Holographic Plasmons

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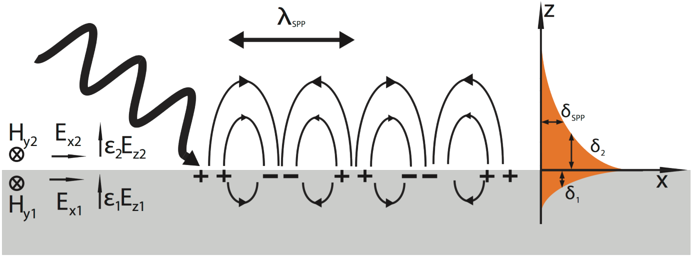
[1712.05672, 1804.02284] *and work in progress*

I. Introduction

Plasmons

a self-sustained oscillation driven by the dynamical polarization of the system



it is encoded in the longitudinal dielectric function ϵ_L

[D. Bohm, D. Pines 1952]

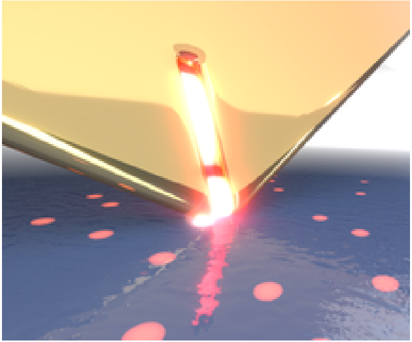
Historic Applications

historic applications of plasmonic effects date back over two millenia, quite prominently in the staining of glass

Modern Plasmonics

plasmons have a shorter wavelength compared to an electromagnetic wave of the same frequency, offering new technological possibilities

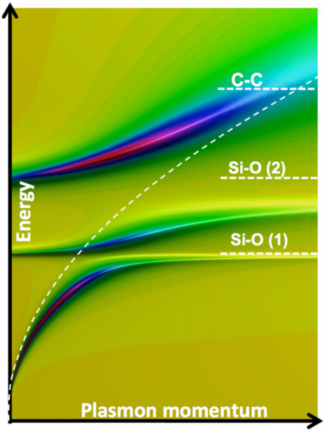
- color engineering
- miniaturization
- biomedical
- spectroscopy
- metamaterials with negative refraction index
- solar cells
- ...



Co-Dimension

qualitatively different characteristics based on co-dimension of the device/system

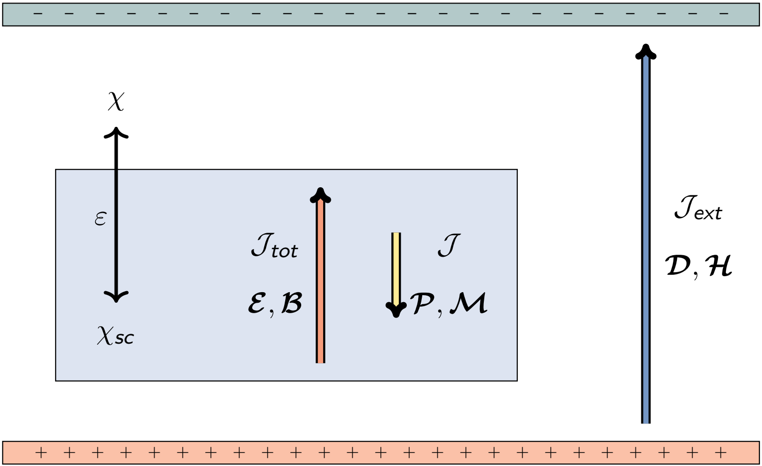
- **co-dim** 0
  bulk plasmon
  $\longrightarrow$ gapped
- **co-dim** 1
  surface plasmon
  $\longrightarrow \ \omega \propto \sqrt{k}$
- **co-dim** > 1
  localized surface plasmon
  …



Nomenclature & Conventions

[The Theory of Quantum Liquids – Pines, Nozières]

