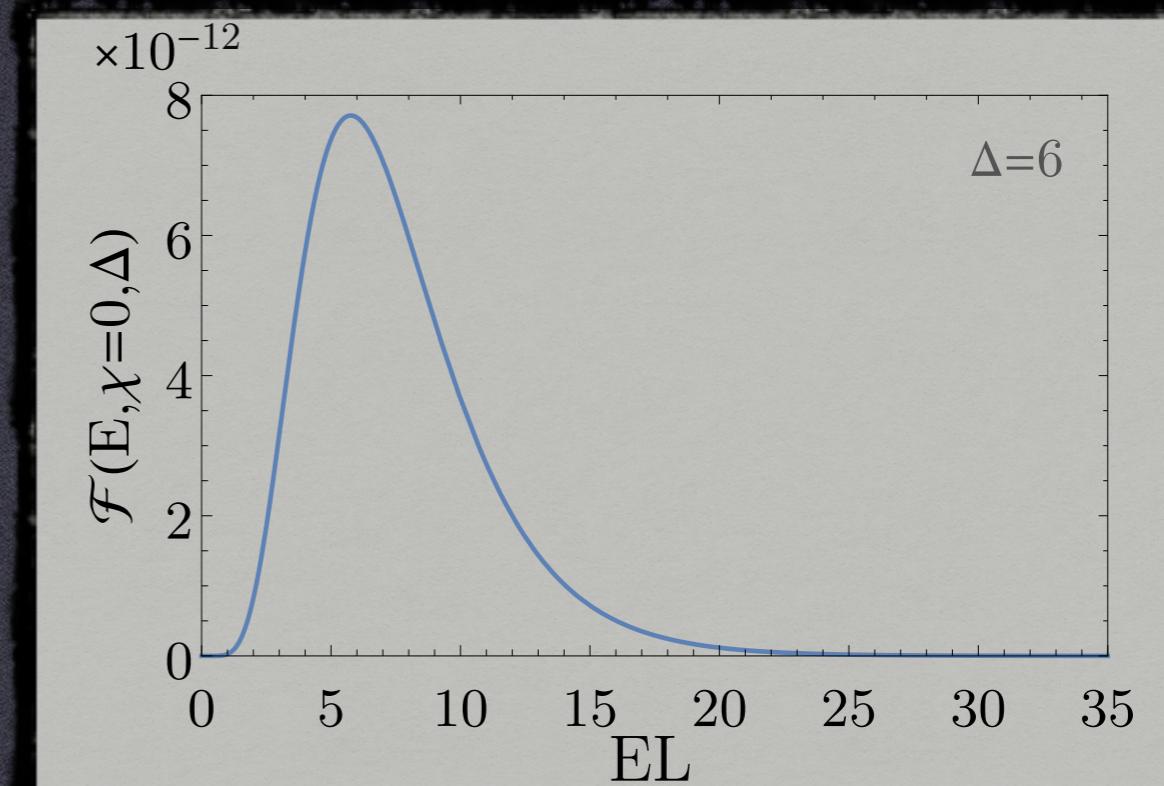
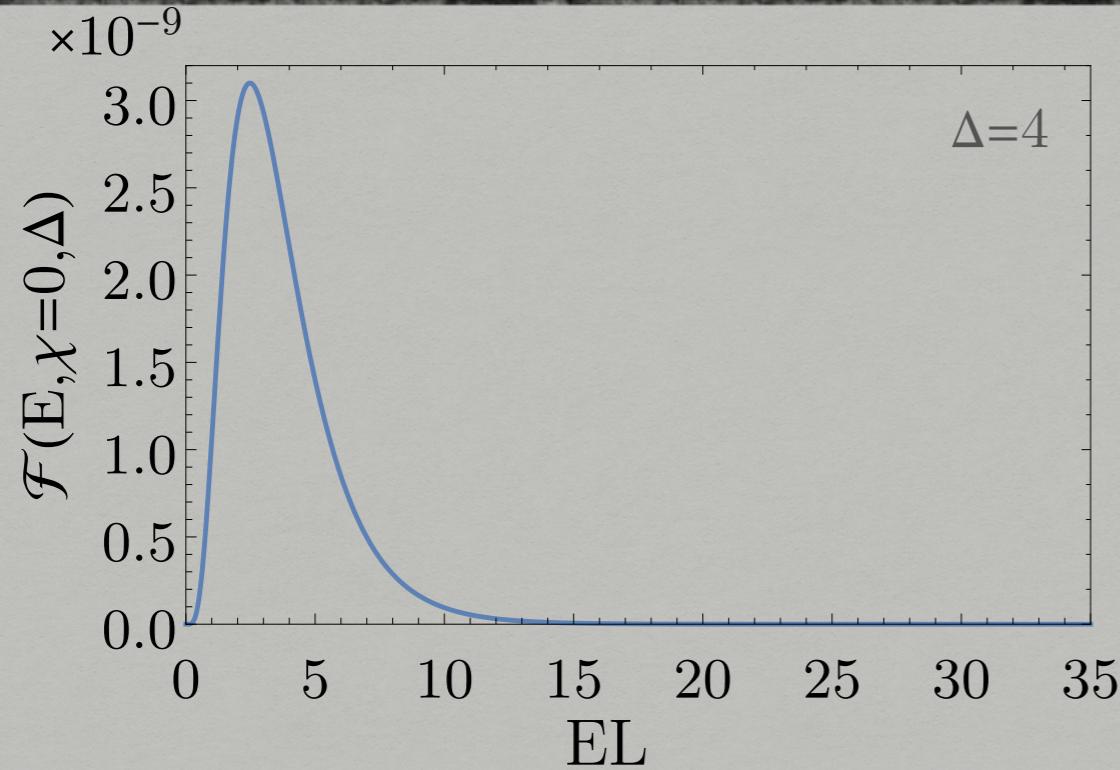
c=20



# On the Line: a view to the spectrum

$$E = \frac{E_a + E_b}{2}$$

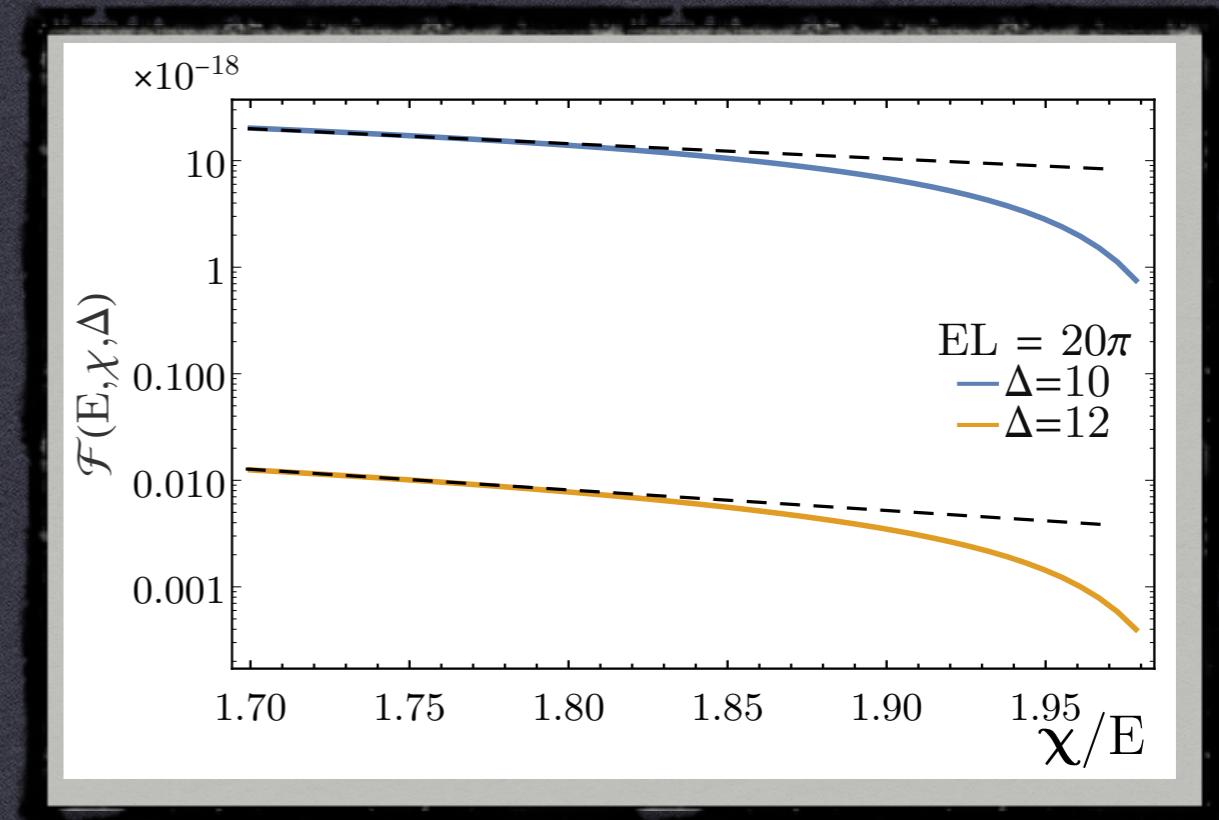
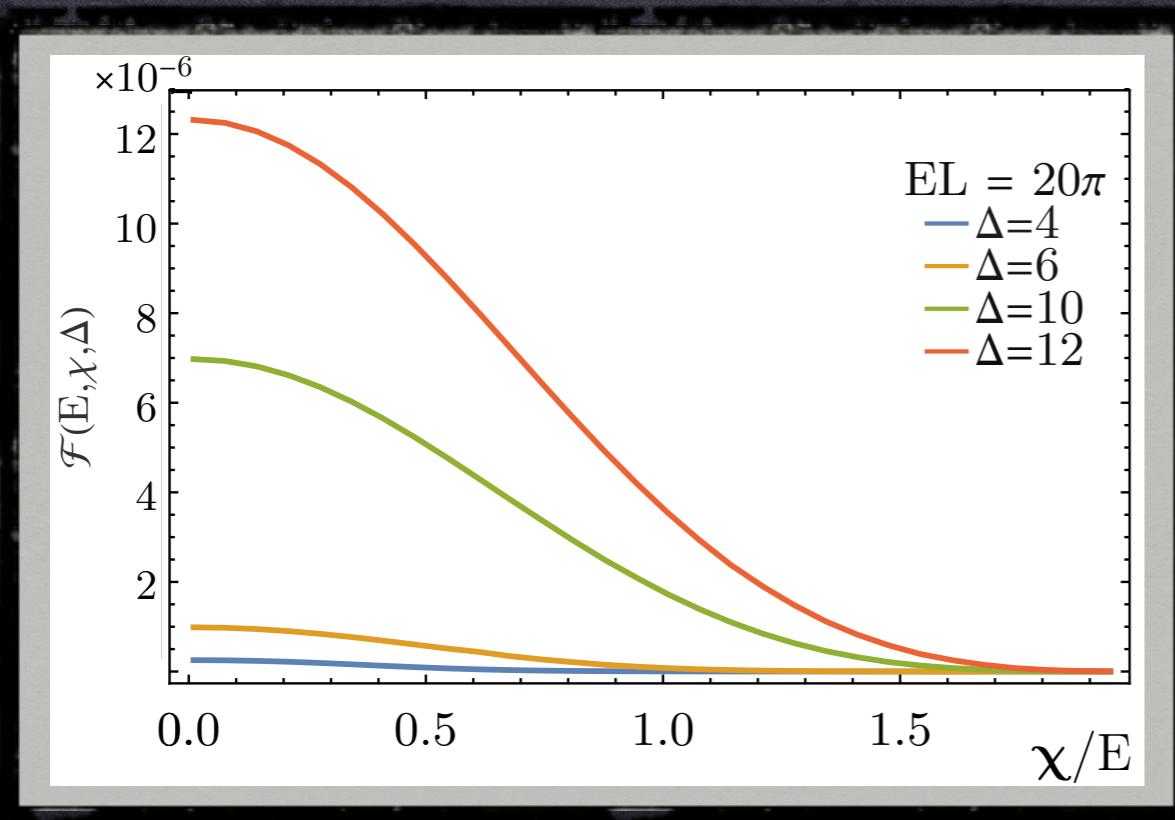
c=20



