Utrecht University

More on
Turbulent Strings in AdS

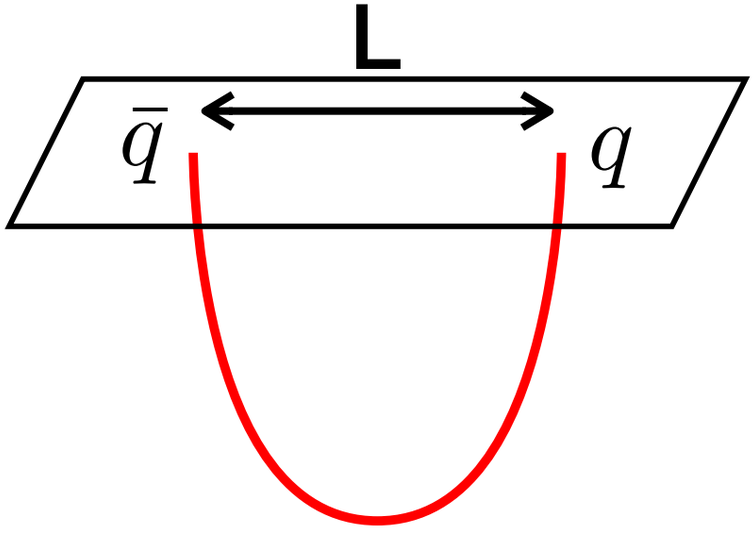
Takaaki Ishii
Utrecht University

1504.02190, and to appear
w/ Keiju Murata, Kentaroh Yoshida

31 Jul 2018@Gauge/Gravity Duality 2018, Wurzburg

Contents

Holographic $q\bar{q}$ potential



Probe open string in AdS

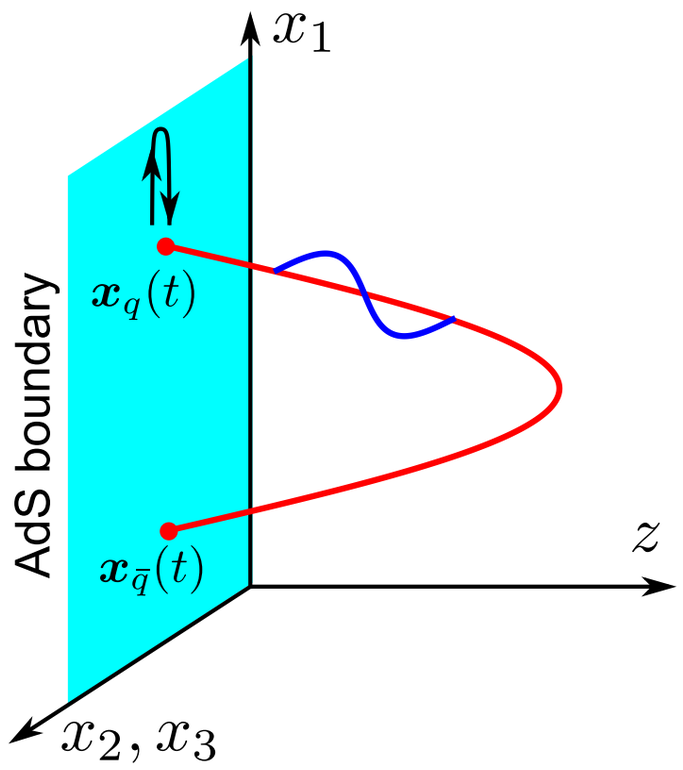
$$S = -\frac{1}{2\pi\alpha'}\int d^2\sigma\sqrt{-\det\gamma_{ab}}$$

Put this in e.g. Poincare AdS

$$ds^2 = \frac{1}{z^2}\left(-dt^2 + dz^2 + d\boldsymbol{x}^2\right)$$

This configuration is stable against linear perturbations.

Turbulent strings

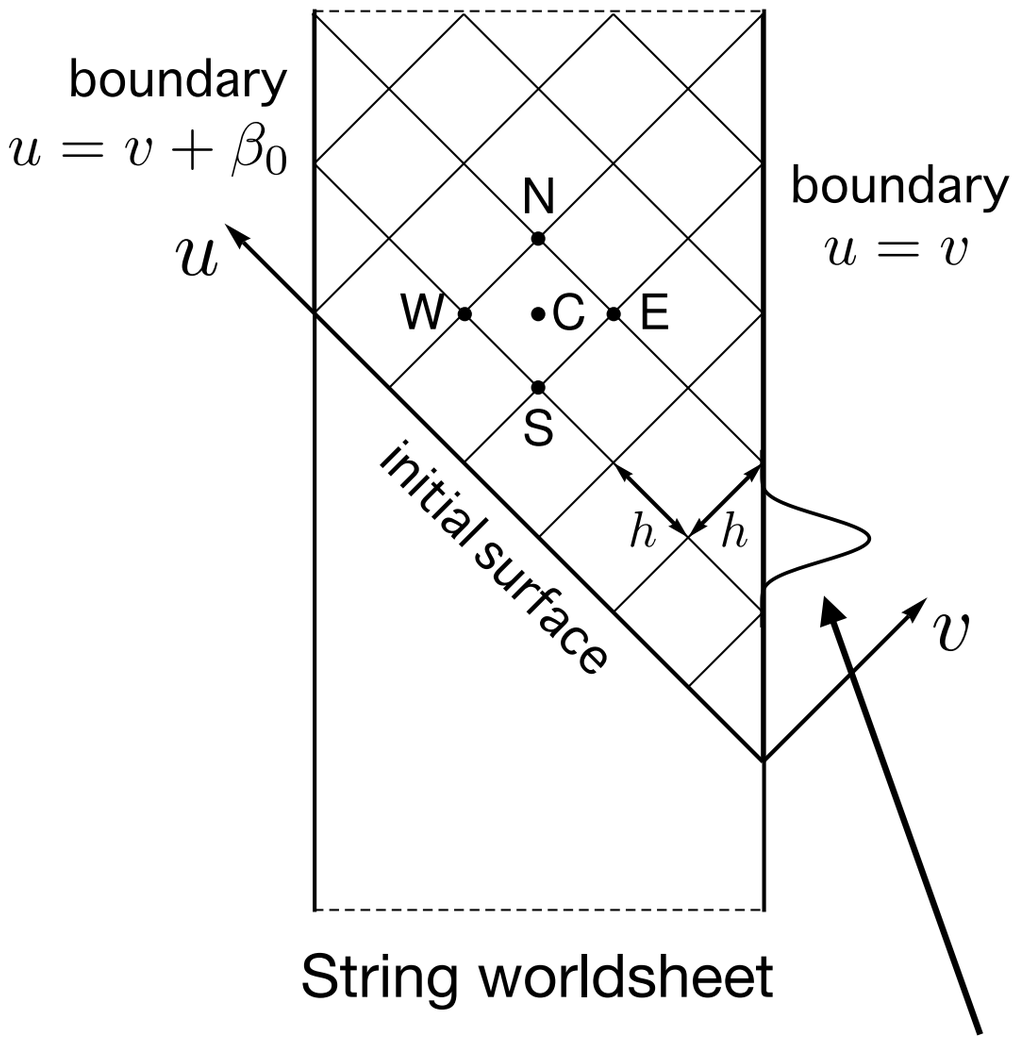


We perturb the qq̄-string **nonlinearly**.

- **Quench** by moving the endpoints.

- Time evolution of **small perturbations**.

Formalism



String worldsheet

Use worldsheet light-cone coord

$$ds^2 = 2\gamma_{uv} du dv$$

and solve the string EOMs

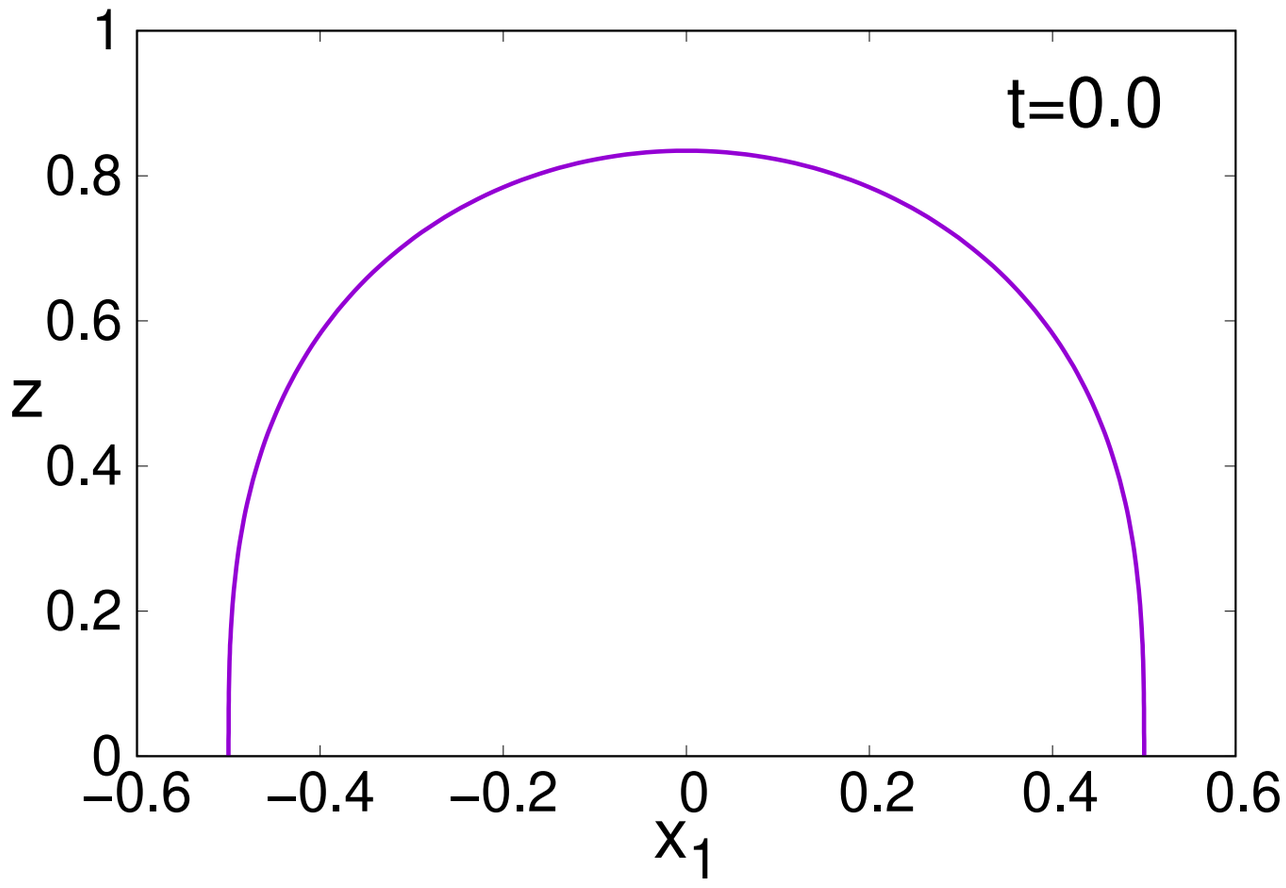
$$T_{,uv} = \frac{1}{Z}(T_{,u}Z_{,v} + T_{,v}Z_{,u})$$

$$Z_{,uv} = \frac{1}{Z}(T_{,u}T_{,v} + Z_{,u}Z_{,v} - \boldsymbol{X}_{,u} \cdot \boldsymbol{X}_{,v})$$

$$\boldsymbol{X}_{,uv} = \frac{1}{Z}(\boldsymbol{X}_{,u}Z_{,v} + \boldsymbol{X}_{,v}Z_{,u})$$

Boundary perturbation in
0<t<Δt with amplitude ε

String time evolution

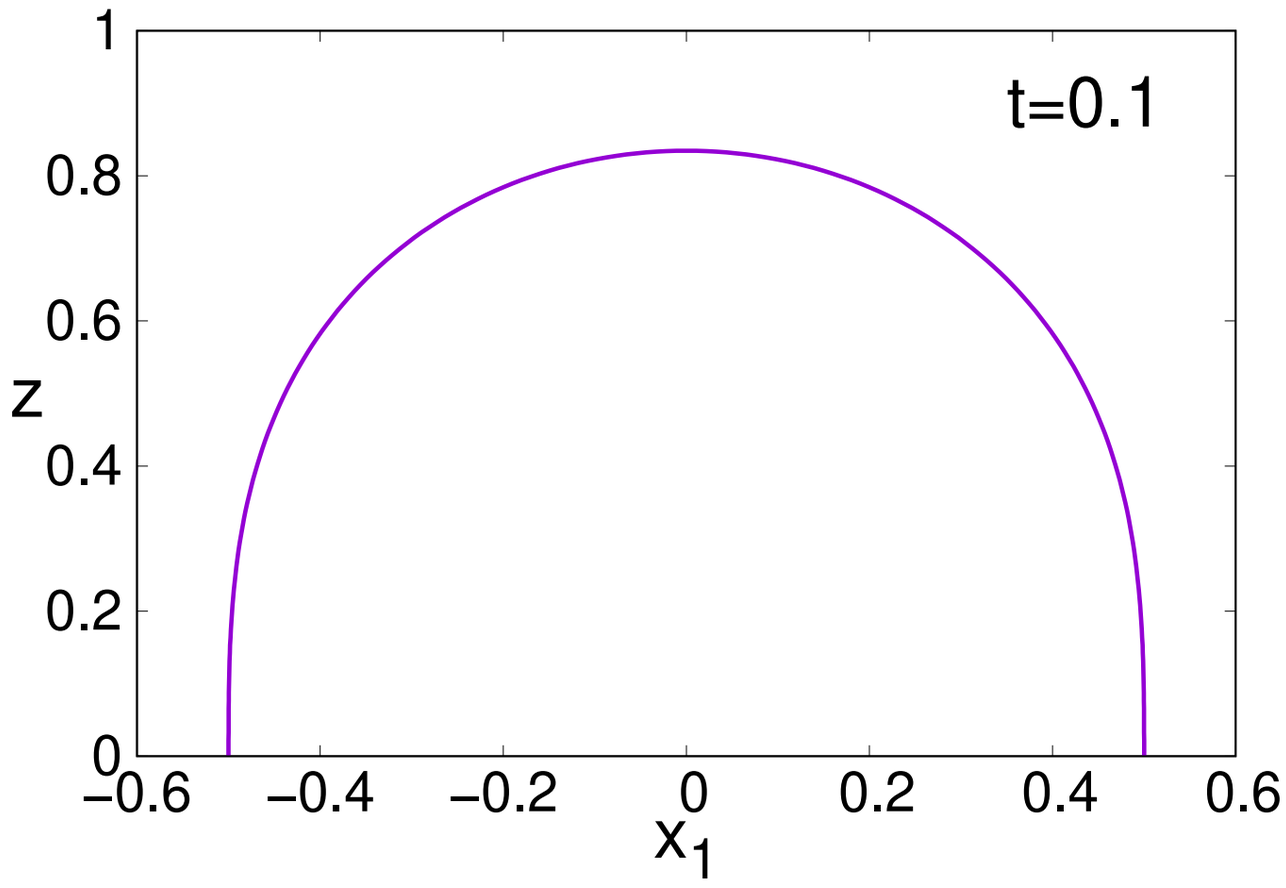


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

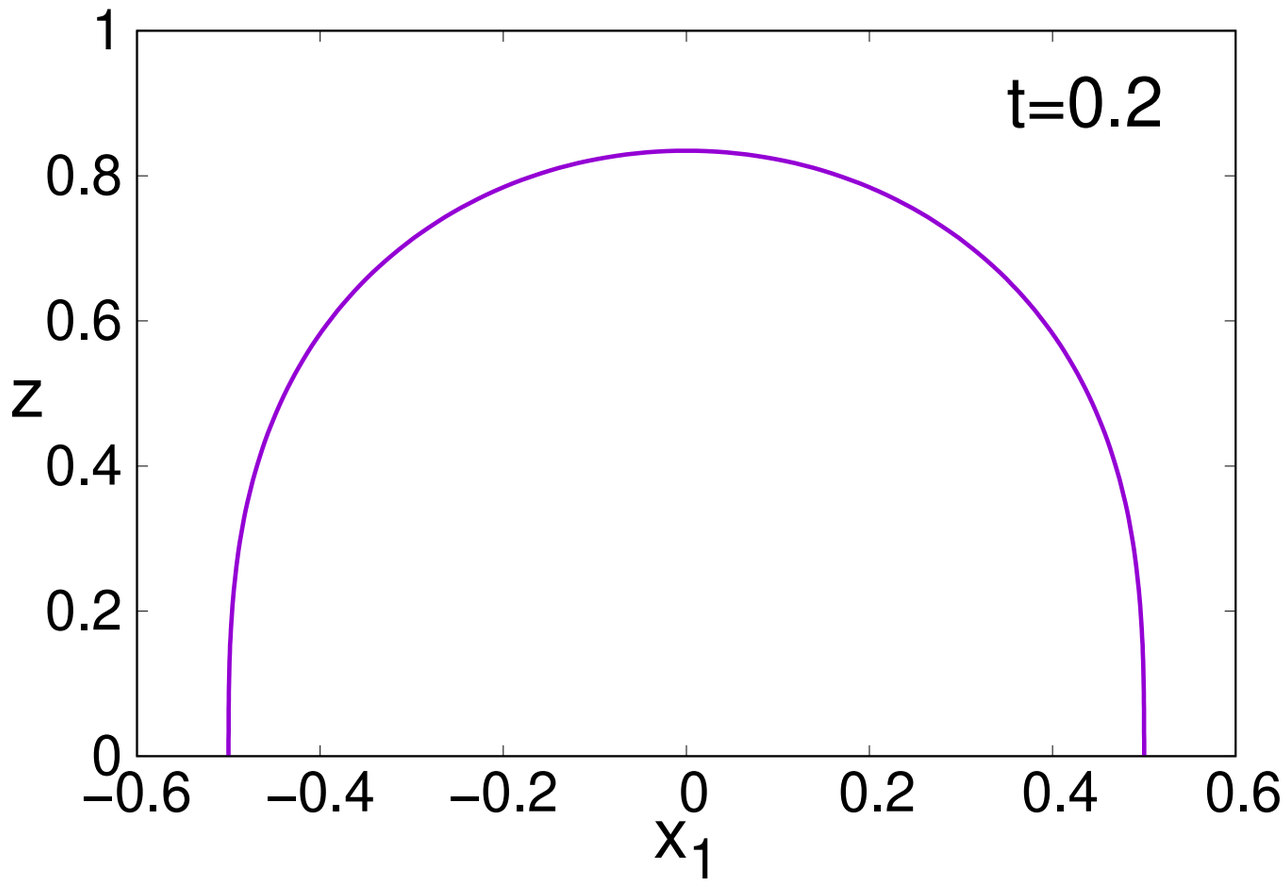


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

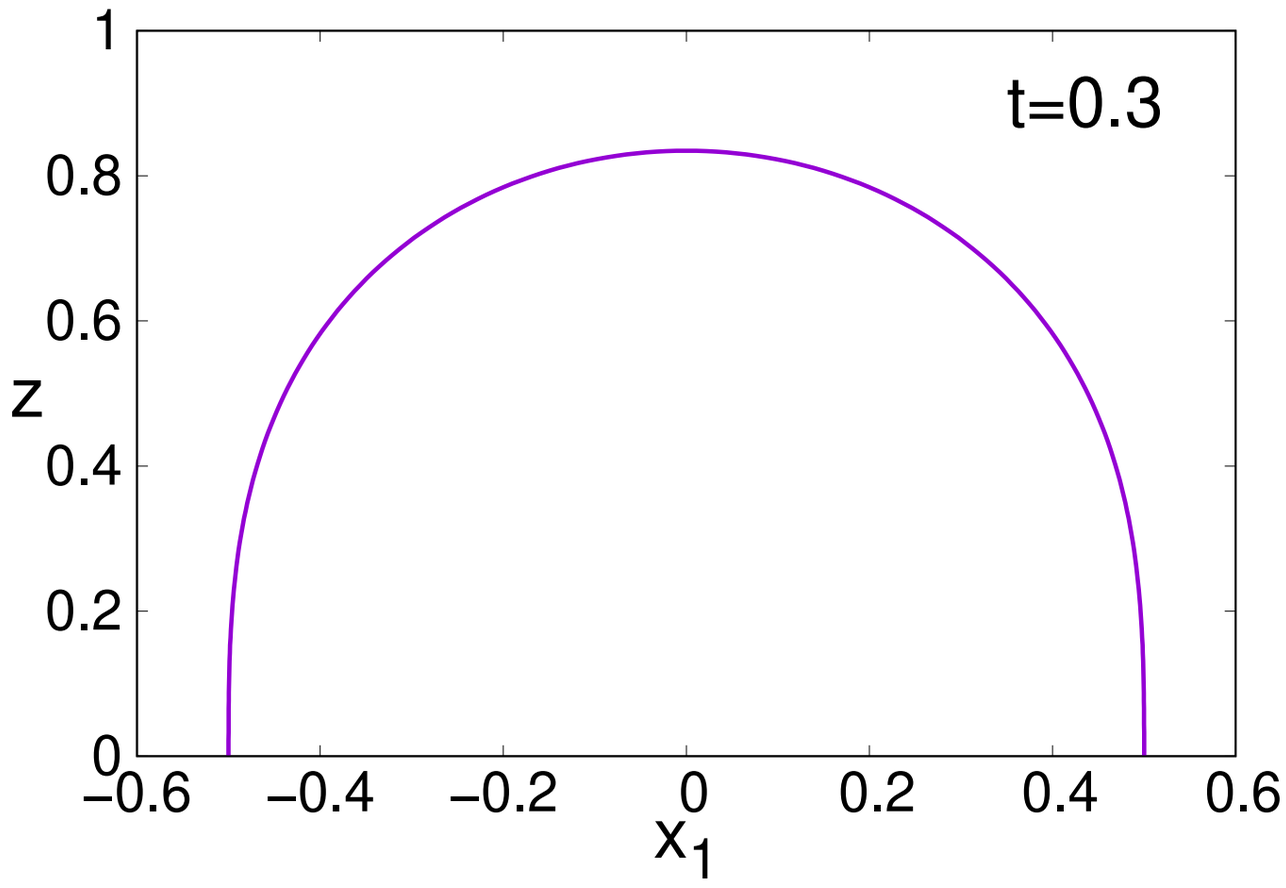


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

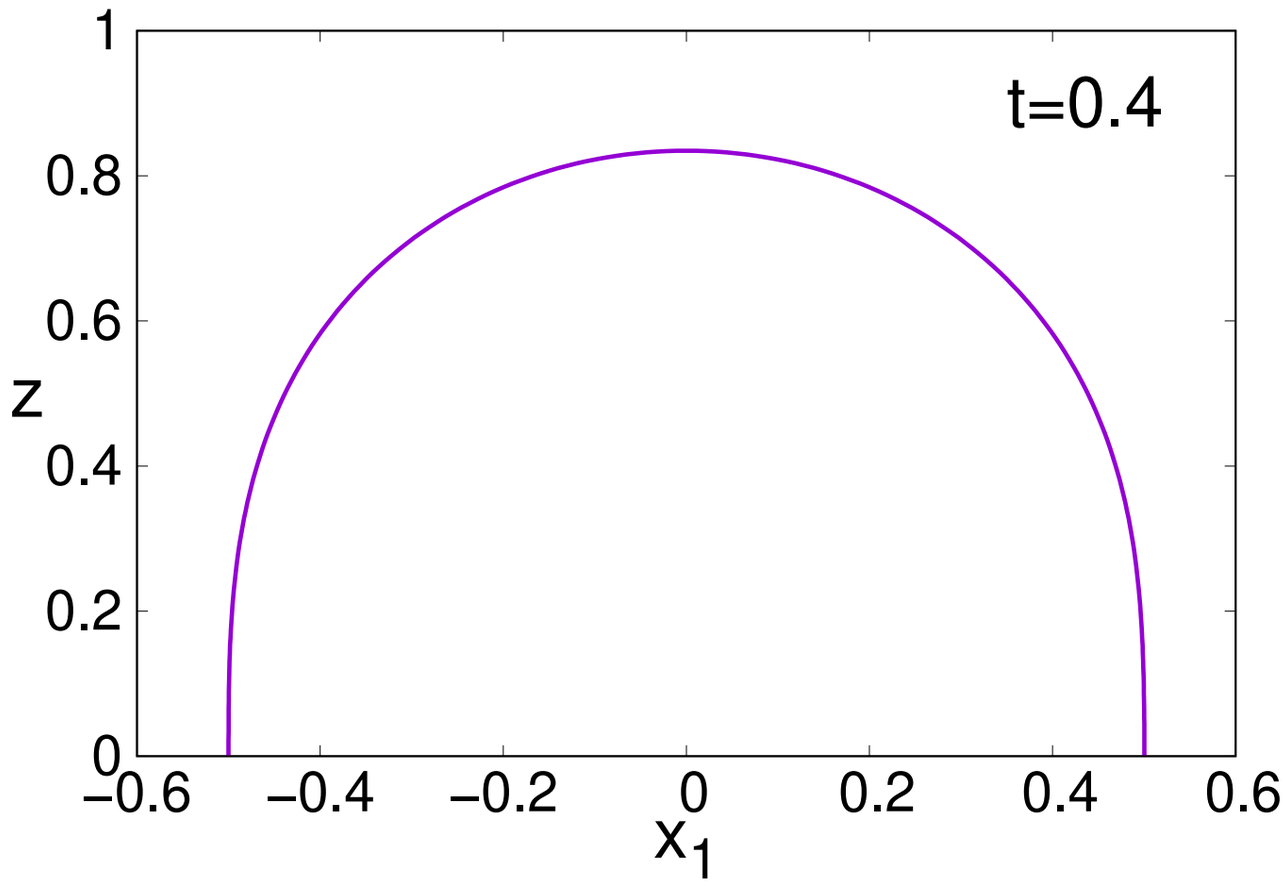


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

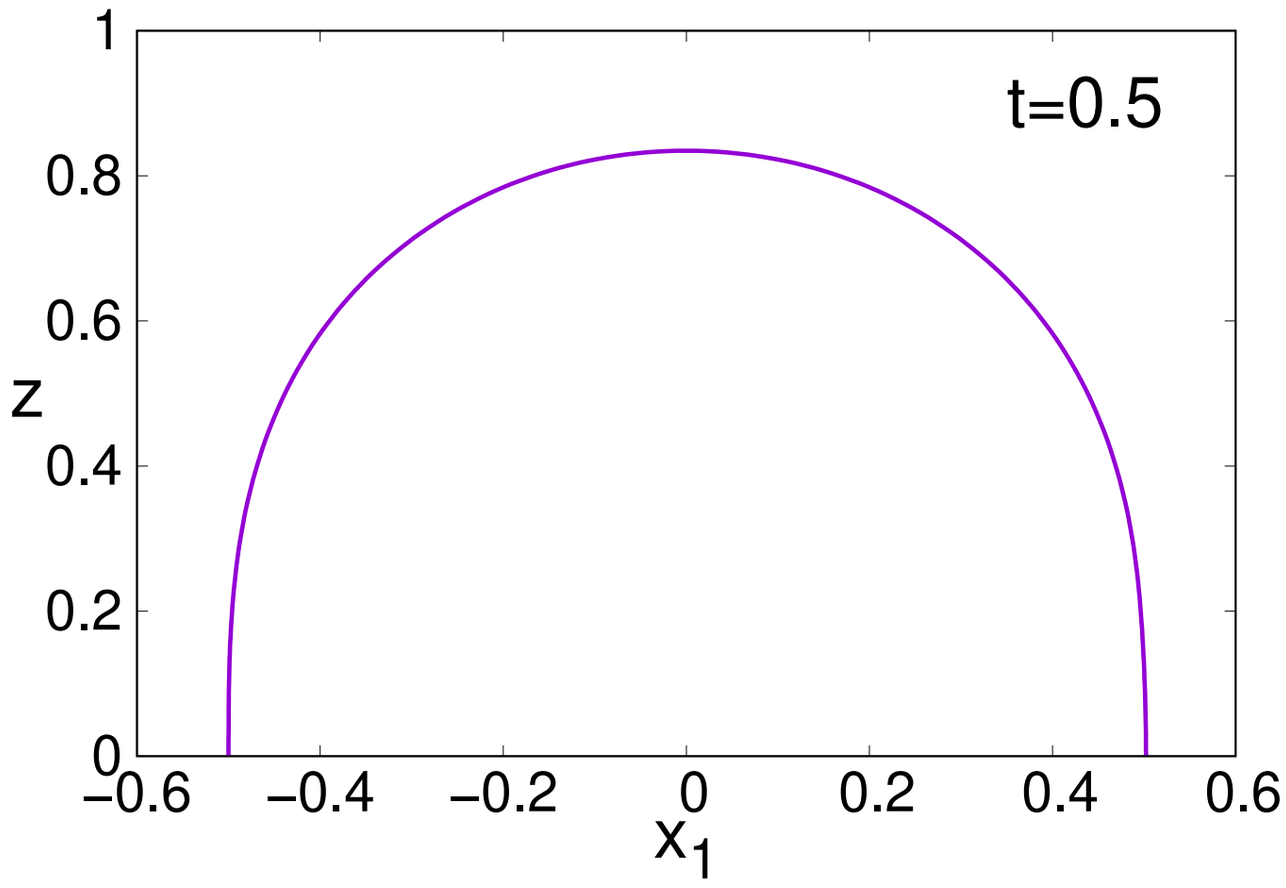


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

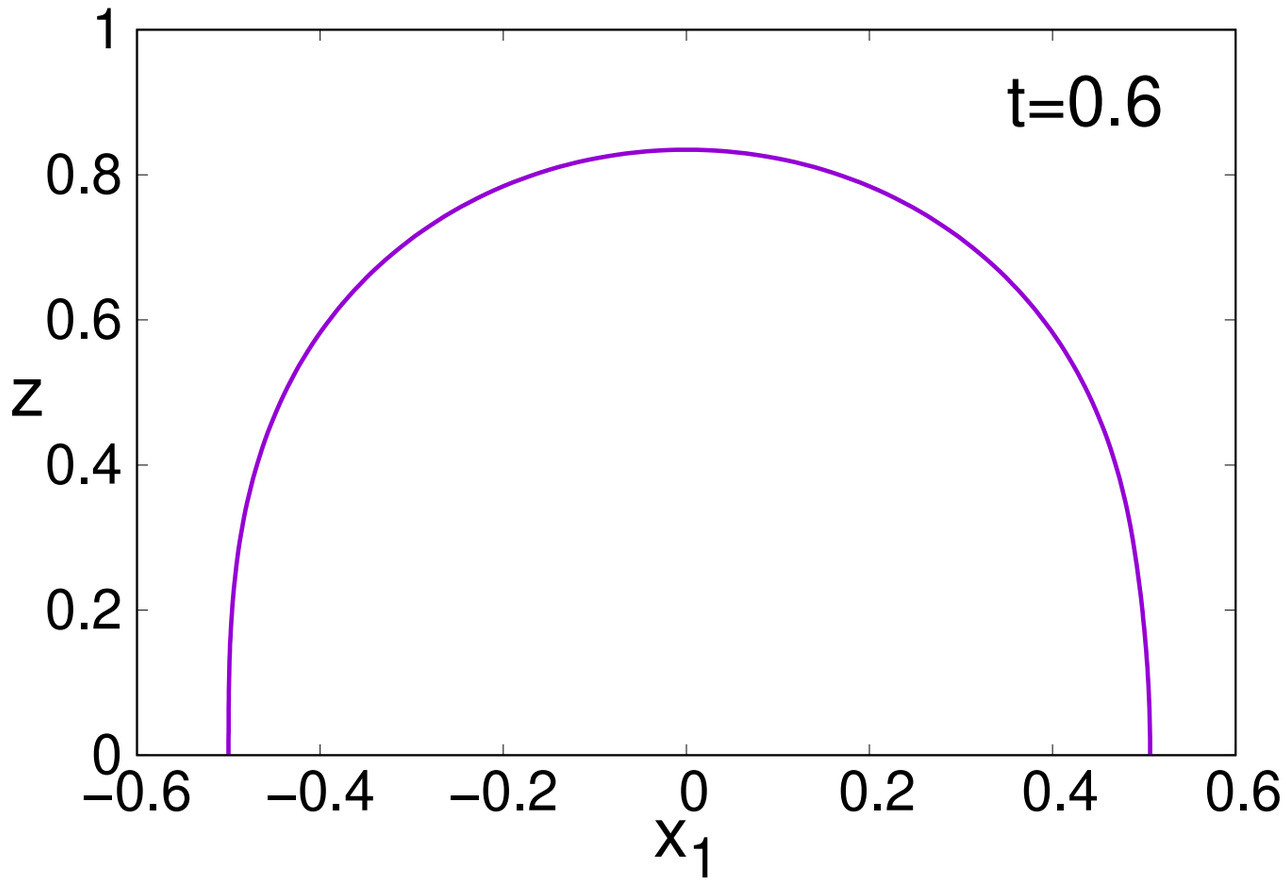


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

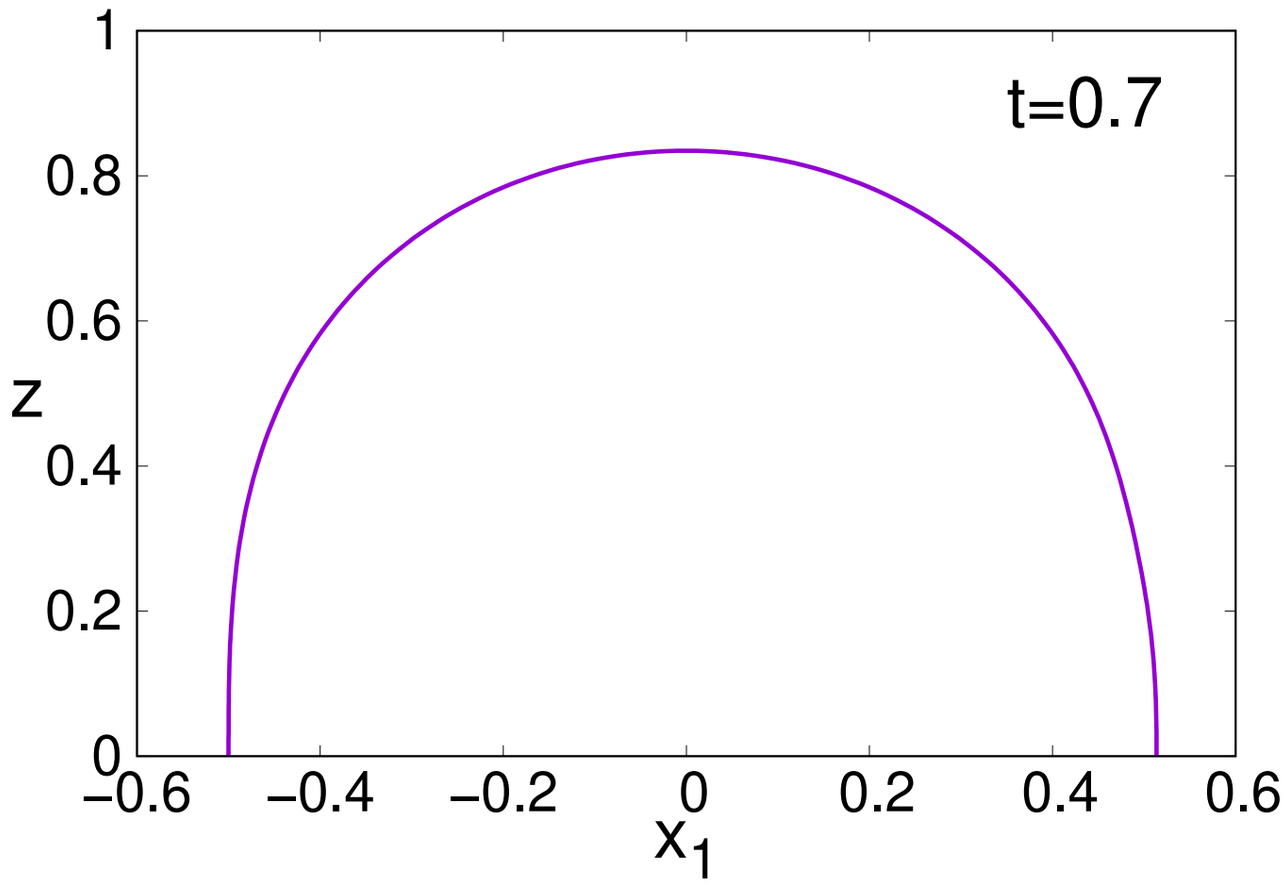


String motion in (t,z,x_1): AdS3 $\subset$ AdS5

Quench: $0<t<2$

Amplitude: $\varepsilon=0.03$

String time evolution

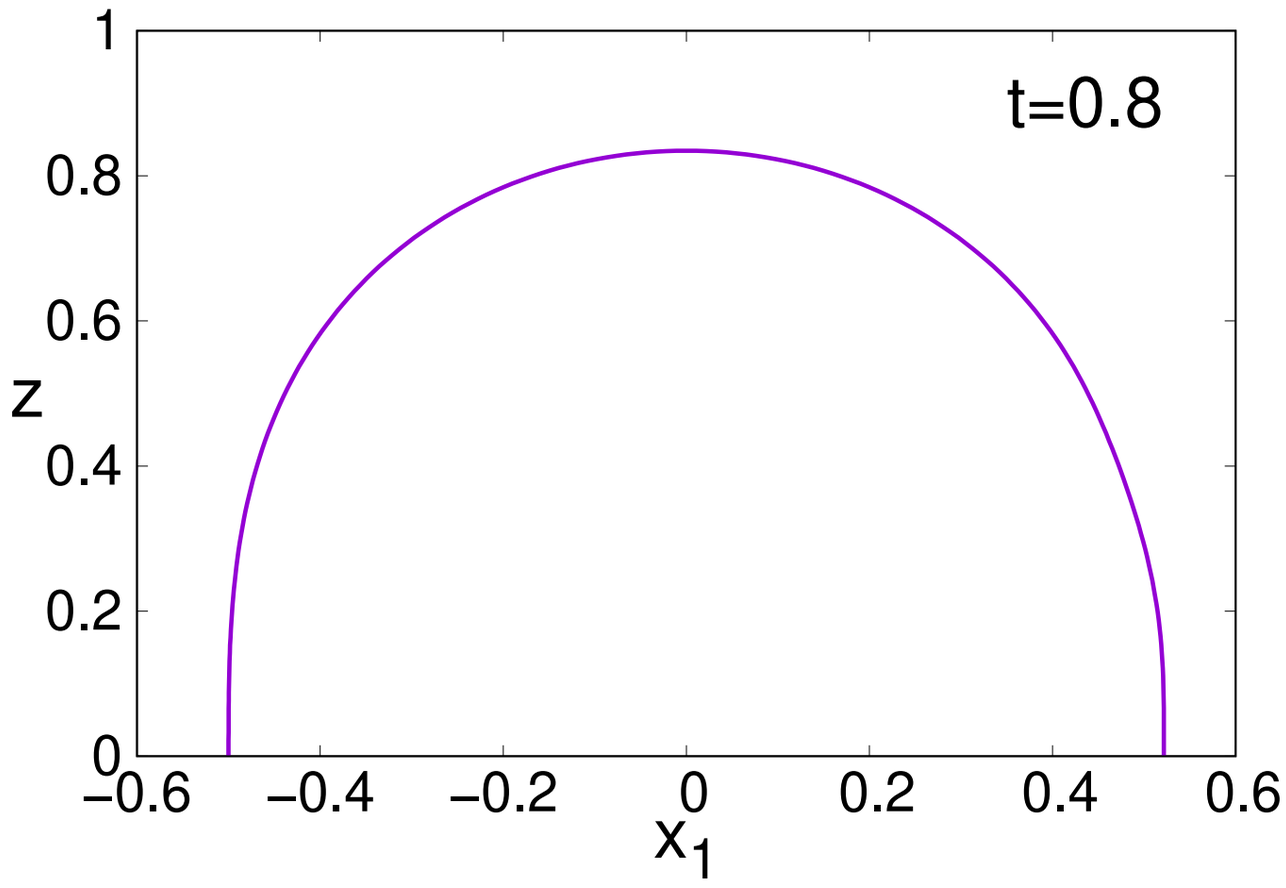


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

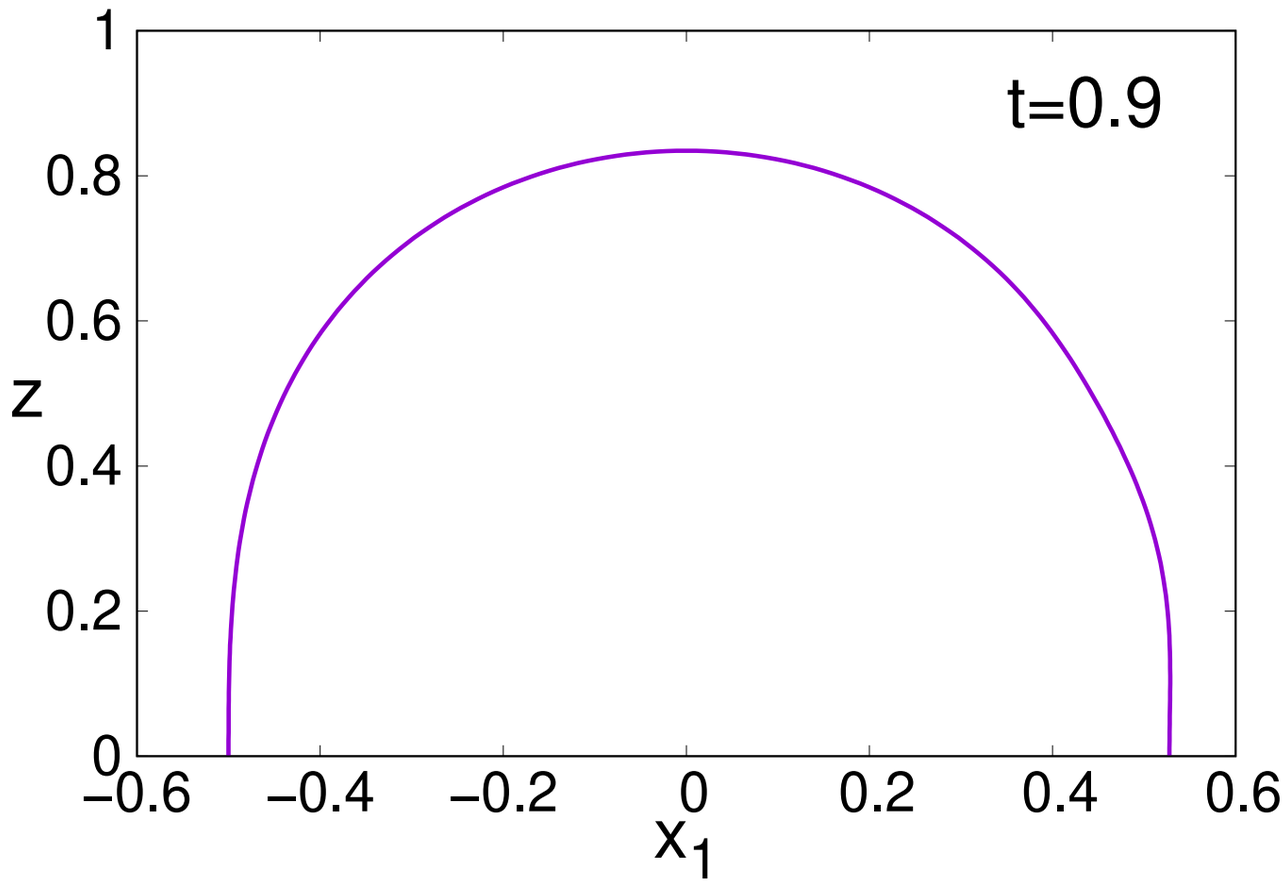


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

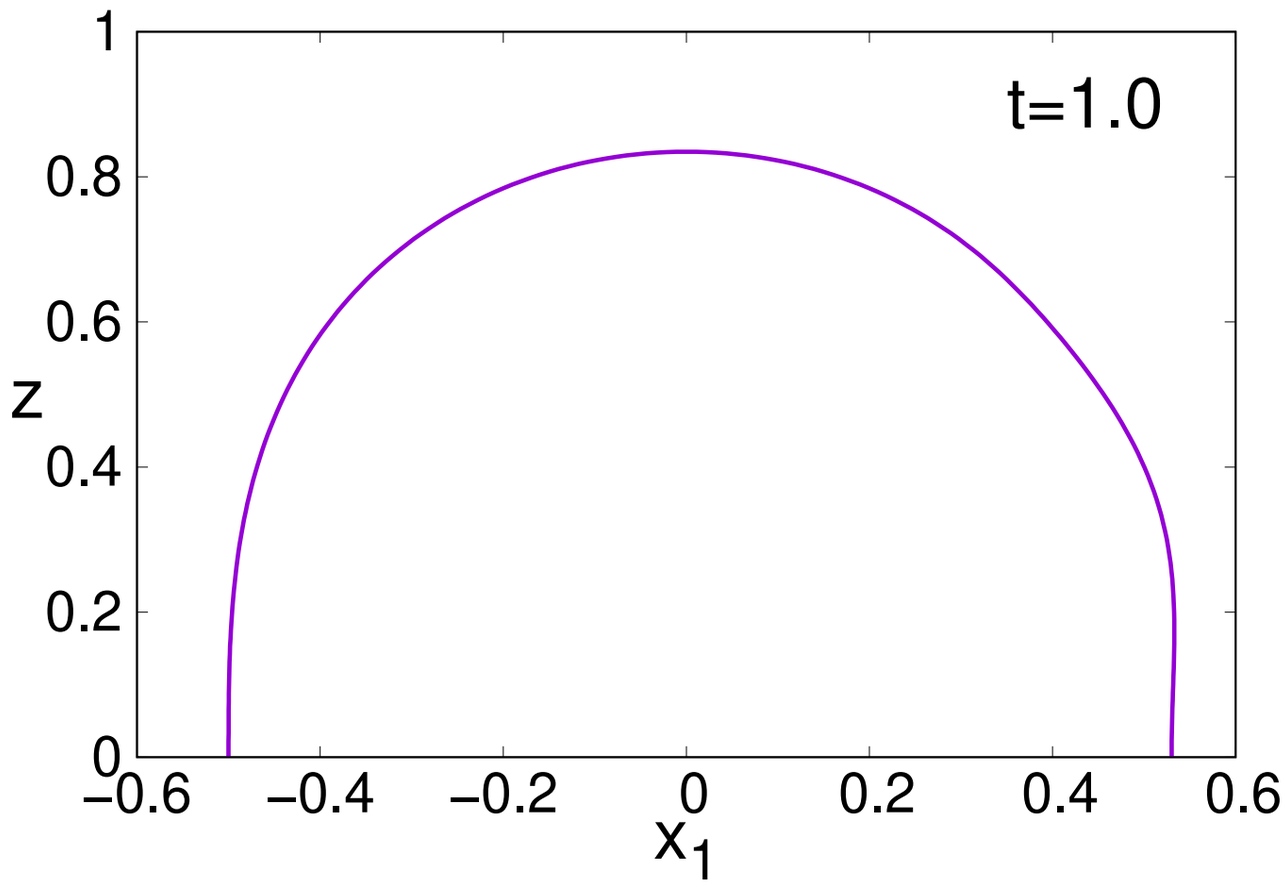


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

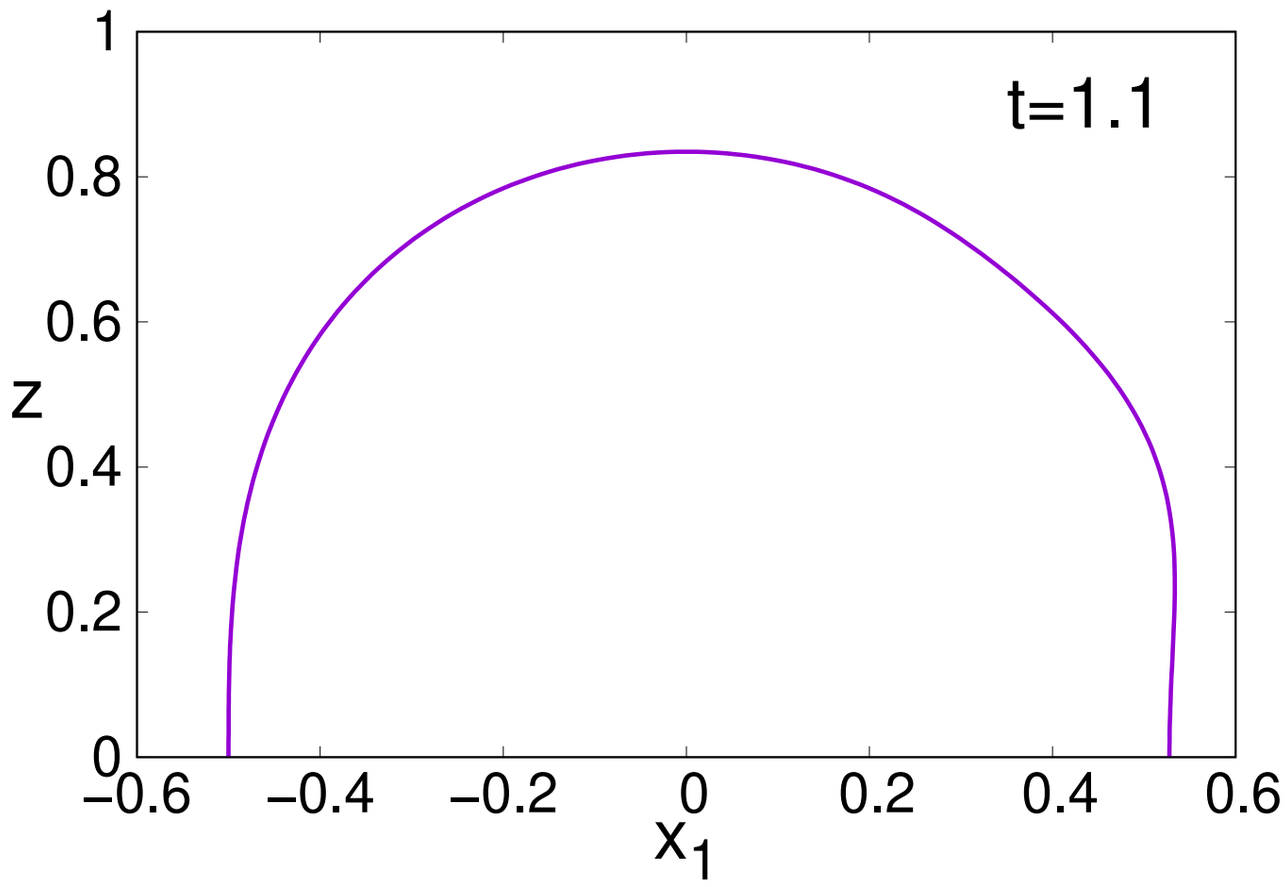


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

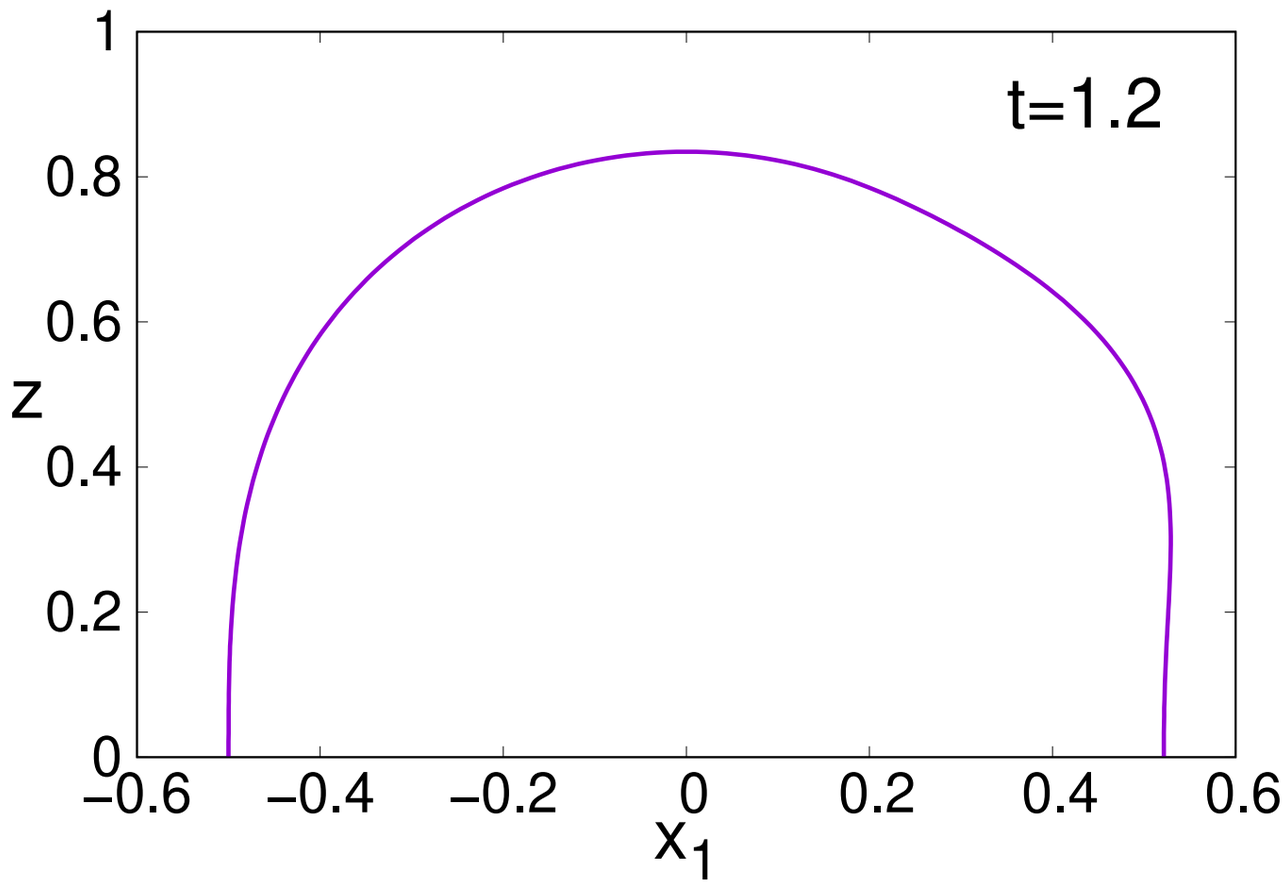


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

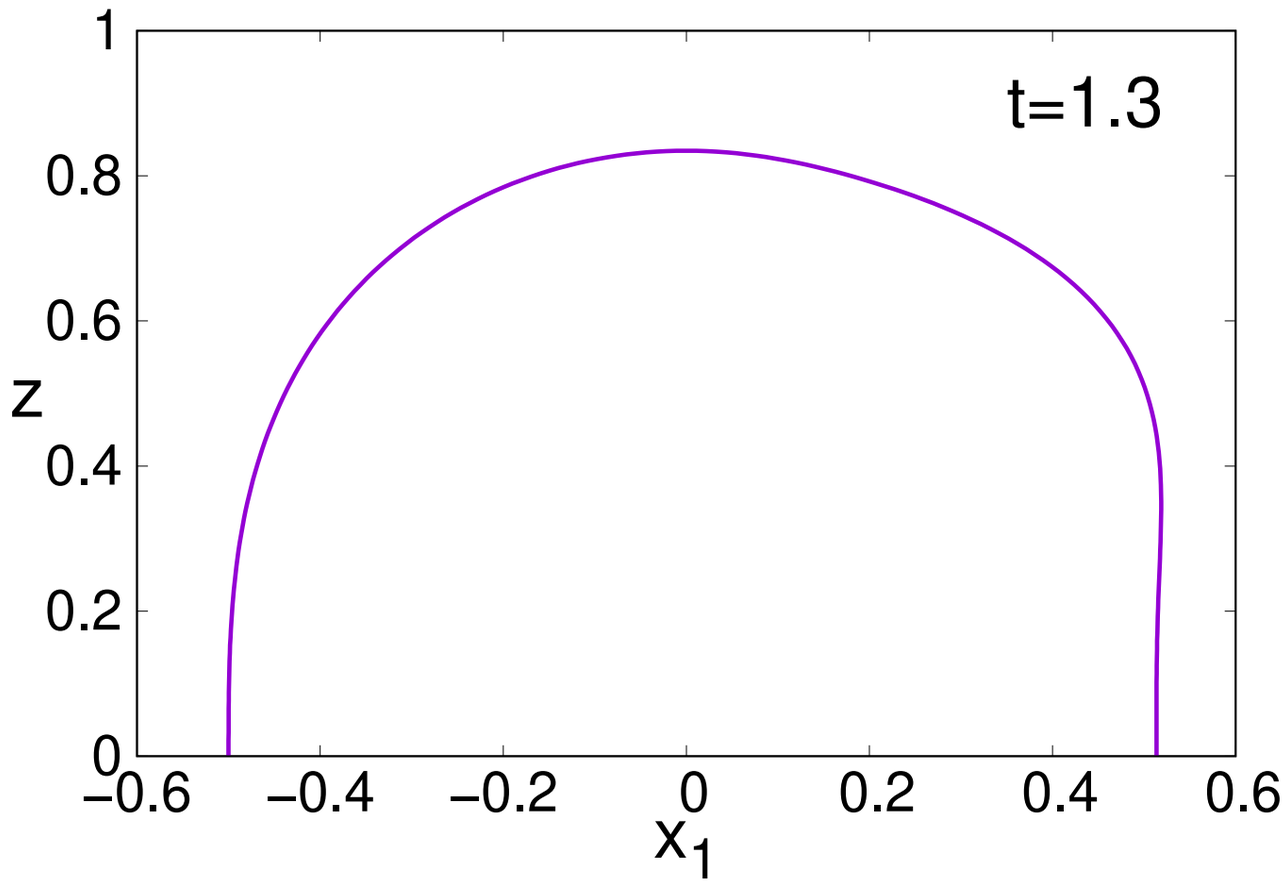


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

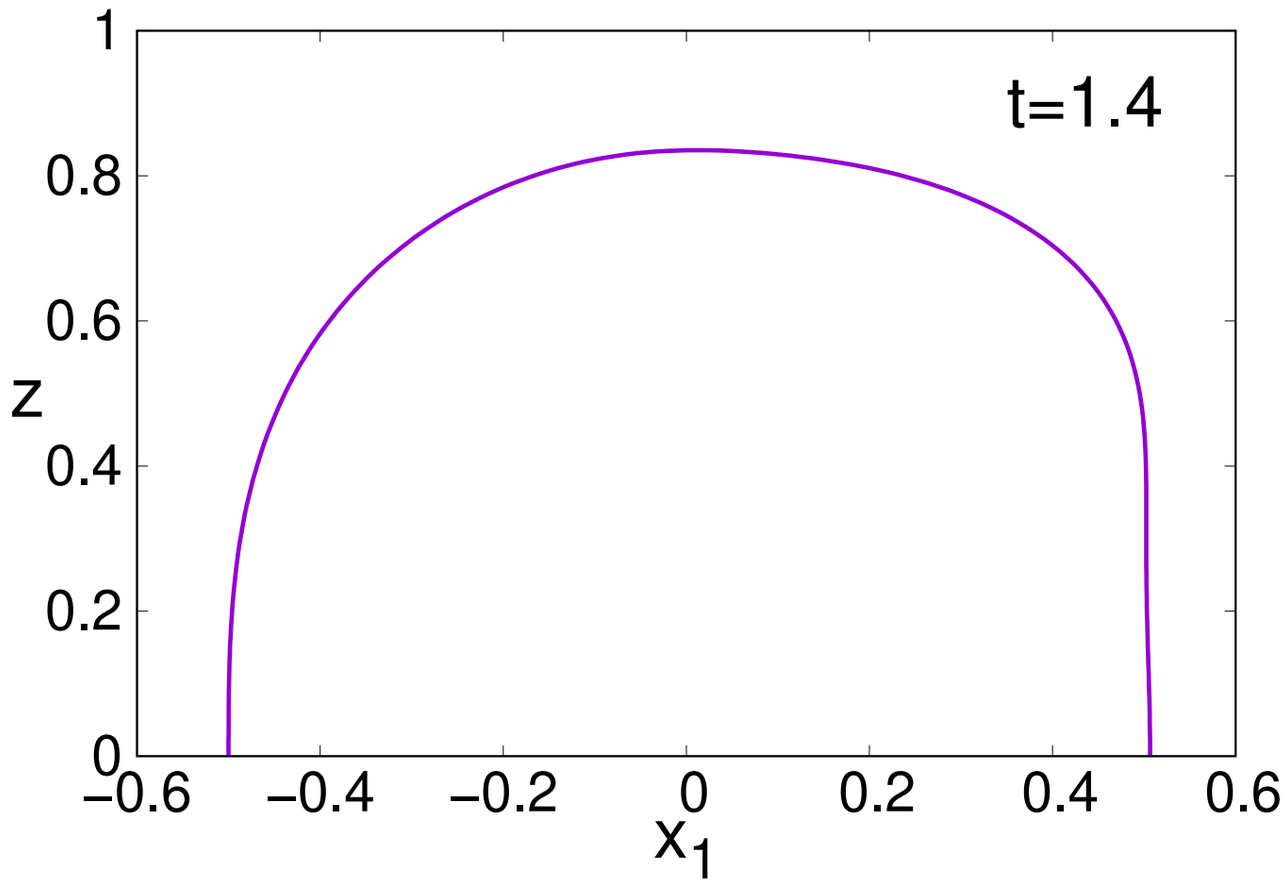


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

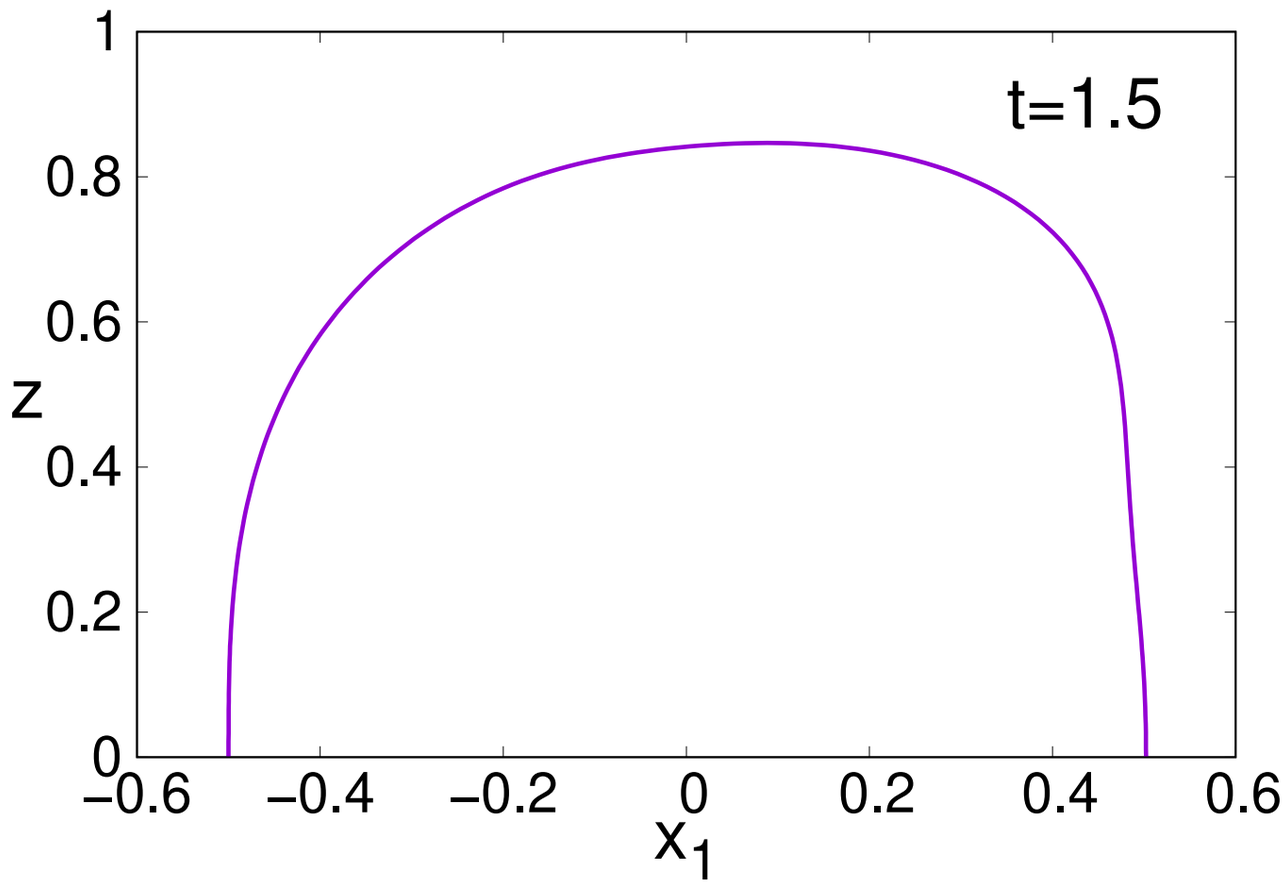


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

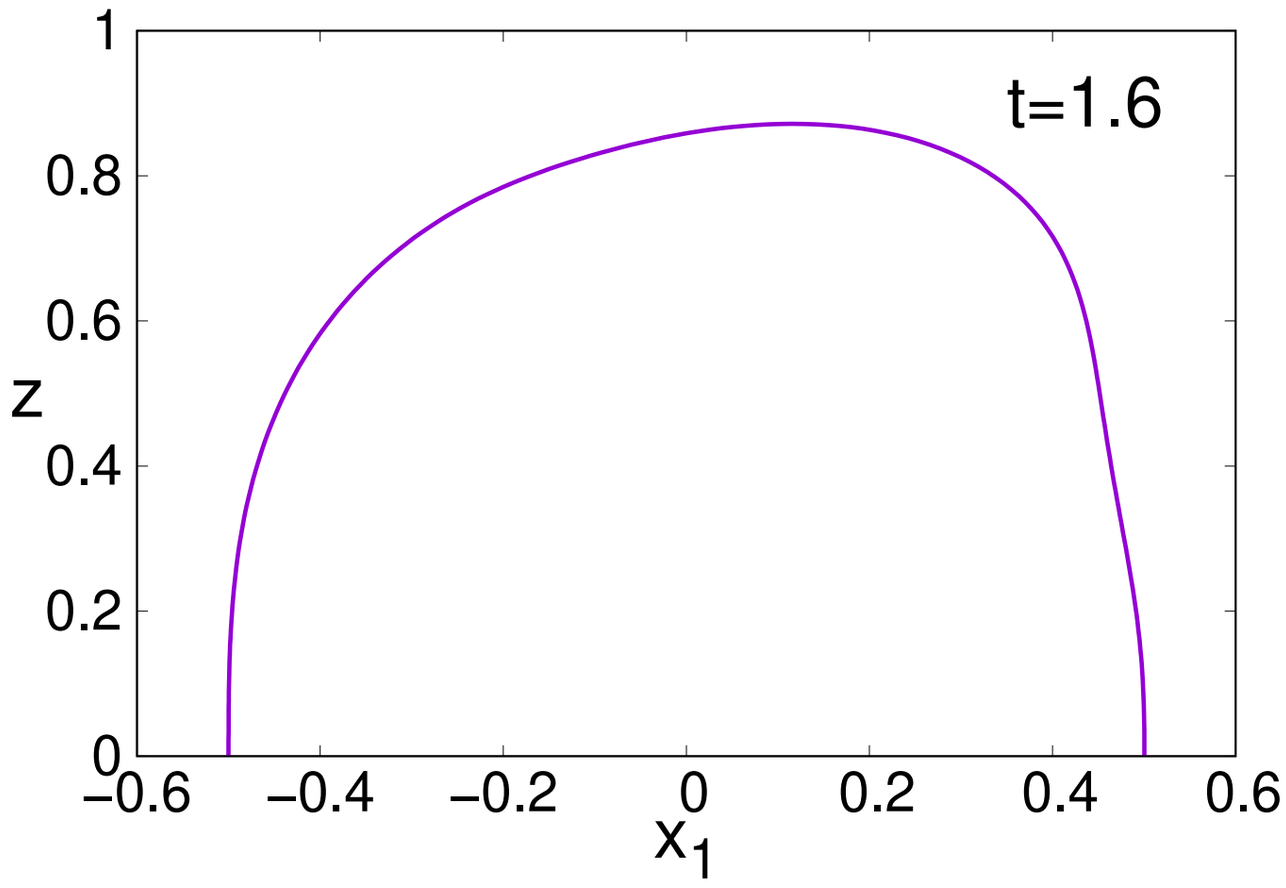


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

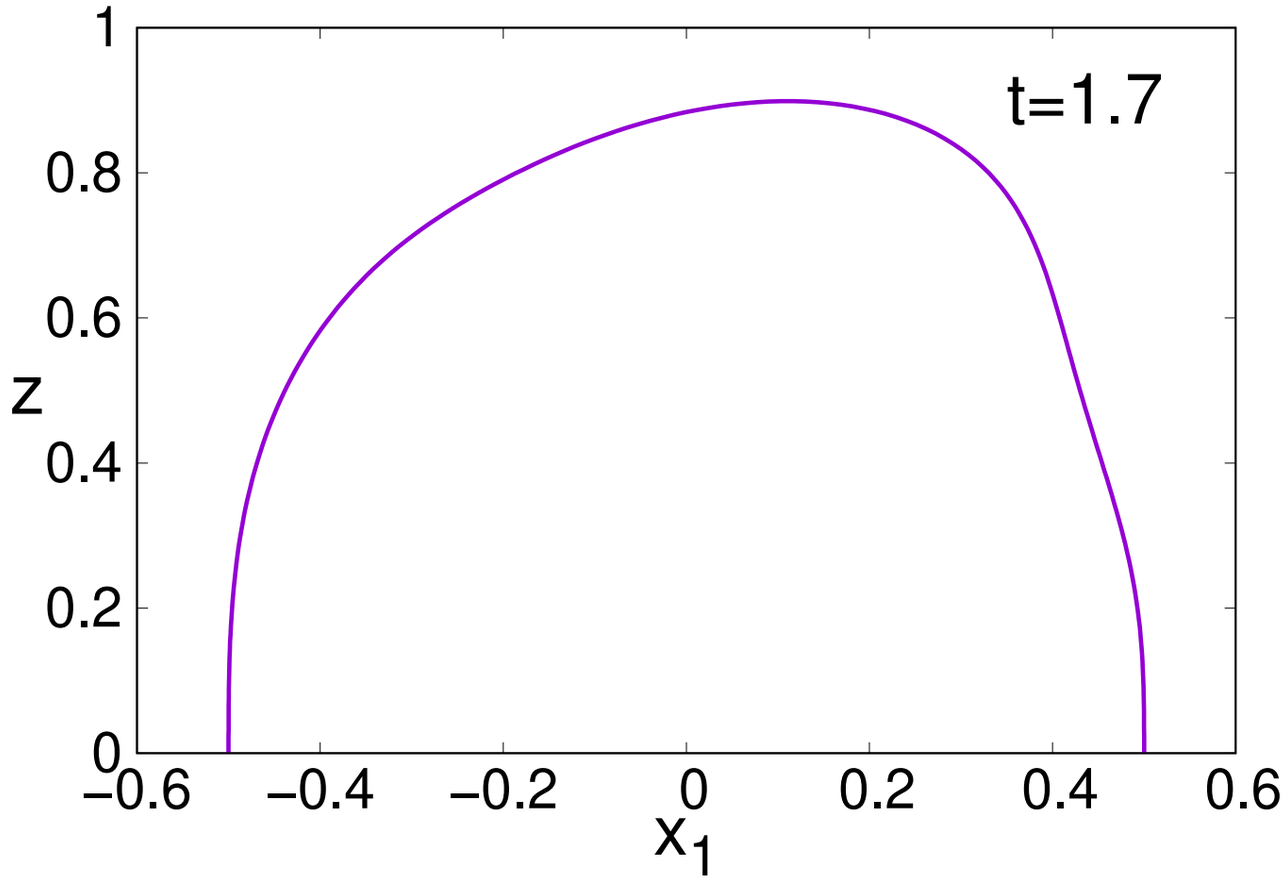


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

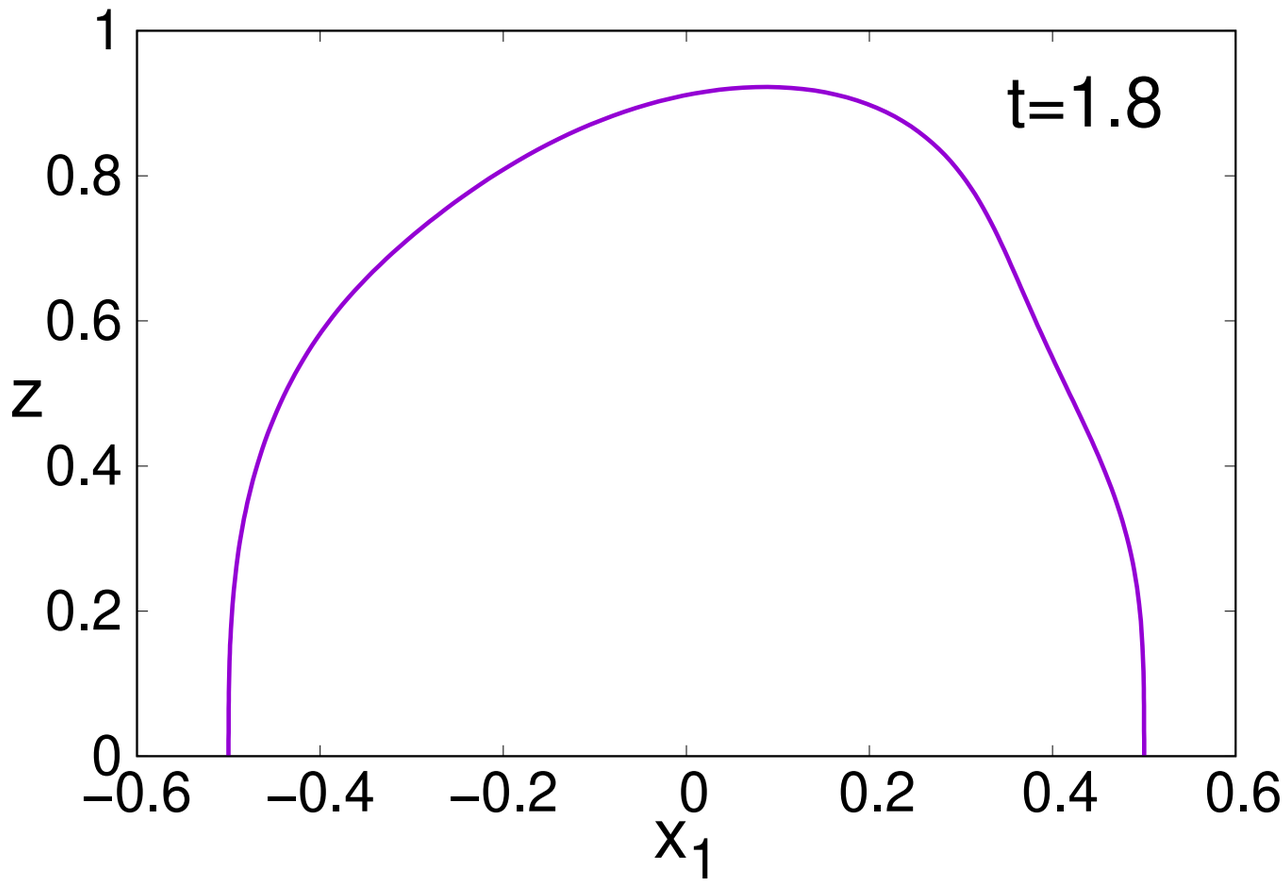


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

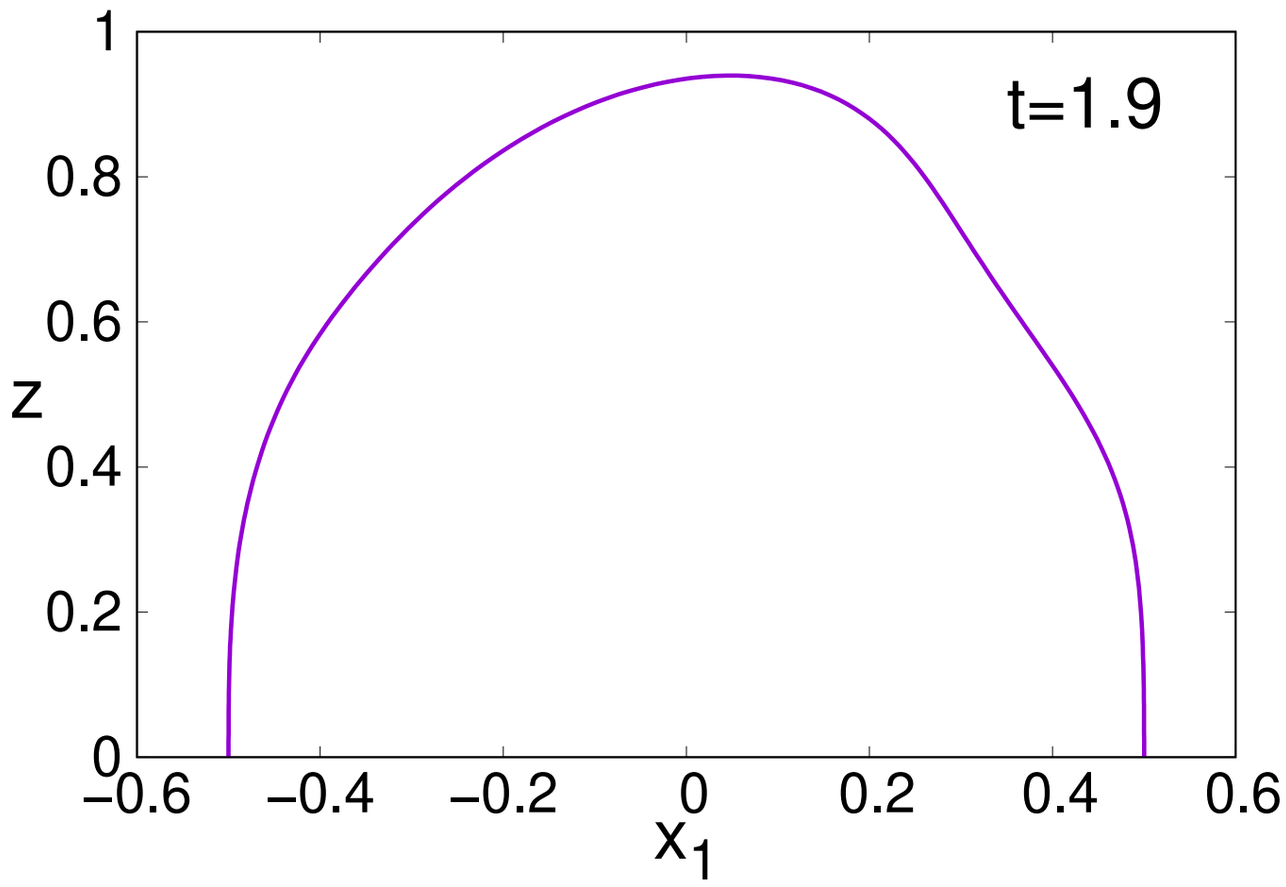


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

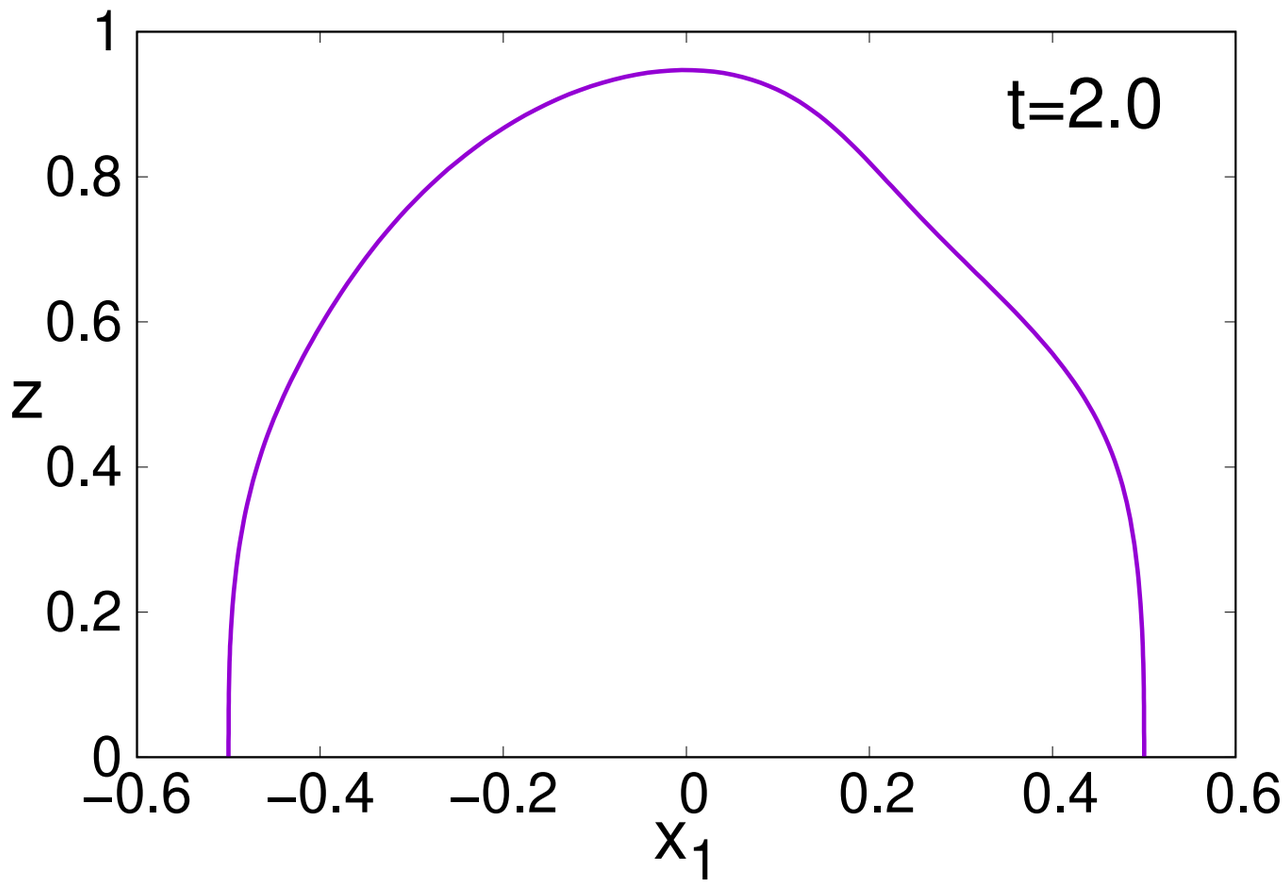


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

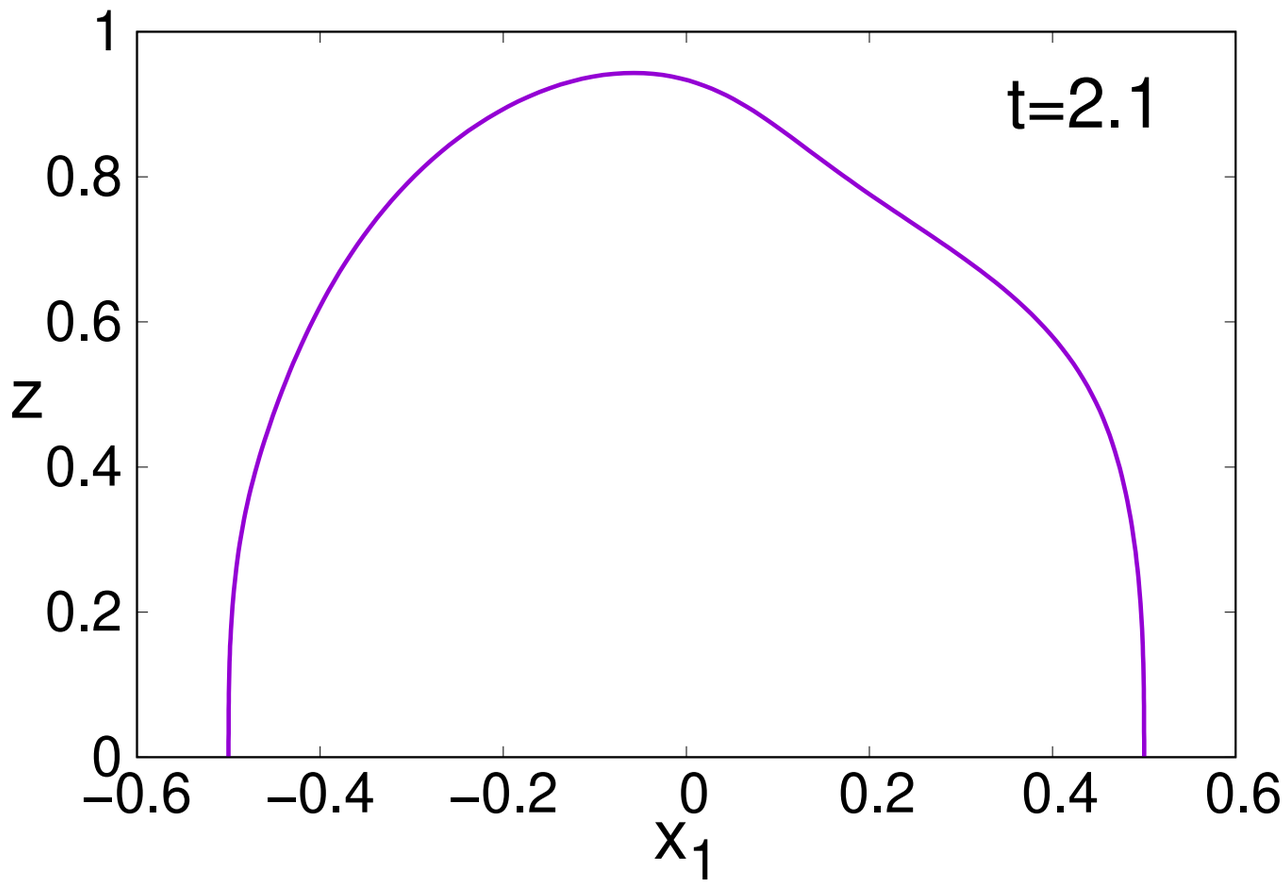


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

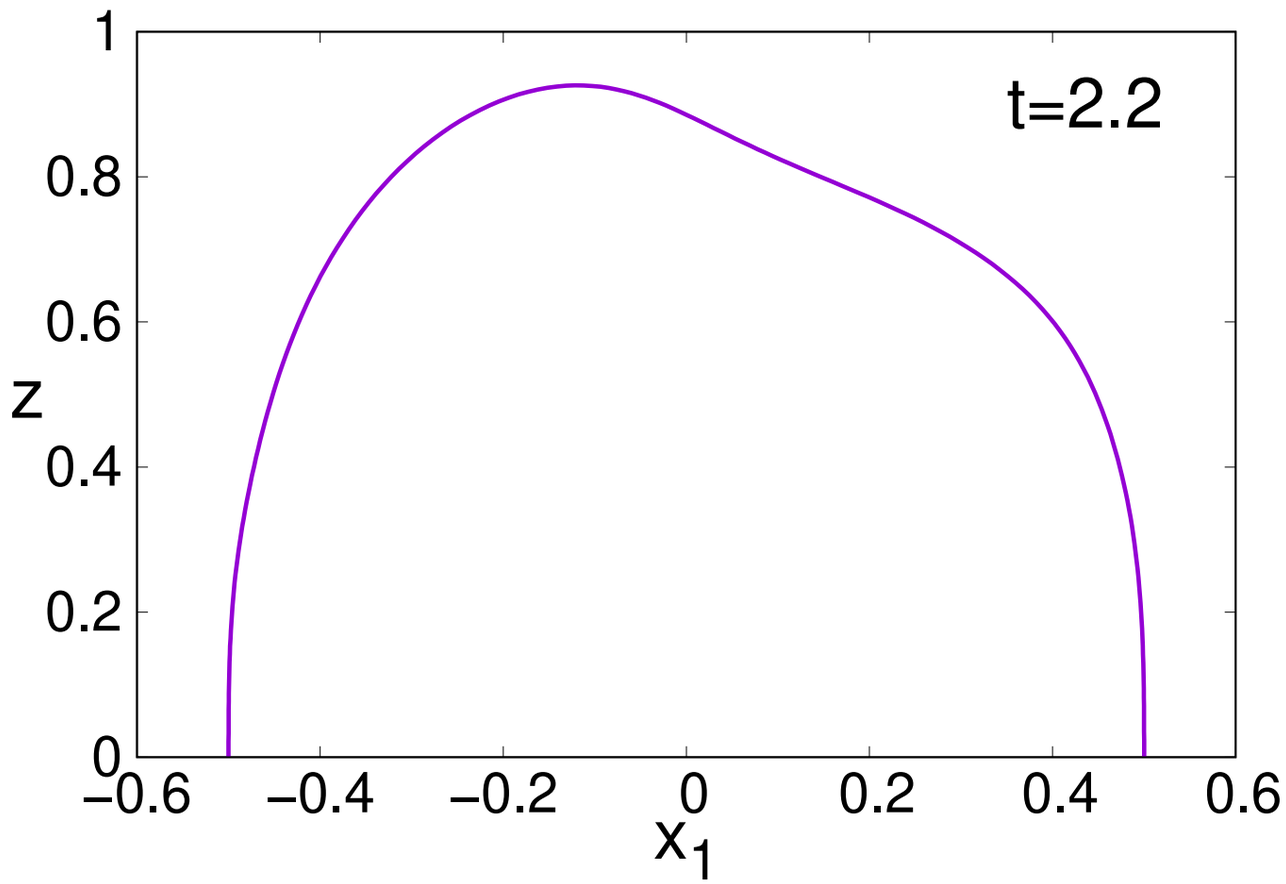


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

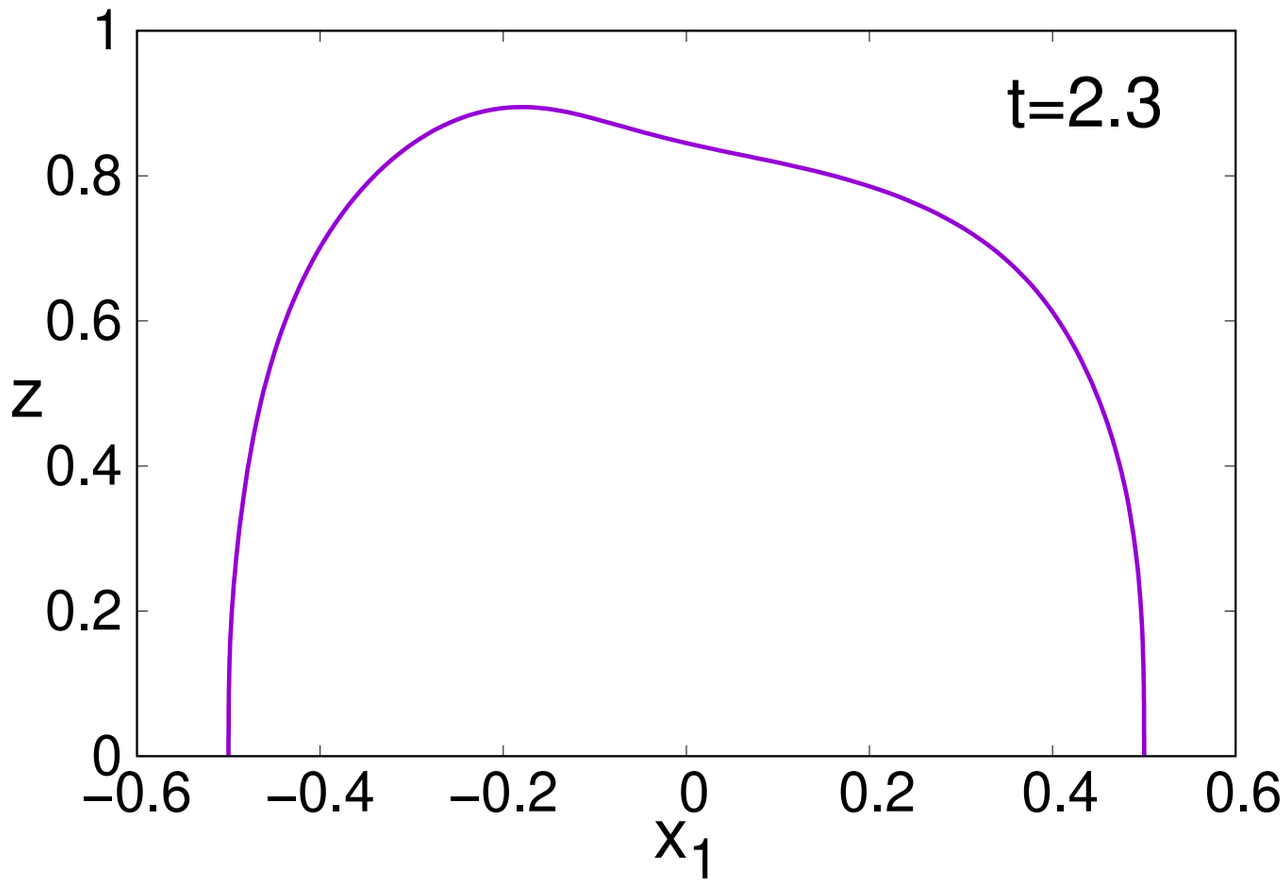


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

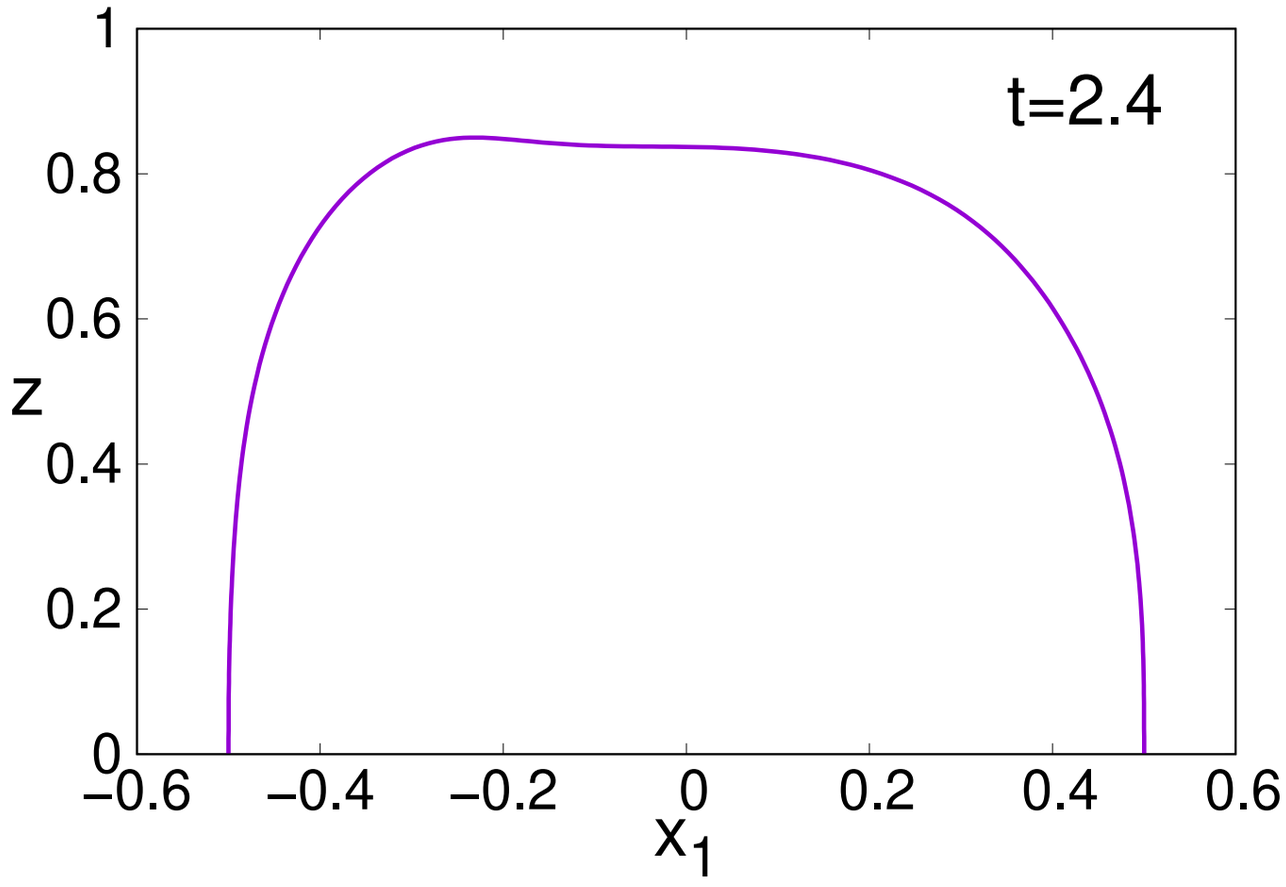


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

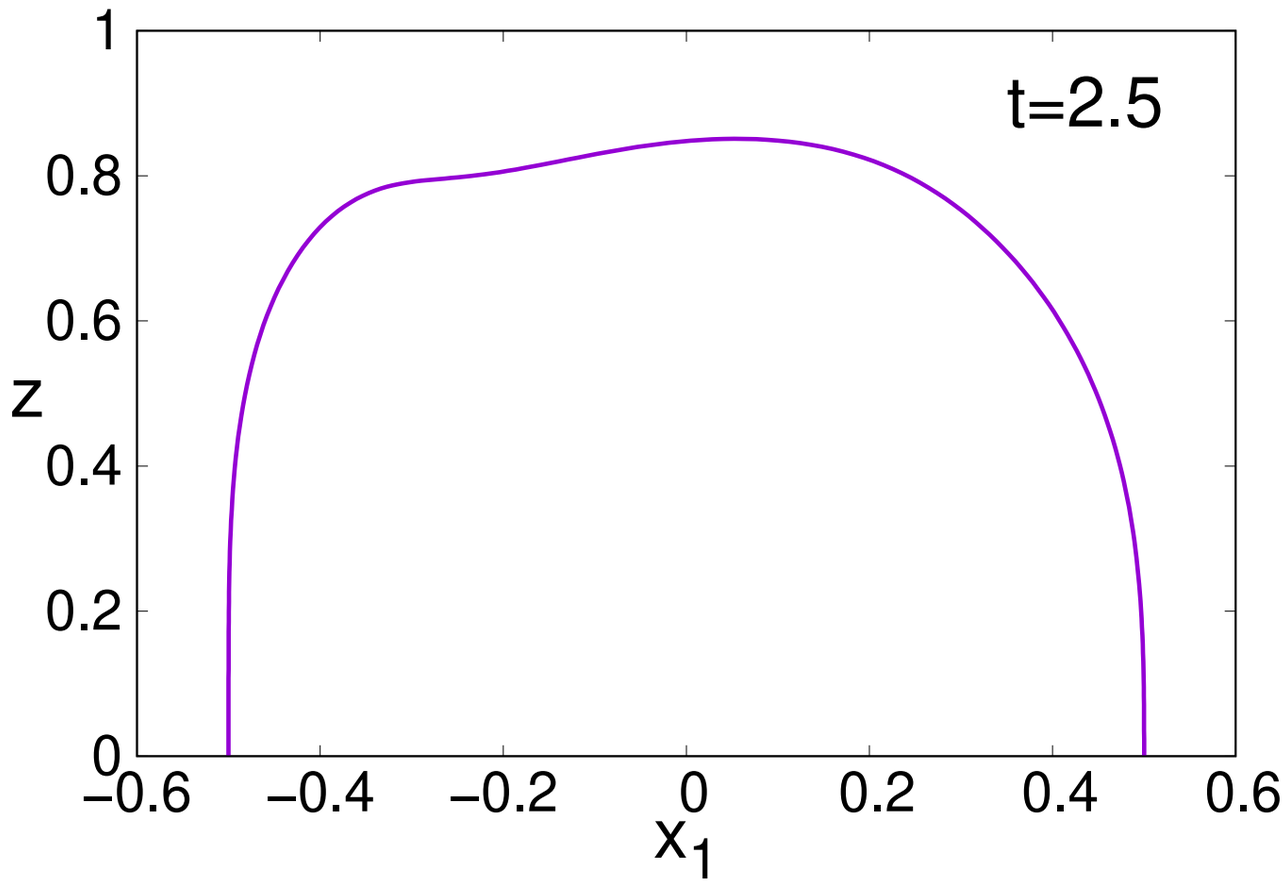


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

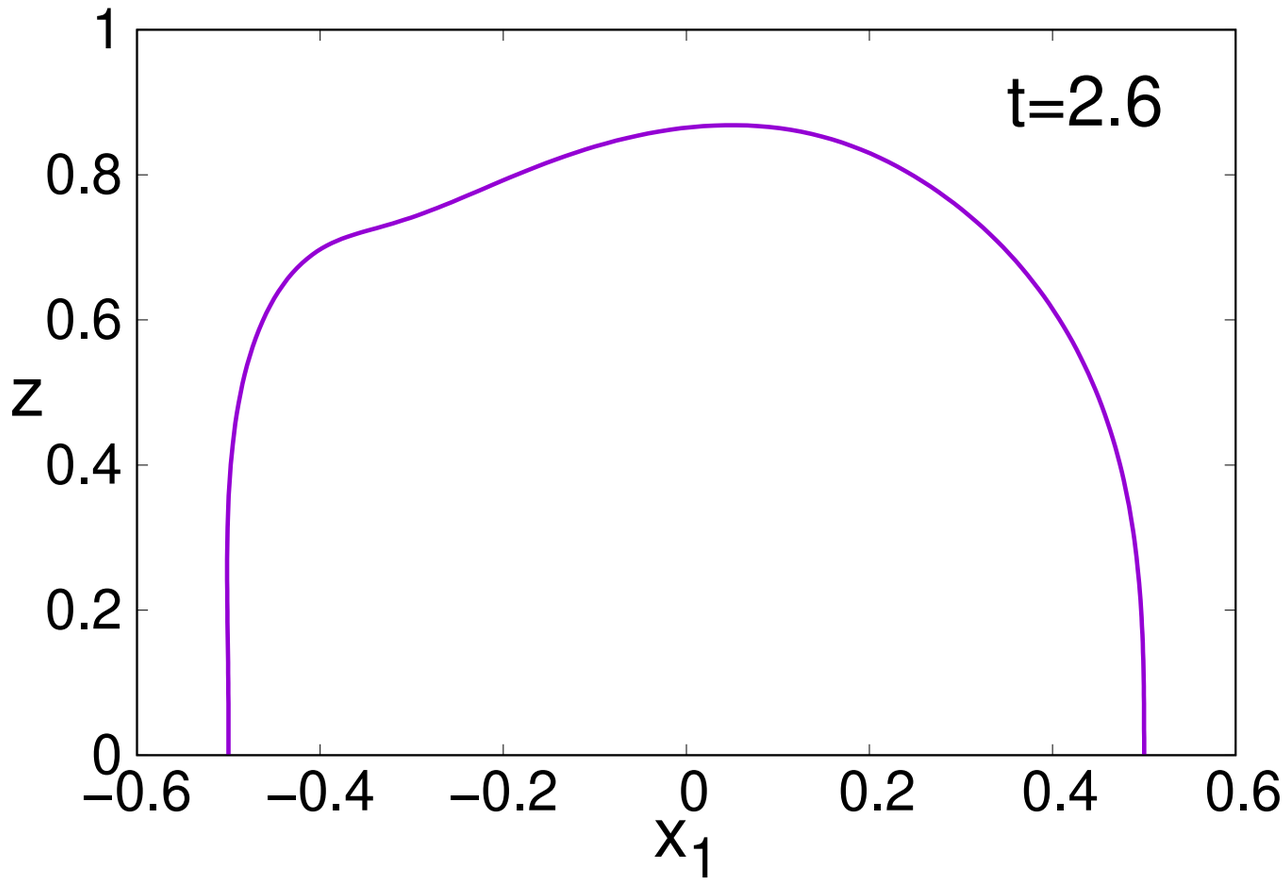


String motion in (t,z,x_1): AdS3 $\subset$ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