[Plasmonics: Fundamentals and Applications – Maier]



Macroscopic Maxwell Equations

equations of motion

$$\begin{aligned} d\mathcal{F} &= 0 \\ d\star\mathcal{W} &= \star\mathcal{J}_{ext} \end{aligned}$$

decomposition

field strength $\mathcal{F}$ describes *'screened'* electric field strength $\boldsymbol{\mathcal{E}}$ and magnetic flux density $\boldsymbol{\mathcal{B}}$

$$\mathcal{F} = \boldsymbol{\mathcal{E}} \wedge dt + \star^{-1}(\boldsymbol{\mathcal{B}} \wedge dt)$$

induction tensor $\mathcal{W} = \star^{-1}\frac{\partial\mathcal{L}}{\partial\mathcal{F}}$ describes *'external'* electric displacement $\boldsymbol{\mathcal{D}}$ and magnetic field strength $\boldsymbol{\mathcal{H}}$

$$\mathcal{W} = \boldsymbol{\mathcal{D}} \wedge dt + \star^{-1}(\boldsymbol{\mathcal{H}} \wedge dt)$$

'internal' current

$$\mathcal{J} = -\langle\rho\rangle\, dt + \boldsymbol{j} = \star^{-1} d\star(\mathcal{F} - \mathcal{W})$$

Linear Response

- Green function

$$\mathcal{J} \;=\; G \cdot \mathcal{A}$$

- conductivity

$$\boldsymbol{j} \;=\; \sigma \cdot \boldsymbol{\mathcal{E}}$$

$$\sigma_{ij} \;=\; -\frac{\langle \boldsymbol{j}_i \, \boldsymbol{j}_j \rangle}{i\omega} \;=\; -\frac{G_{ij}}{i\omega}$$

- *'screened'* density-density response

$$\chi_{sc} \;=\; \langle \rho \, \rho \rangle \;=\; G_{00}$$

- *'physical'* density-density response

$$\chi \;=\; \frac{\chi_{sc}}{\epsilon_L}$$

- dielectric tensor

$$\boldsymbol{\mathcal{D}} \;=\; \epsilon \cdot \boldsymbol{\mathcal{E}}$$

Plasmon Condition and Physical Modes

'plasmon condition'

'internal' effects inside the system, $\mathcal{J}_{ext} = 0$, $\boldsymbol{\mathcal{D}}_L = 0$, but $\boldsymbol{\mathcal{E}}_L \neq 0$

$$\Longrightarrow \quad \epsilon_L(\omega, k) \;=\; 0$$

'physical' mode

pole in the response function to *'external'* fields

$$\chi \;=\; \frac{\chi_{sc}}{\epsilon_L} \;=\; \frac{k^2}{i\omega}\frac{\sigma_L}{\epsilon_L}$$

plasmon condition identifies 'physical' modes

poles of σ_L are poles of ϵ_L

$$\left(\epsilon - 1 + \frac{\sigma}{i\omega}\right) \cdot \boldsymbol{\mathcal{E}} \;=\; \frac{\boldsymbol{k} \times \boldsymbol{\mathcal{M}}}{\omega}$$

II. Induction Tensor & Plasmons in Holography

Holographic Correspondence

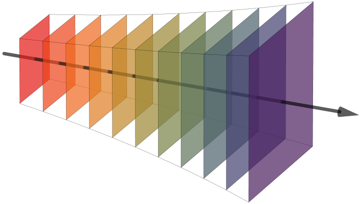
action $\int_M \mathcal{L}[A, F, g, ...]$

bulk e.o.m. :