# On the Line: a view to the spectrum

$$E = \frac{E_a + E_b}{2}$$

c=20



ETH?

## 4- Disentangling experiment

Hard cutoff in the spectrum:

$$|\Lambda\rangle \equiv \frac{1}{\sqrt{Z_\Lambda}} \sum_{a|E_a \leq \Lambda} e^{-\beta E_a/2} |a\rangle |a\rangle$$

$$Z_\Lambda \equiv \sum_{a|E_a \leq \Lambda} e^{-\beta E_a} = \int_0^\Lambda dE \rho(E) e^{-\beta E}$$

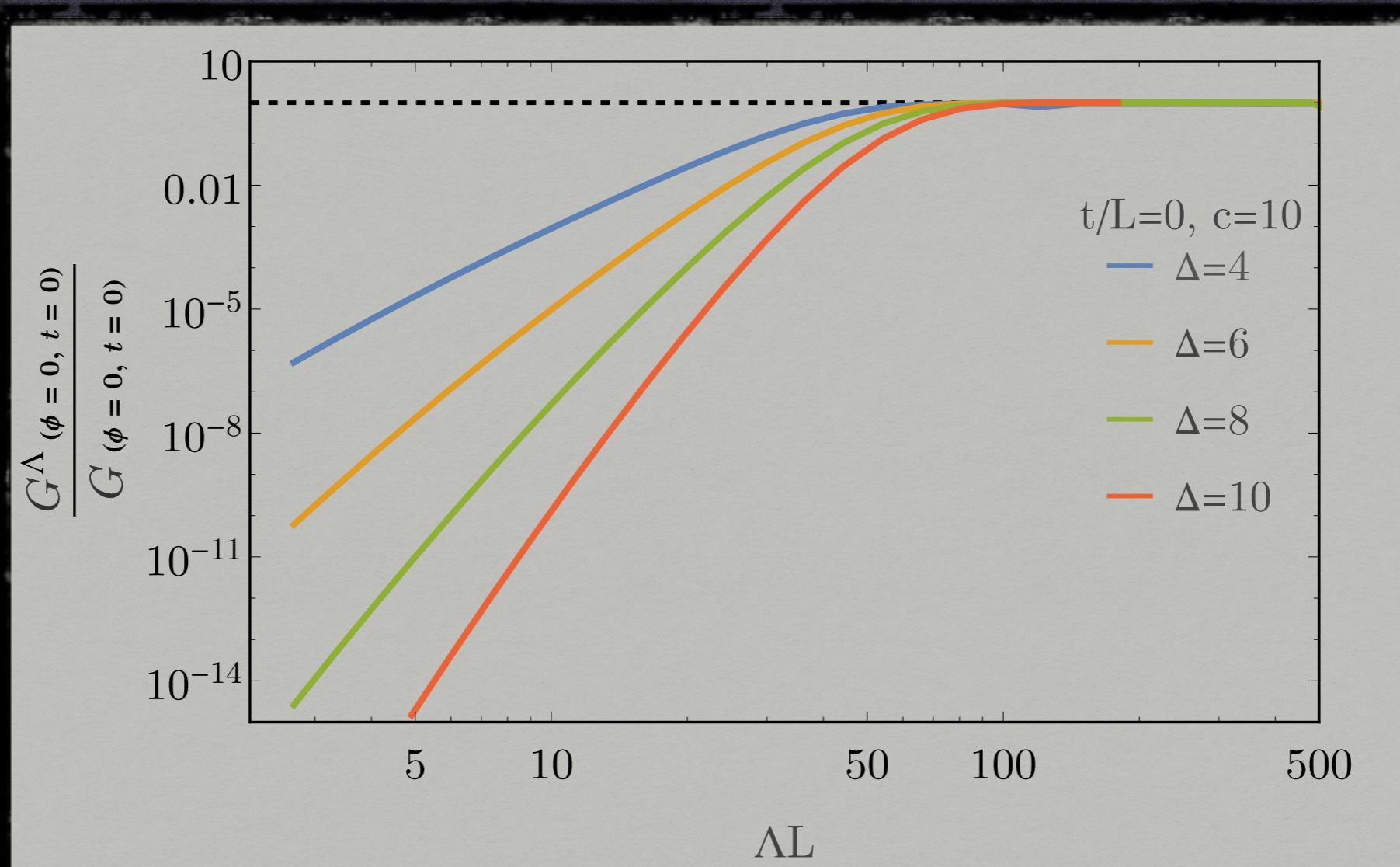
$$\langle \text{TFD} | \mathcal{O}(t_1) \otimes \mathcal{O}(t_2) | \text{TFD} \rangle \rightarrow \langle \Lambda | \mathcal{O}(t_1) \otimes \mathcal{O}(t_2) | \Lambda \rangle \equiv G_L^\Lambda(t_{12}, \phi_{12})$$

$$G_L^\Lambda(t_{12}, \phi_{12}) \equiv \frac{1}{Z^\Lambda(\beta)} \int_{-\Lambda}^{\Lambda} d\chi \int_{\chi/2}^{\Lambda - \chi/2} dE e^{i\chi t_{12}} e^{-\beta E} \mathcal{F}(E, \chi, \Delta)$$

## 4- Disentangling experiment

$$G_L^\Lambda(t_{12}, \phi_{12}) \equiv \frac{1}{Z^\Lambda(\beta)} \int\limits_{-\Lambda}^{\Lambda} d\chi \int\limits_{\chi/2}^{\Lambda - \chi/2} dE e^{i\chi t_{12}} e^{-\beta E} \mathcal{F}(E, \chi, \Delta)$$