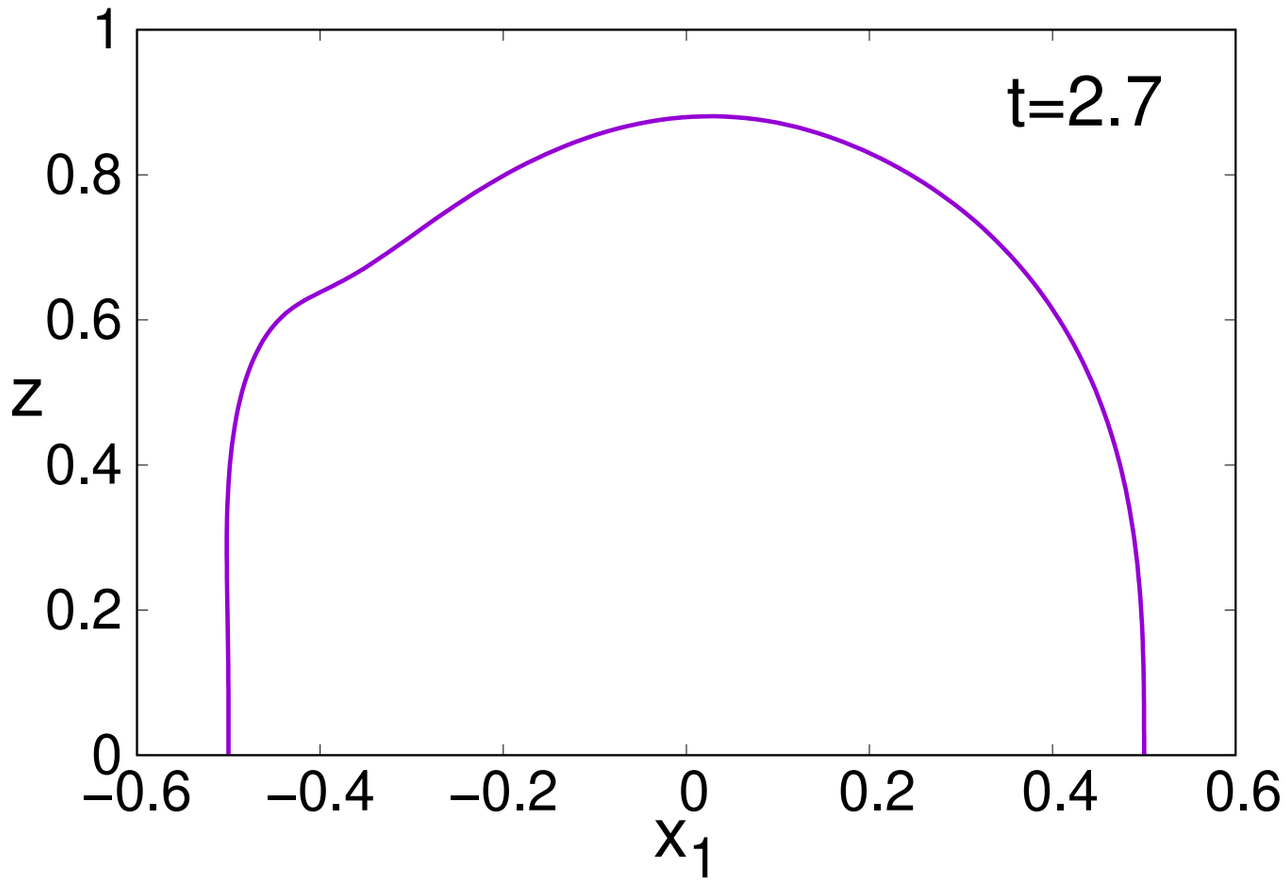


String motion in (t,z,x_1): AdS3 $\subset$ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

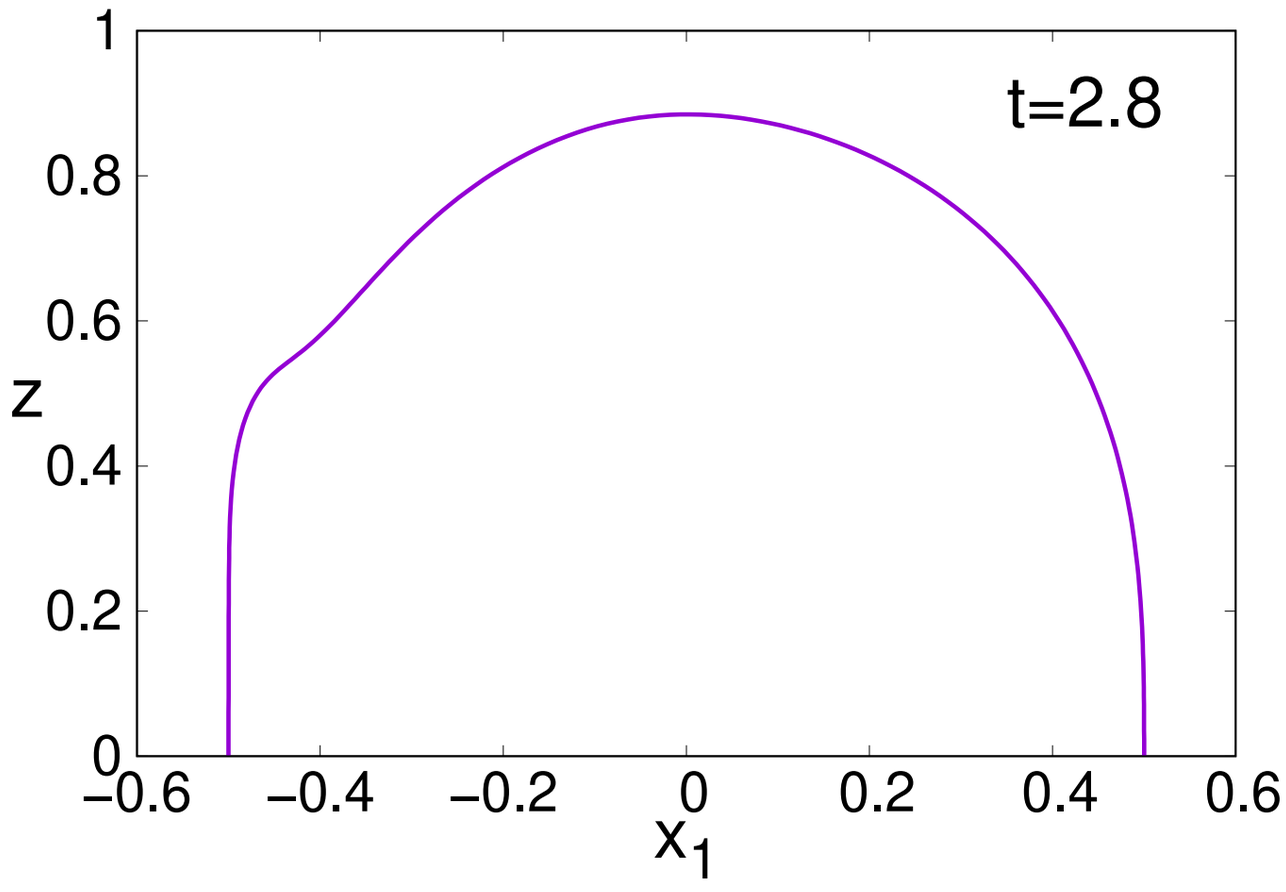


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

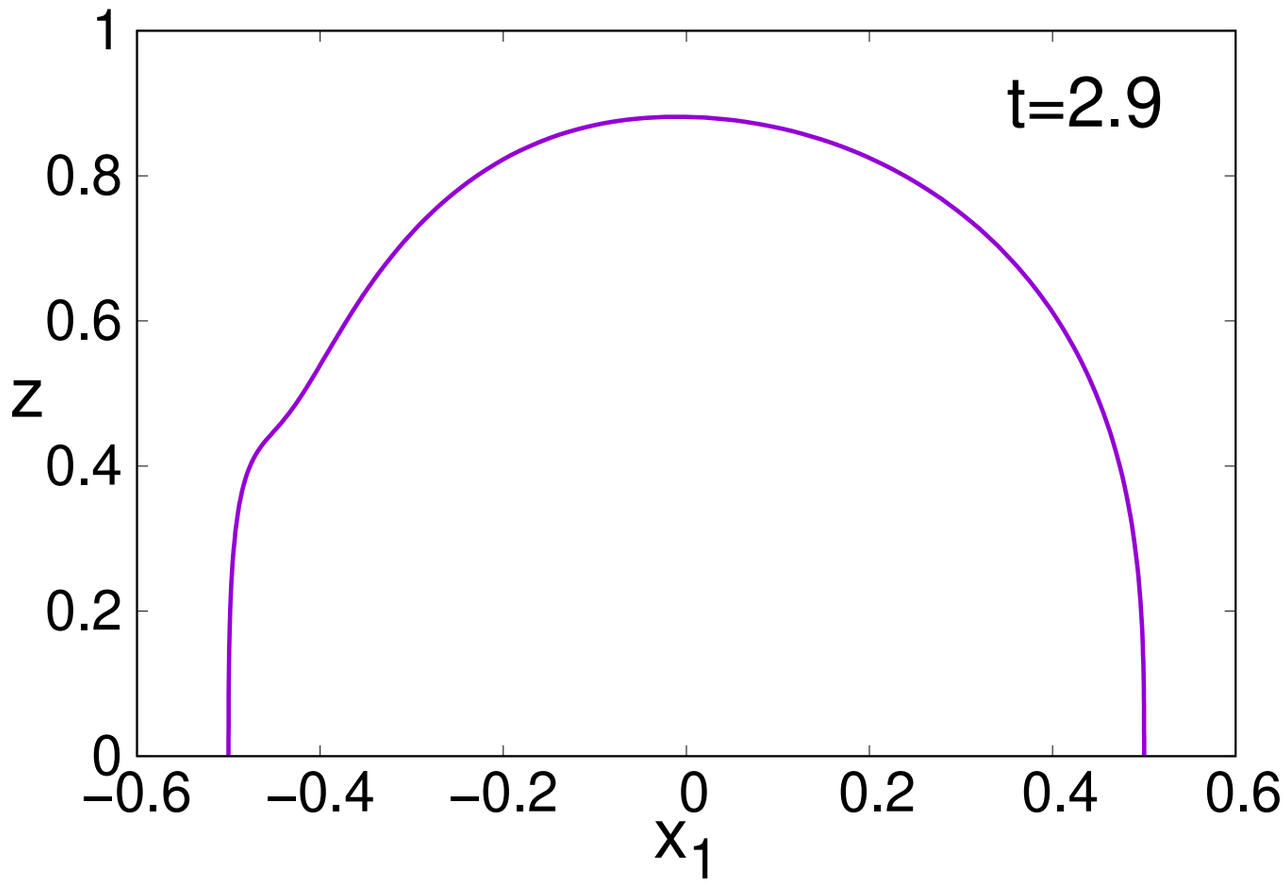


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

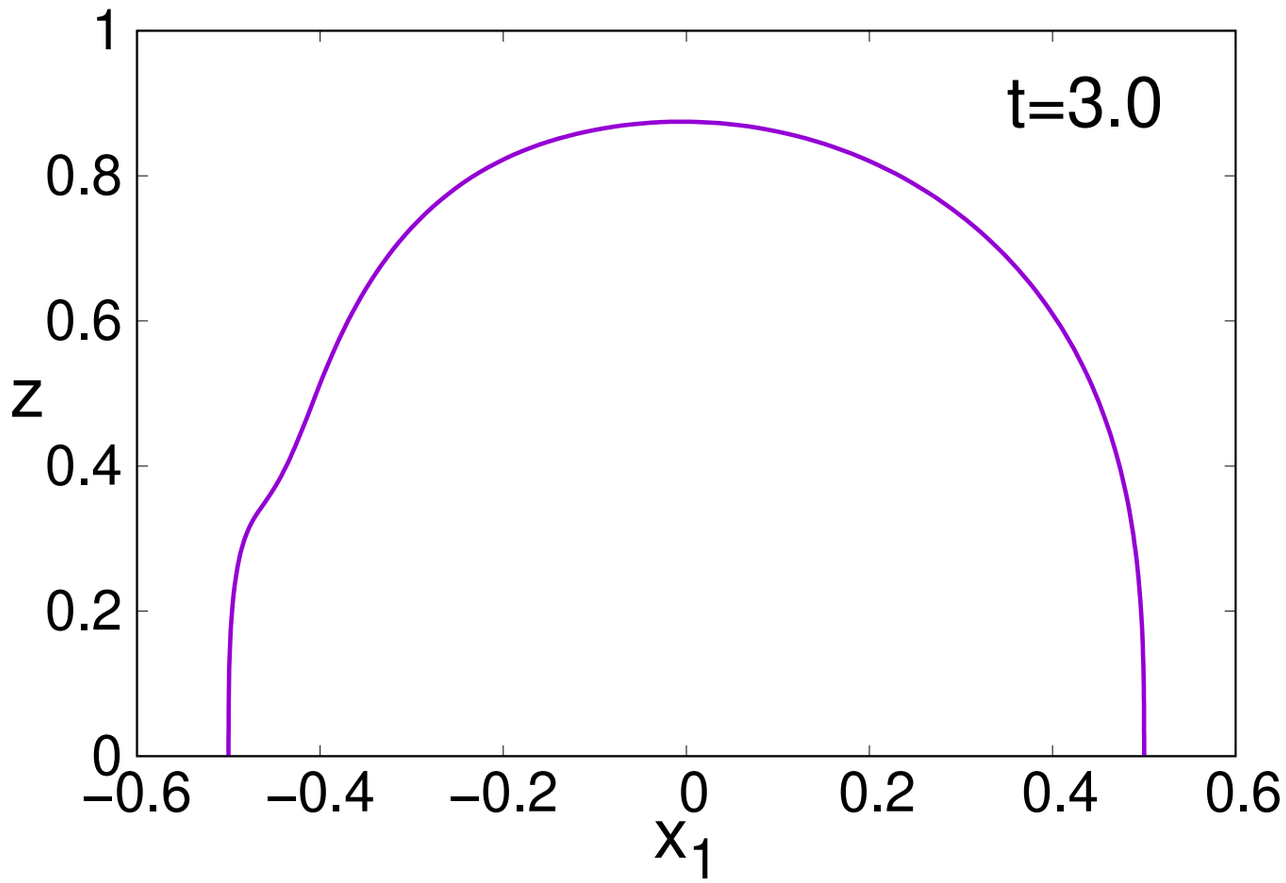


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

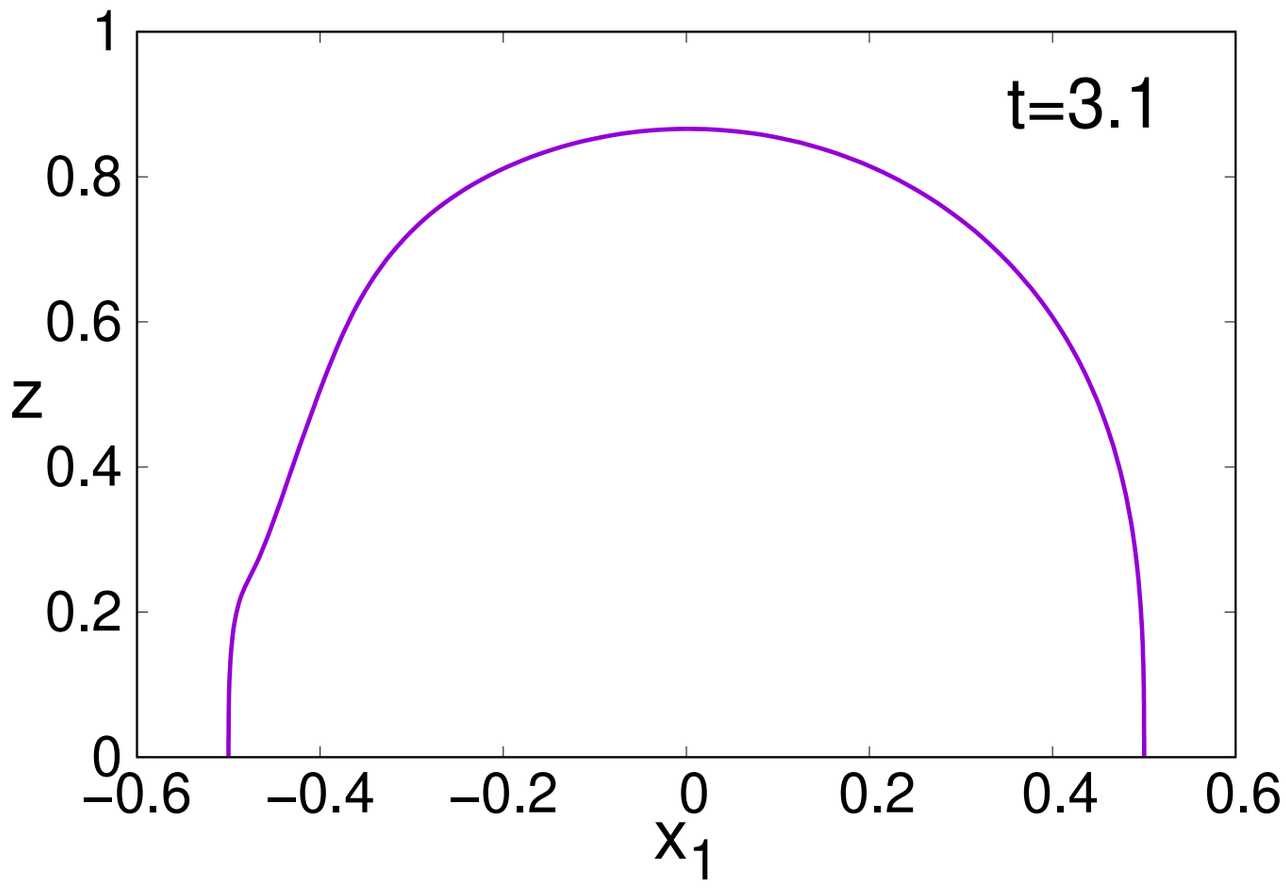


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

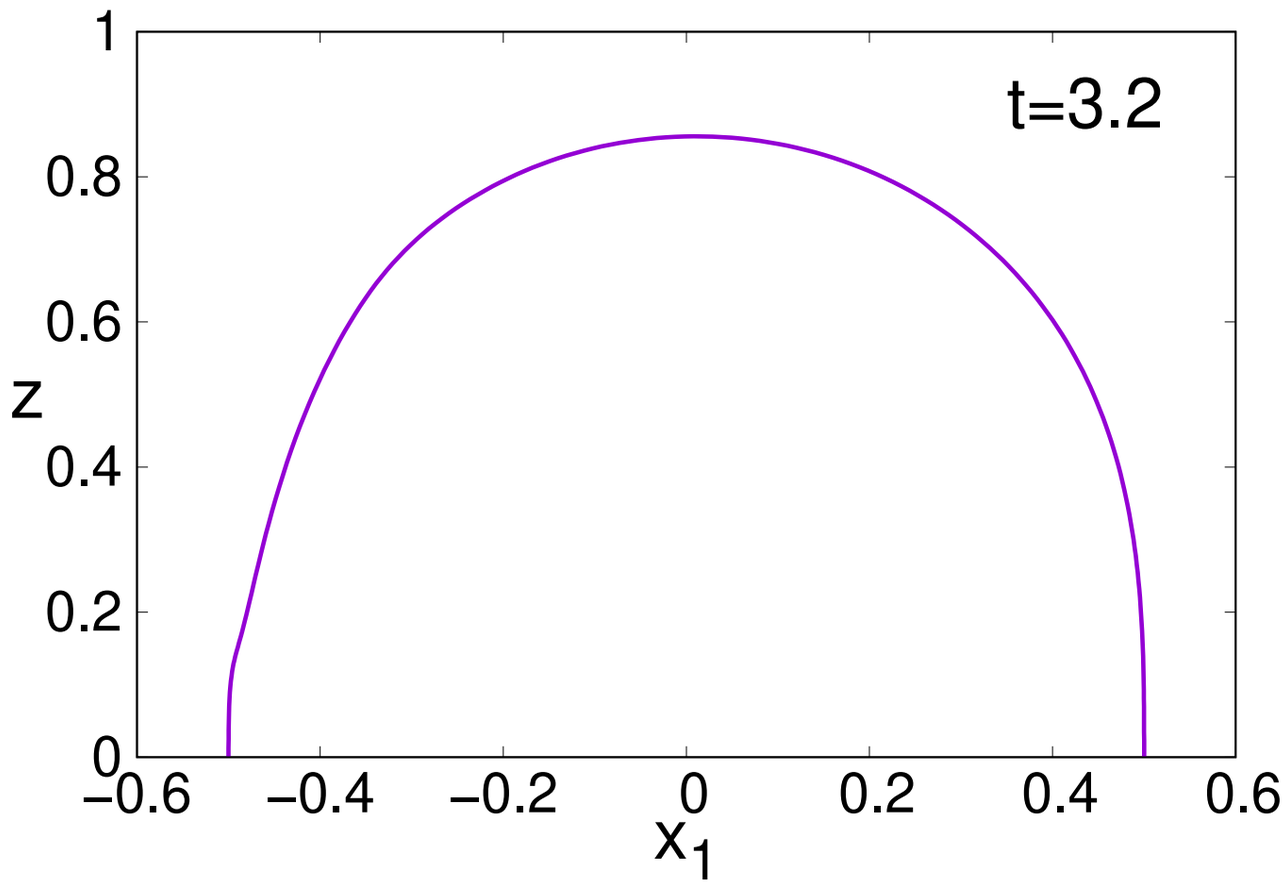


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

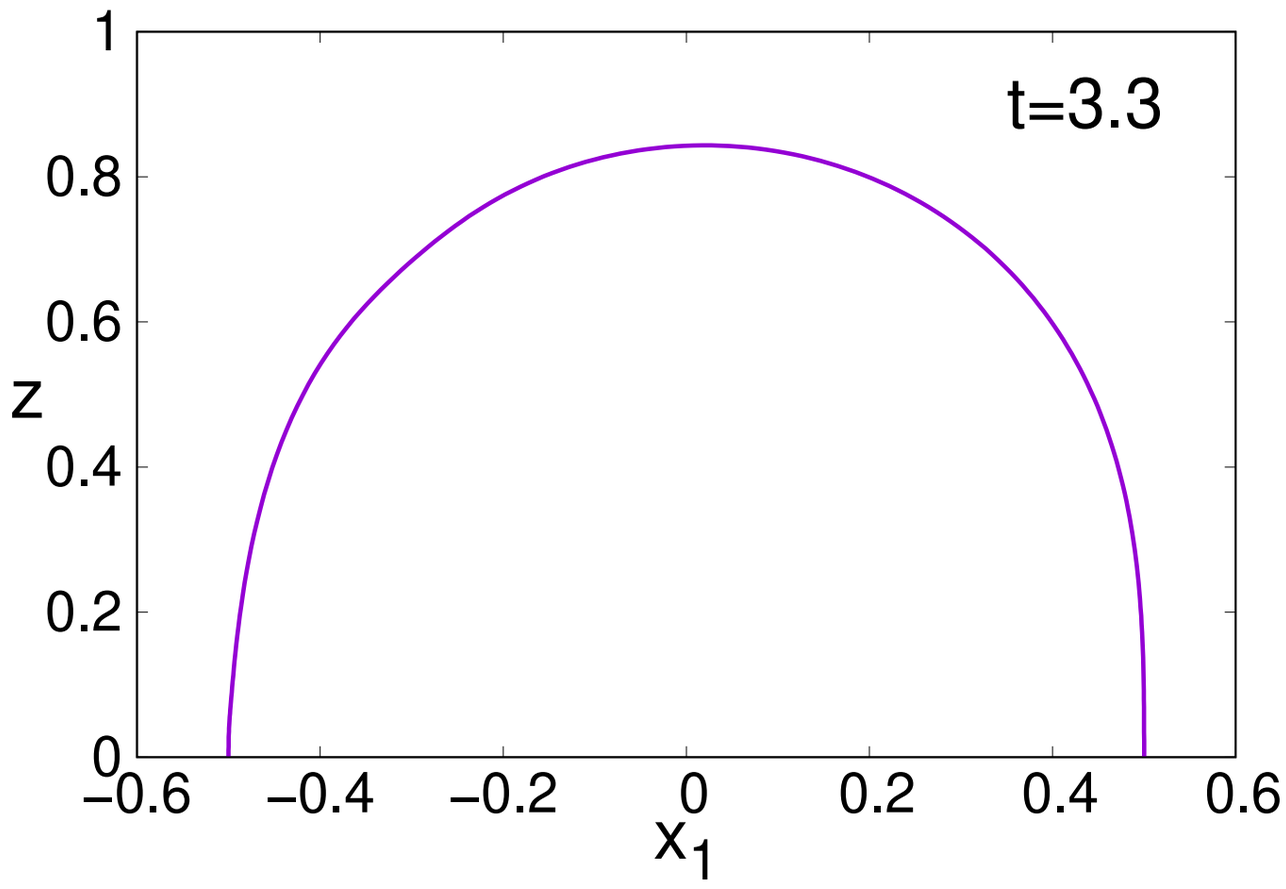


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

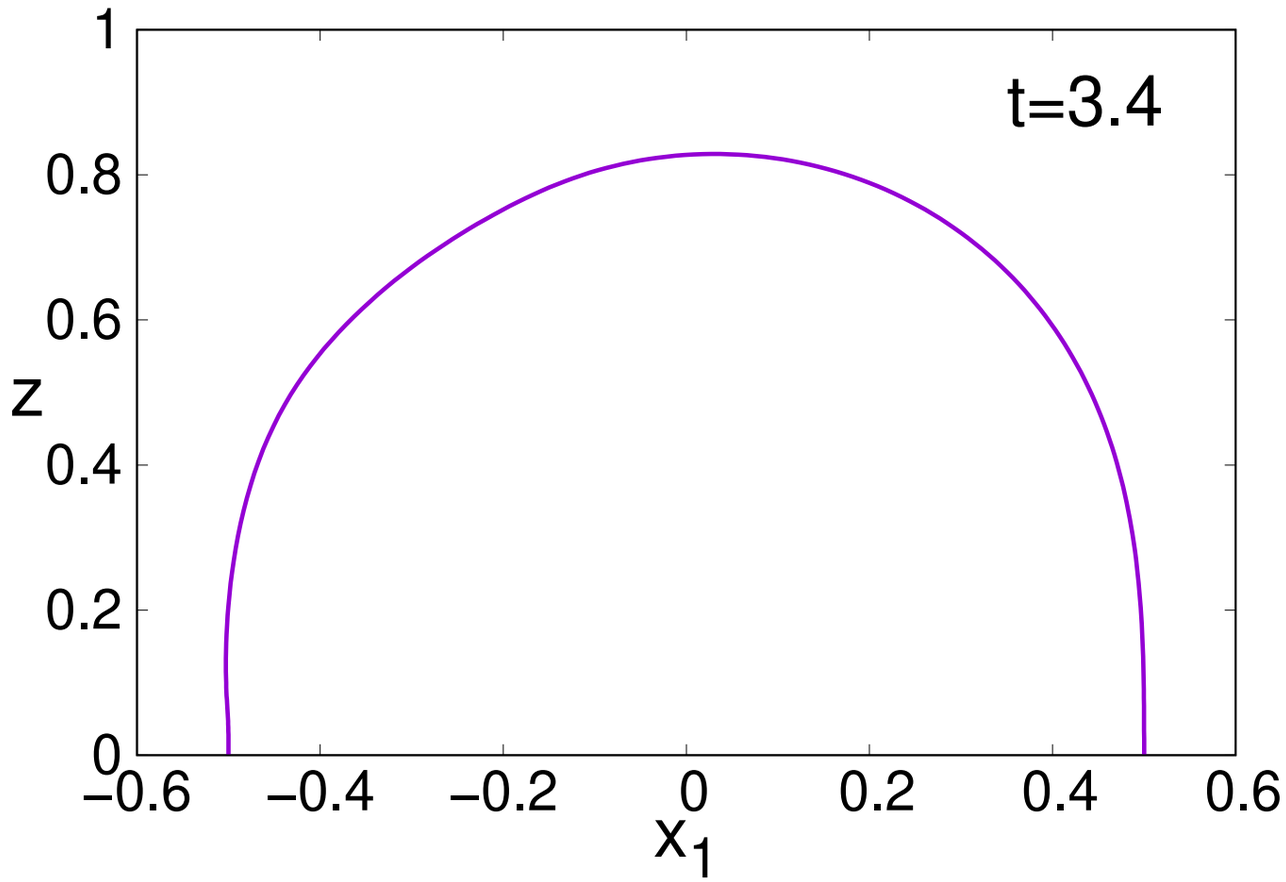


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

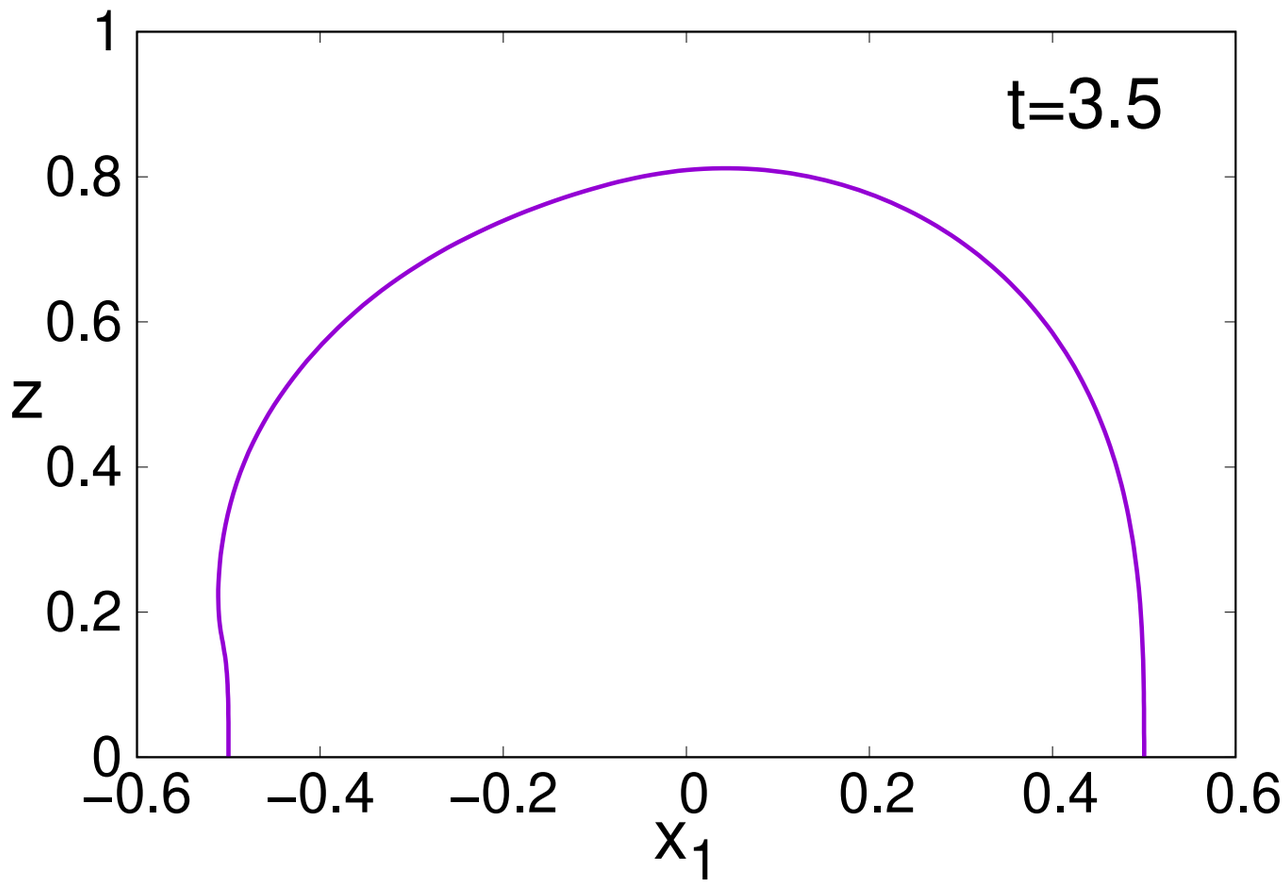


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

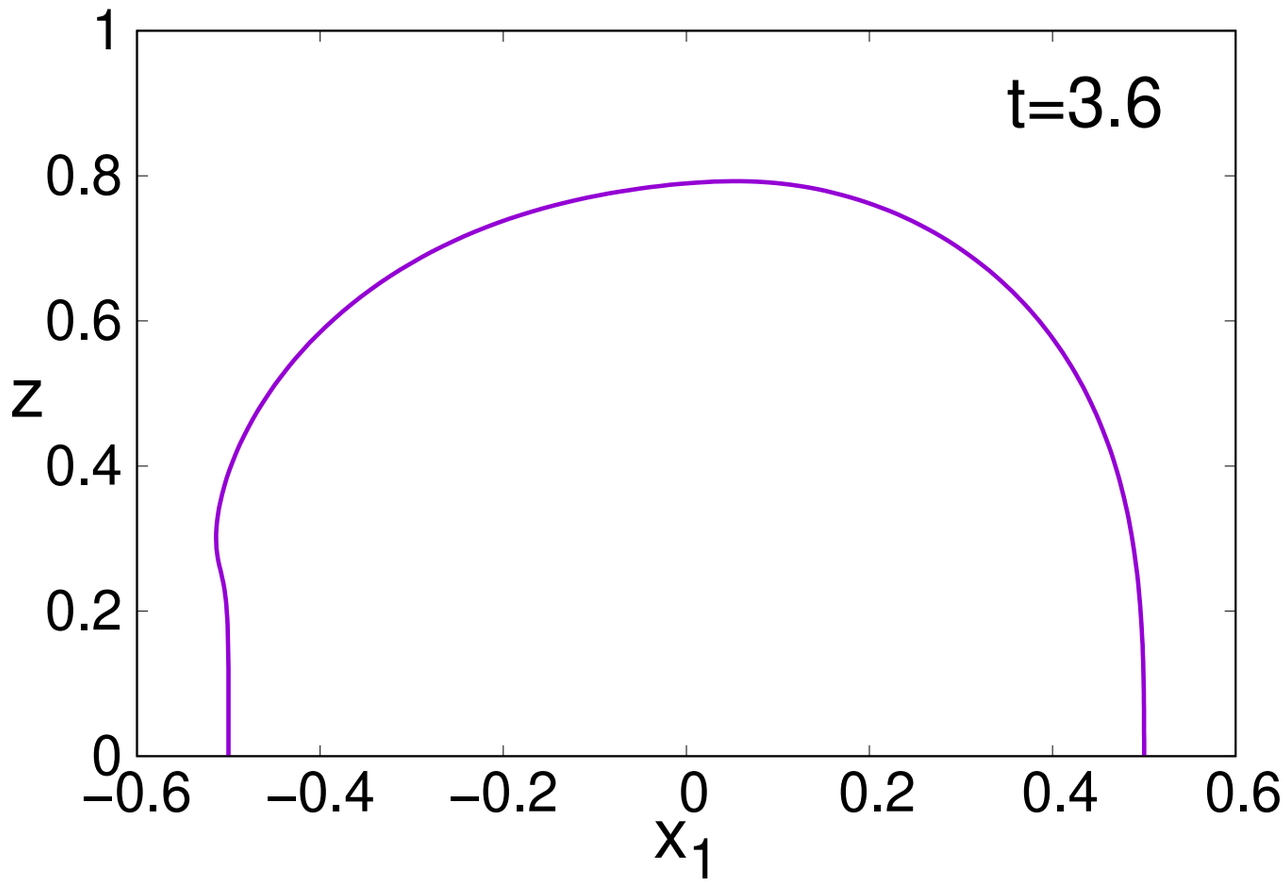


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

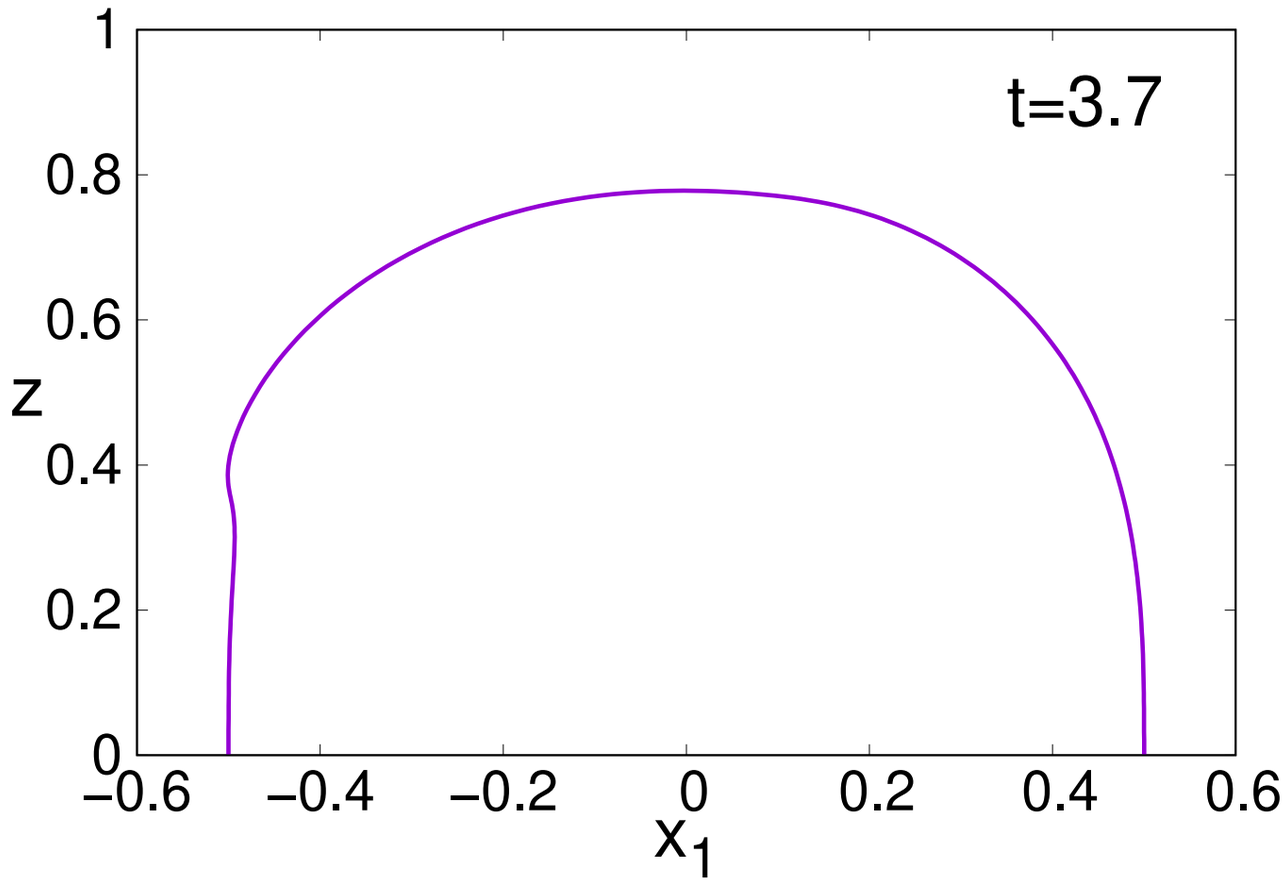


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

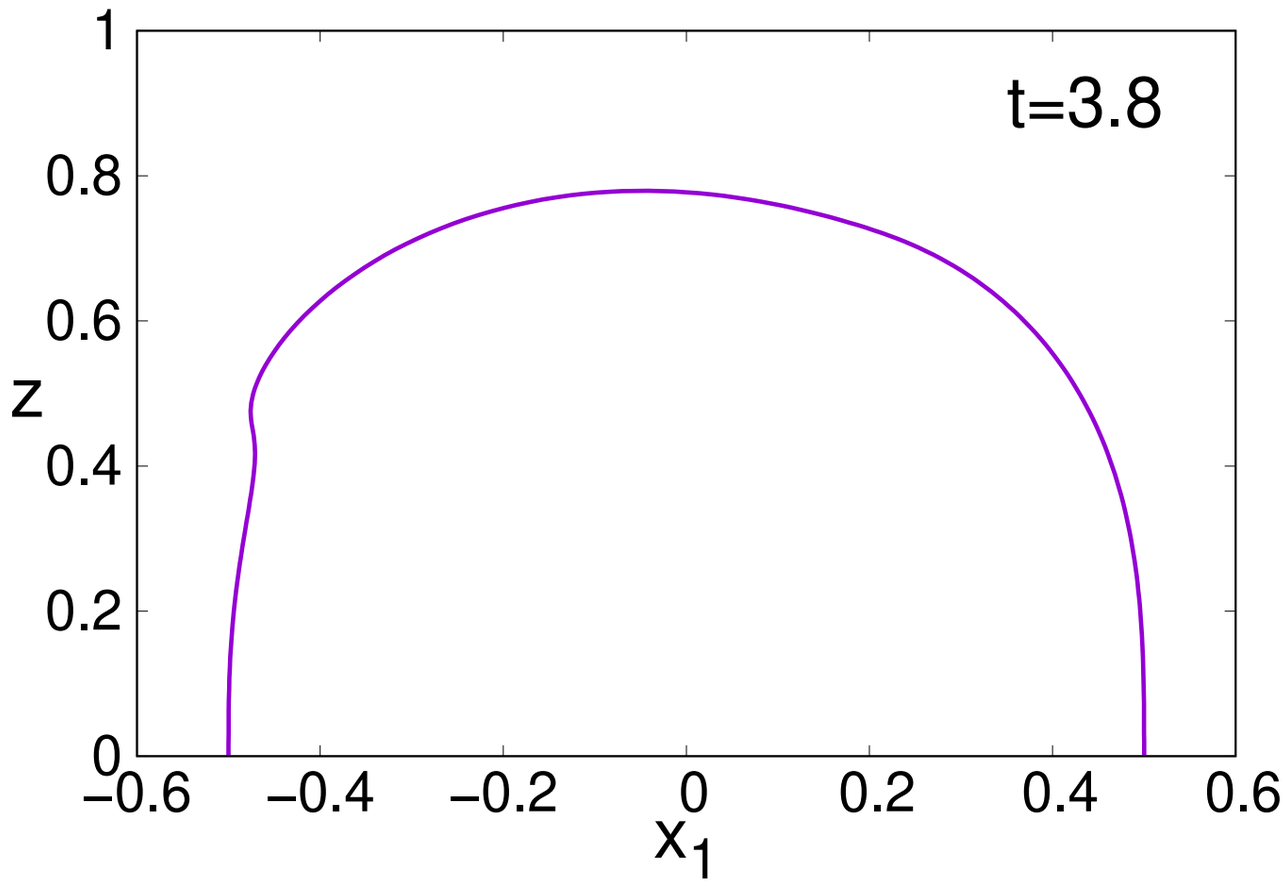


String motion in (t,z,x_1): AdS3 $\subset$ AdS5

Quench: $0<t<2$

Amplitude: $\varepsilon=0.03$

String time evolution

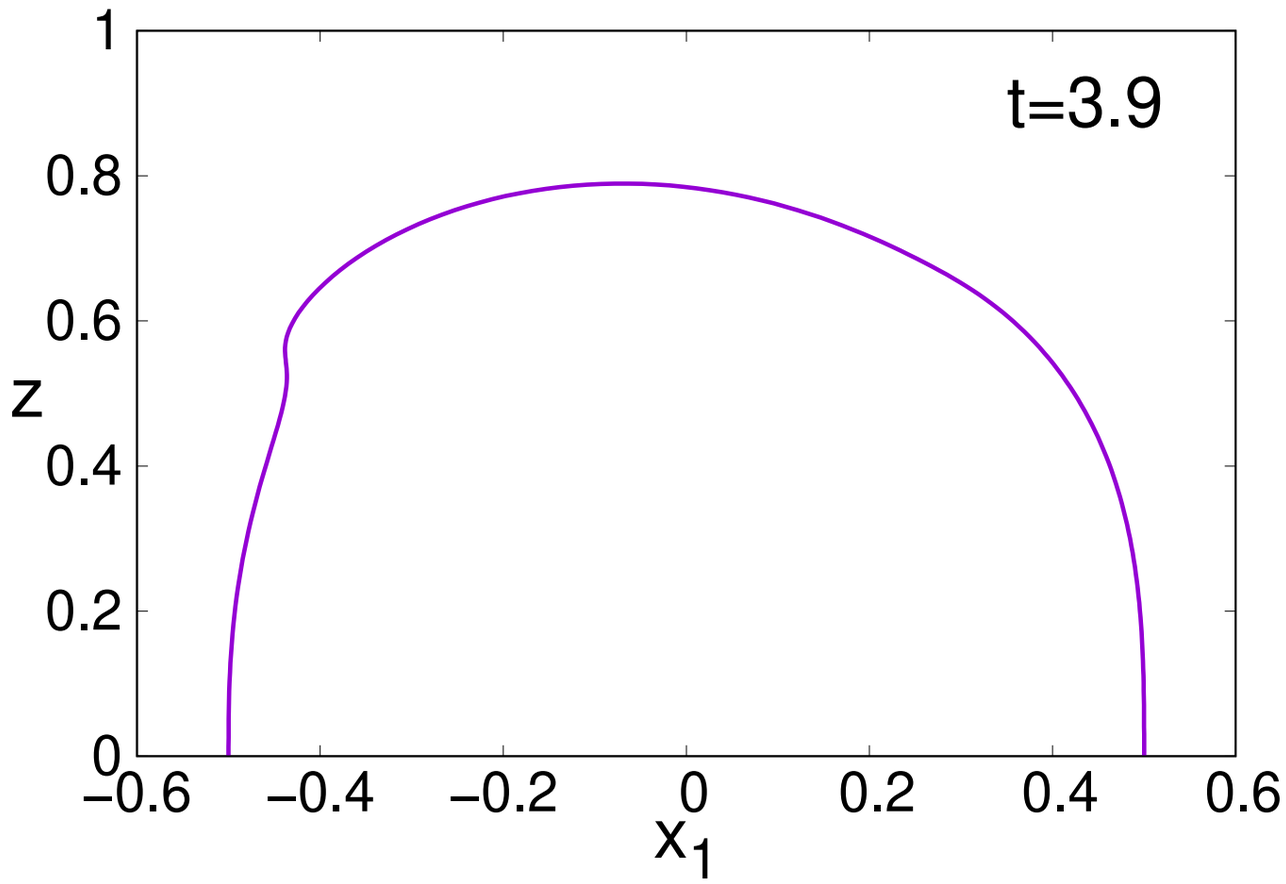


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

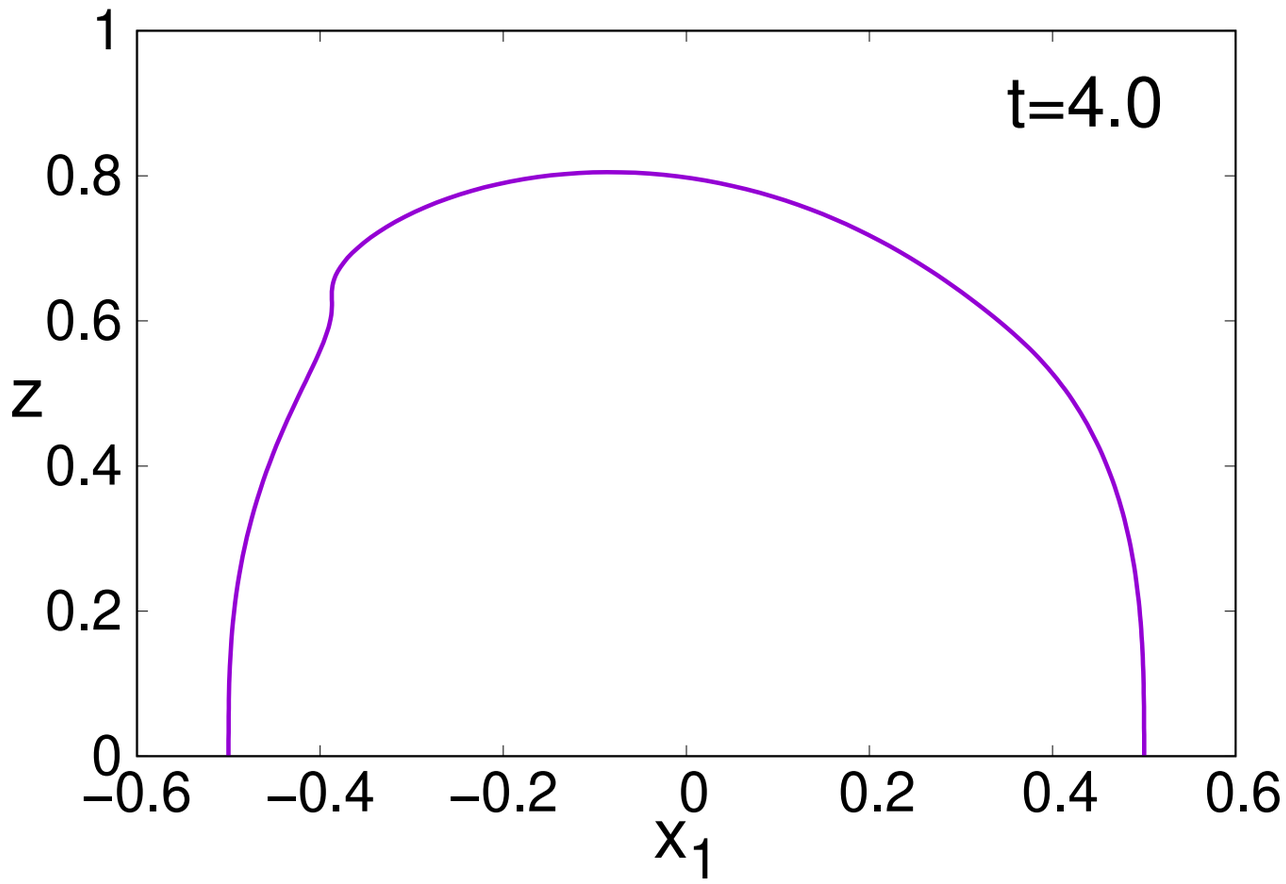


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

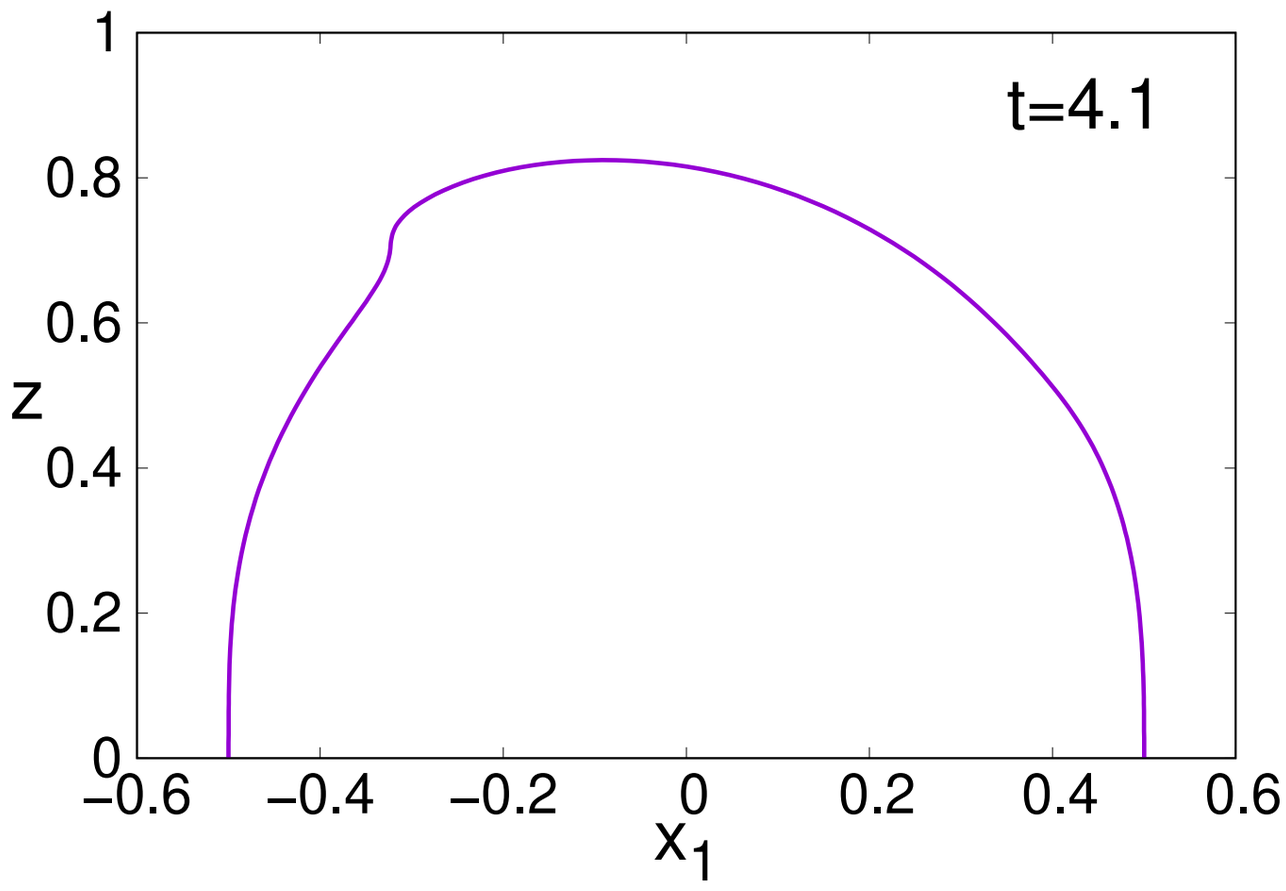


String motion in (t,z,x_1): AdS3 $\subset$ AdS5

Quench: $0<t<2$

Amplitude: $\varepsilon=0.03$

String time evolution

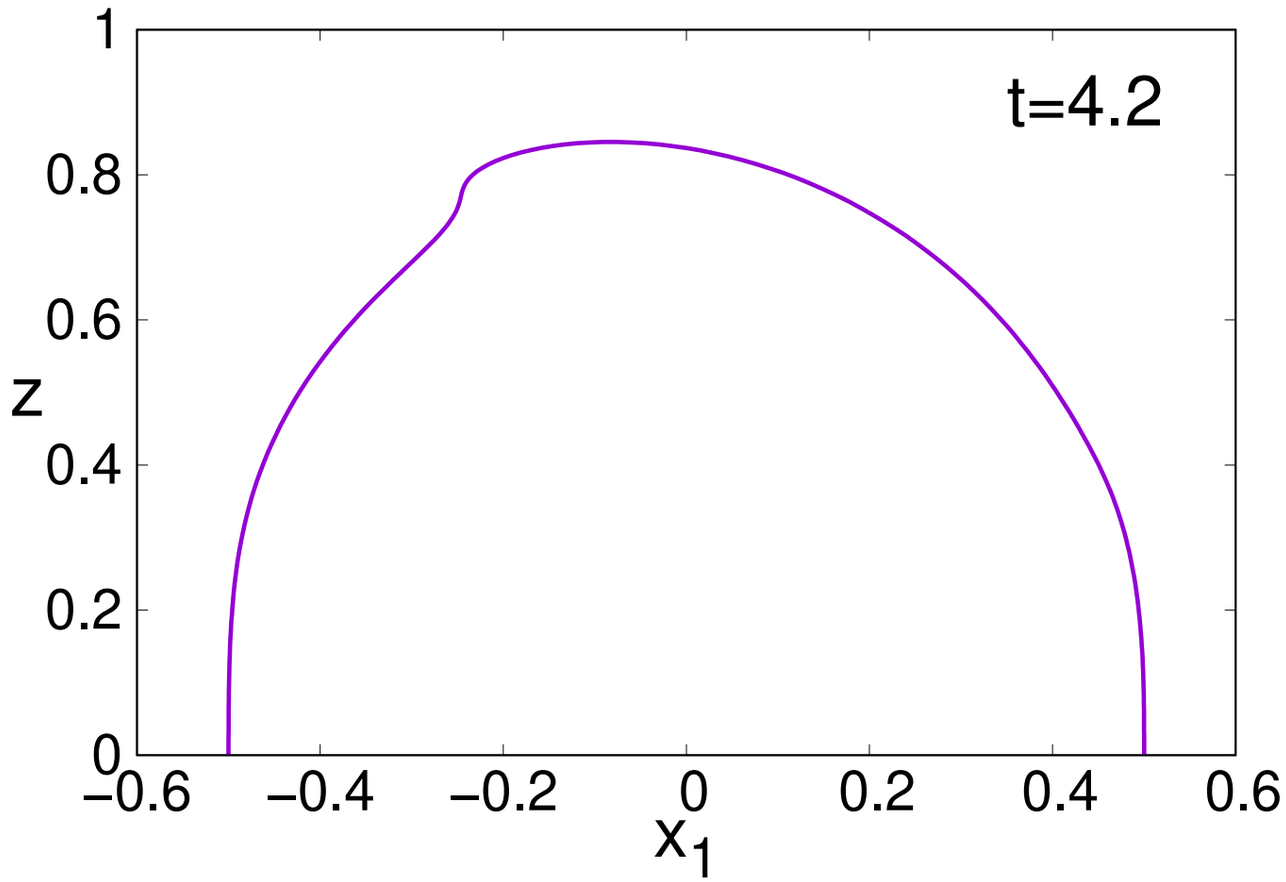


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

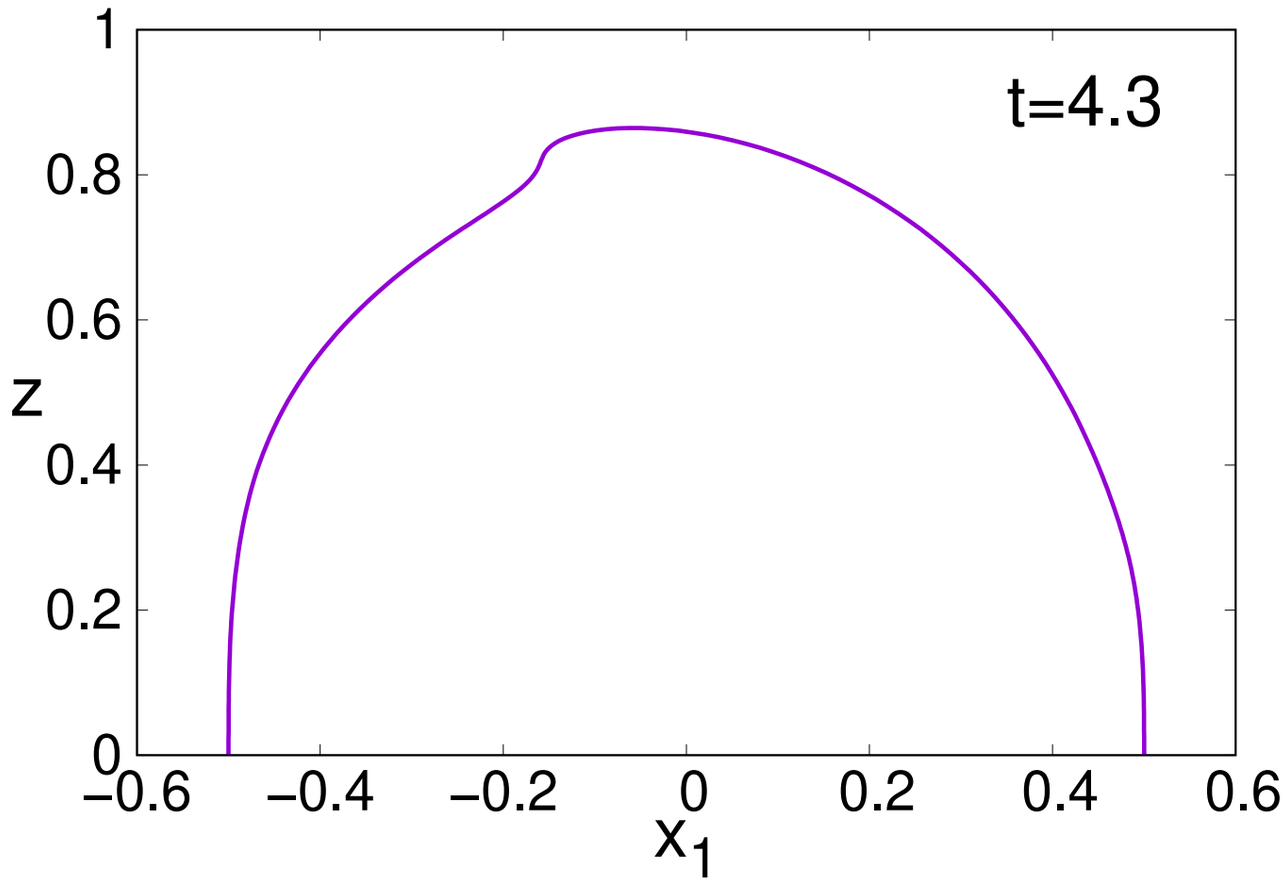


String motion in (t,z,x_1): AdS3 $\subset$ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