$dF_{bulk} = 0$

$d{*}W_{bulk} = {*}J_{bulk}$

$+$ *other fields*



boundary field strength and conserved current

$$\mathcal{F} \sim F_{bulk}\big|_{\partial M} \quad \longleftrightarrow \quad \mathcal{A} \sim A_{bulk}\big|_{\partial M}, \; \imath_n A_{bulk} \equiv 0$$

$$\mathcal{J} \sim \imath_n W_{bulk}$$

regularity condition in the IR: $\Phi[\mathcal{A}, \mathcal{J}, \dots] \equiv 0$

$$\longrightarrow \quad \mathcal{J}^i[\mathcal{A}_0 + \delta\mathcal{A}] = \mathcal{J}^i[\mathcal{A}_0] + G^{ij}\delta\mathcal{A}_j + C^{ijk}\delta\mathcal{A}_j\delta\mathcal{A}_k + ...$$

Boundary Induction Tensor

- electromagnetism is *per definitionem* a theory for a dynamical $U(1)$ gauge field, $\mathcal{S} = \int_{\mathcal{M}} \mathcal{L}[d\mathcal{A}, \mathcal{A}, \ldots]$
- Euler – Lagrange eq. for such a theory are generic

$$d\mathcal{F} \;=\; 0\,, \qquad d\star\mathcal{W} \;=\; \star\mathcal{J}_{ext}$$

consistency with established holographic correspondence for $\mathcal{F}$ and $\mathcal{J}$ demands to identify $\mathcal{W}$ via

$$\mathcal{J} \;=\; \star^{-1} d\star(\mathcal{F} - \mathcal{W})$$

[U. Gran, M. Aronsson, TZ 2017]

- reproduces identities like $\left(\epsilon - 1 + \frac{\sigma}{i\omega}\right)\cdot\boldsymbol{\mathcal{E}} \;=\; \frac{\boldsymbol{k}\times\boldsymbol{\mathcal{M}}}{\omega}$
- compatible with ϵ_L from *holographic optics*

[A. Amariti, D. Forcella, A. Mariotti, and G. Policastro 2010]
[D. Forcella, A. Mezzalira, and D. Musso 2014]
[L. Liu and H. Liu 2016]

Holographic Plasmons

Plasmon Condition

$$\epsilon_{xx}(\omega, k) \; = \; 0 \;\; \longleftrightarrow \;\; \boldsymbol{\mathcal{D}}_x = 0\,, \; \boldsymbol{\mathcal{E}}_x \neq 0$$

due to Maxwell equations, equivalent to

$$\boldsymbol{j}_x \; = \; \dot{\boldsymbol{\mathcal{E}}}_x$$

in terms of boundary quantities

$$\omega^2 \mathcal{A}_x - \mathcal{J}_x\big|_{\partial M} \; = \; 0$$

$\mathcal{J}$ is model-dependent in general, but in most holographic setups considered, $\mathcal{J} \sim \mathfrak{L}_n \mathcal{A}$

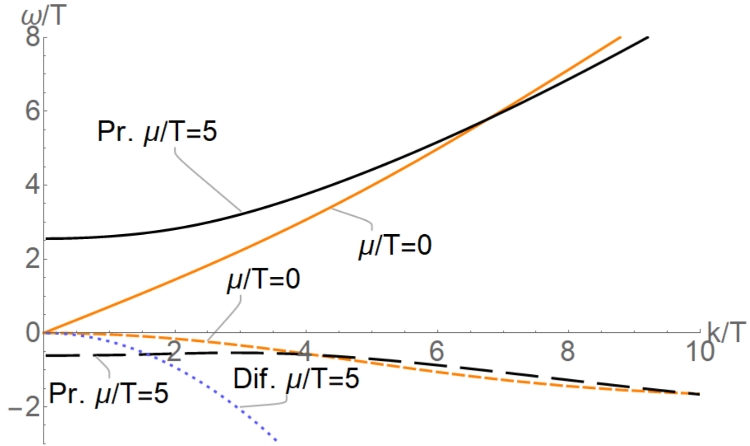
'*Holographic*' Plasmon Condition

$$\omega^2 \mathcal{A}_x - p(\omega, k) \cdot \mathcal{A}'_x\big|_{\partial M} \; = \; 0$$