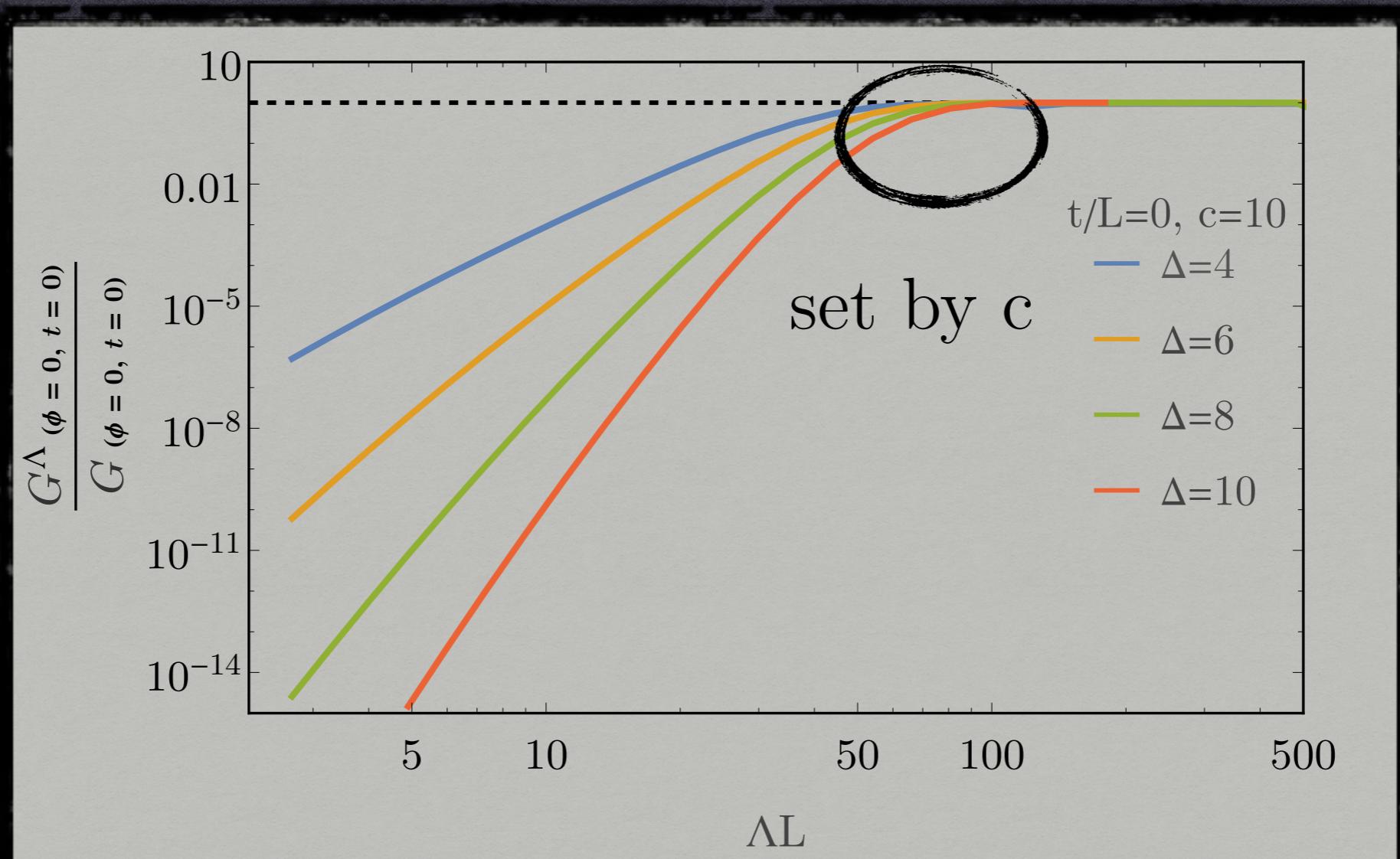
$t=0$ ,  $\beta$  fixed, cutoff dependence



## 4- Disentangling experiment

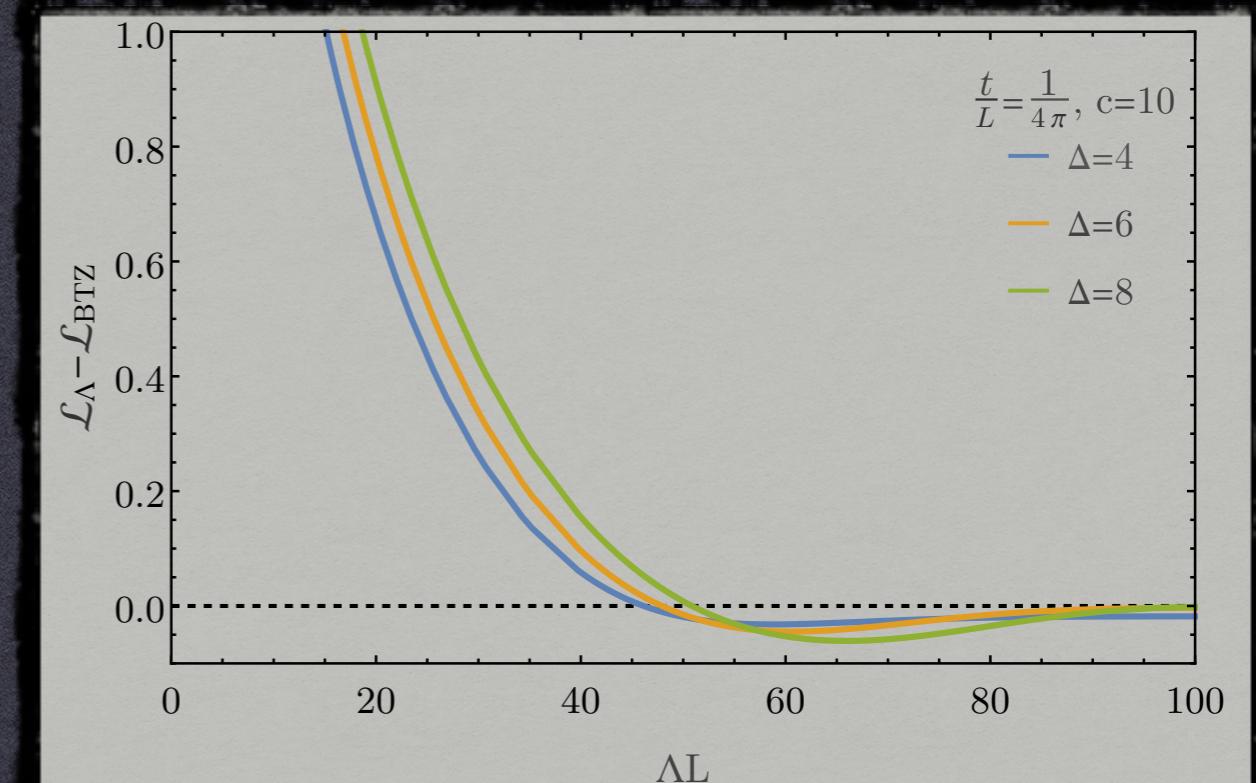
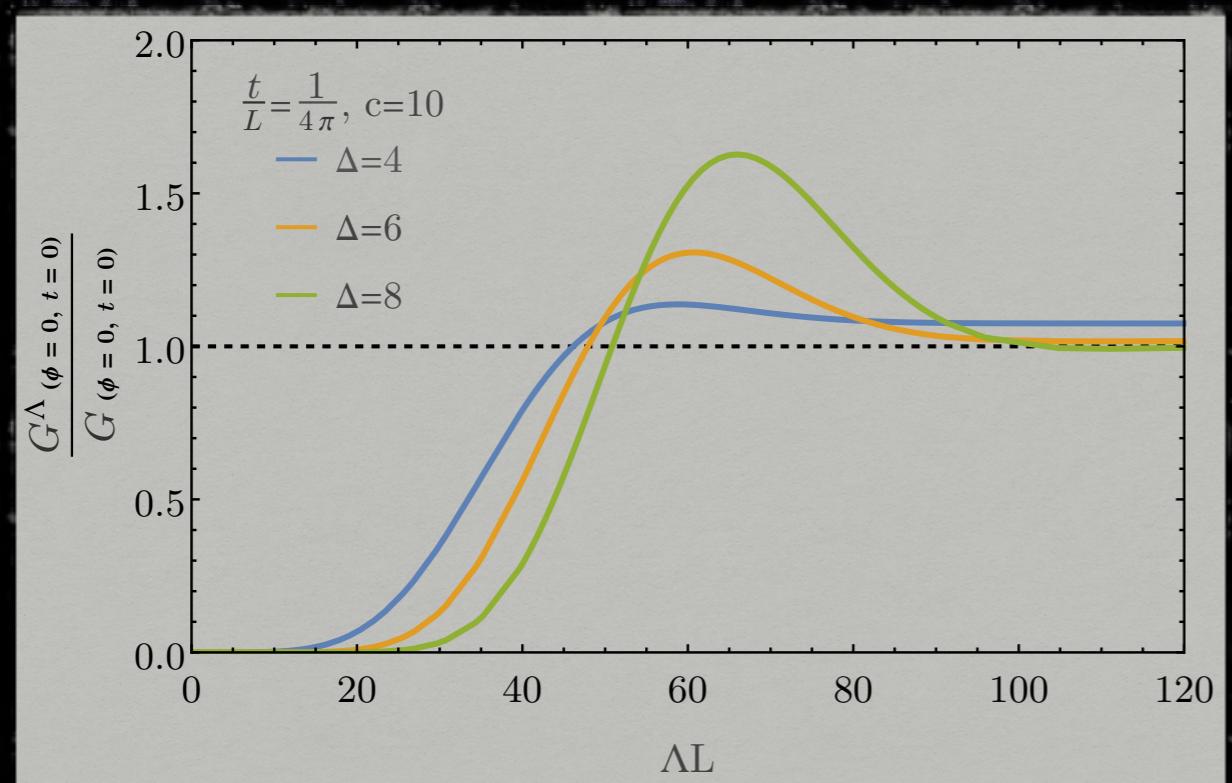
$$G_L^\Lambda(t_{12}, \phi_{12}) \equiv \frac{1}{Z^\Lambda(\beta)} \int_{-\Lambda}^{\Lambda} d\chi \int_{\chi/2}^{\Lambda - \chi/2} dE e^{i\chi t_{12}} e^{-\beta E} \mathcal{F}(E, \chi, \Delta)$$