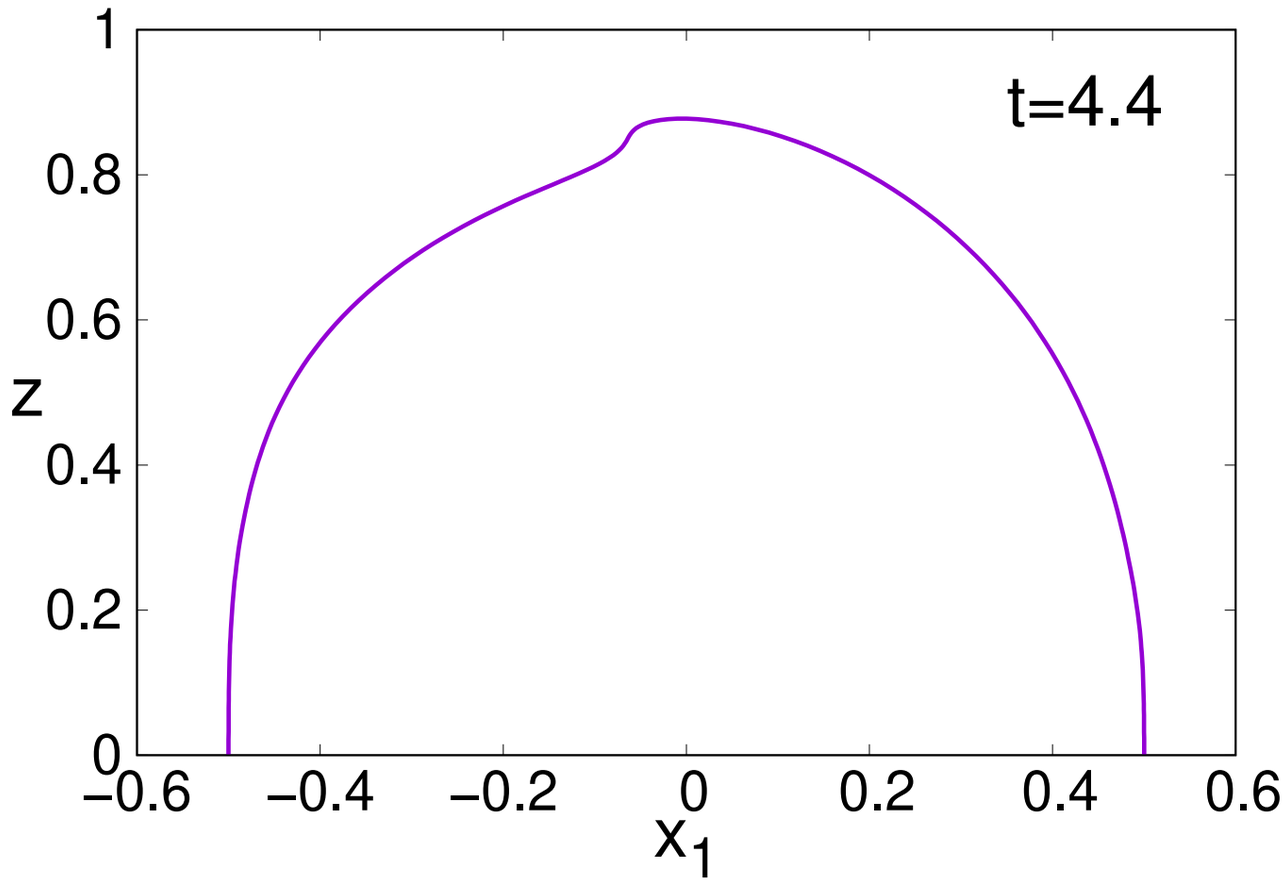


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

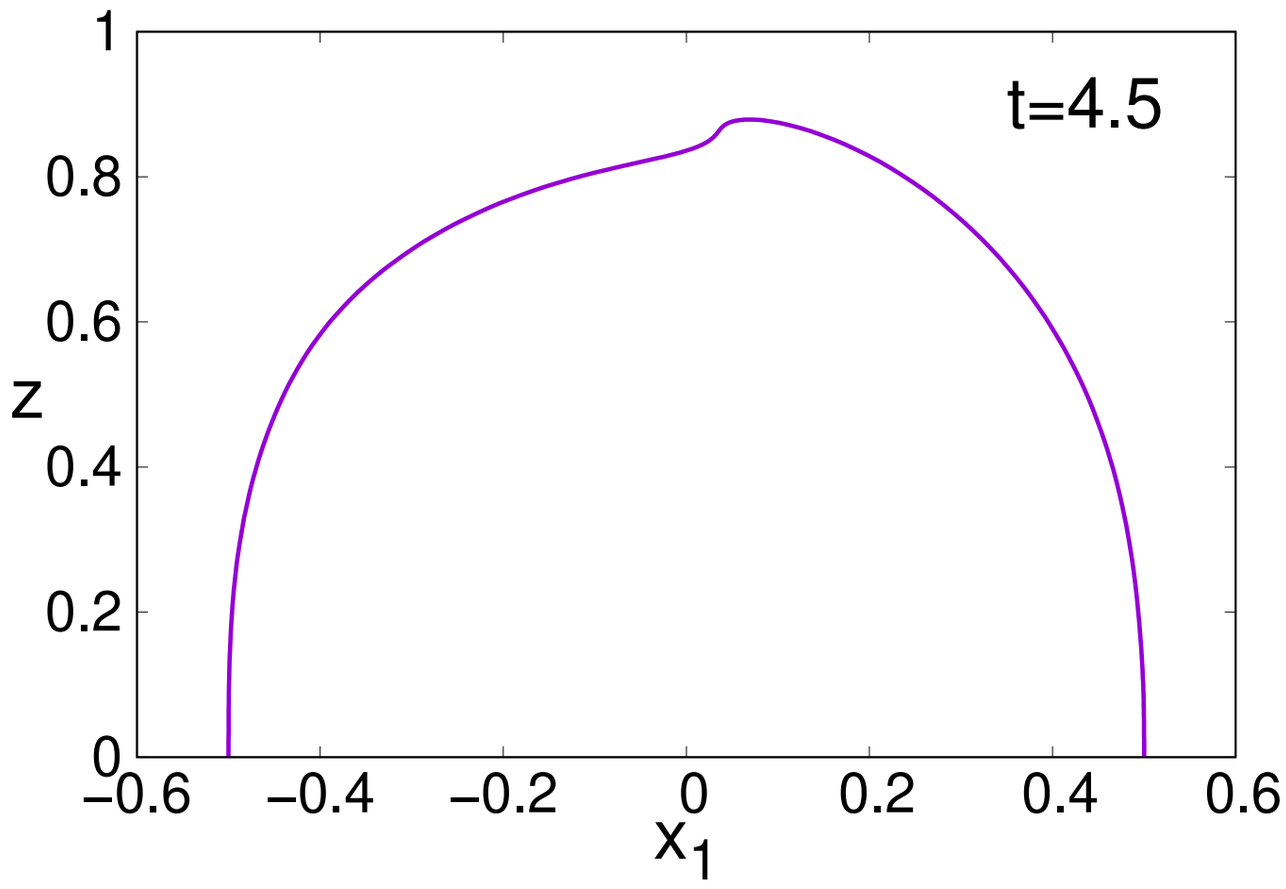


String motion in (t,z,x_1): AdS3 $\subset$ AdS5

Quench: $0<t<2$

Amplitude: $\varepsilon=0.03$

String time evolution

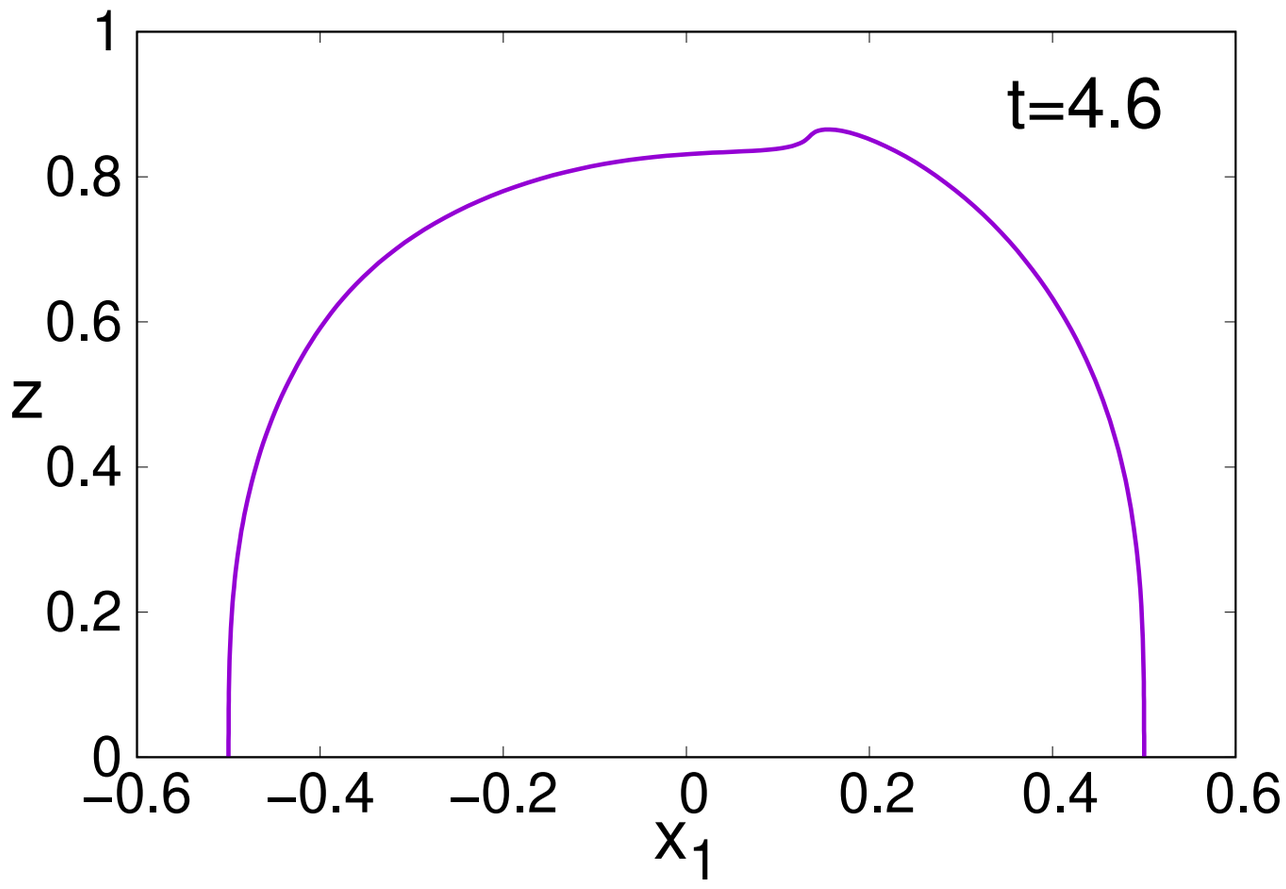


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

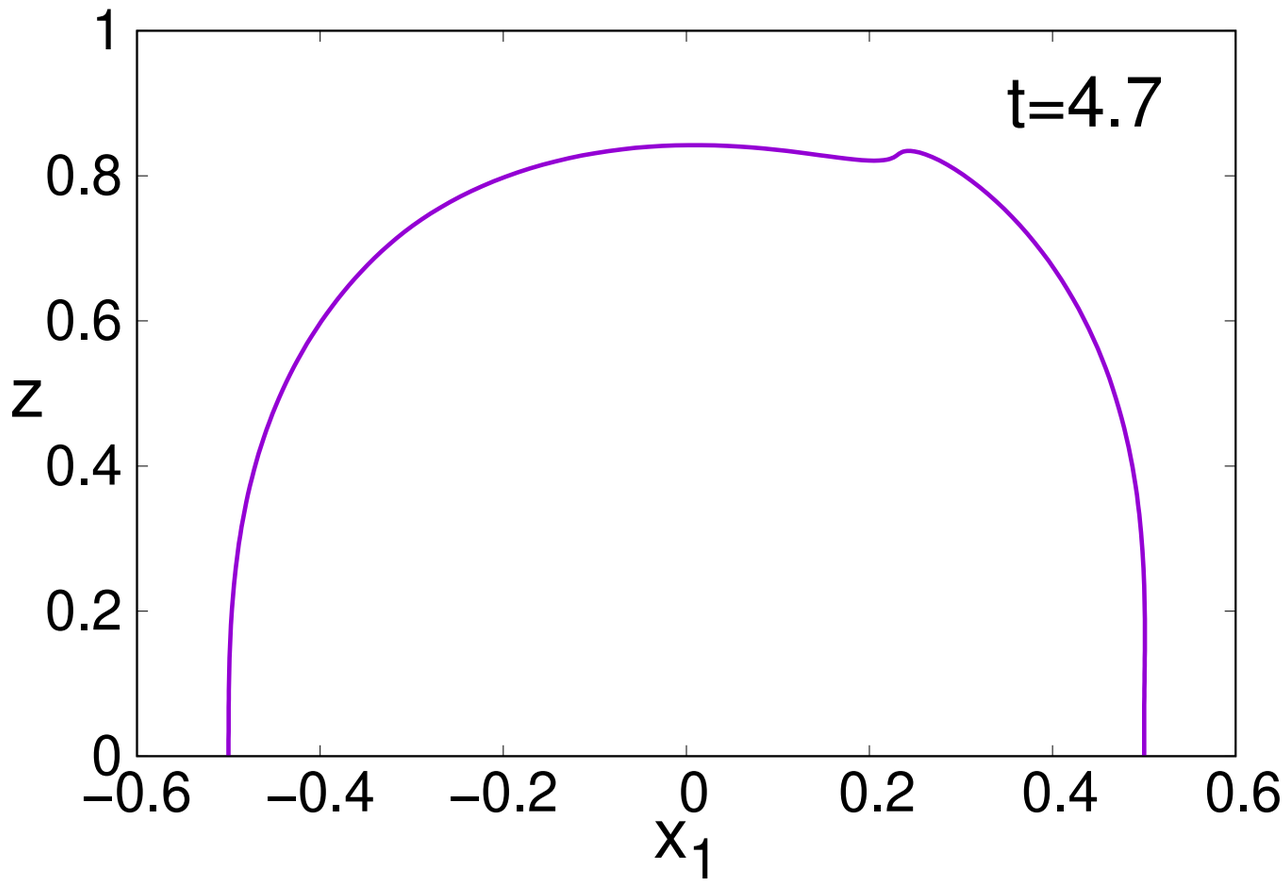


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

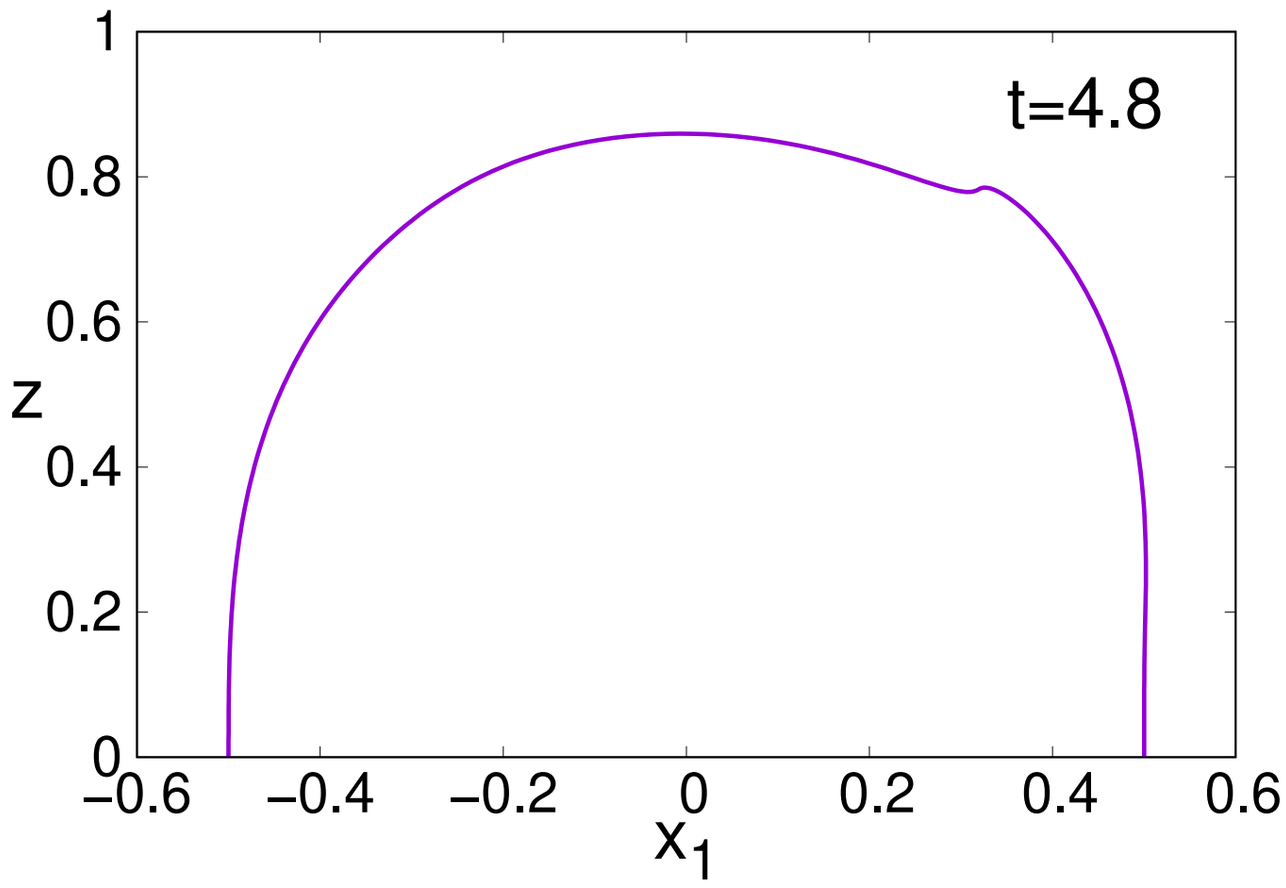


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

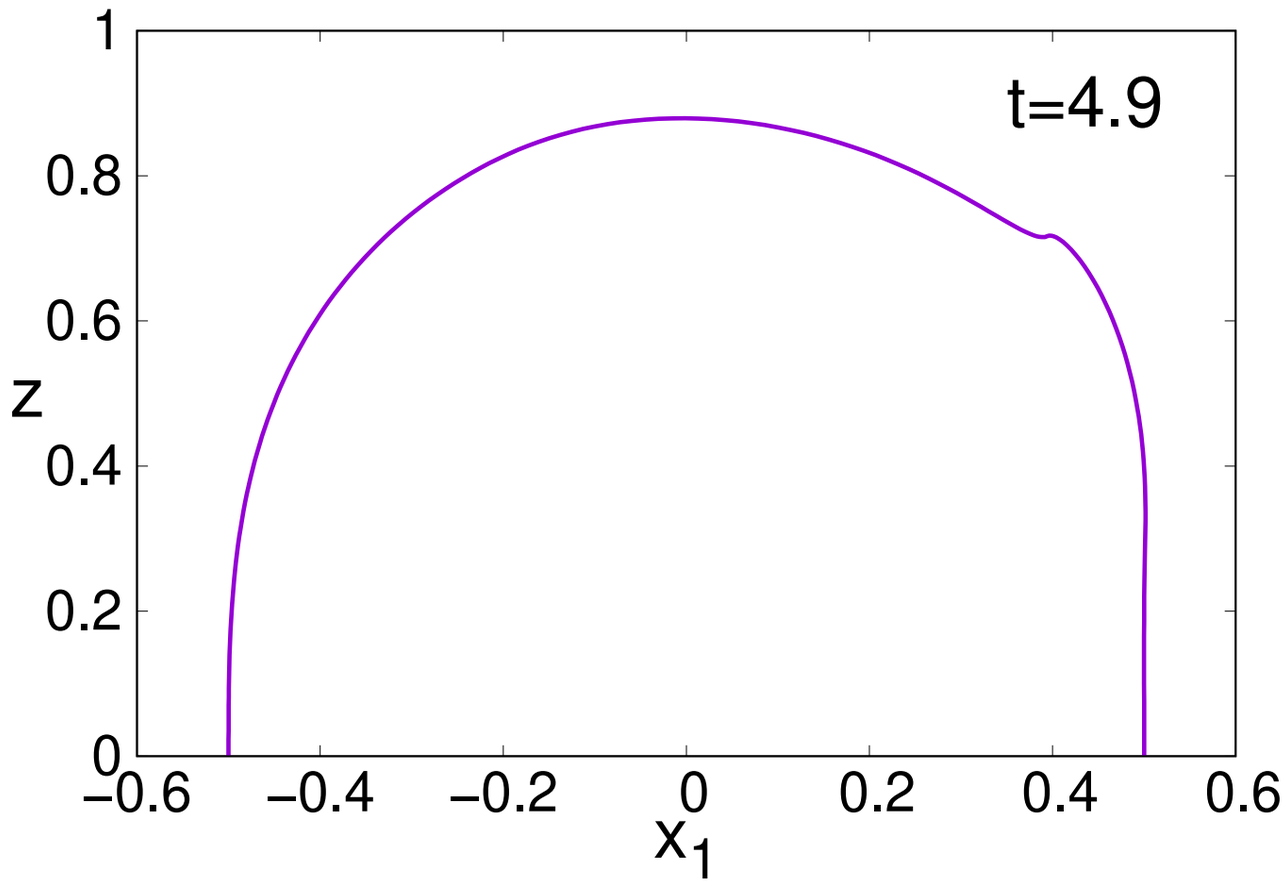


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

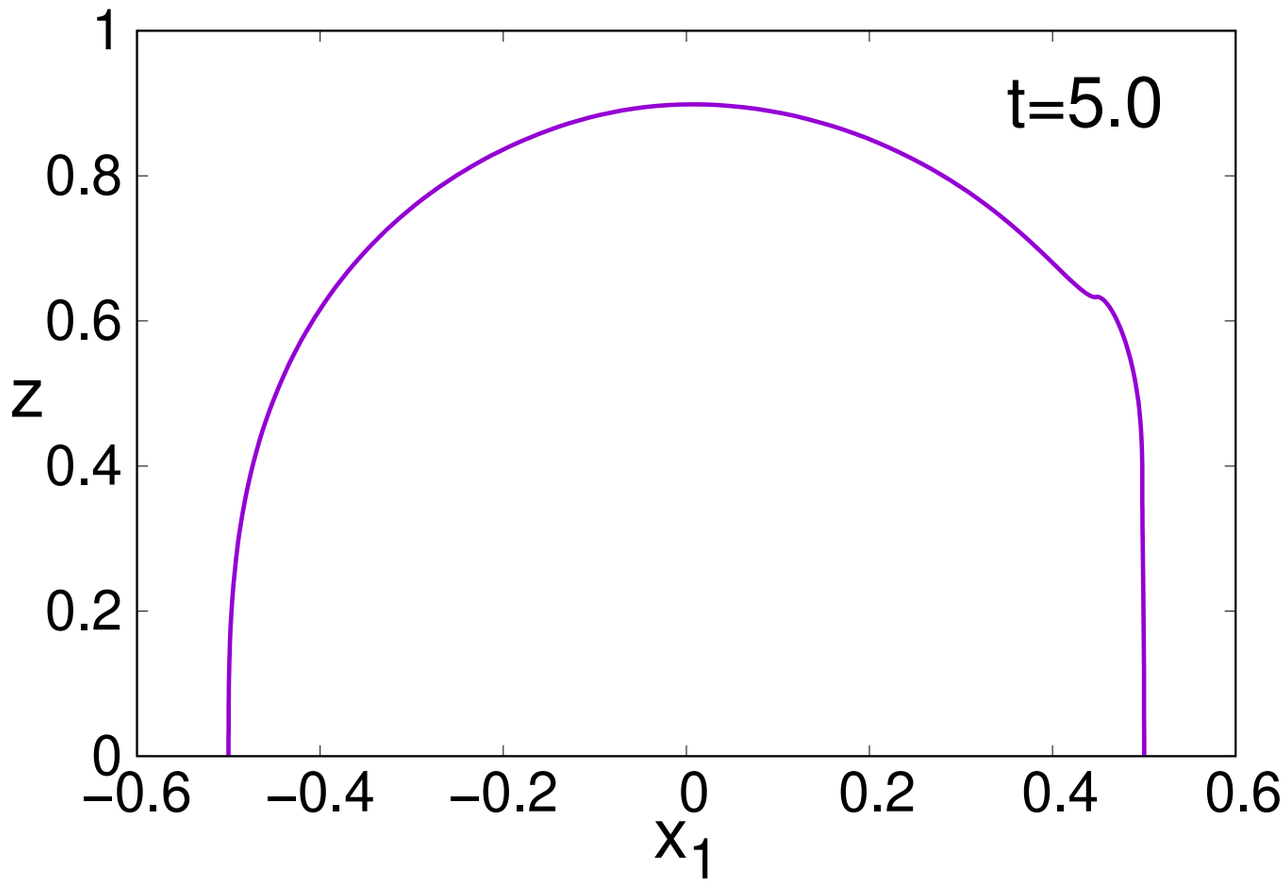


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

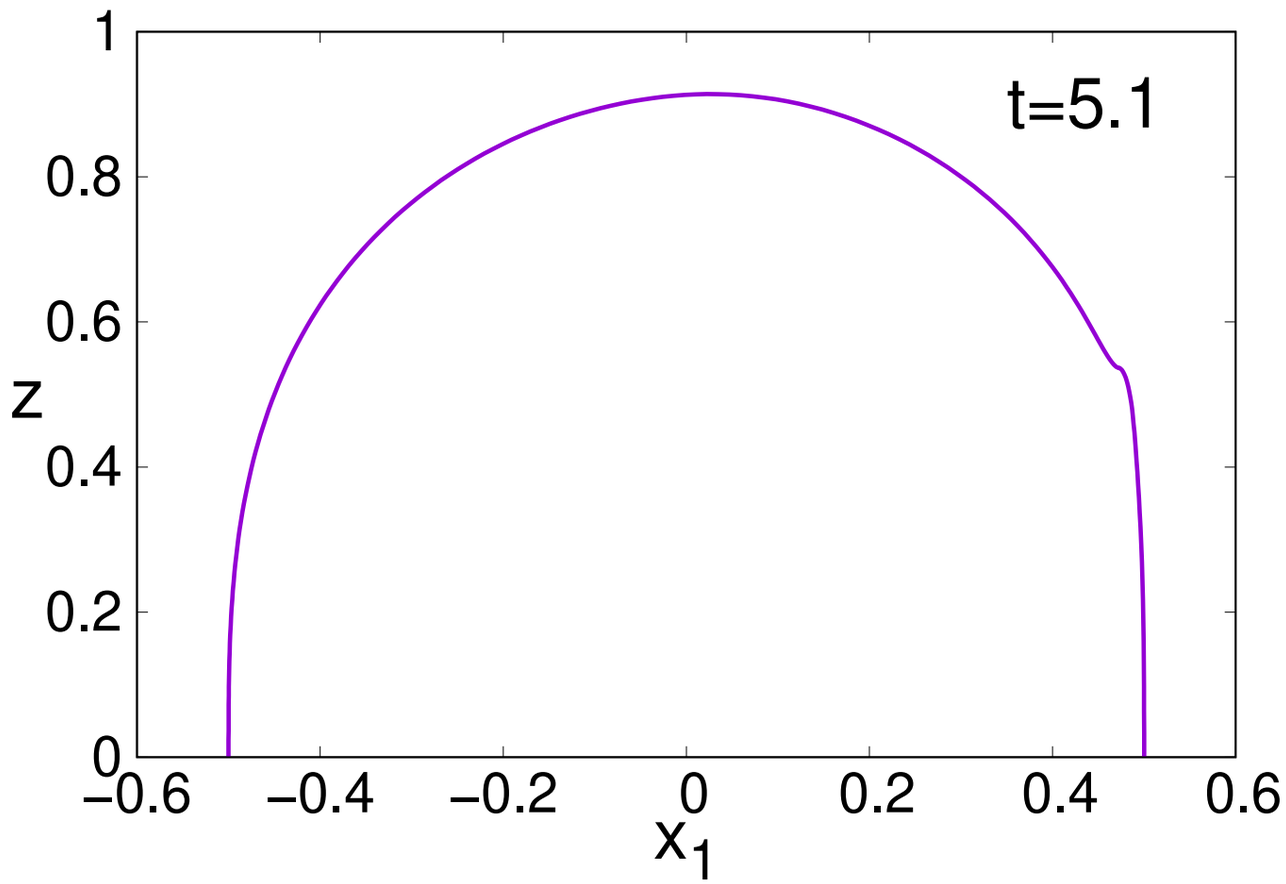


String motion in (t,z,x_1): AdS3 $\subset$ AdS5

Quench: $0<t<2$

Amplitude: $\varepsilon=0.03$

String time evolution

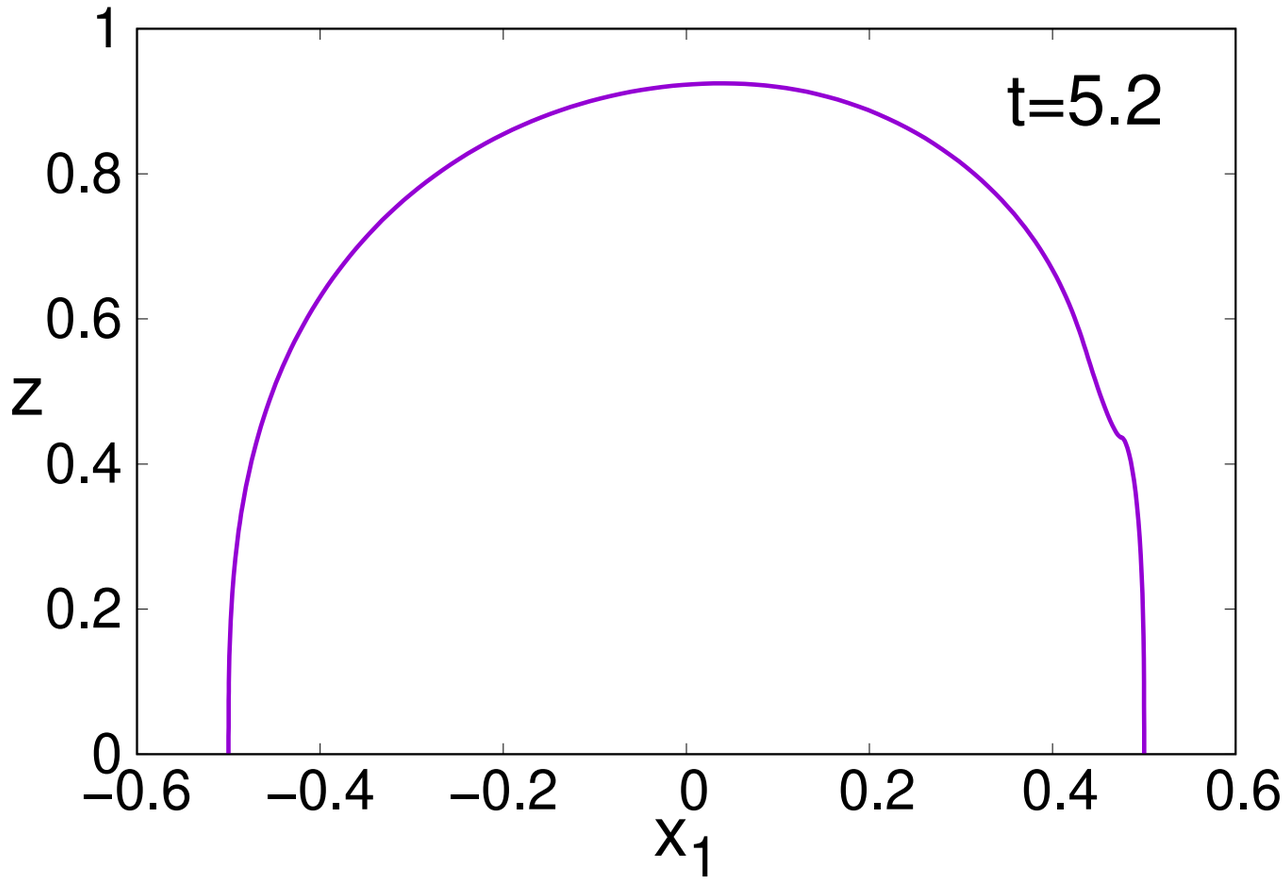


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

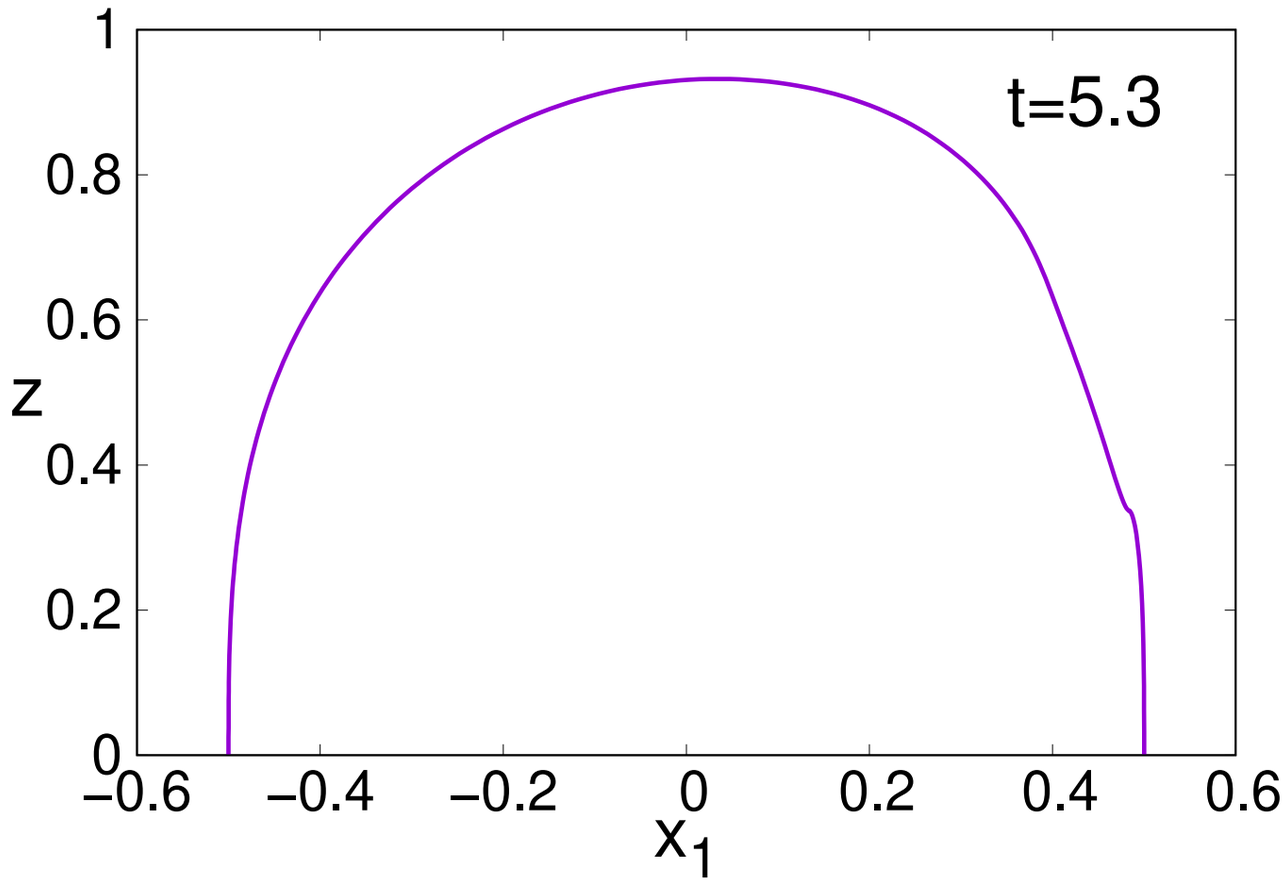


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

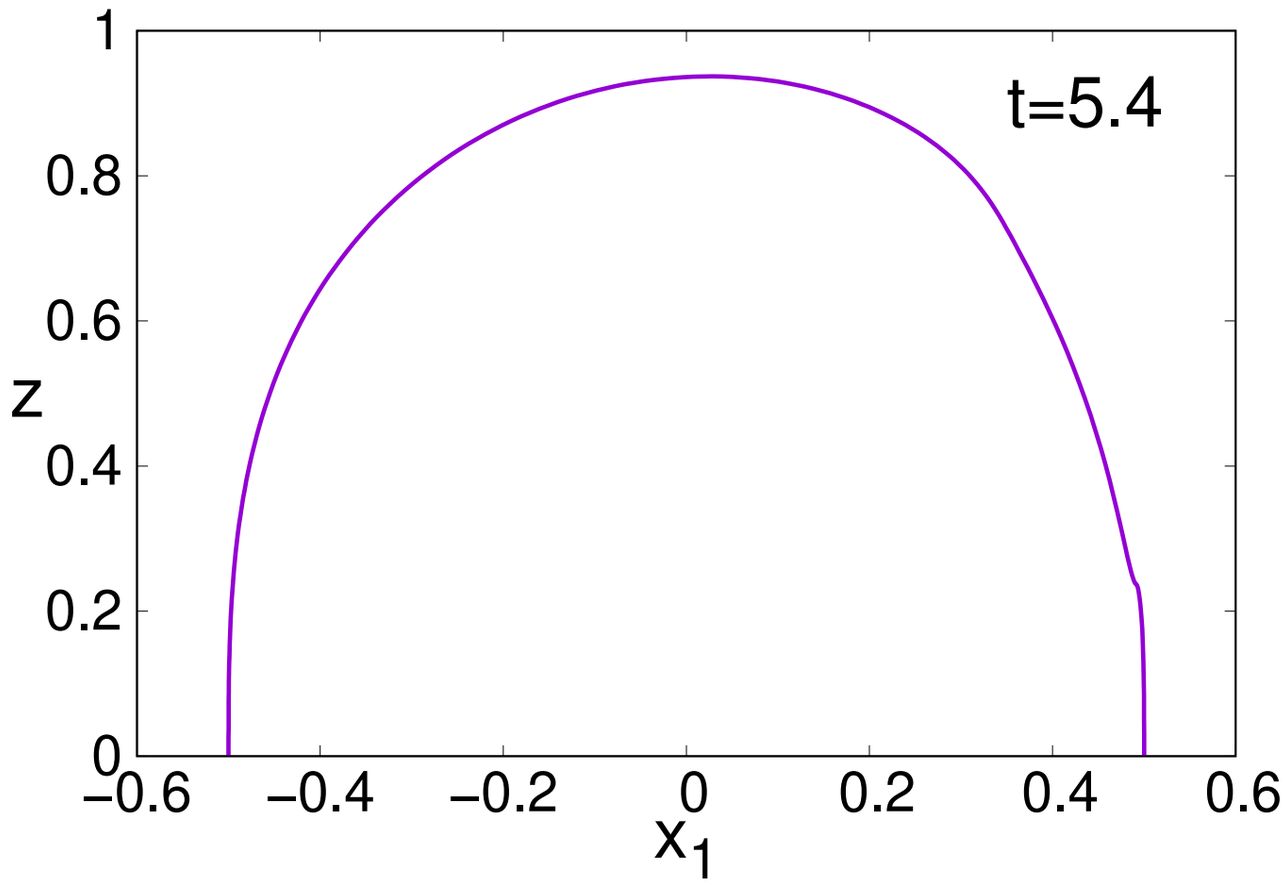


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

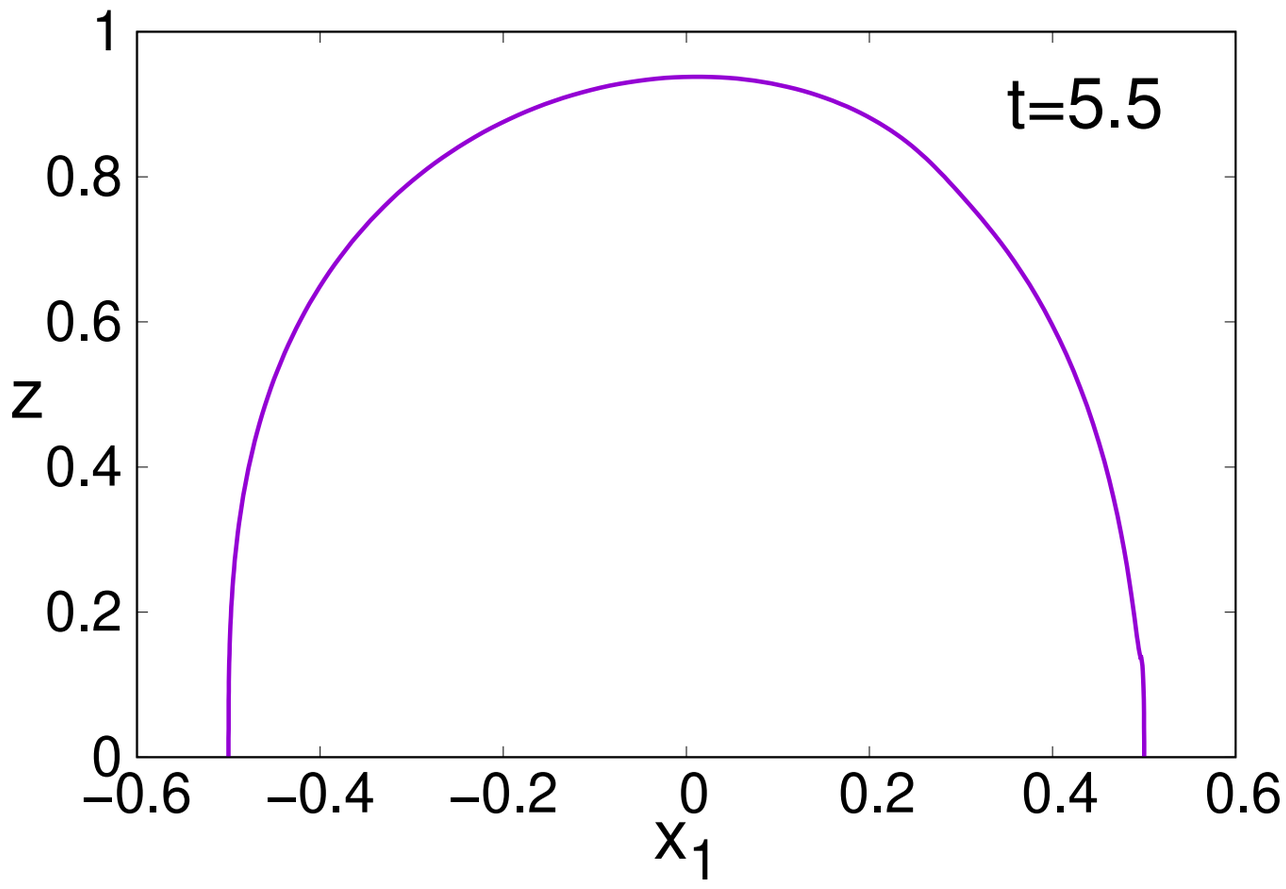


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

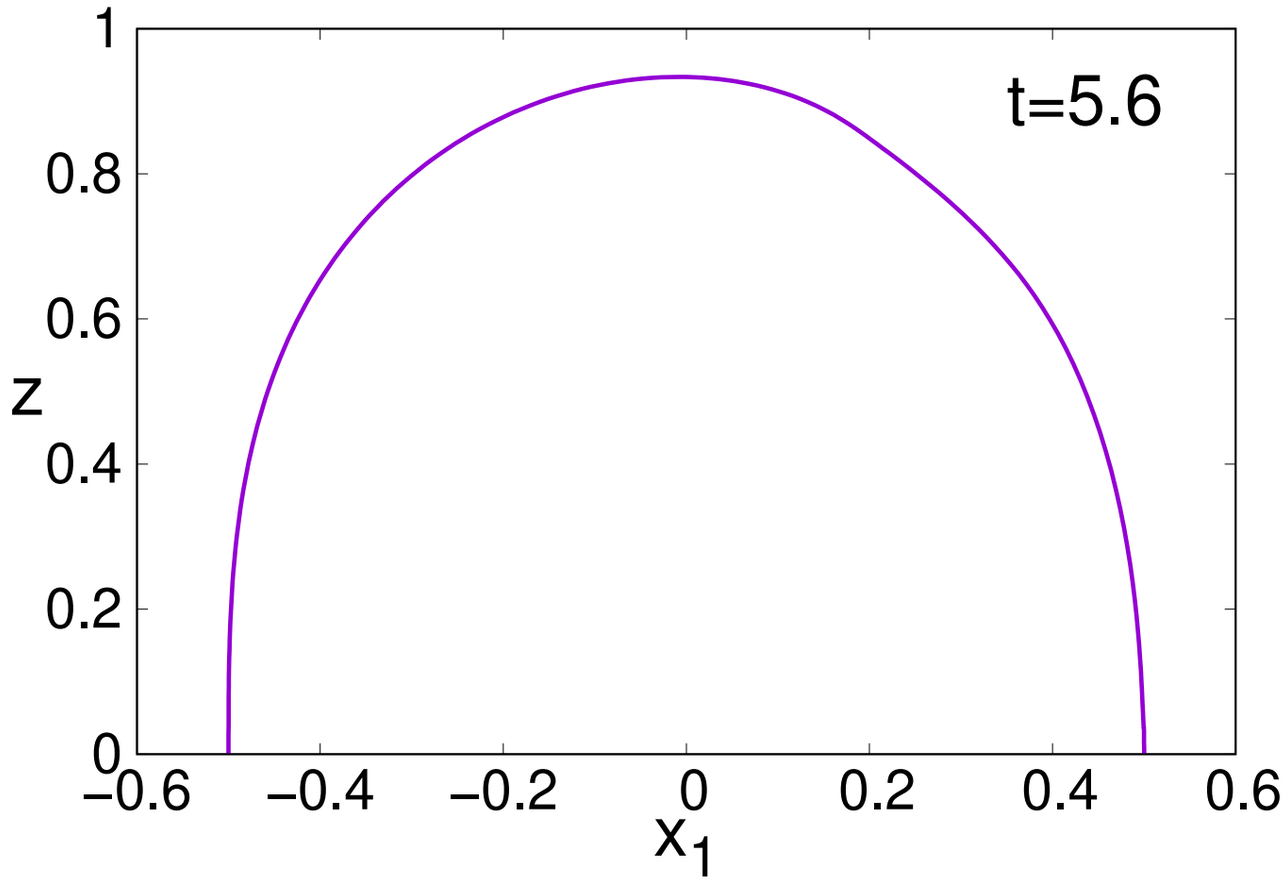


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

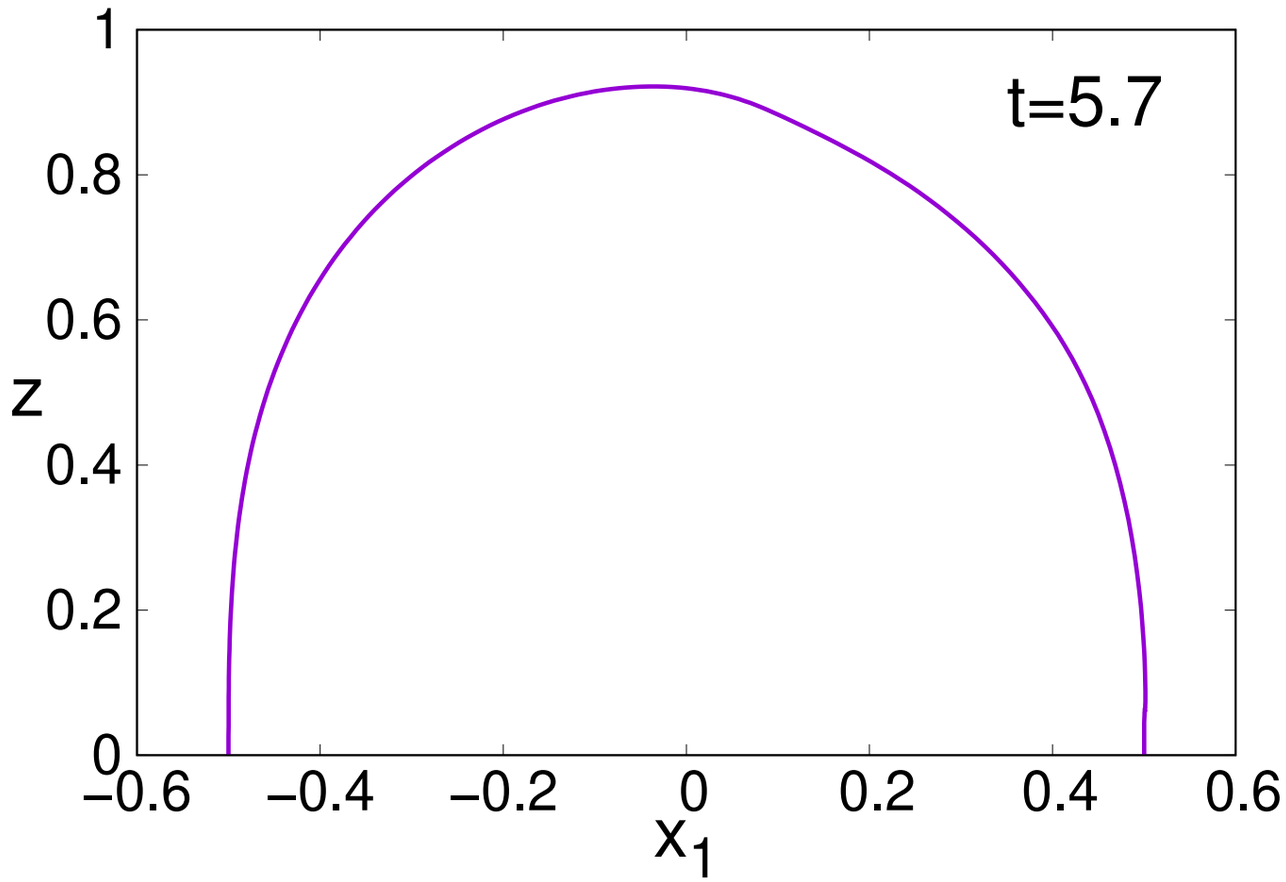


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

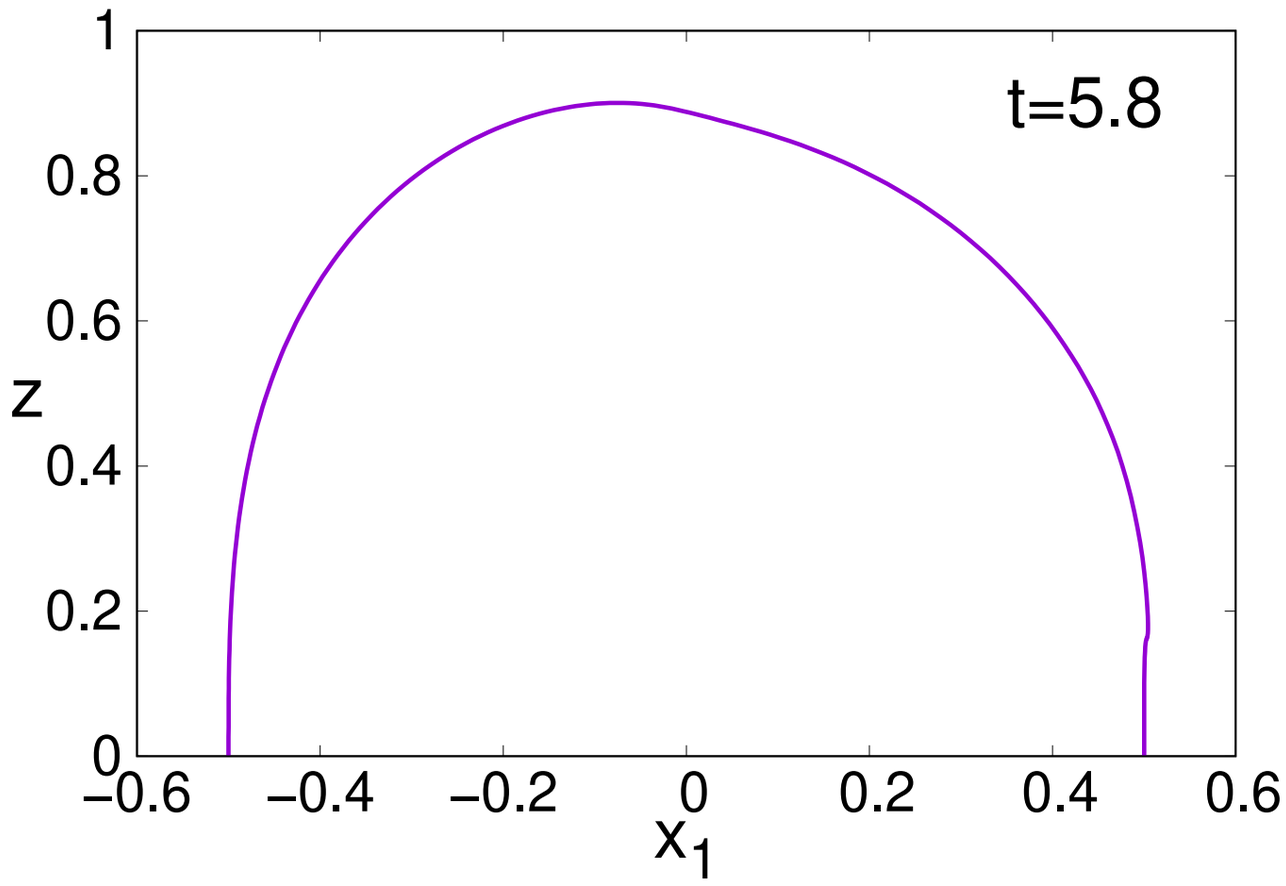


String motion in (t,z,x_1): AdS3 $\subset$ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

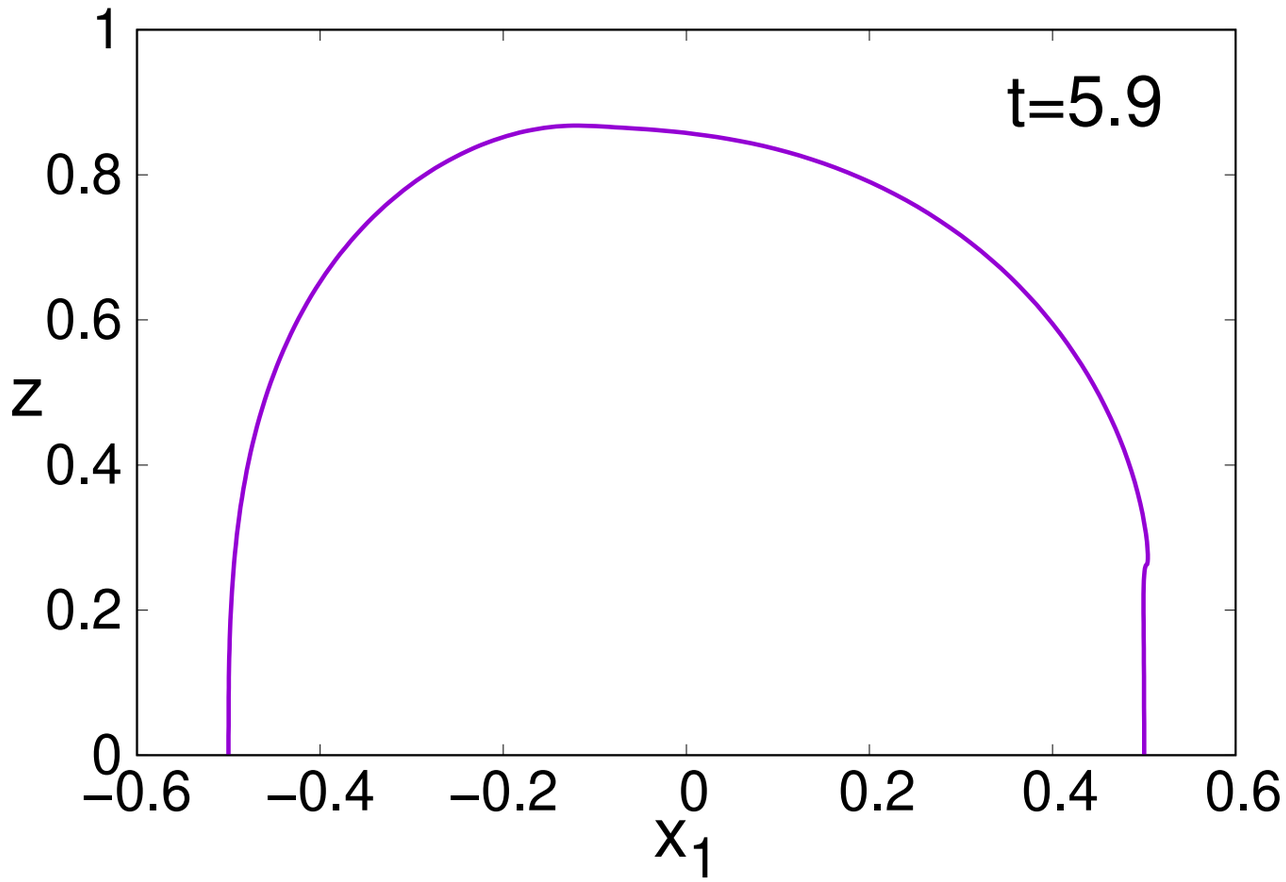


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

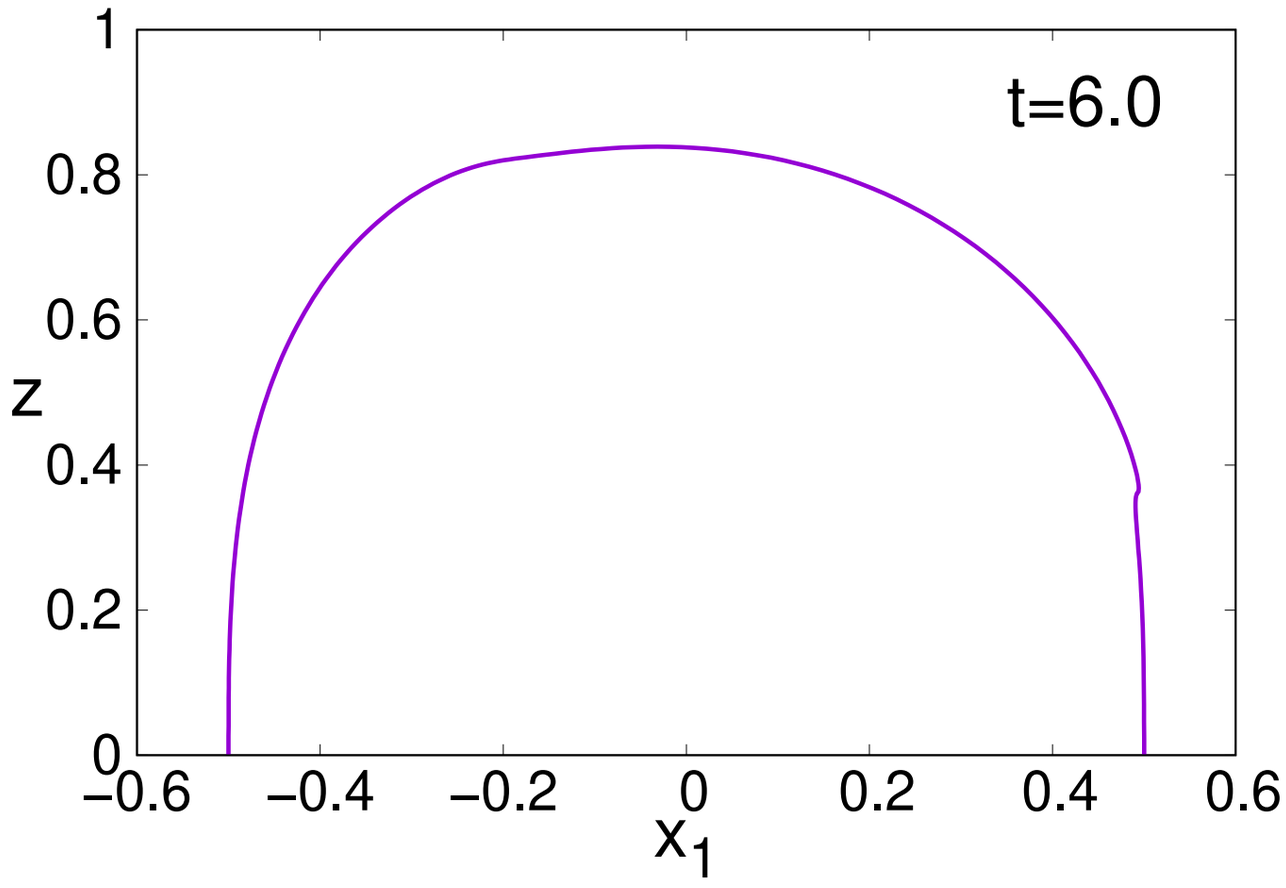


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

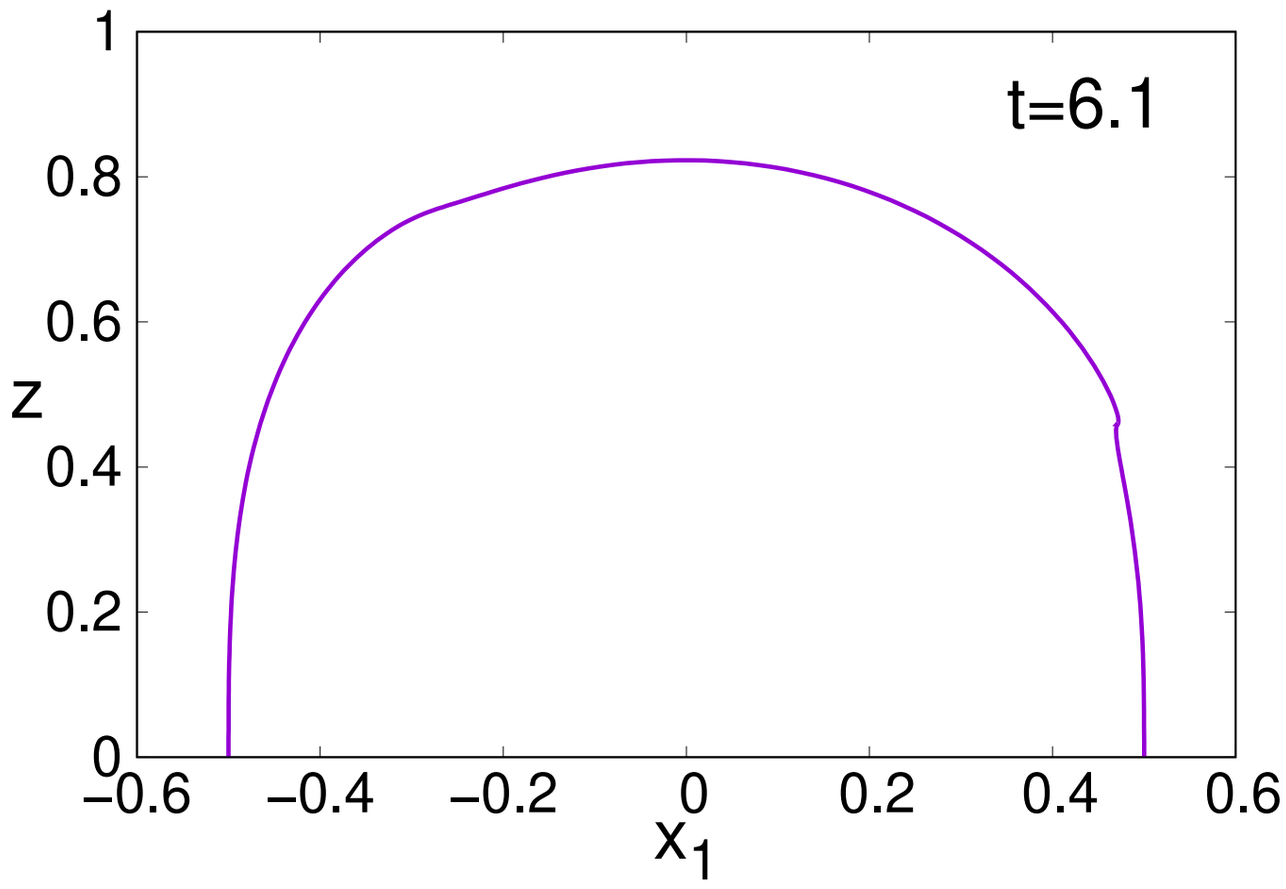


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

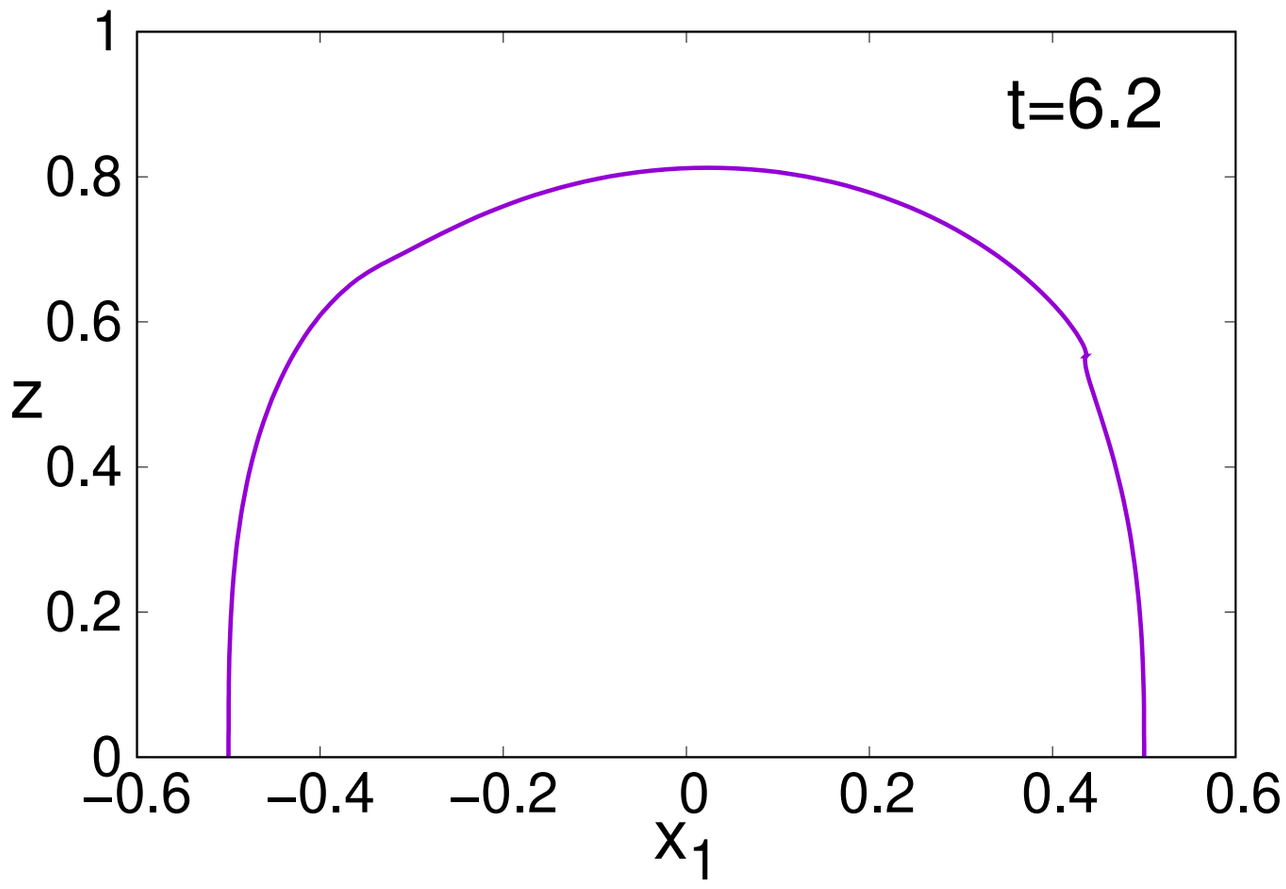


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

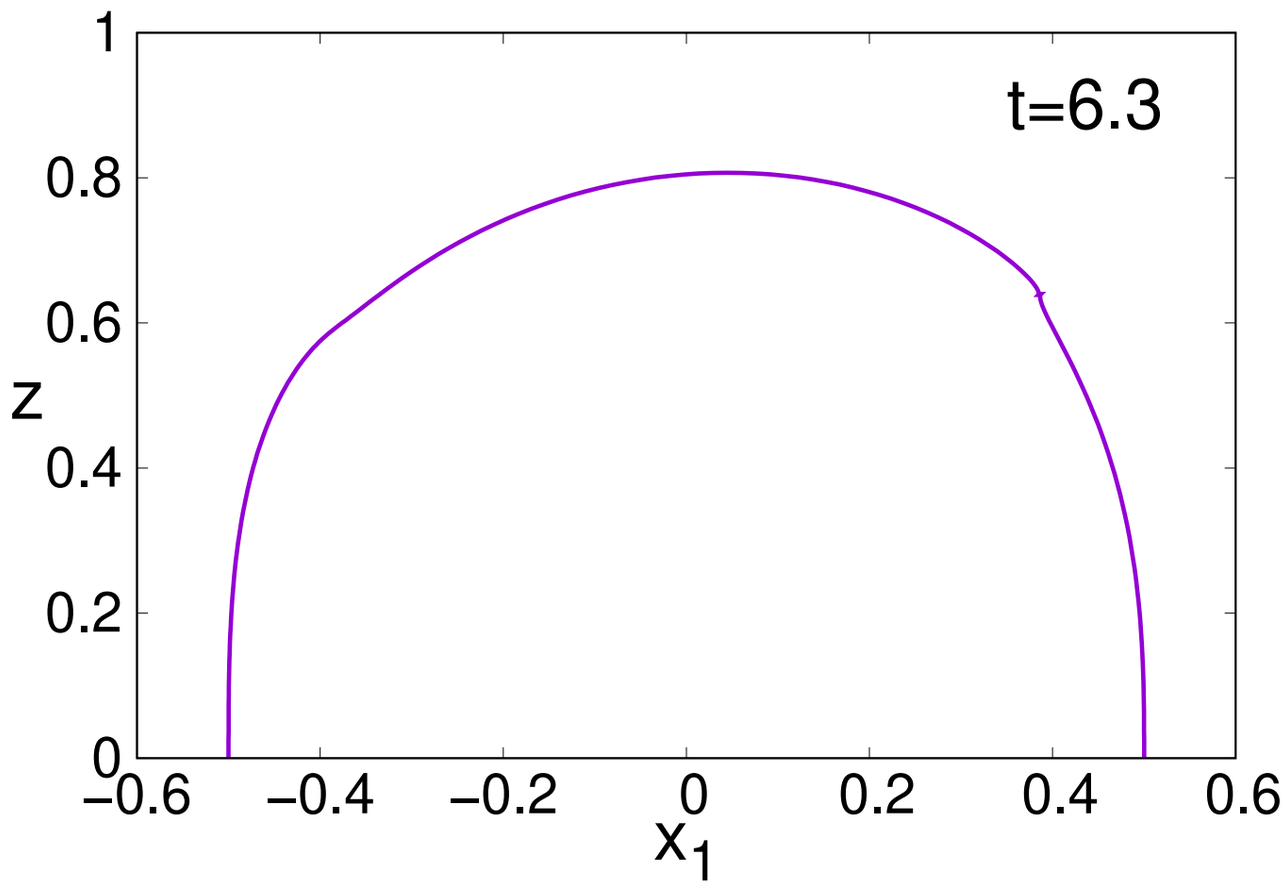


String motion in (t,z,x_1): AdS3 $\subset$ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

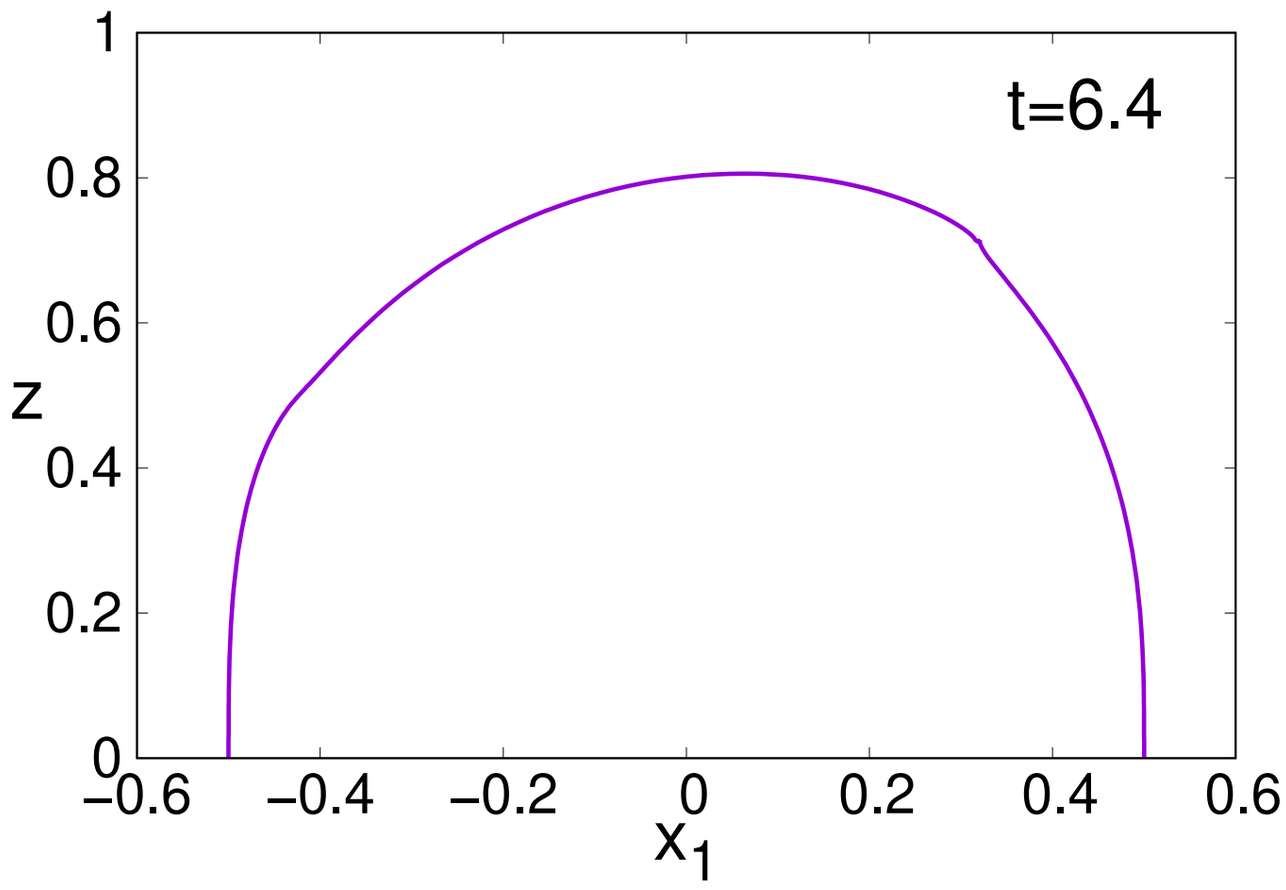


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

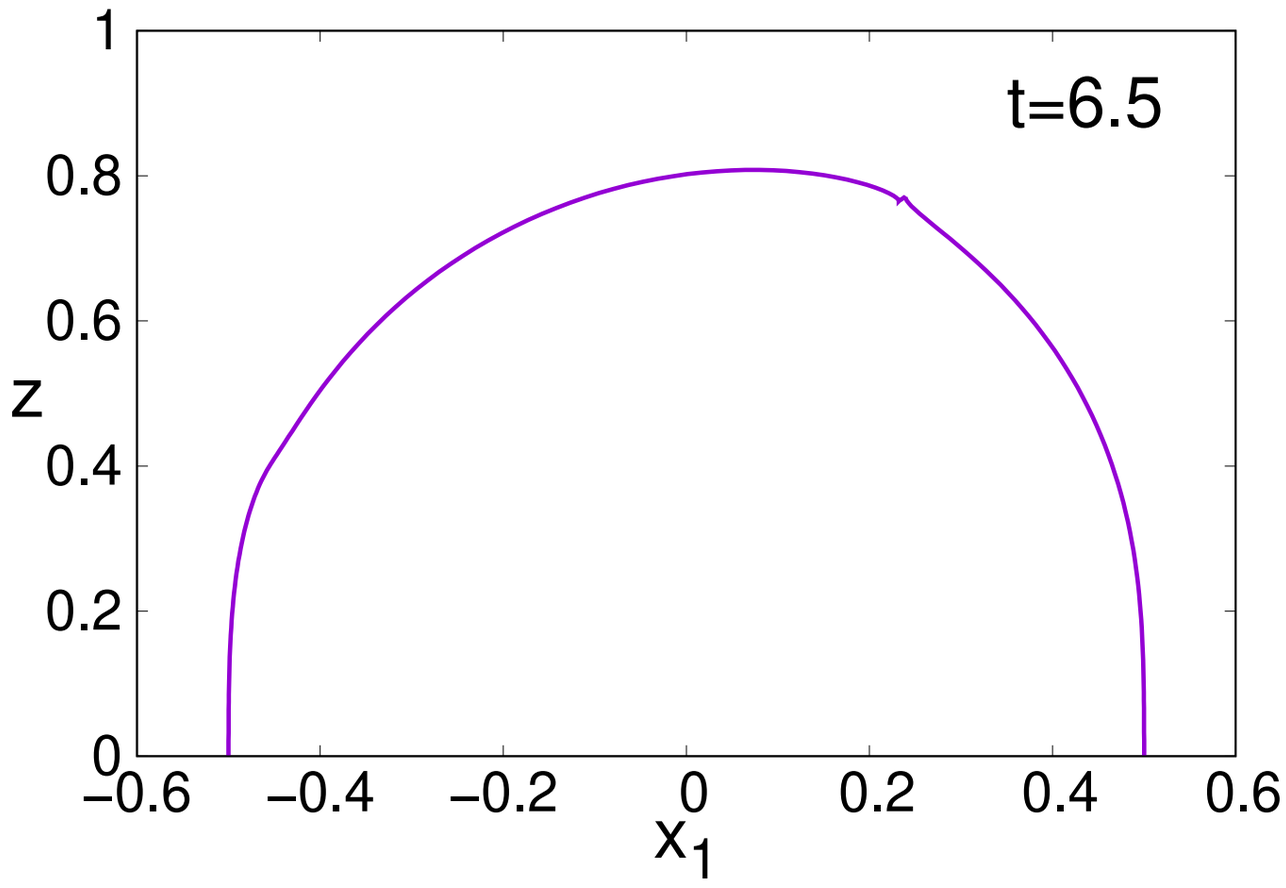


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

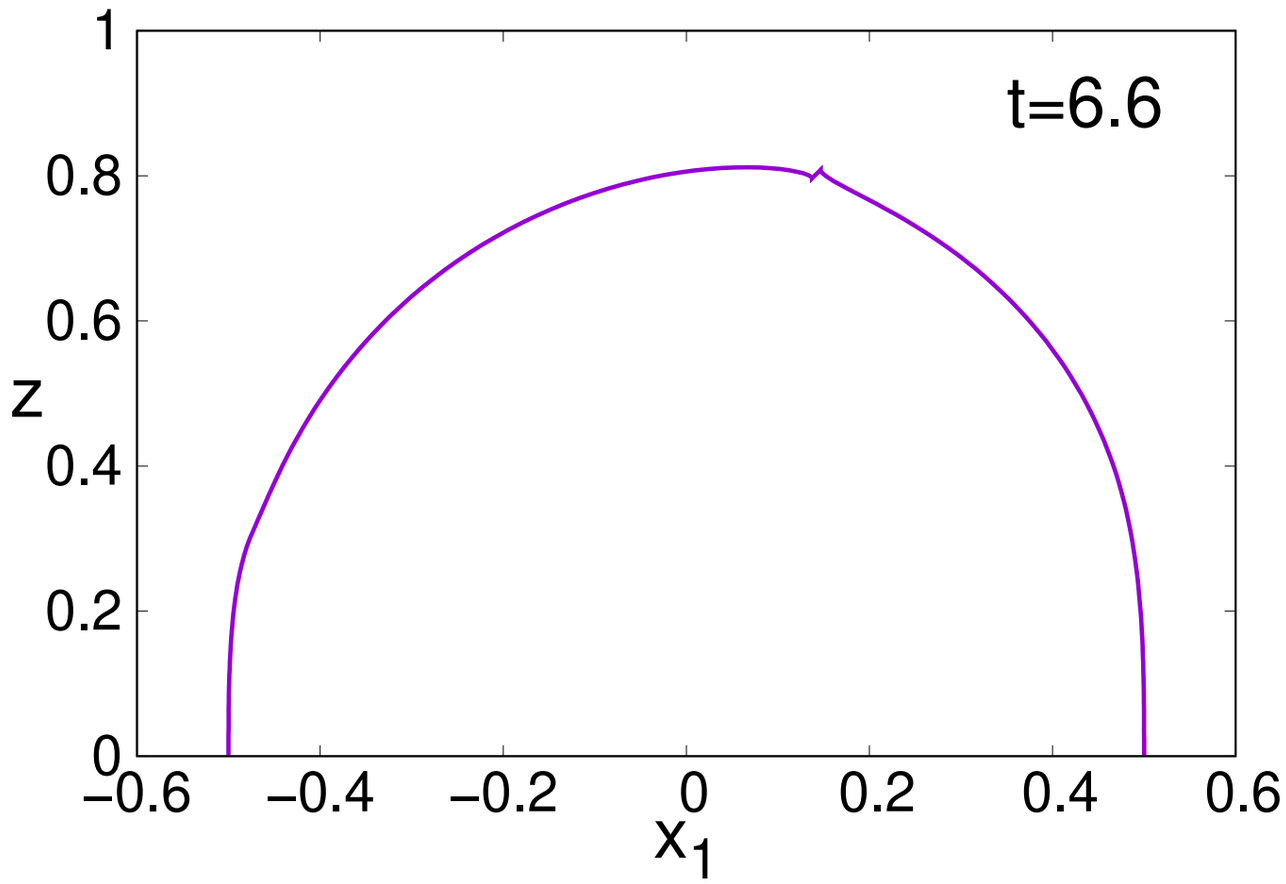


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

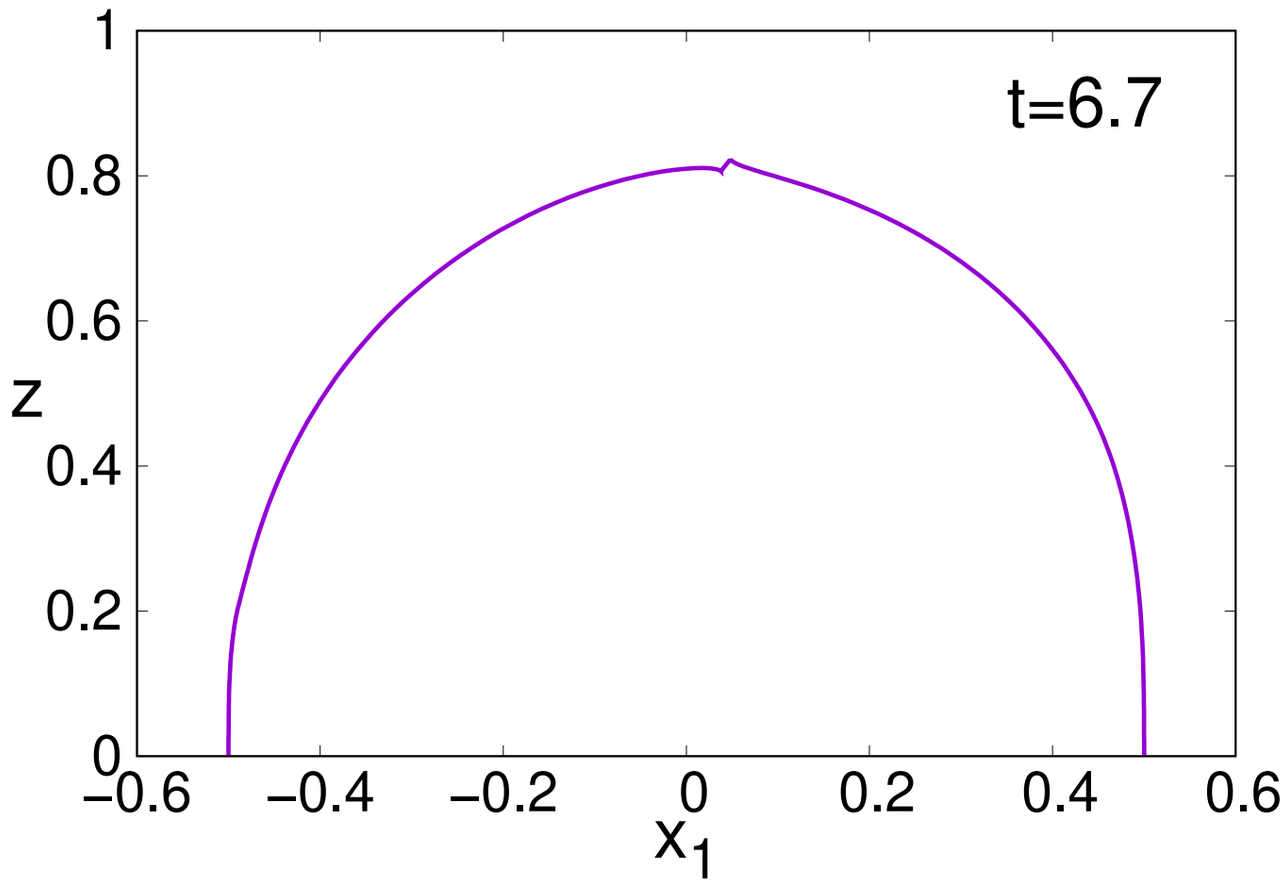


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

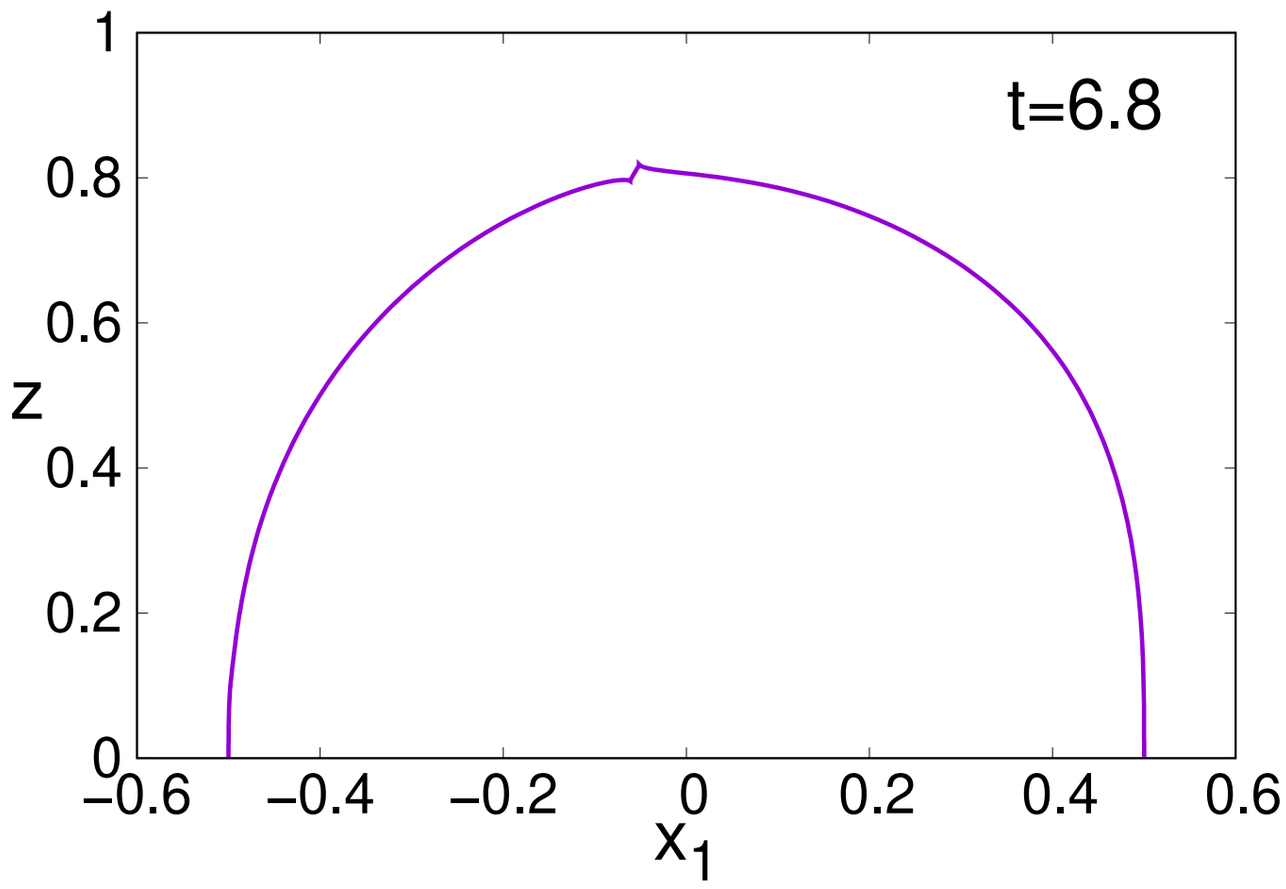


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

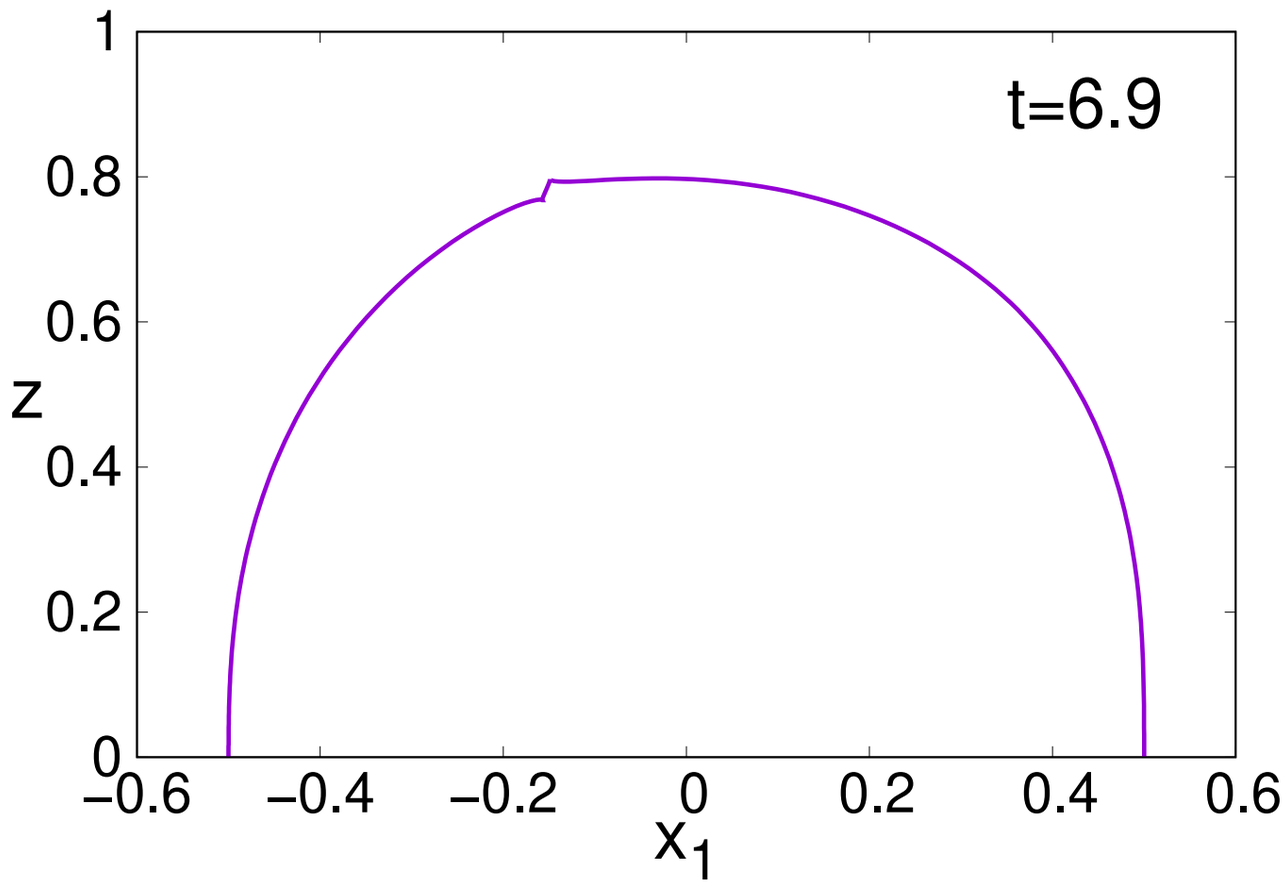


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

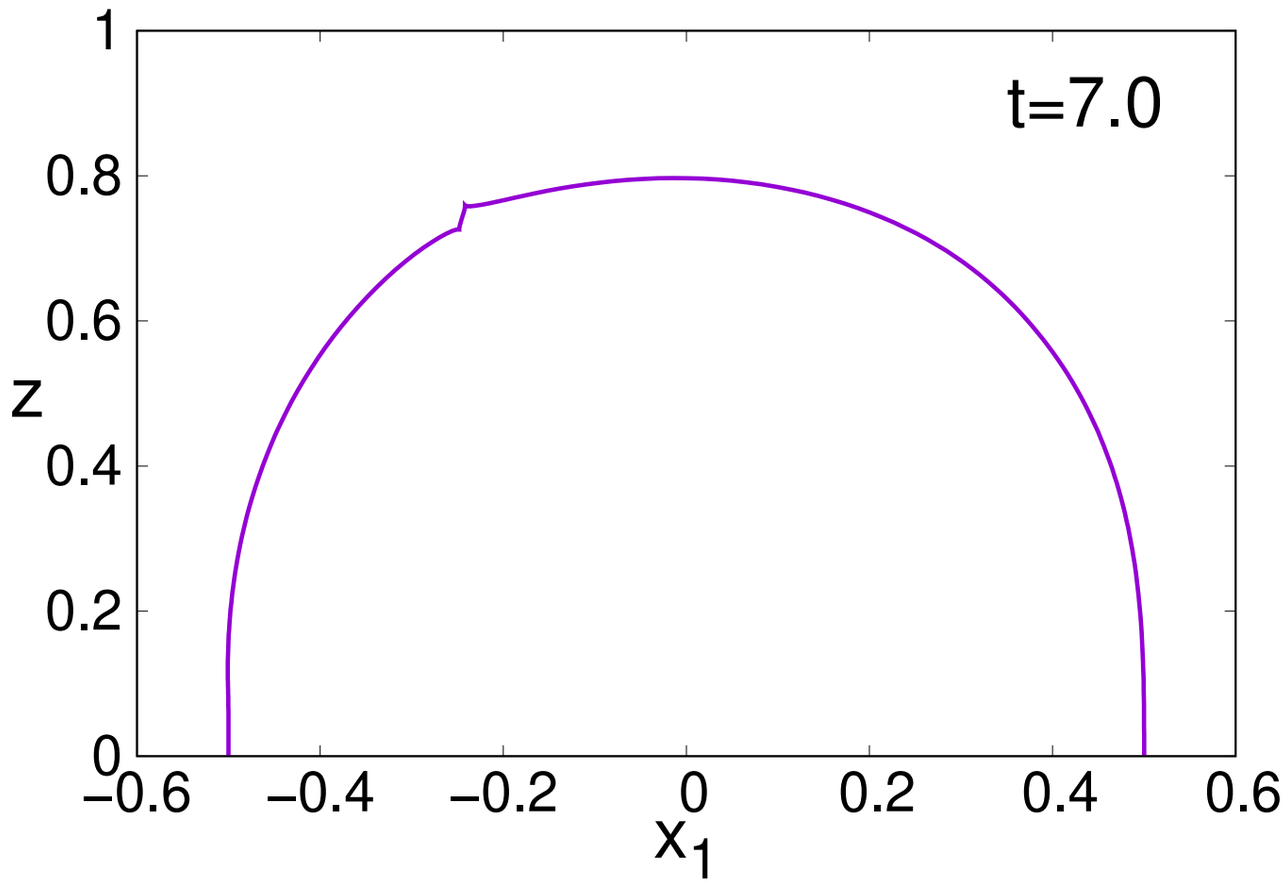


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

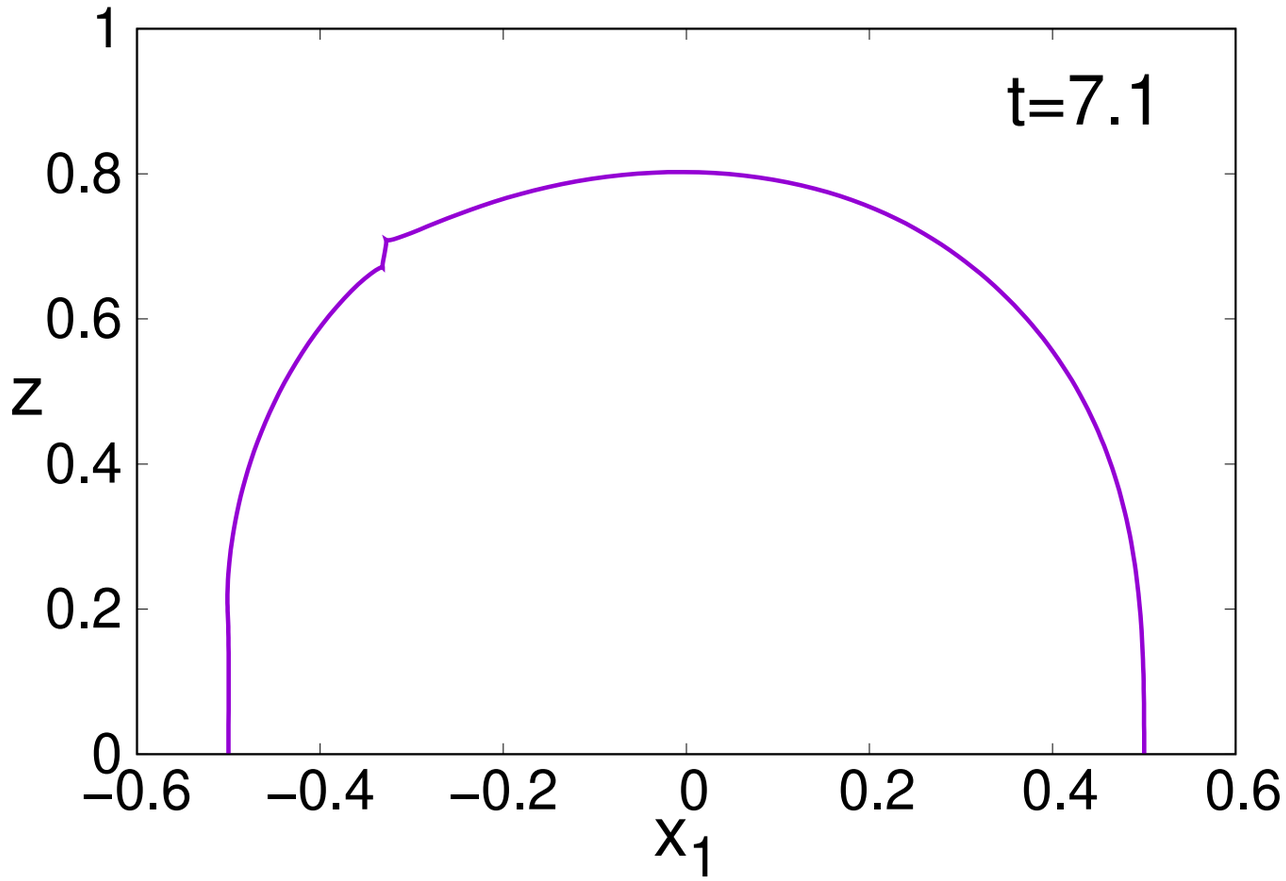


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

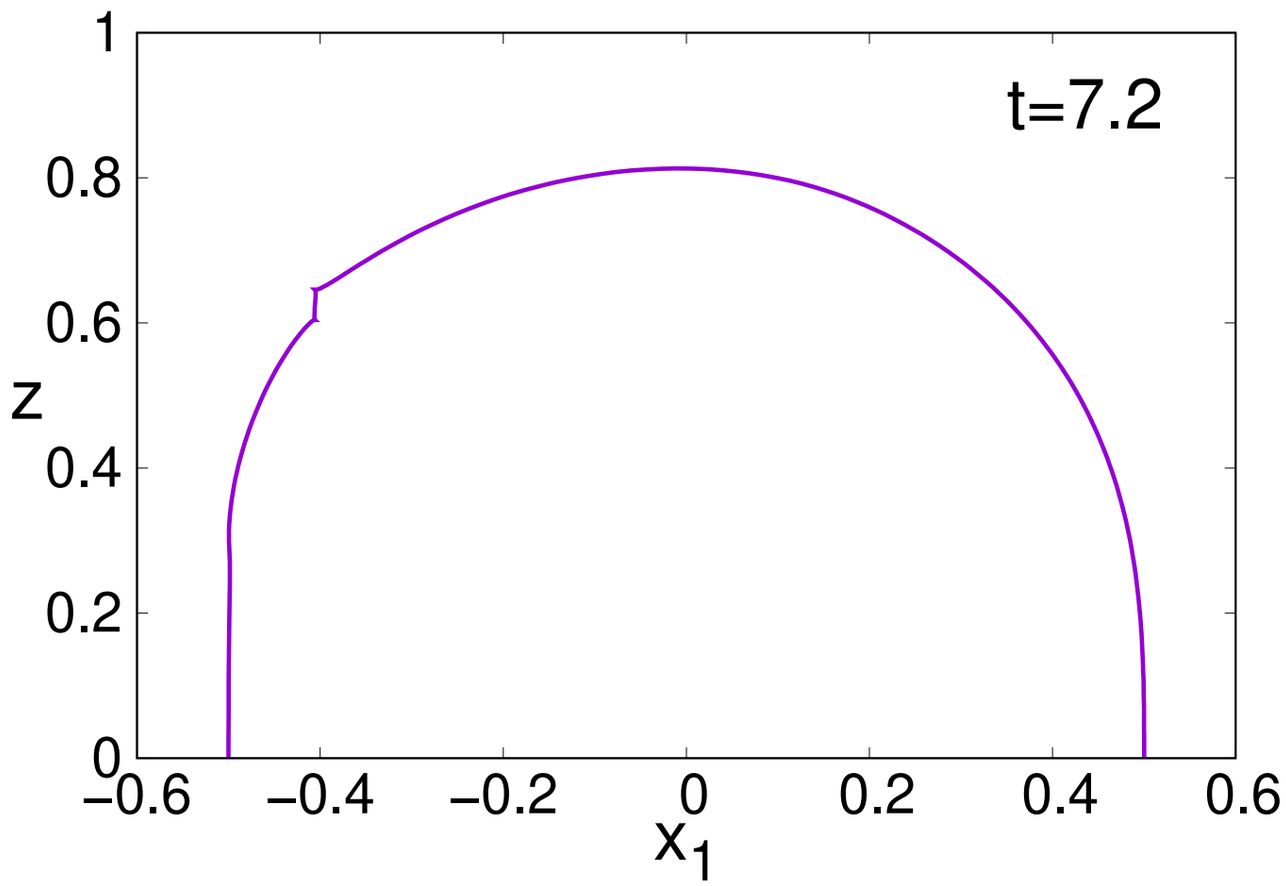


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

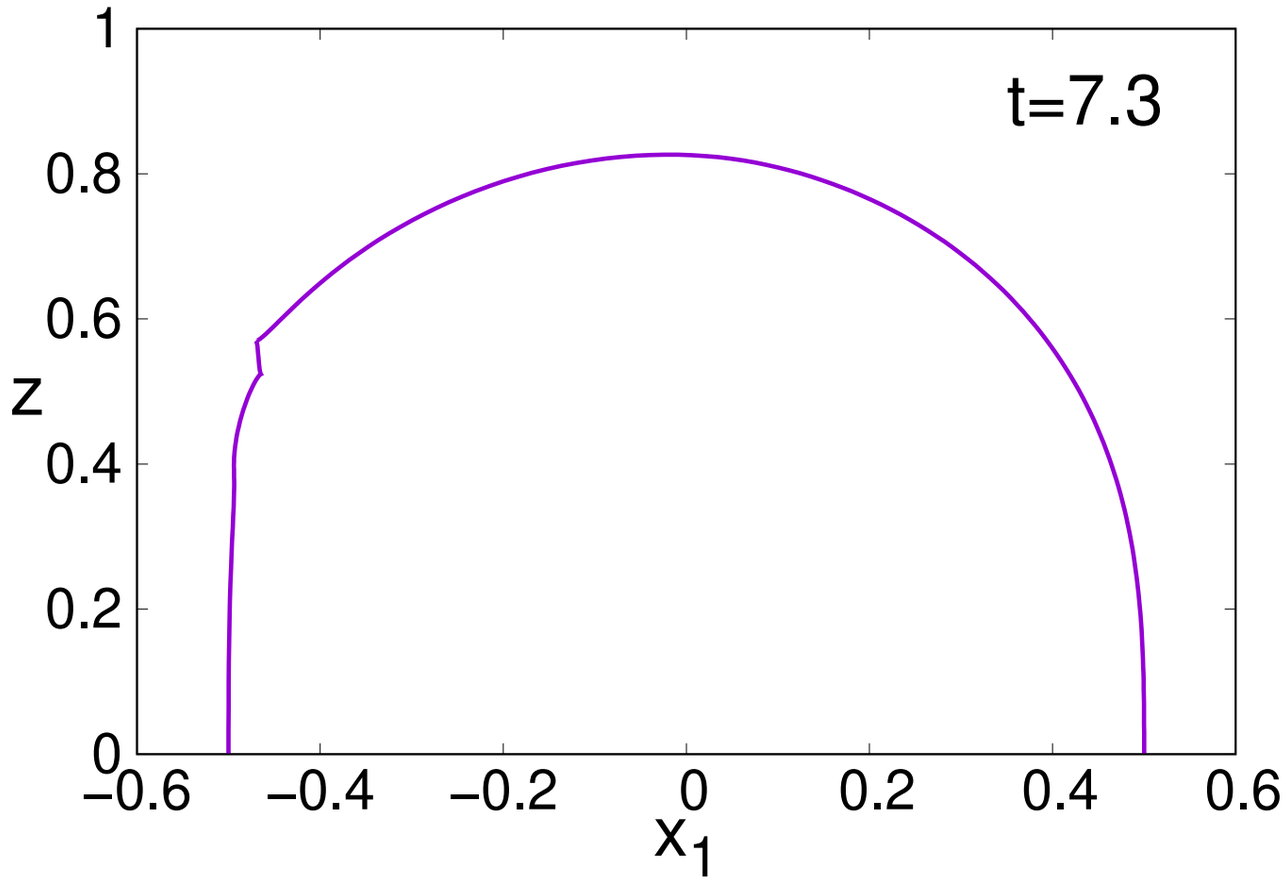


String motion in (t,z,x_1): AdS3 $\subset$ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

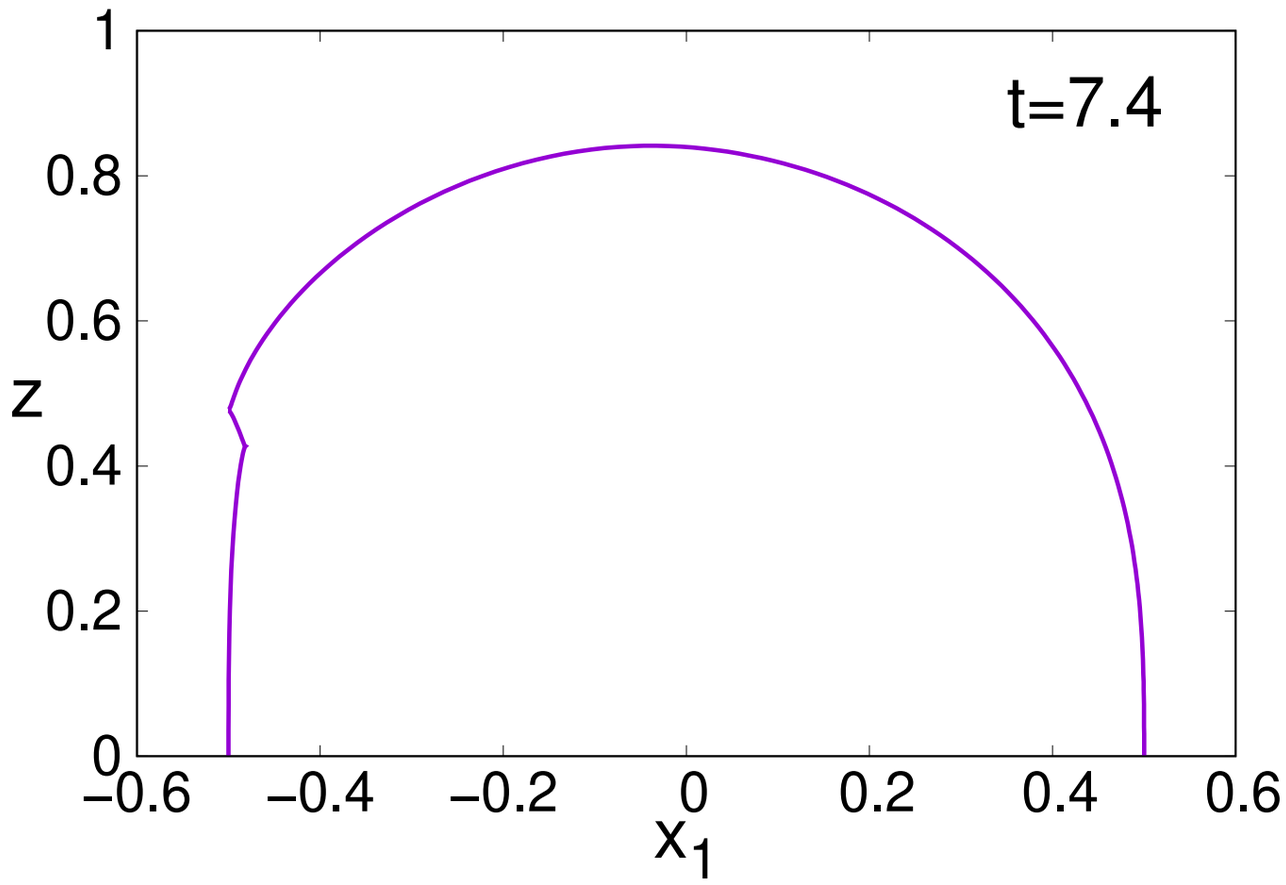


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

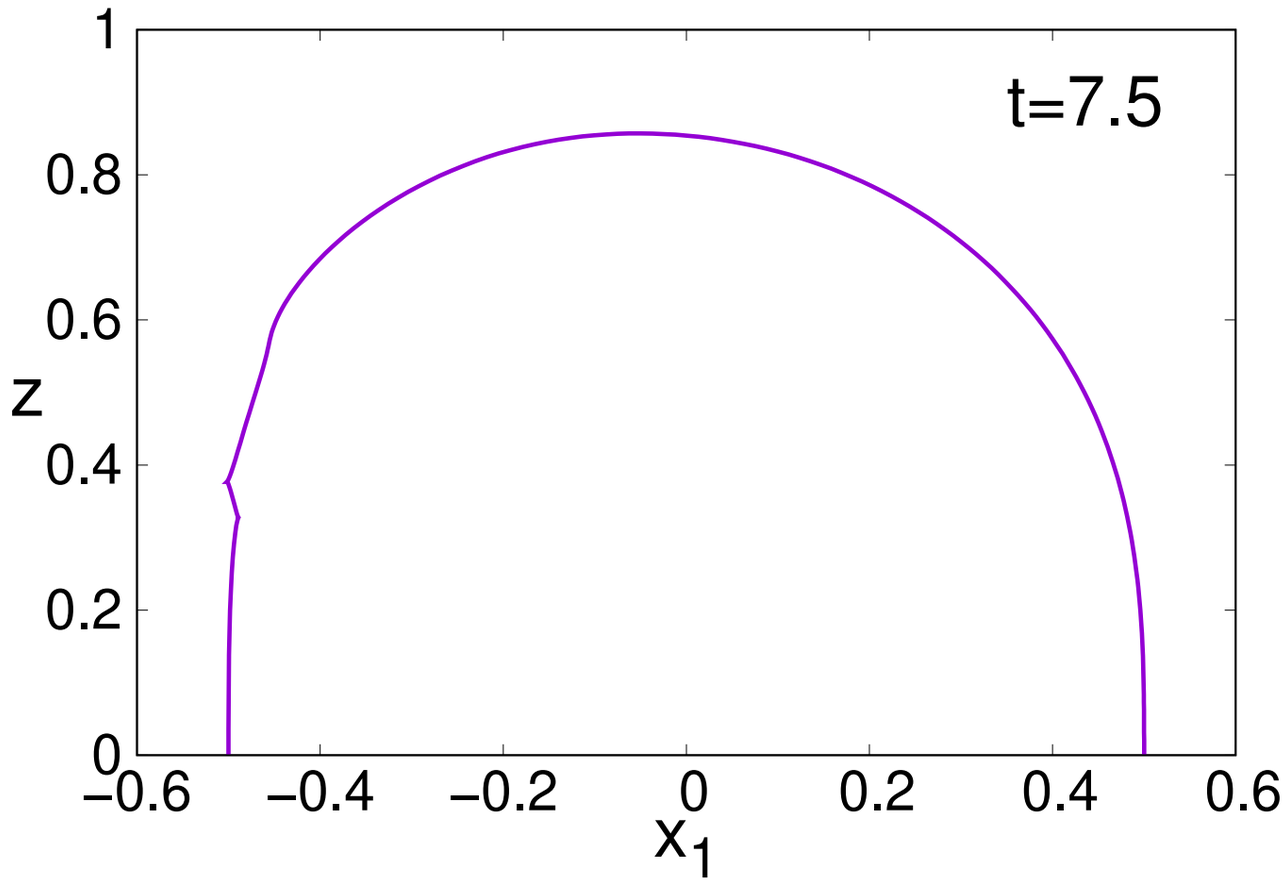


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

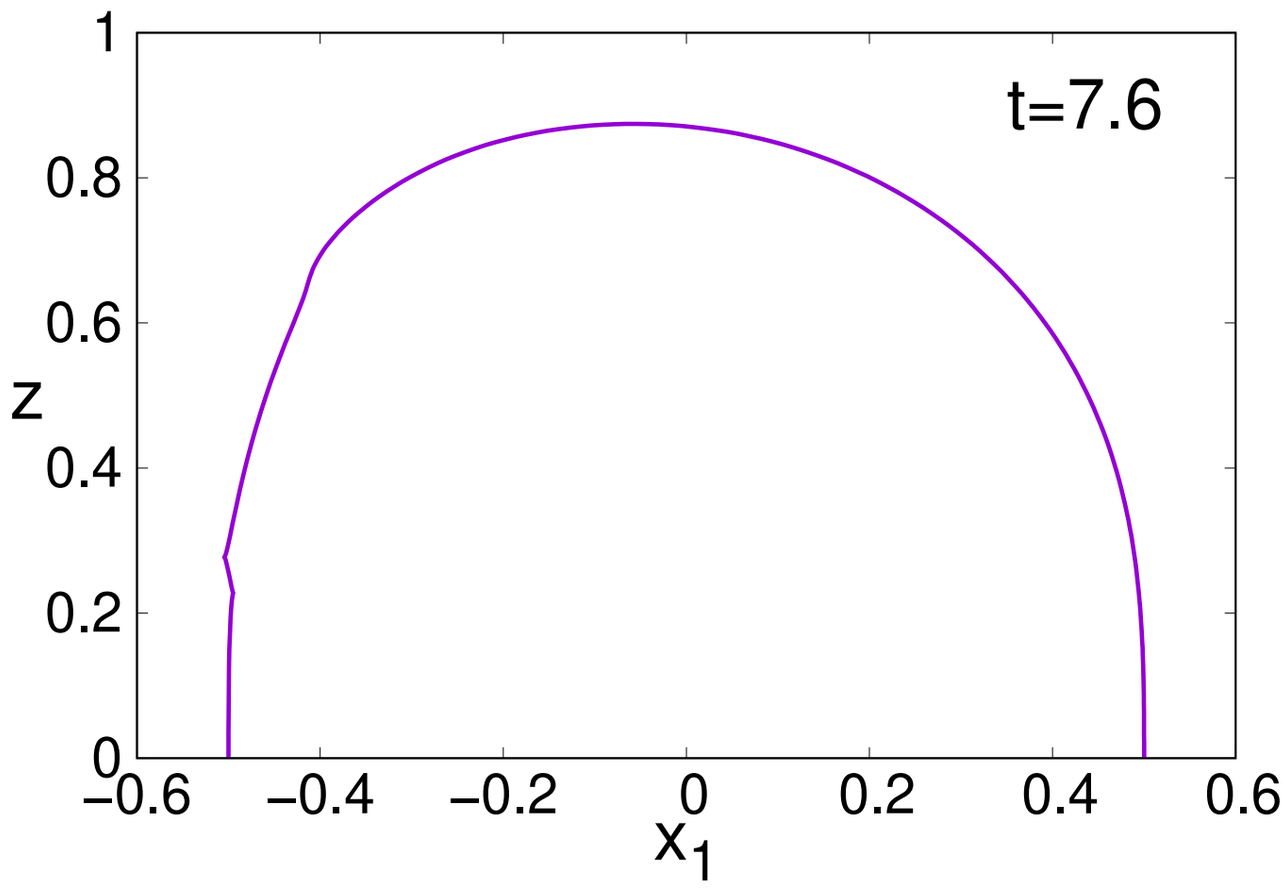


String motion in (t,z,x_1): AdS3 $\subset$ AdS5

Quench: $0<t<2$

Amplitude: $\varepsilon=0.03$

String time evolution

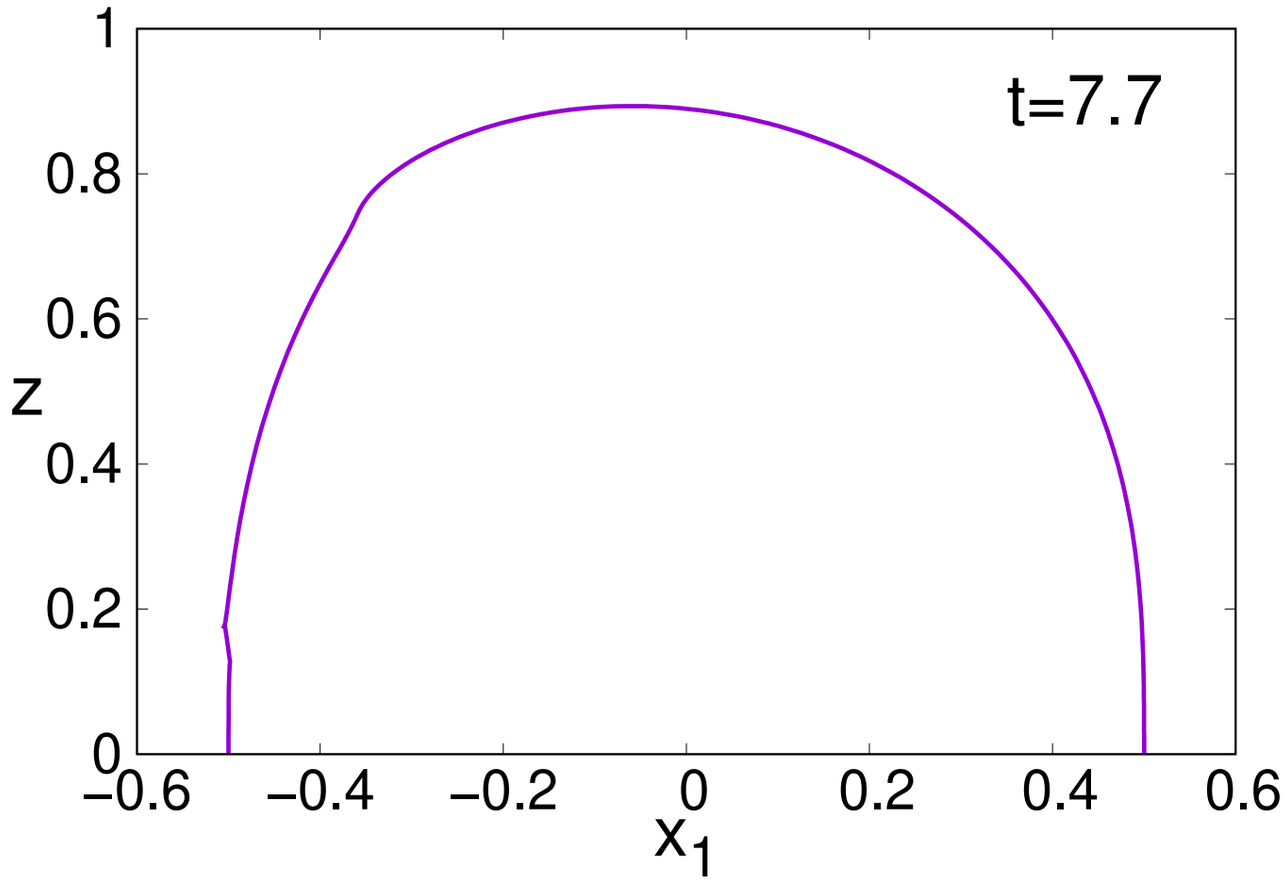


String motion in (t,z,x_1): AdS3 $\subset$ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

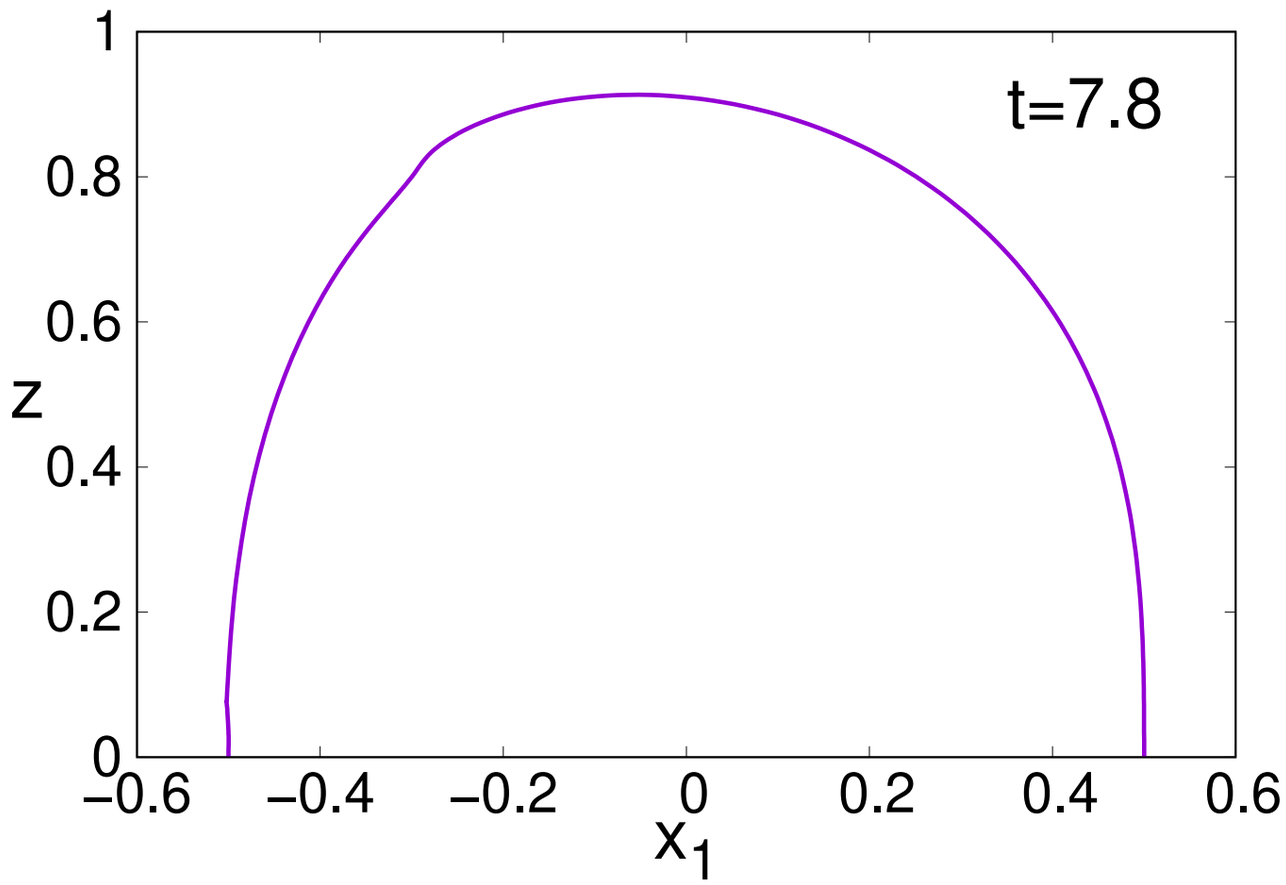


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

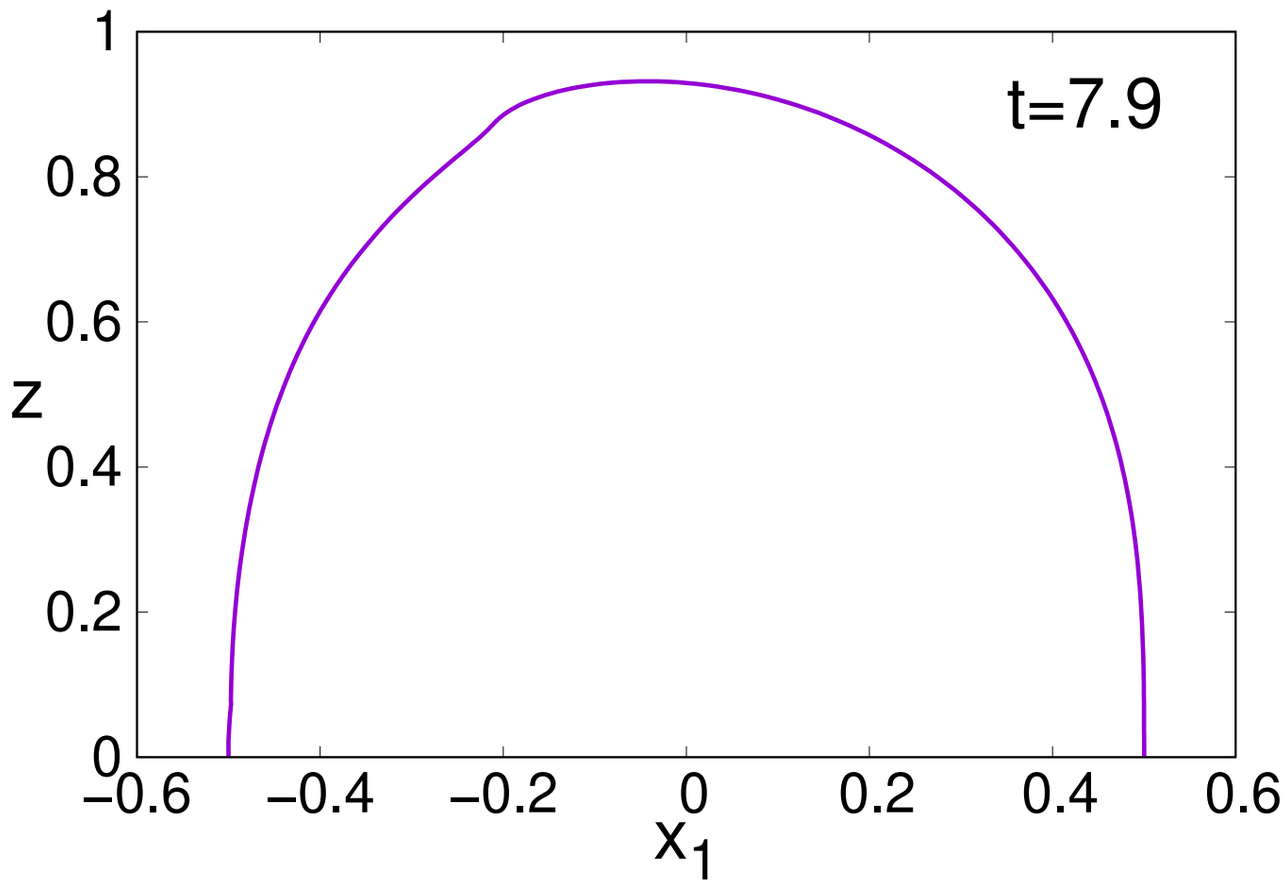


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

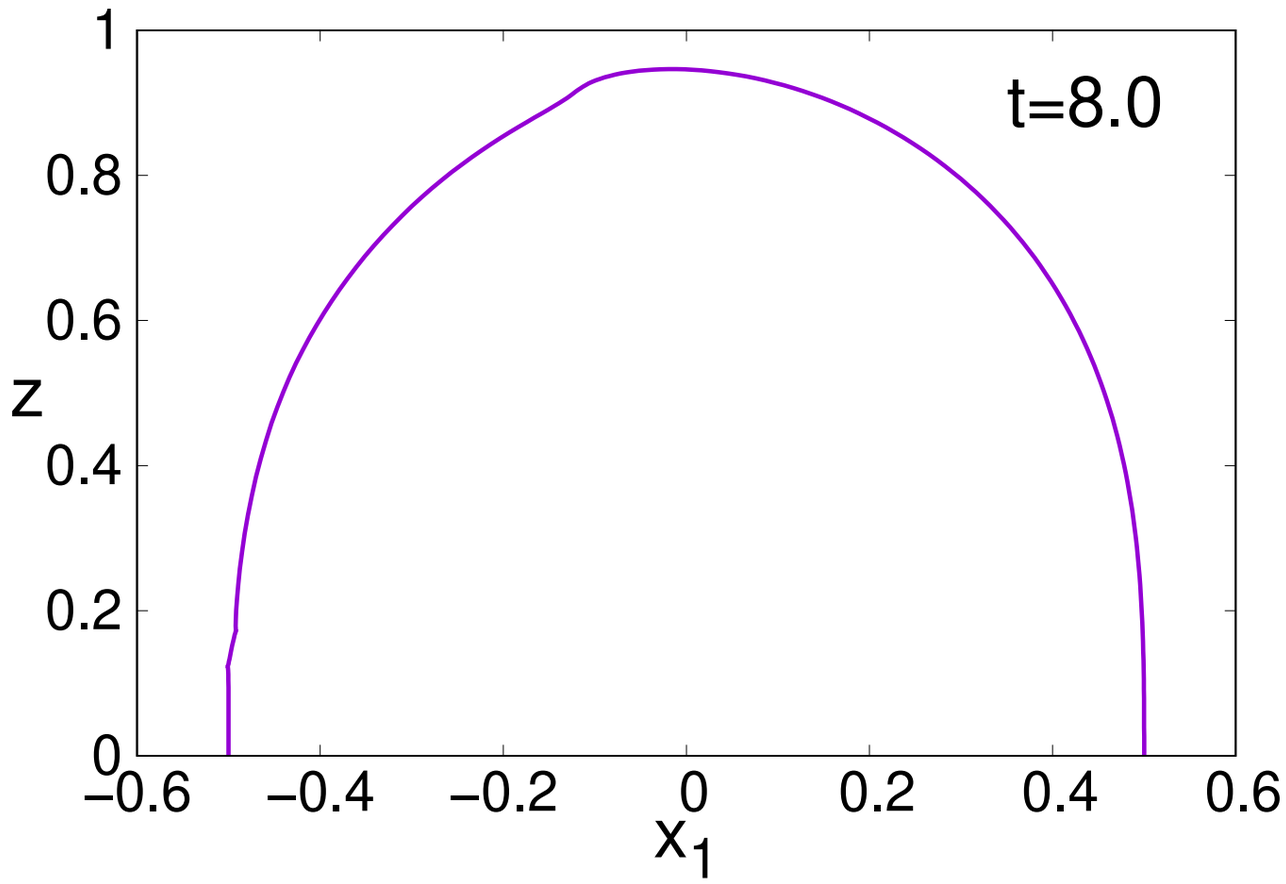


String motion in (t,z,x_1): AdS3 ⊂ AdS5

Quench: 0<t<2

Amplitude: ε=0.03

String time evolution

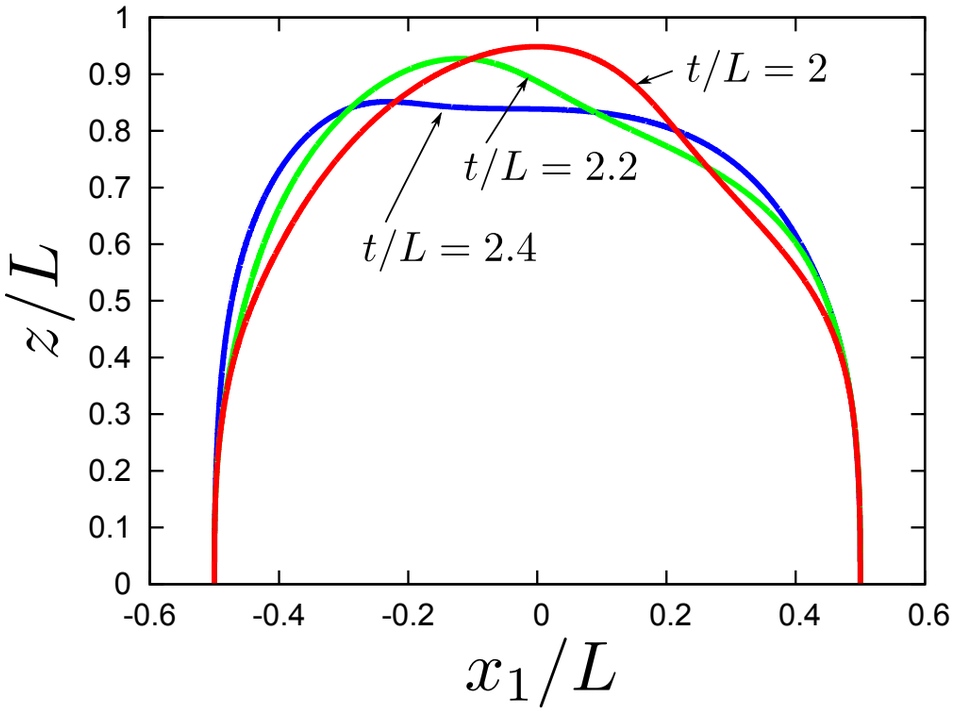


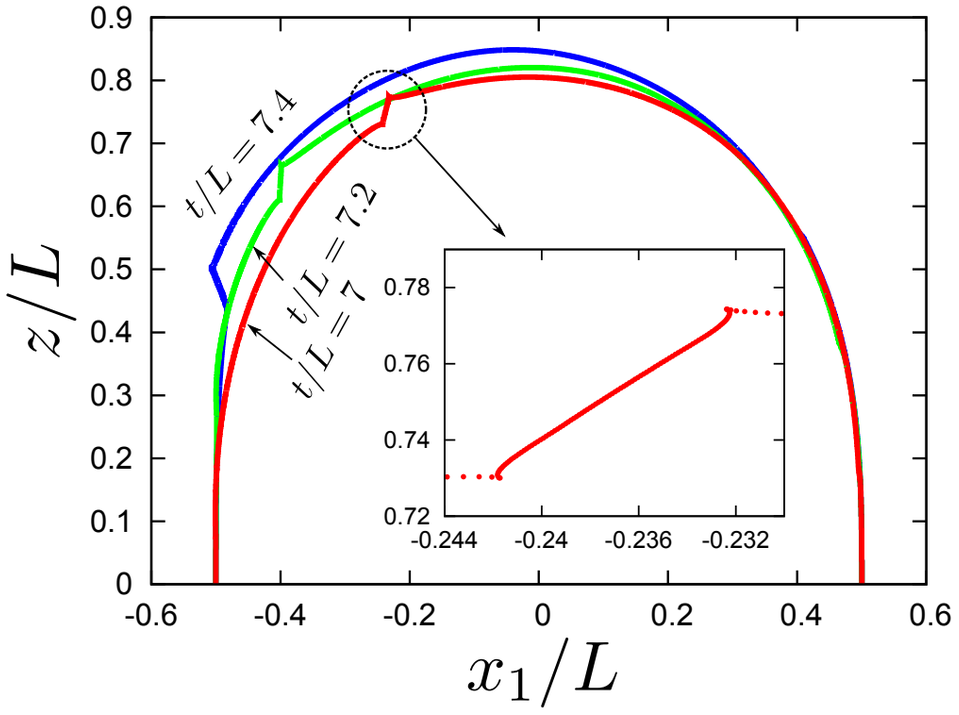
String motion in (t,z,x_1): AdS3 $\subset$ AdS5

Quench: $0<t<2$

Amplitude: $\varepsilon=0.03$

Cusp formation





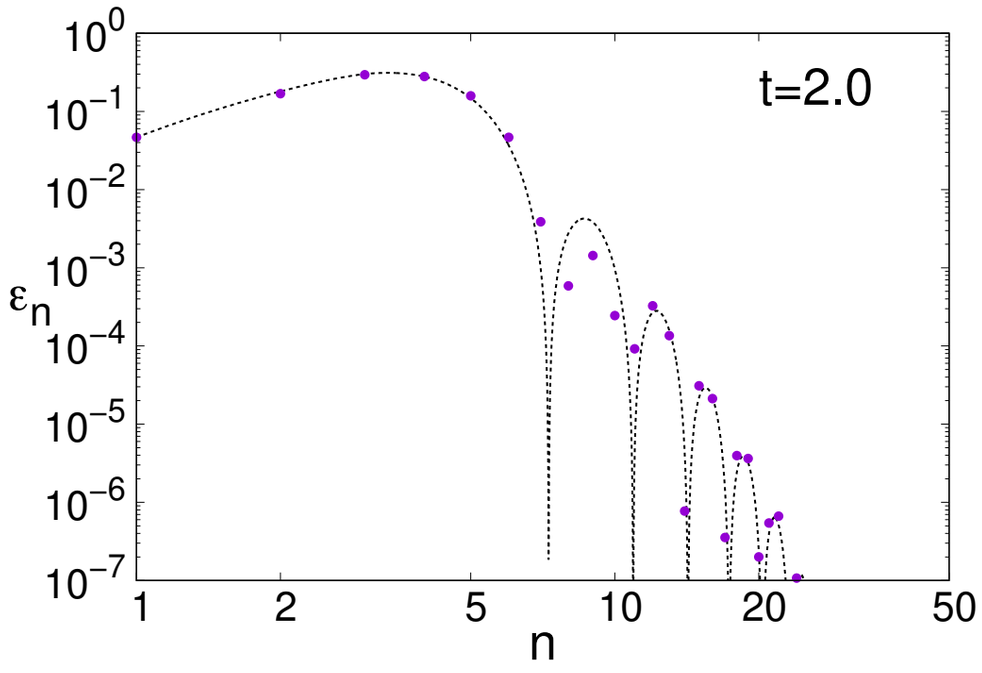
The waves on the string become sharper.