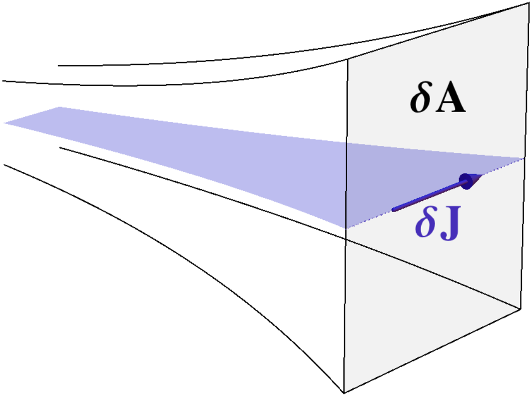
[U. Gran, M. Aronsson, TZ 2017]

III. Applications & Results

Holographic 'Bulk Plasmons' (RN)



'Lessons' for Holographic Graphene



$\longrightarrow$ if relevant, b.c. has to incorporate that $\delta\mathcal{A}$ can permeate more dimensions than $\delta\mathcal{J}$

Toy Model for Holographic Surface Plasmons

- a perturbation of the internal charge density $\delta\rho$ must be related to a change in the potential

$$\delta\phi(t, \boldsymbol{r}) \;=\; \int \mathrm{d}^3 r' \, \frac{\delta\rho(t, \boldsymbol{r}')}{4\pi |\boldsymbol{r} - \boldsymbol{r}'|}$$

- after Fourier transforming

$$\delta\phi(\omega, \boldsymbol{k}) \;=\; \frac{1}{\boldsymbol{k}^2}\delta\rho(\omega, \boldsymbol{k}) \iff \omega^2 \delta A_x + \delta J_x \;=\; 0$$

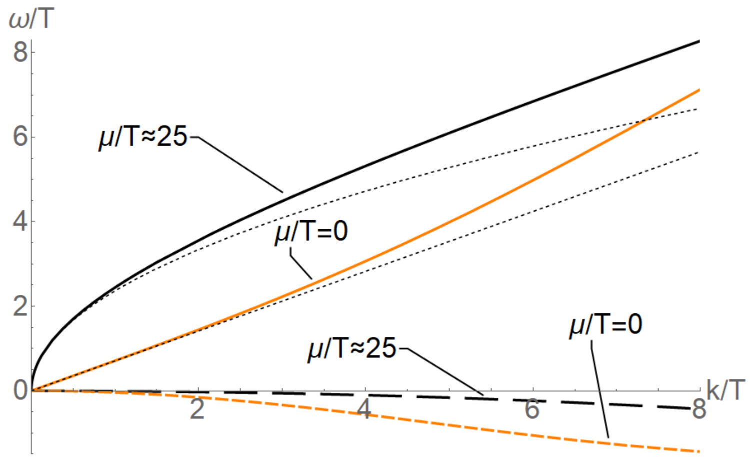
- same calculation for $\delta\rho = \delta(z)\delta\sigma$

$$\delta\phi(\omega, \boldsymbol{k}) \;=\; \frac{1}{2|\boldsymbol{k}|}\delta\sigma(\omega, \boldsymbol{k})$$

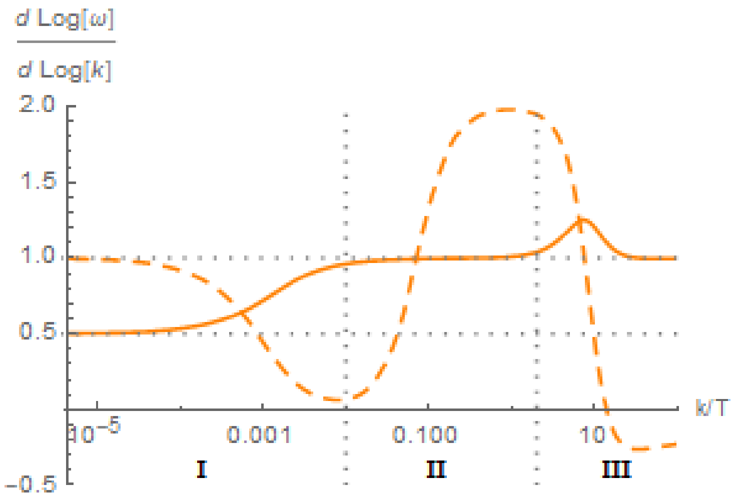
$\longrightarrow$ include 'correction factor' $p = \frac{|\boldsymbol{k}|}{2}$ into plasmon condition