$t=0$ ,  $\beta$  fixed, cutoff dependence



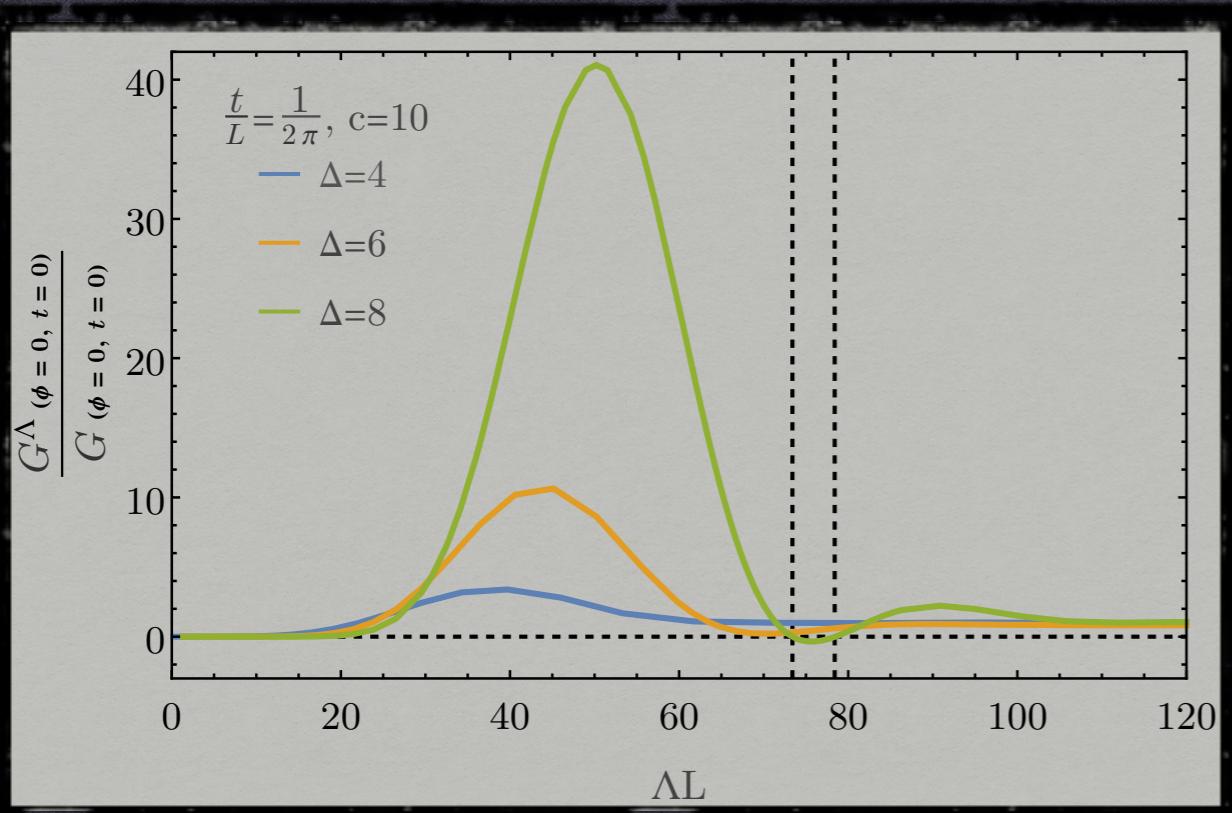
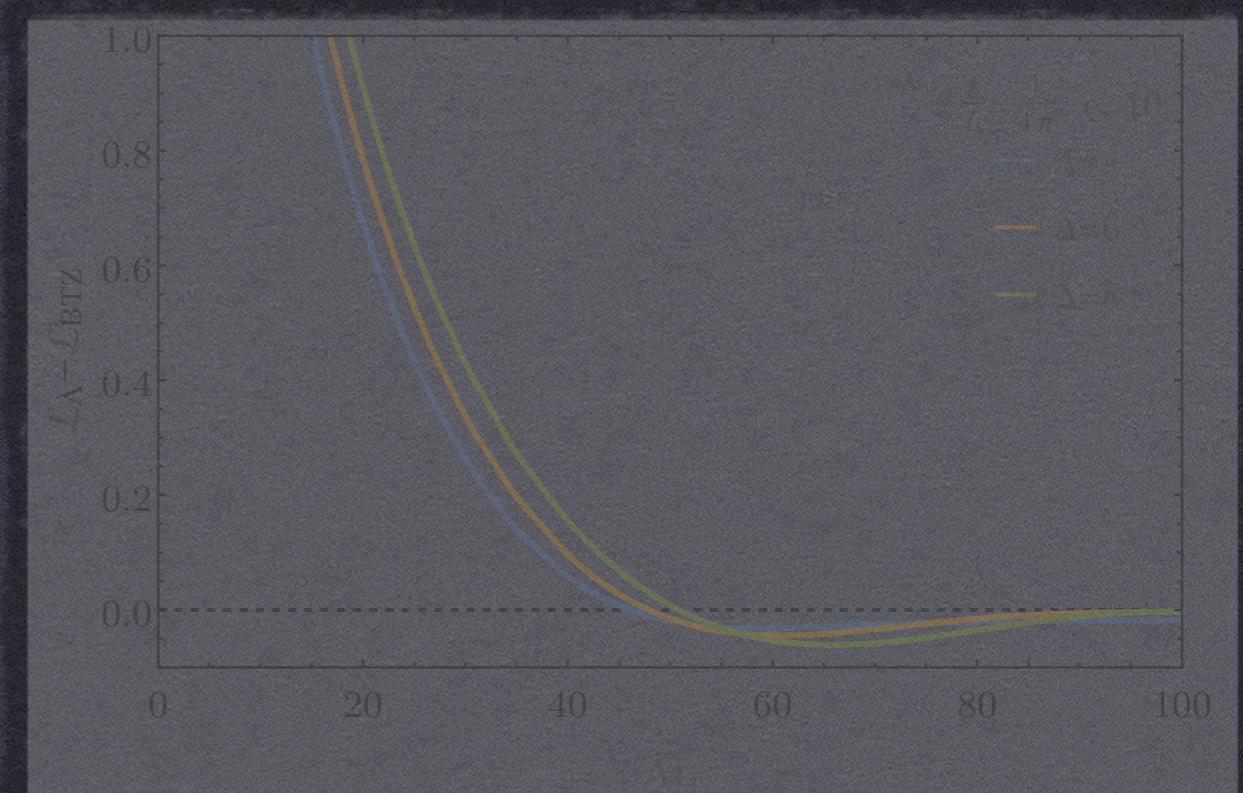
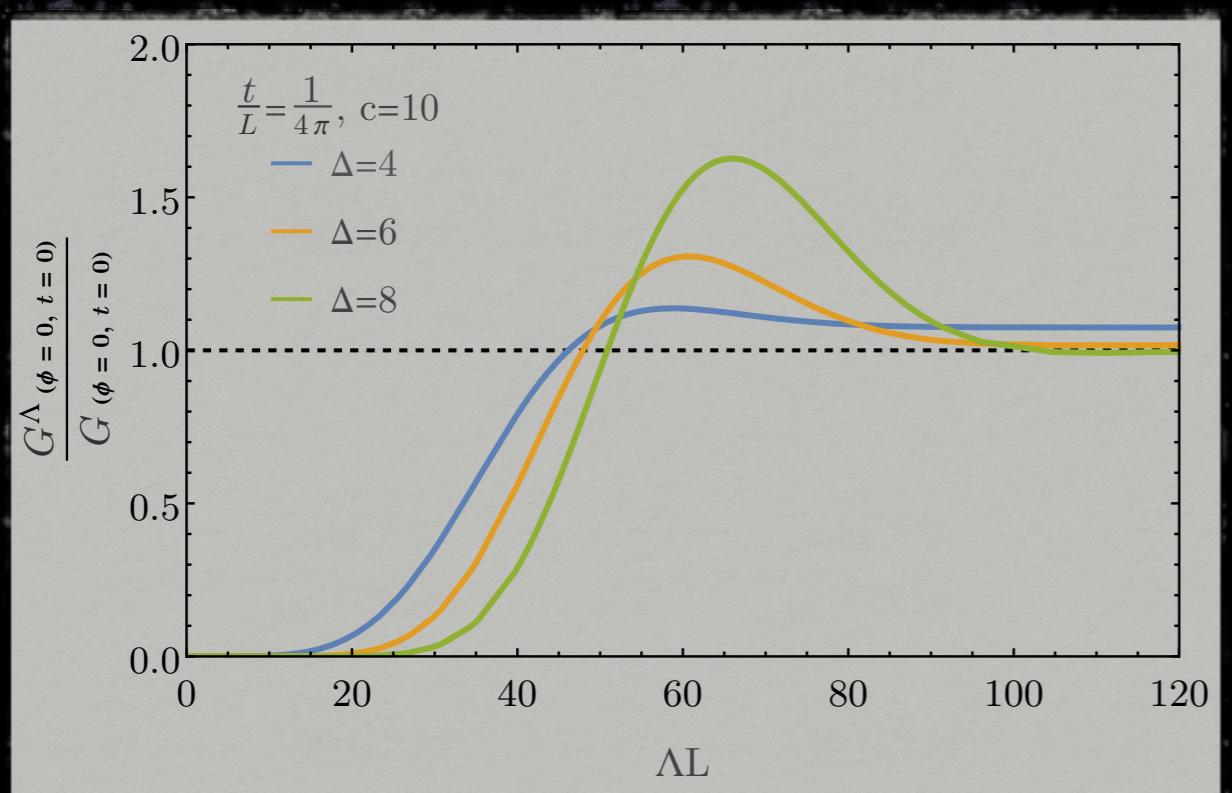
# 4- Disentangling experiment