Cusp pairs are created.

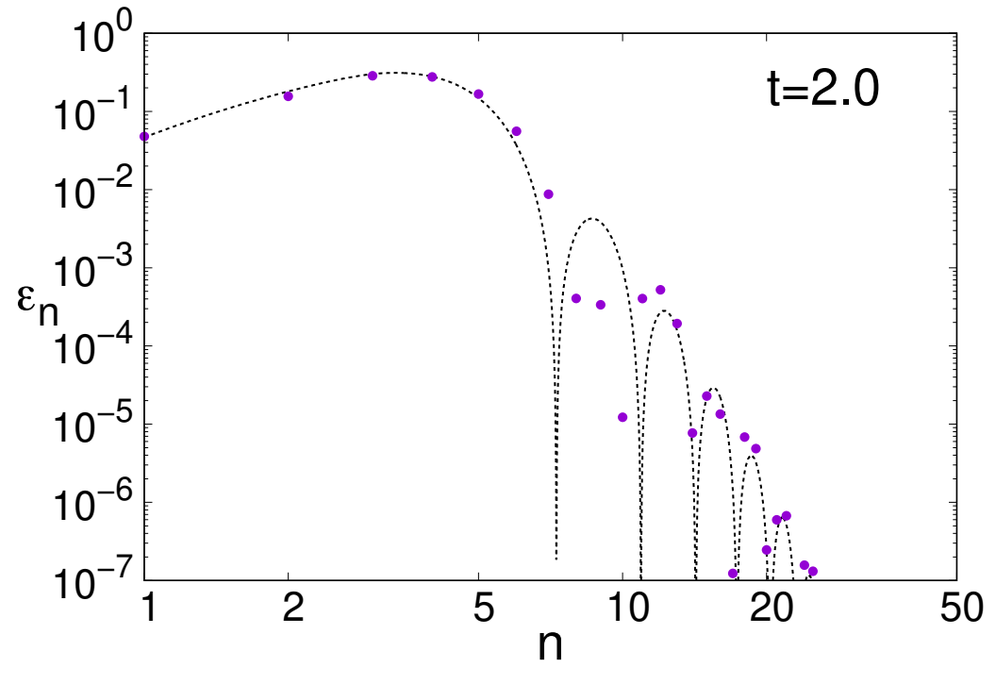
There is a nonzero minimal ε for the cusp formation.

Energy spectrum

We expand the nonlinear solutions by linear eigenmodes and compute the energy spectrum.



ε=0.005 (no cusp)



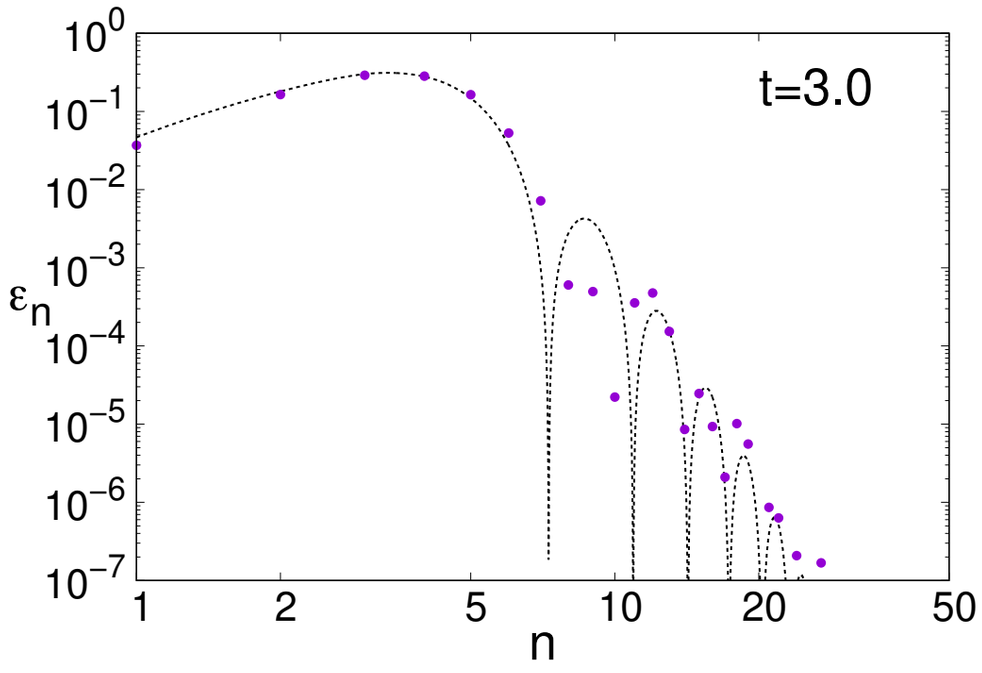
ε=0.01 (cusps t~27)

*log-log plot

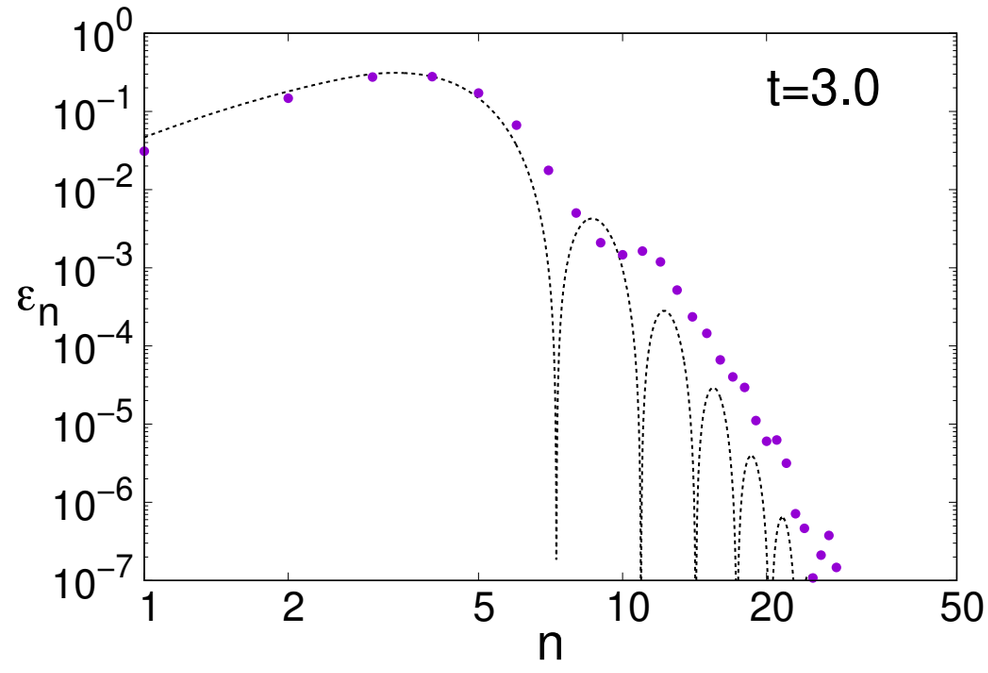
*Dashed line: linearized theory

Energy spectrum

We expand the nonlinear solutions by linear eigenmodes and compute the energy spectrum.



ε=0.005 (no cusp)



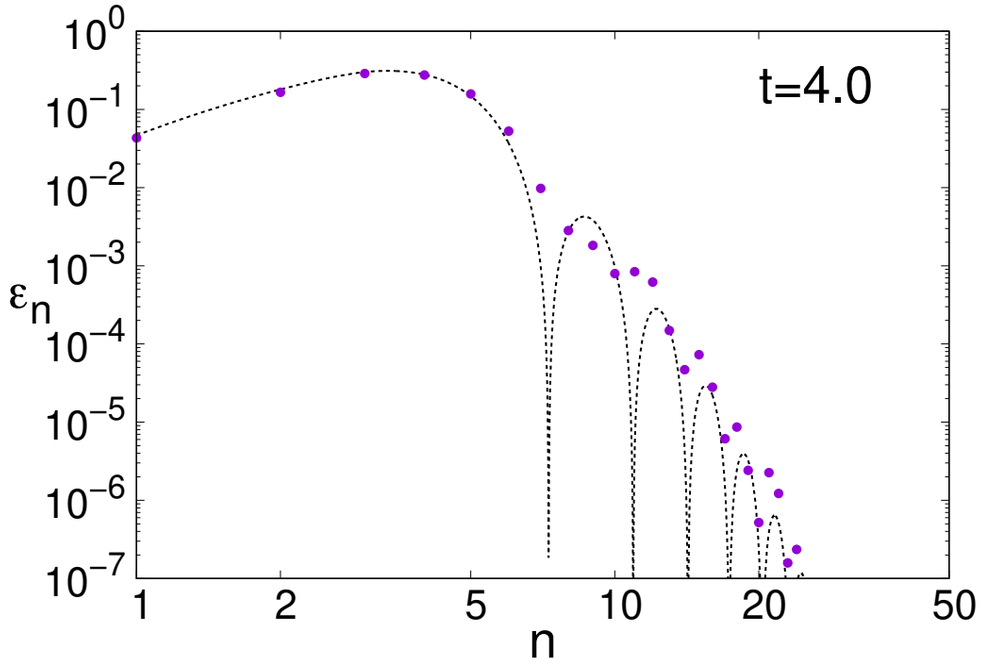
ε=0.01 (cusps t~27)

*log-log plot

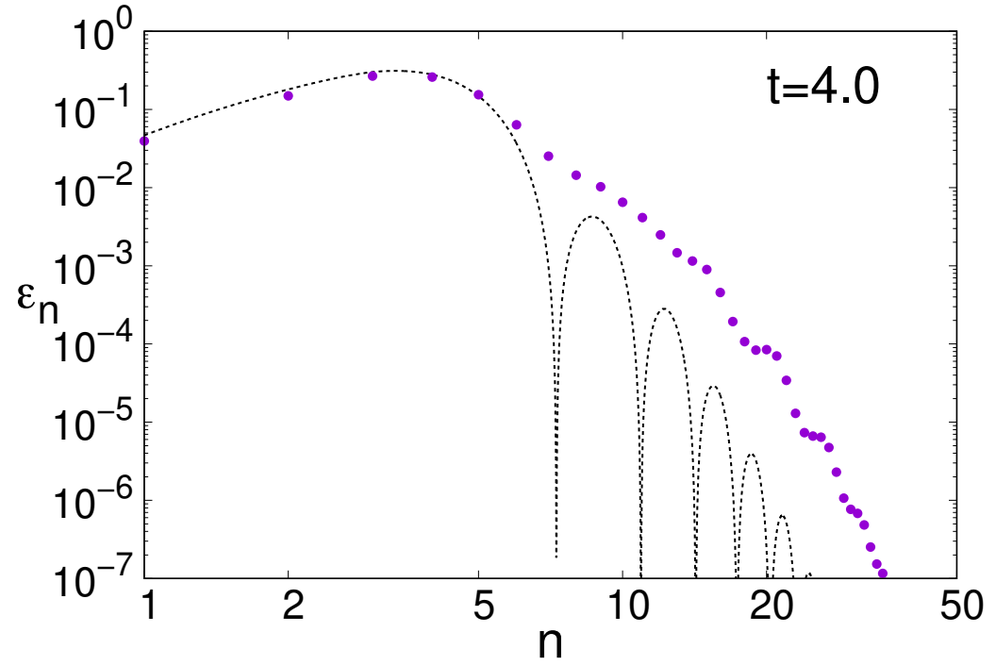
*Dashed line: linearized theory

Energy spectrum

We expand the nonlinear solutions by linear eigenmodes and compute the energy spectrum.



ε=0.005 (no cusp)



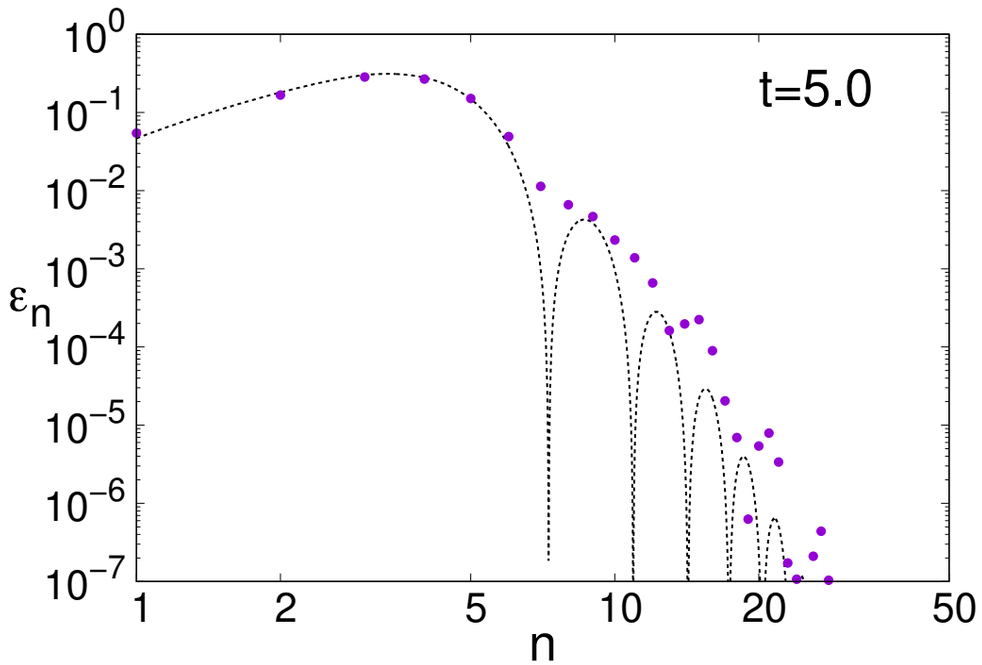
ε=0.01 (cusps t~27)

*log-log plot

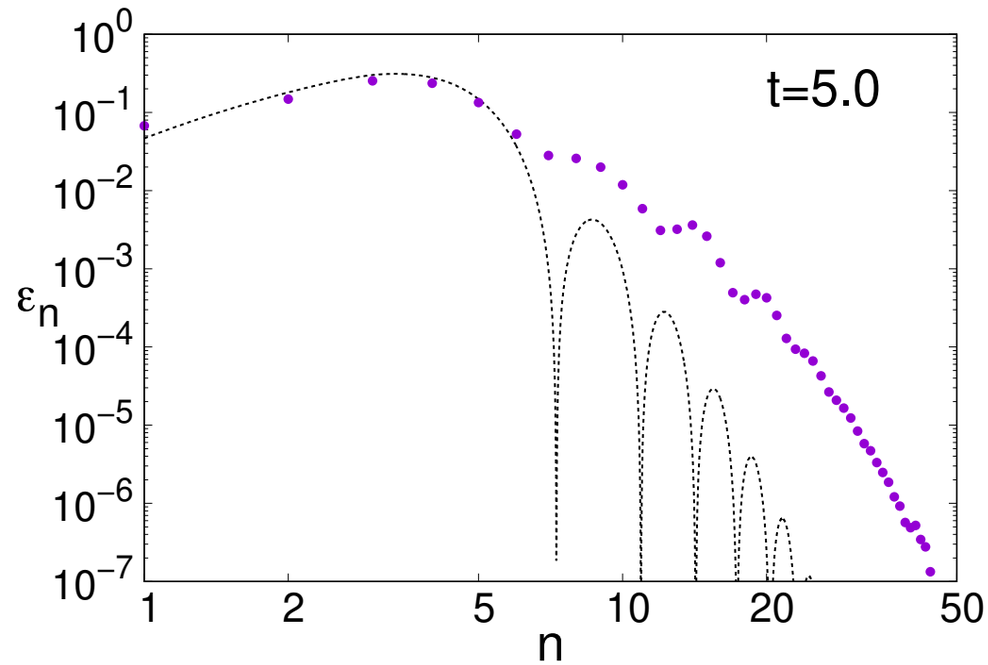
*Dashed line: linearized theory

Energy spectrum

We expand the nonlinear solutions by linear eigenmodes and compute the energy spectrum.



ε=0.005 (no cusp)



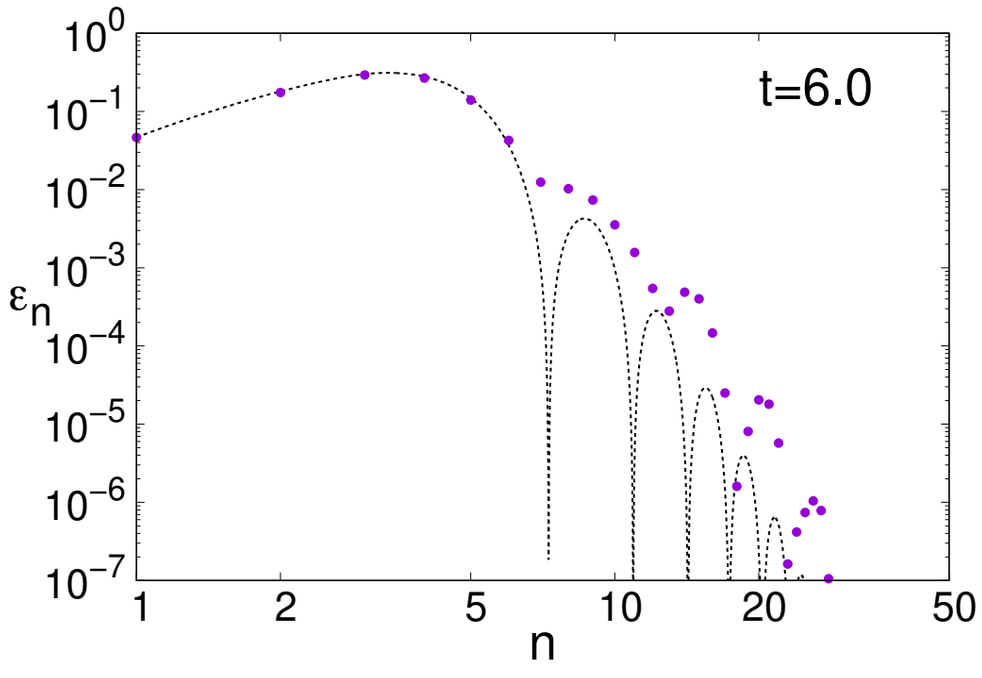
ε=0.01 (cusps t~27)

*log-log plot

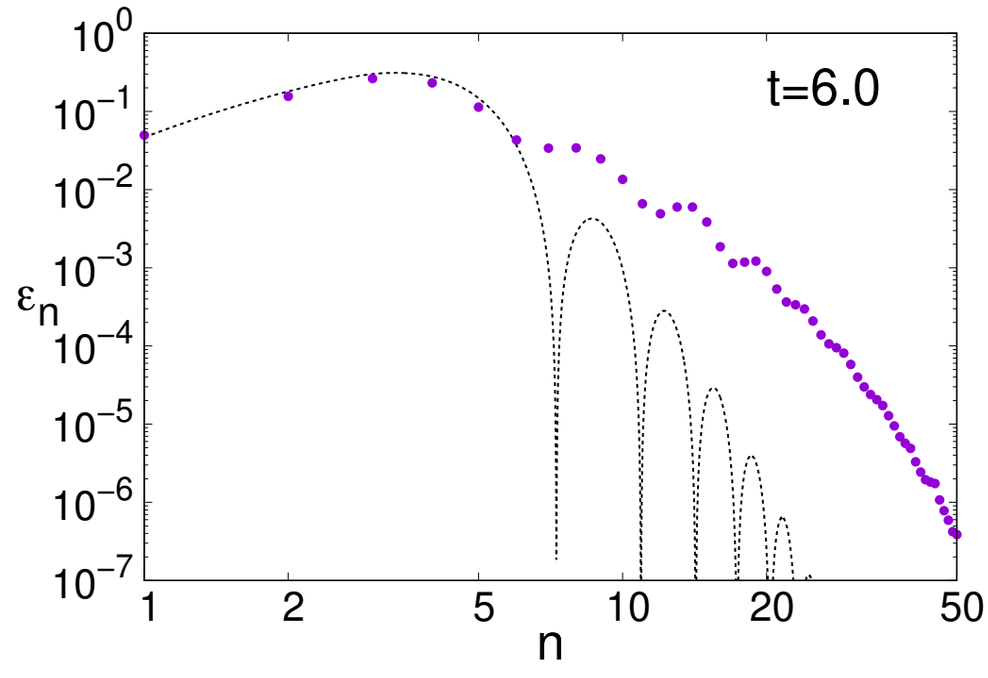
*Dashed line: linearized theory

Energy spectrum

We expand the nonlinear solutions by linear eigenmodes and compute the energy spectrum.



ε=0.005 (no cusp)



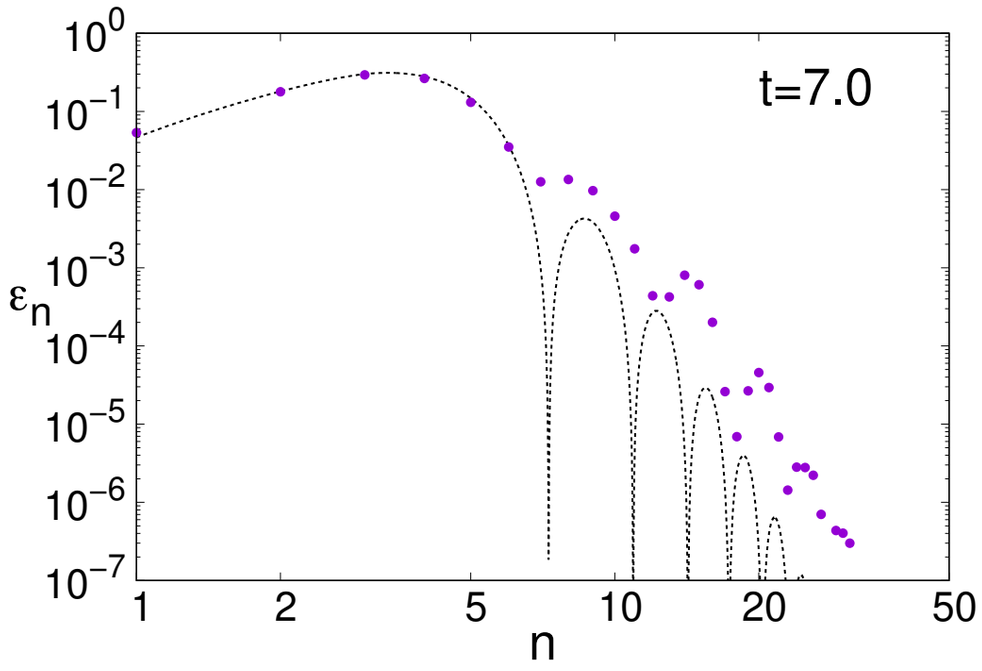
ε=0.01 (cusps t~27)

*log-log plot

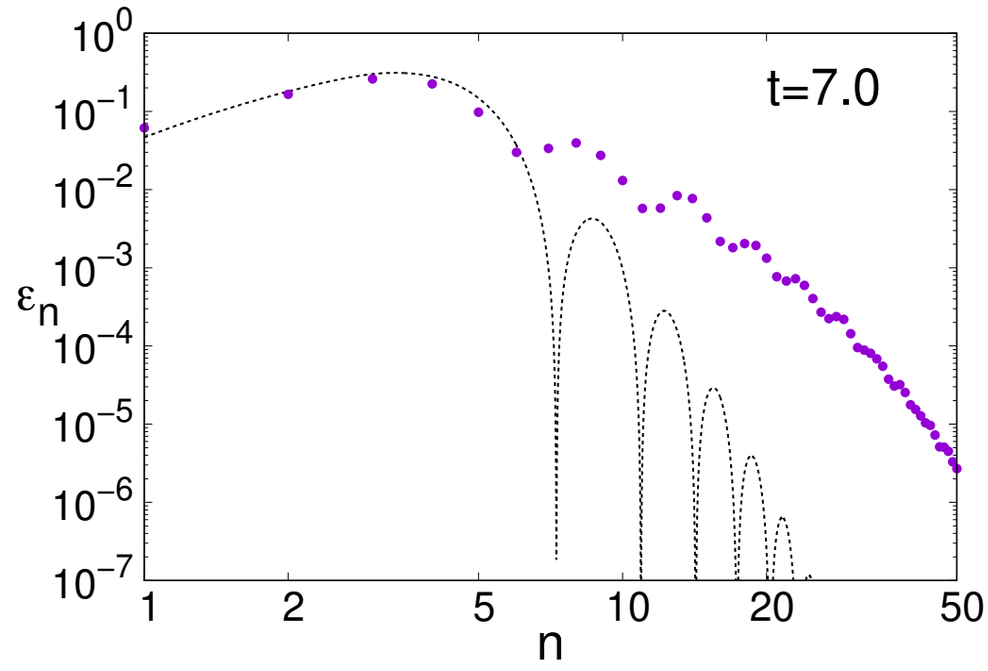
*Dashed line: linearized theory

Energy spectrum

We expand the nonlinear solutions by linear eigenmodes and compute the energy spectrum.



ε=0.005 (no cusp)



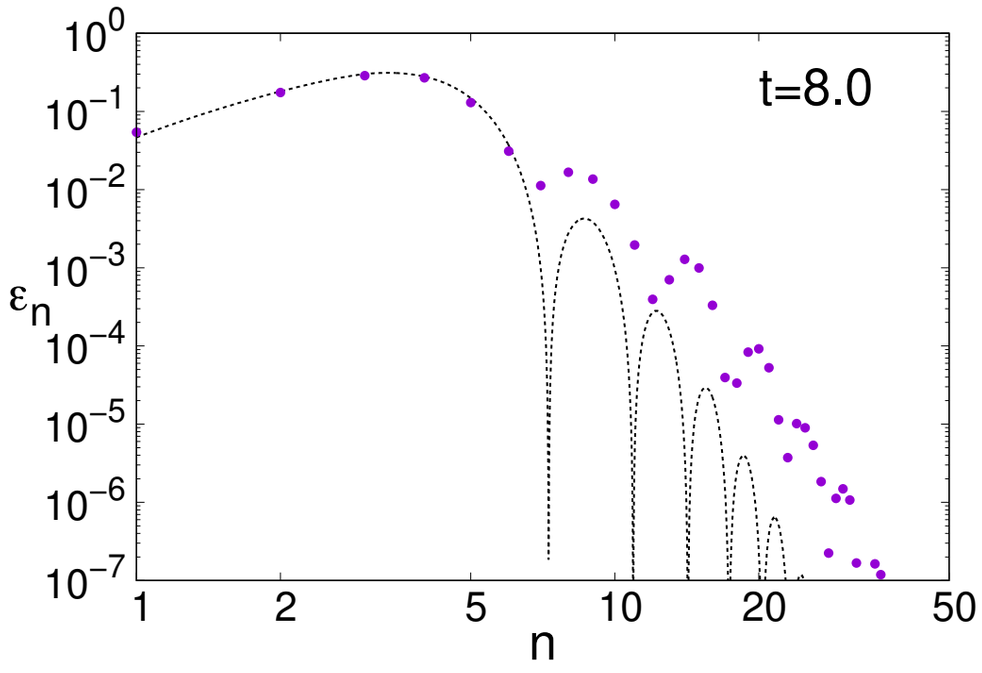
ε=0.01 (cusps t~27)

*log-log plot

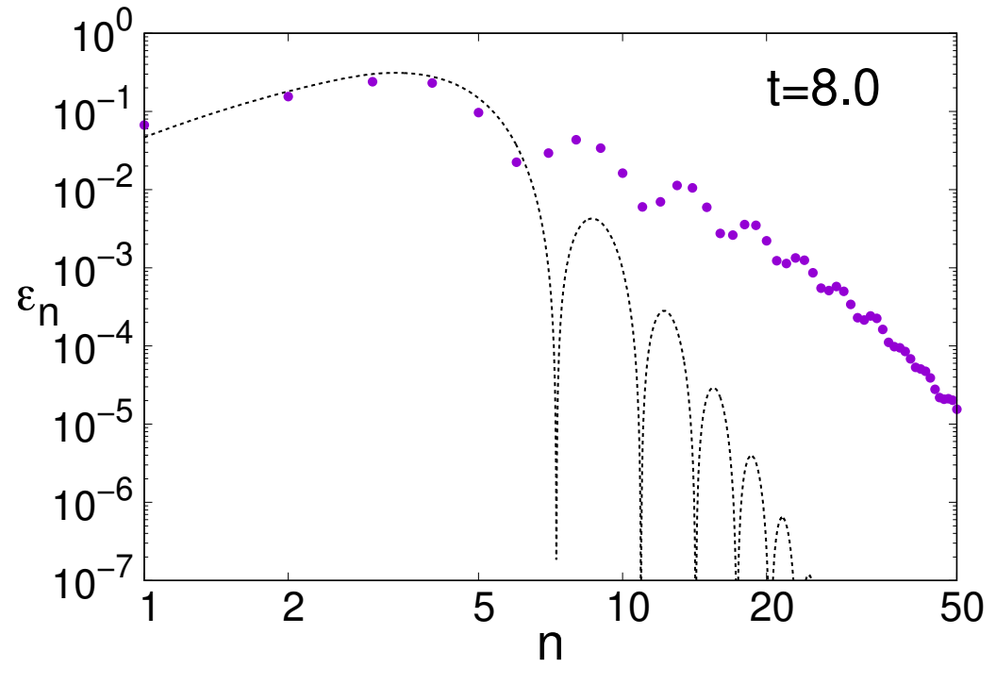
*Dashed line: linearized theory

Energy spectrum

We expand the nonlinear solutions by linear eigenmodes and compute the energy spectrum.



ε=0.005 (no cusp)



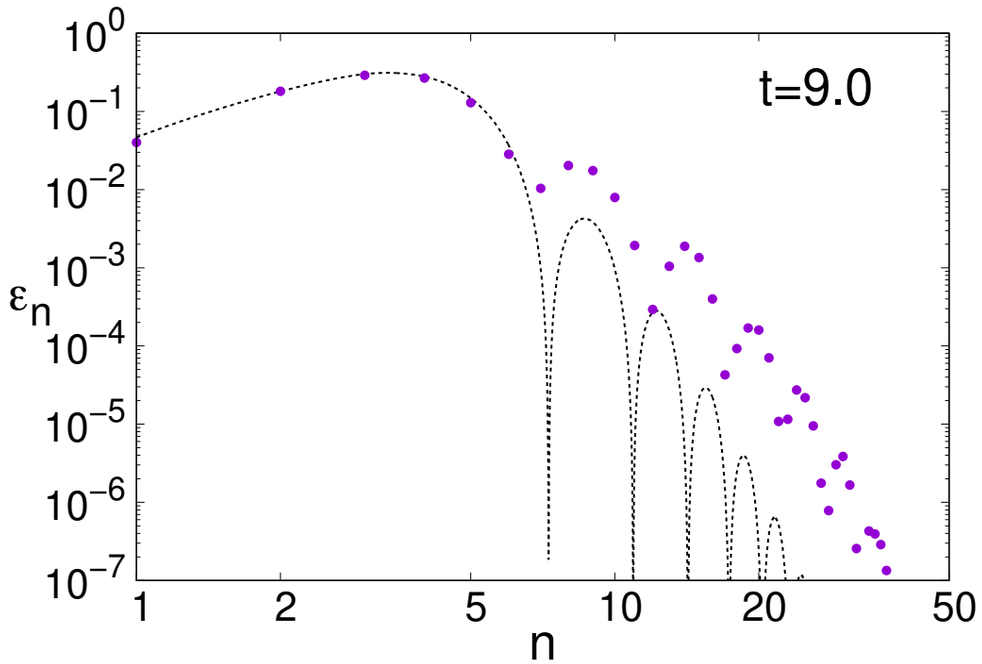
ε=0.01 (cusps t~27)

*log-log plot

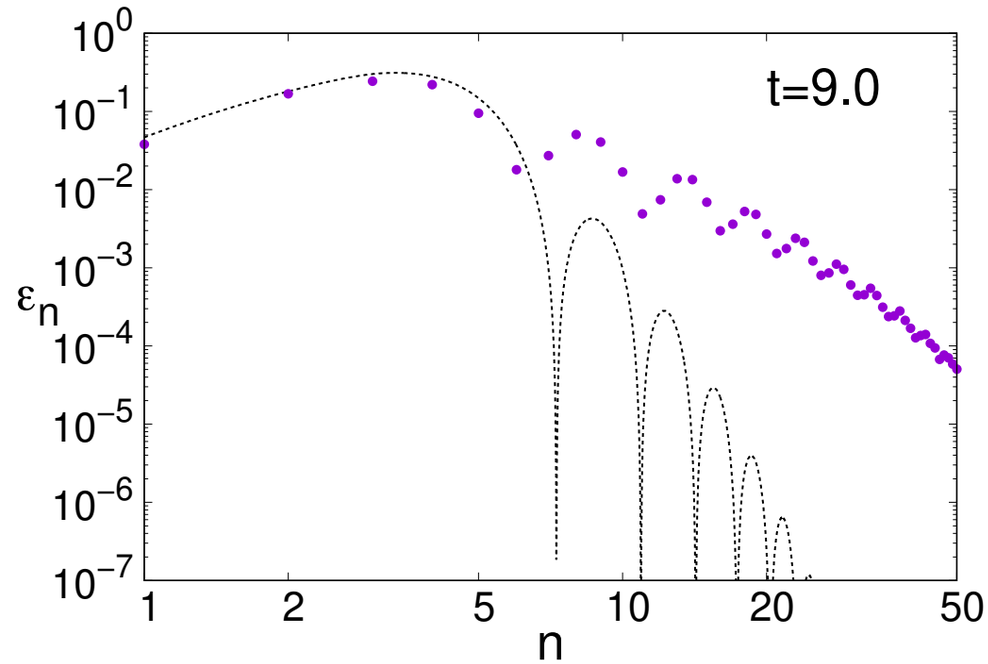
*Dashed line: linearized theory

Energy spectrum

We expand the nonlinear solutions by linear eigenmodes and compute the energy spectrum.



ε=0.005 (no cusp)



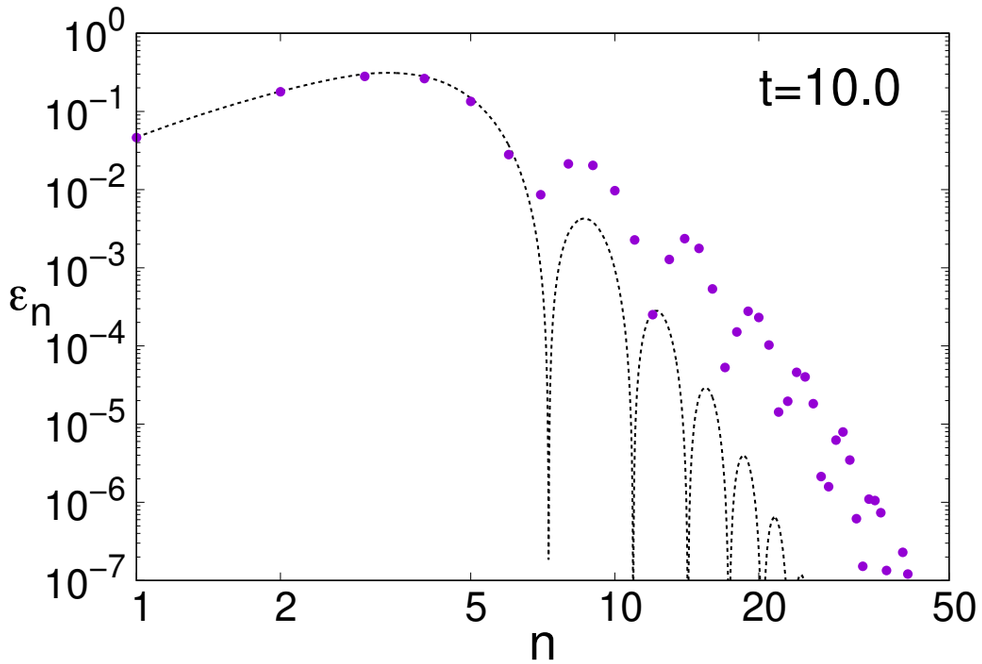
ε=0.01 (cusps t~27)

*log-log plot

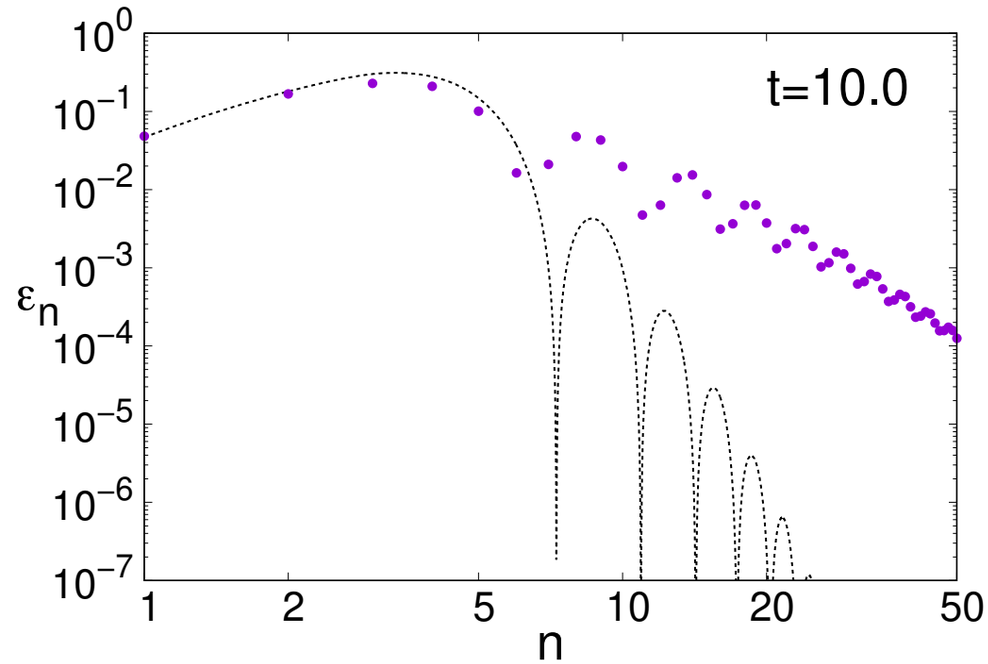
*Dashed line: linearized theory

Energy spectrum

We expand the nonlinear solutions by linear eigenmodes and compute the energy spectrum.



ε=0.005 (no cusp)



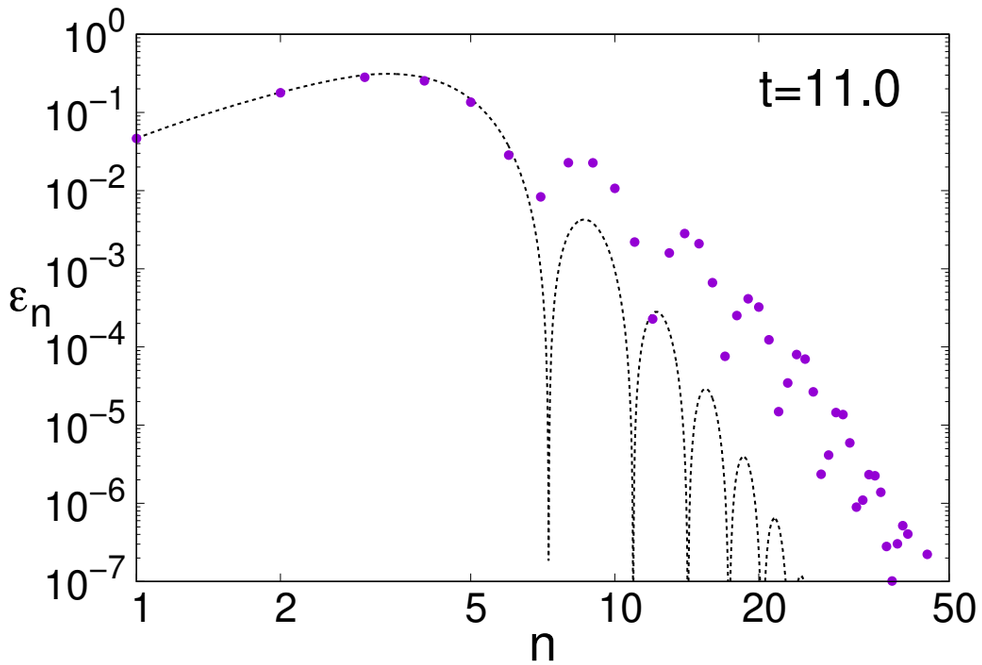
ε=0.01 (cusps t~27)

*log-log plot

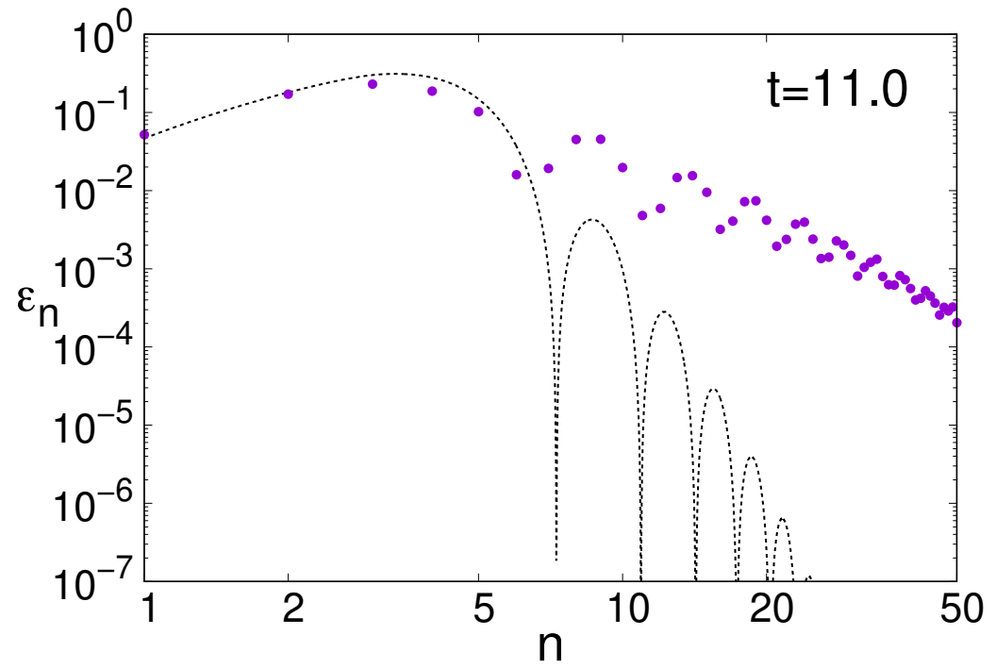
*Dashed line: linearized theory

Energy spectrum

We expand the nonlinear solutions by linear eigenmodes and compute the energy spectrum.



ε=0.005 (no cusp)



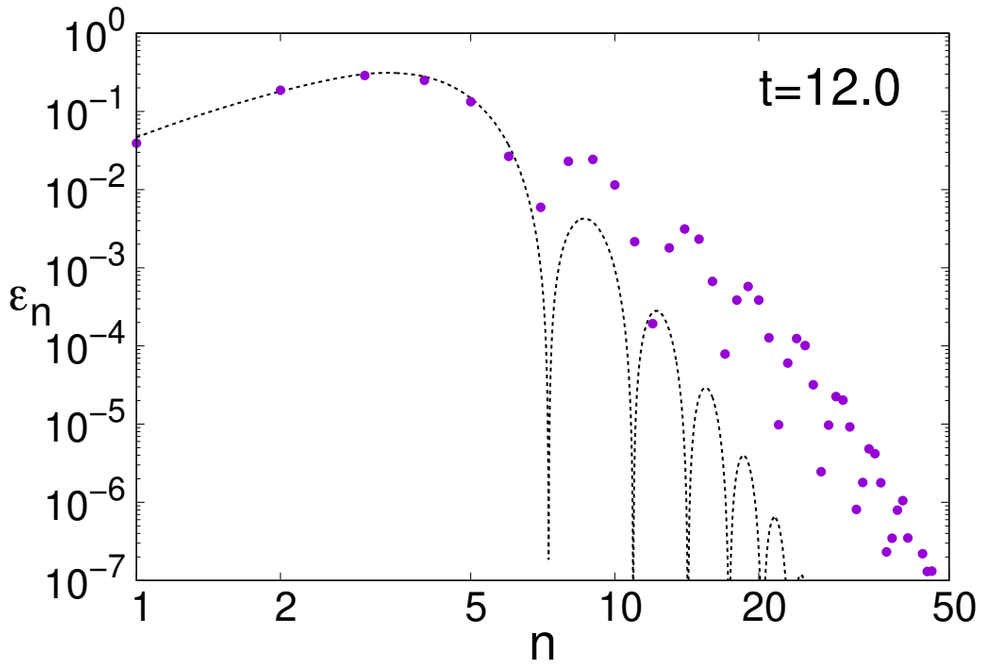
ε=0.01 (cusps t~27)

*log-log plot

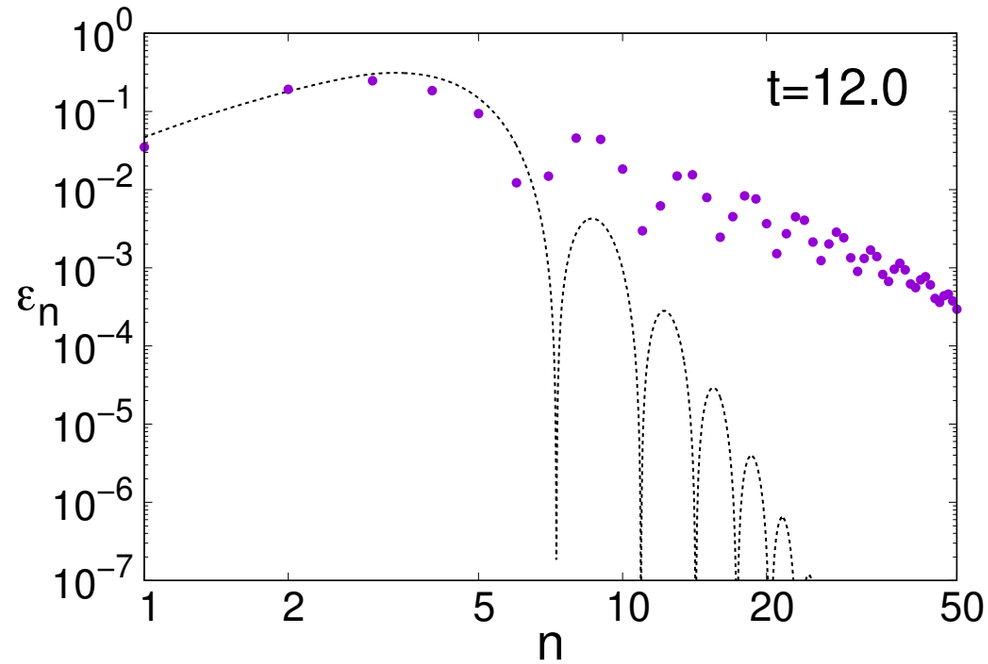
*Dashed line: linearized theory

Energy spectrum

We expand the nonlinear solutions by linear eigenmodes and compute the energy spectrum.



ε=0.005 (no cusp)



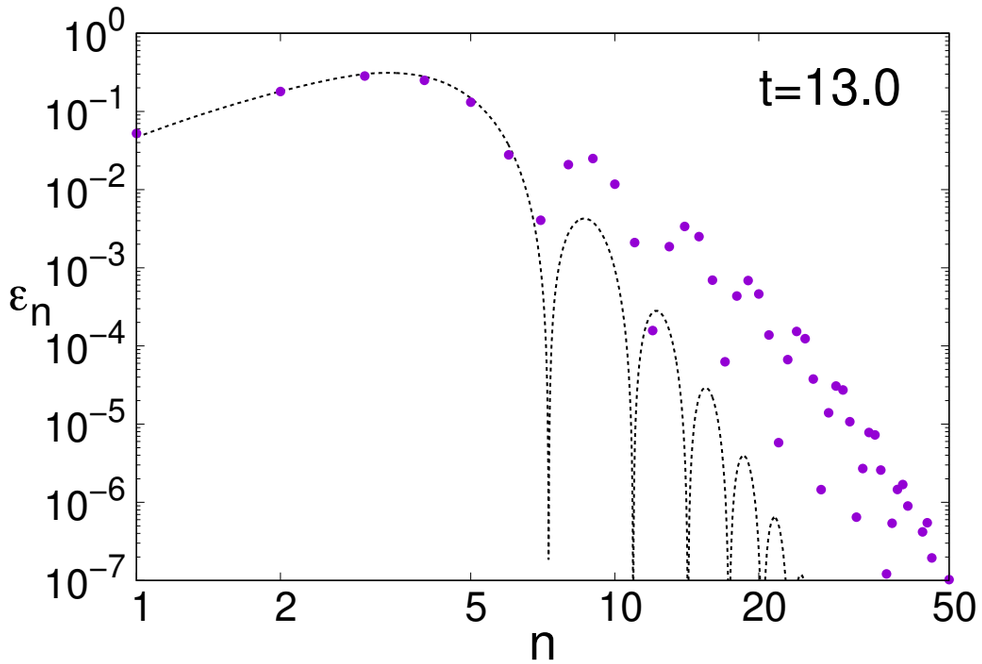
ε=0.01 (cusps t~27)

*log-log plot

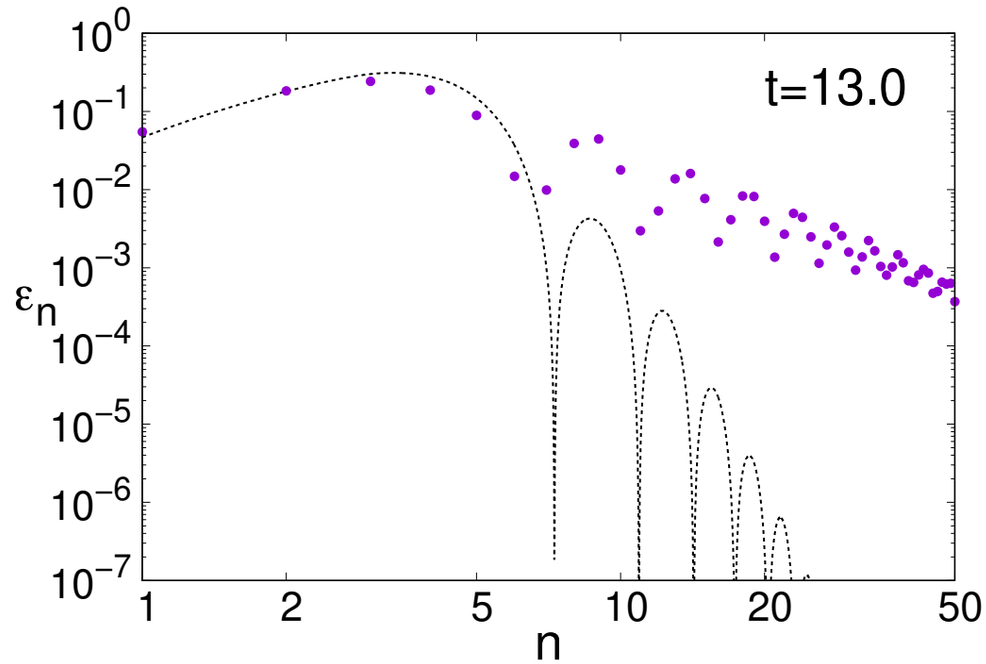
*Dashed line: linearized theory

Energy spectrum

We expand the nonlinear solutions by linear eigenmodes and compute the energy spectrum.



ε=0.005 (no cusp)



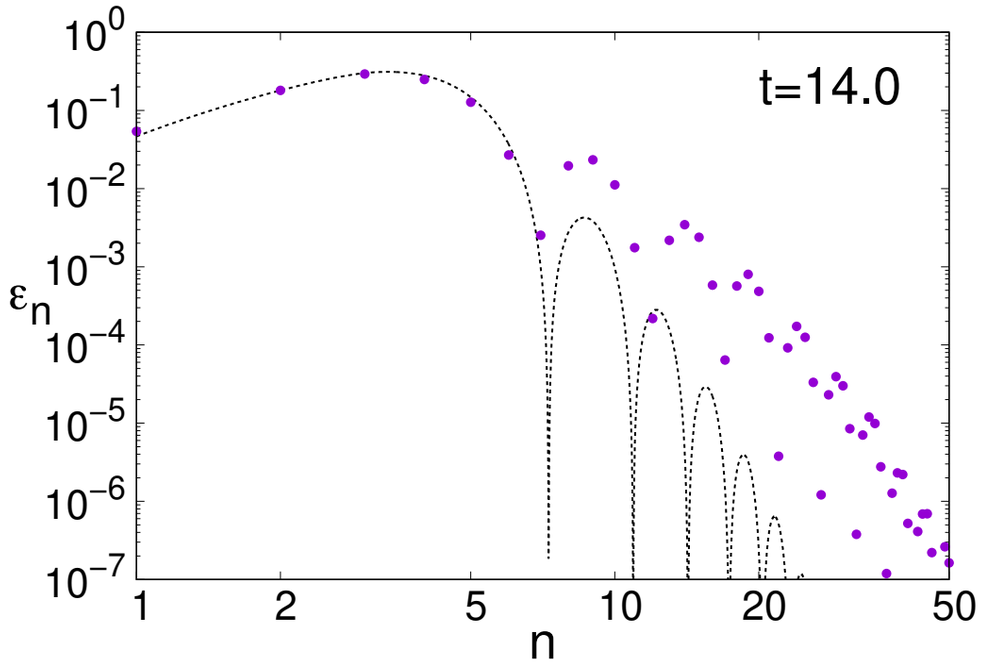
ε=0.01 (cusps t~27)

*log-log plot

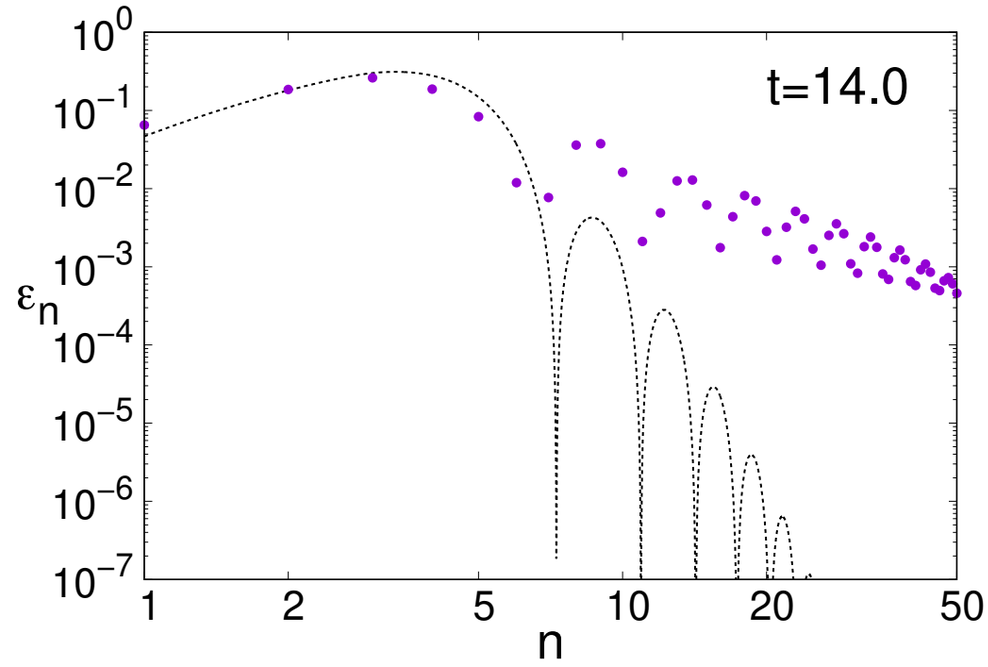
*Dashed line: linearized theory

Energy spectrum

We expand the nonlinear solutions by linear eigenmodes and compute the energy spectrum.



ε=0.005 (no cusp)



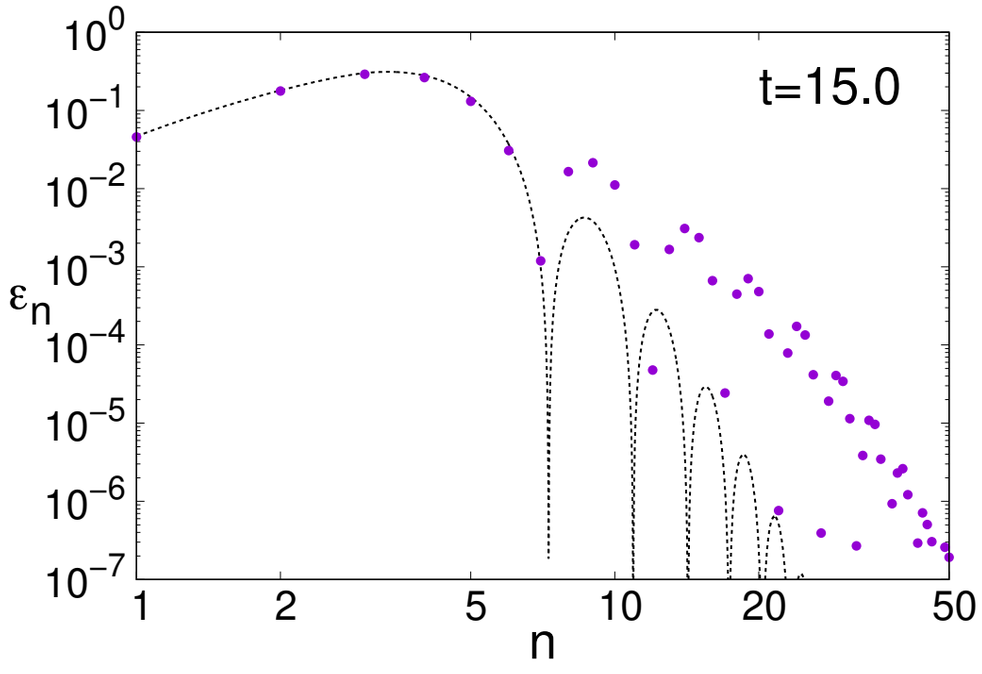
ε=0.01 (cusps t~27)

*log-log plot

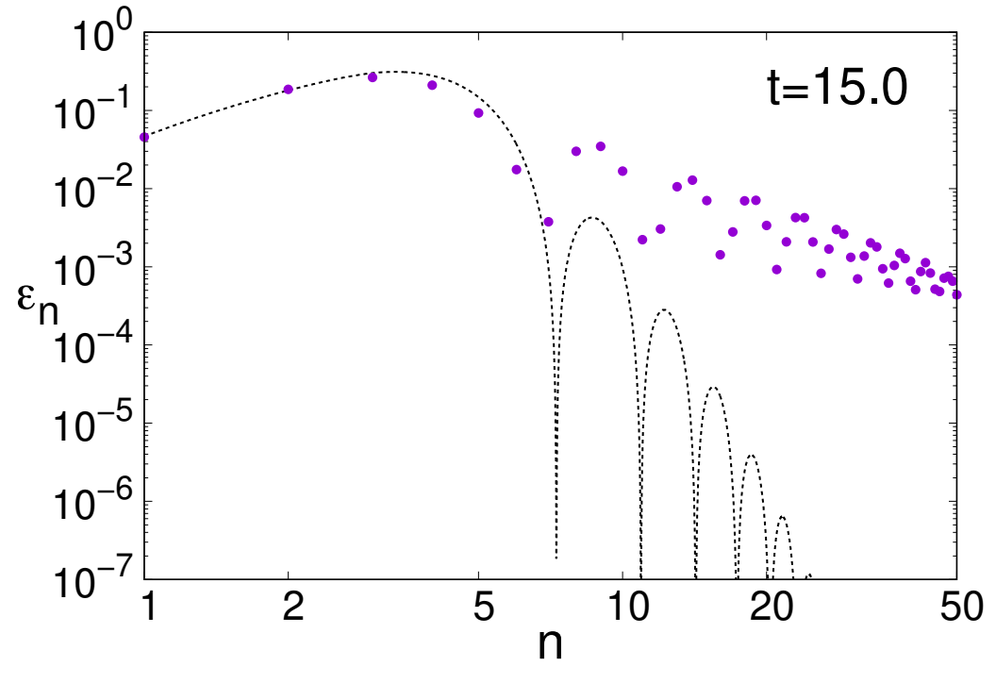
*Dashed line: linearized theory

Energy spectrum

We expand the nonlinear solutions by linear eigenmodes and compute the energy spectrum.



ε=0.005 (no cusp)



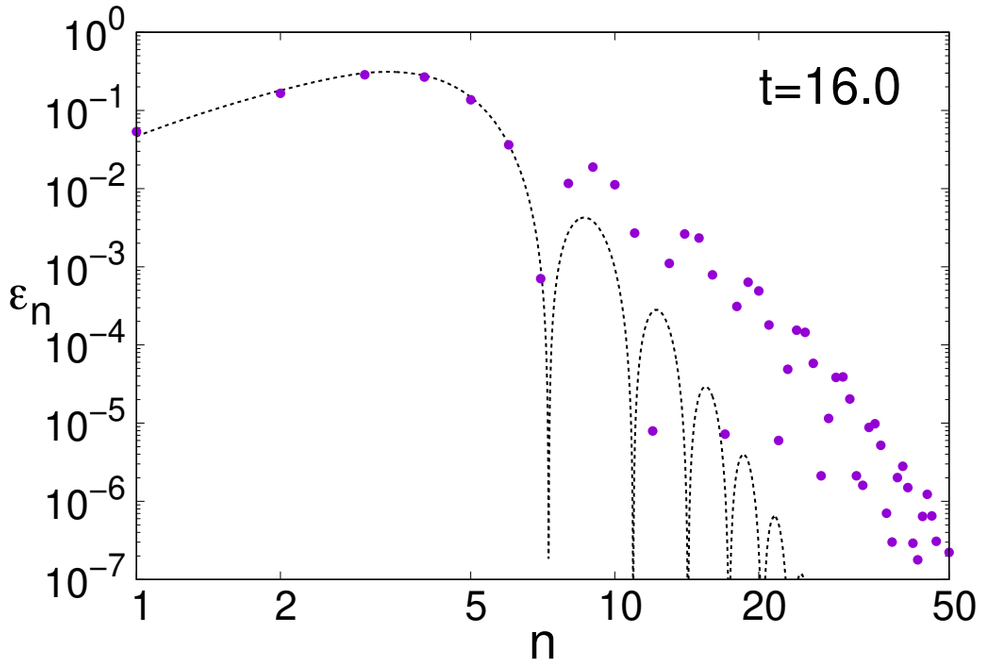
ε=0.01 (cusps t~27)

*log-log plot

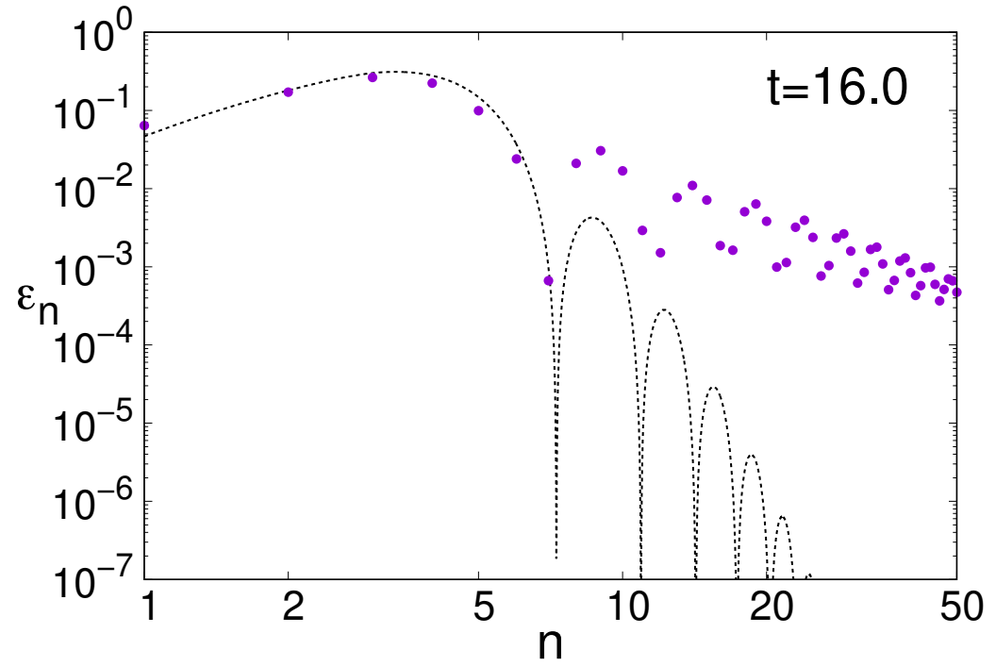
*Dashed line: linearized theory

Energy spectrum

We expand the nonlinear solutions by linear eigenmodes and compute the energy spectrum.



ε=0.005 (no cusp)



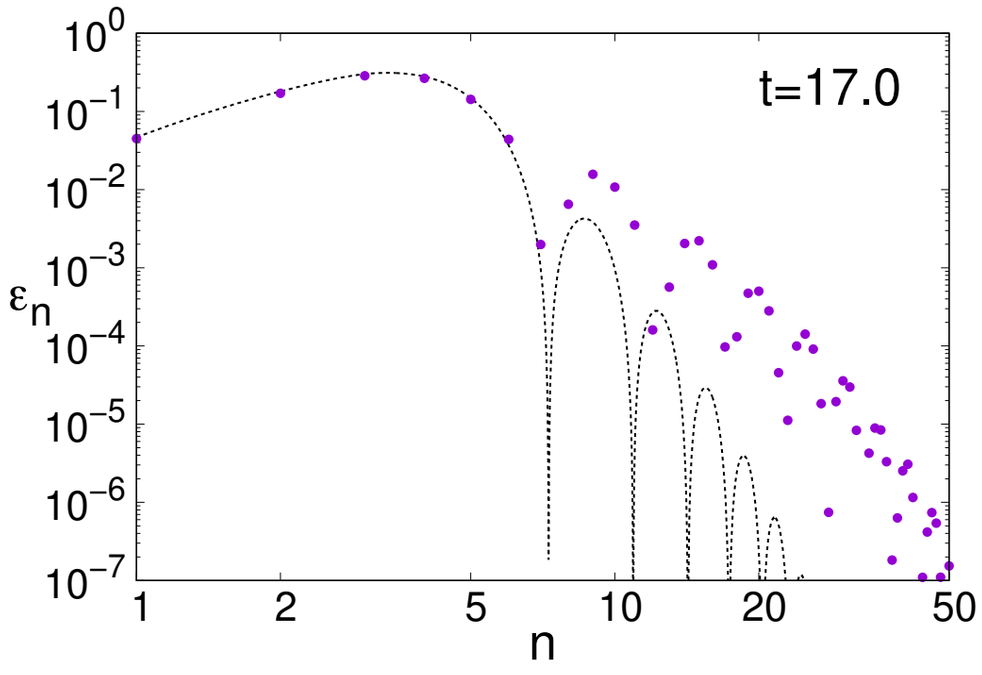
ε=0.01 (cusps t~27)

*log-log plot

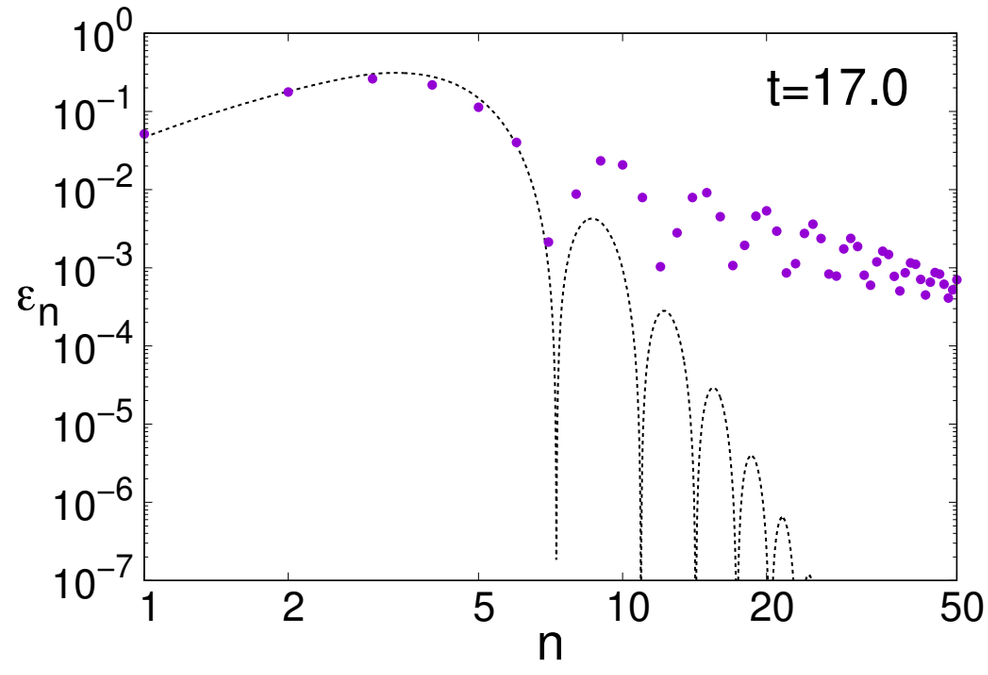
*Dashed line: linearized theory

Energy spectrum

We expand the nonlinear solutions by linear eigenmodes and compute the energy spectrum.



ε=0.005 (no cusp)



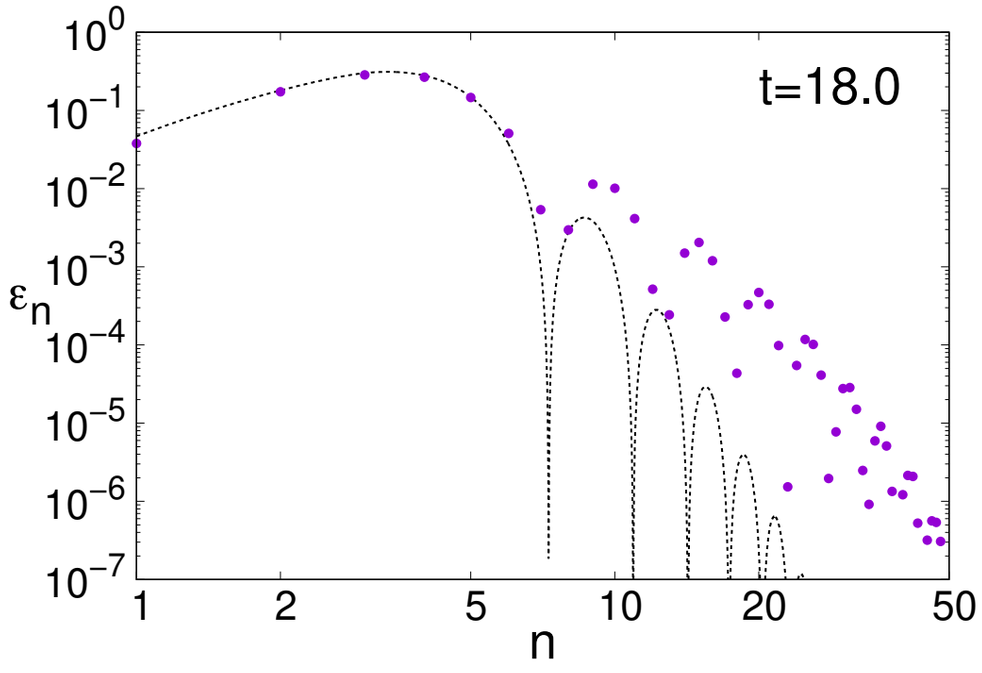
ε=0.01 (cusps t~27)

*log-log plot

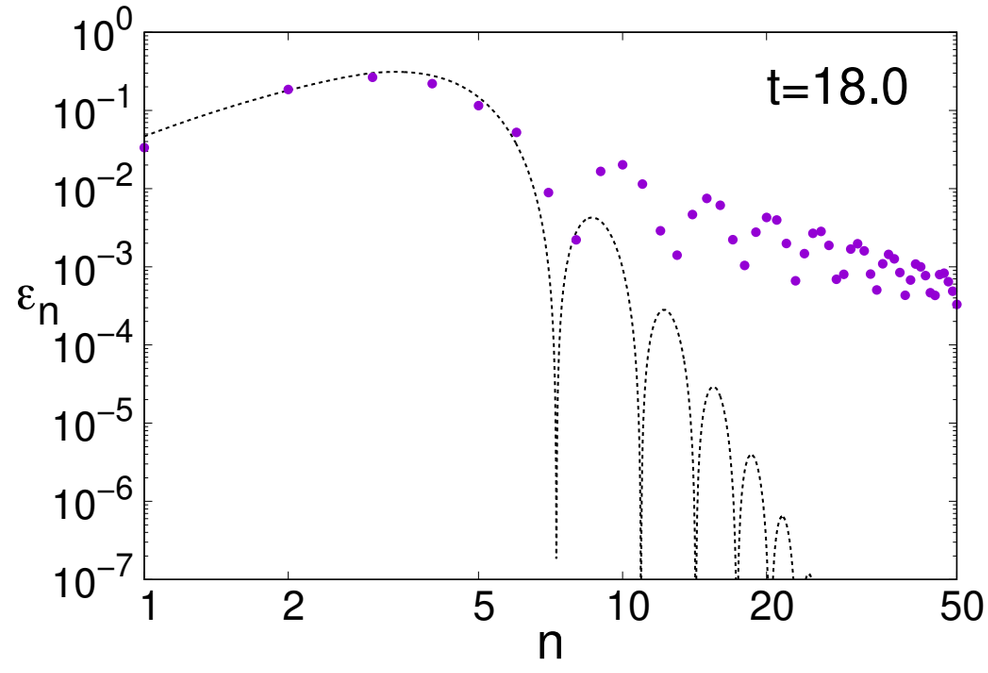
*Dashed line: linearized theory

Energy spectrum

We expand the nonlinear solutions by linear eigenmodes and compute the energy spectrum.



ε=0.005 (no cusp)



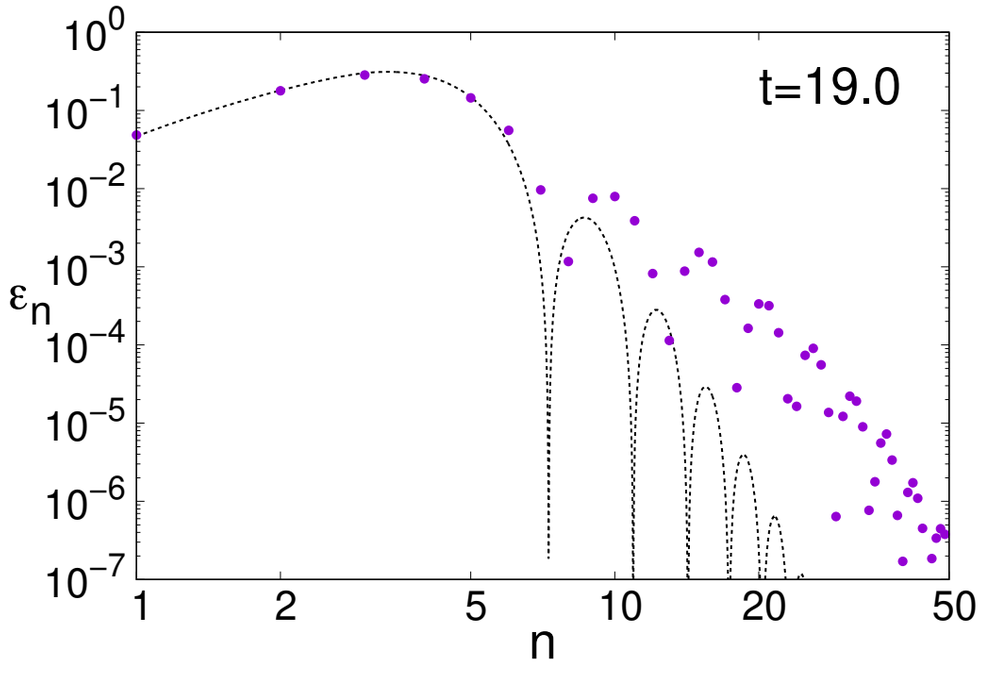
ε=0.01 (cusps t~27)

*log-log plot

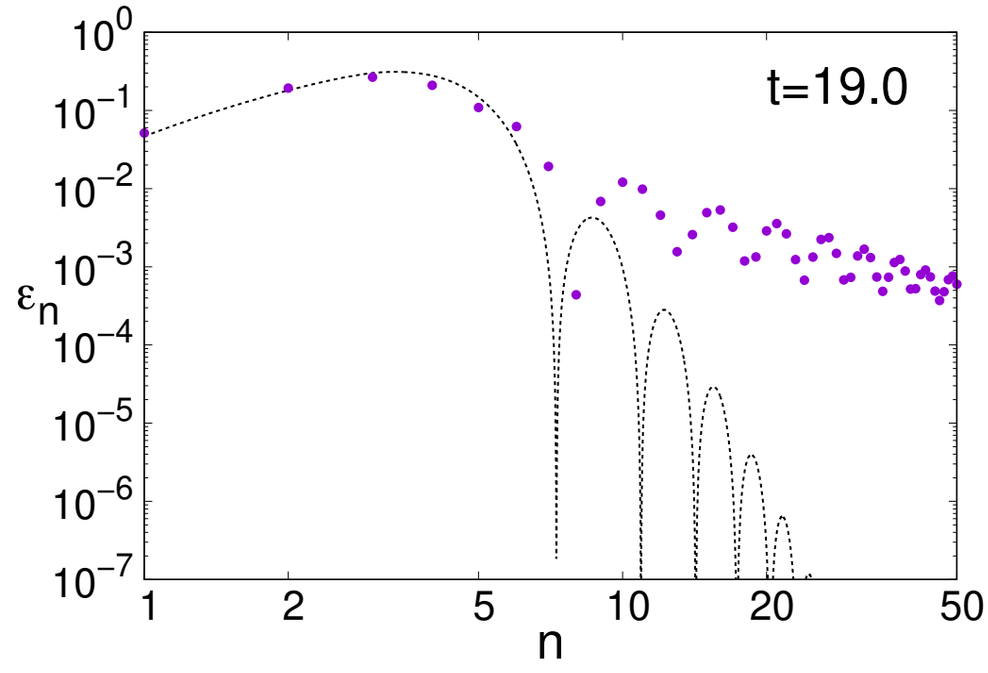
*Dashed line: linearized theory

Energy spectrum

We expand the nonlinear solutions by linear eigenmodes and compute the energy spectrum.



ε=0.005 (no cusp)



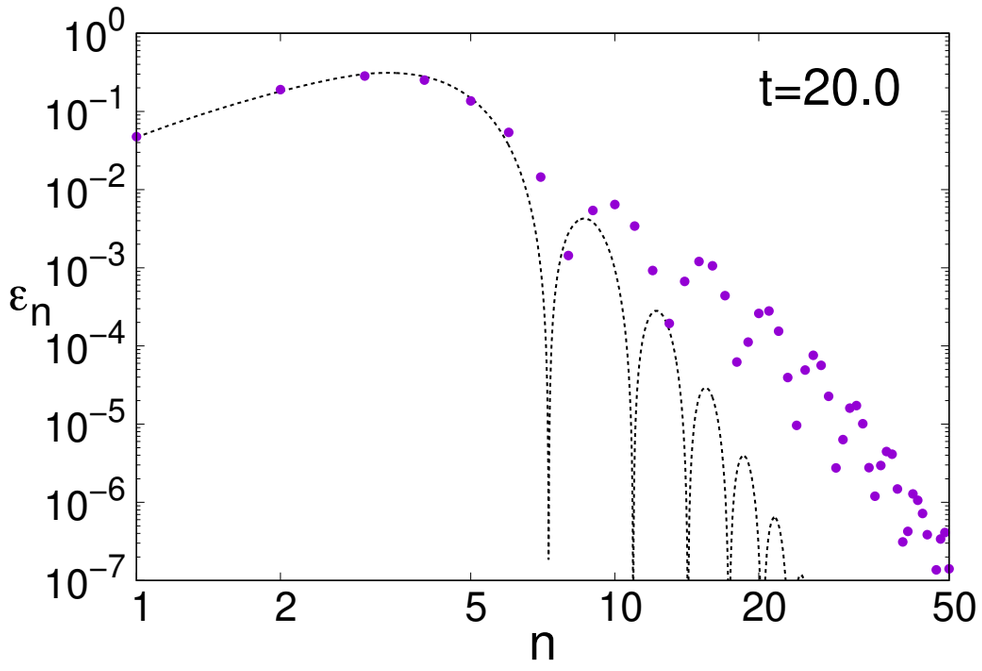
ε=0.01 (cusps t~27)

*log-log plot

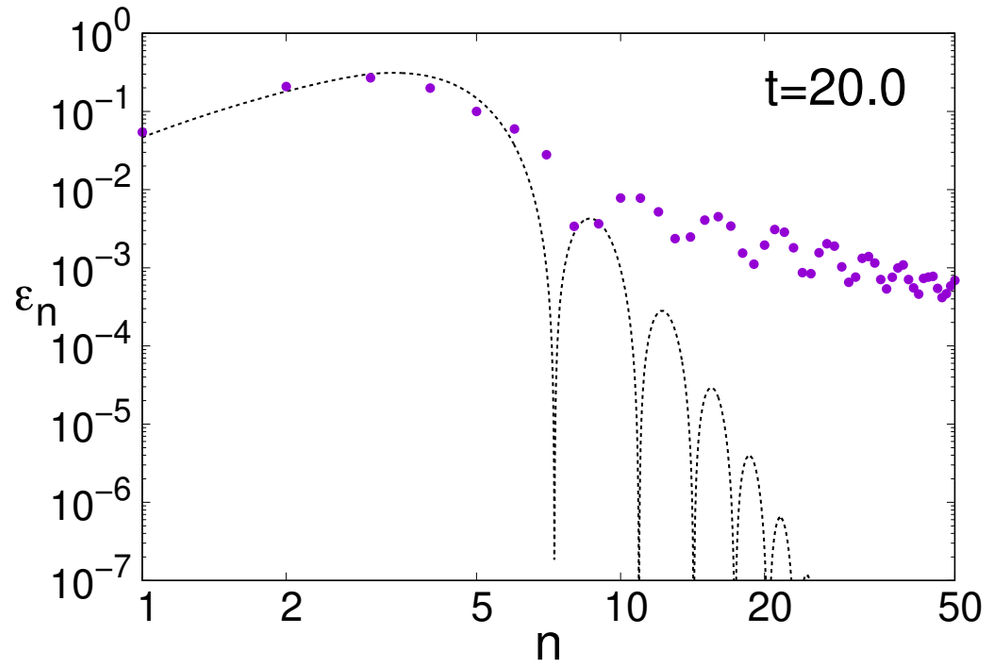
*Dashed line: linearized theory

Energy spectrum

We expand the nonlinear solutions by linear eigenmodes and compute the energy spectrum.



ε=0.005 (no cusp)



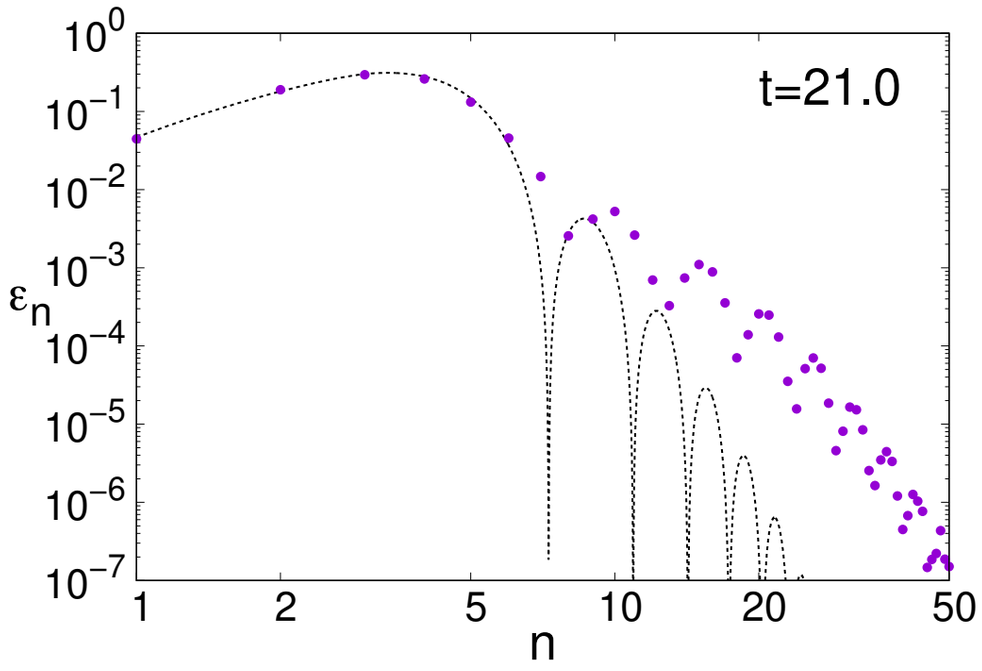
ε=0.01 (cusps t~27)

*log-log plot

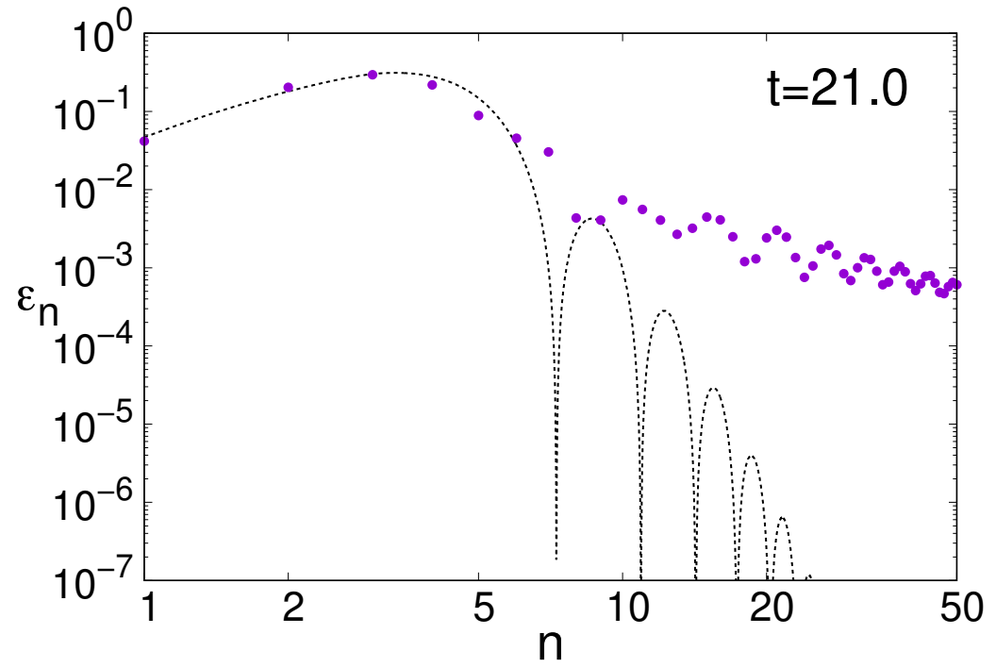
*Dashed line: linearized theory

Energy spectrum

We expand the nonlinear solutions by linear eigenmodes and compute the energy spectrum.