[U. Gran, M. Aronsson, TZ 2018]

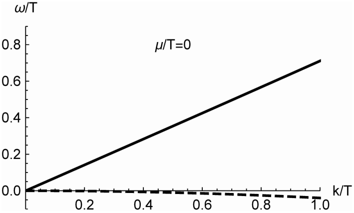
co-dim 1 Dispersion Relation

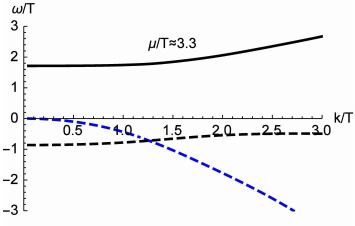


co-dim 1 Dispersion Relation

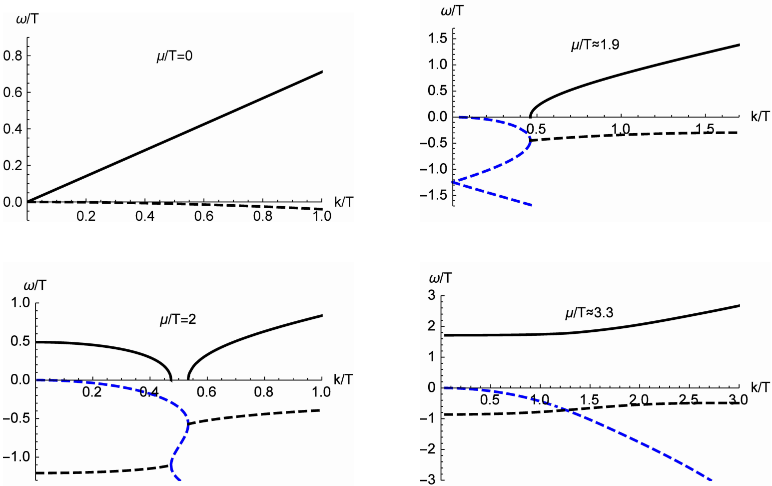


Transition to Gapped Dispersion





Transition to Gapped Dispersion



cf. Quantum Dissipation ?

1^{st} *order approximation* + *corrections*

- dispersion from 'naive' Caldeira – Leggett model approach

$$\overbrace{\omega^2 - c^2 k^2}^{\text{kinetic term}} + \underbrace{i\Gamma(T, \mu, \ldots)\omega}_{\text{damping}} + \overbrace{\upsilon(T, \mu, \ldots)}^{\text{potential}}$$

  damping/dissipation due to coupling to a bath (of harmonic oscillators) which is integrated out

- 'exotic' dispersion in transitional stages would require a more intricate combination of bath and potential, *e.g.*

$$\omega^2 - c^2 k^2 + \frac{\upsilon + i\Gamma\omega}{1 + i\Upsilon\omega}$$

IV. Summary & Outlook

Summary

- extended the holographic dictionary to consistently identify polarization and magnetization in the effective dual field theory on the boundary
- plasmons, *i.e.* poles in the 'physical' response function χ
- identified ways to improve the holographic description of 2-dim. (resp. co-dim. 1) systems

Outlook

- further refinements of the model
  - more sophisticated bulk setup
  - effect of couplings to A and F, or polarization, in the bulk
- investigate the 'microscopics' of the boundary theory
  - form factors
  - sum rules