$t \neq 0$ ,  $\beta$  fixed, cutoff dependence



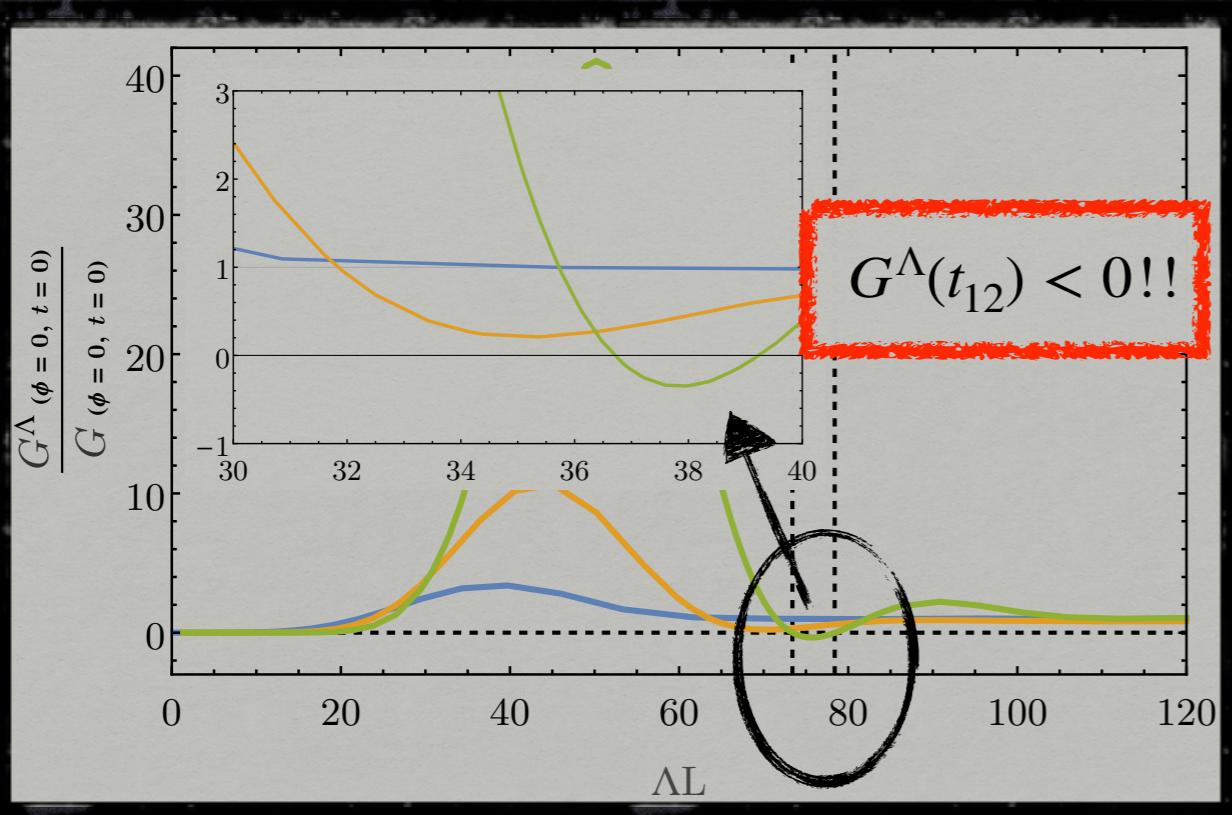
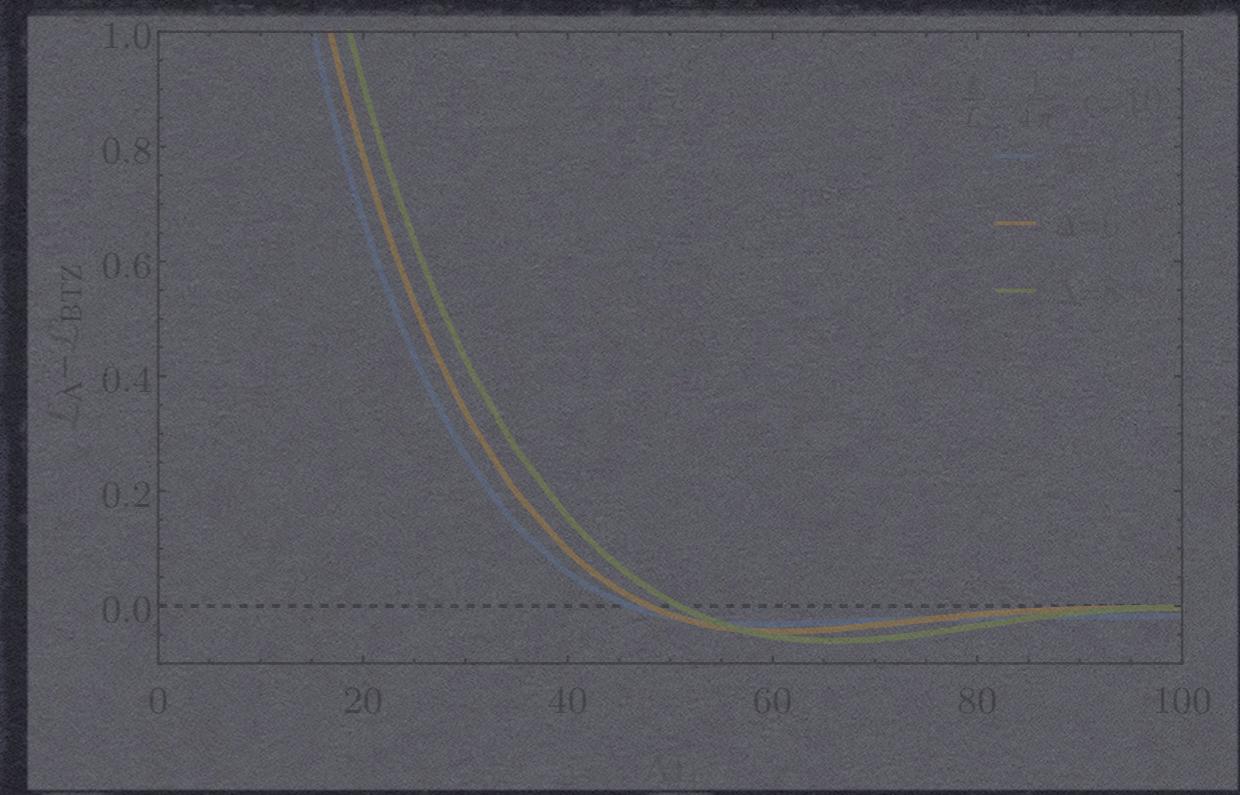
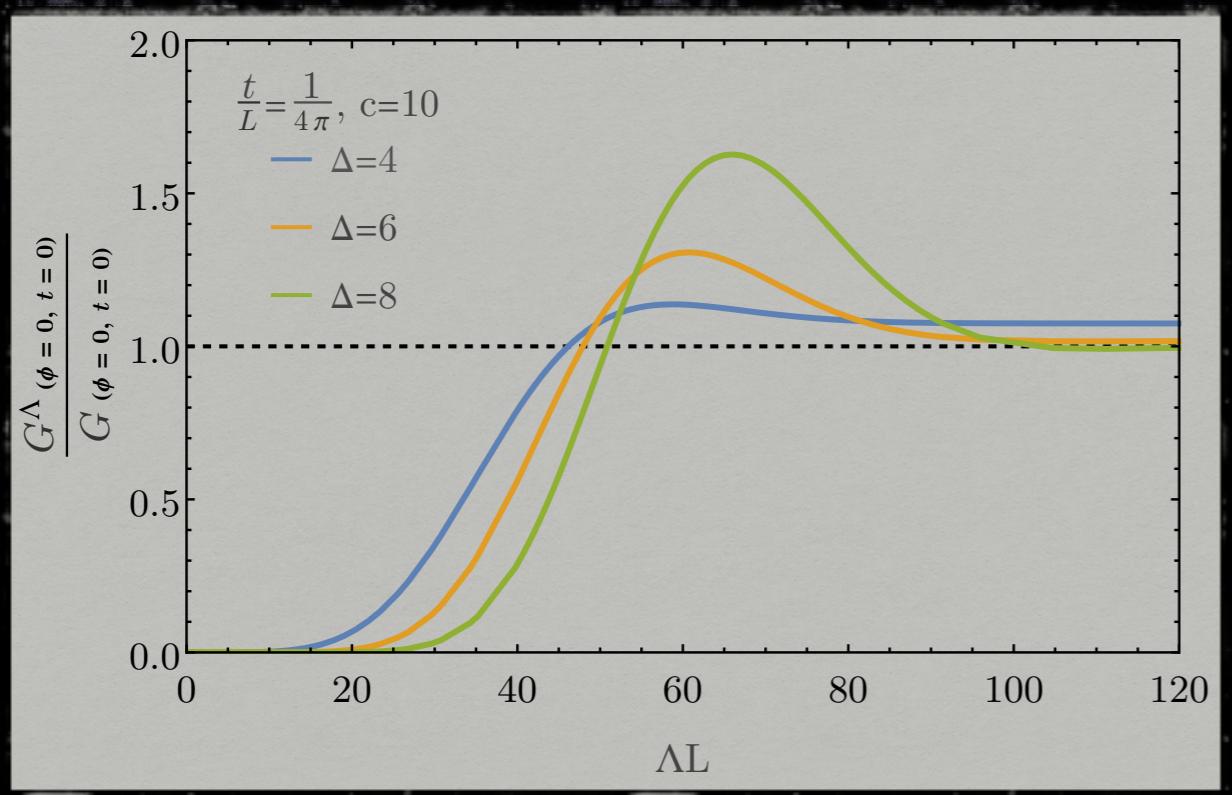
# 4- Disentangling experiment