ε=0.005 (no cusp)



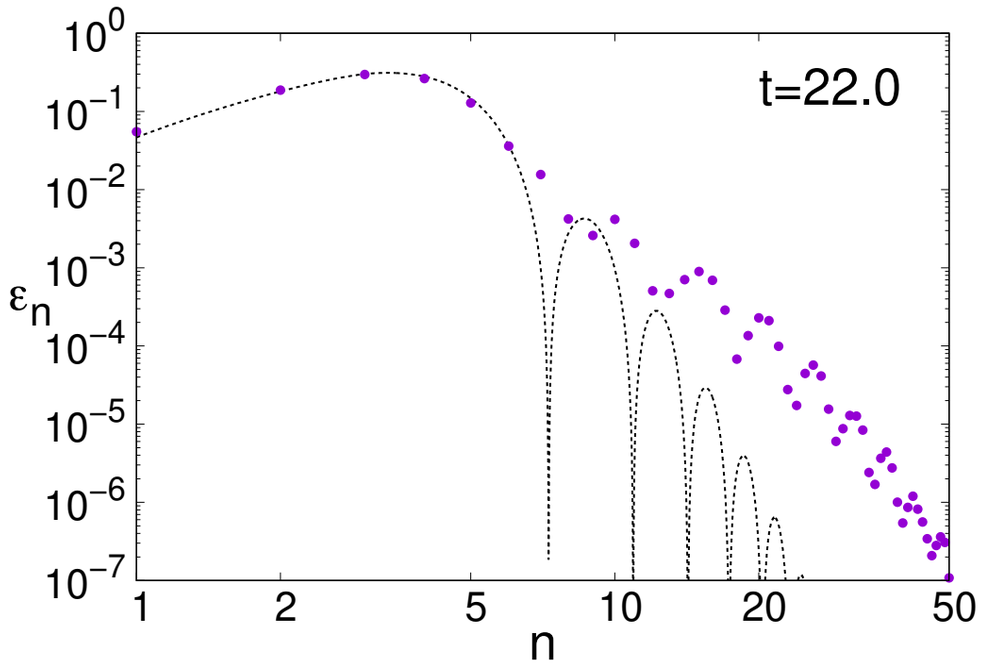
ε=0.01 (cusps t~27)

*log-log plot

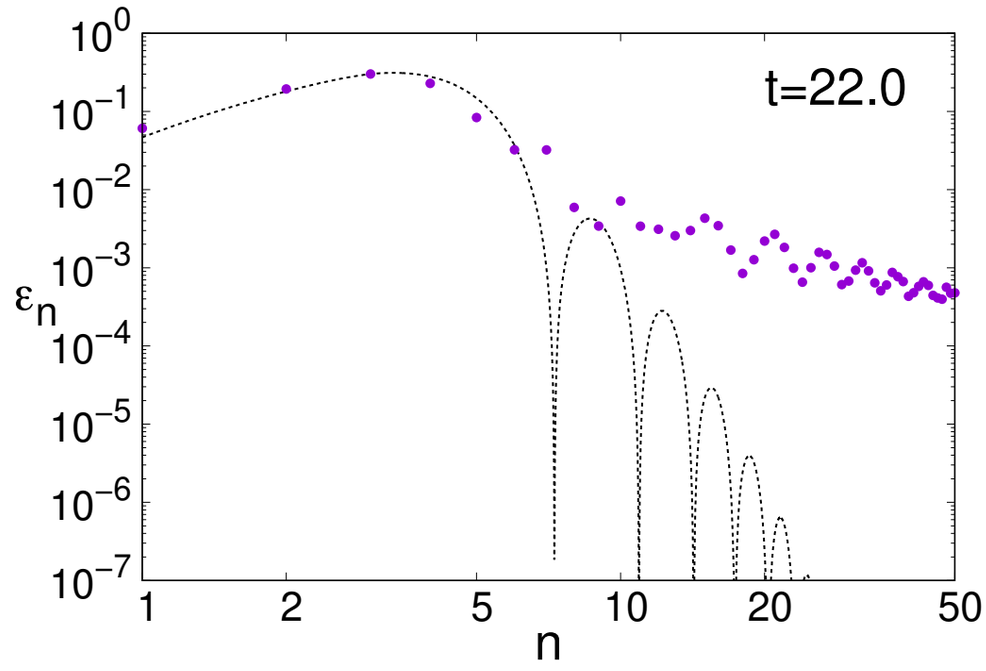
*Dashed line: linearized theory

Energy spectrum

We expand the nonlinear solutions by linear eigenmodes and compute the energy spectrum.



ε=0.005 (no cusp)



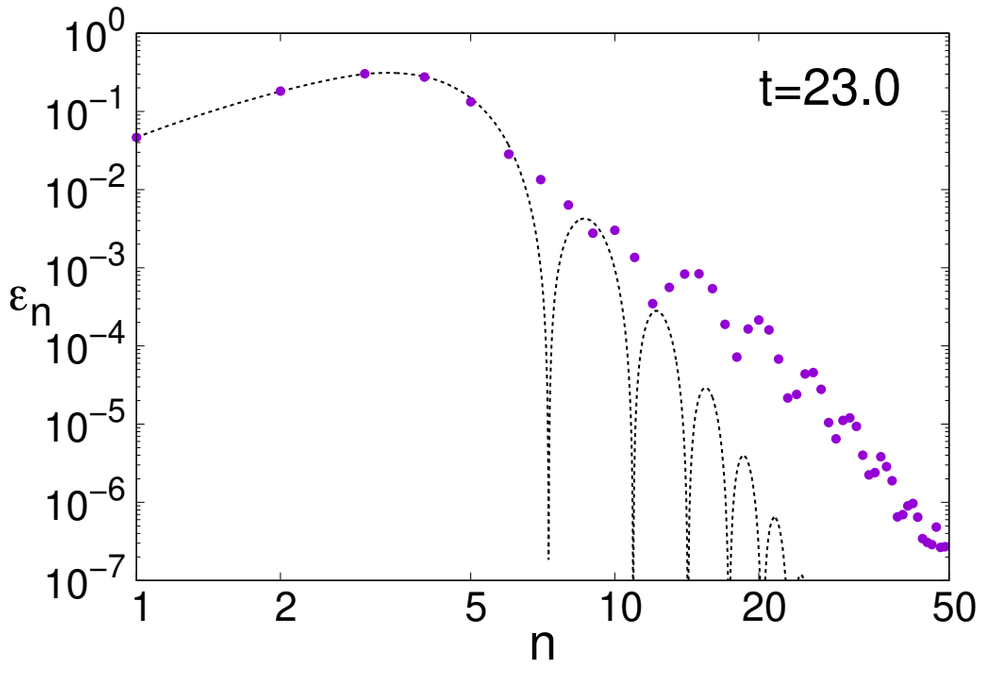
ε=0.01 (cusps t~27)

*log-log plot

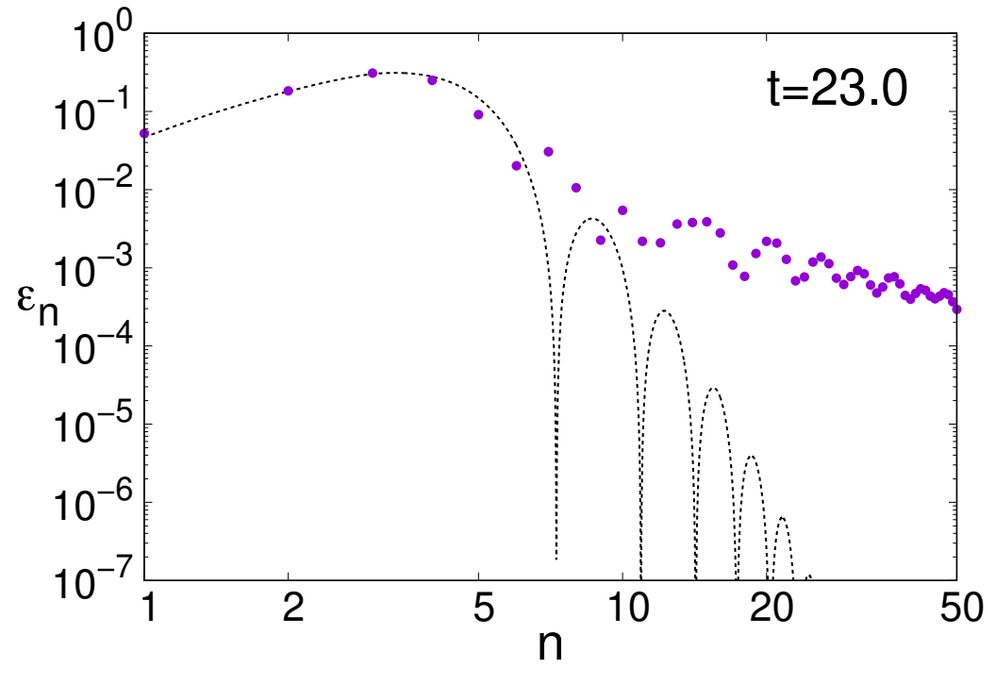
*Dashed line: linearized theory

Energy spectrum

We expand the nonlinear solutions by linear eigenmodes and compute the energy spectrum.



ε=0.005 (no cusp)



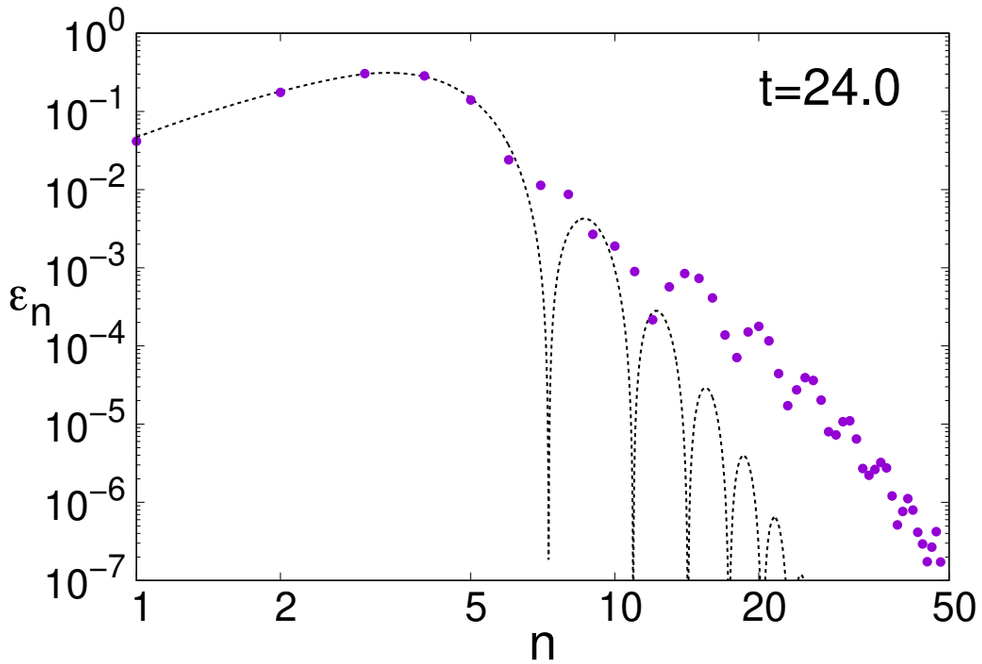
ε=0.01 (cusps t~27)

*log-log plot

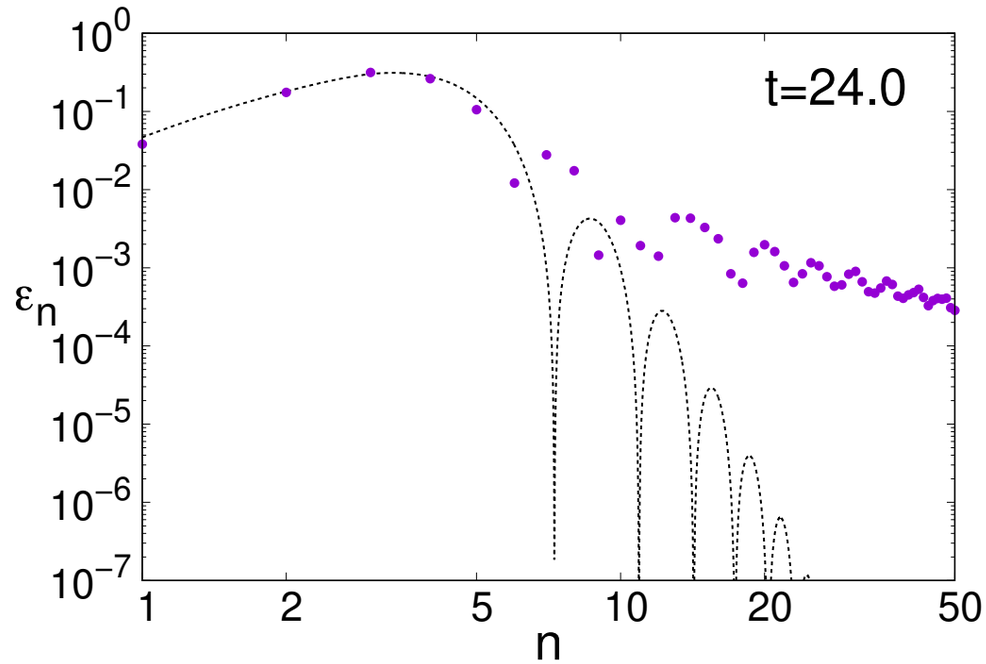
*Dashed line: linearized theory

Energy spectrum

We expand the nonlinear solutions by linear eigenmodes and compute the energy spectrum.



ε=0.005 (no cusp)



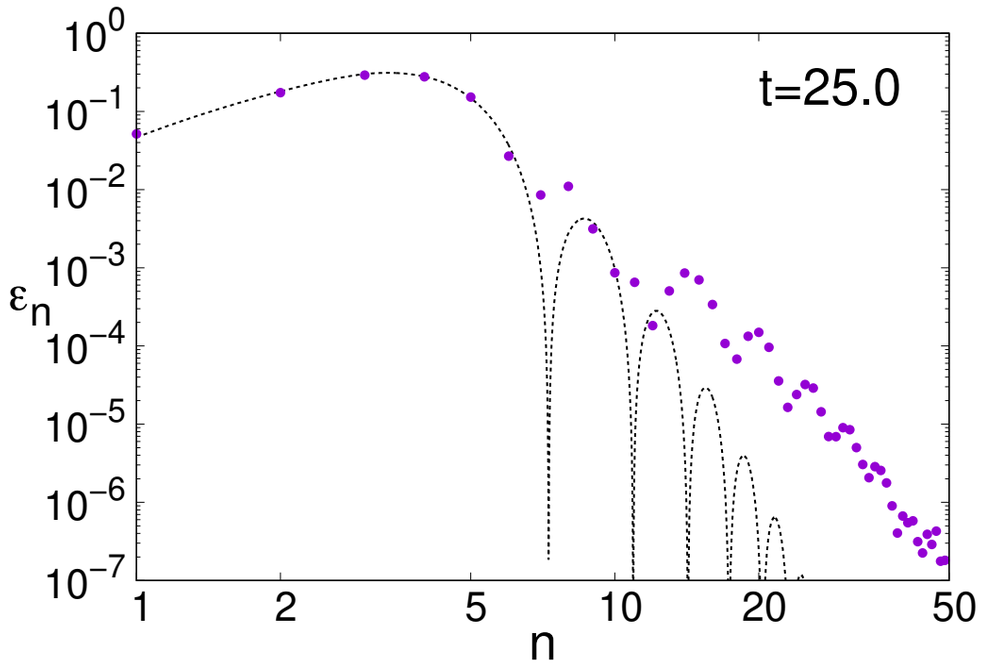
ε=0.01 (cusps t~27)

*log-log plot

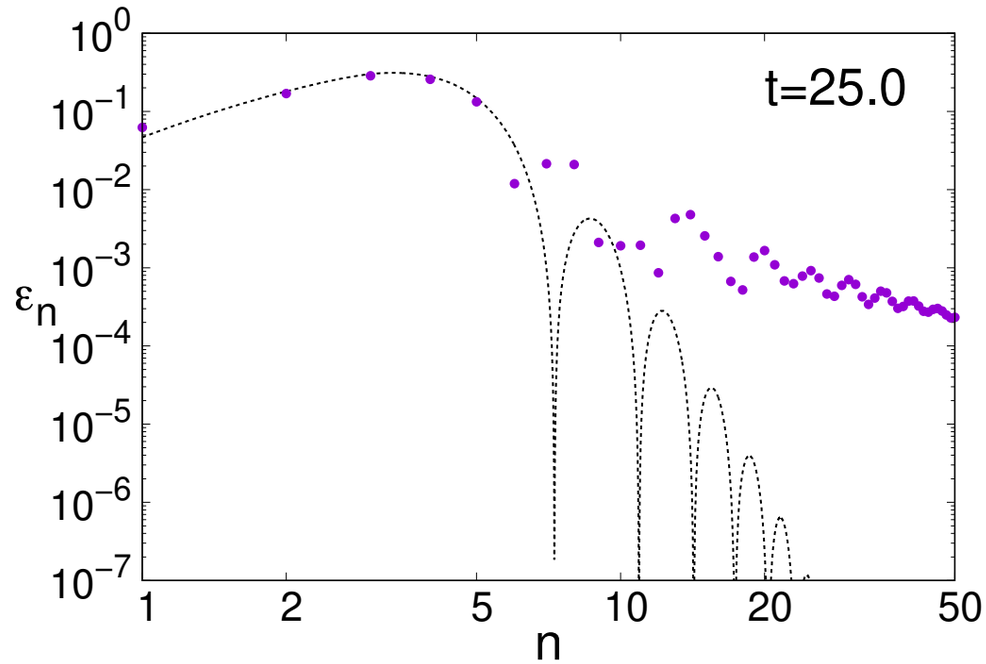
*Dashed line: linearized theory

Energy spectrum

We expand the nonlinear solutions by linear eigenmodes and compute the energy spectrum.



ε=0.005 (no cusp)



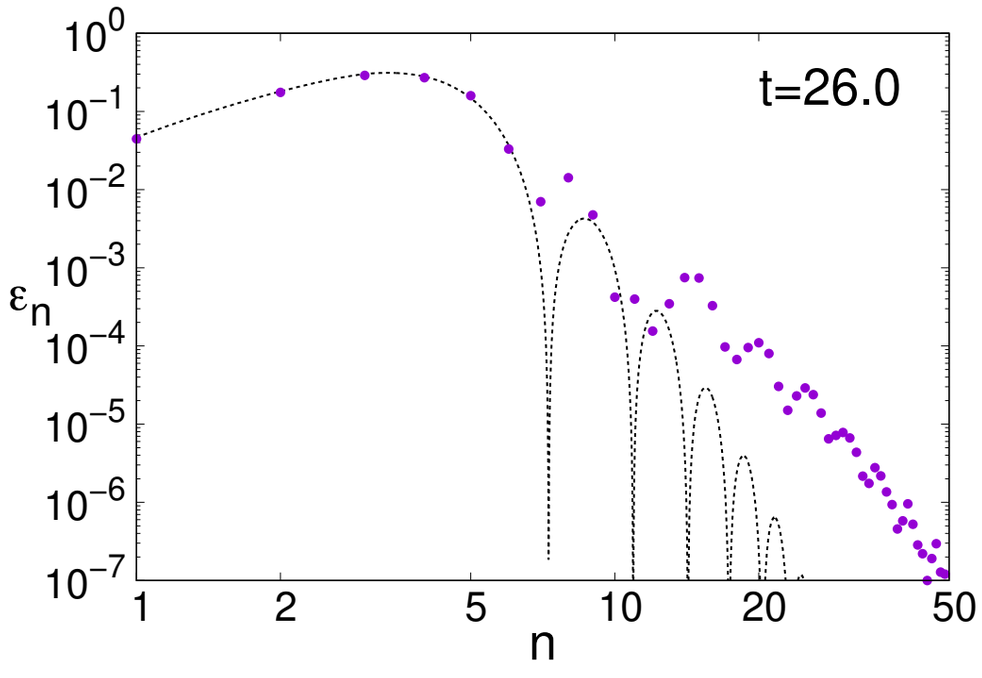
ε=0.01 (cusps t~27)

*log-log plot

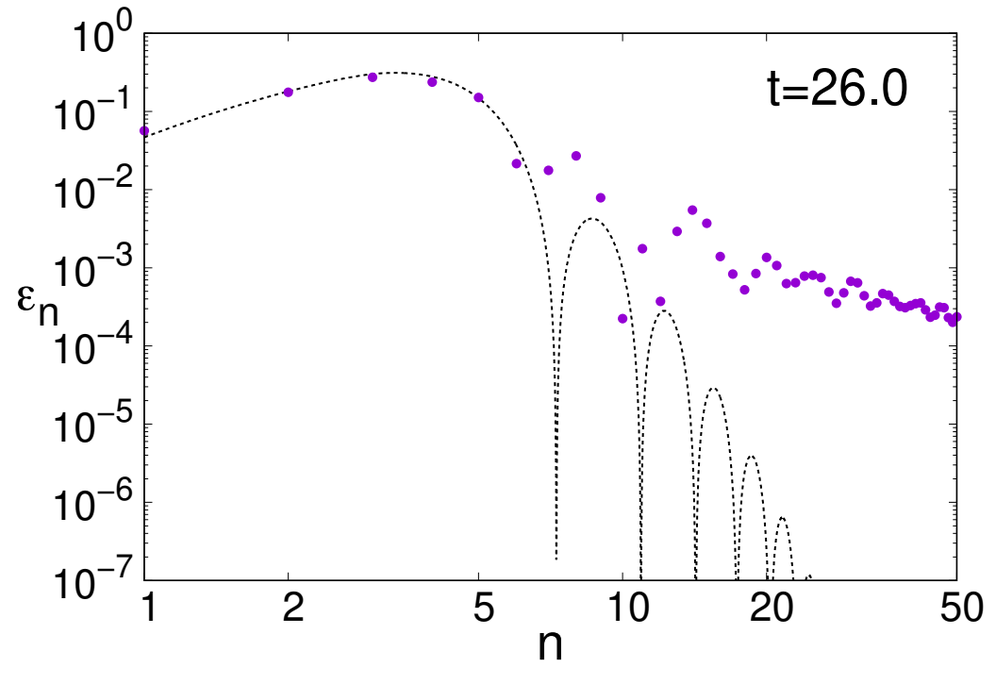
*Dashed line: linearized theory

Energy spectrum

We expand the nonlinear solutions by linear eigenmodes and compute the energy spectrum.



ε=0.005 (no cusp)



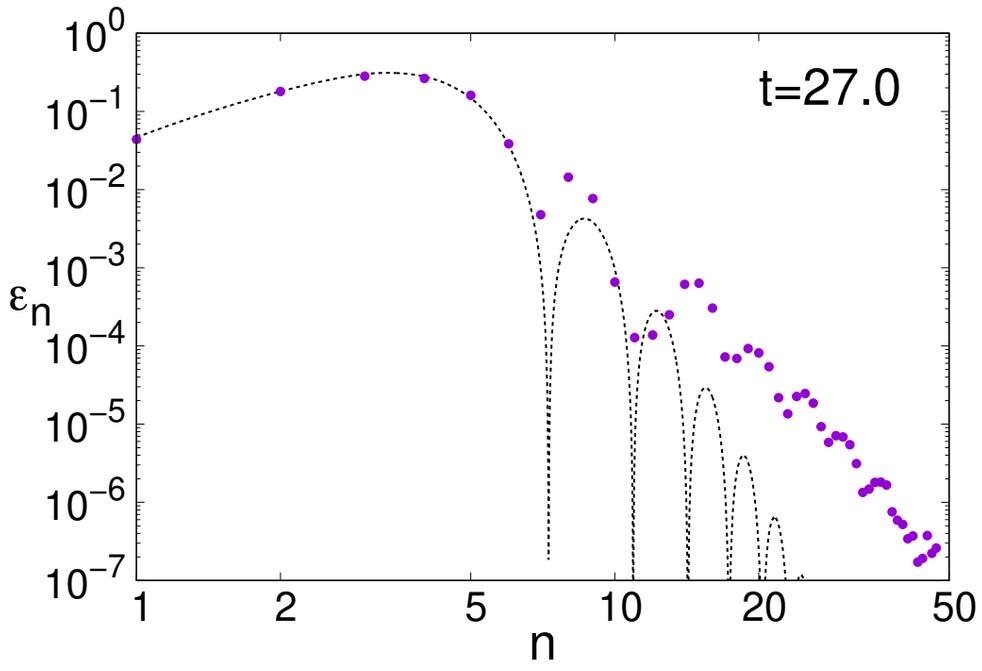
ε=0.01 (cusps t~27)

*log-log plot

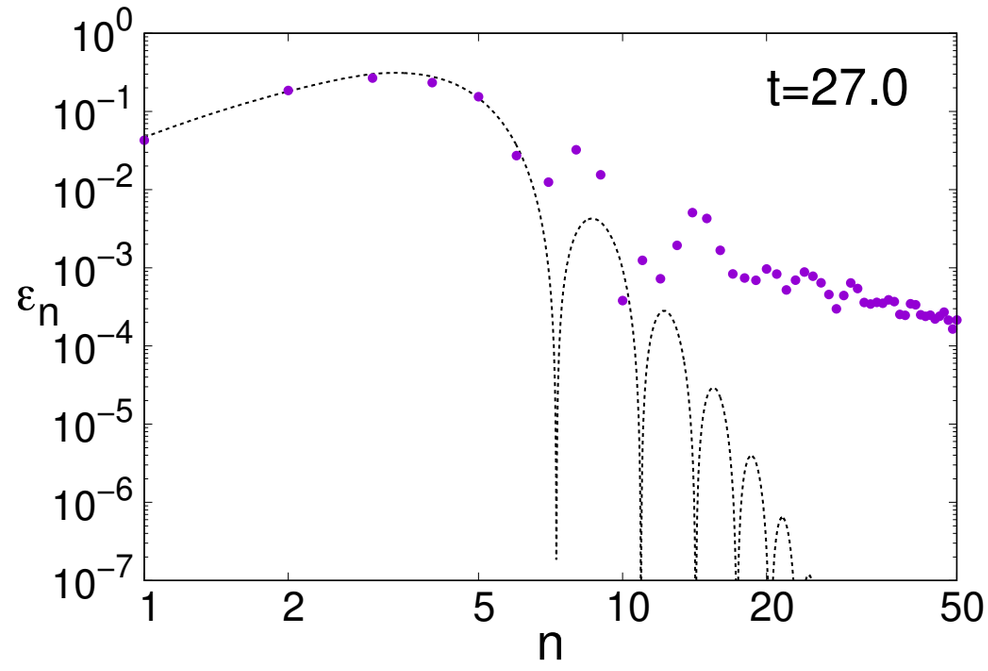
*Dashed line: linearized theory

Energy spectrum

We expand the nonlinear solutions by linear eigenmodes and compute the energy spectrum.



ε=0.005 (no cusp)

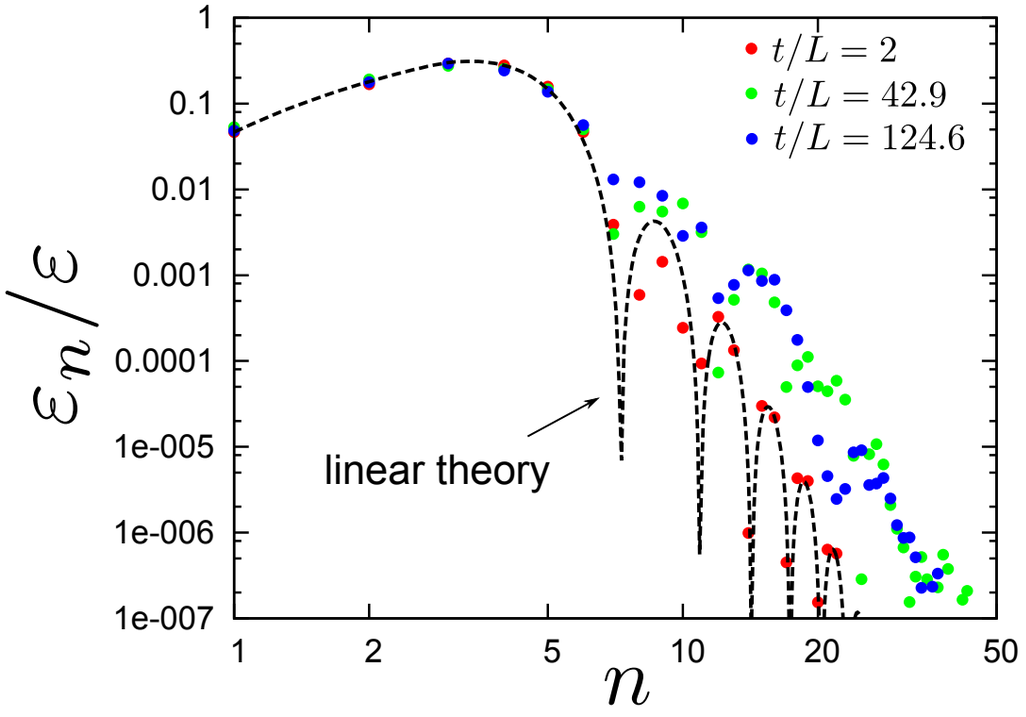


ε=0.01 (cusps t~27)

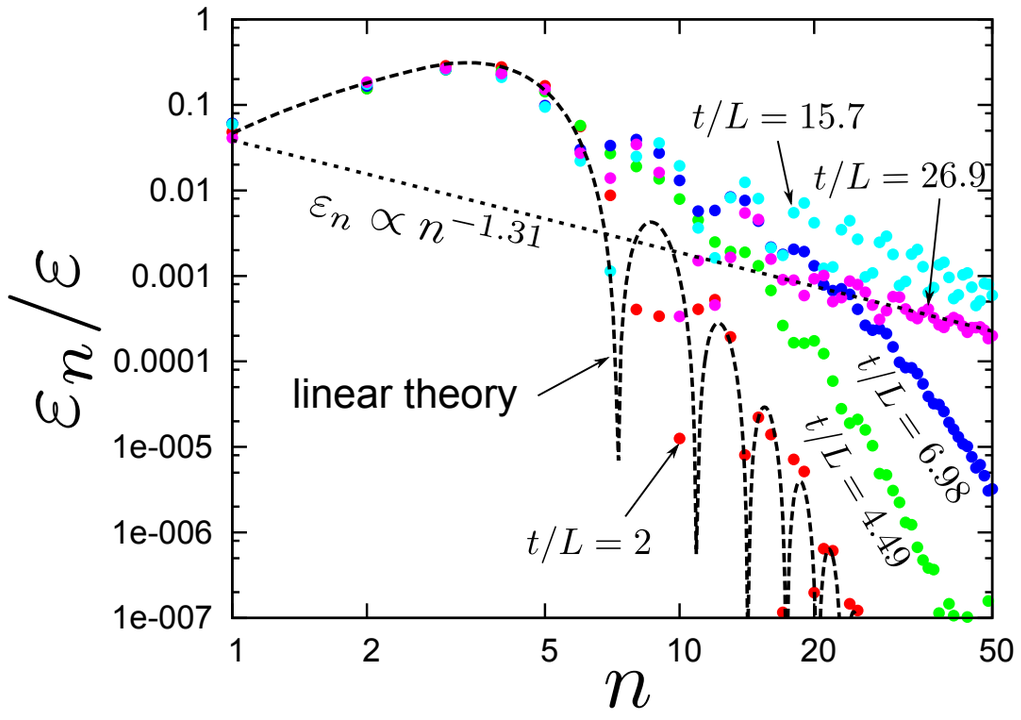
*log-log plot

*Dashed line: linearized theory

Energy cascade



ε=0.005 (no cusp)



ε=0.01 (cusps t~27)

The case of cusp formation: direct energy cascade results in the power-law energy spectrum.

Questions

String theory in AdS is integrable, and turbulence/cusp formation might not happen?

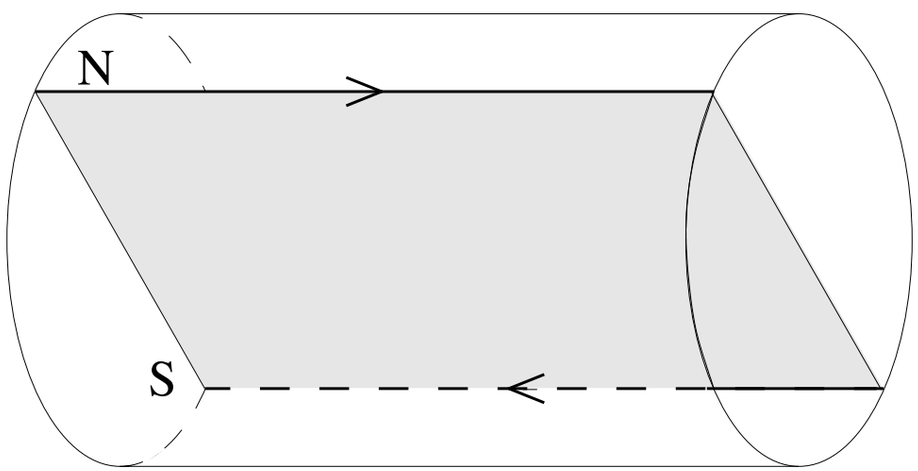
c.f.) chaotic strings in non-integrable background [TI-Murata-Yoshida]

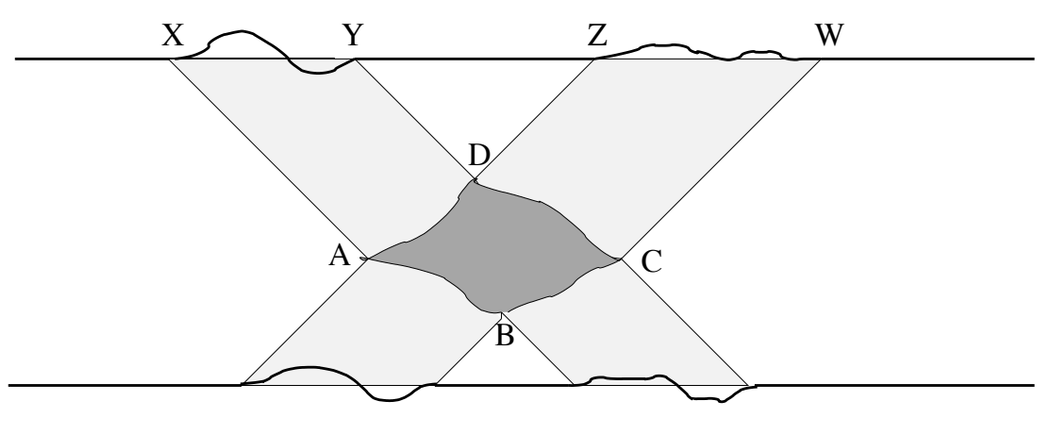
Segmented strings [Gubser et al,Vegh] utilizes AdS string's integrability.

What do our turbulent strings mean?

Nonlinear waves on an AdS string

[Mikhailov]





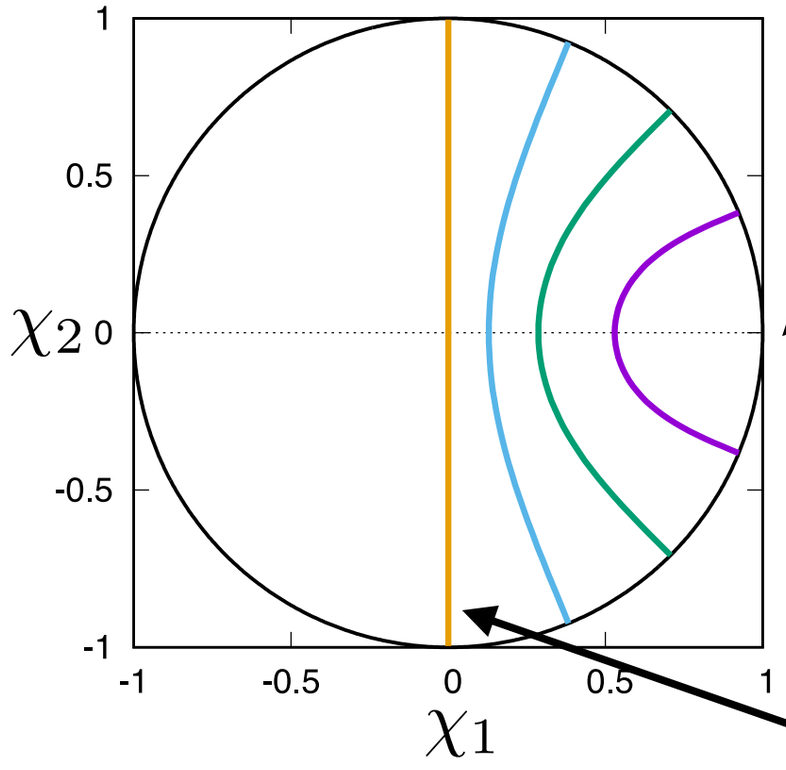
Analytic wave solution has been found in the case of AdS2 embedding $\subset$ global AdS5.

This solution does not look turbulent.

Pictures taken from hep-th/0305169

Holographic qq̄ potential in global AdS

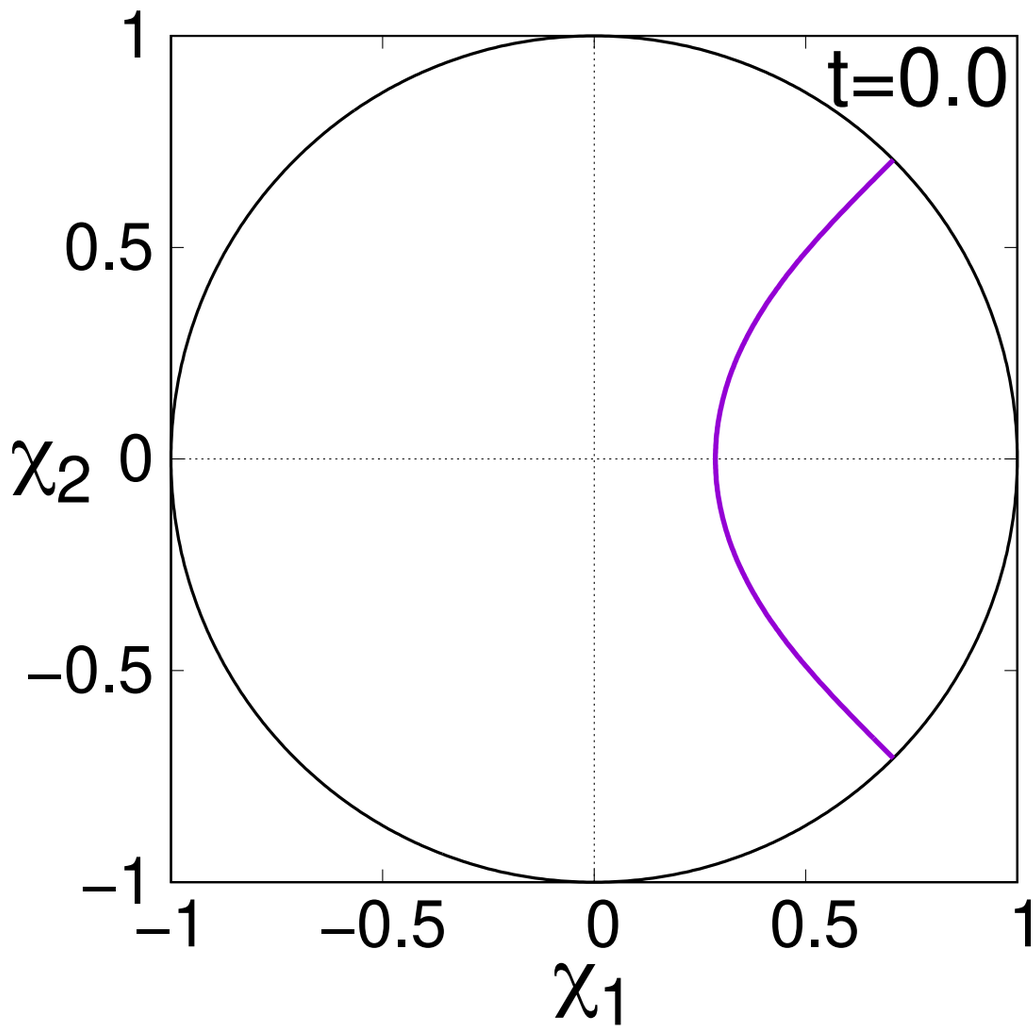
$$ds^2 = -\left(\frac{1+\chi^2}{1-\chi^2}\right) + \frac{4}{(1-\chi^2)^2} d\boldsymbol{\chi}^2$$



Small separation (ψ_b=0) reduces to the string in Poincare AdS.

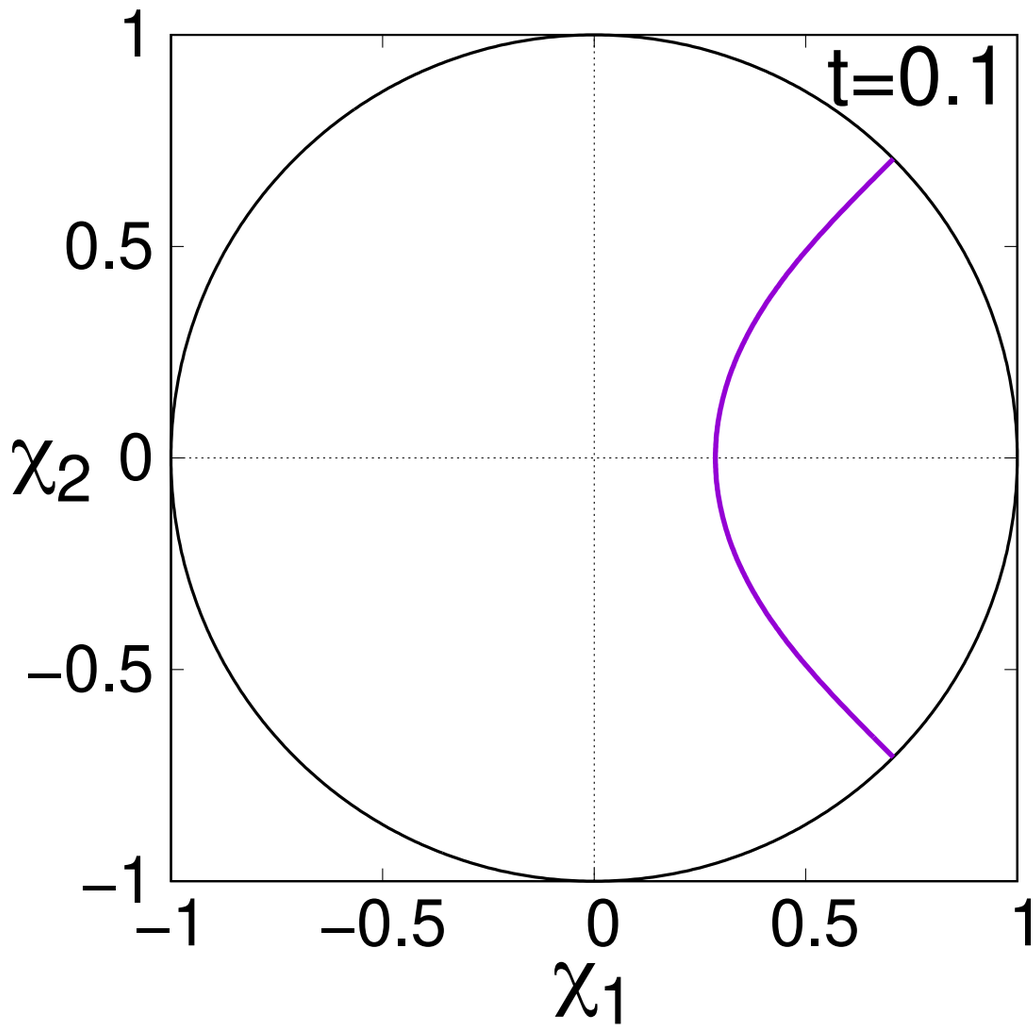
Antipodal string (ψ_b=π/2) has AdS2 induced metric.

$\psi_b=\pi/4$



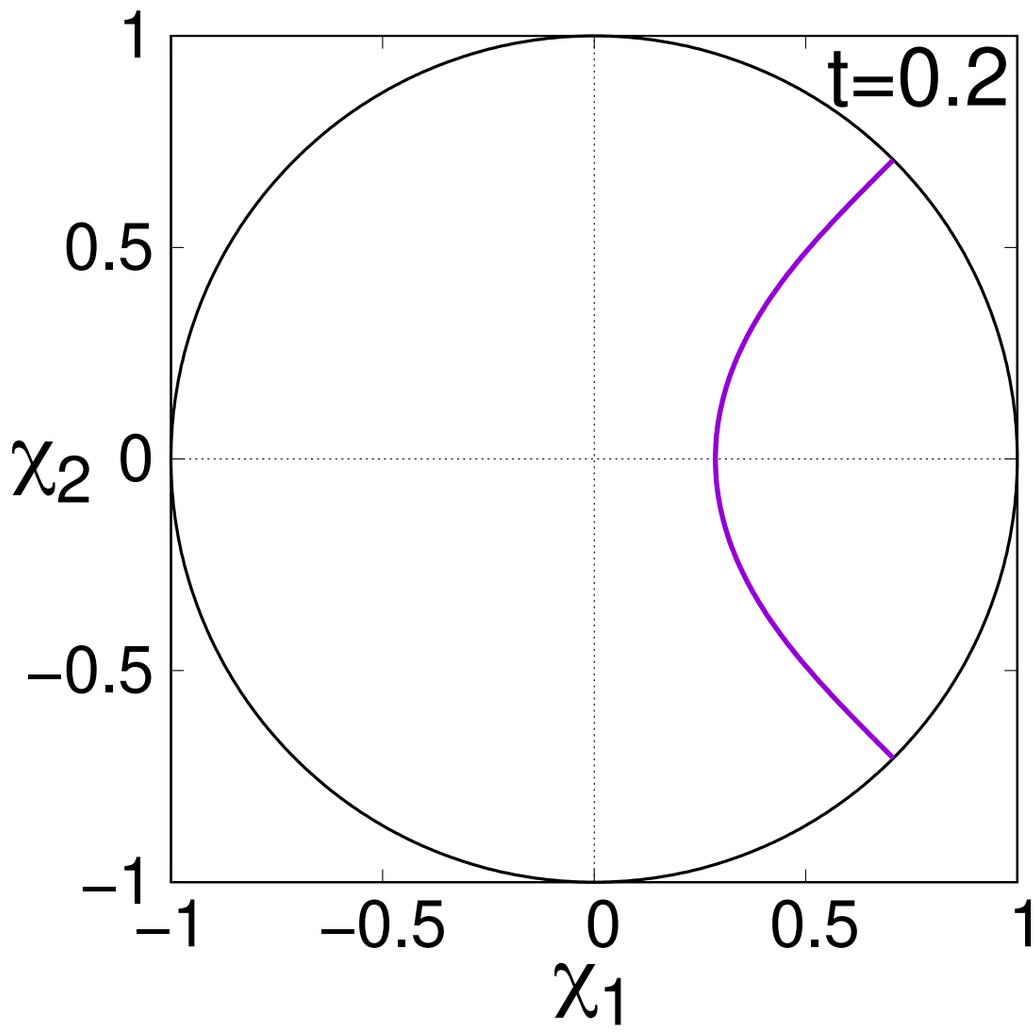
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



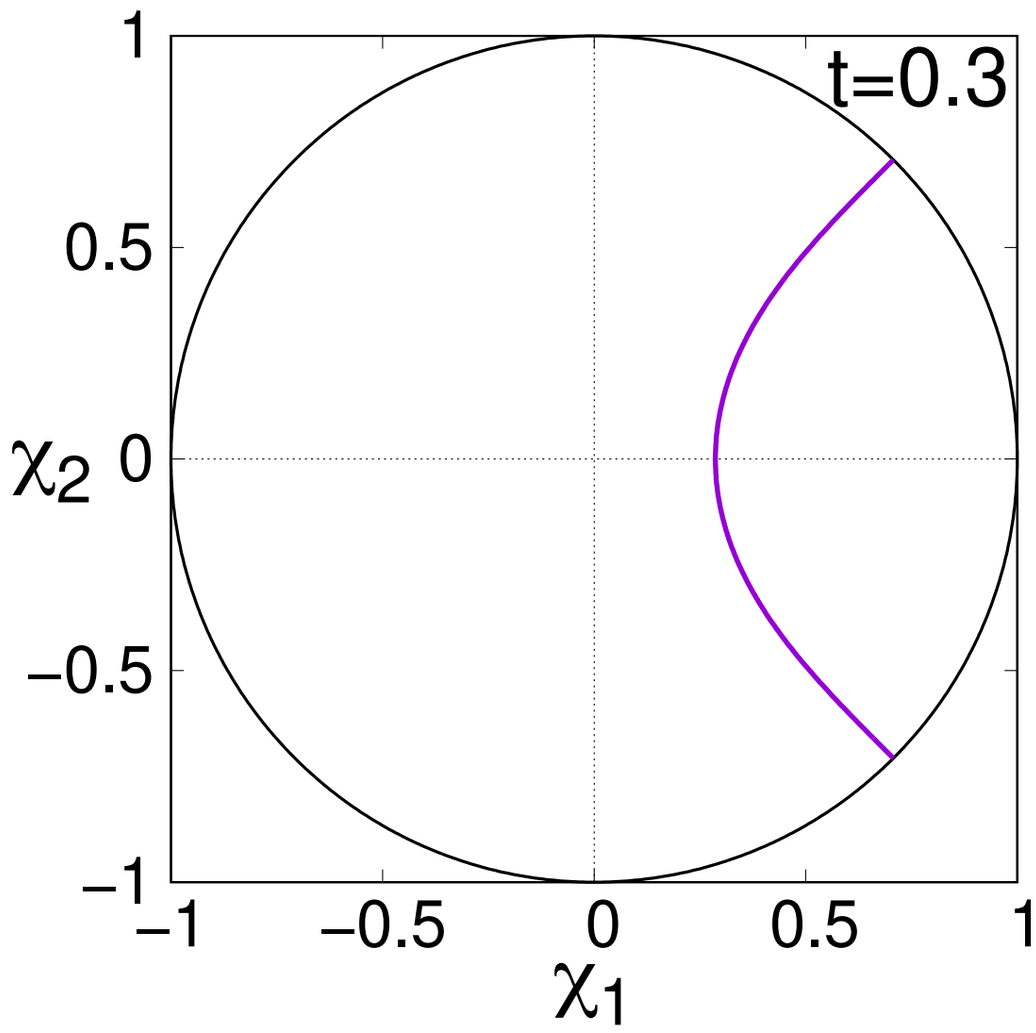
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



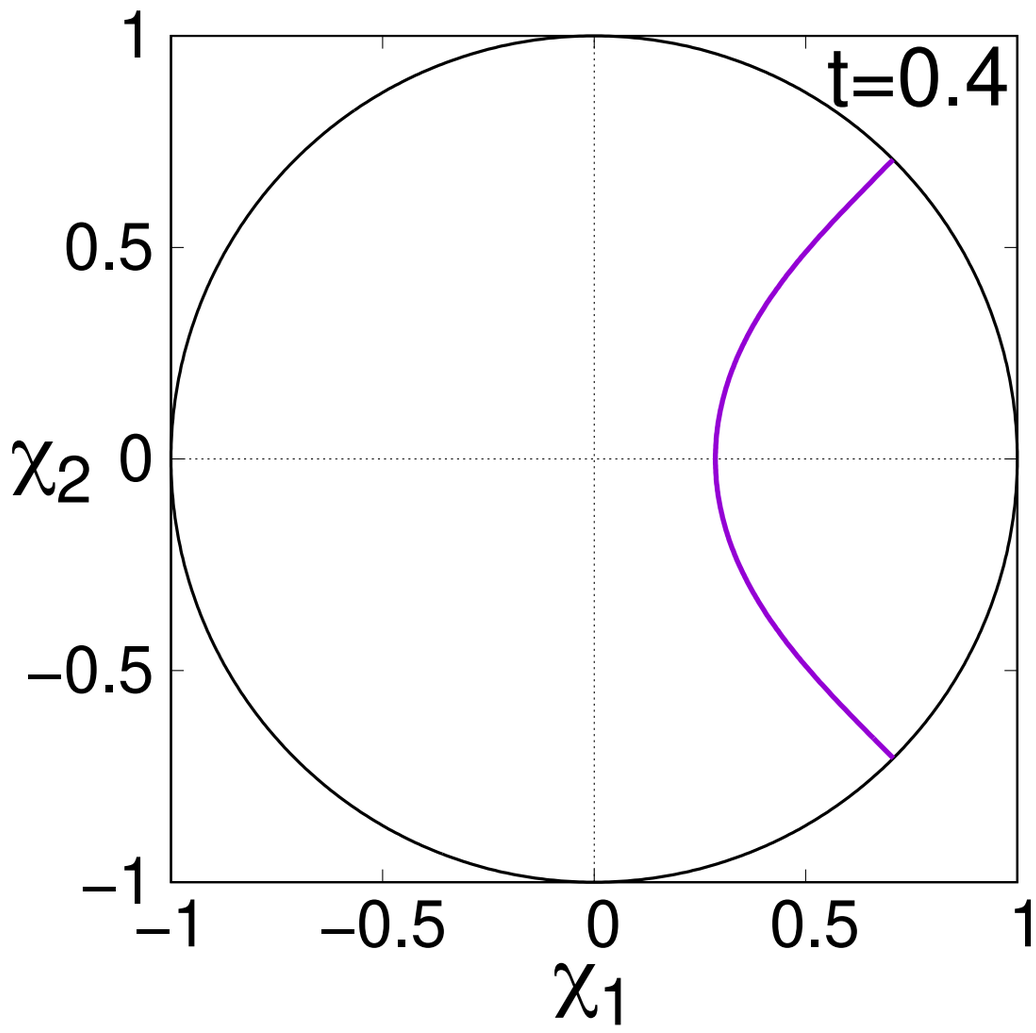
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



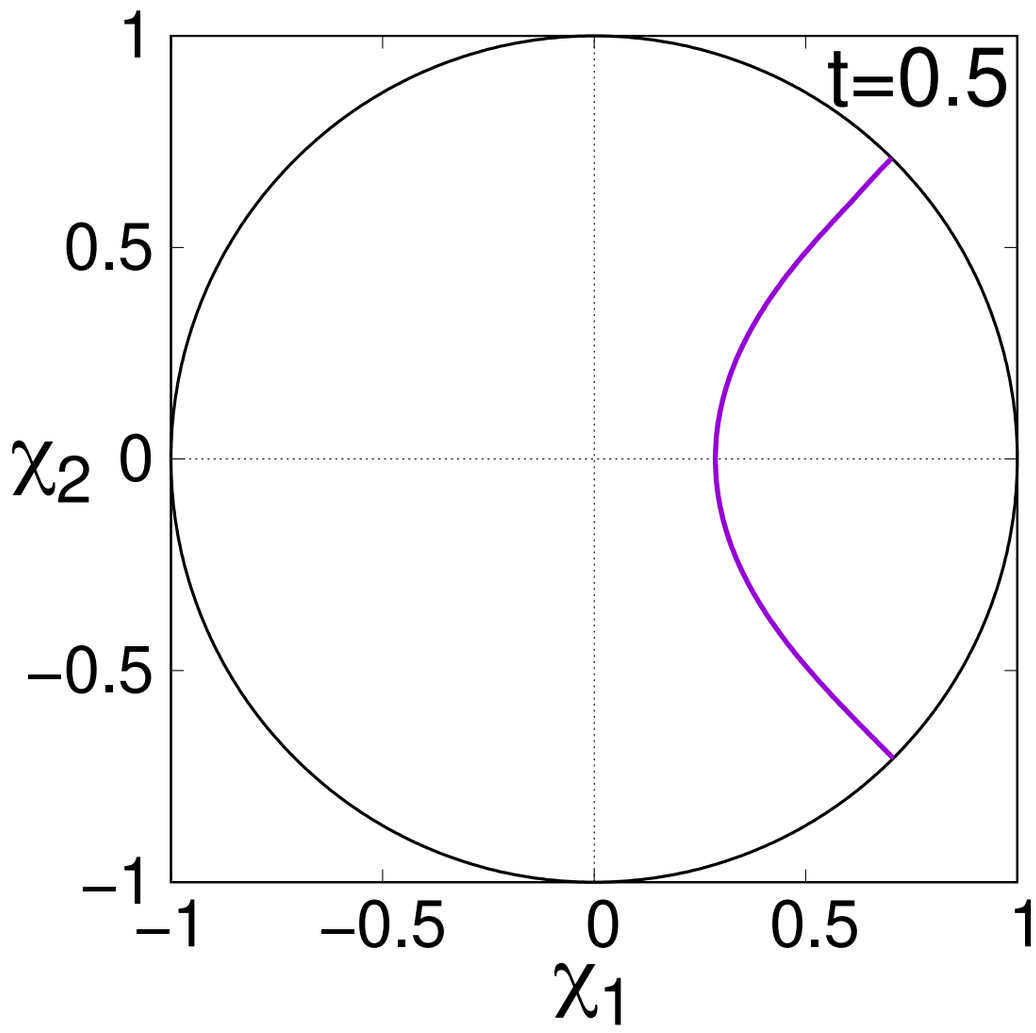
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



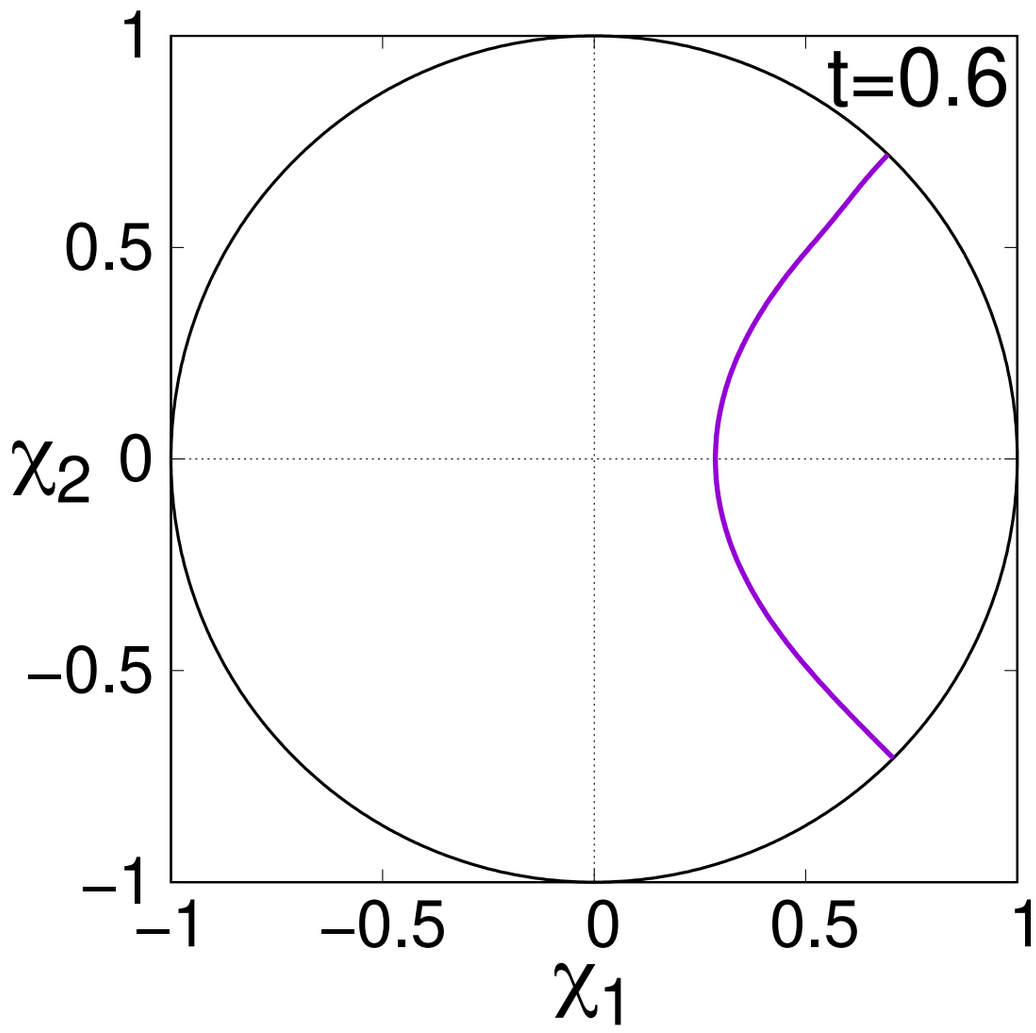
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



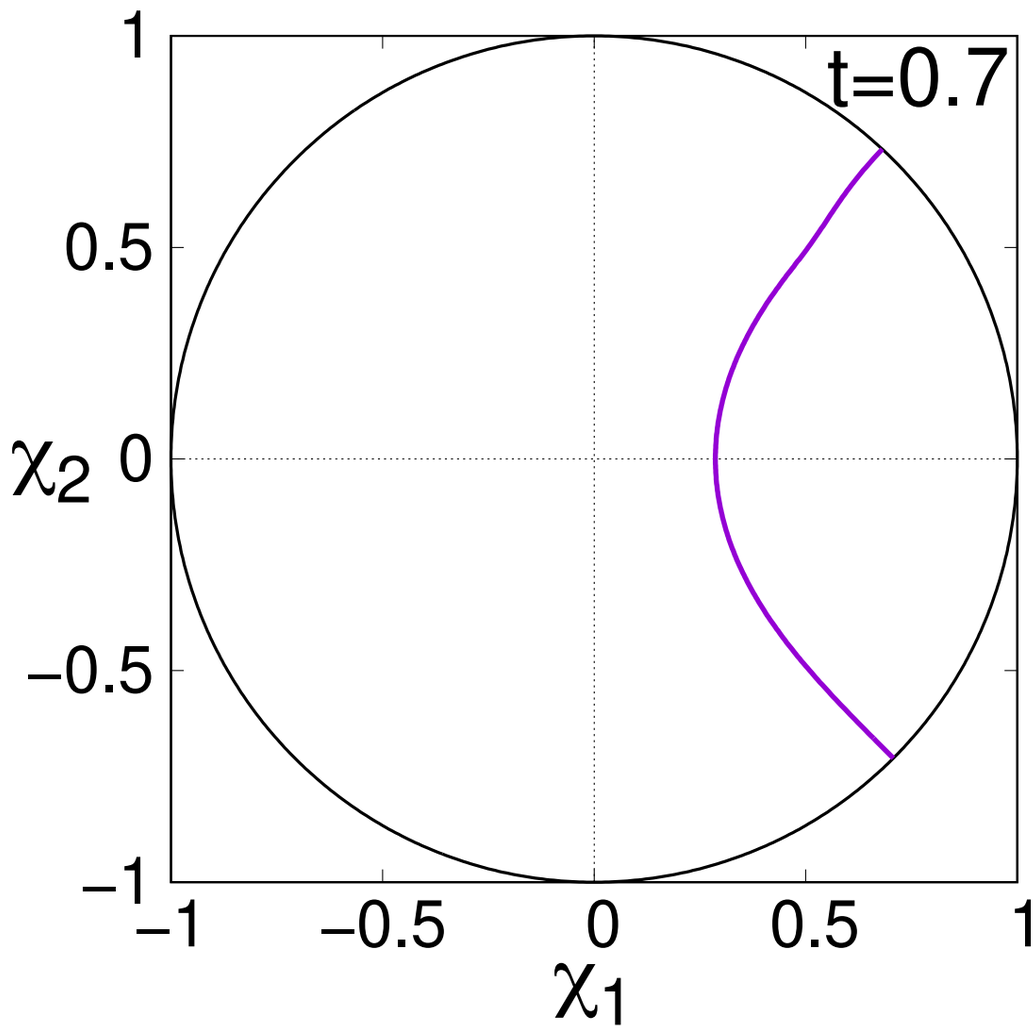
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



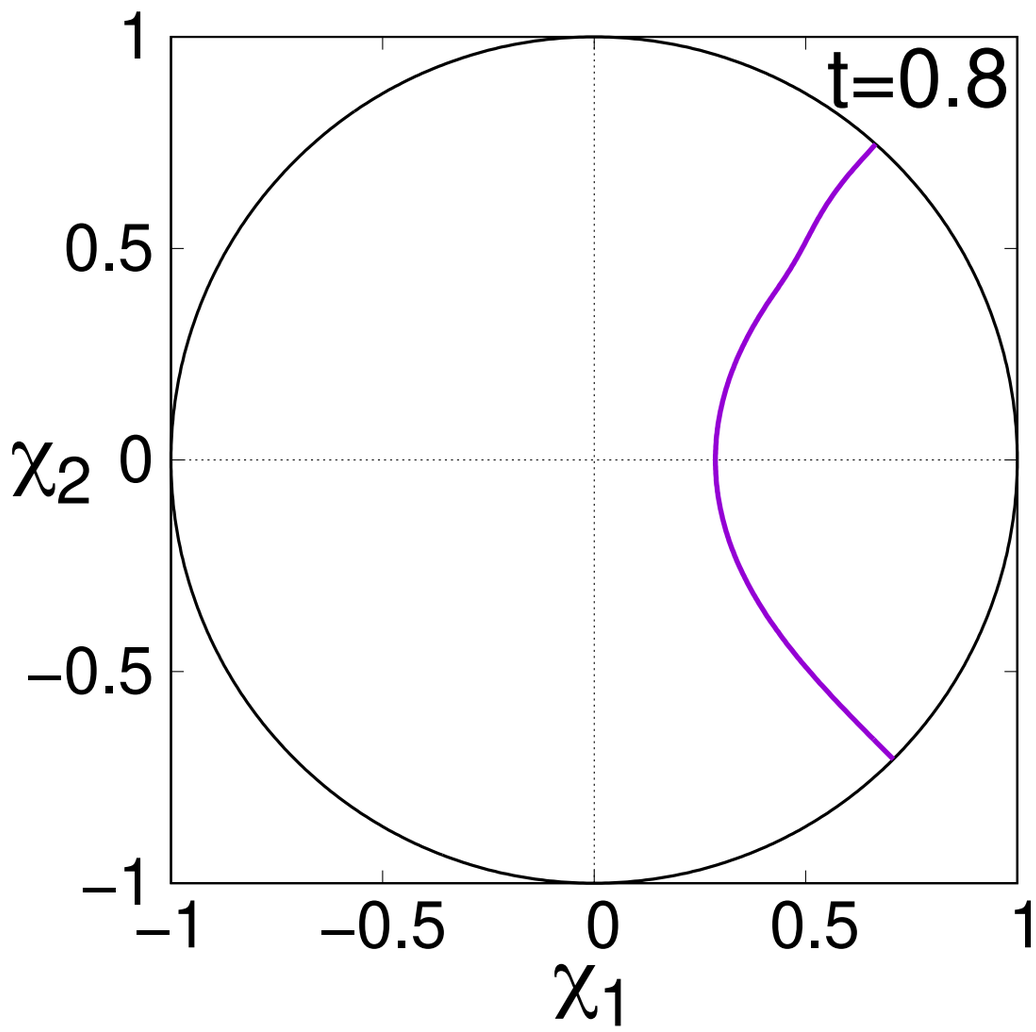
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



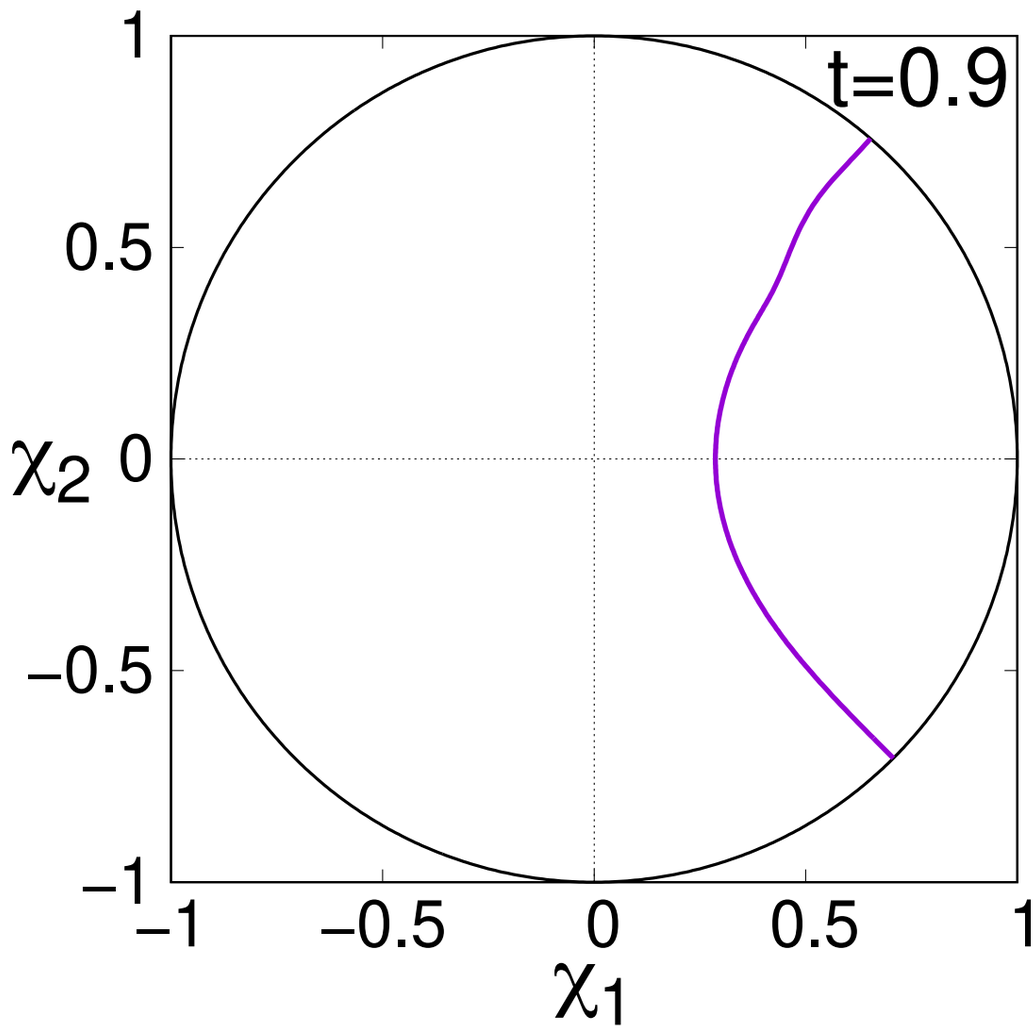
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



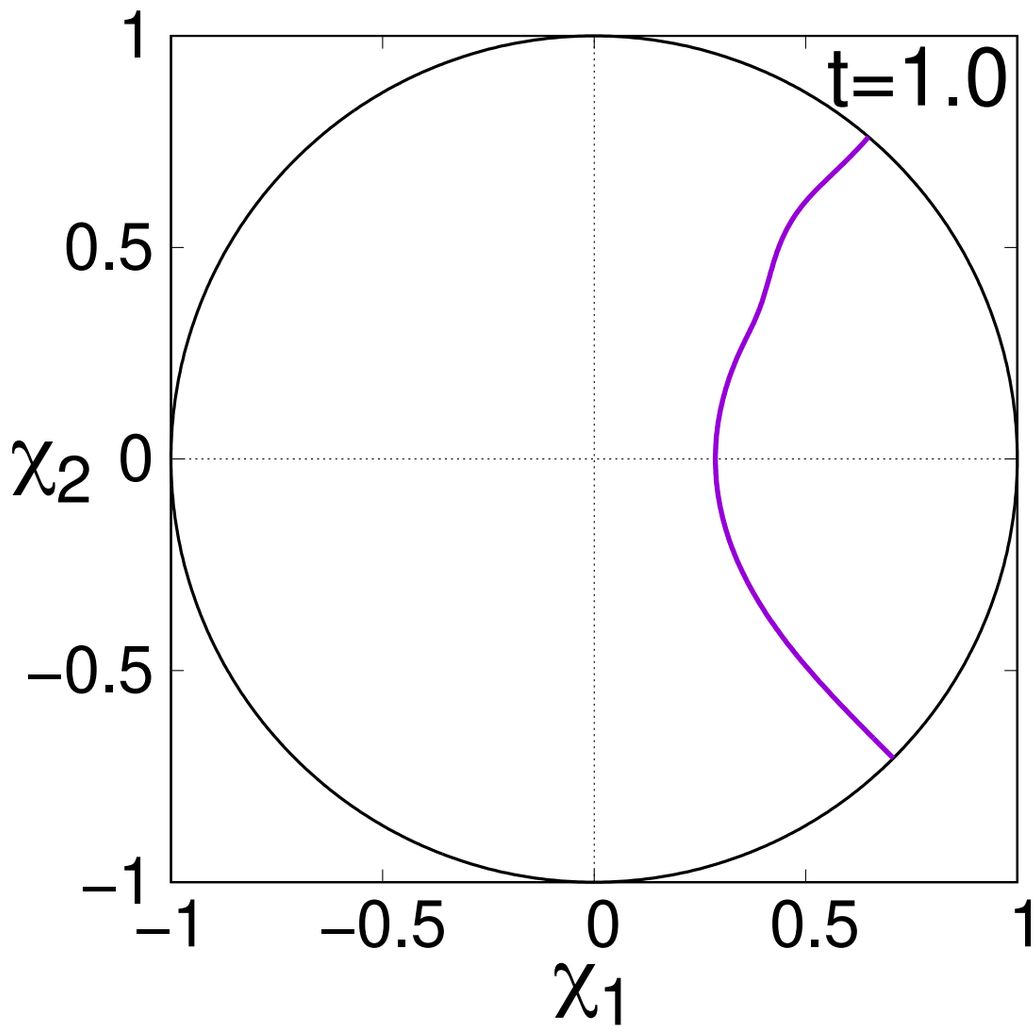
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



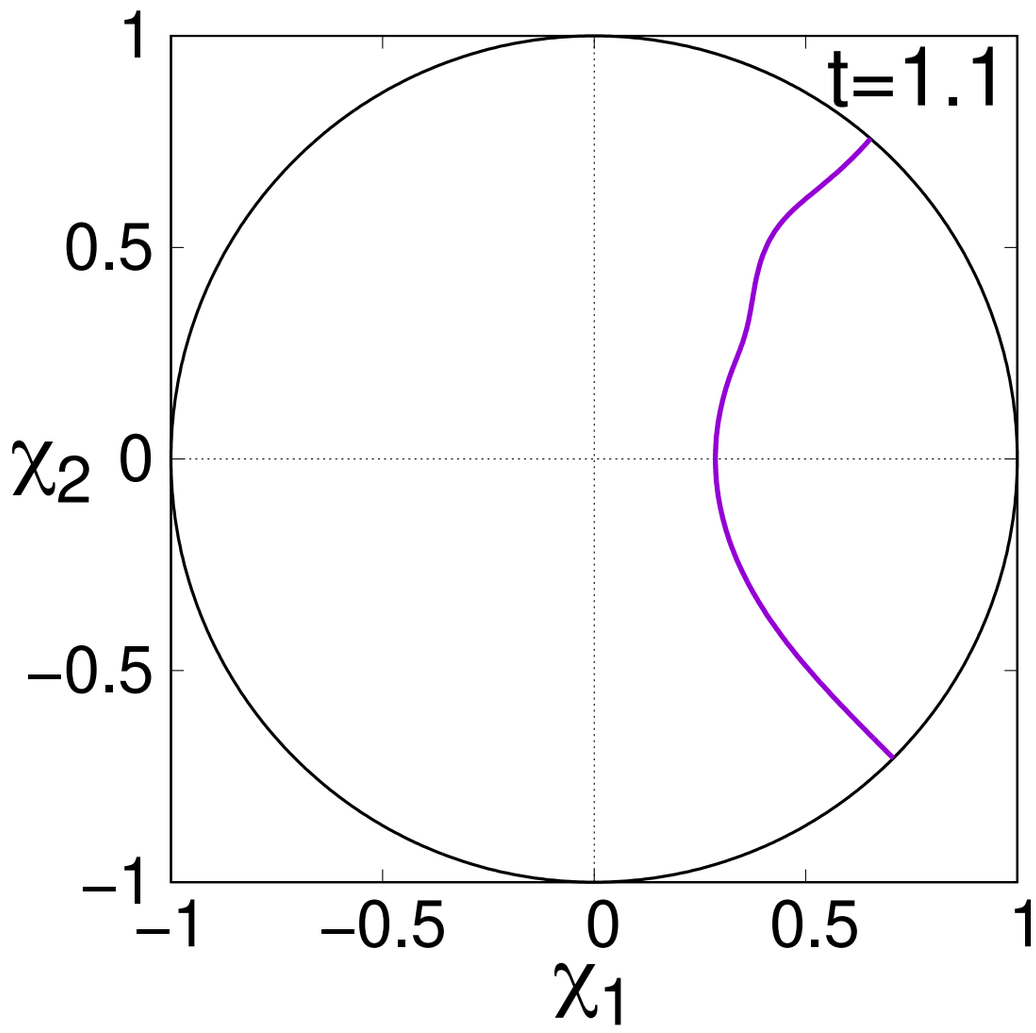
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



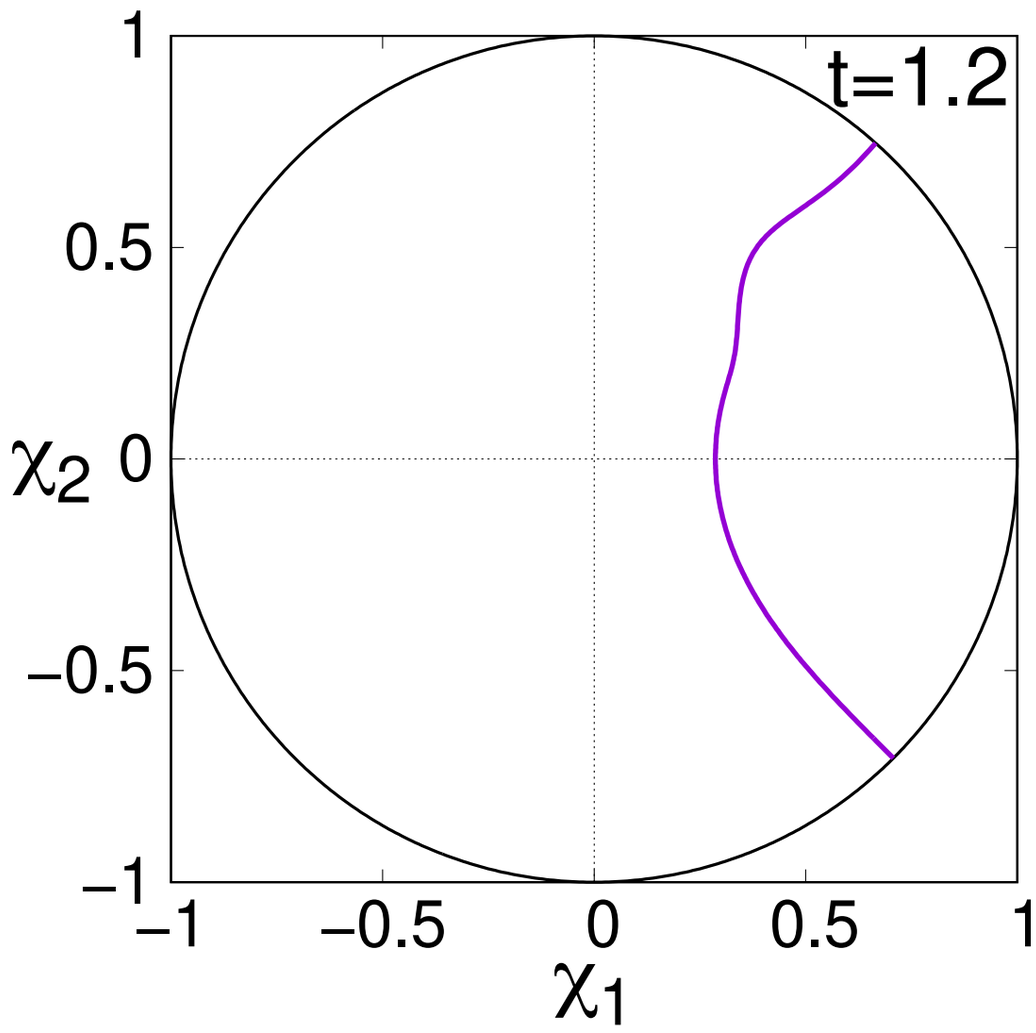
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



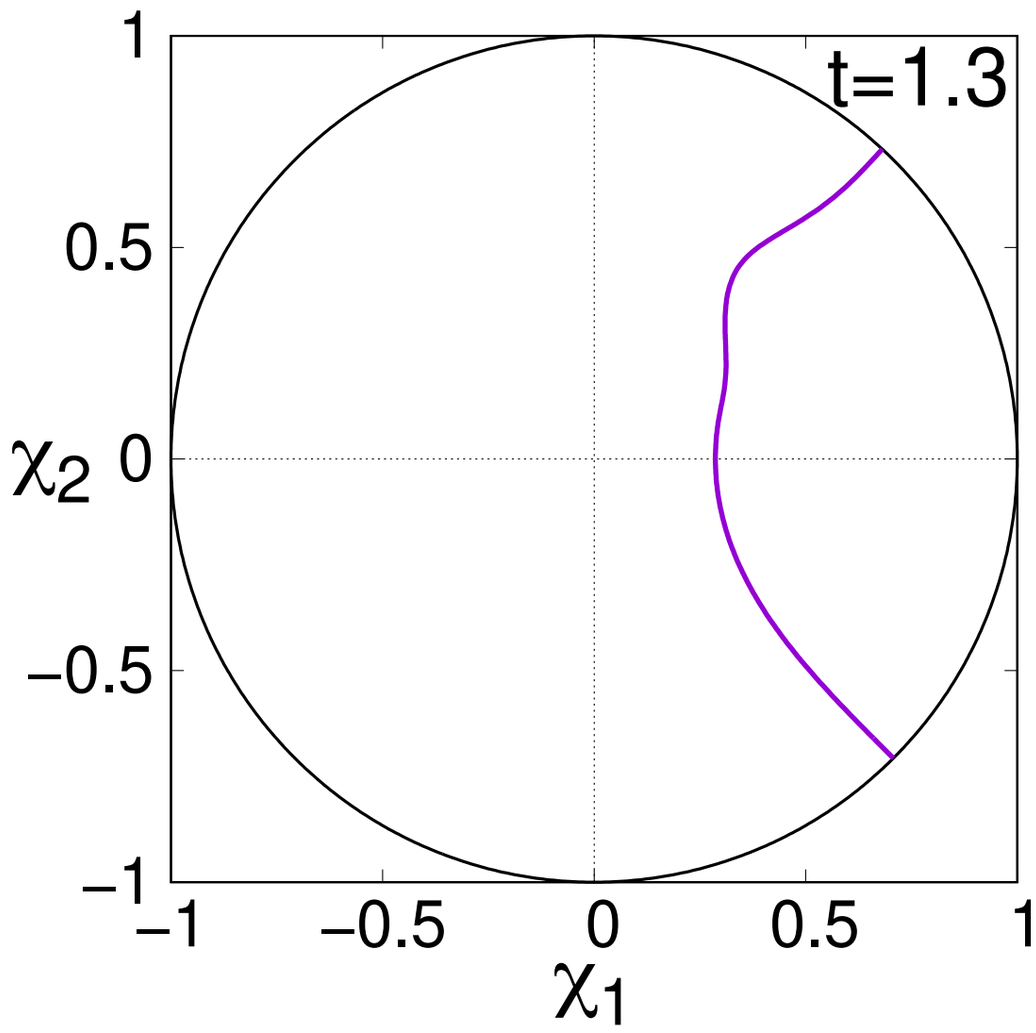
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



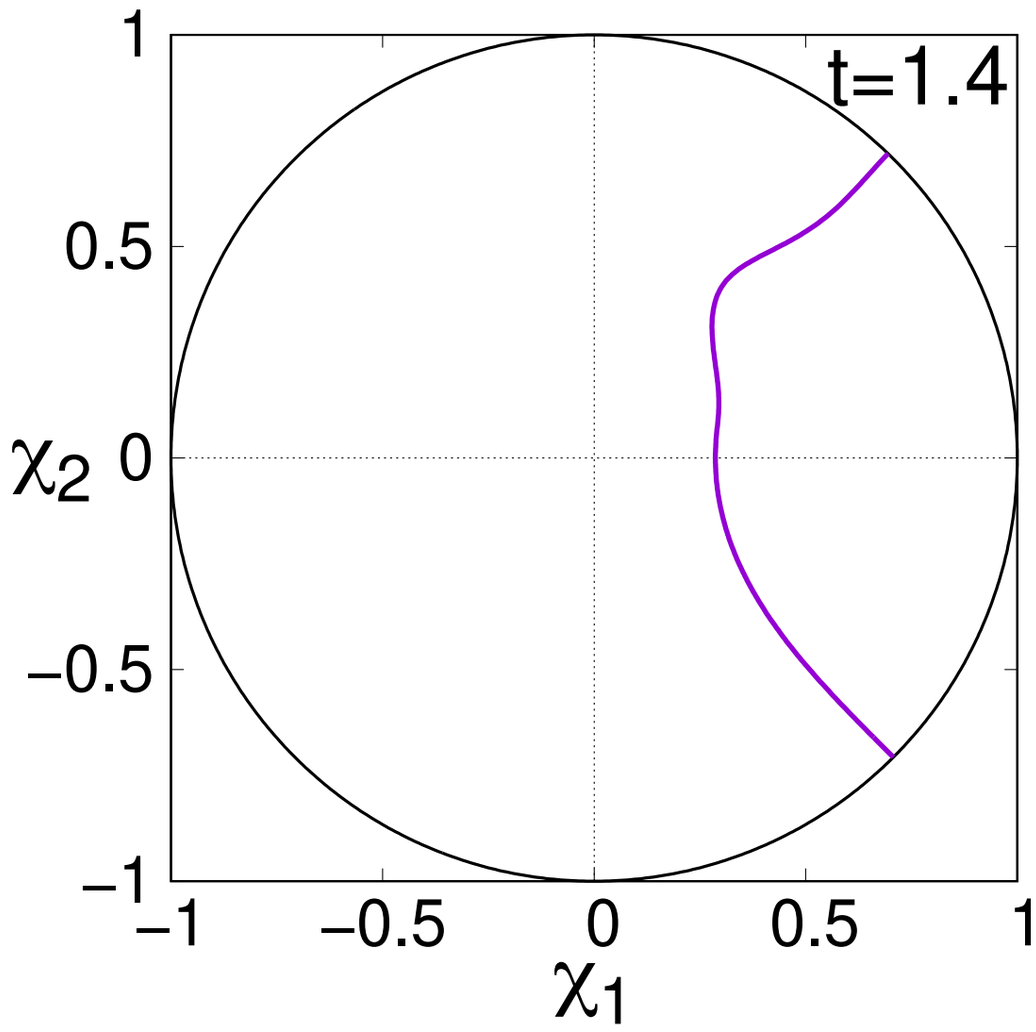
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



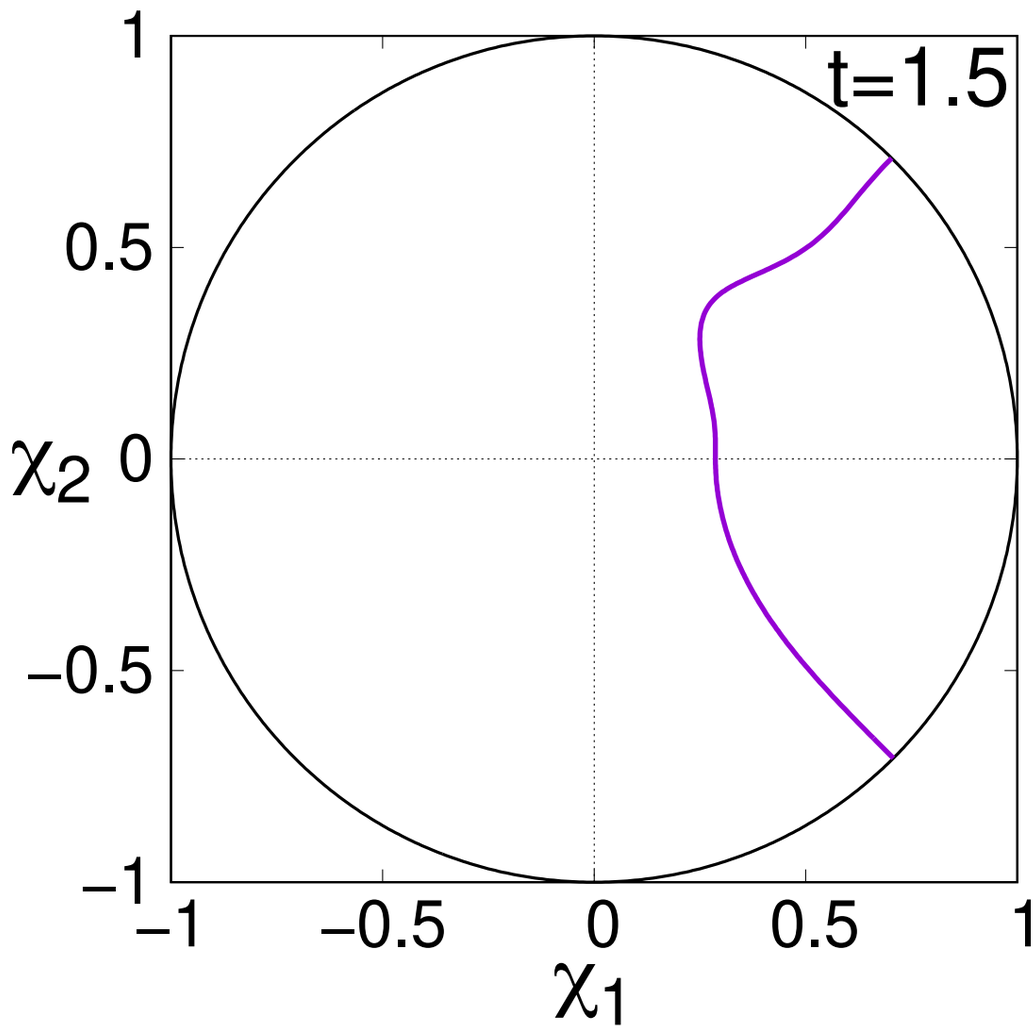
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



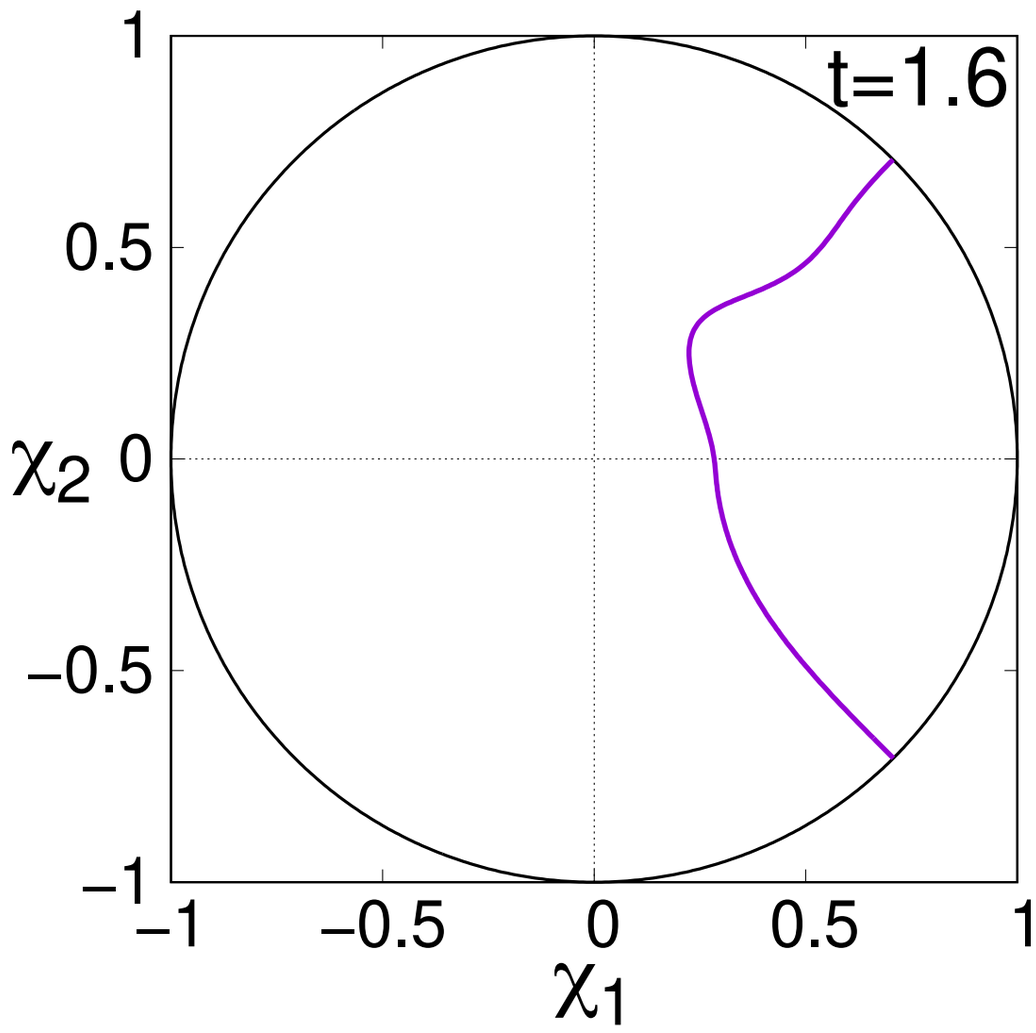
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



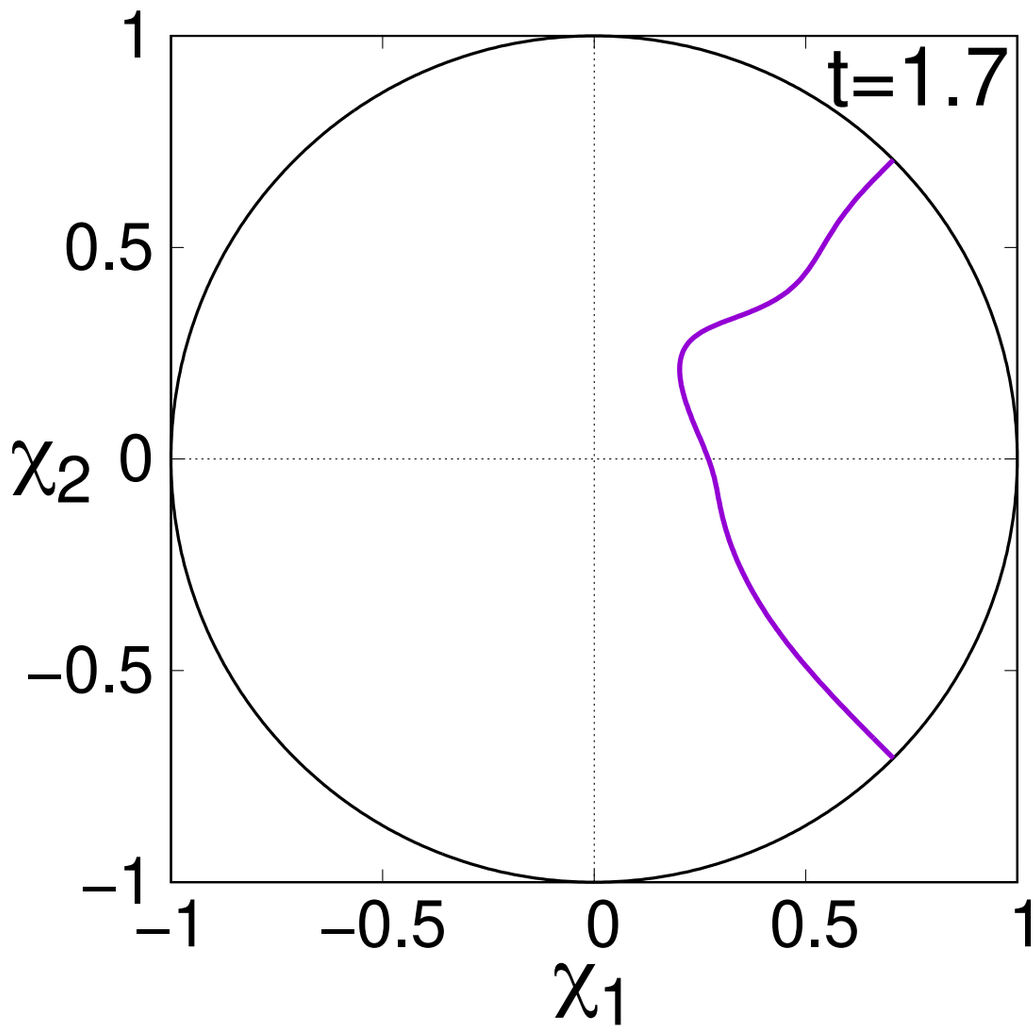
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