$t \neq 0$ ,  $\beta$  fixed, cutoff dependence



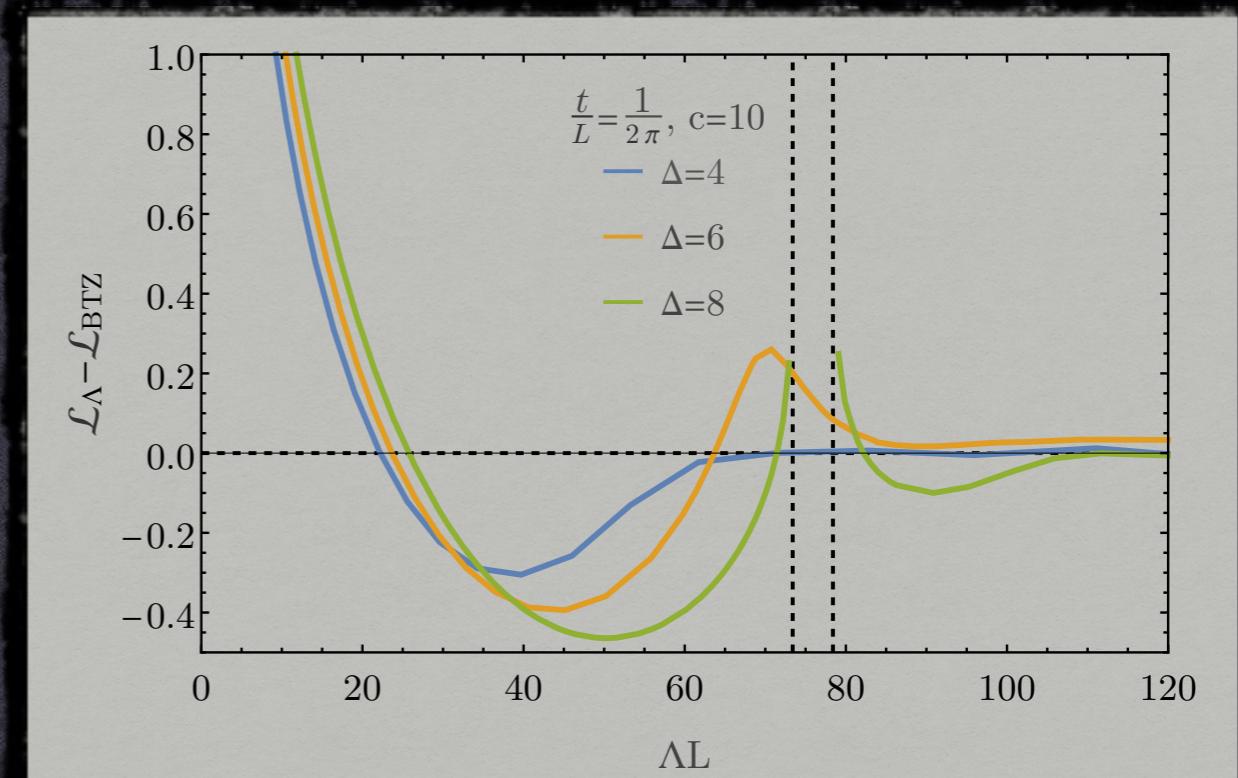
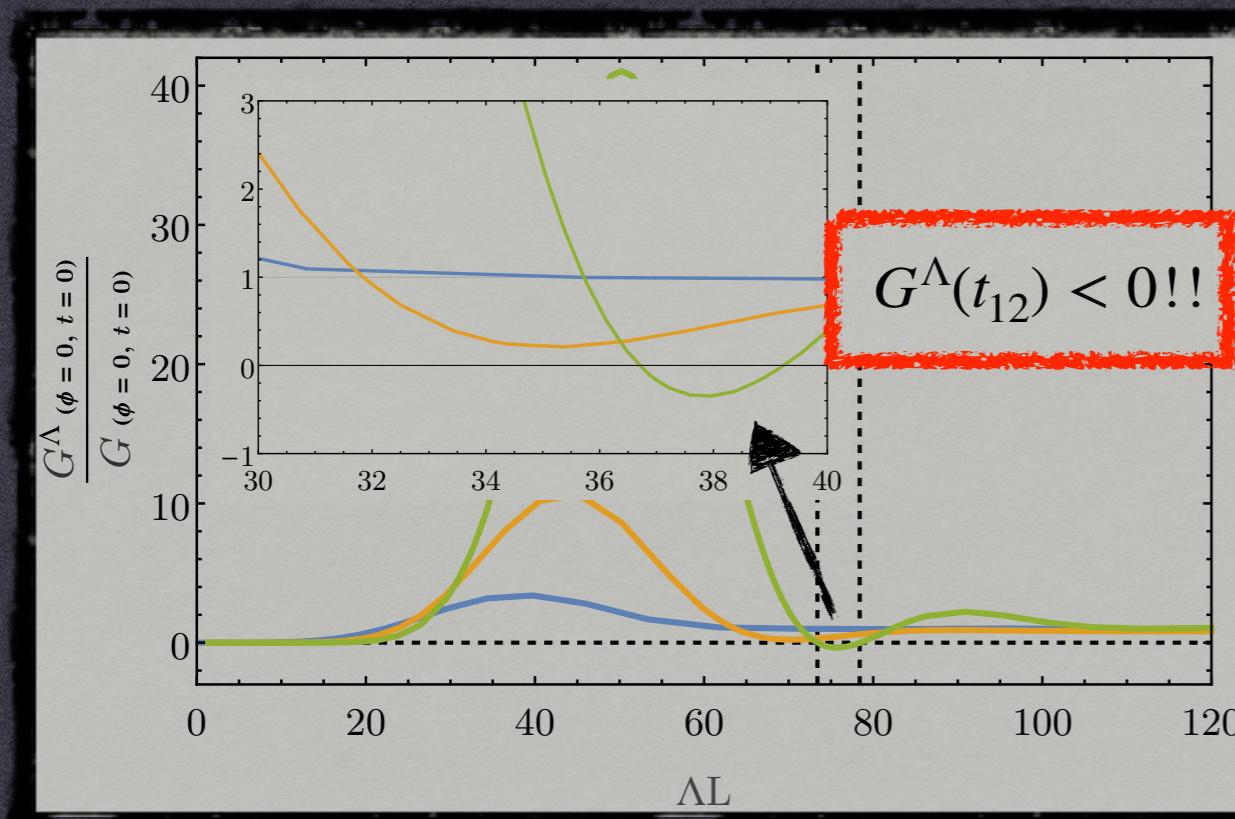
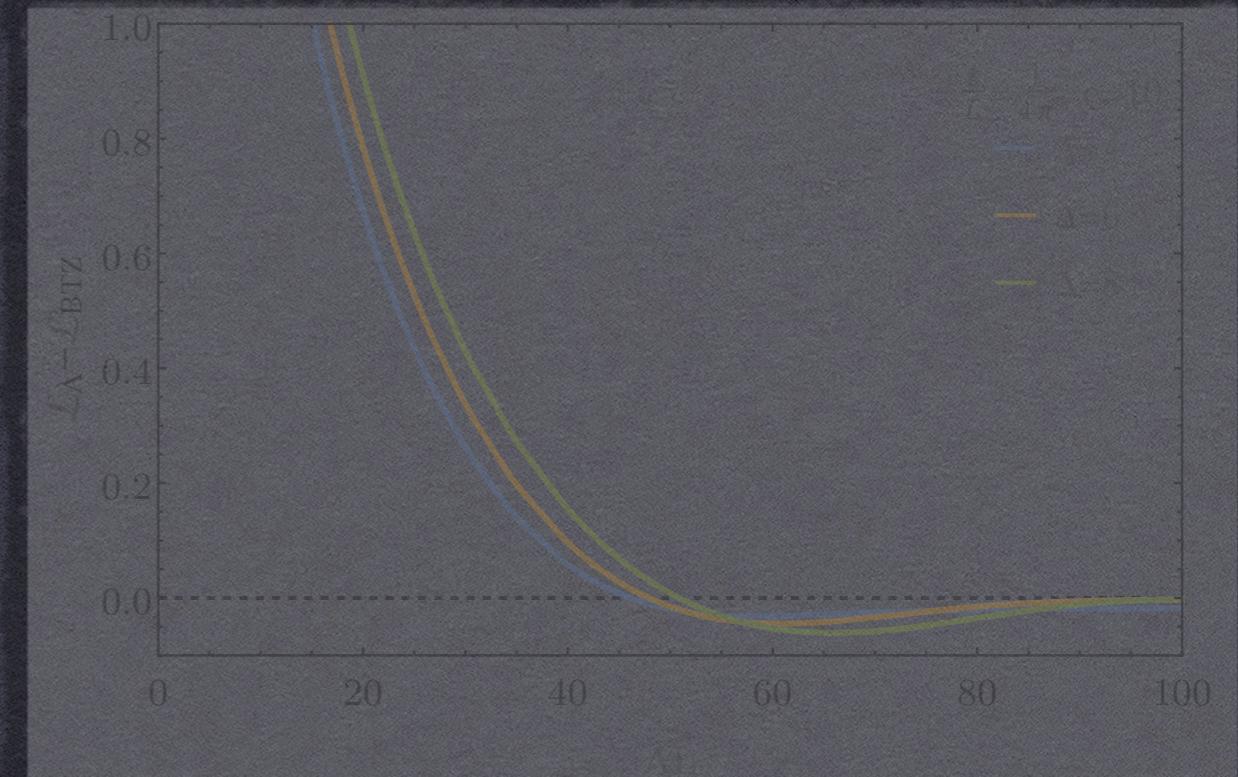
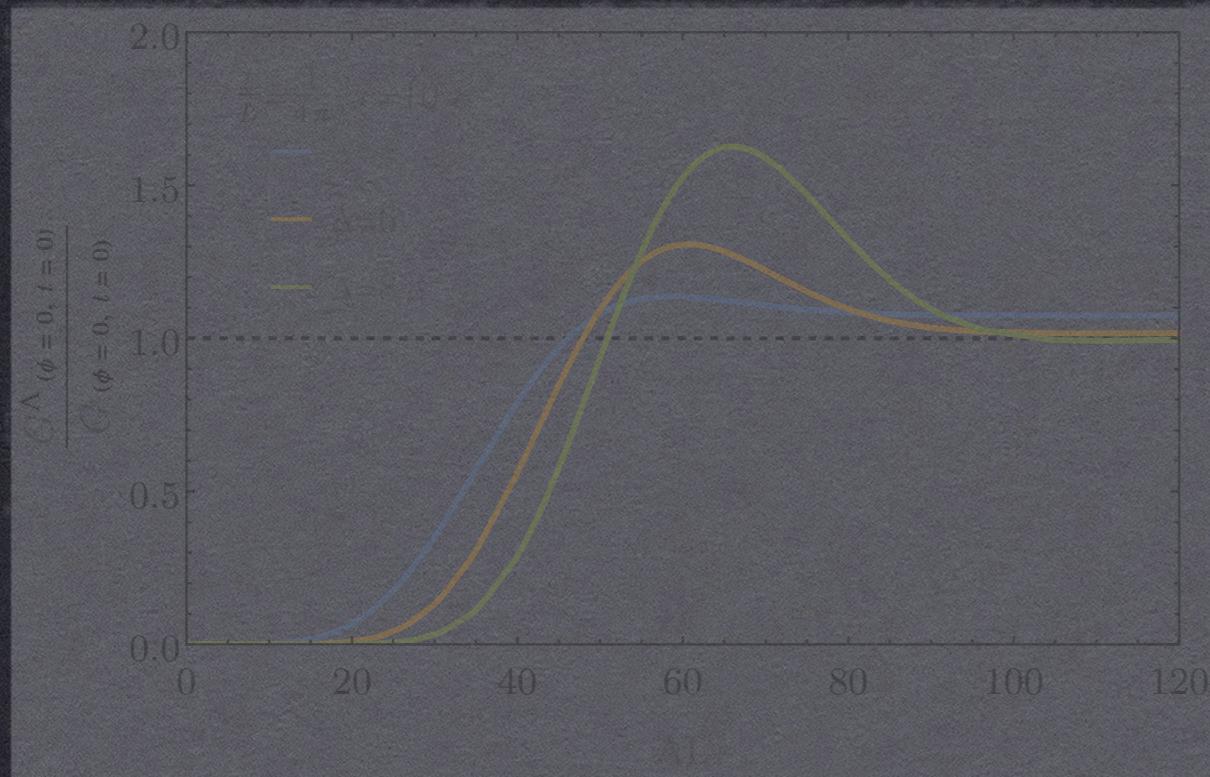
# 4- Disentangling experiment

$t \neq 0, \beta$  fixed, cutoff dependence



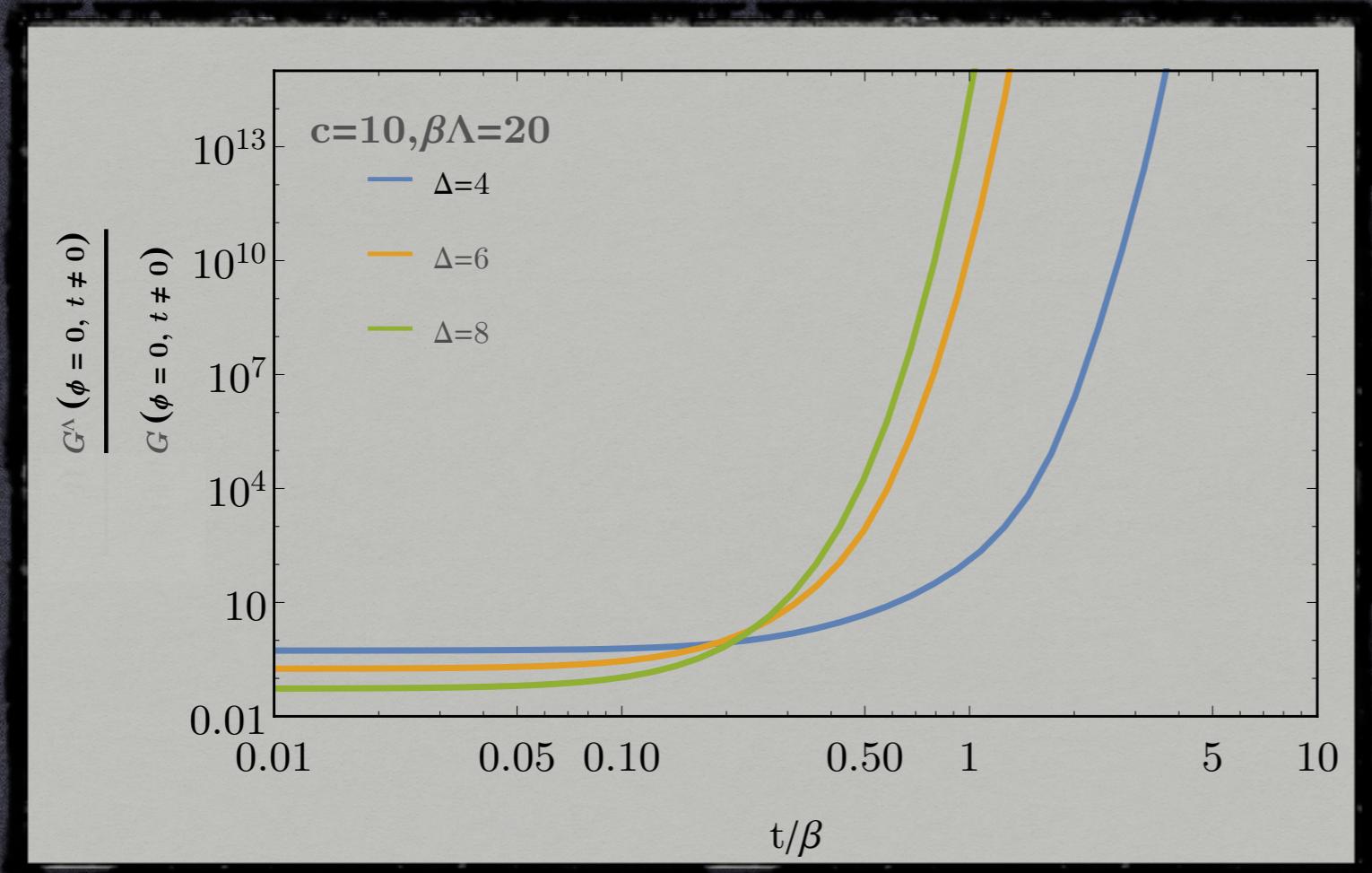
# 4- Disentangling experiment

$t \neq 0$ ,  $\beta$  fixed, cutoff dependence



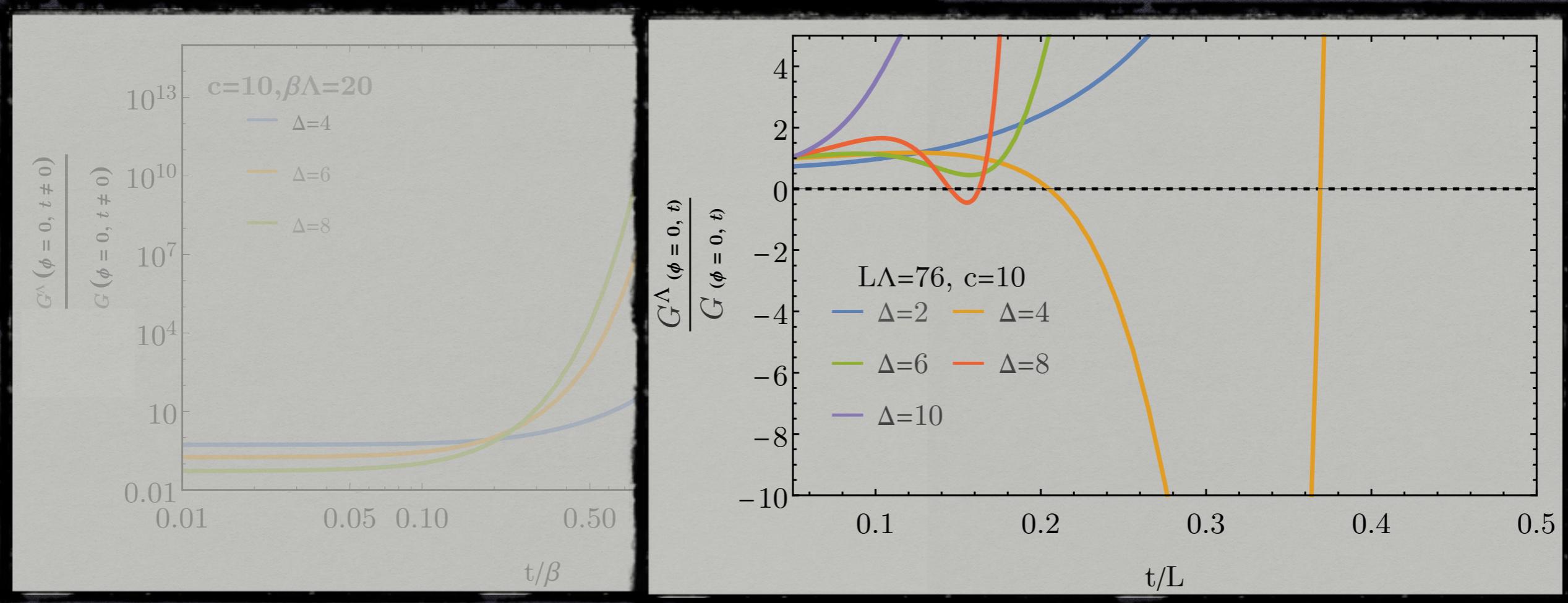
# 4- Disentangling experiment

$\beta\Lambda$  fixed, time dependence



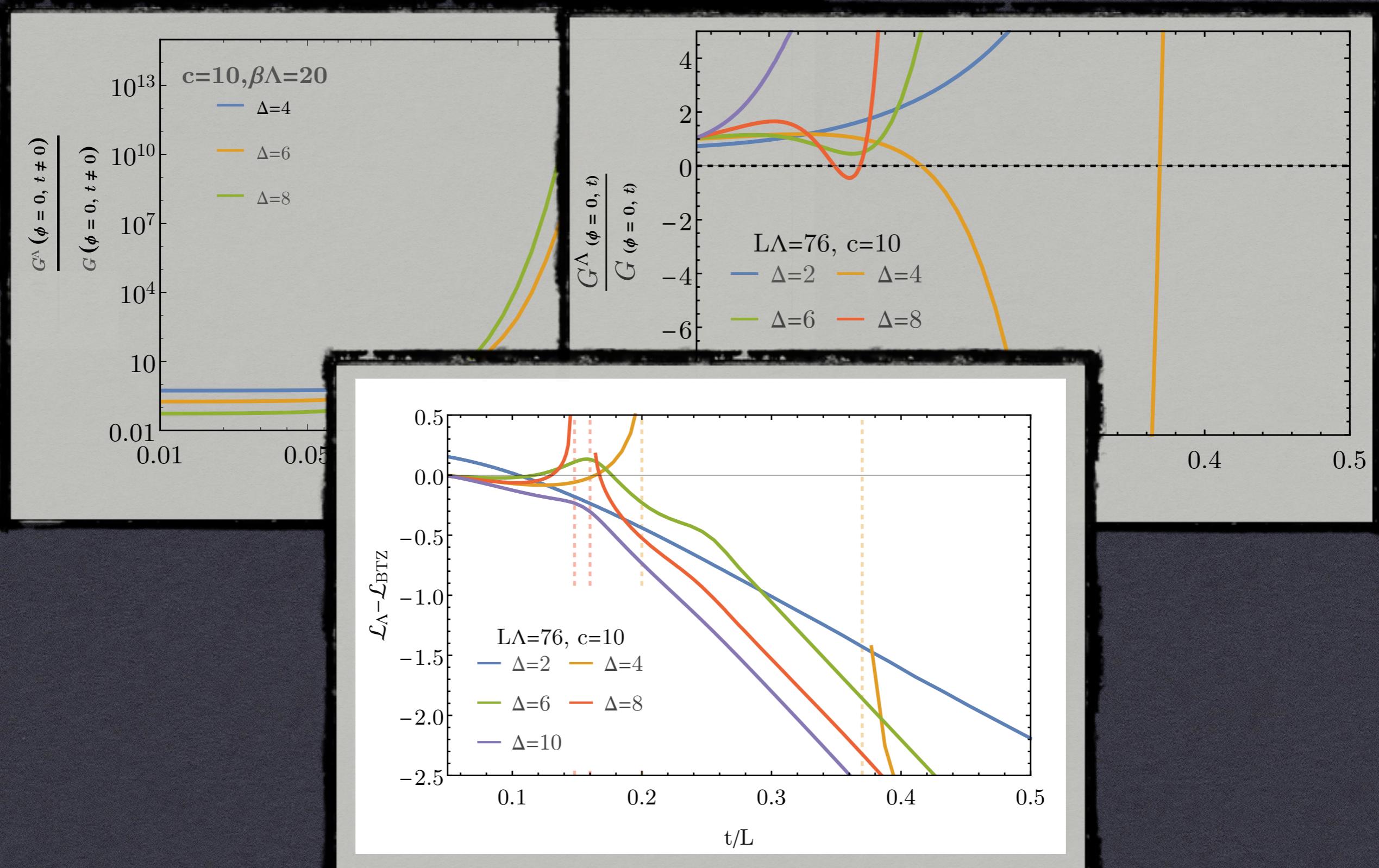
# 4- Disentangling experiment

$\beta\Lambda$  fixed, time dependence



# 4- Disentangling experiment

$\beta\Lambda$  fixed, time dependence



# Wightman with arbitrary entanglement

$$|\Lambda\rangle \equiv \frac{1}{\sqrt{Z_\Lambda}} \sum_{a|E_a \leq \Lambda} e^{-\beta E_a/2} |a\rangle |a\rangle \rightarrow |\Psi\rangle \equiv \frac{1}{\sqrt{Z_\Psi}} \sum_a \Psi(E_a) |a\rangle |a\rangle$$

$$Z_\Psi = \sum_a |\Psi(E_a)|^2 < \infty$$

# Wightman with arbitrary entanglement

$$|\Lambda\rangle \equiv \frac{1}{\sqrt{Z_\Lambda}} \sum_{a|E_a \leq \Lambda} e^{-\beta E_a/2} |a\rangle |a\rangle \rightarrow |\Psi\rangle \equiv \frac{1}{\sqrt{Z_\Psi}} \sum_a \Psi(E_a) |a\rangle |a\rangle$$

$$Z_\Psi = \sum_a |\Psi(E_a)|^2 < \infty$$

$$\langle \Psi | \mathcal{O}(t) \otimes \mathcal{O}(0) | \Psi \rangle$$