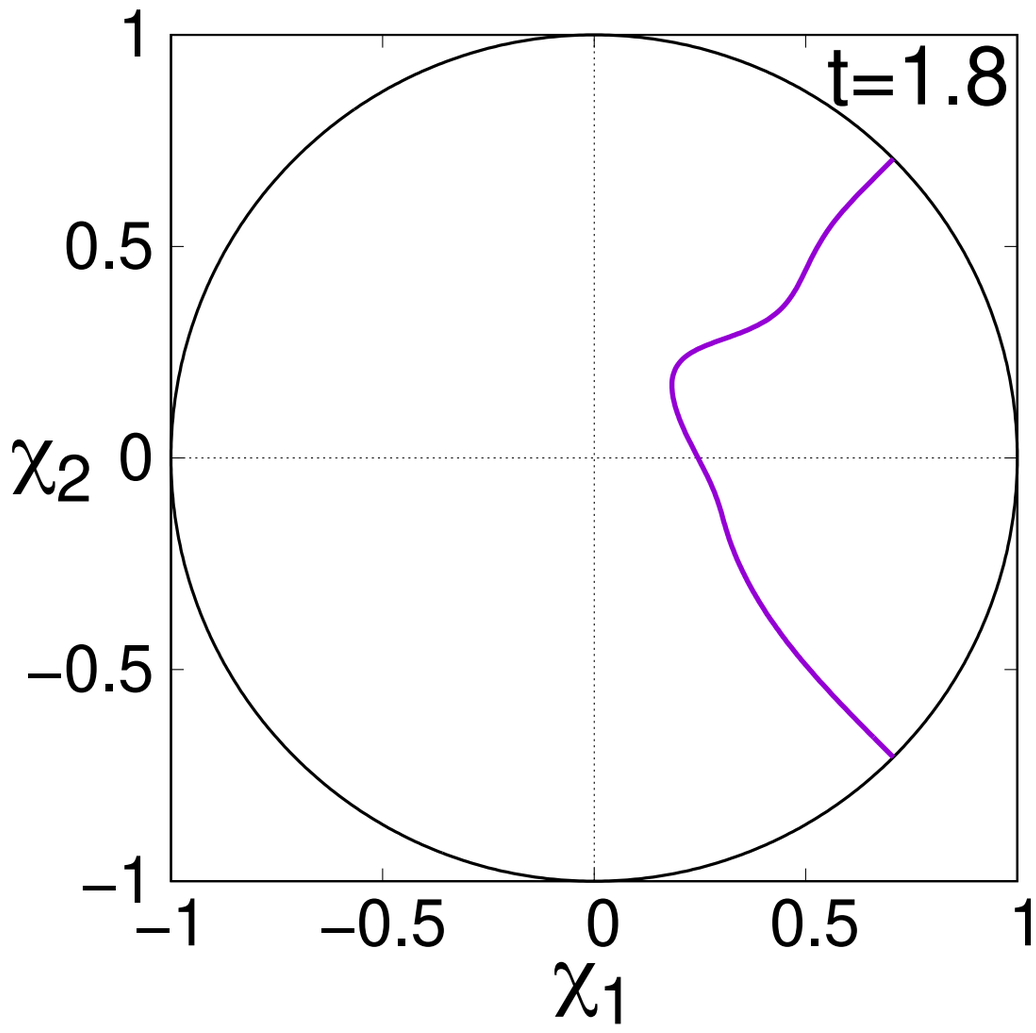
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



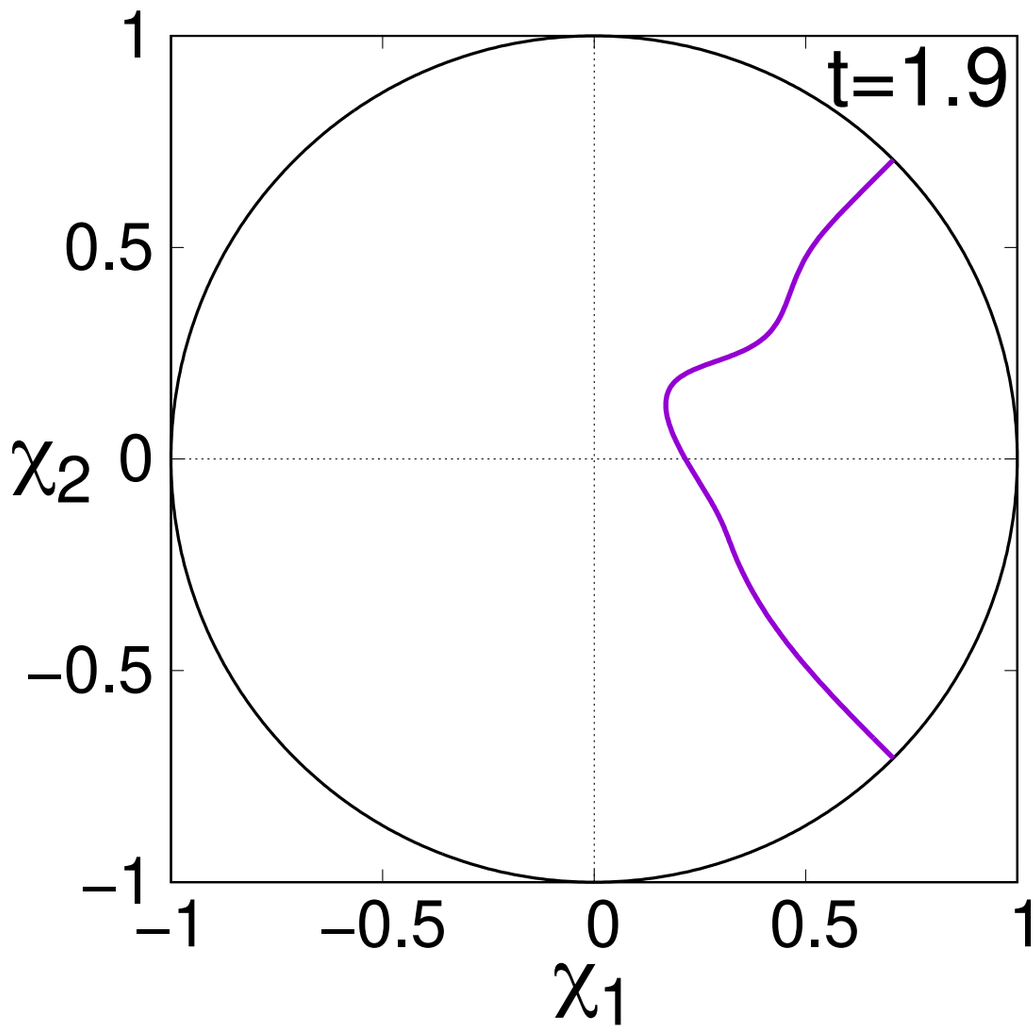
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



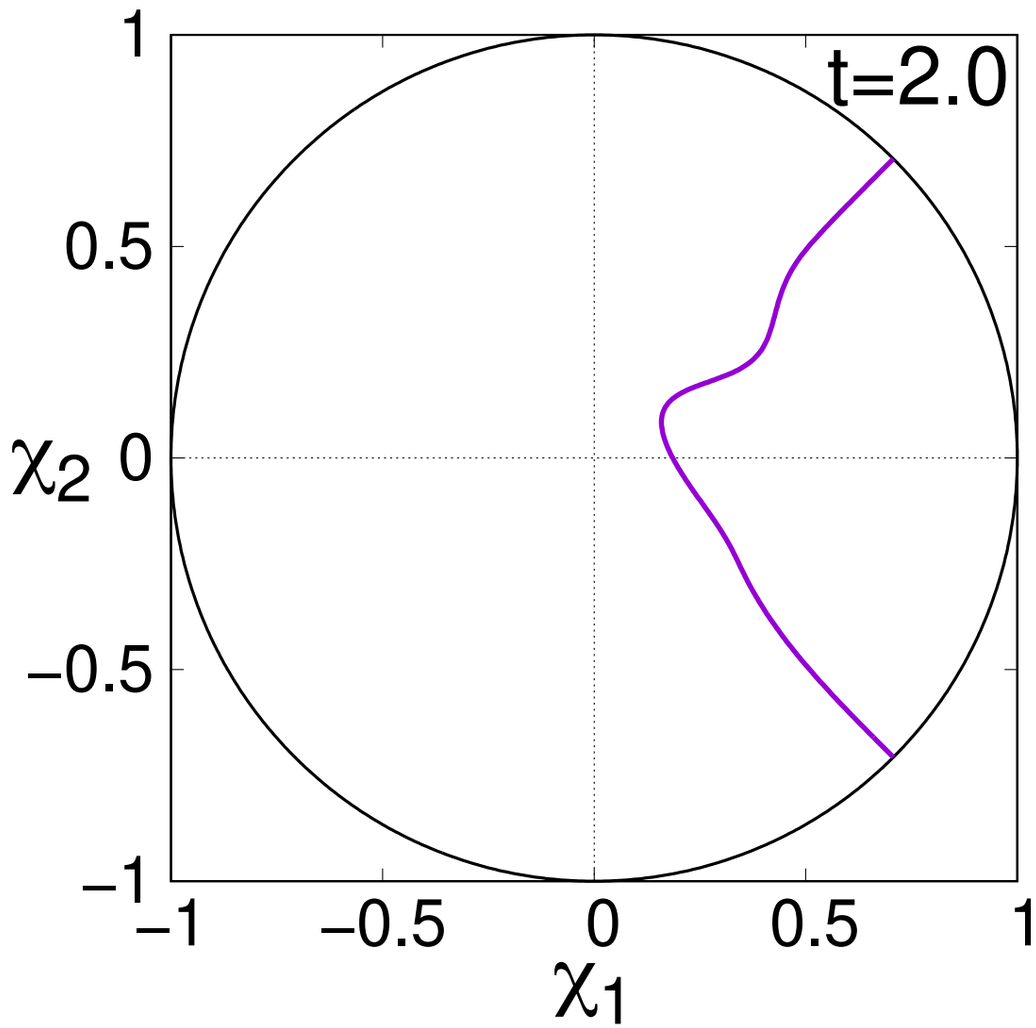
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



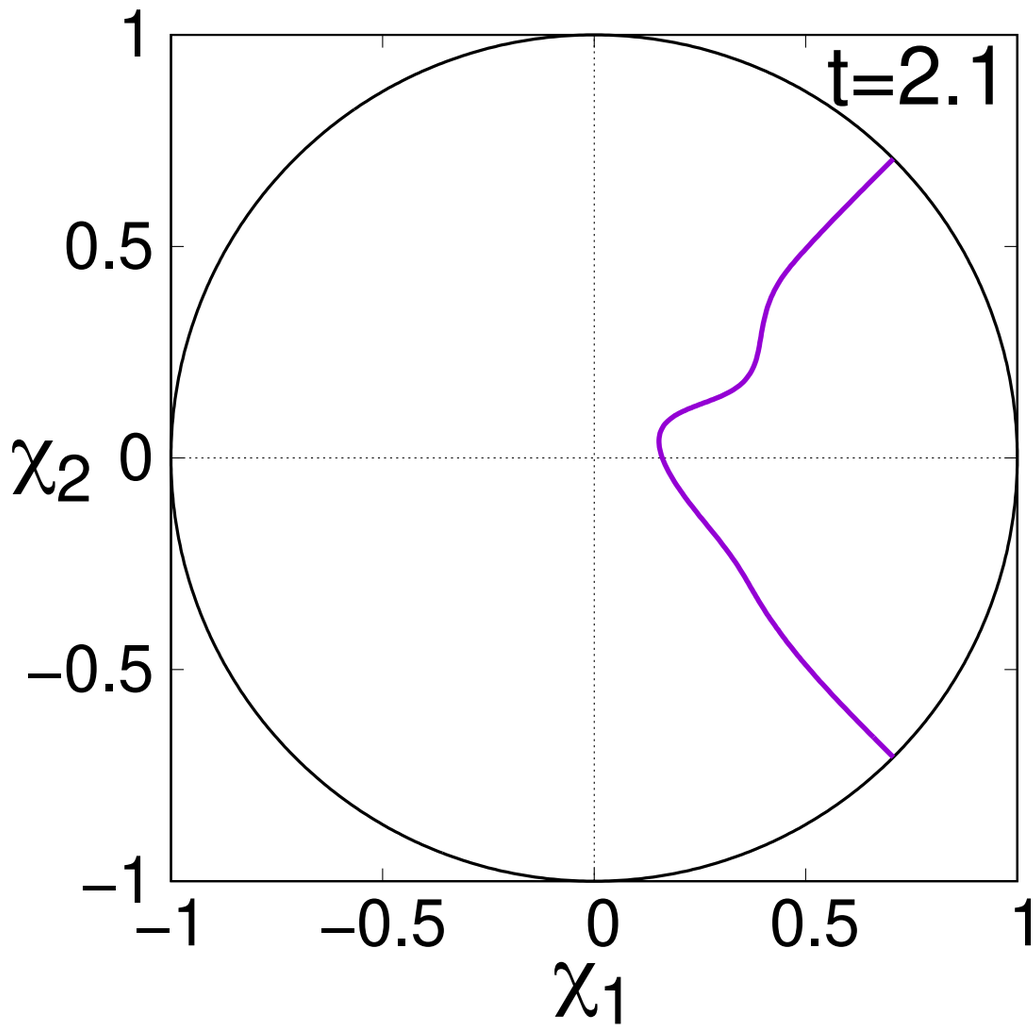
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



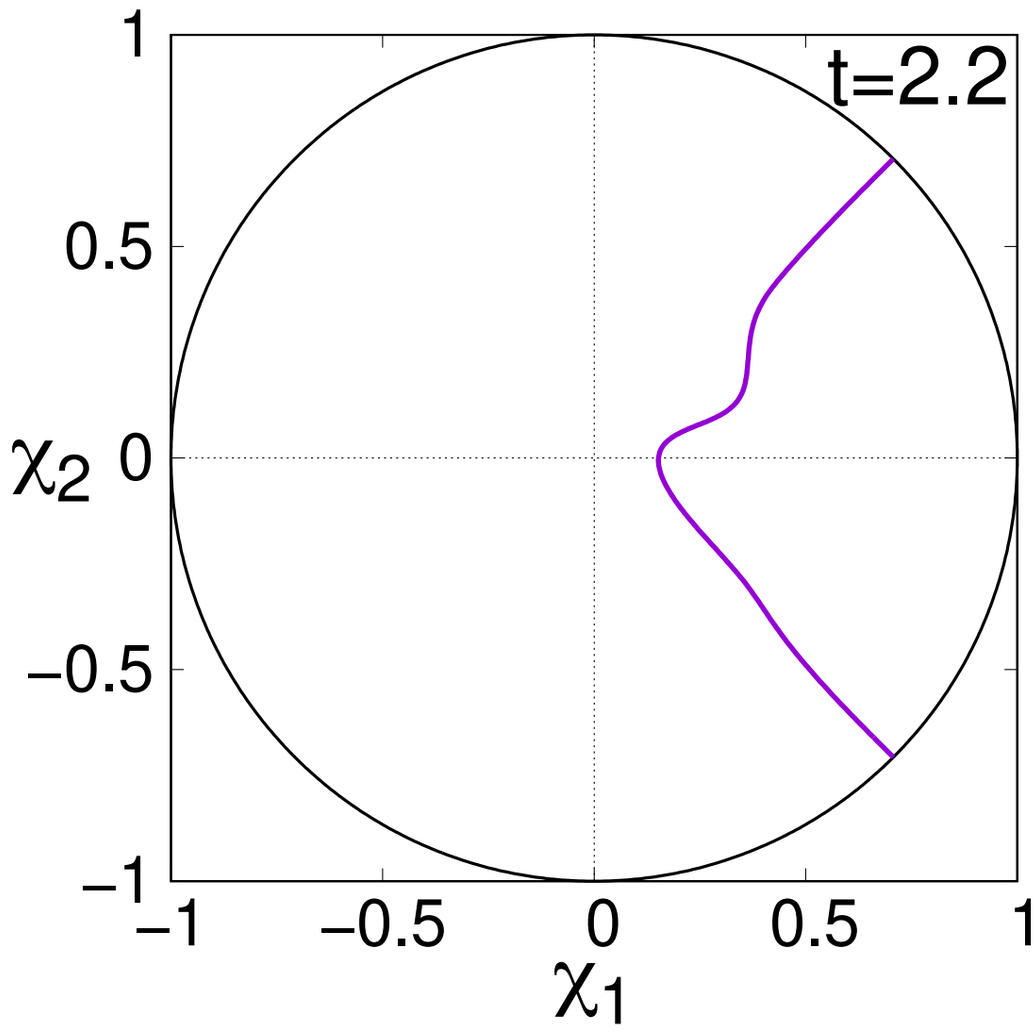
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



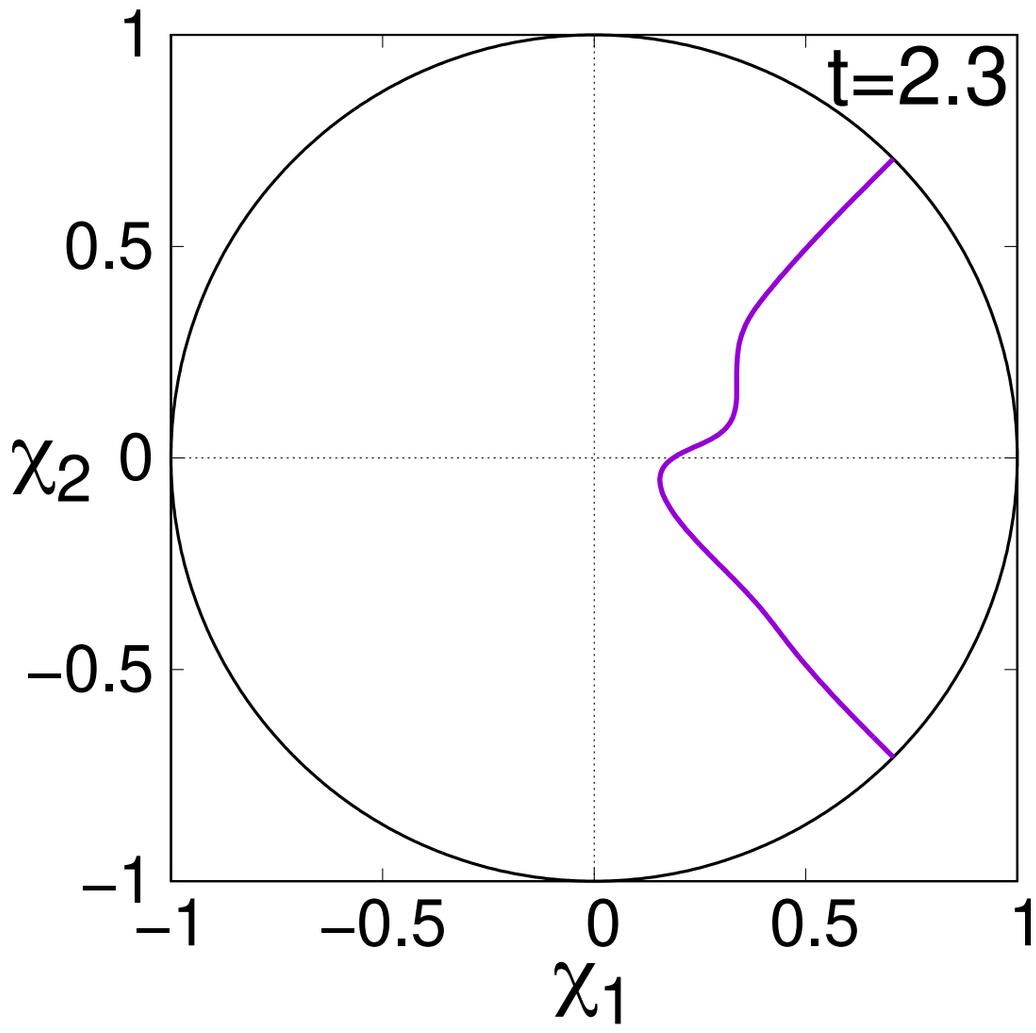
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



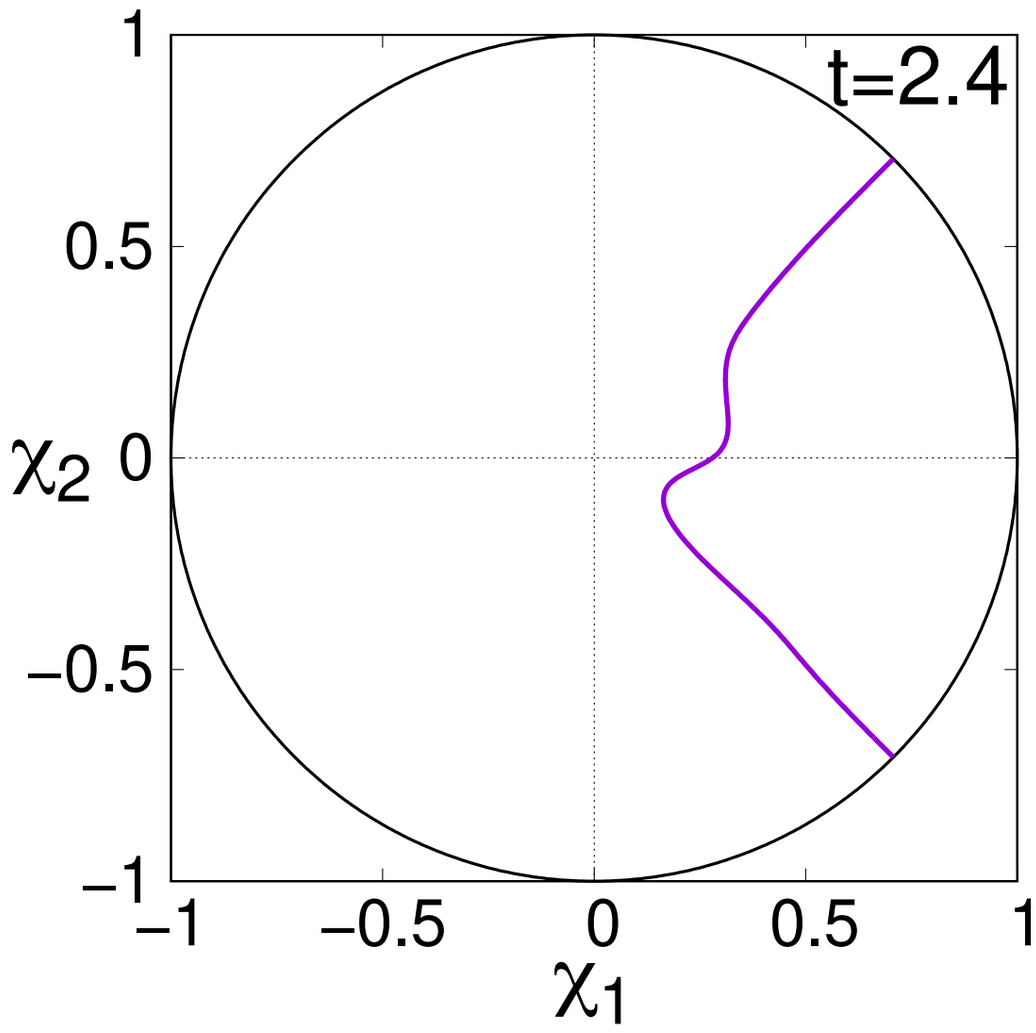
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



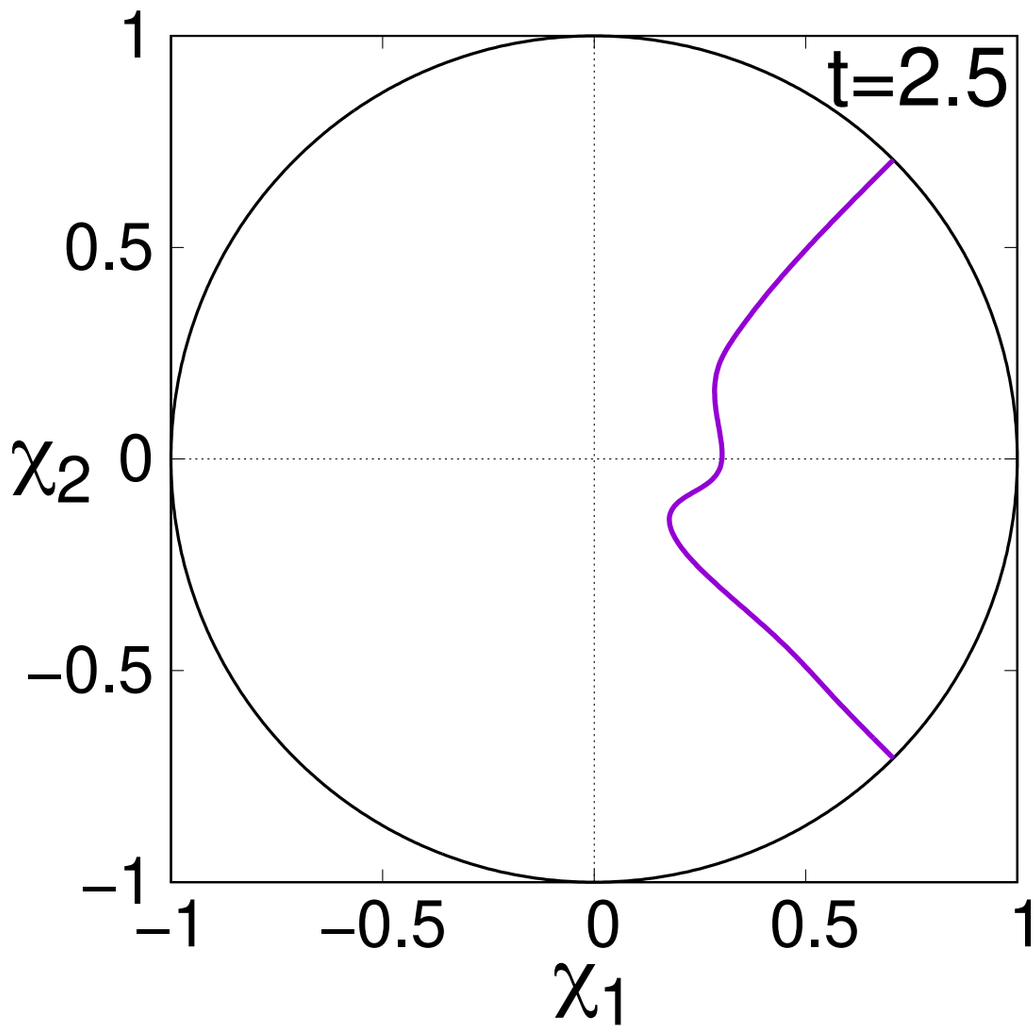
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



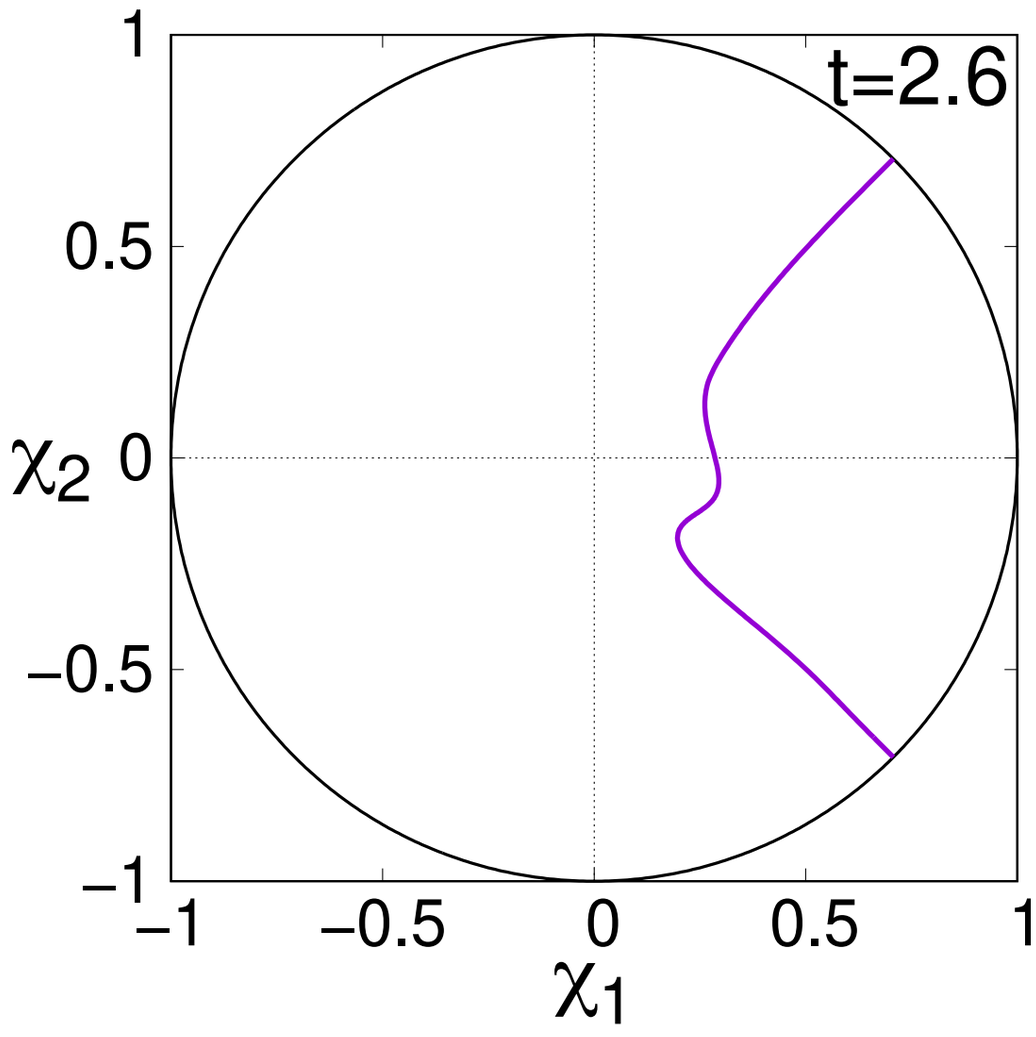
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



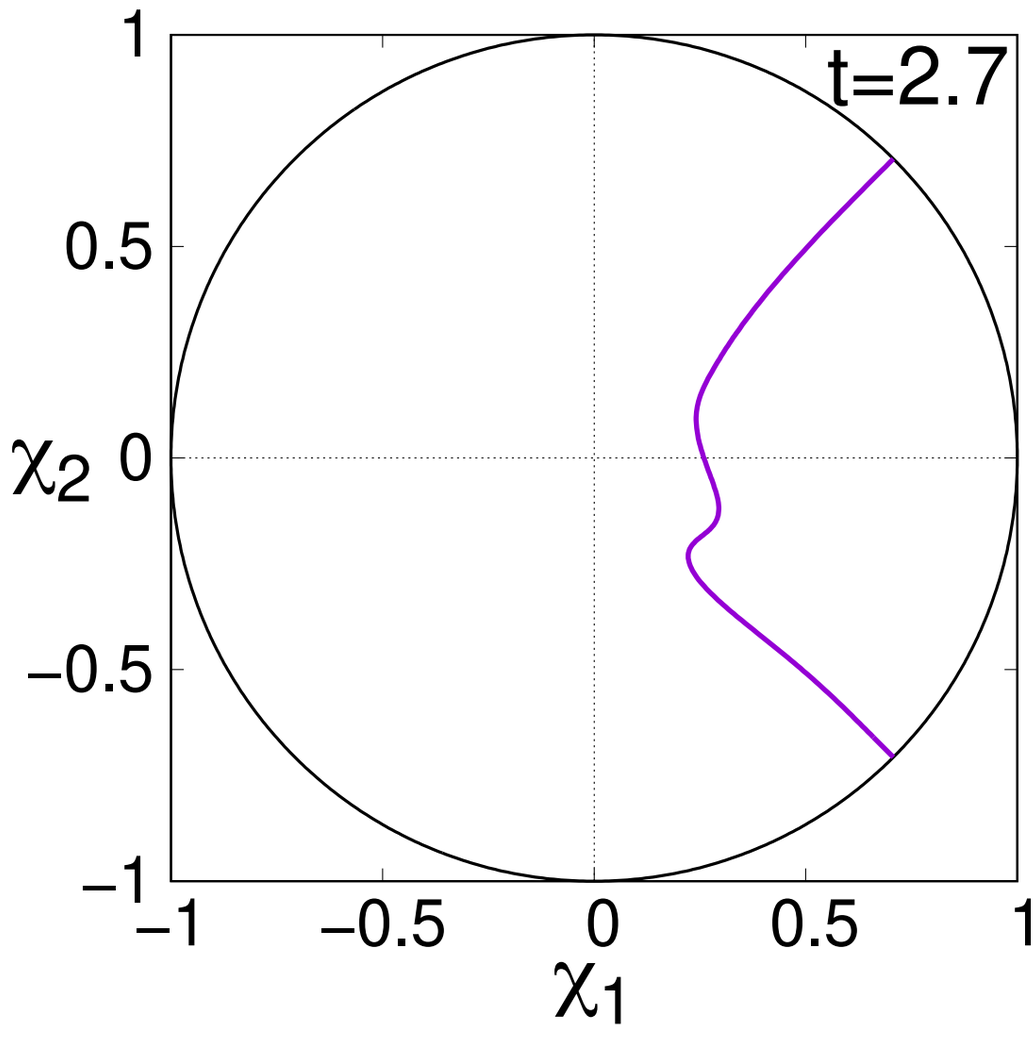
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



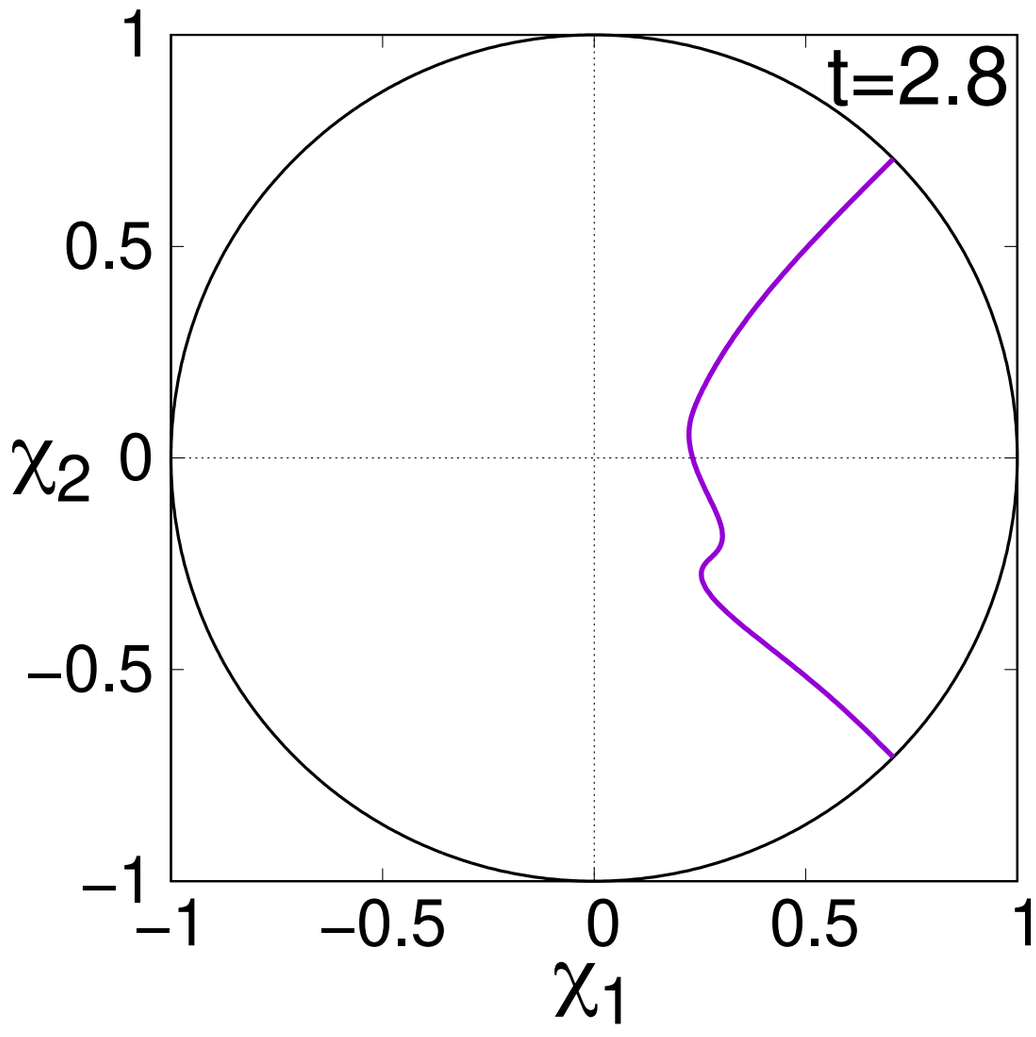
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



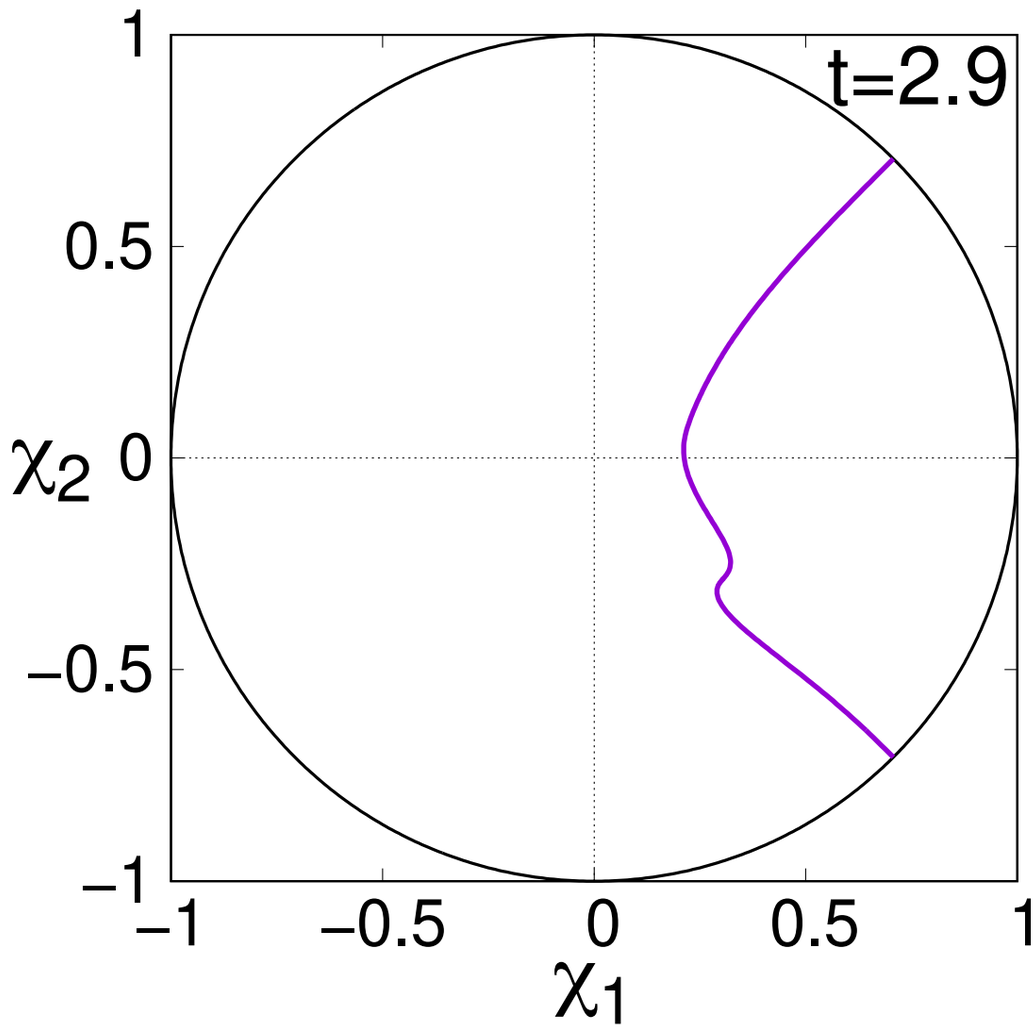
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



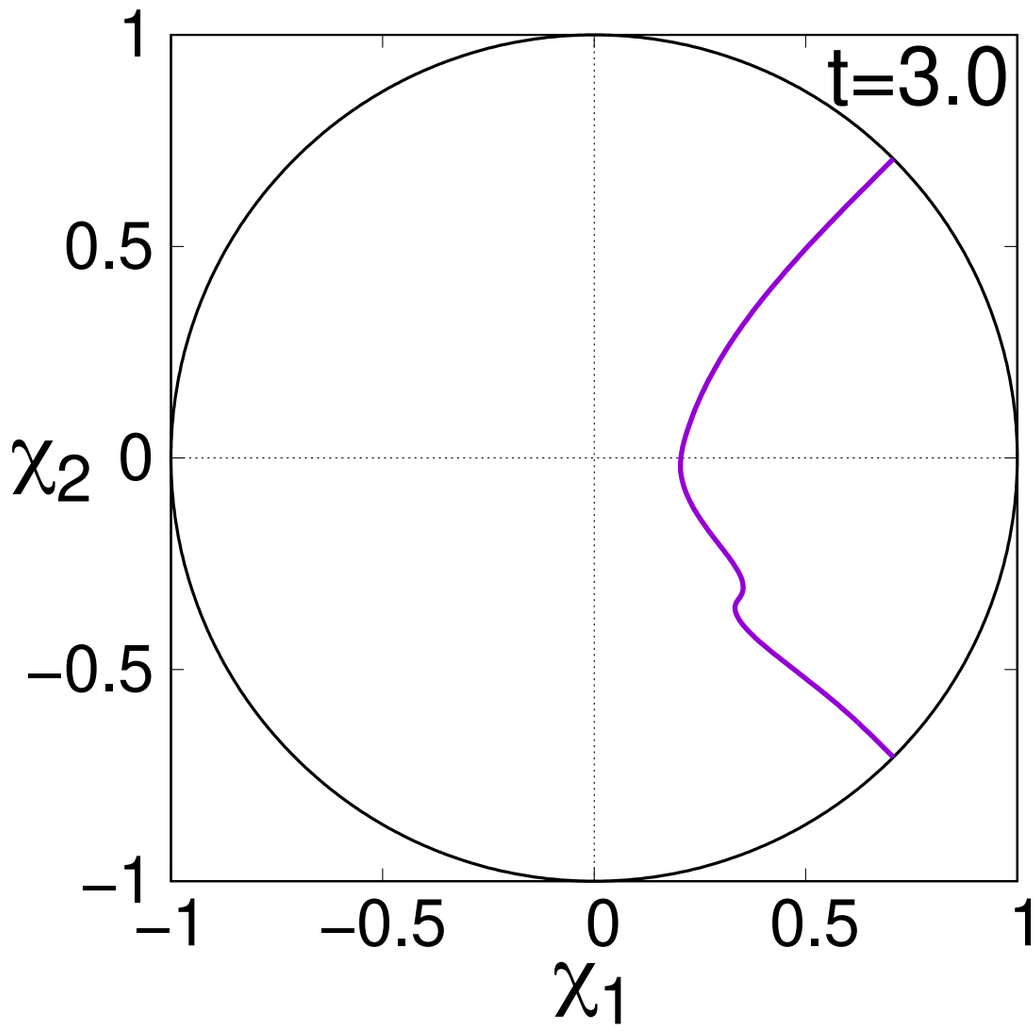
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



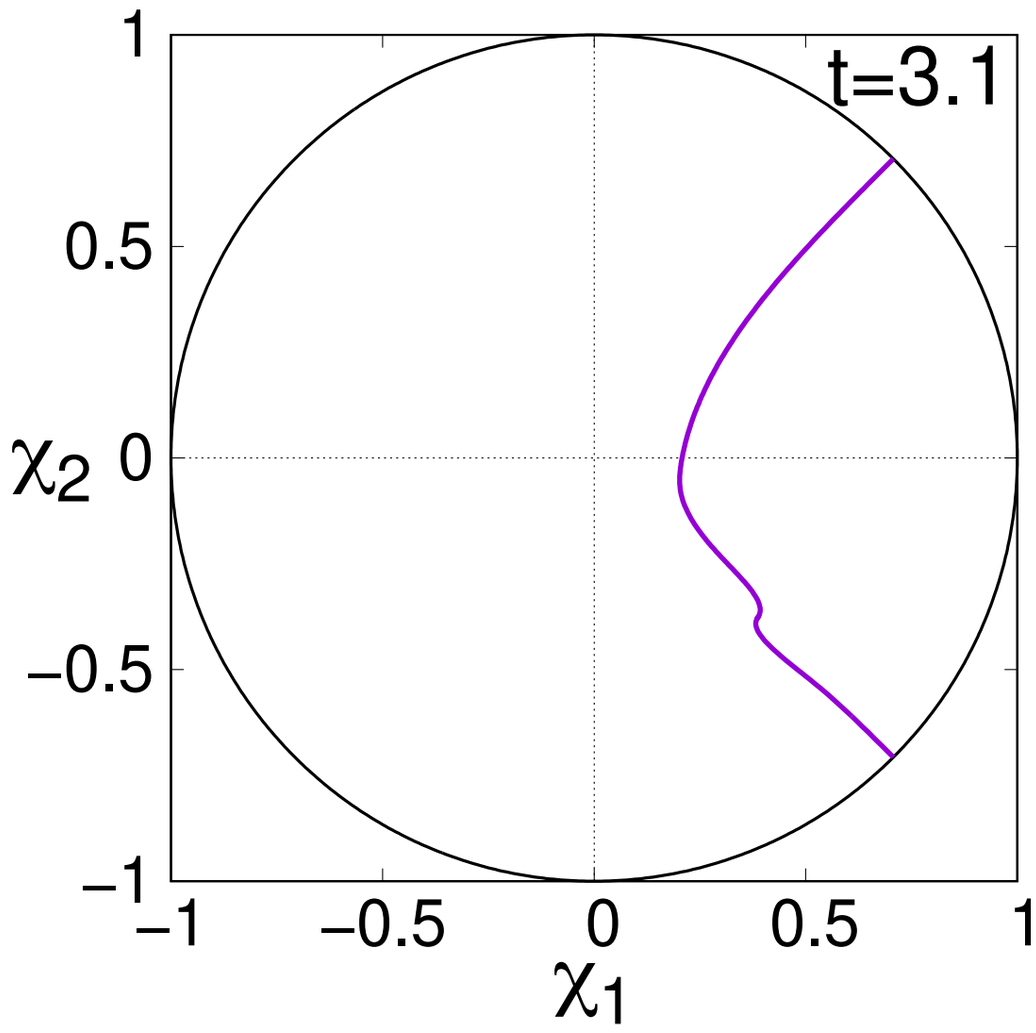
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



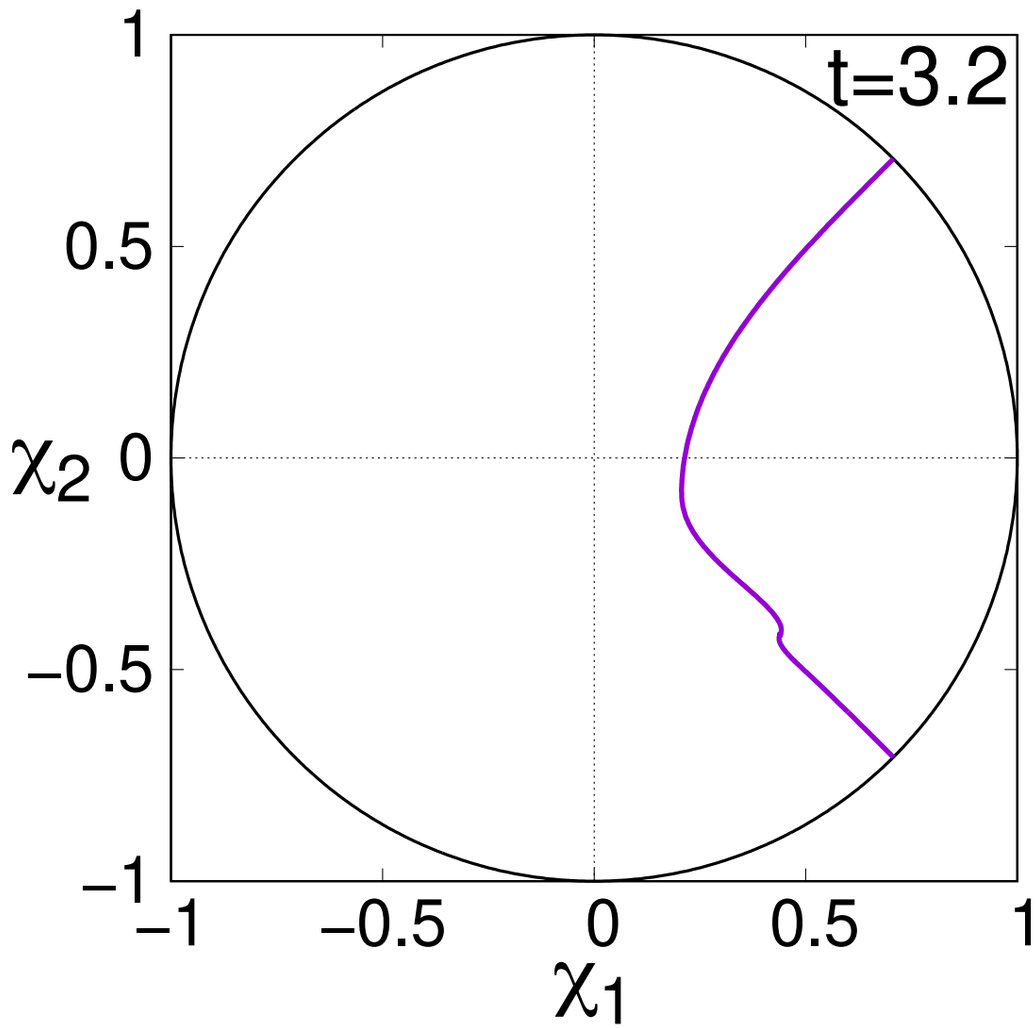
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



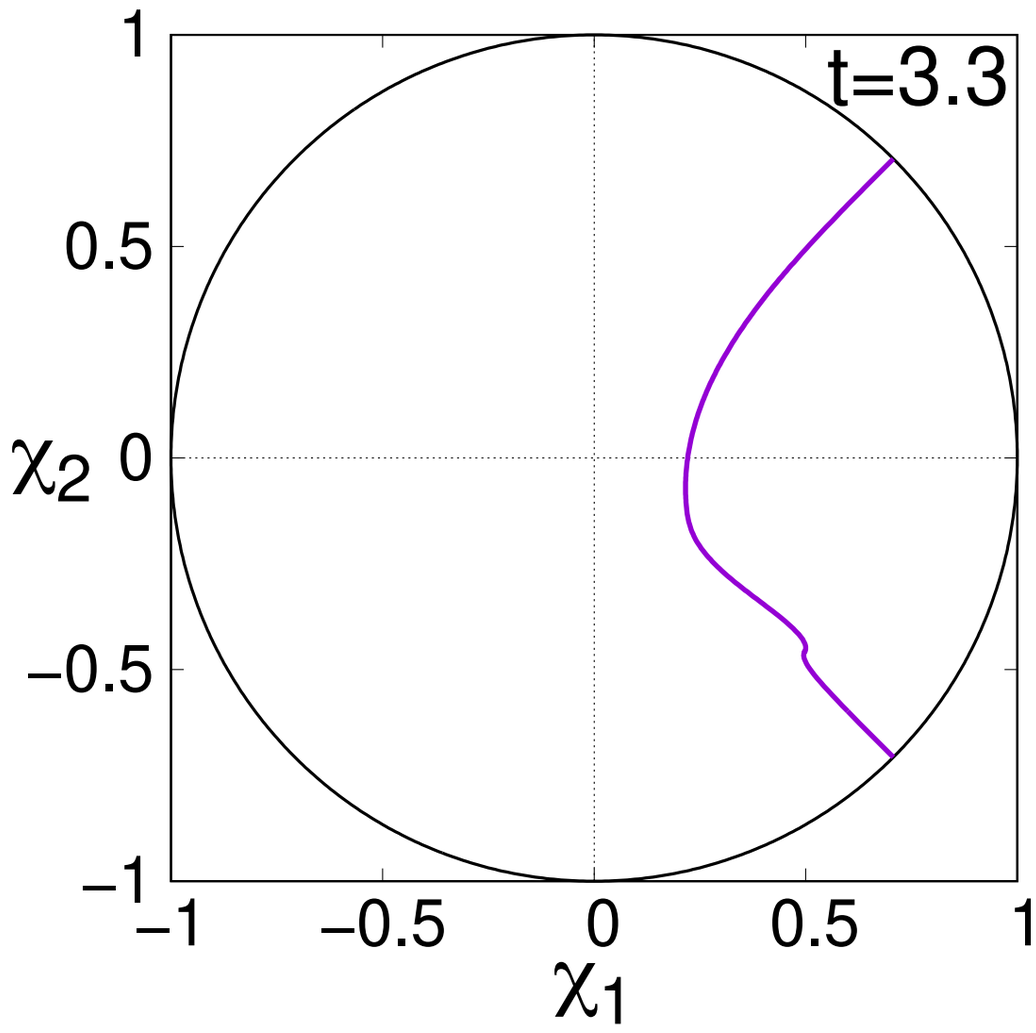
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



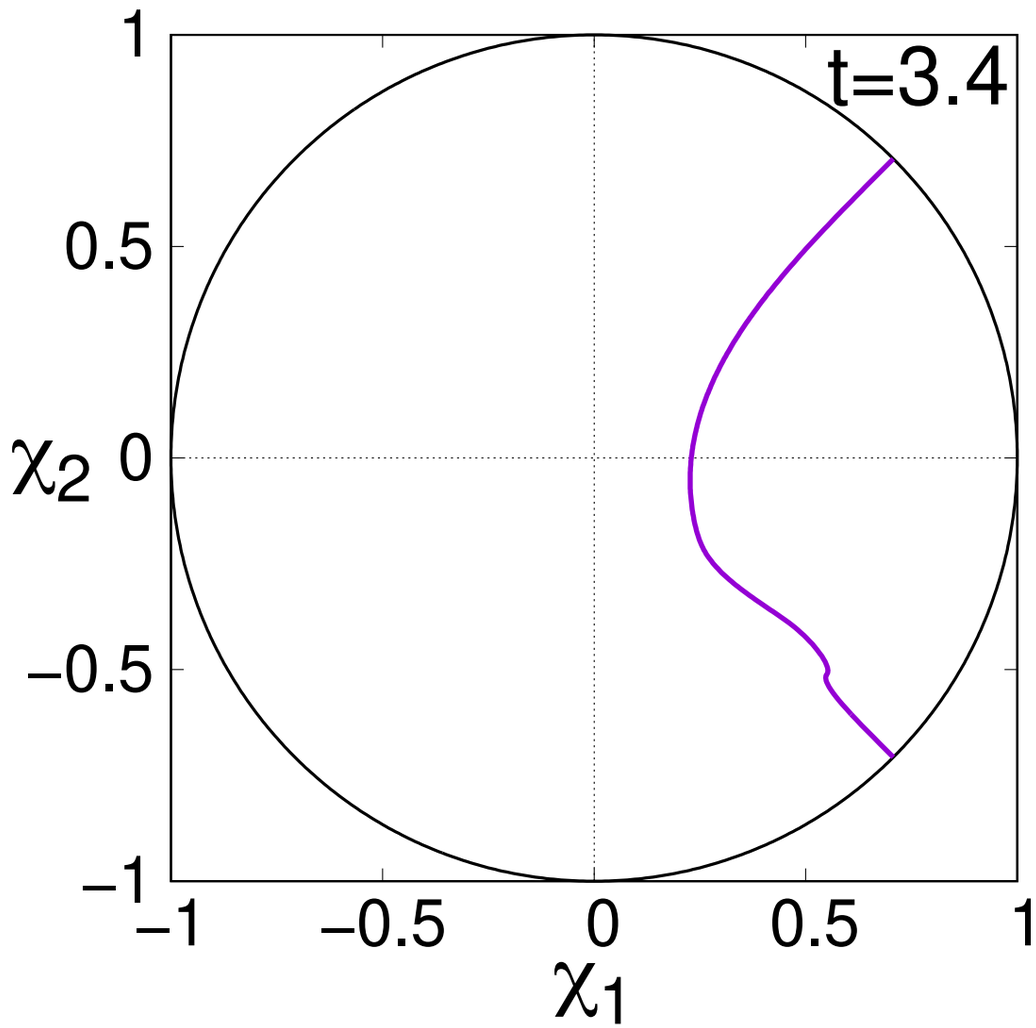
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



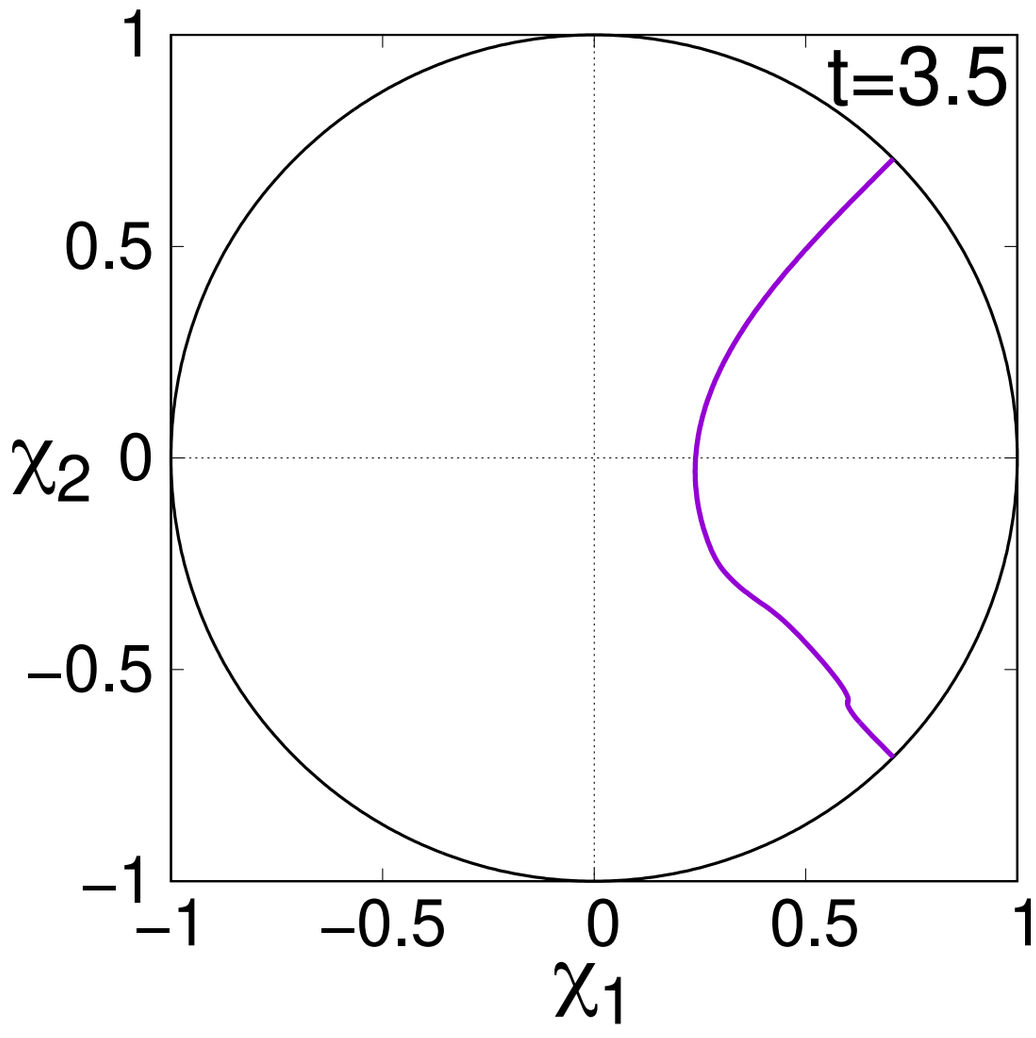
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



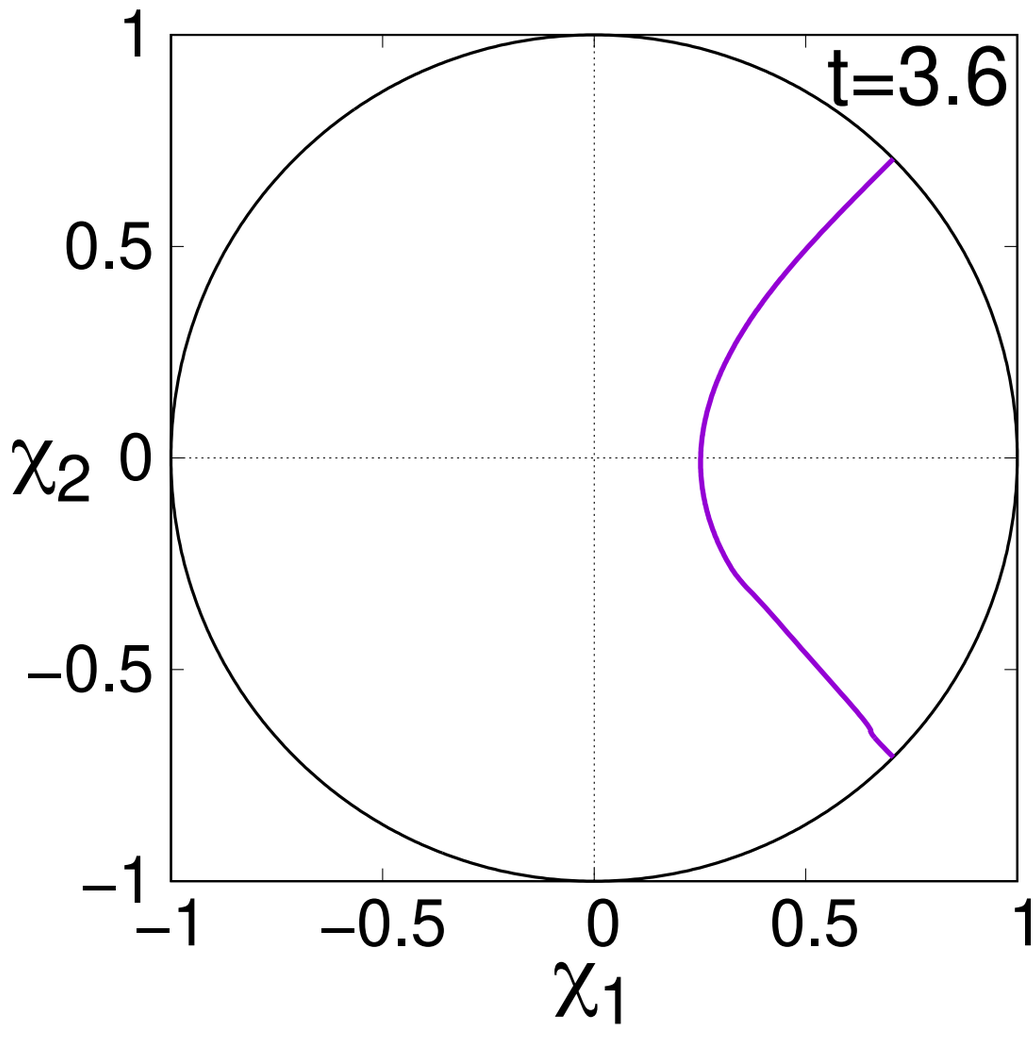
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



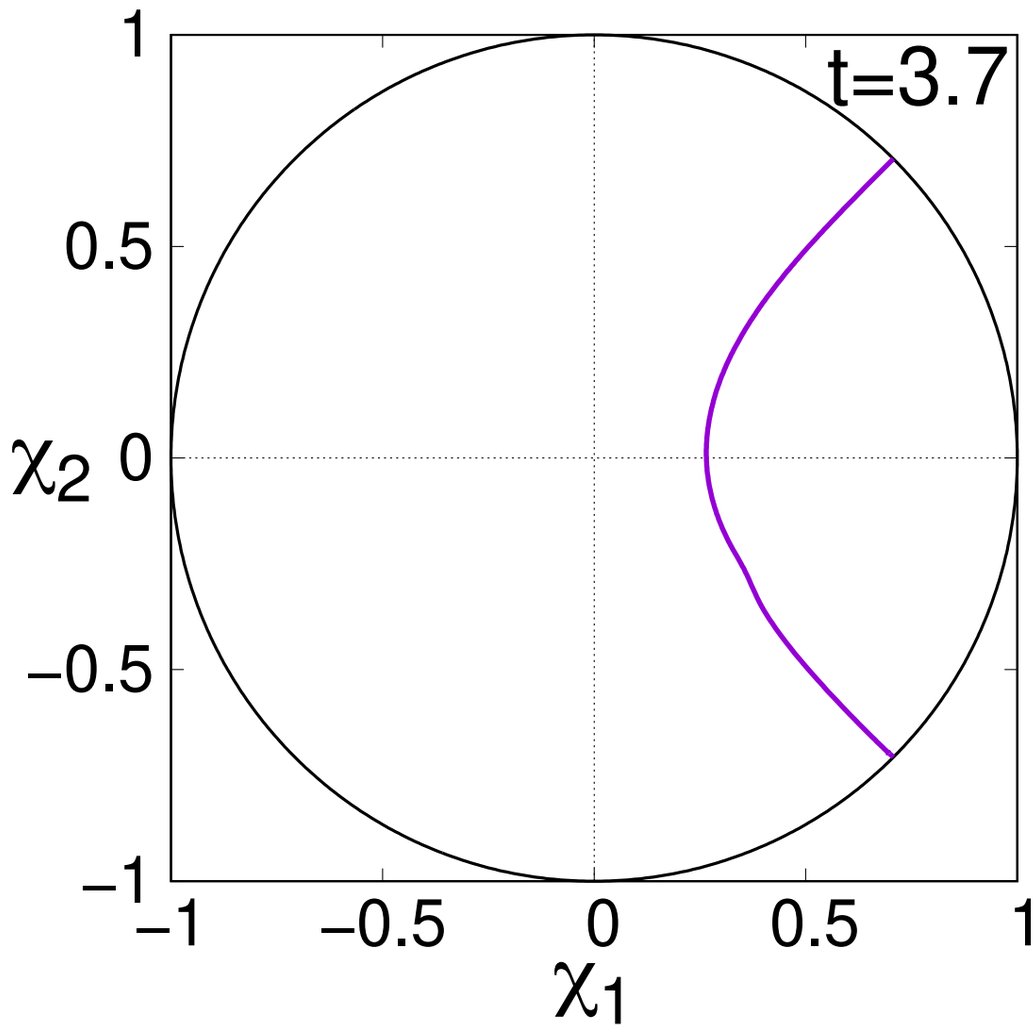
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



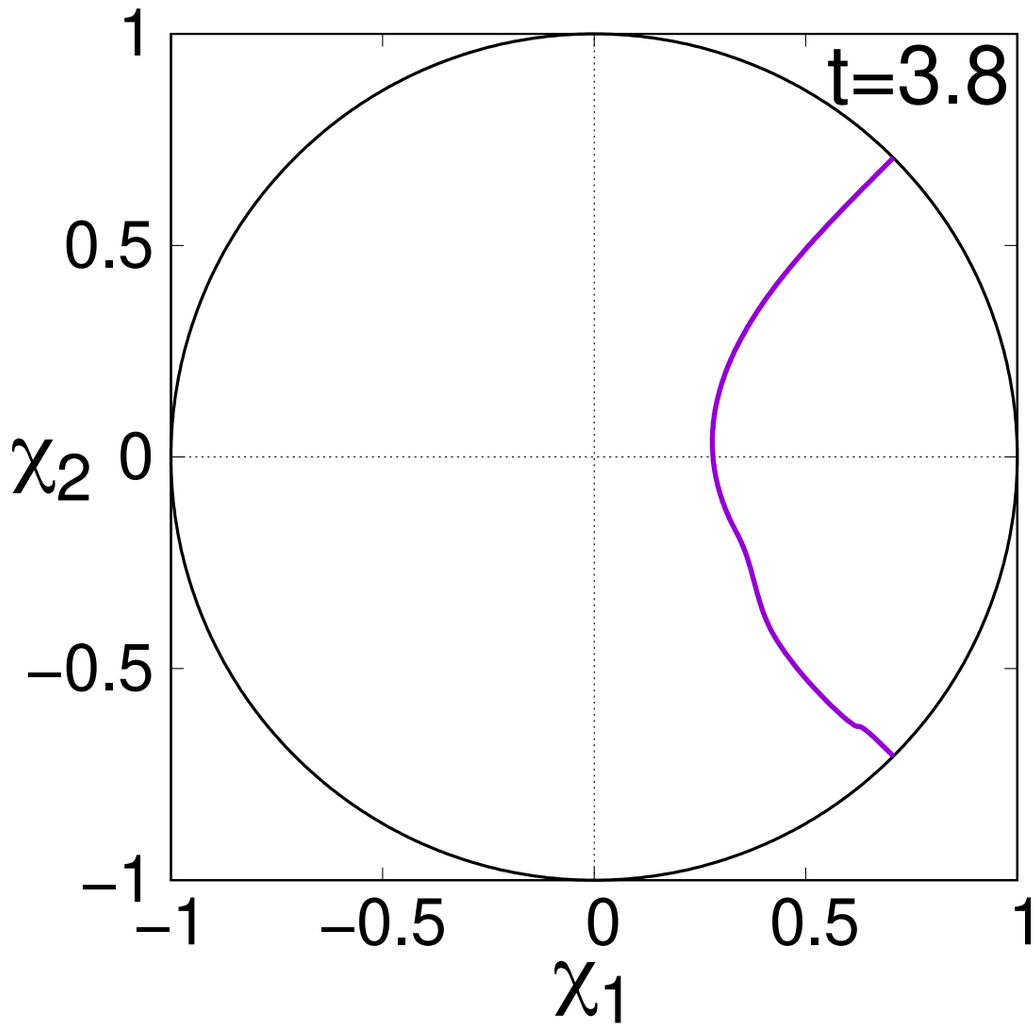
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



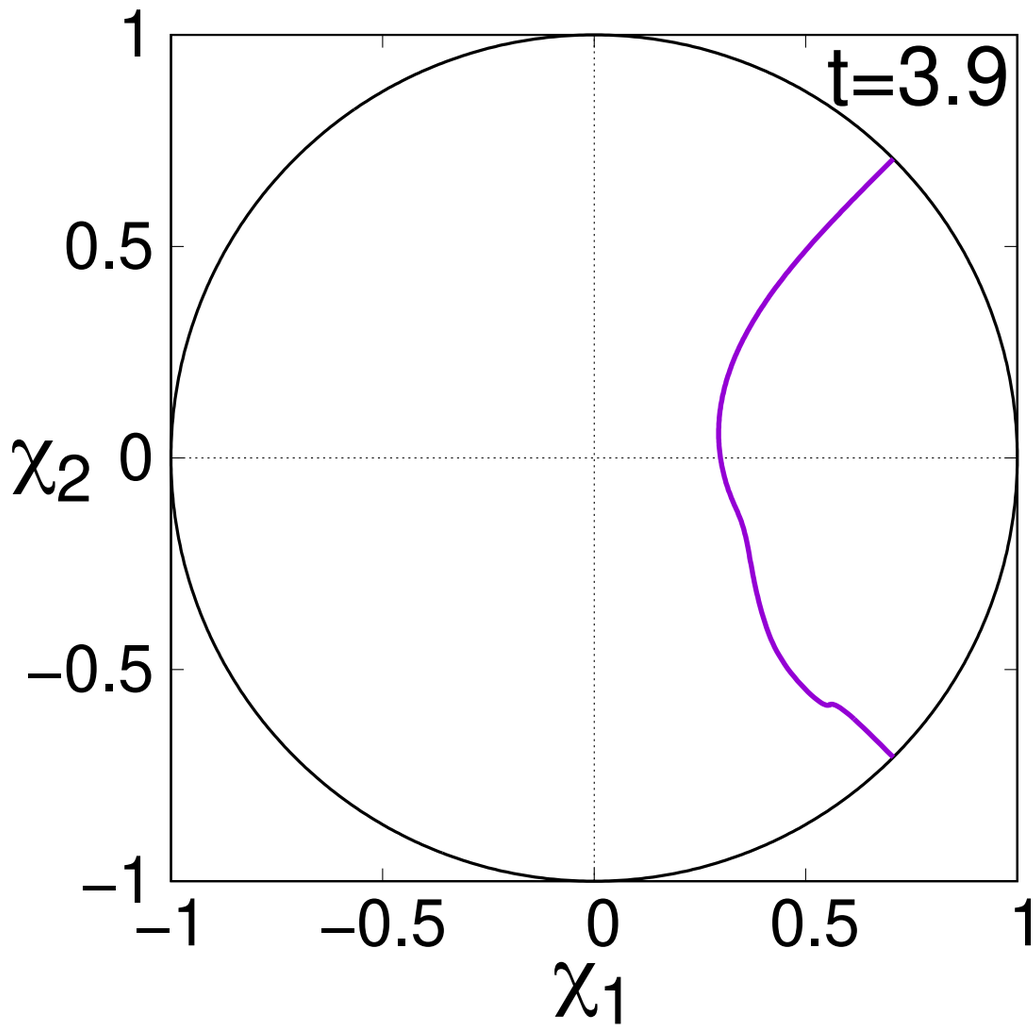
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



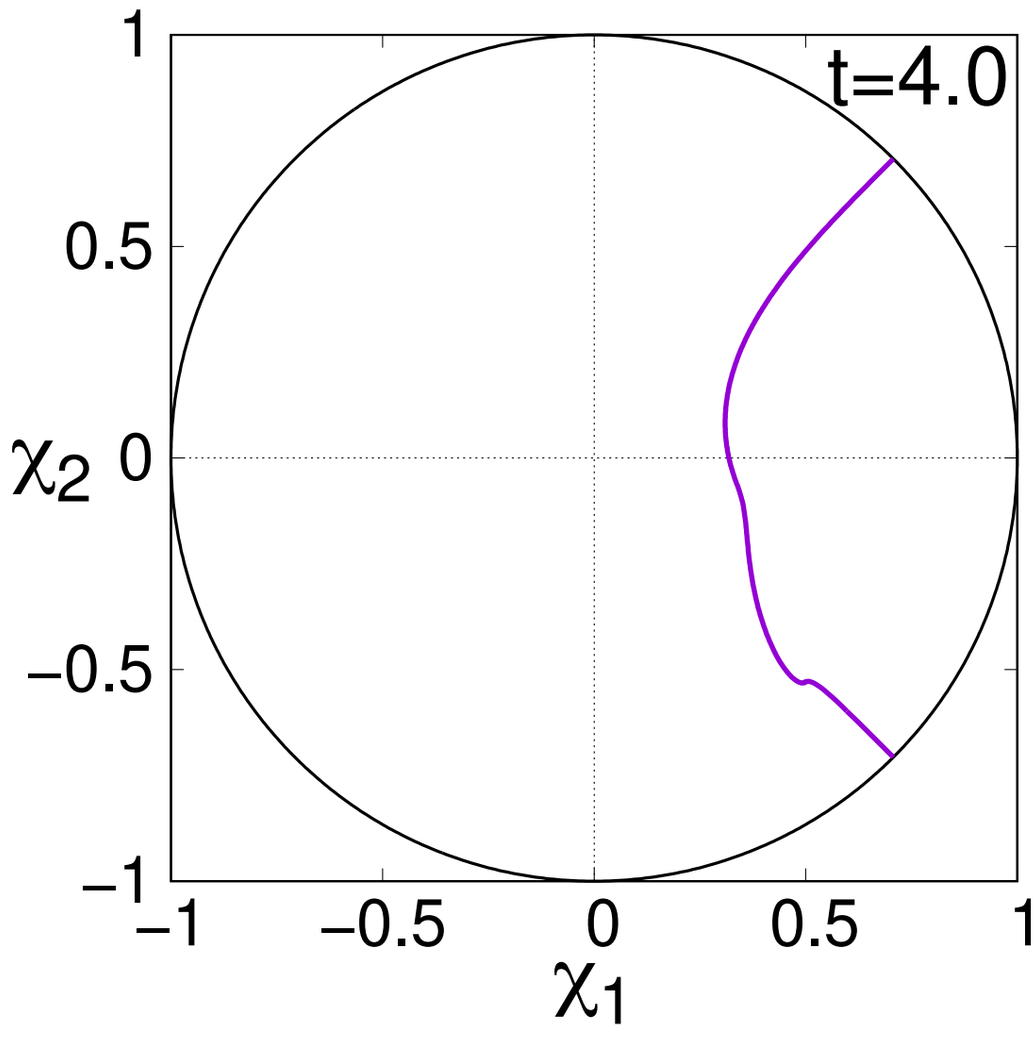
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



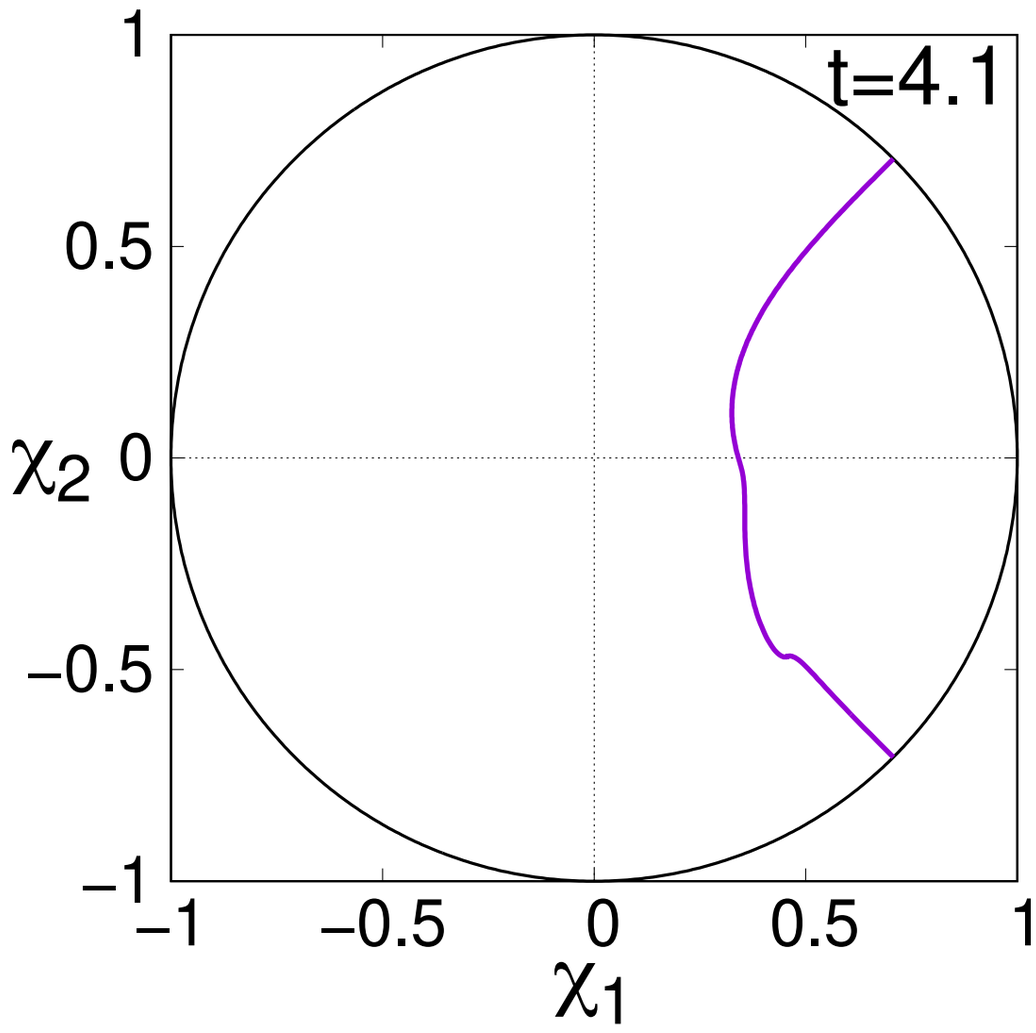
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



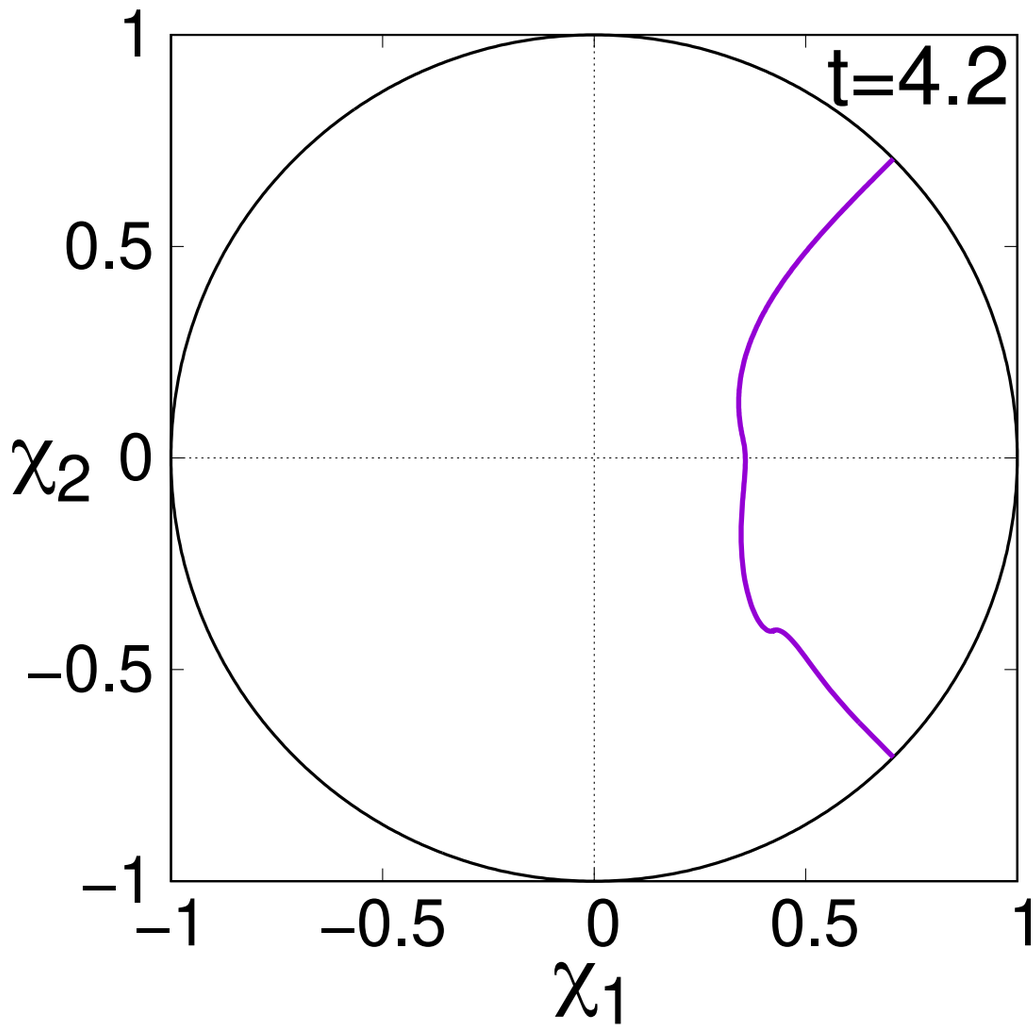
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



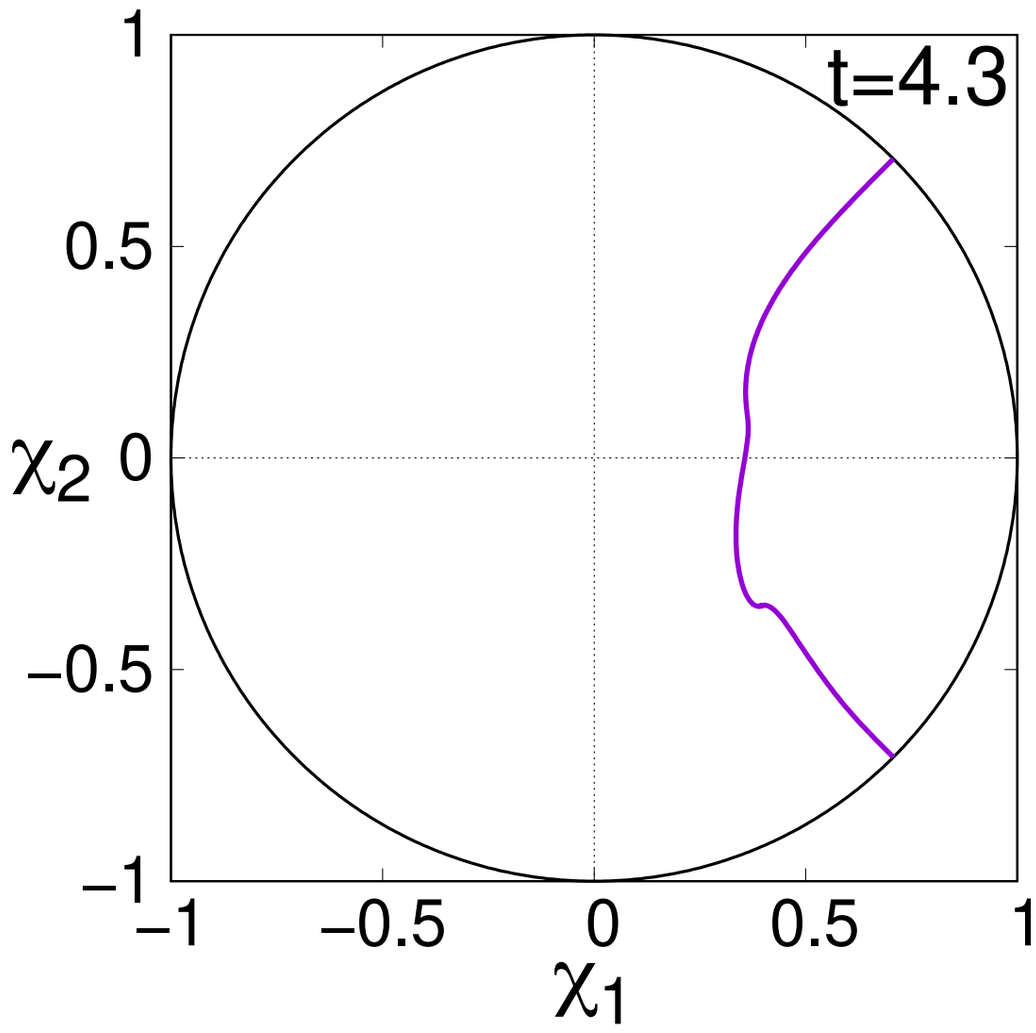
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



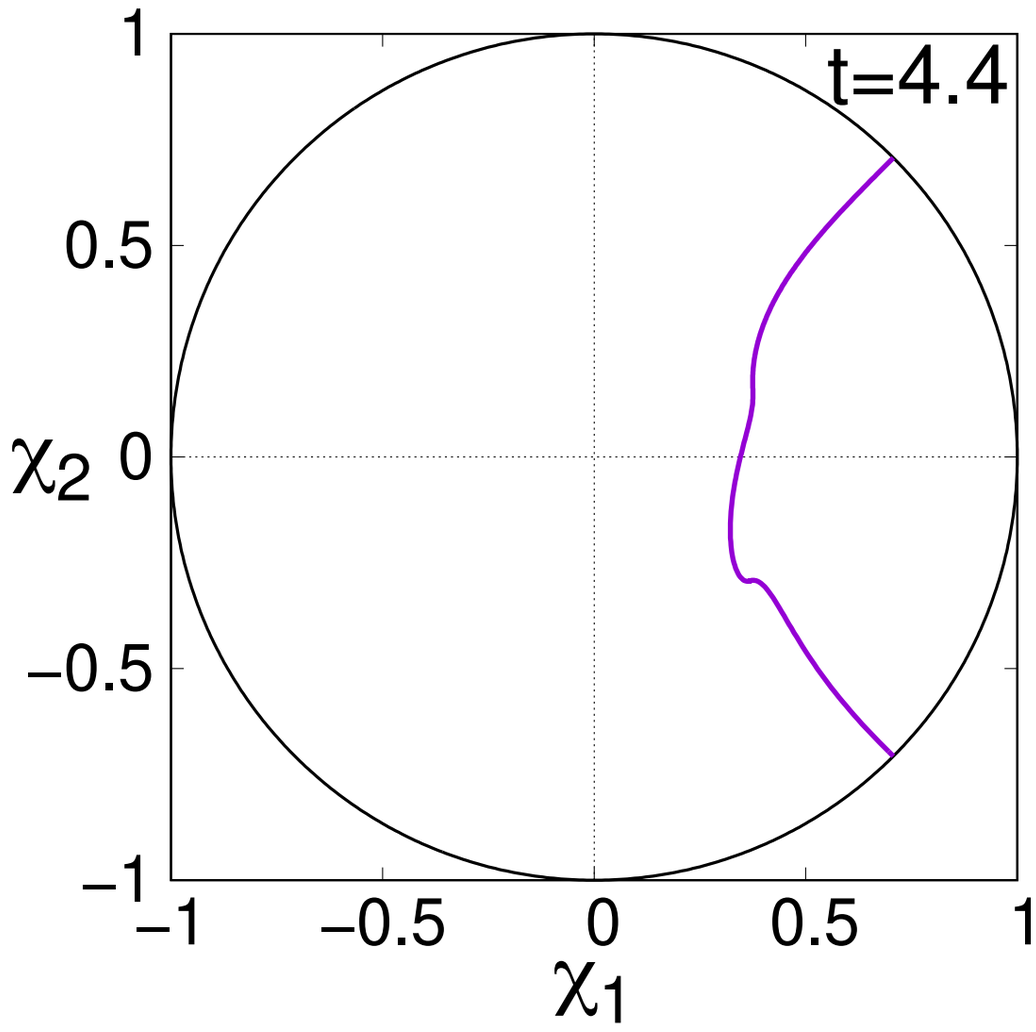
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



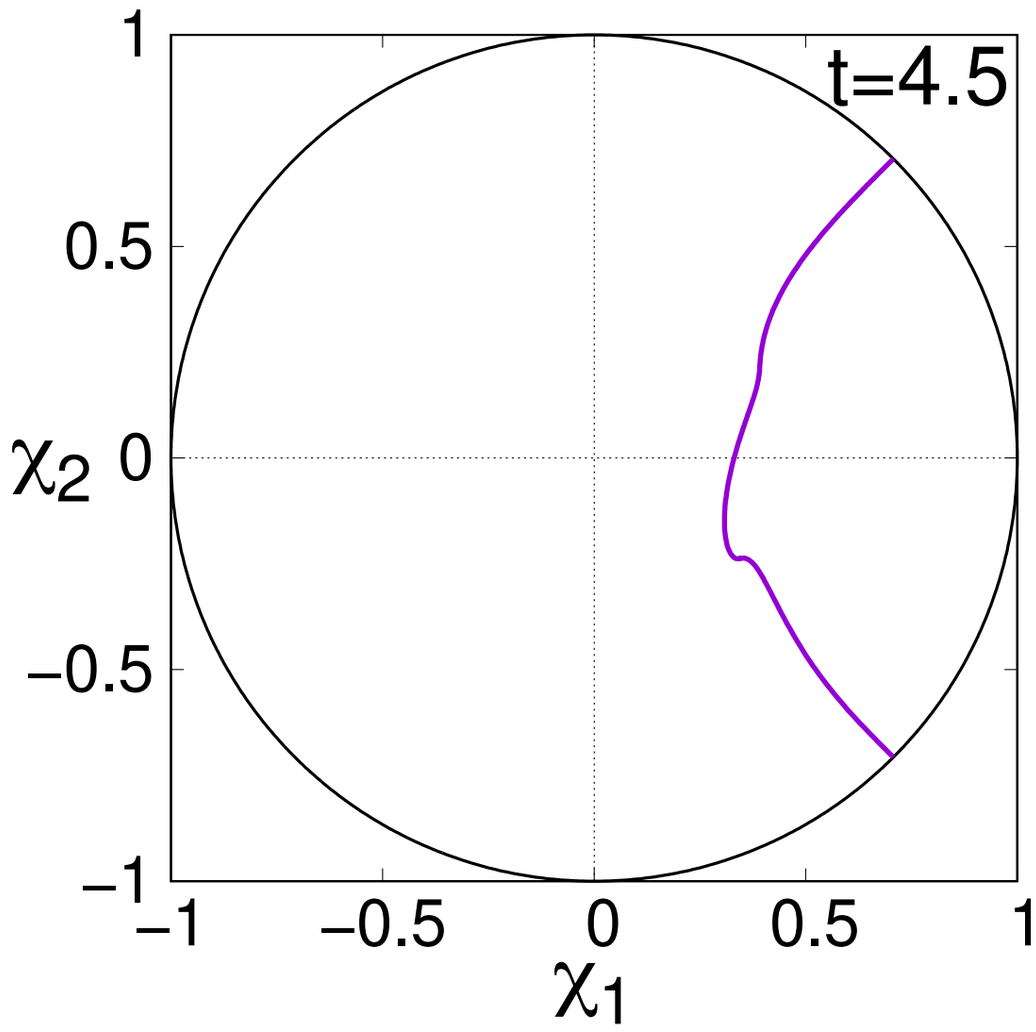
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



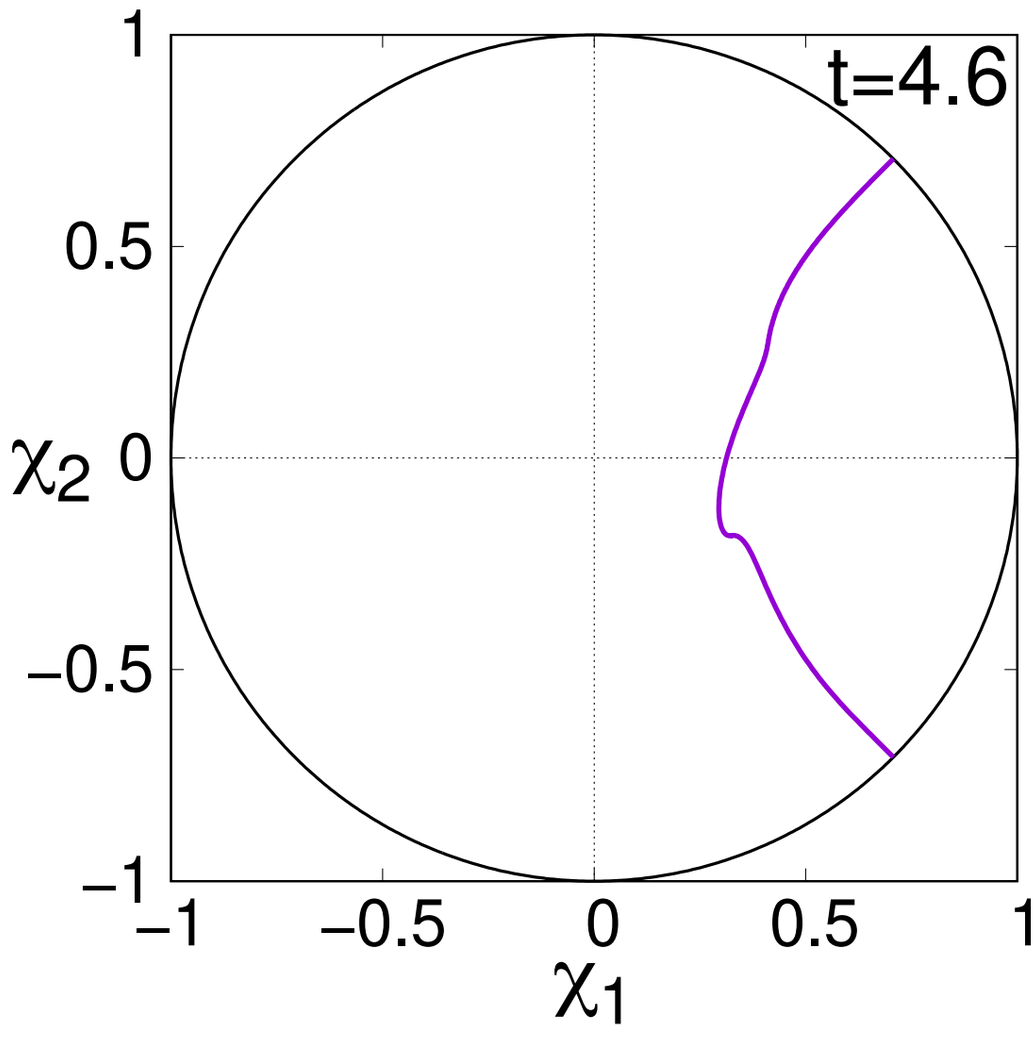
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



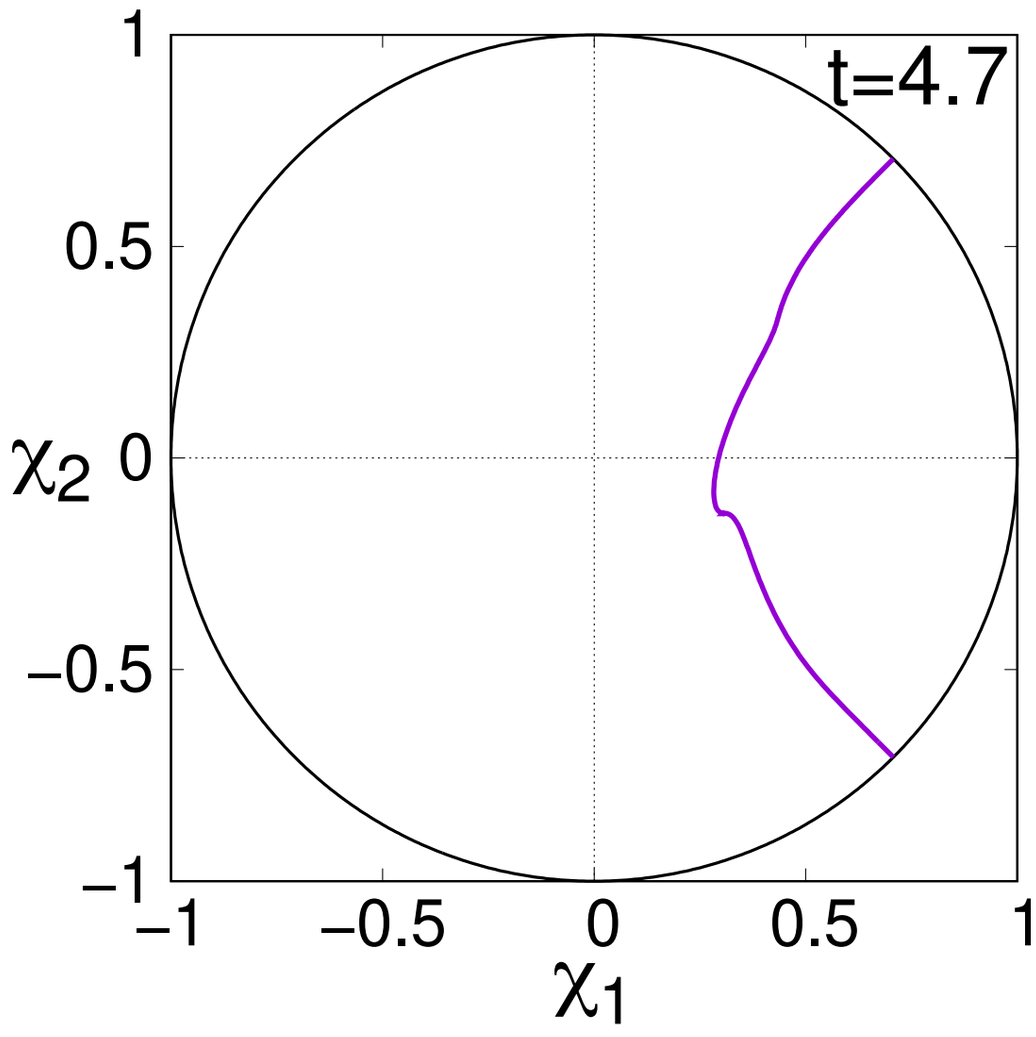
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



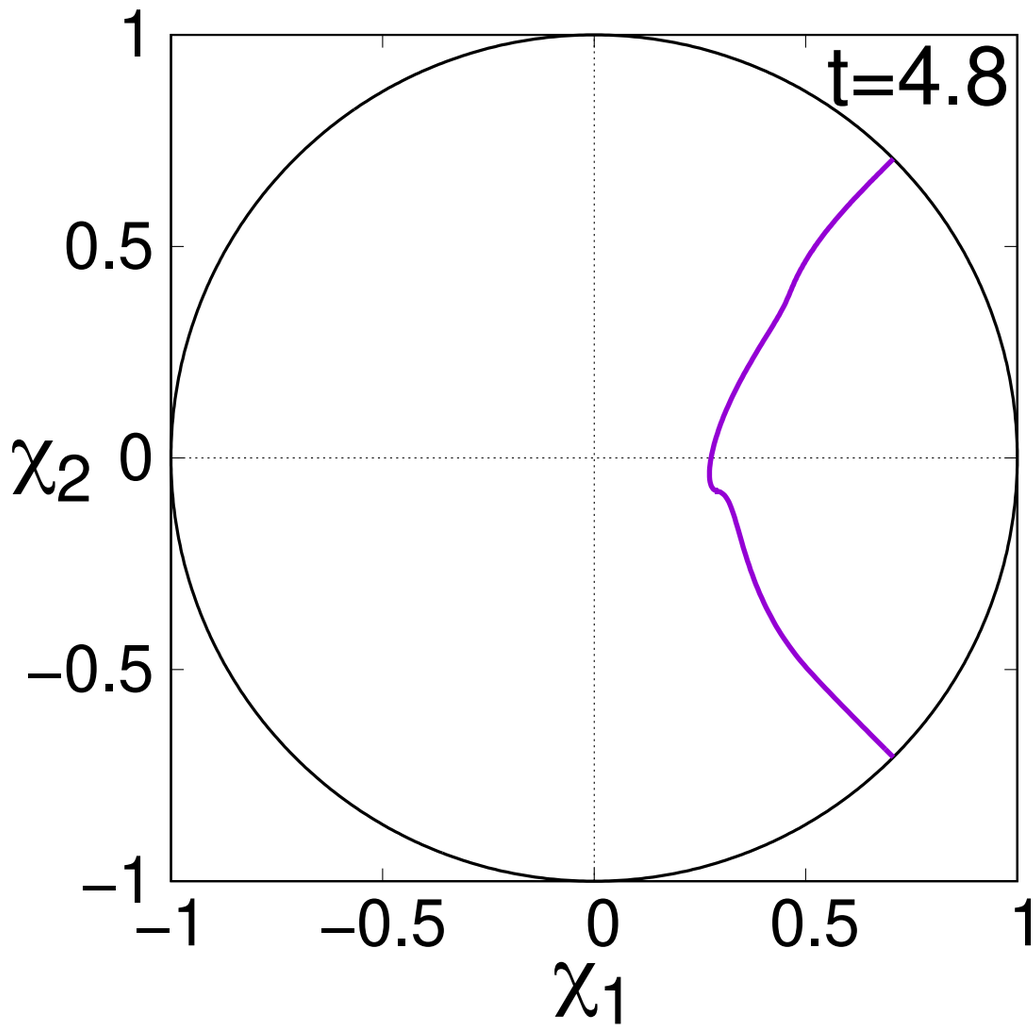
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



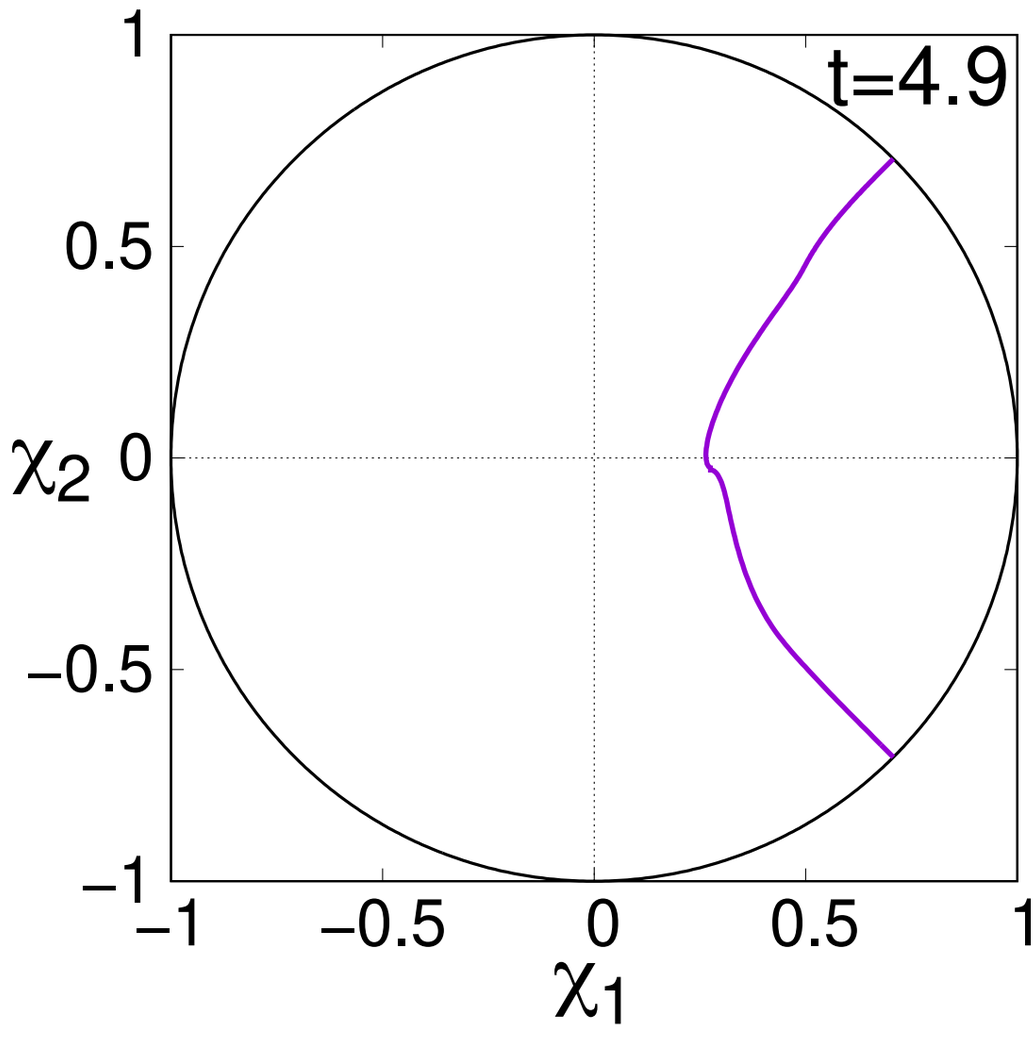
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



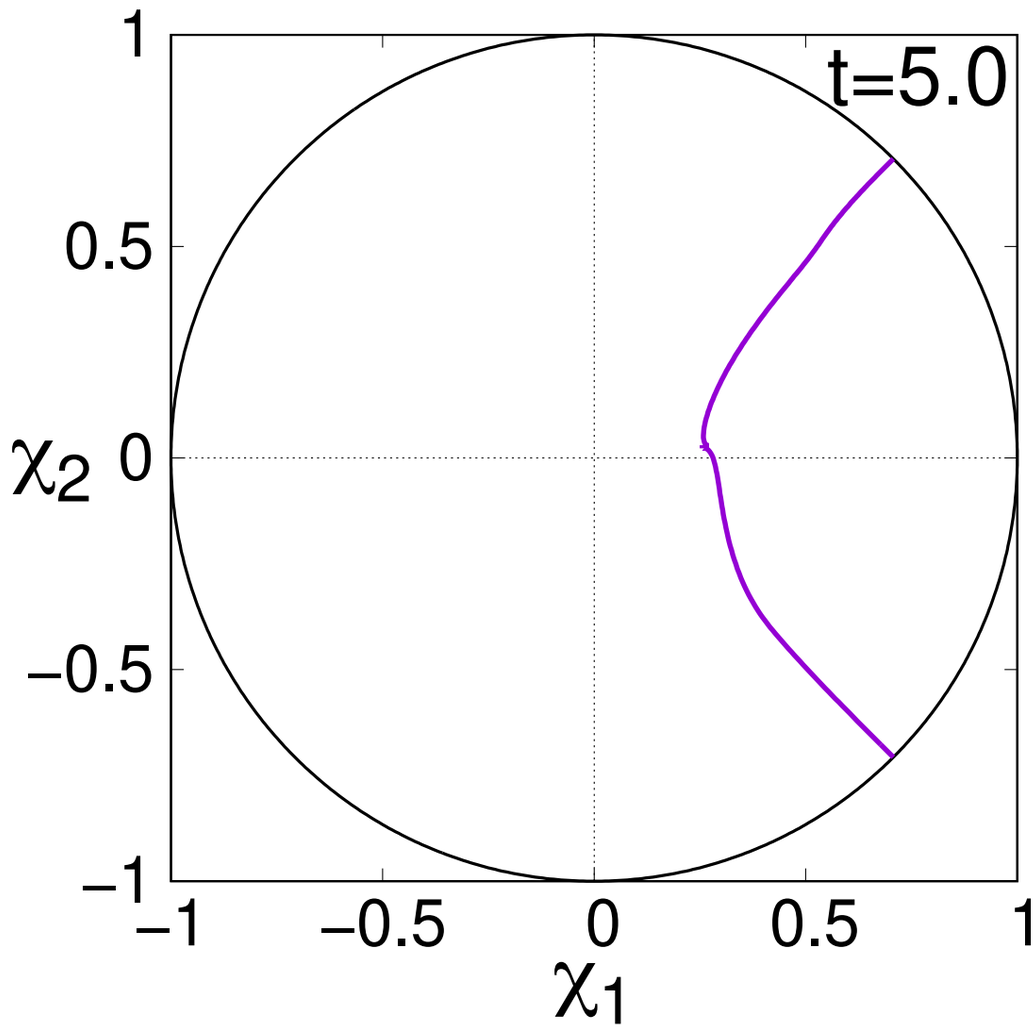
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



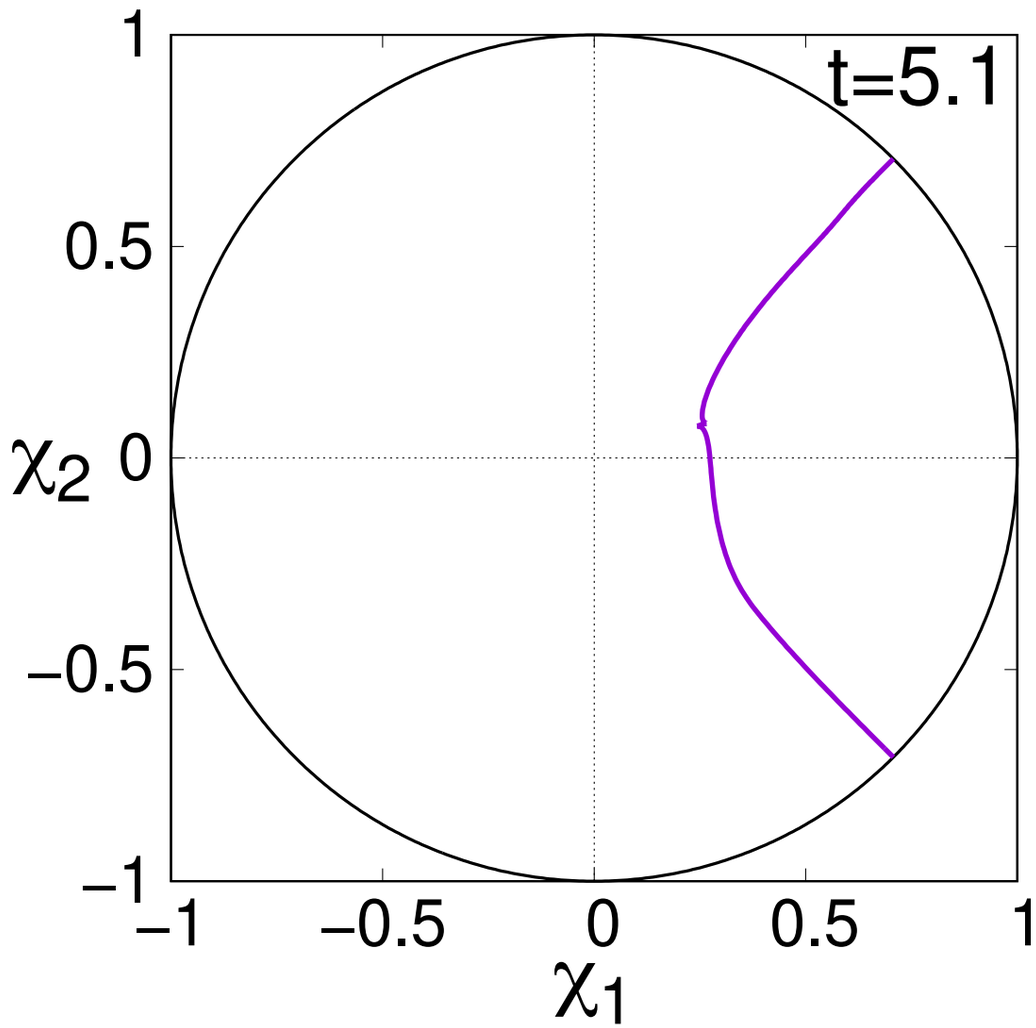
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



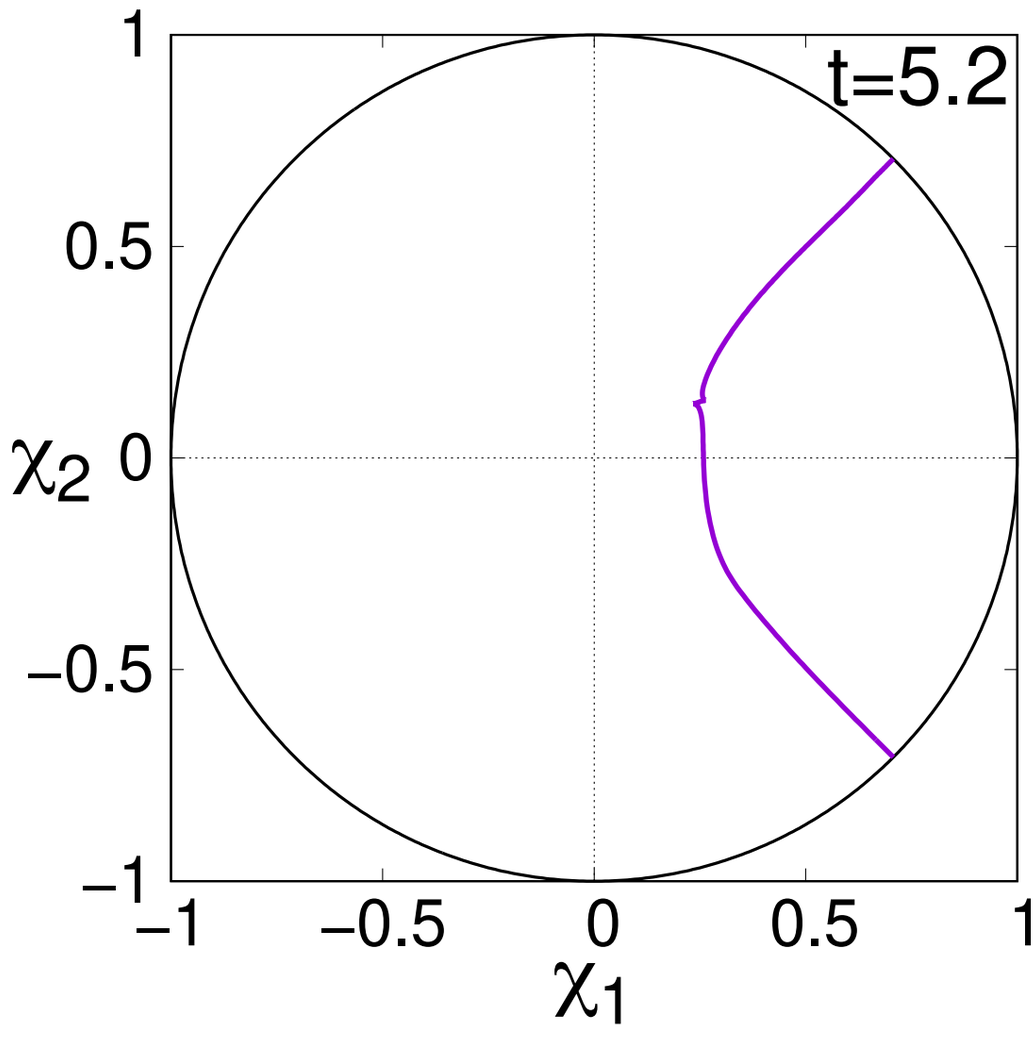
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



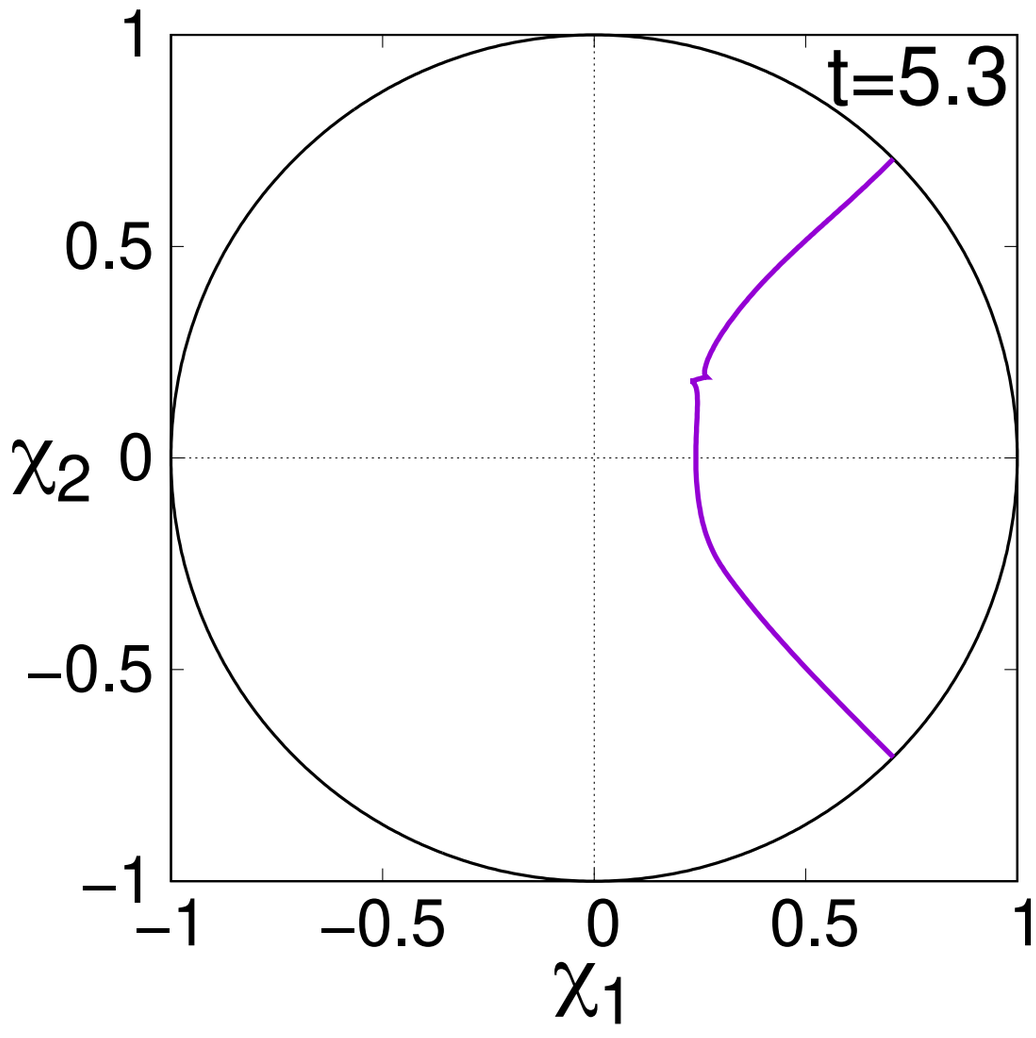
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



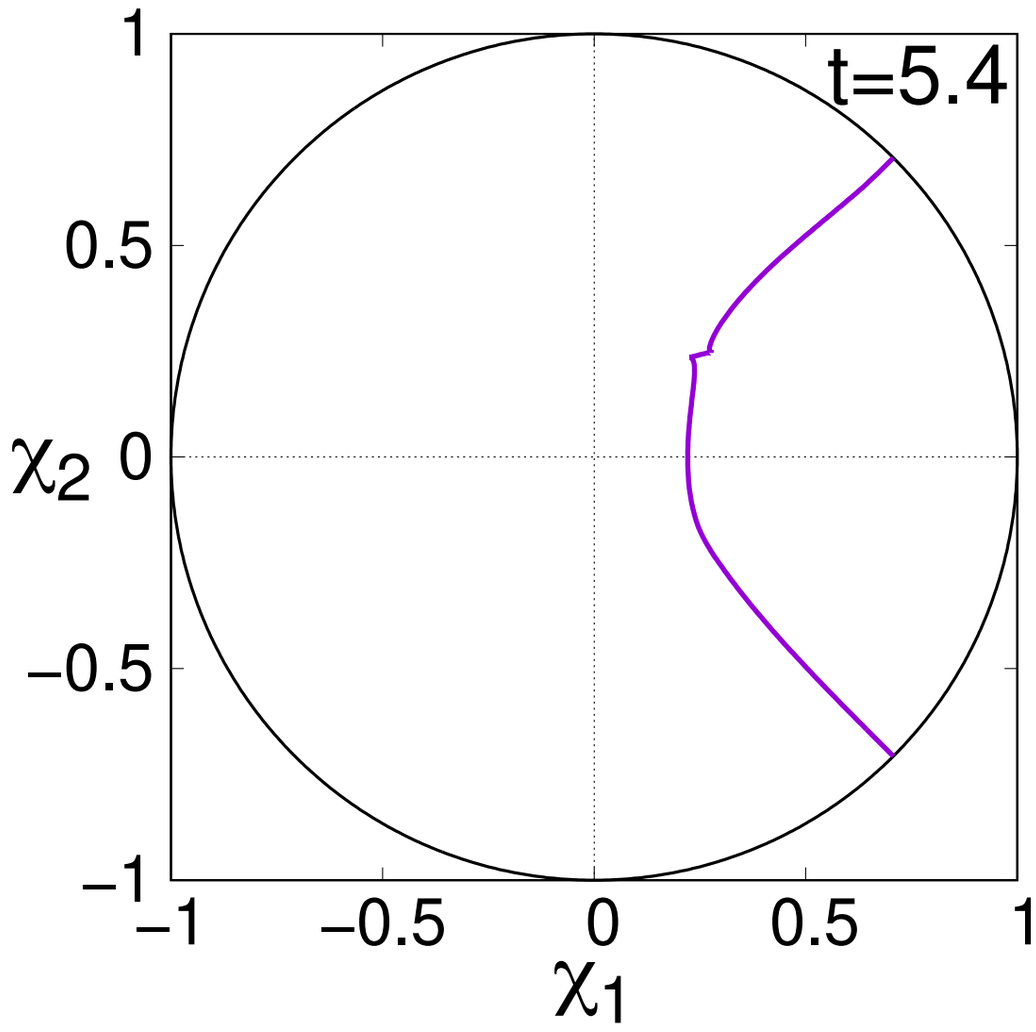
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



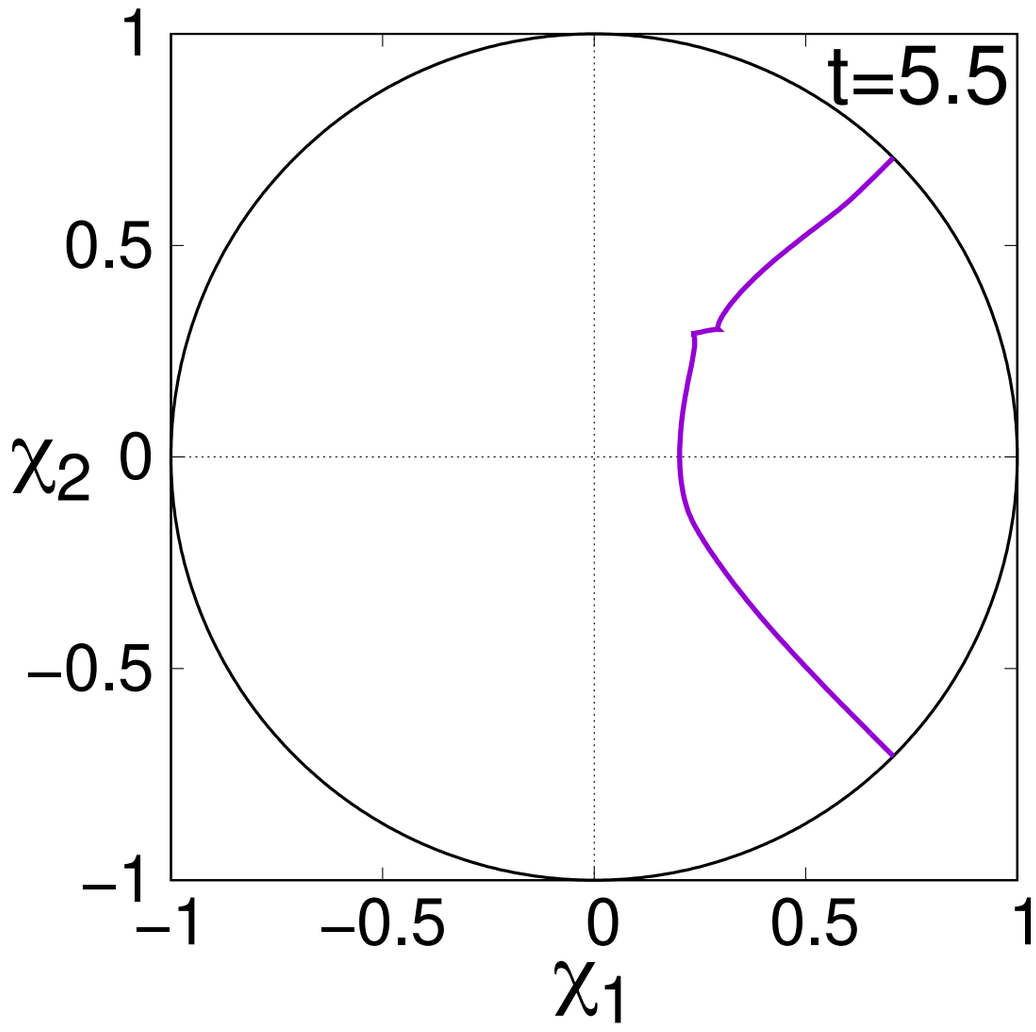
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



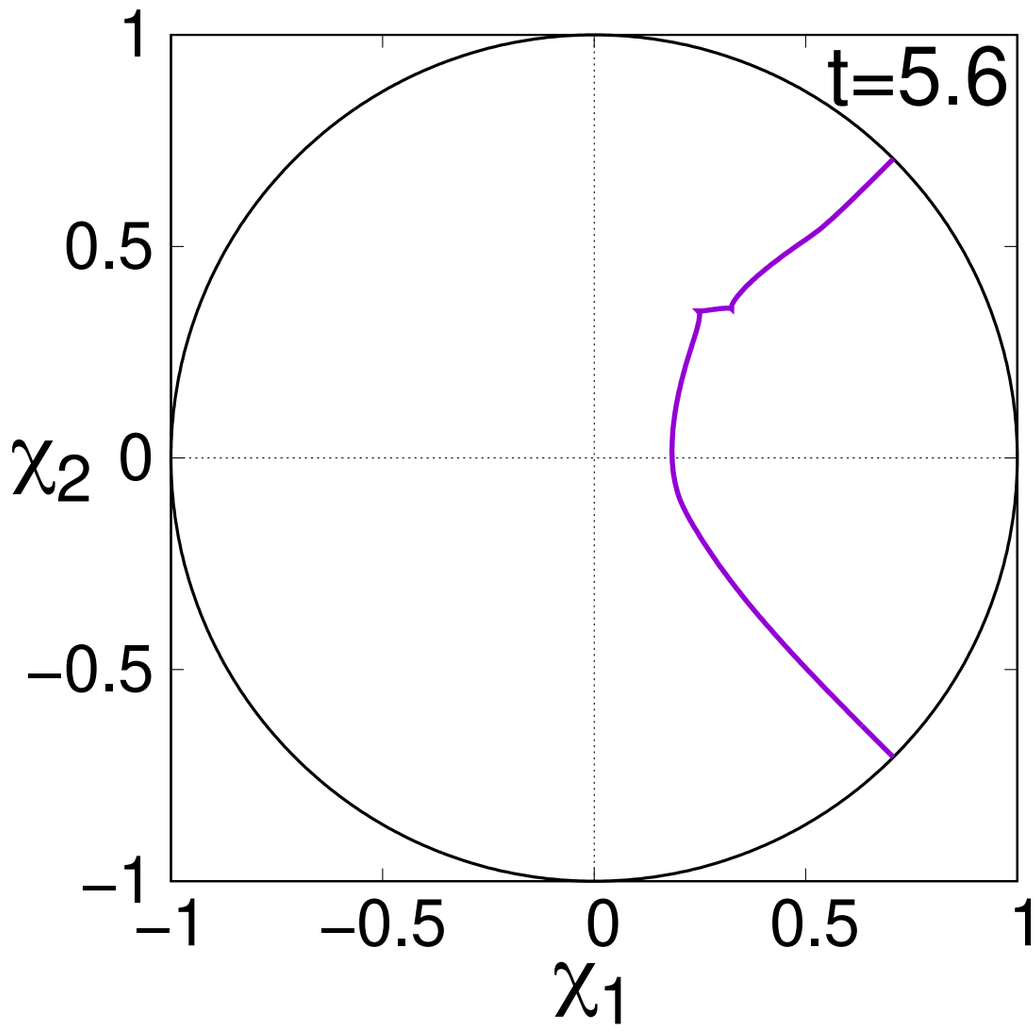
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



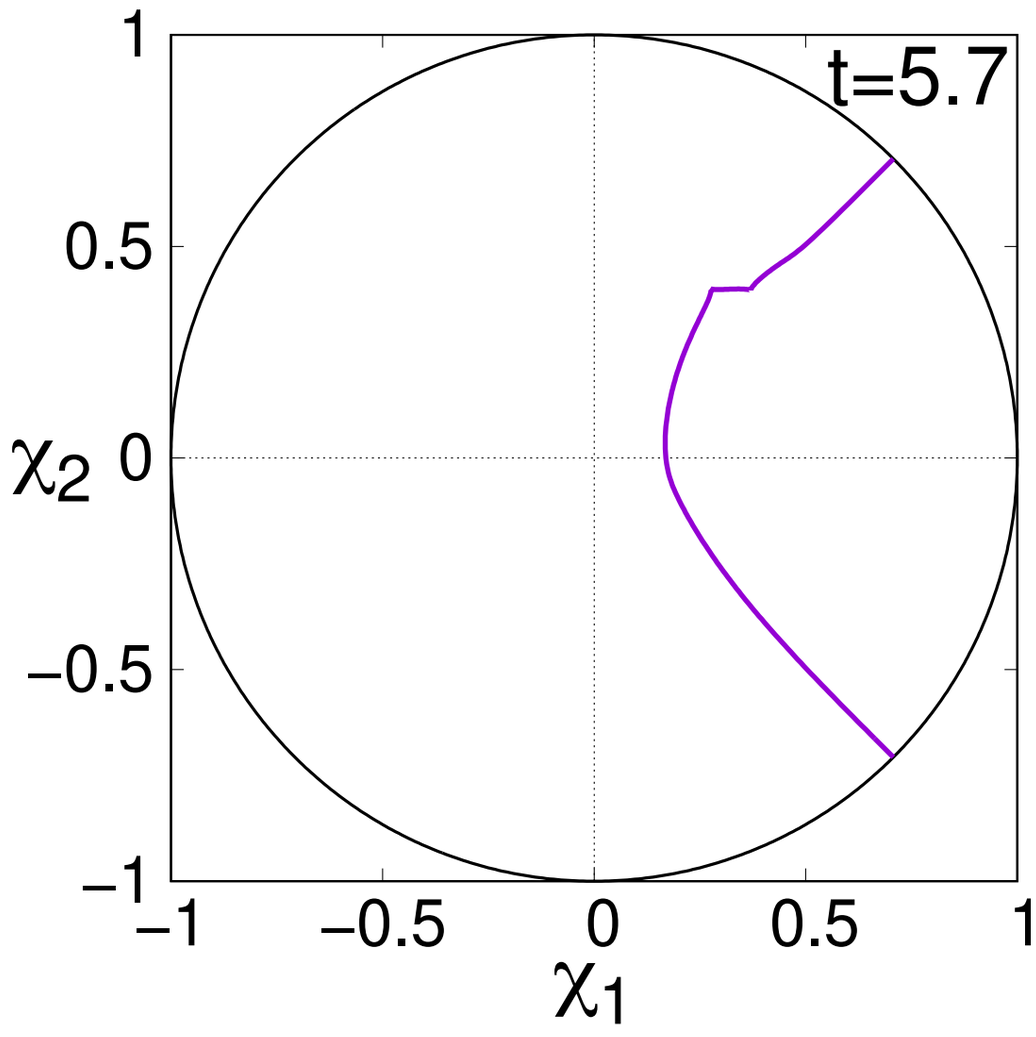
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



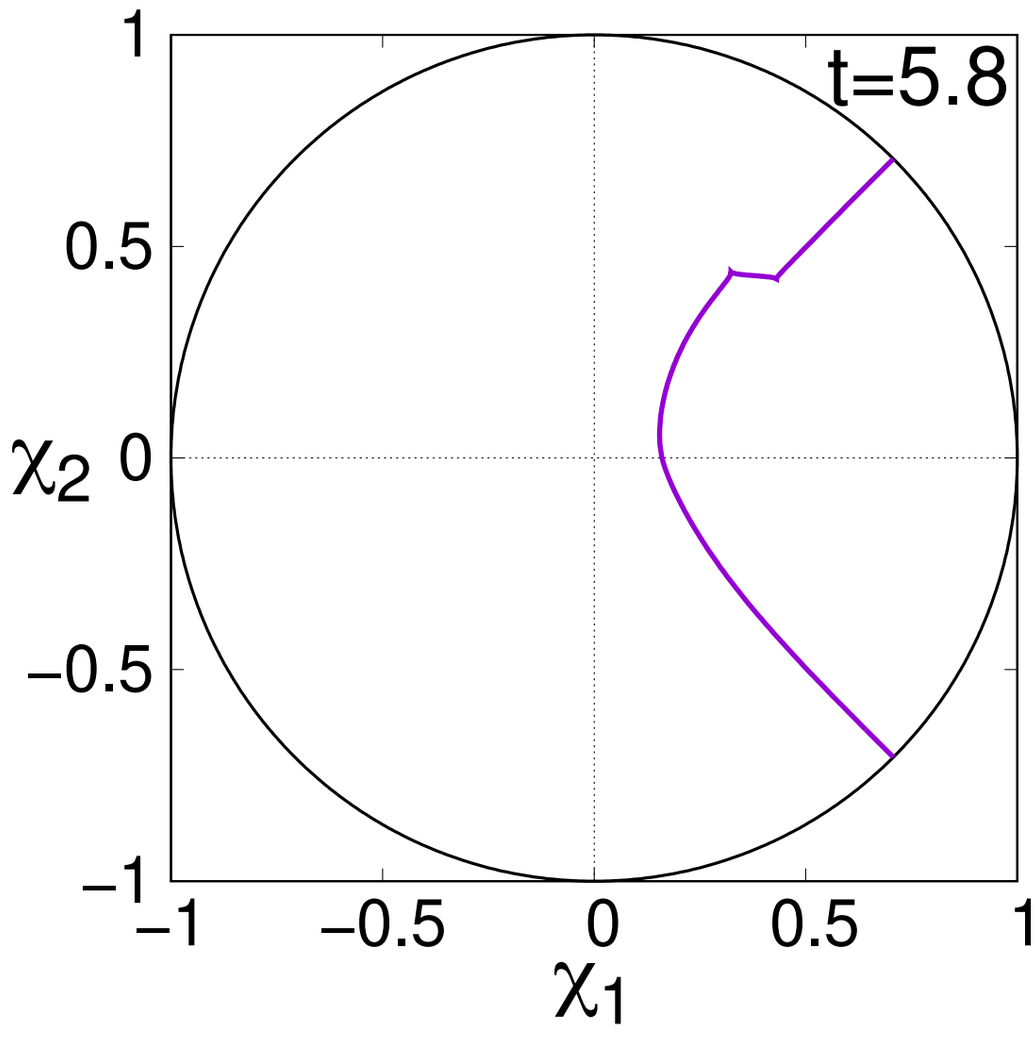
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



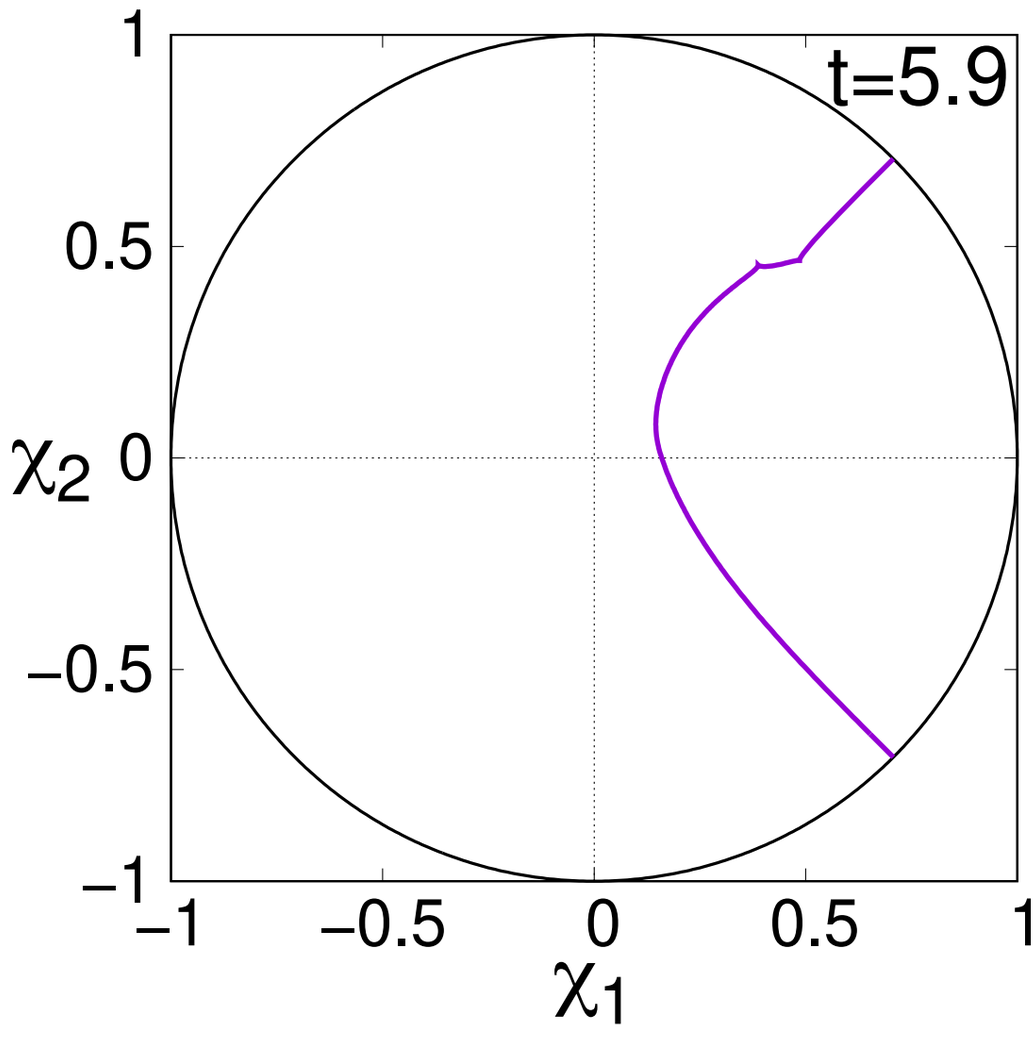
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



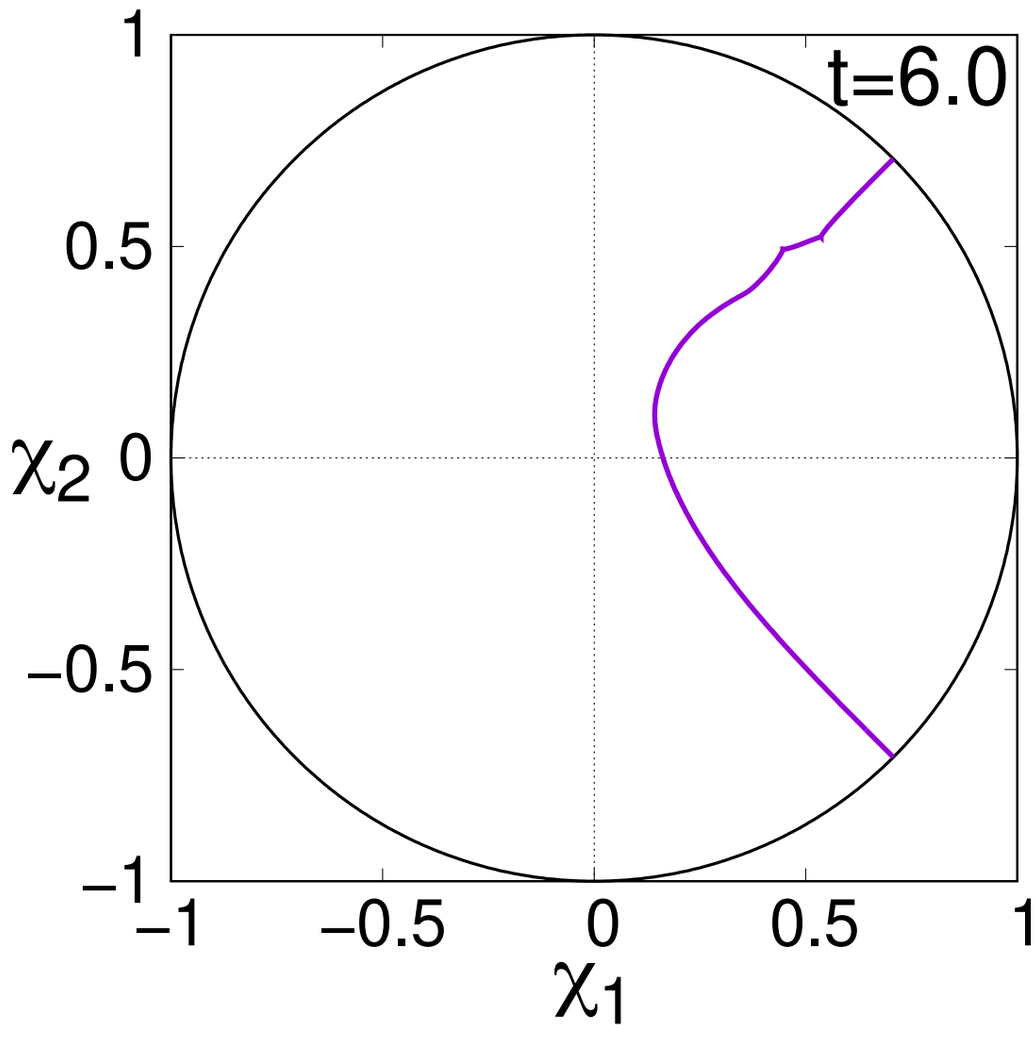
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



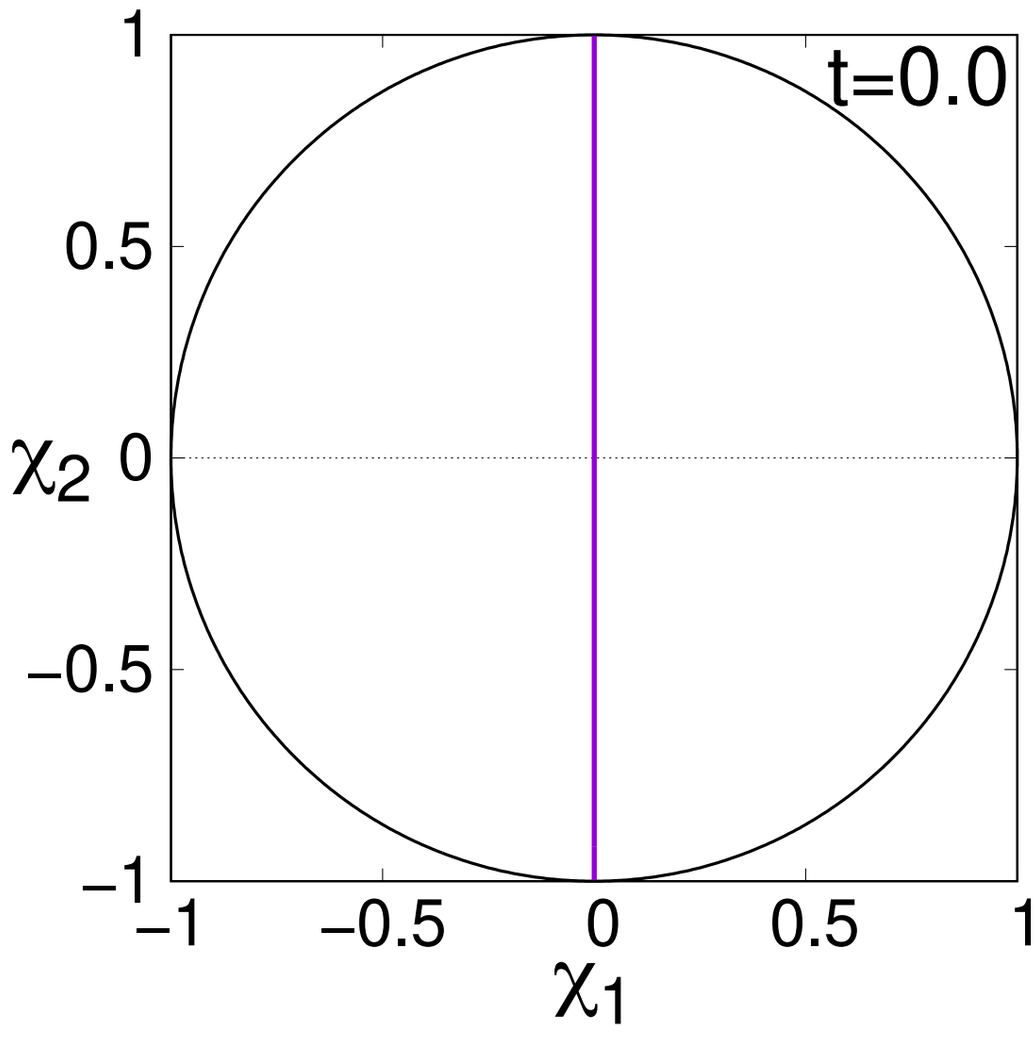
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/4$



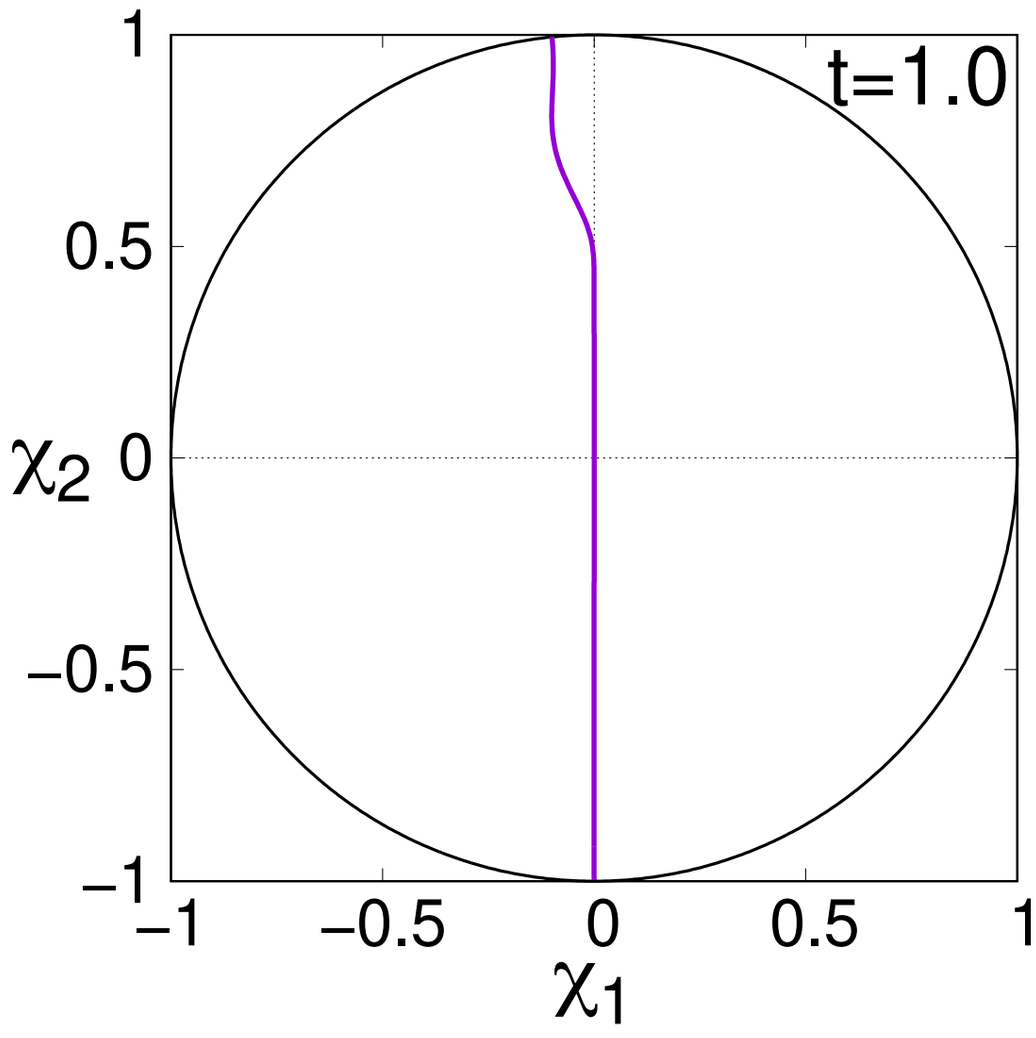
$\Delta t=2$, $\varepsilon=0.08$

$\psi_b=\pi/2$ (antipodal)



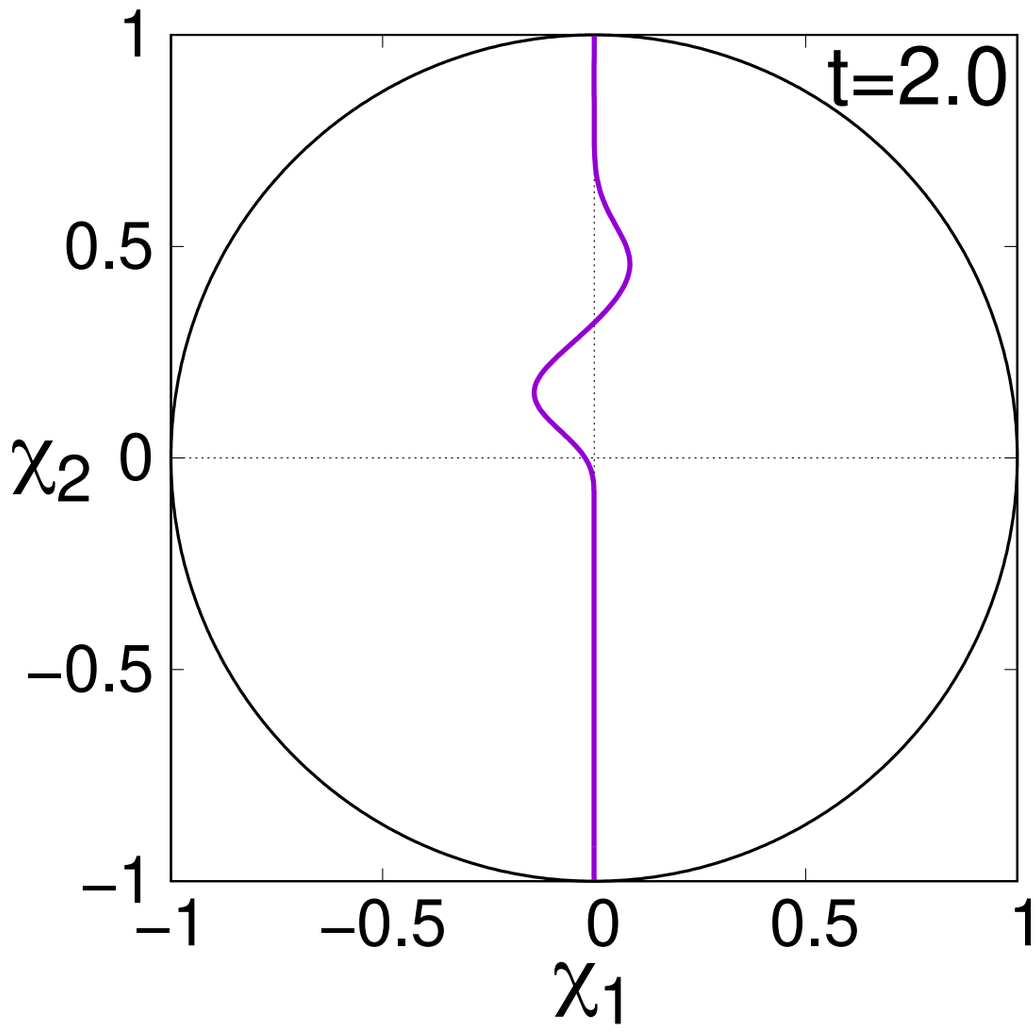
$\Delta t=2$, $\varepsilon=0.1$

$\psi_b=\pi/2$ (antipodal)



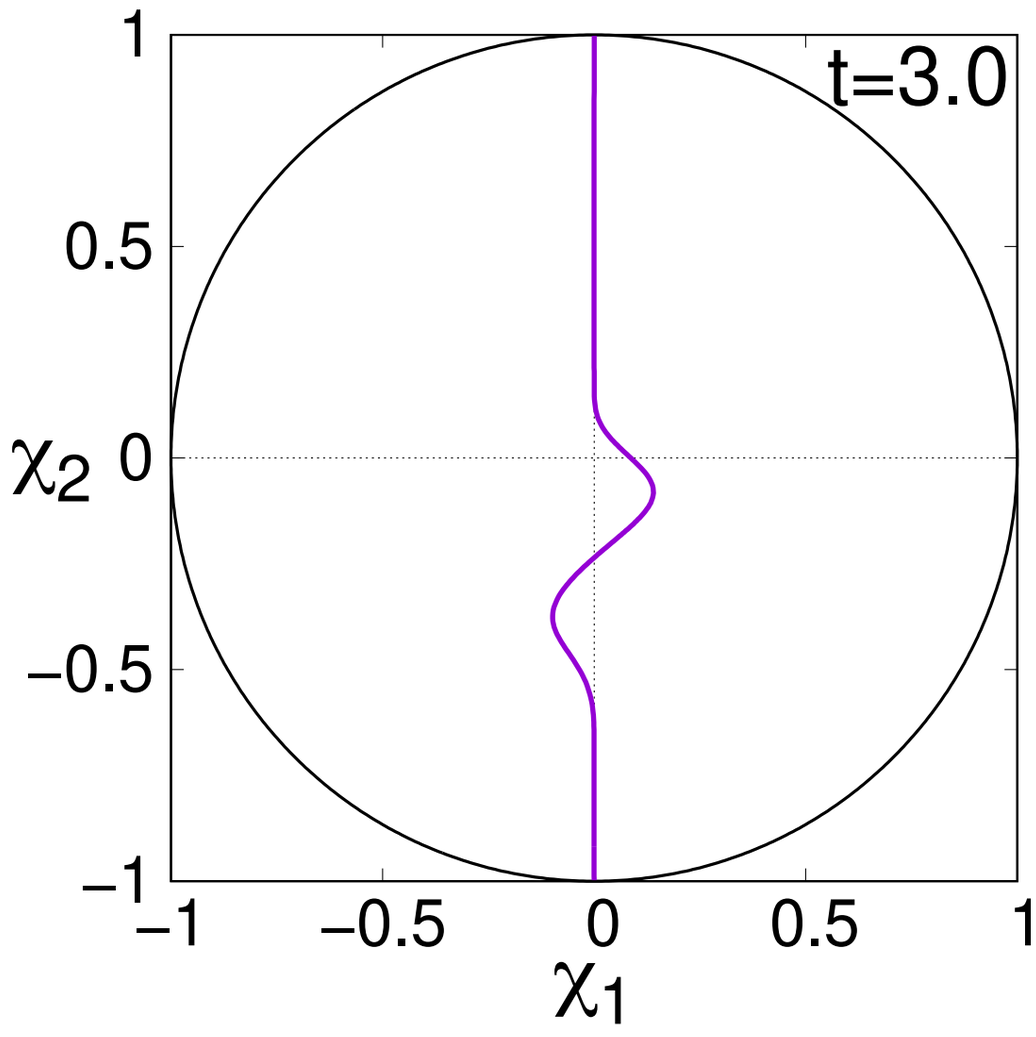
$\Delta t=2$, $\varepsilon=0.1$

$\psi_b=\pi/2$ (antipodal)



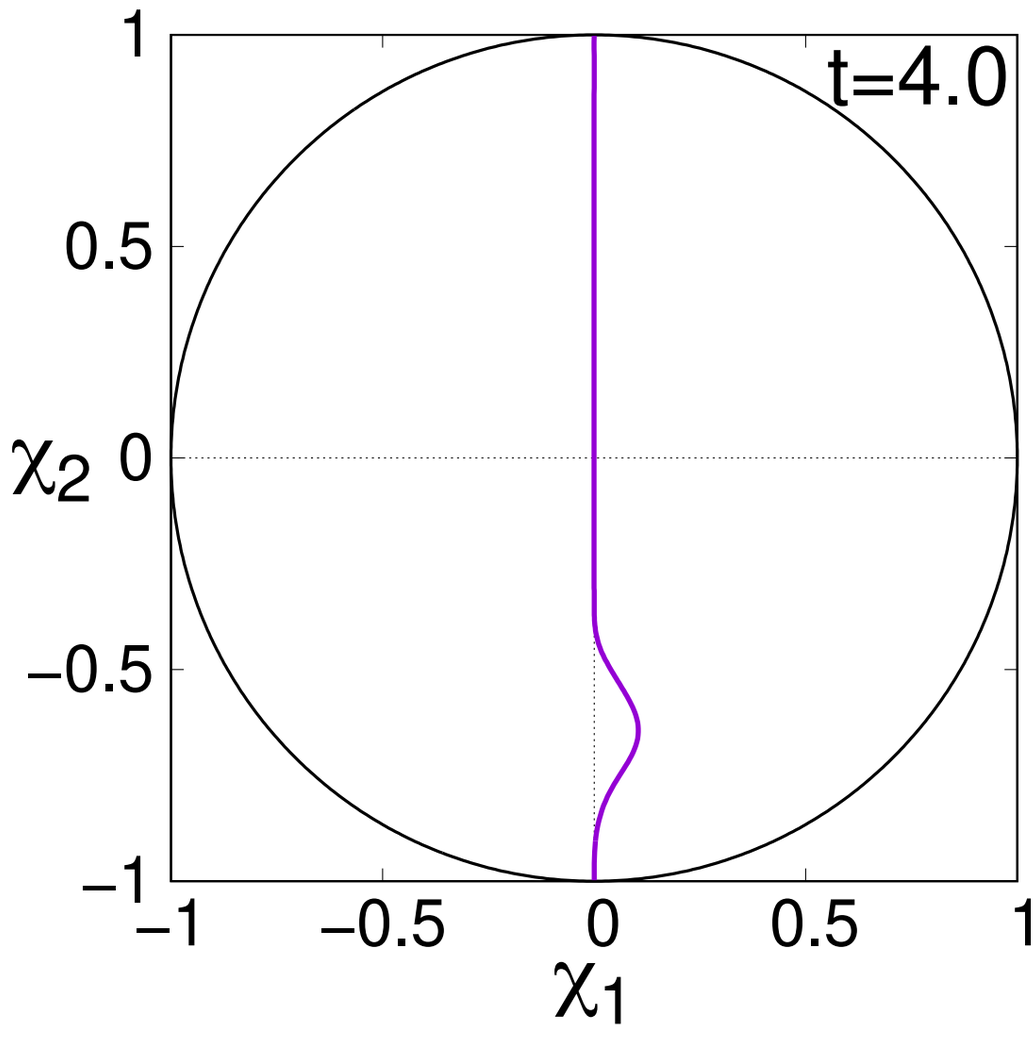
$\Delta t=2$, $\varepsilon=0.1$

$\psi_b=\pi/2$ (antipodal)



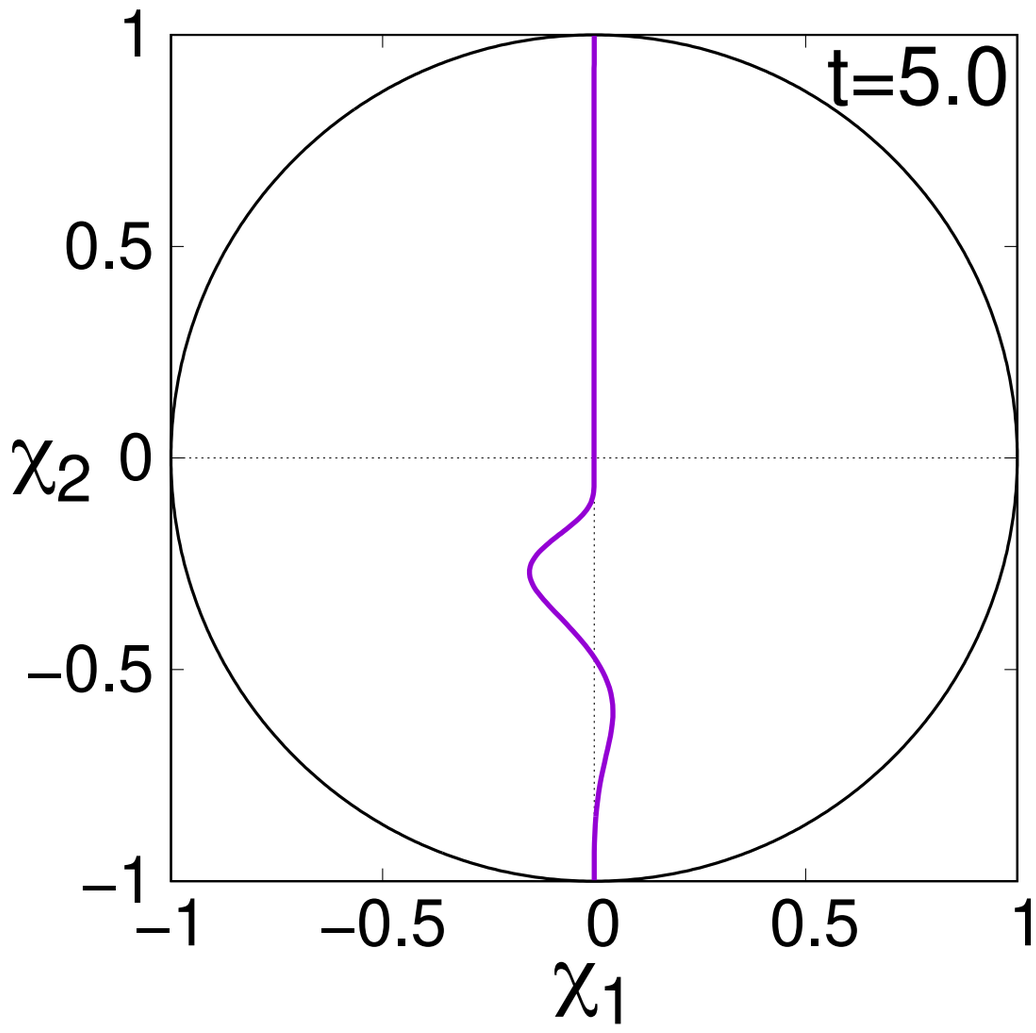
$\Delta t=2$, $\varepsilon=0.1$

$\psi_b=\pi/2$ (antipodal)



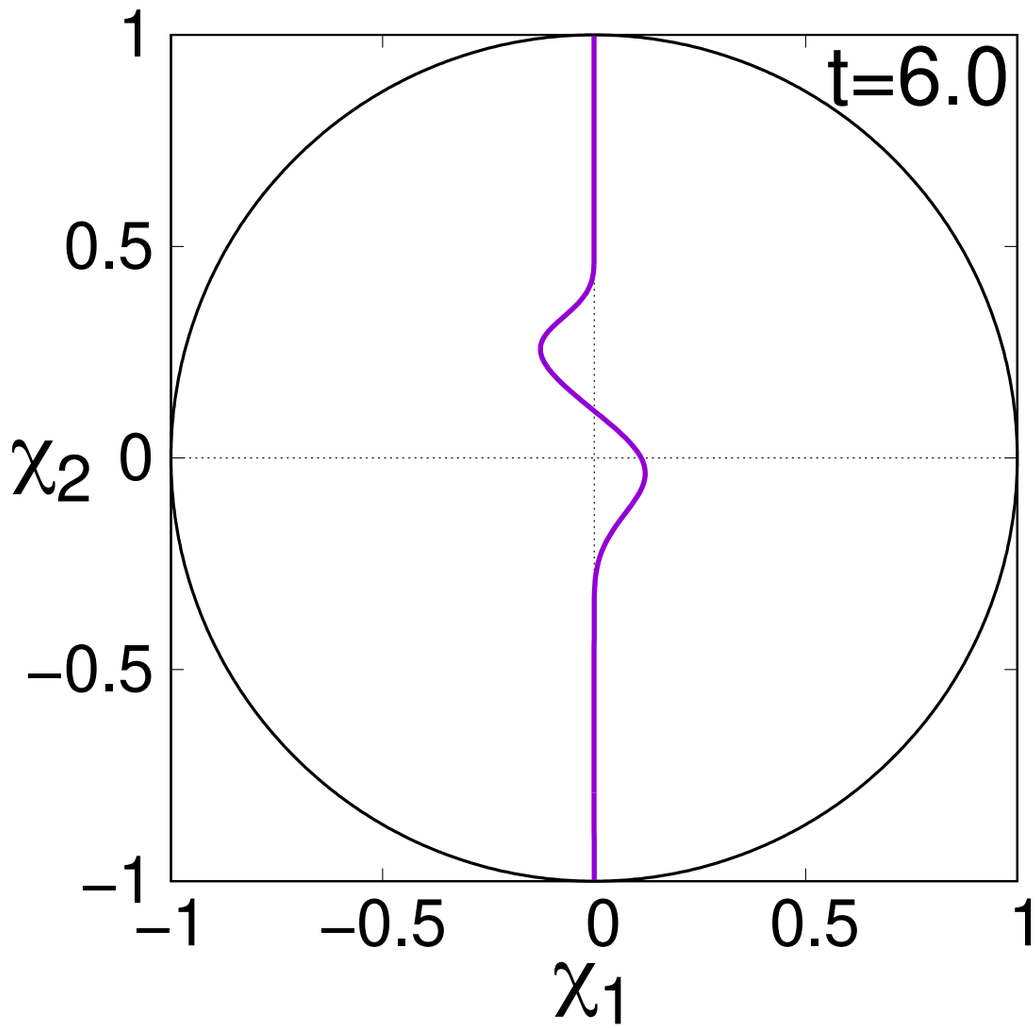
$\Delta t=2$, $\varepsilon=0.1$

$\psi_b=\pi/2$ (antipodal)



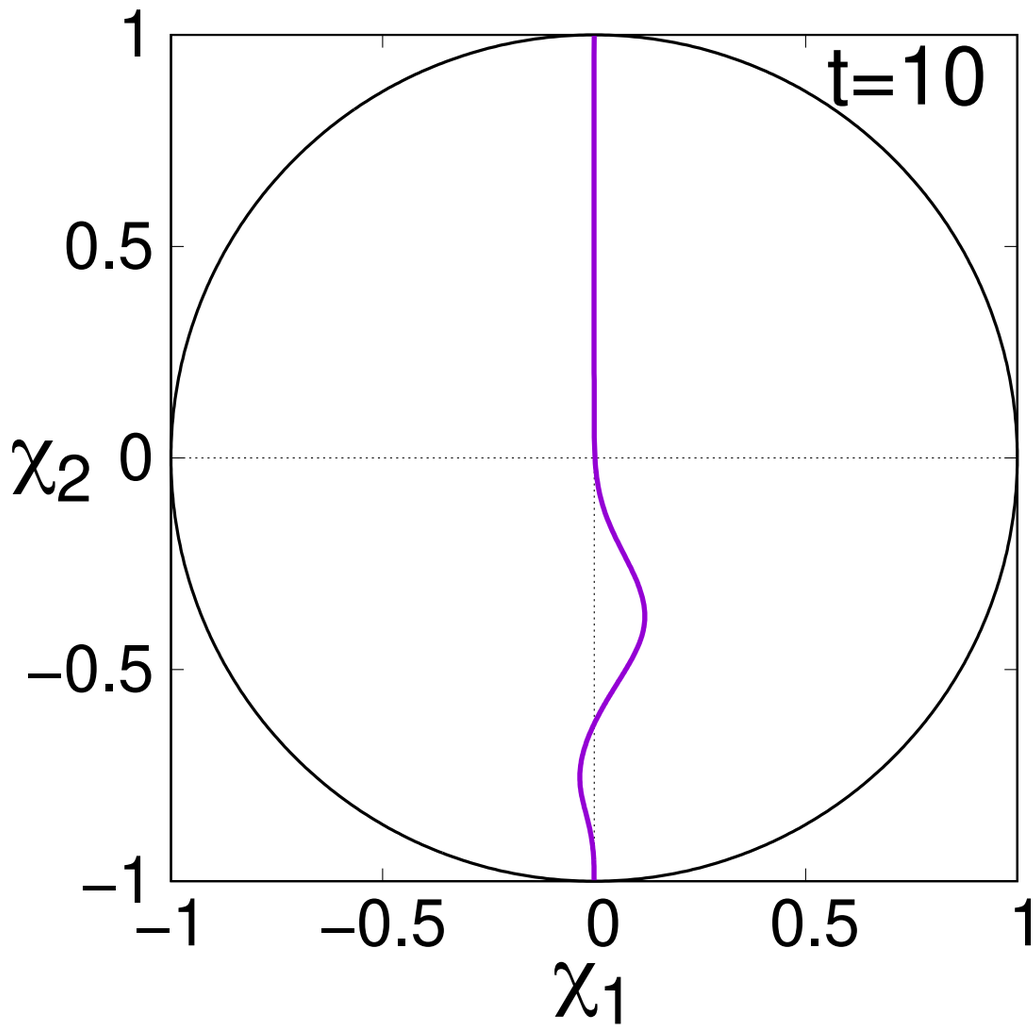
$\Delta t=2$, $\varepsilon=0.1$

$\psi_b=\pi/2$ (antipodal)



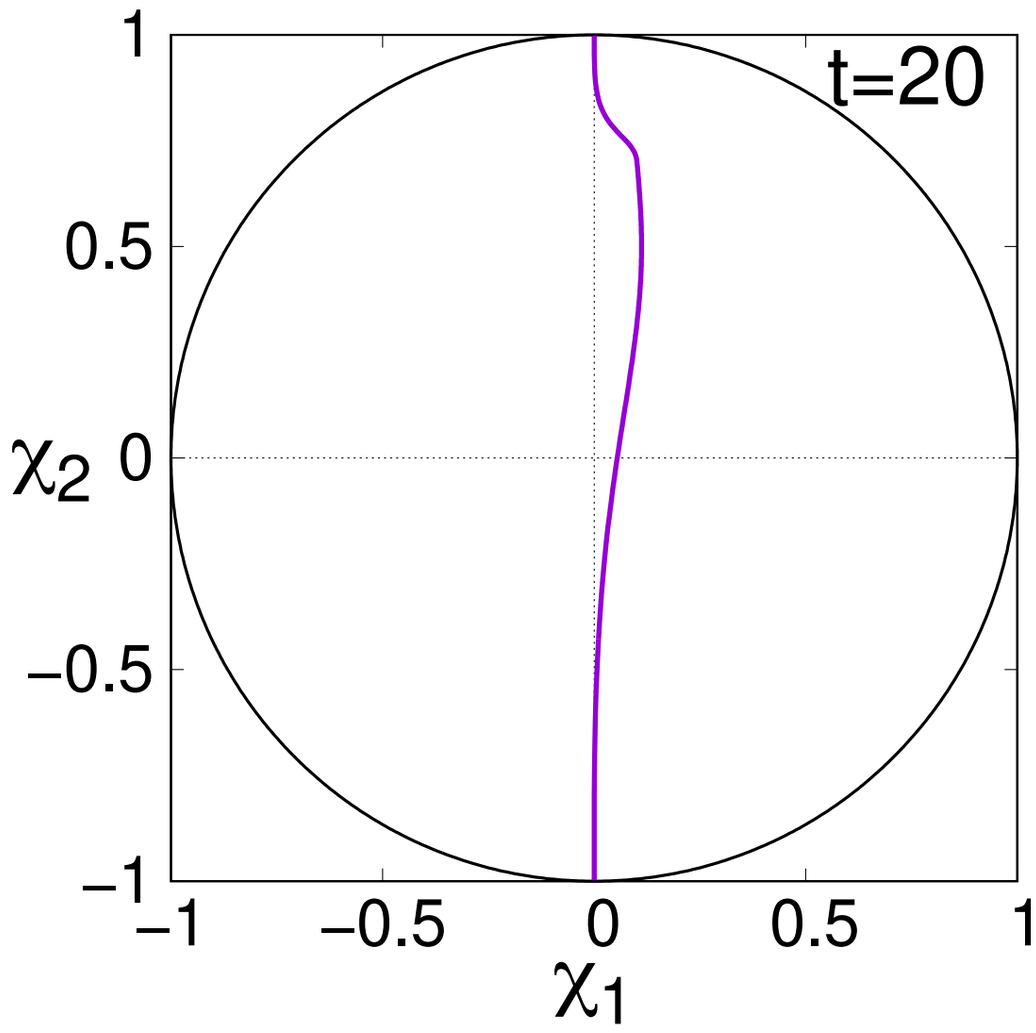
$\Delta t=2$, $\varepsilon=0.1$

$\psi_b=\pi/2$ (antipodal)



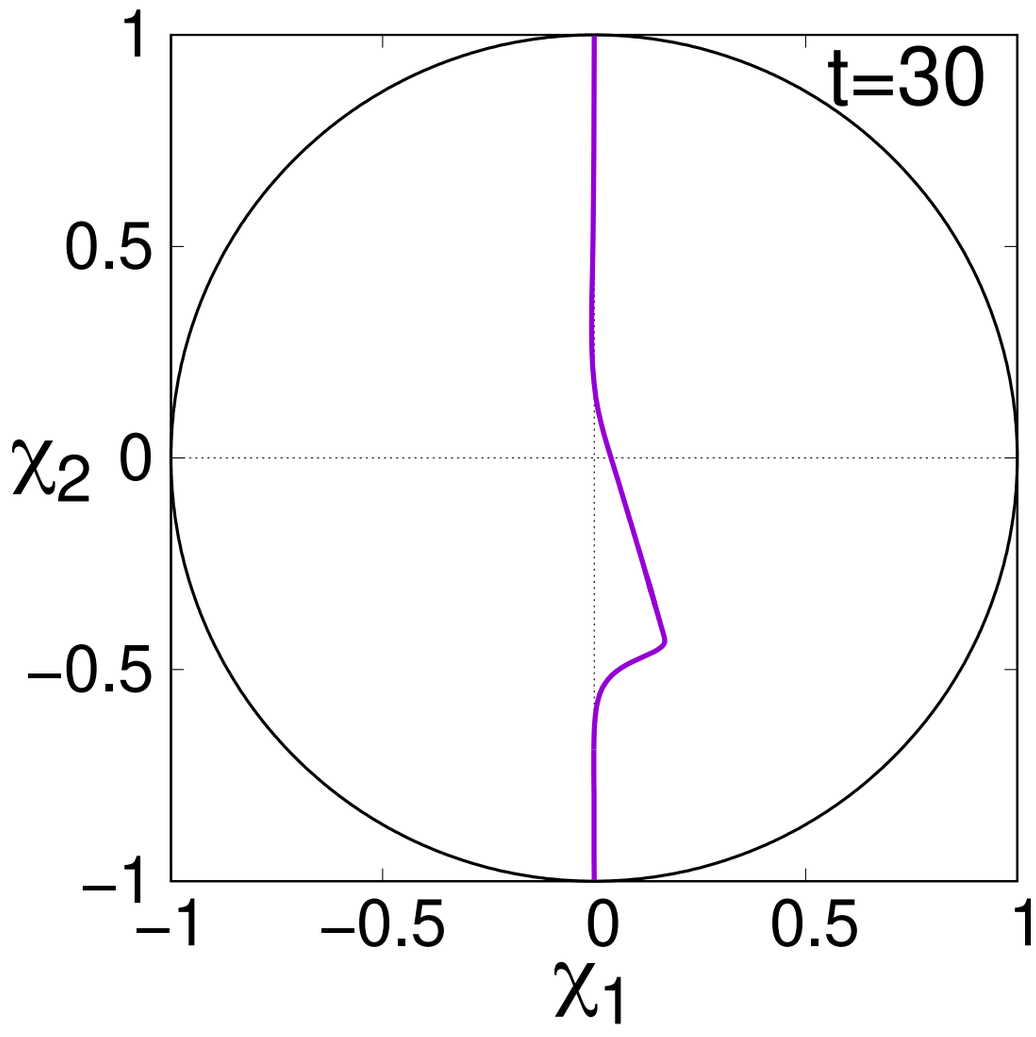
$\Delta t=2$, $\varepsilon=0.1$

$\psi_b=\pi/2$ (antipodal)



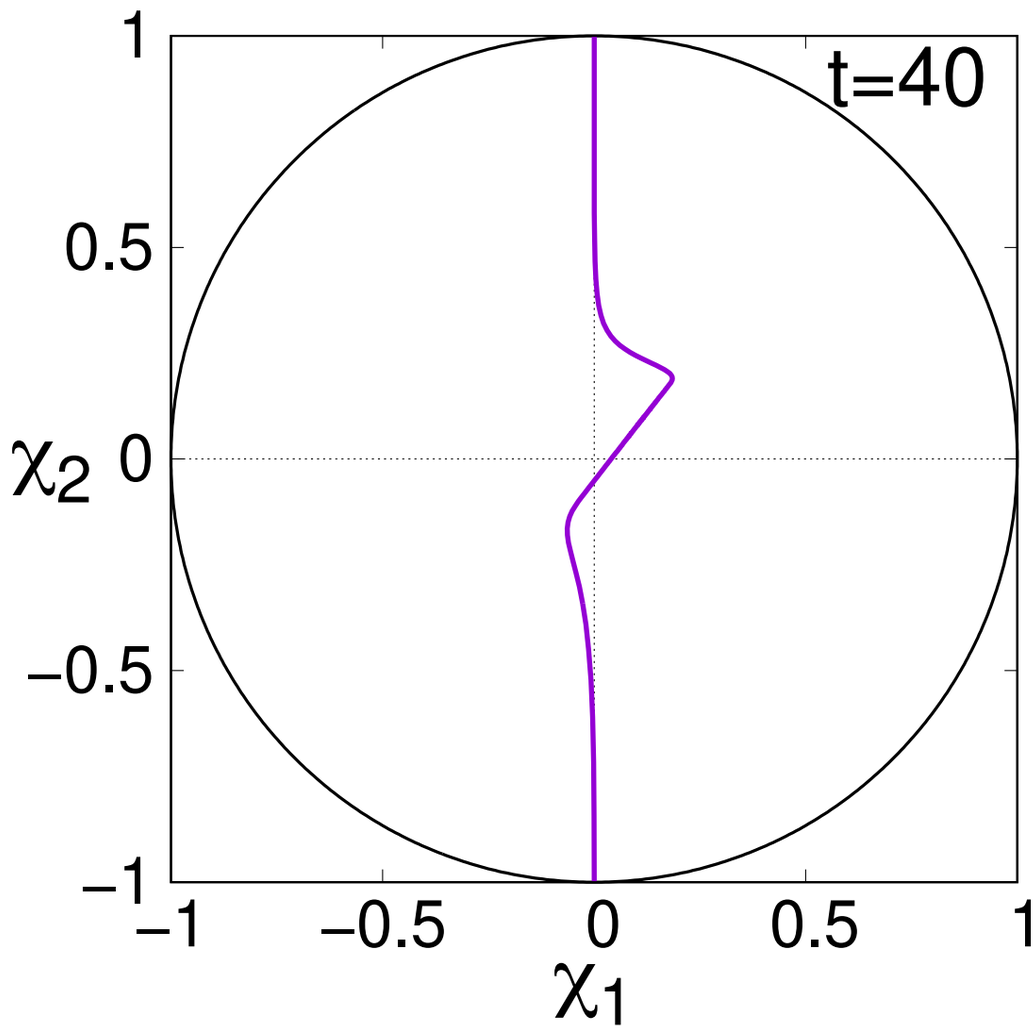
$\Delta t=2$, $\varepsilon=0.1$

$\psi_b=\pi/2$ (antipodal)



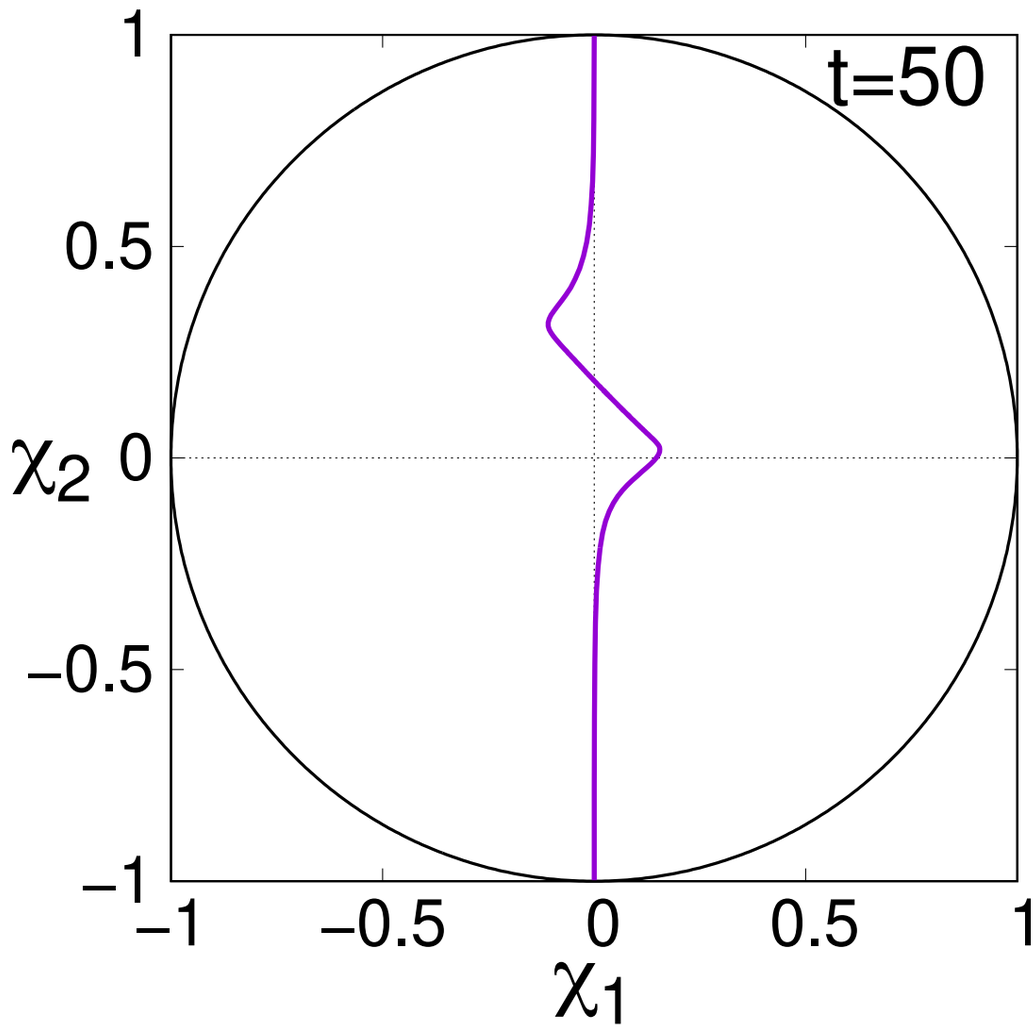
$\Delta t=2$, $\varepsilon=0.1$

$\psi_b=\pi/2$ (antipodal)



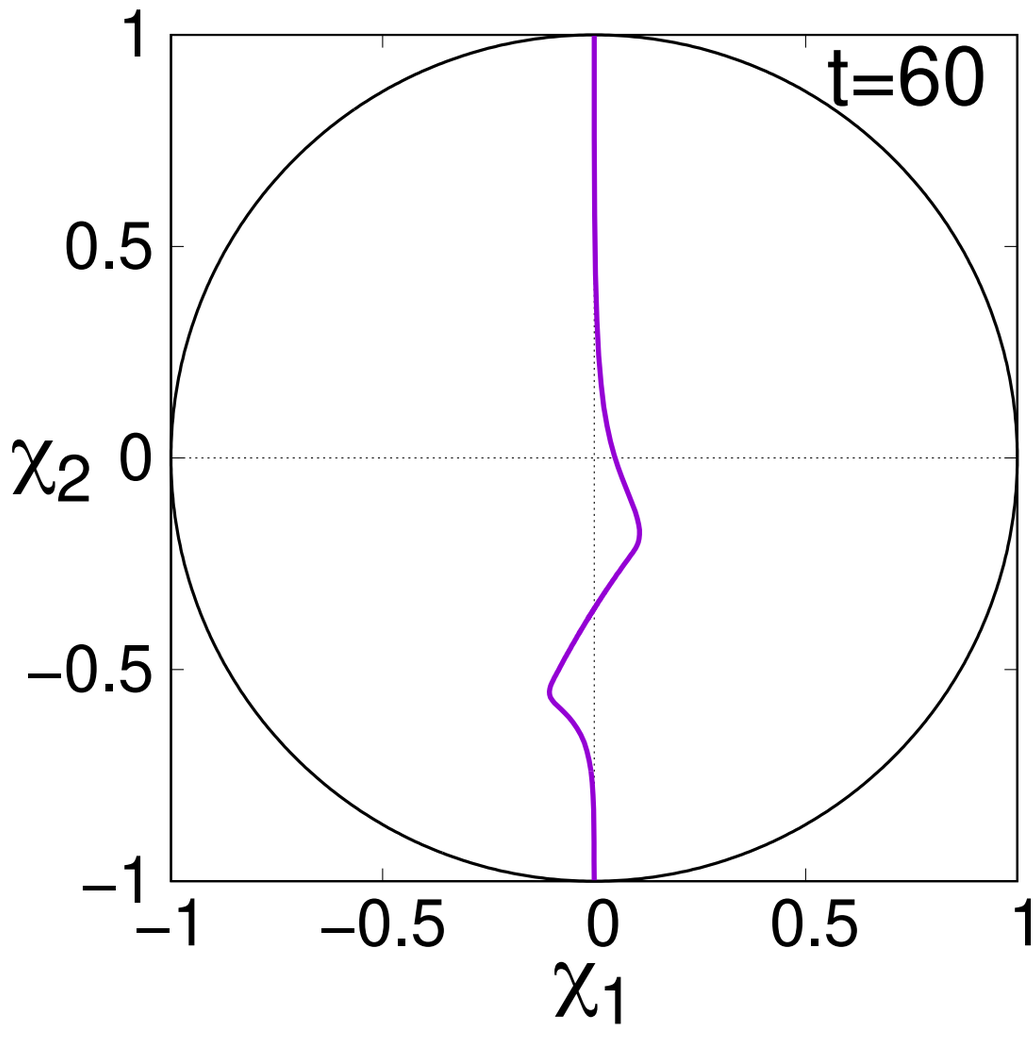
$\Delta t=2$, $\varepsilon=0.1$

$\psi_b=\pi/2$ (antipodal)



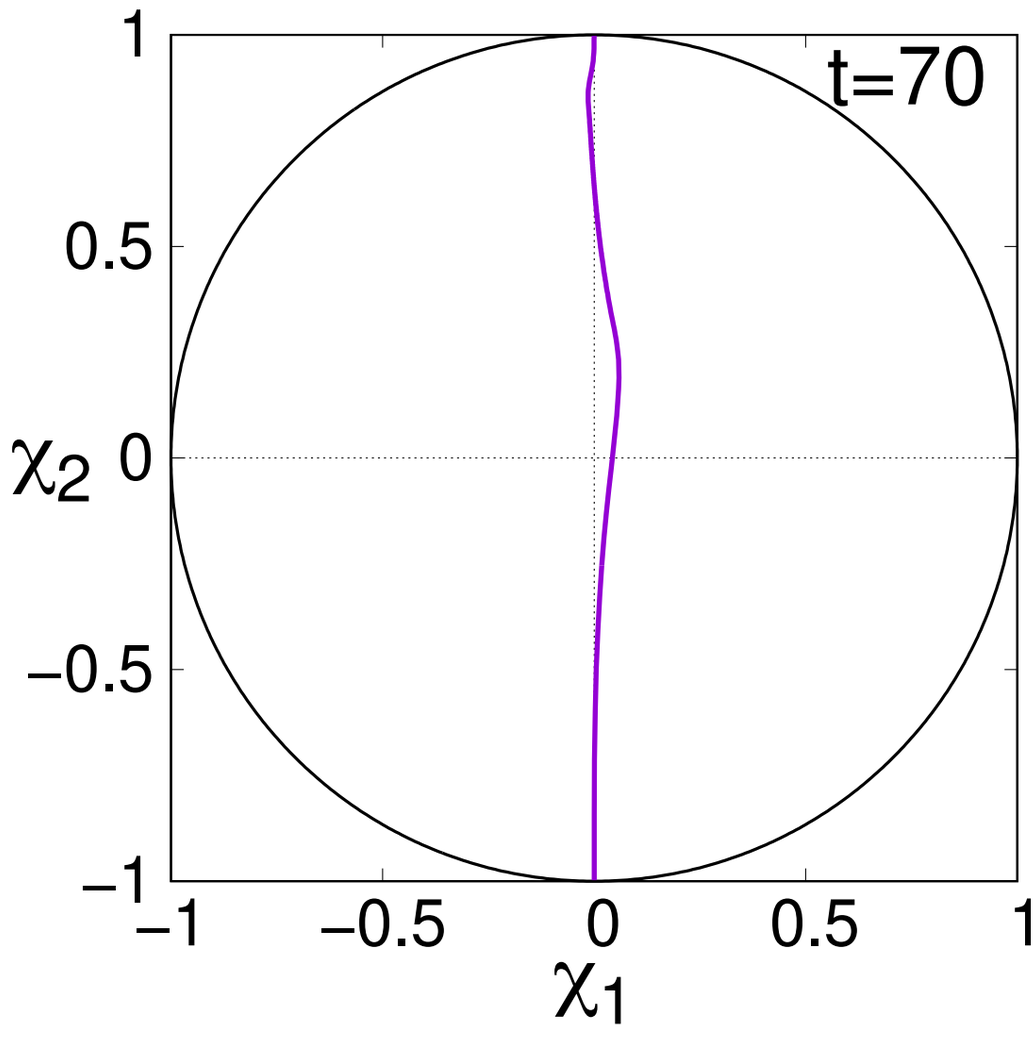
$\Delta t=2$, $\varepsilon=0.1$

$\psi_b=\pi/2$ (antipodal)



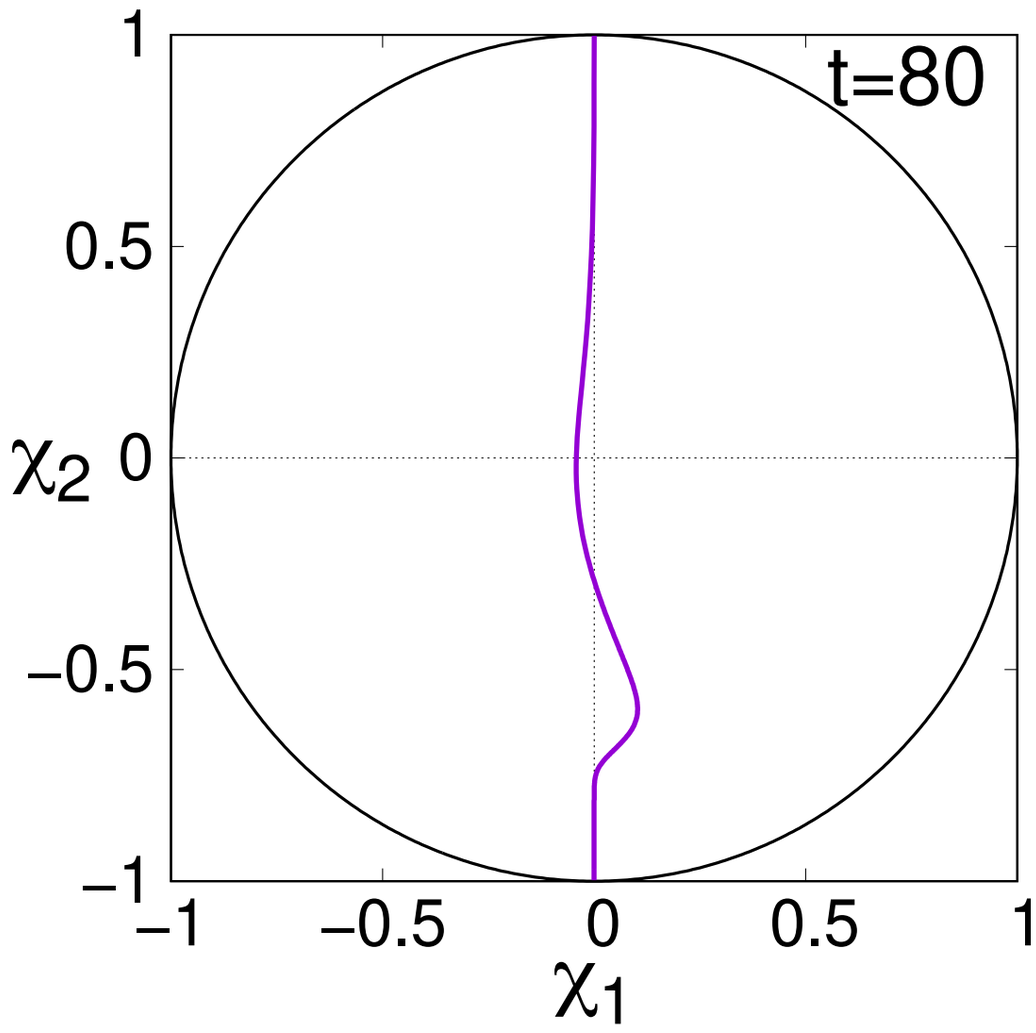
$\Delta t=2$, $\varepsilon=0.1$

$\psi_b=\pi/2$ (antipodal)



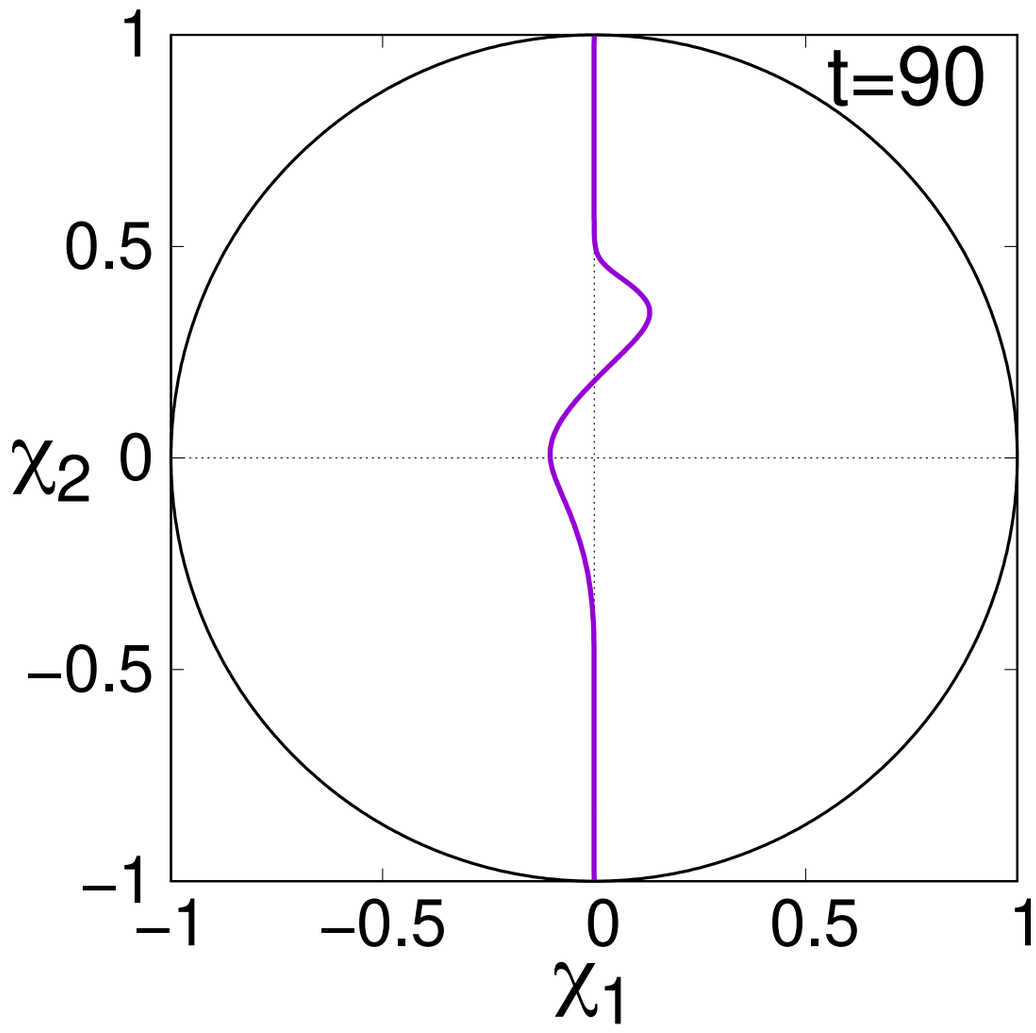
$\Delta t=2$, $\varepsilon=0.1$

$\psi_b=\pi/2$ (antipodal)