$$= \frac{1}{Z_\Psi} \int\limits_{-\infty}^\infty d\chi e^{-i\chi t} \int\limits_{|\chi|/2}^\infty dE \; \Psi^* \left( E + \frac{\chi}{2} \right) \Psi \left( E - \frac{\chi}{2} \right) \rho \left( E + \frac{\chi}{2} \right) \rho \left( E - \frac{\chi}{2} \right) \mathcal{F}(E, \chi, \Delta)$$

# CONCLUSIONS

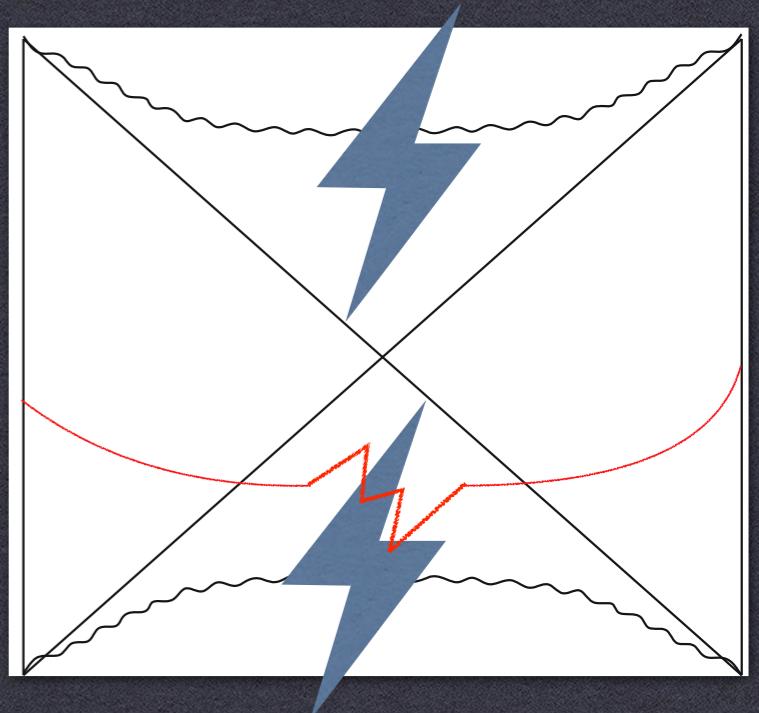
- ◆ Obtained  $\overline{\langle a | O | b \rangle^2} \equiv \mathcal{F}(E, \chi, \Delta)$   
If ETH is at work  $\rho(E)\mathcal{F}(E, \chi, \Delta) = O(E)^2\delta_{\hat{\delta}}(\chi) + f(E, \chi)^2R_{ab}$

- ◆ Disentangling the TFD:

$$G^\Lambda(t_{12}, \phi_{12}) = \frac{1}{Z^\Lambda(\beta)} \sum_{ab} e^{-\beta \frac{E_a + E_b}{2}} e^{-it_{12}E_{ab}} |\langle a | O | b \rangle|^2$$
$$E_a, E_b < \Lambda$$

Dramatic behaviour of the geometry as measured using the geodesic approximation with  $G^\Lambda(t_{12}, \phi_{12})$

- ◆ Change in the geometry must be nonlocal due to Killing symmetry



# CONCLUSIONS

What else?

- ◆ Repeat on the circle?
- ◆ Use  $\overline{\langle a | O | b \rangle^2} \equiv \mathcal{F}(E, \chi, \Delta)$  for some time evolution/quench?
- ◆ Interpretation of ‘disentangling experiment’ in terms of geometric objects?

Perhaps recasting this with conformal blocks and use interpretation in terms of geodesic Witten diagrams