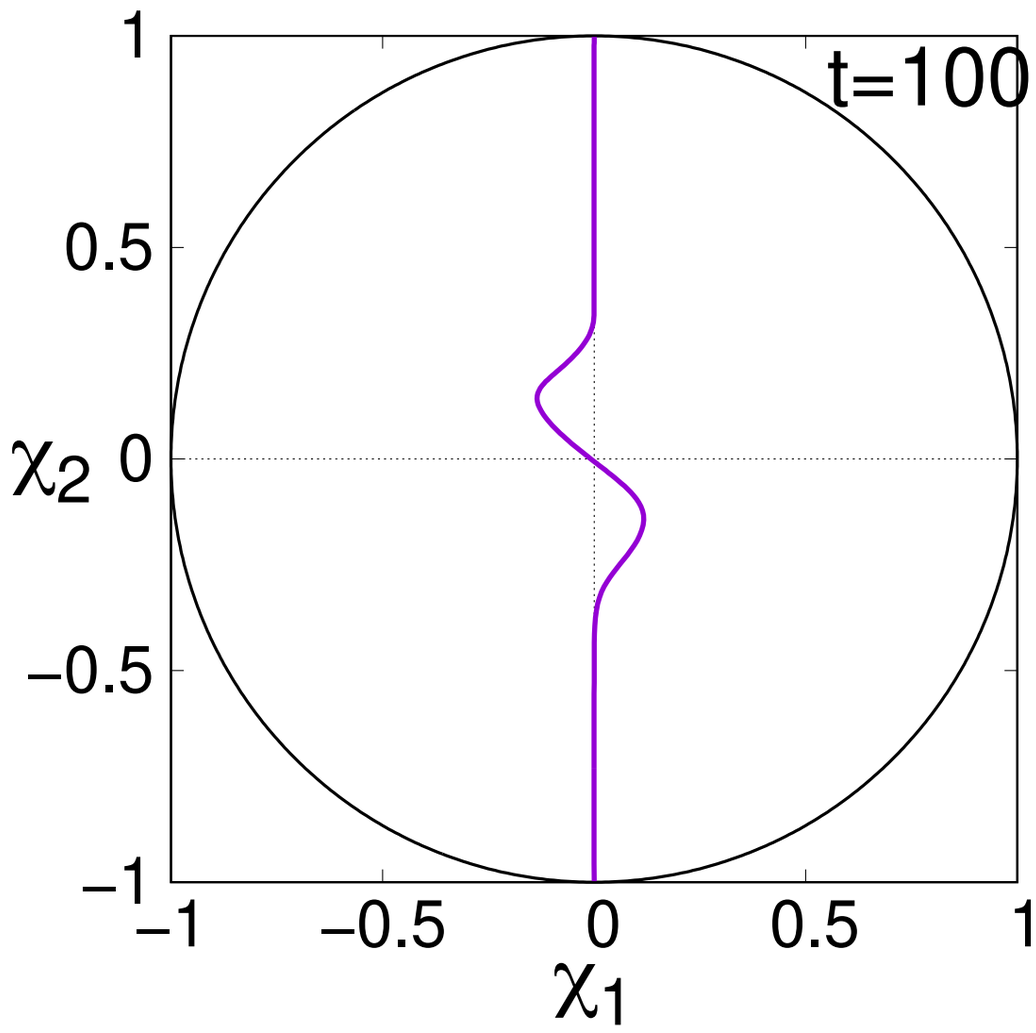
$\Delta t=2$, $\varepsilon=0.1$

$\psi_b=\pi/2$ (antipodal)



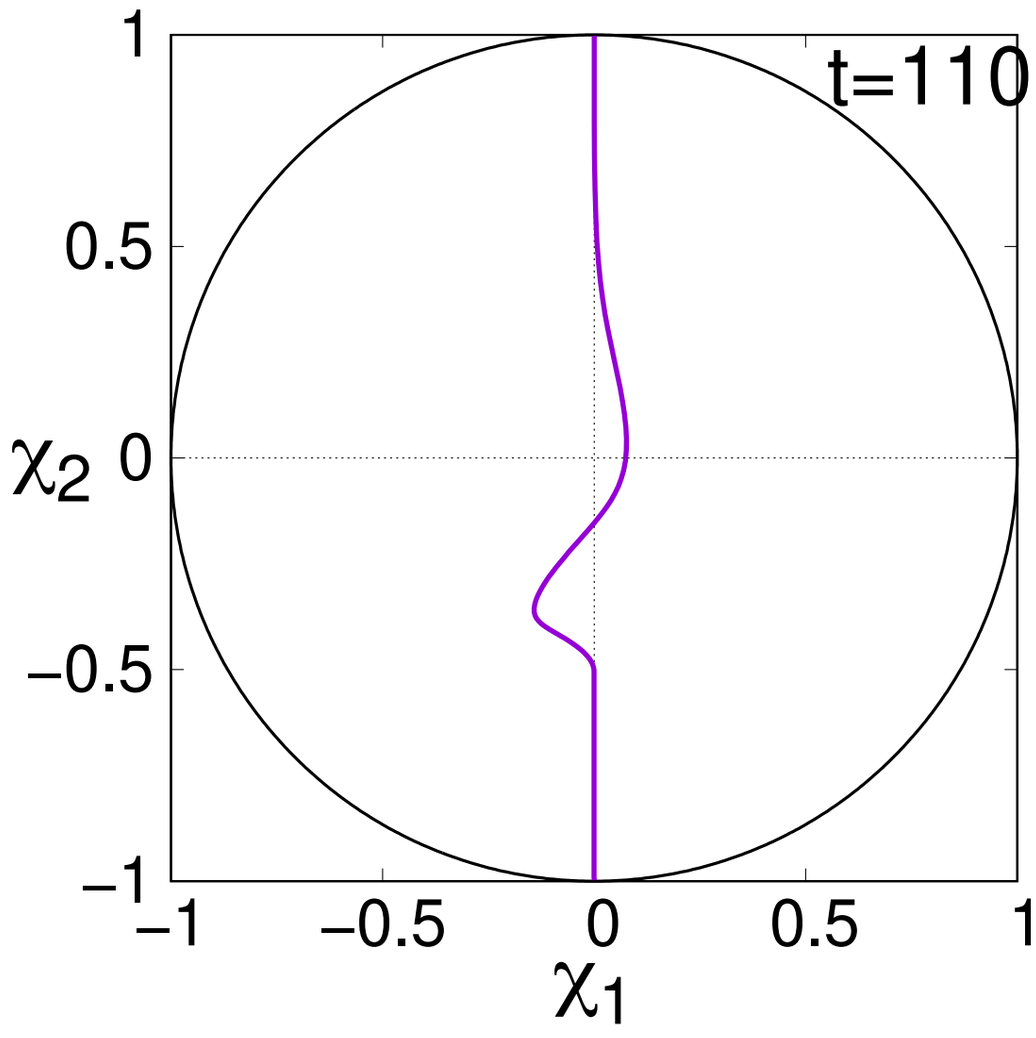
$\Delta t=2$, $\varepsilon=0.1$

$\psi_b=\pi/2$ (antipodal)



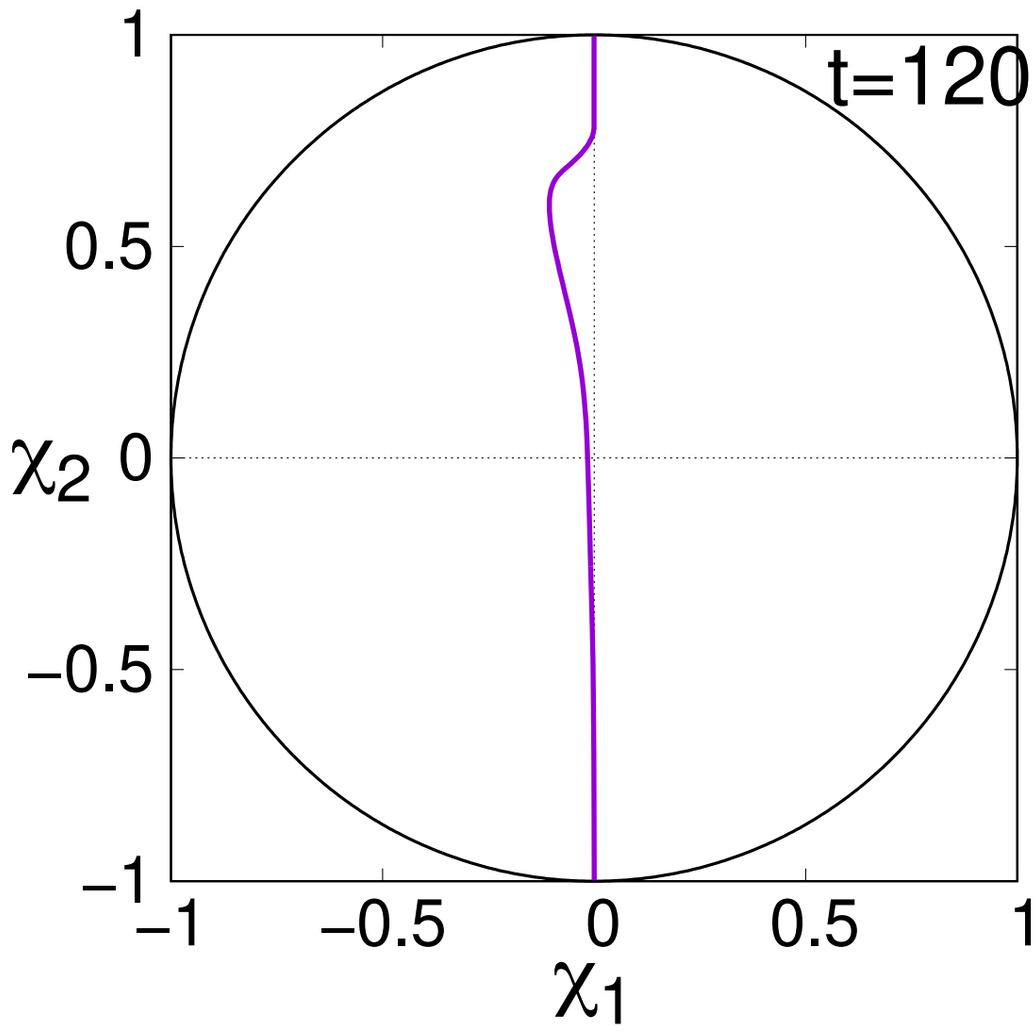
$\Delta t=2$, $\varepsilon=0.1$

$\psi_b=\pi/2$ (antipodal)



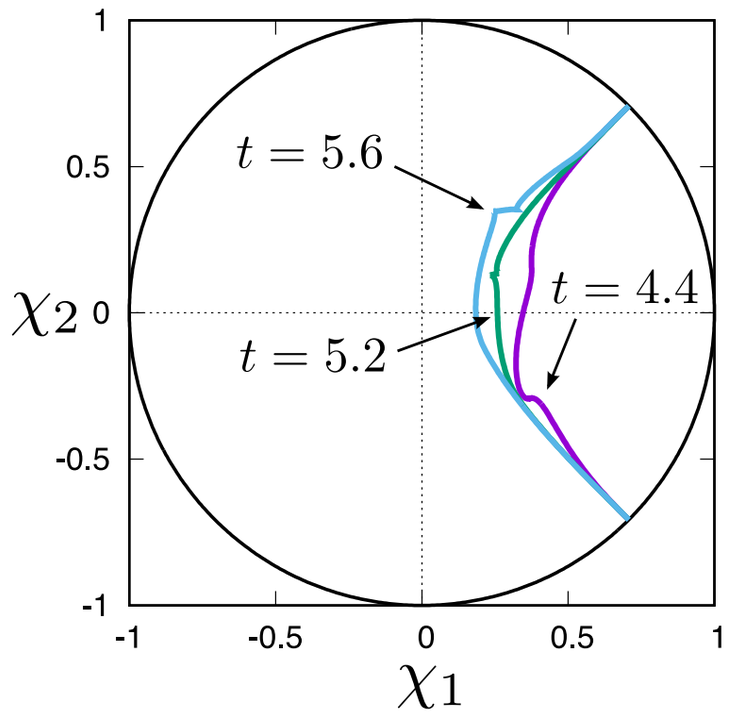
$\Delta t=2$, $\varepsilon=0.1$

$\psi_b=\pi/2$ (antipodal)

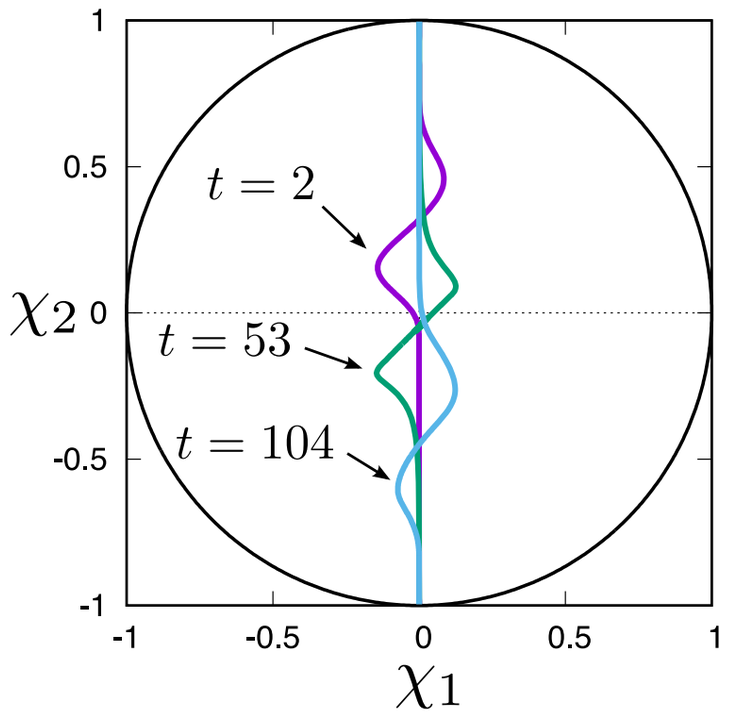


$\Delta t=2$, $\varepsilon=0.1$

Cusps or no cusps

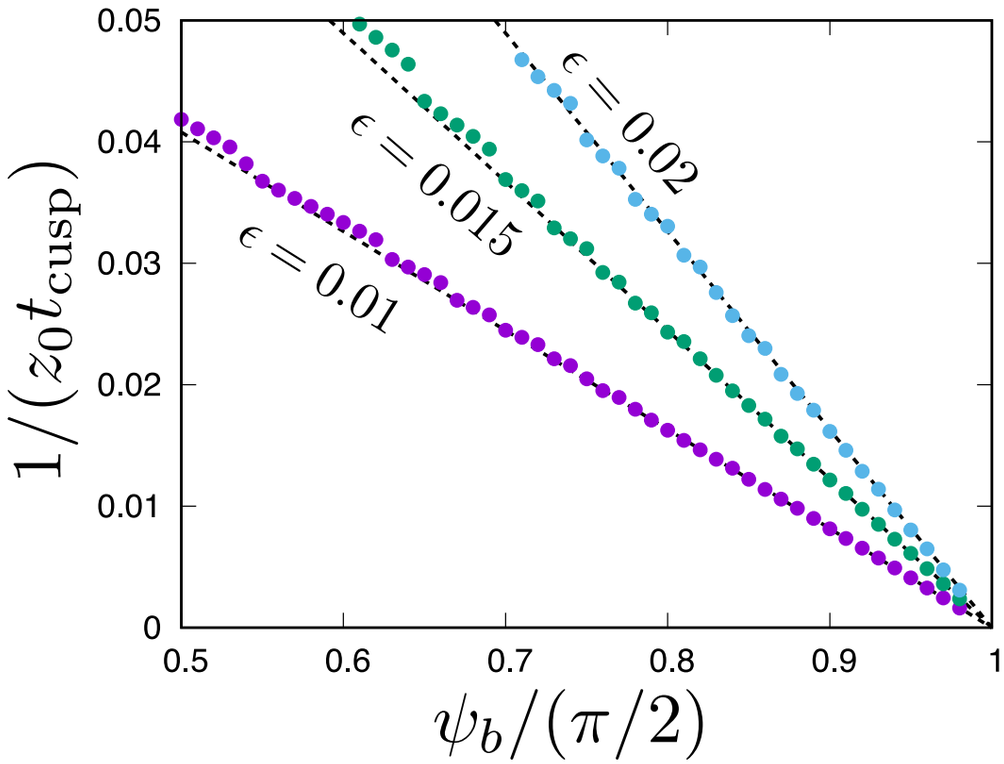


Cusp formation



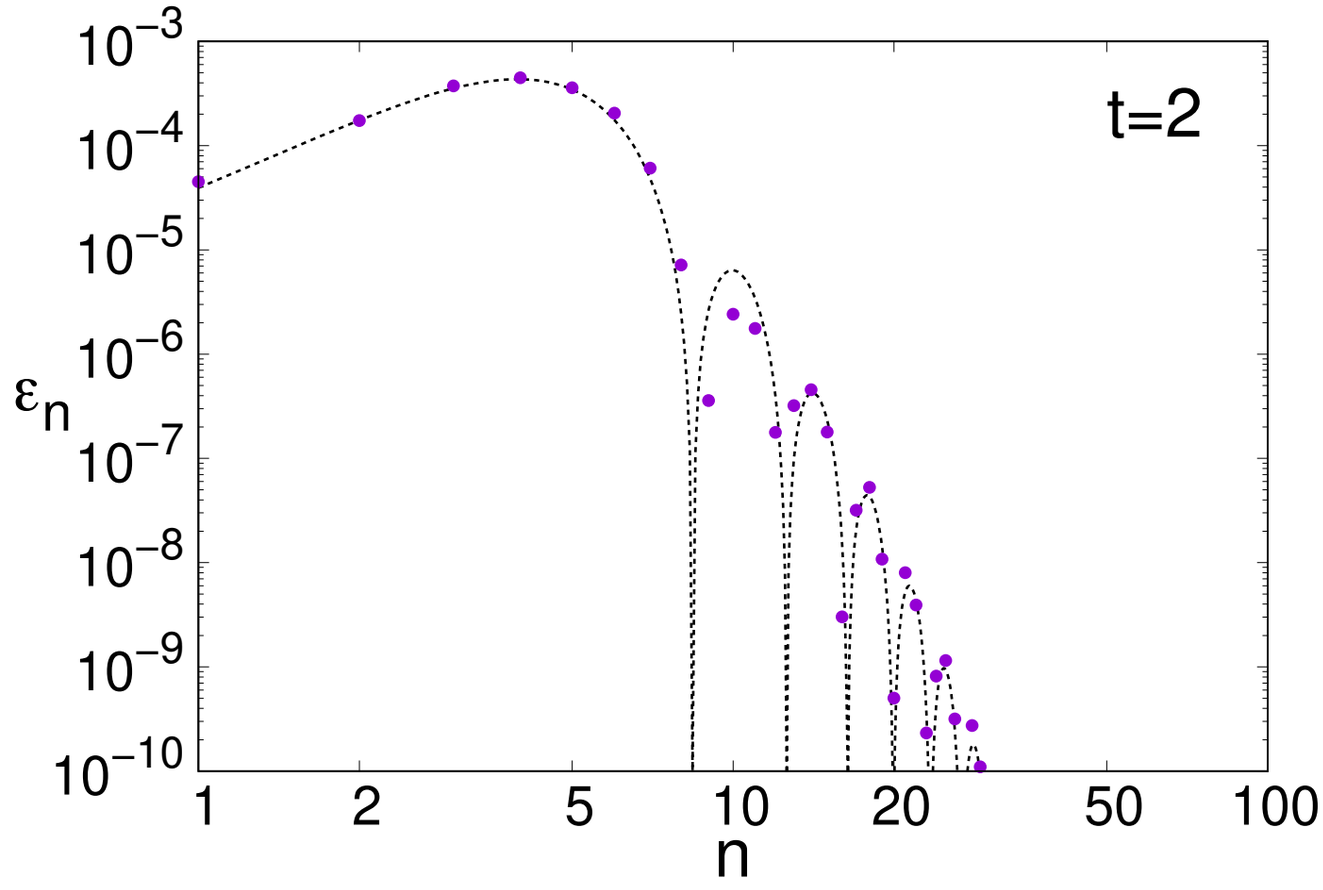
Antipodal: No cusps

Cusp formation time scale



$$t_{\rm cusp} \sim \epsilon^{-1}(\psi_b - \pi/2)^{-1}$$

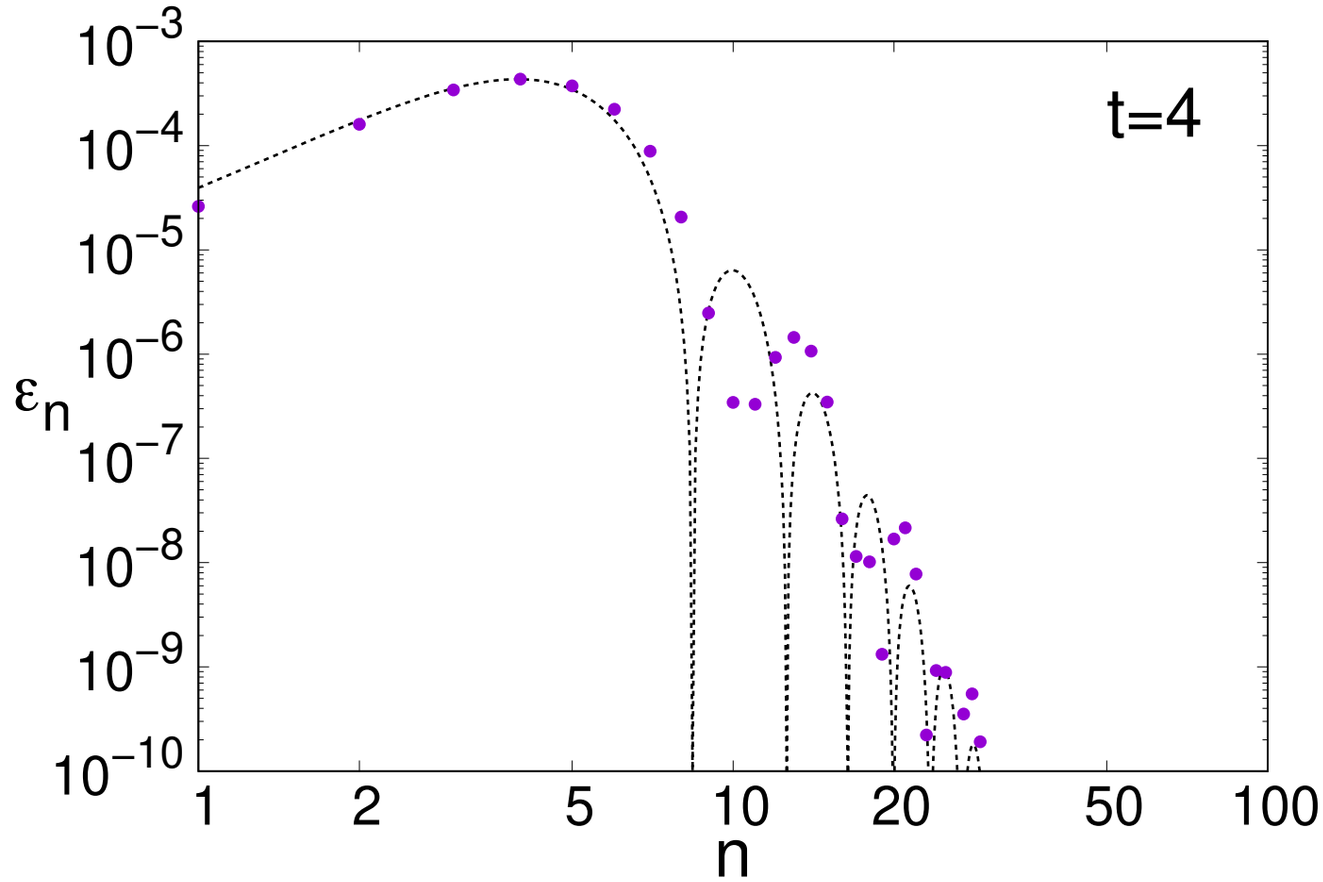
Energy spectrum: $\psi_b=\pi/4$



$\Delta t=2$, $\varepsilon=0.01$

*log-log plot

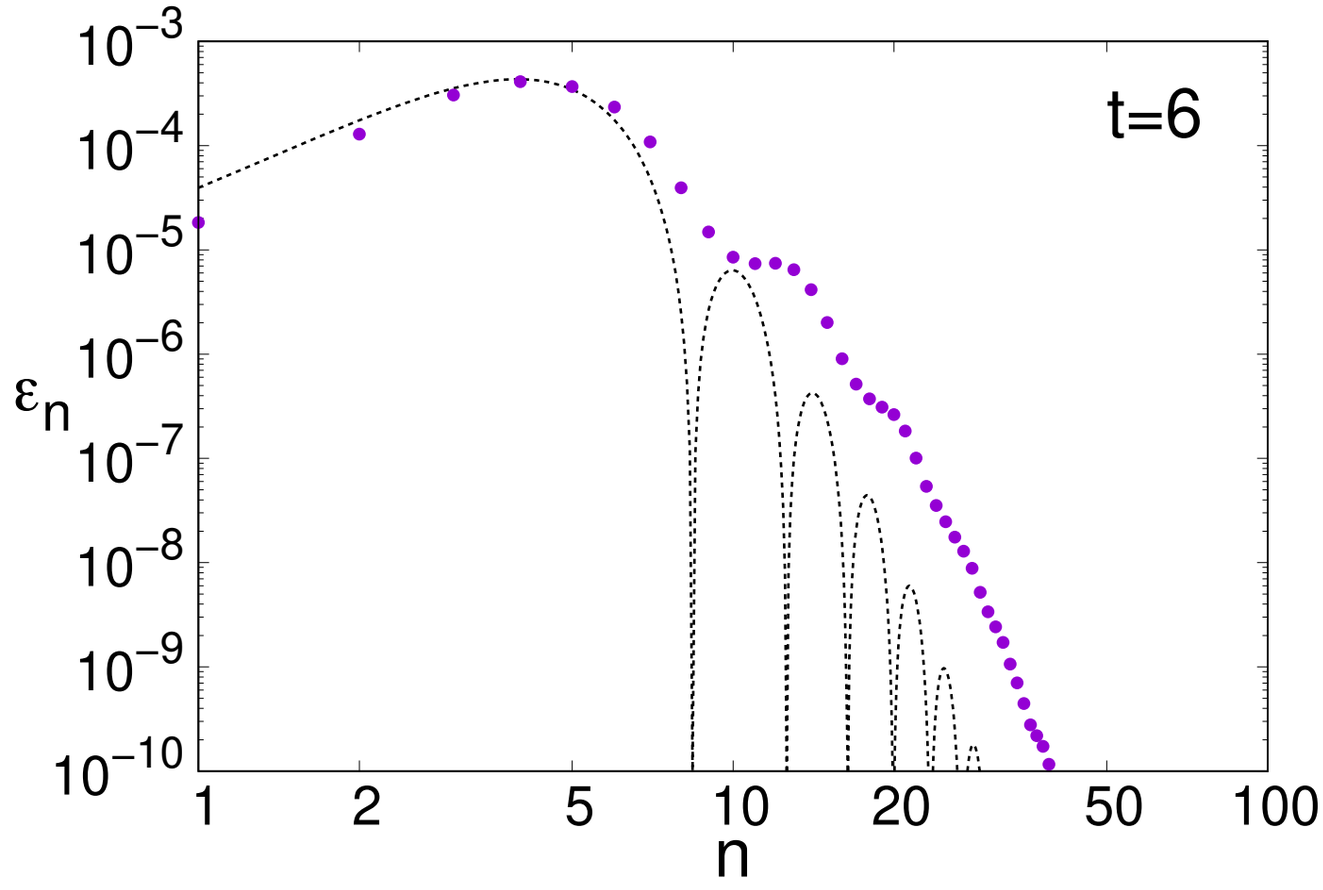
Energy spectrum: $\psi_b=\pi/4$



$\Delta t=2$, $\varepsilon=0.01$

*log-log plot

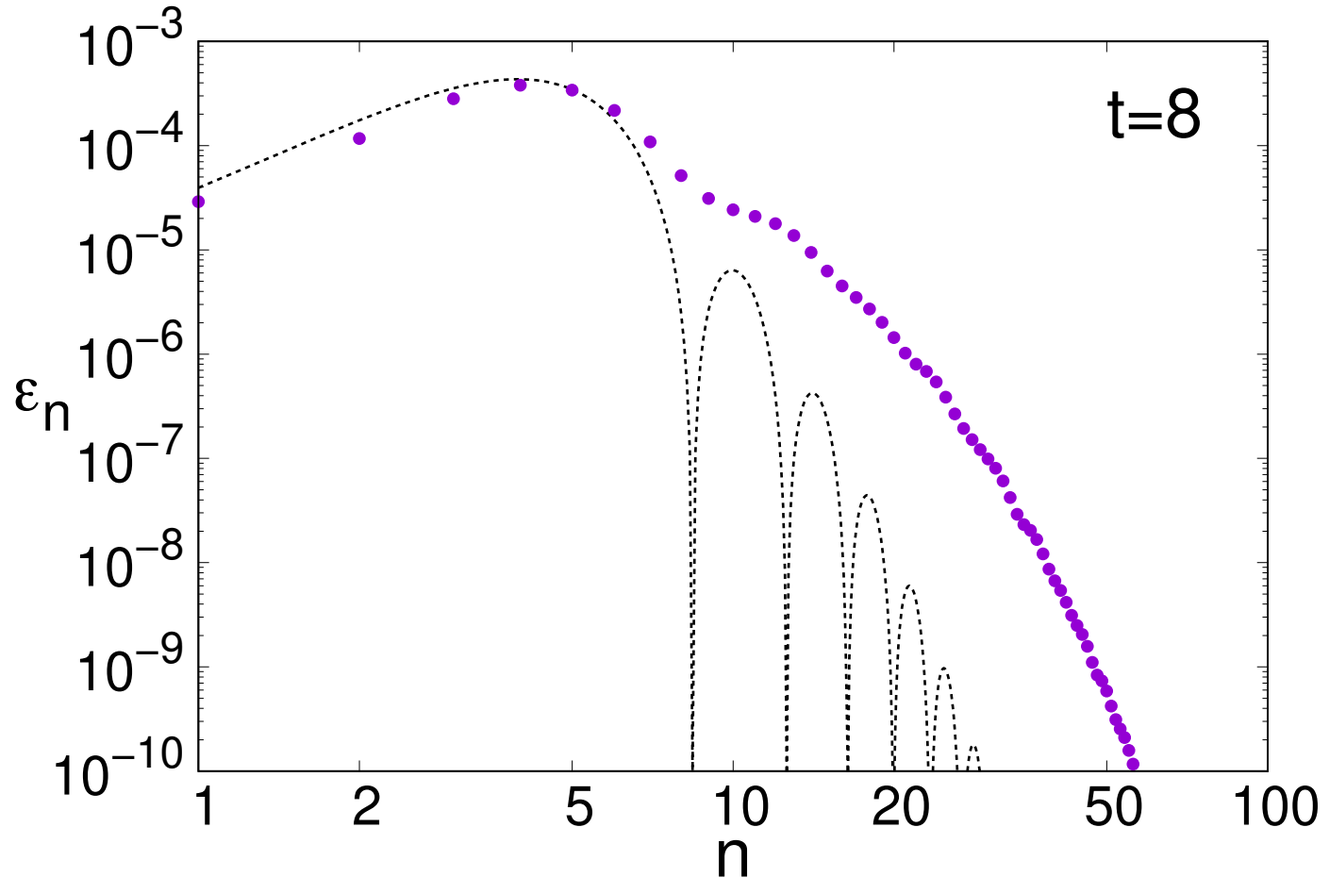
Energy spectrum: $\psi_b=\pi/4$



$\Delta t=2$, $\varepsilon=0.01$

*log-log plot

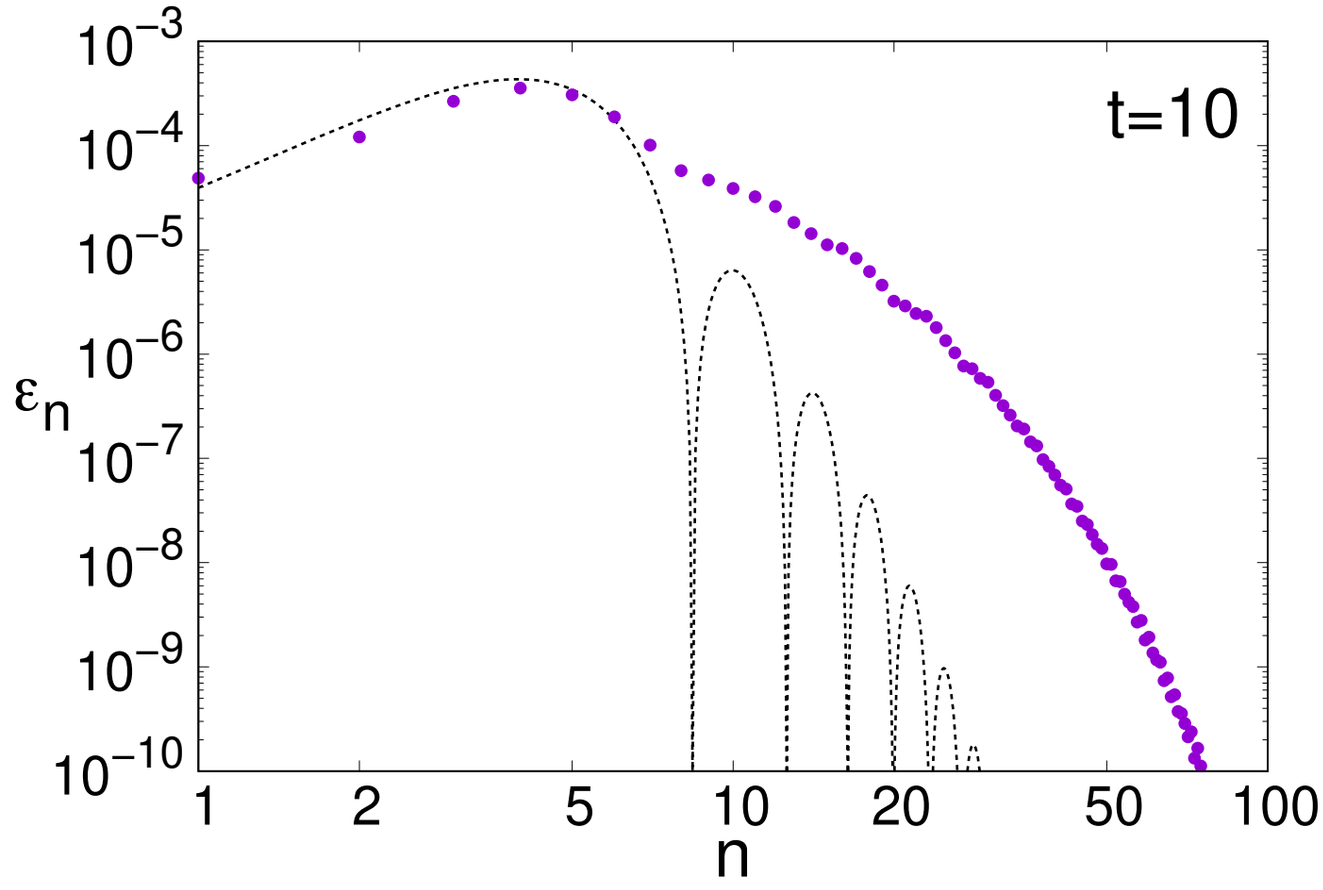
Energy spectrum: $\psi_b=\pi/4$



$\Delta t=2$, $\varepsilon=0.01$

*log-log plot

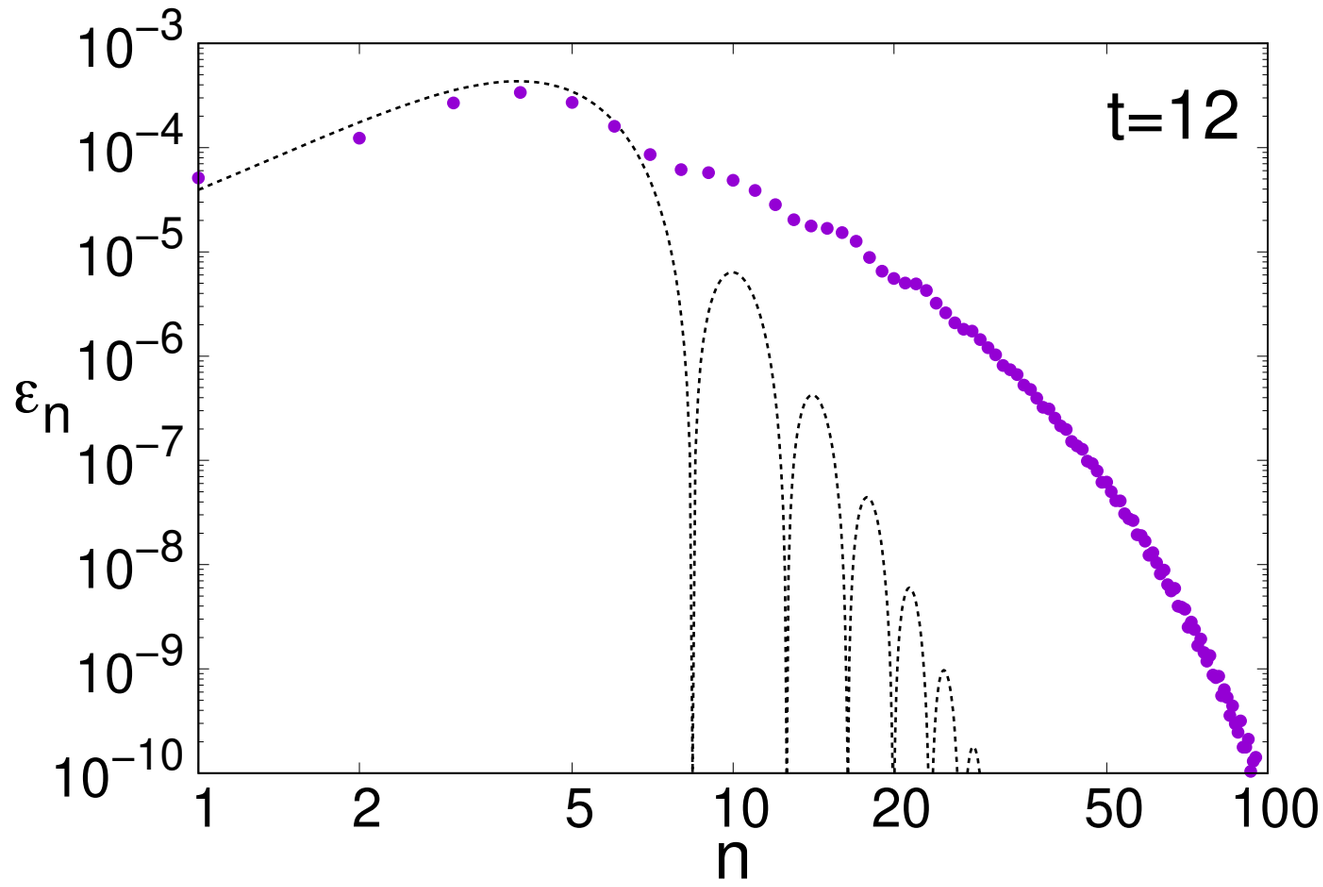
Energy spectrum: $\psi_b=\pi/4$



$\Delta t=2$, $\varepsilon=0.01$

*log-log plot

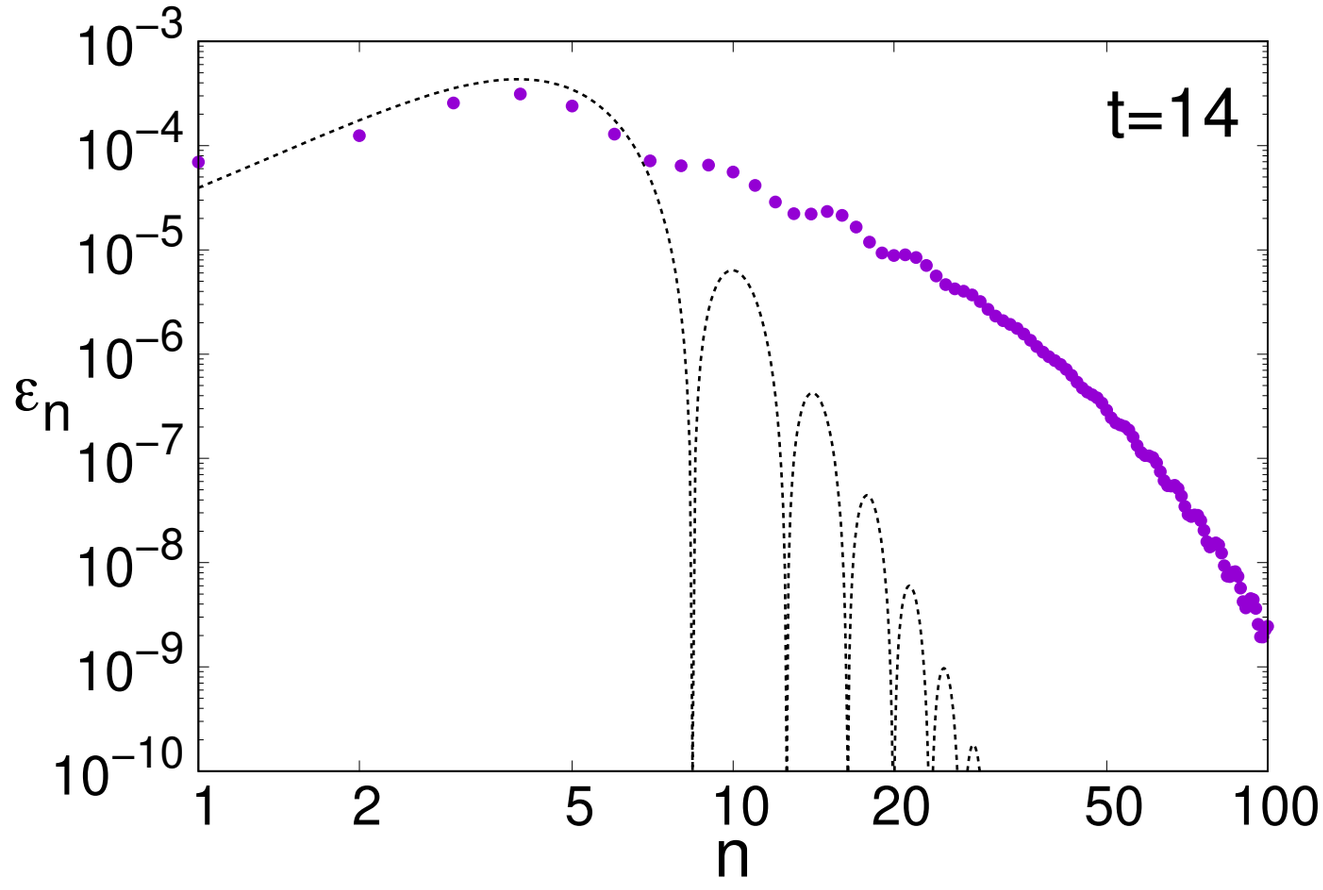
Energy spectrum: $\psi_b=\pi/4$



$\Delta t=2$, $\varepsilon=0.01$

*log-log plot

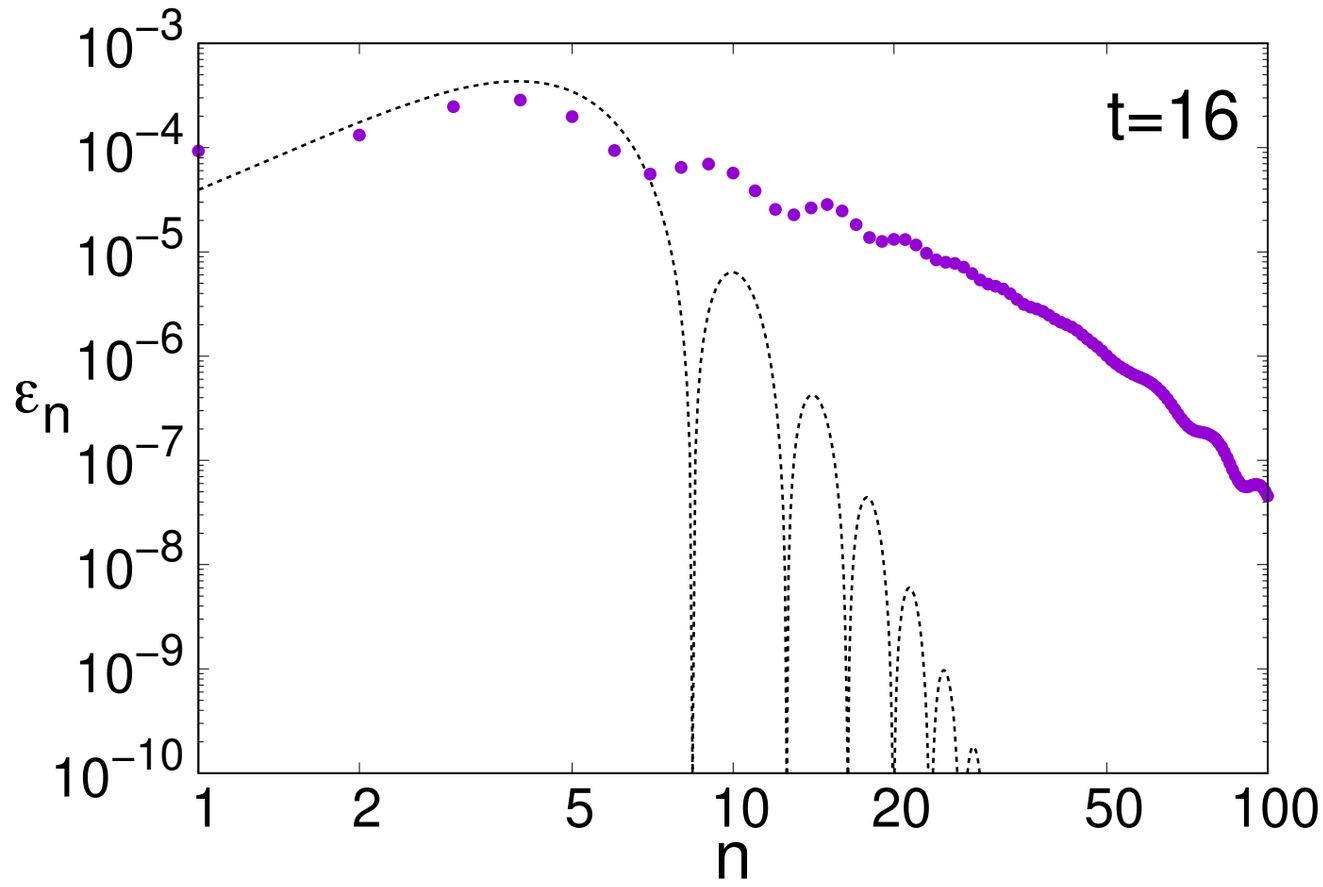
Energy spectrum: $\psi_b=\pi/4$



$\Delta t=2$, $\varepsilon=0.01$

*log-log plot

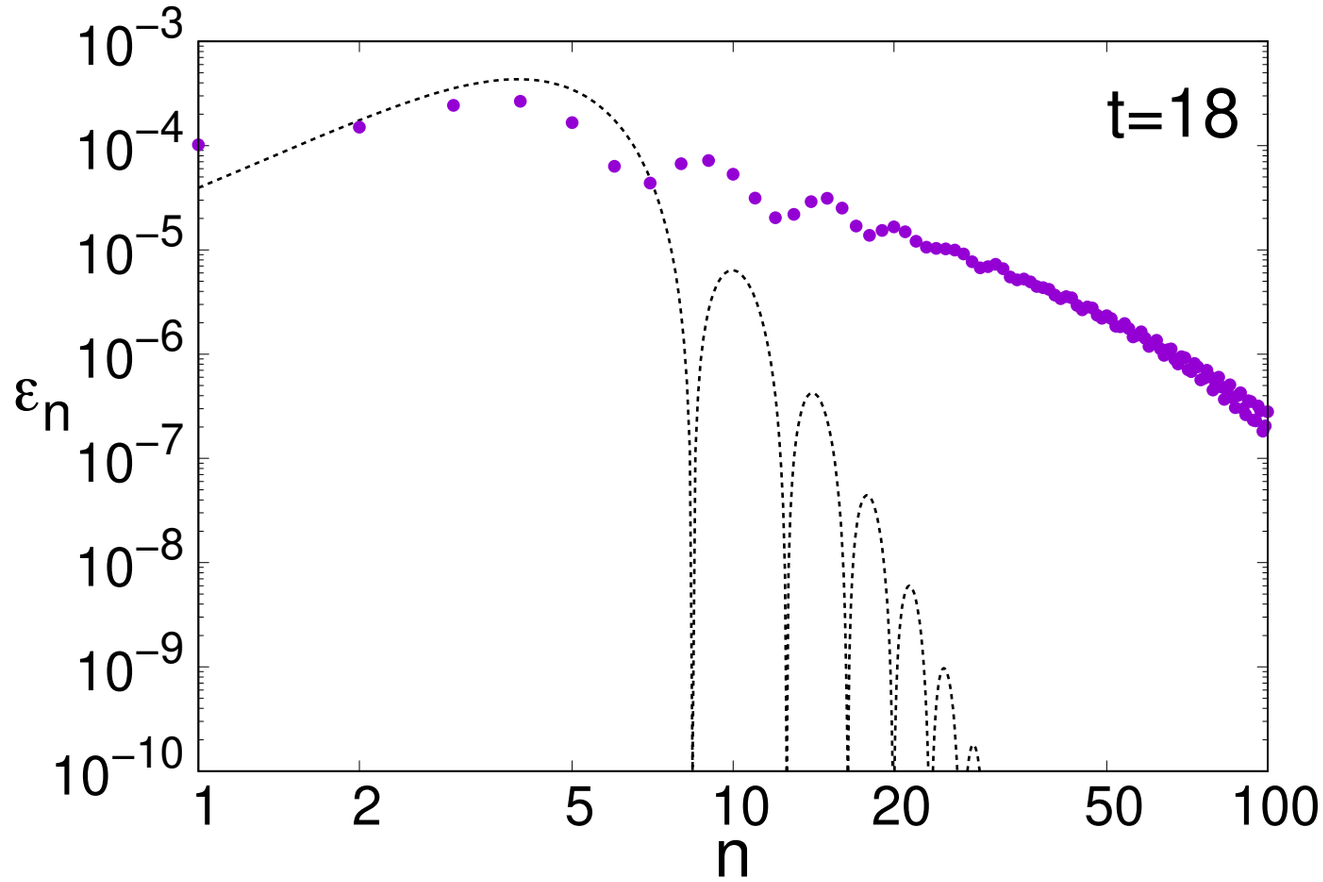
Energy spectrum: $\psi_b=\pi/4$



$\Delta t=2$, $\varepsilon=0.01$

*log-log plot

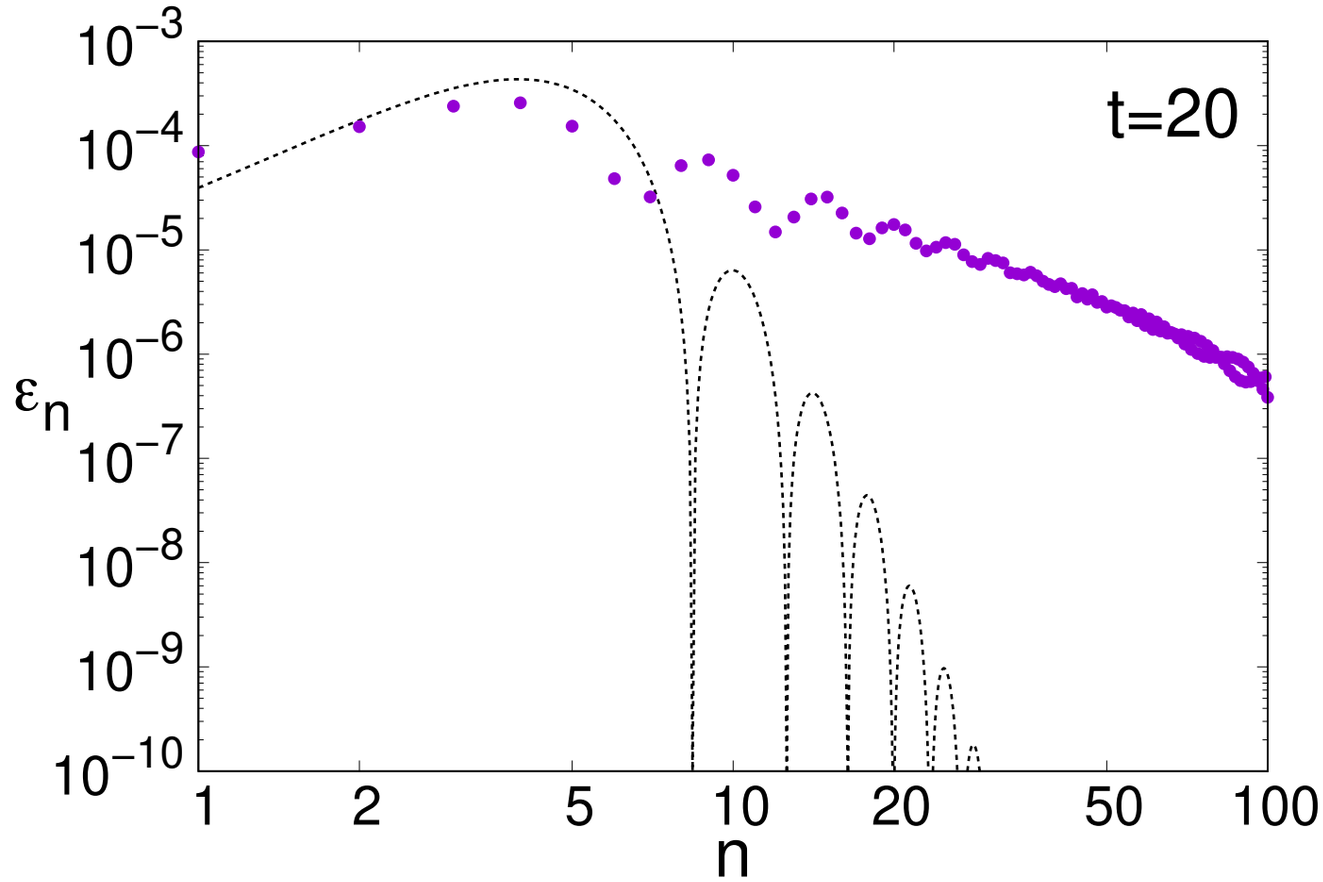
Energy spectrum: $\psi_b=\pi/4$



$\Delta t=2$, $\varepsilon=0.01$

*log-log plot

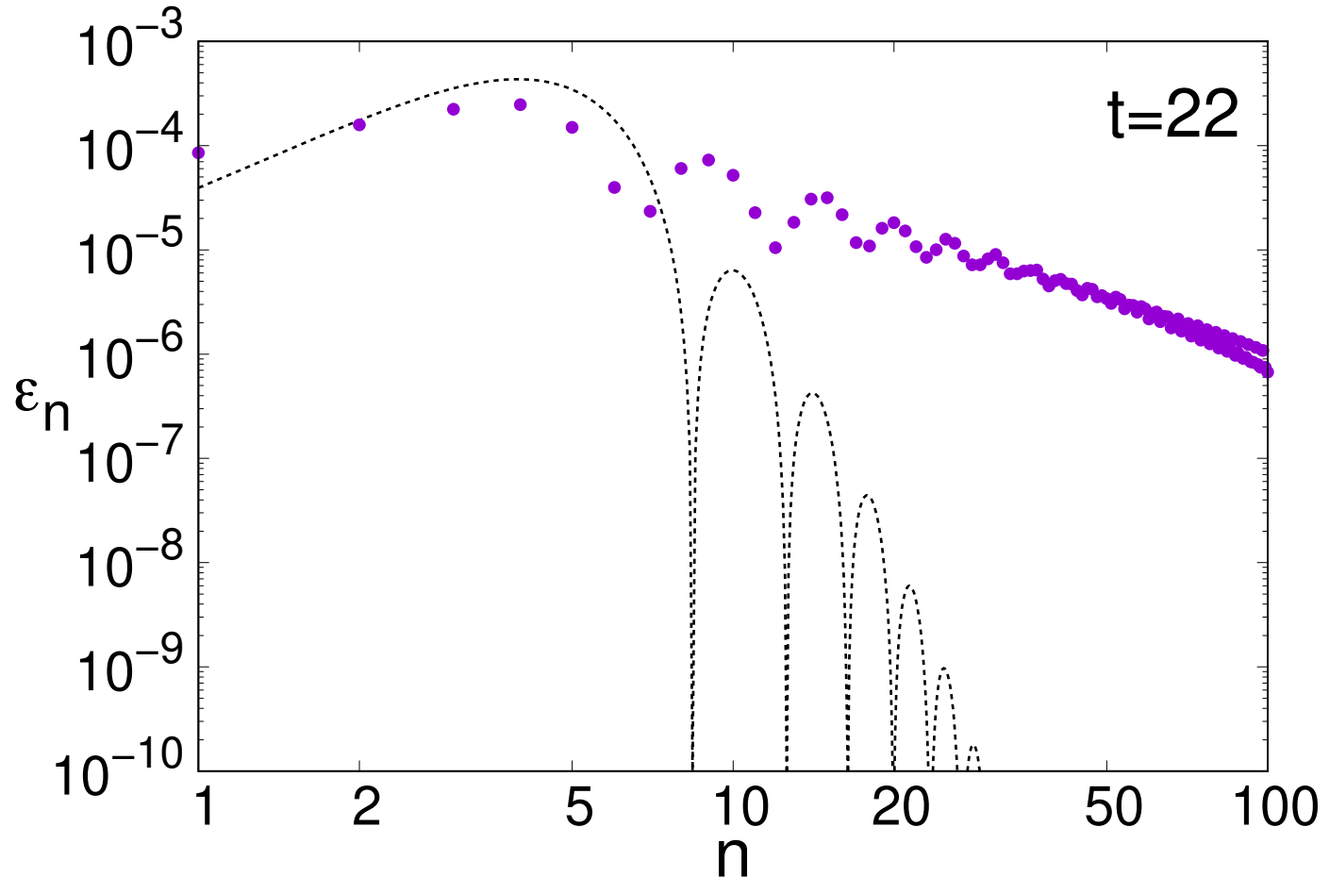
Energy spectrum: $\psi_b=\pi/4$



$\Delta t=2$, $\varepsilon=0.01$

*log-log plot

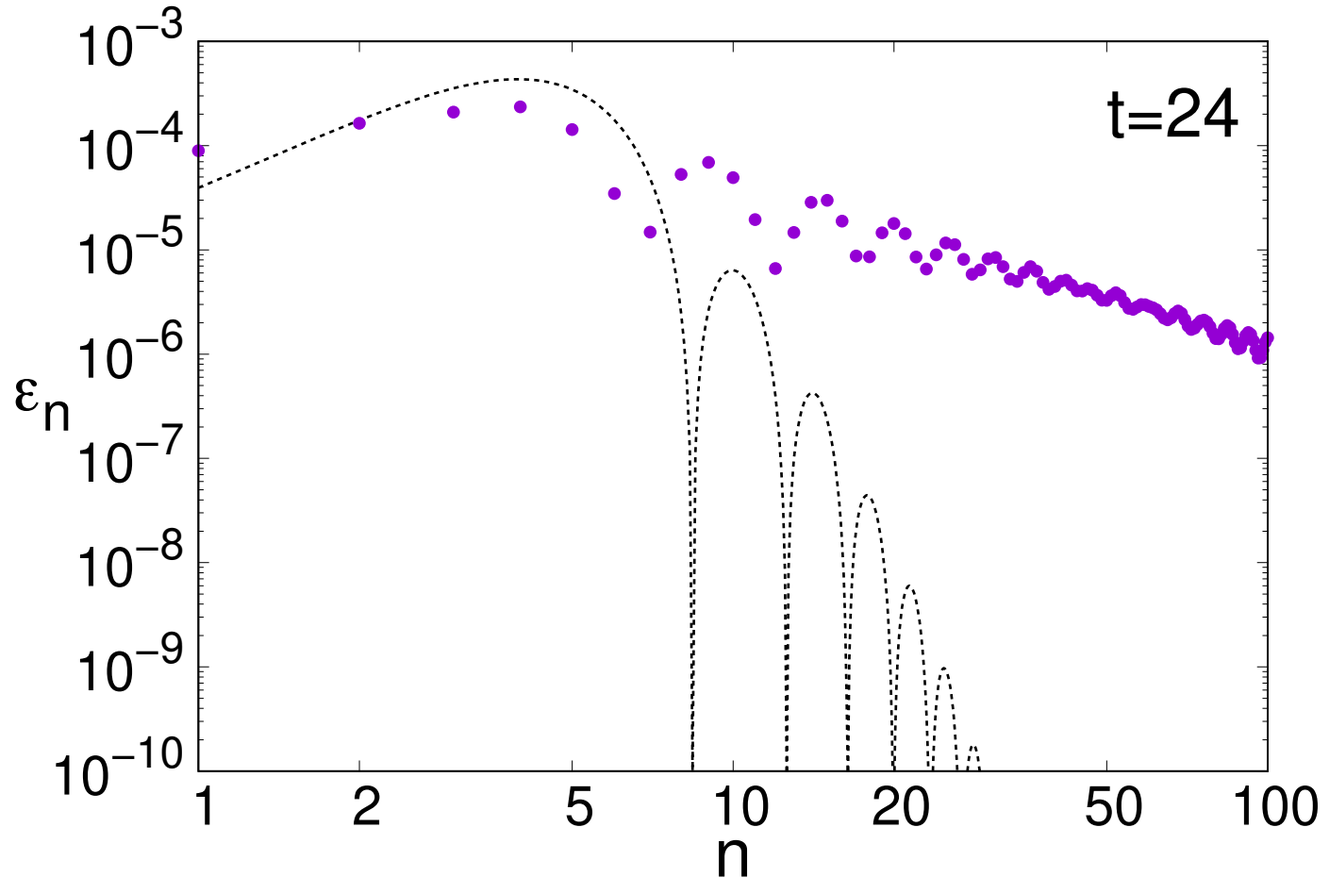
Energy spectrum: $\psi_b=\pi/4$



$\Delta t=2$, $\varepsilon=0.01$

*log-log plot

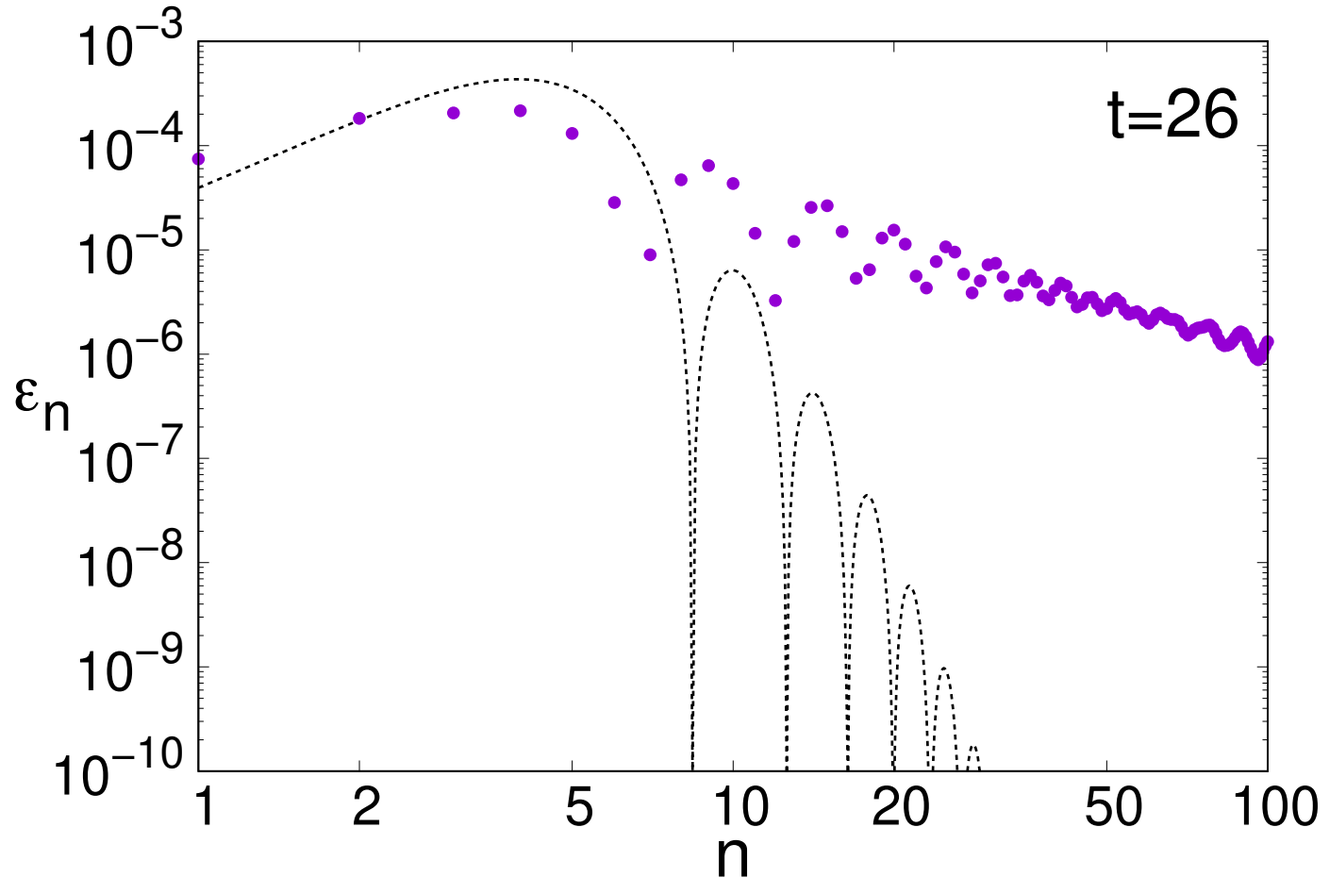
Energy spectrum: $\psi_b=\pi/4$



$\Delta t=2$, $\varepsilon=0.01$

*log-log plot

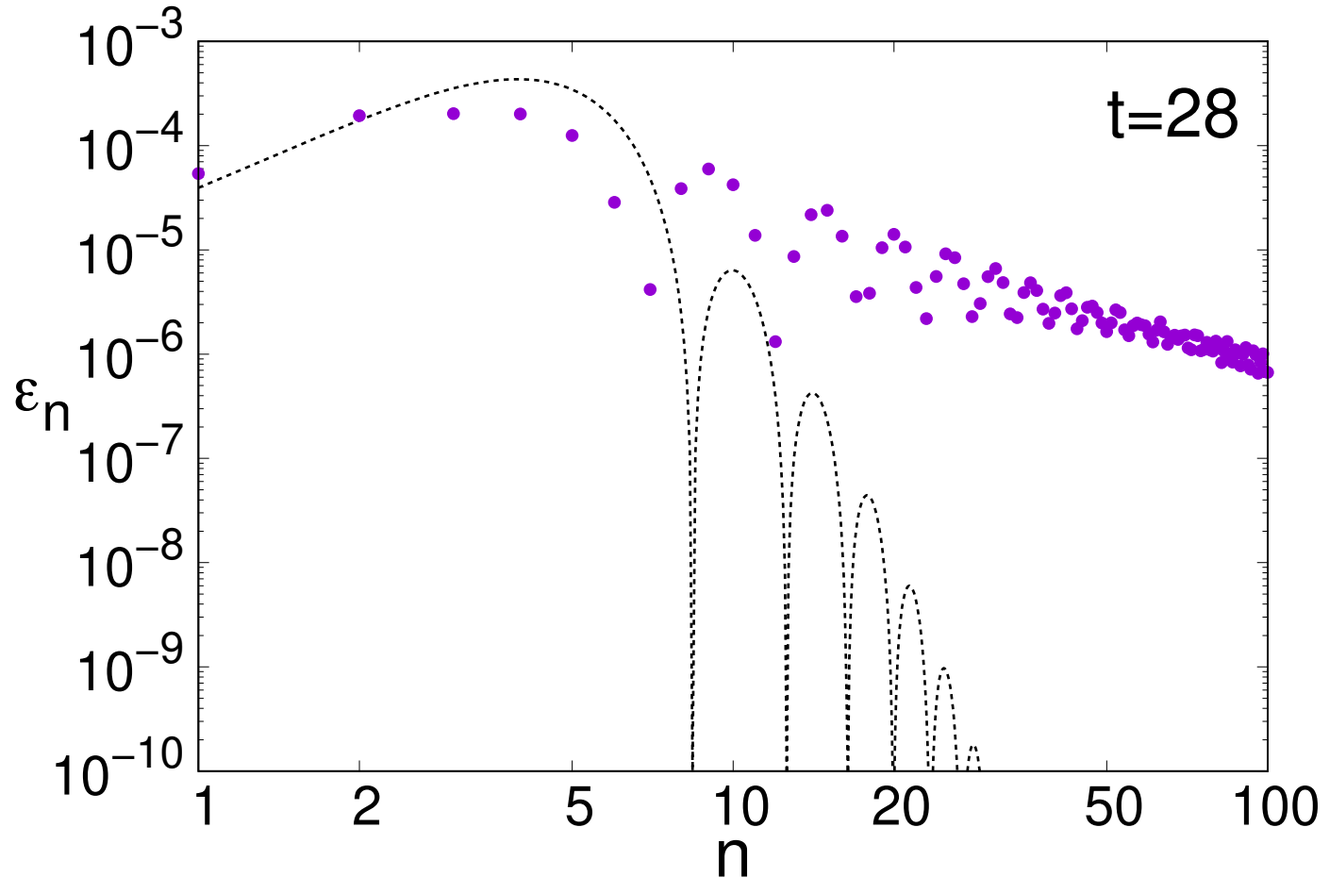
Energy spectrum: $\psi_b=\pi/4$



$\Delta t=2$, $\varepsilon=0.01$

*log-log plot

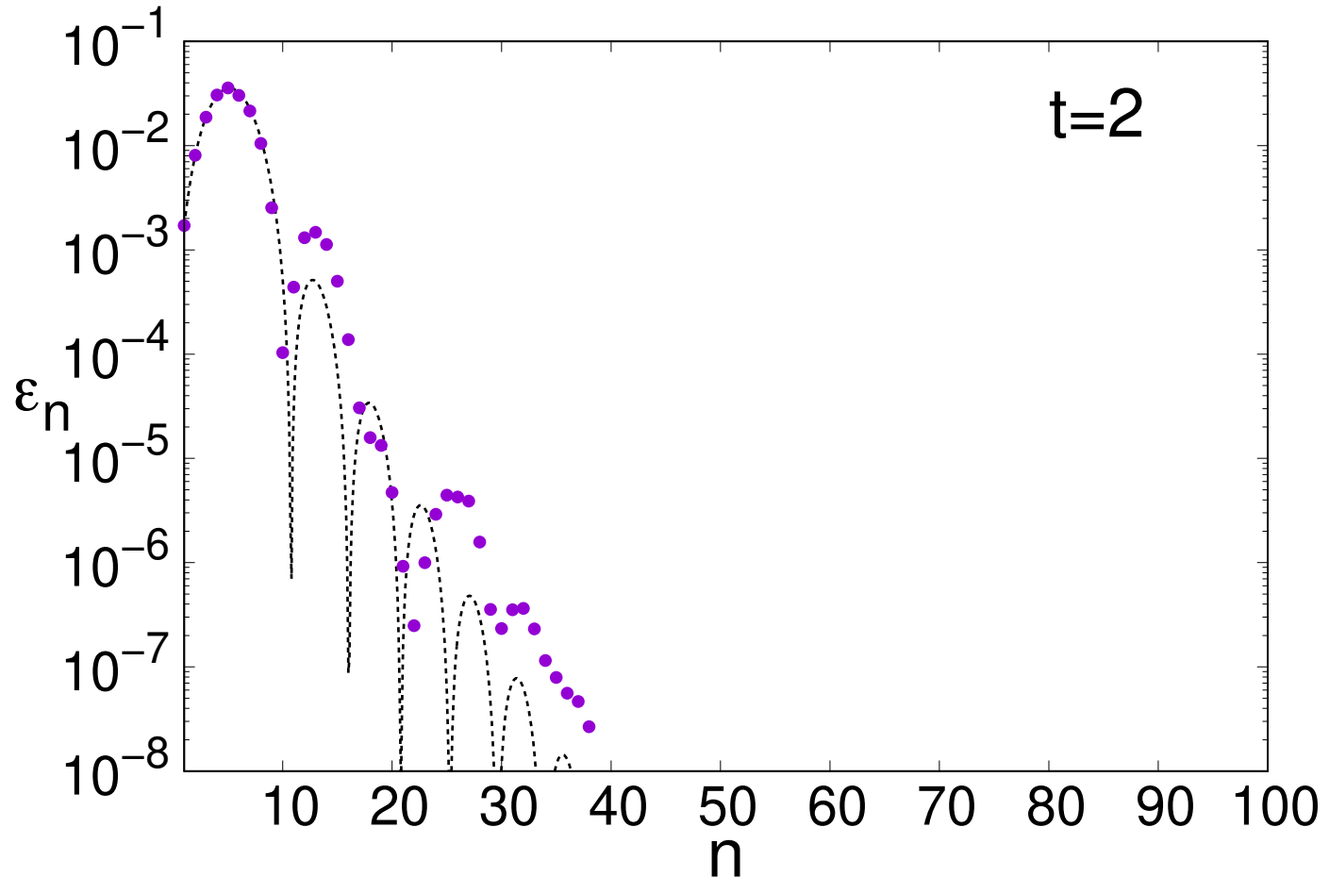
Energy spectrum: $\psi_b=\pi/4$



$\Delta t=2$, $\varepsilon=0.01$

*log-log plot

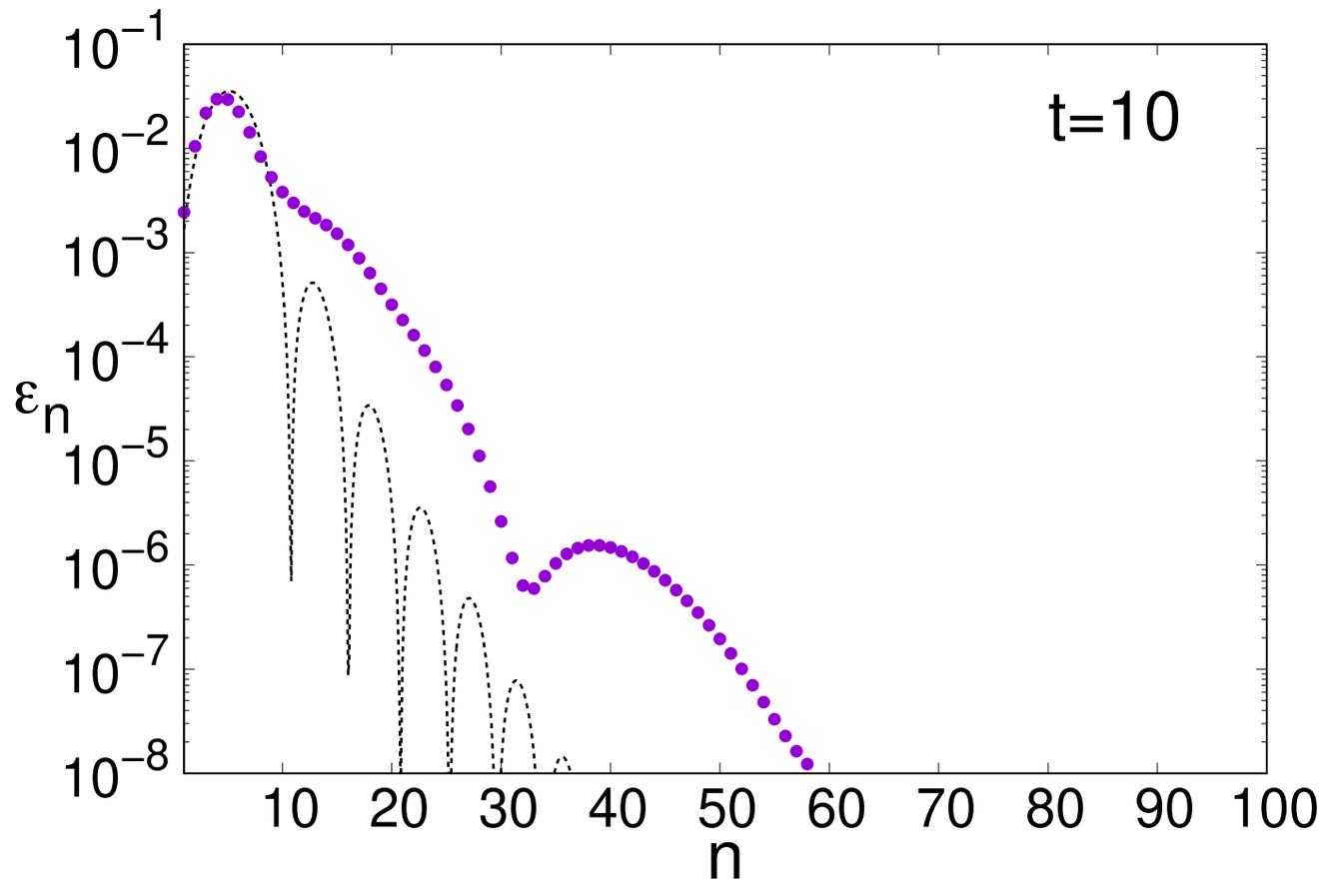
Energy spectrum: $\psi_b=\pi/2$



$\Delta t=2$, $\varepsilon=0.1$

*log plot

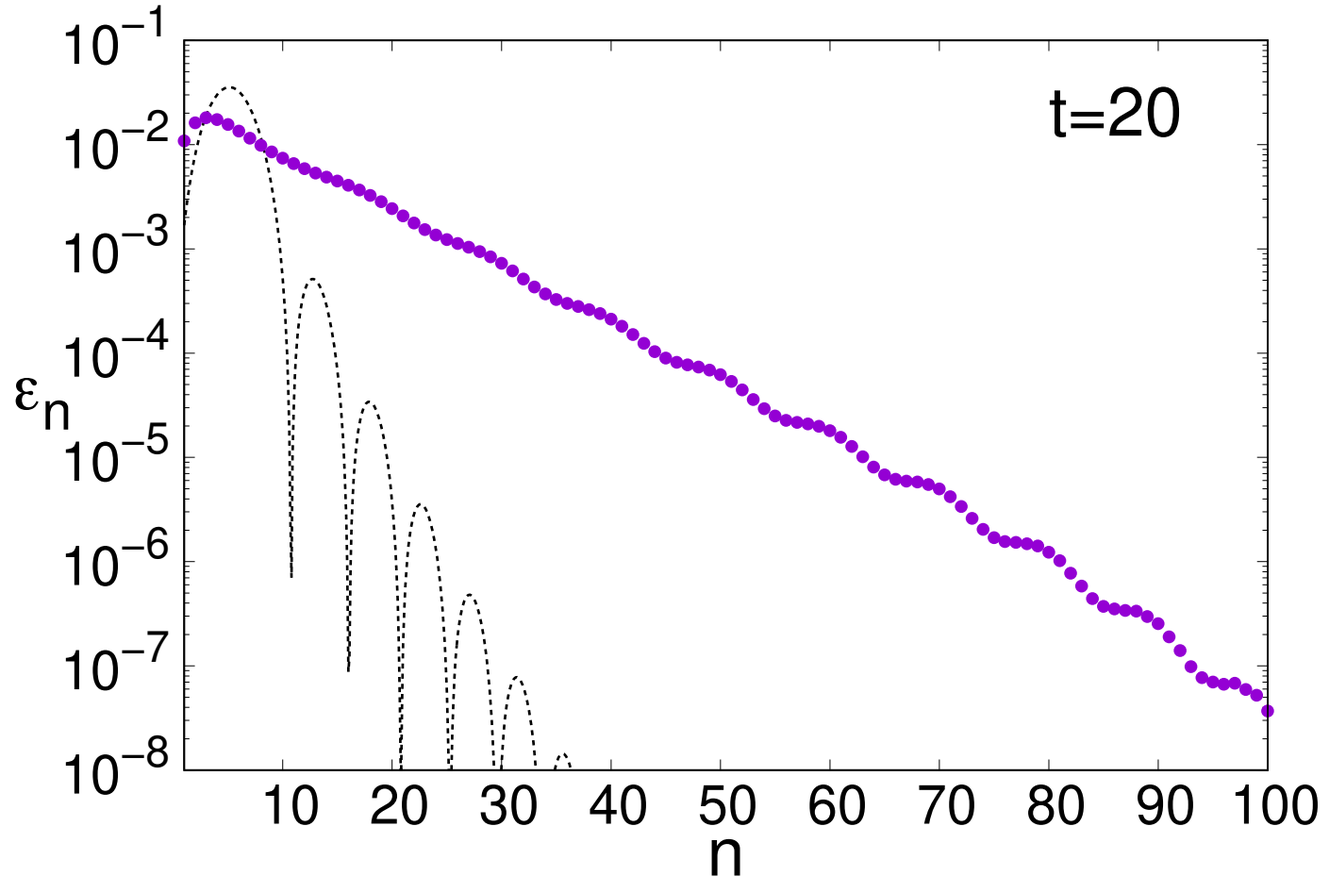
Energy spectrum: $\psi_b=\pi/2$



$\Delta t=2$, $\varepsilon=0.1$

*log plot

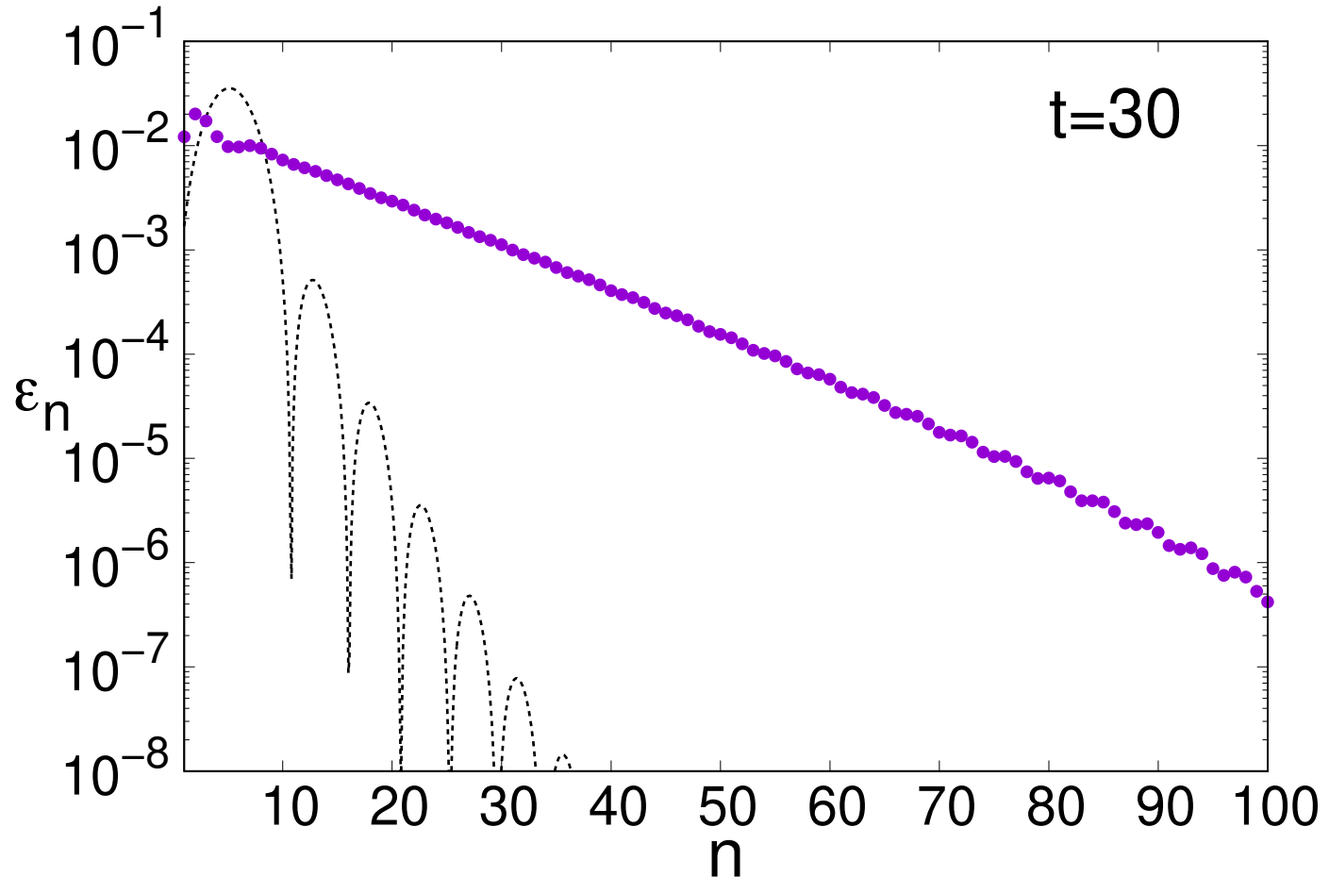
Energy spectrum: $\psi_b=\pi/2$



$\Delta t=2$, $\varepsilon=0.1$

*log plot

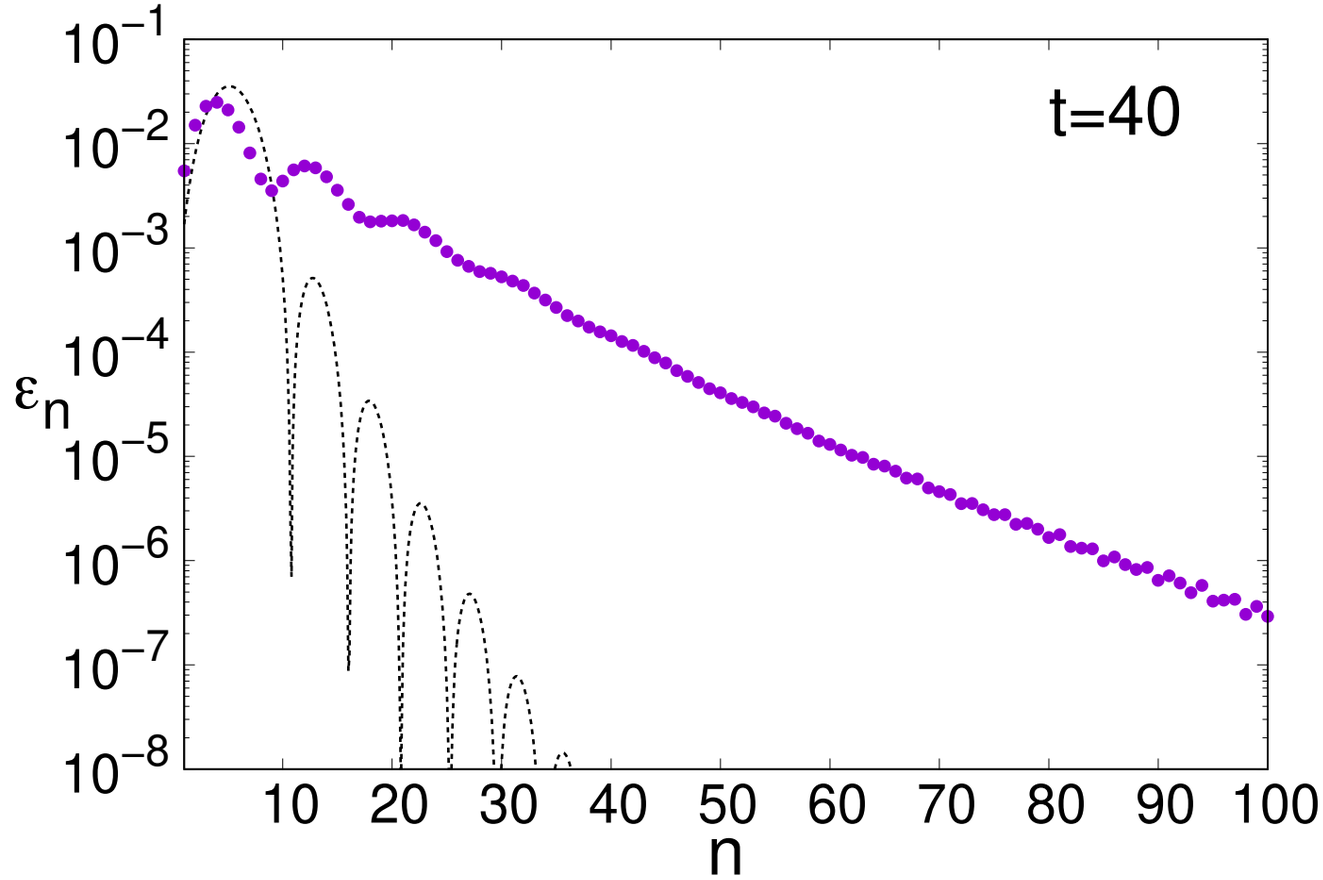
Energy spectrum: $\psi_b=\pi/2$



$\Delta t=2$, $\varepsilon=0.1$

*log plot

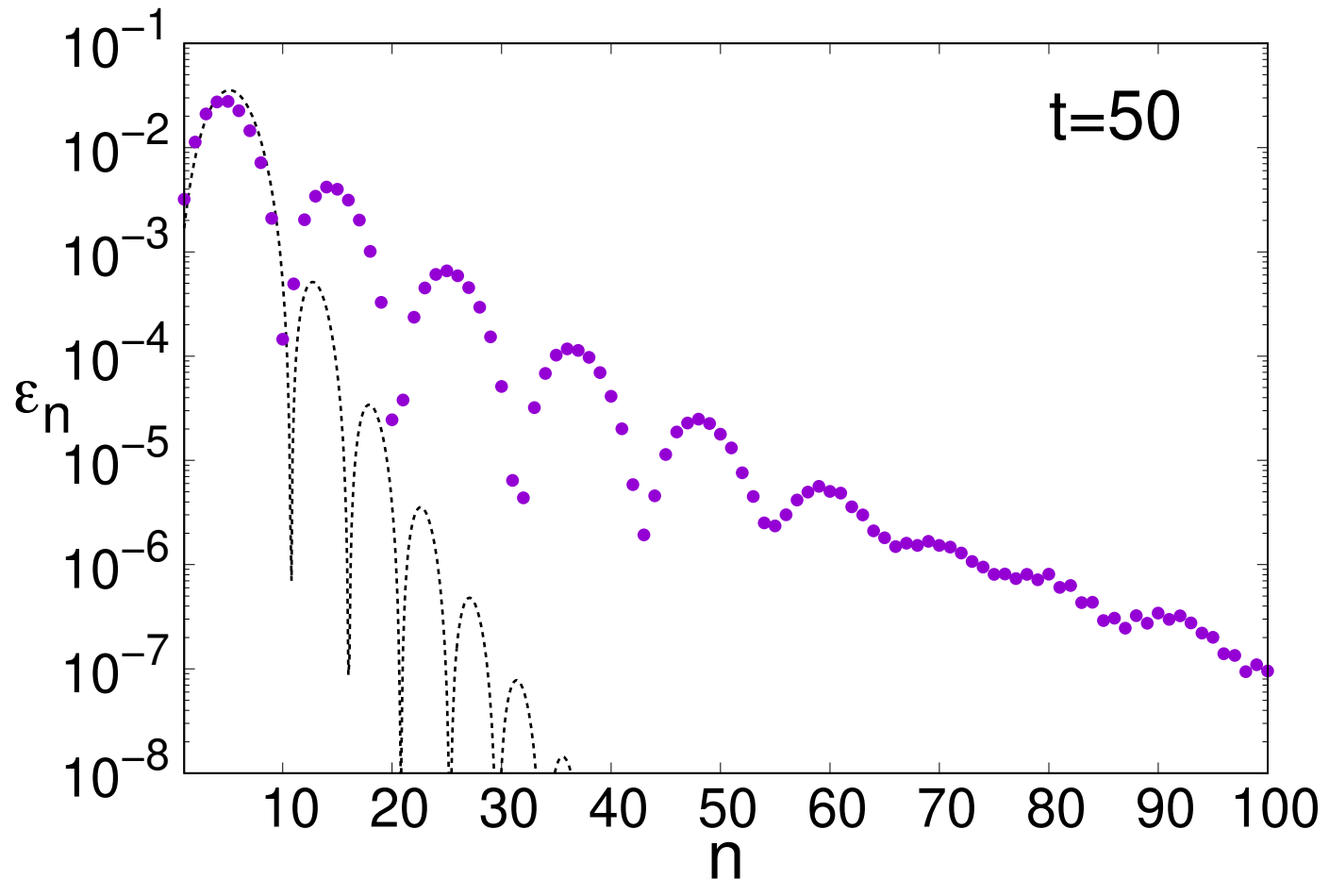
Energy spectrum: $\psi_b=\pi/2$



$\Delta t=2$, $\varepsilon=0.1$

*log plot

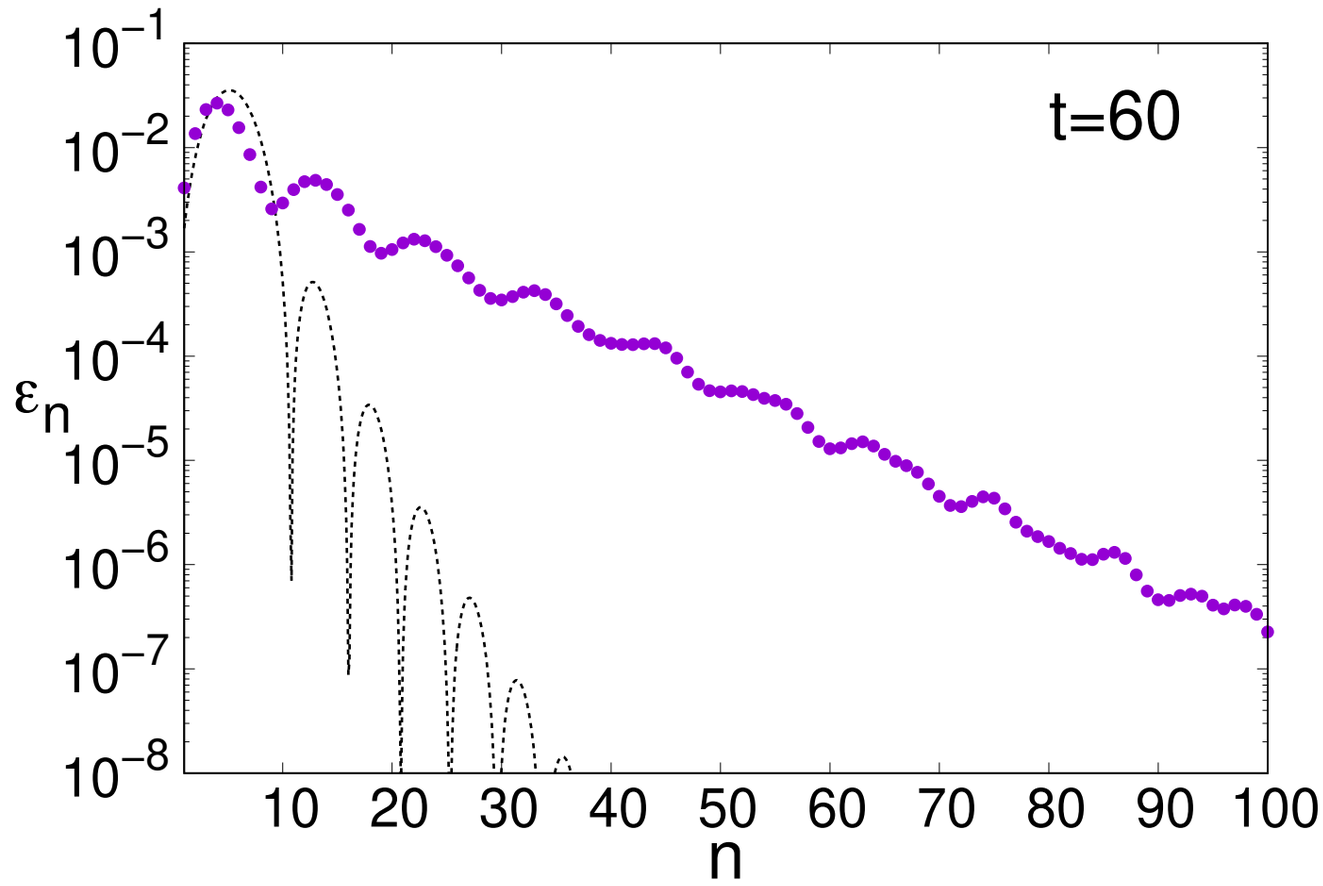
Energy spectrum: $\psi_b=\pi/2$



$\Delta t=2$, $\varepsilon=0.1$

*log plot

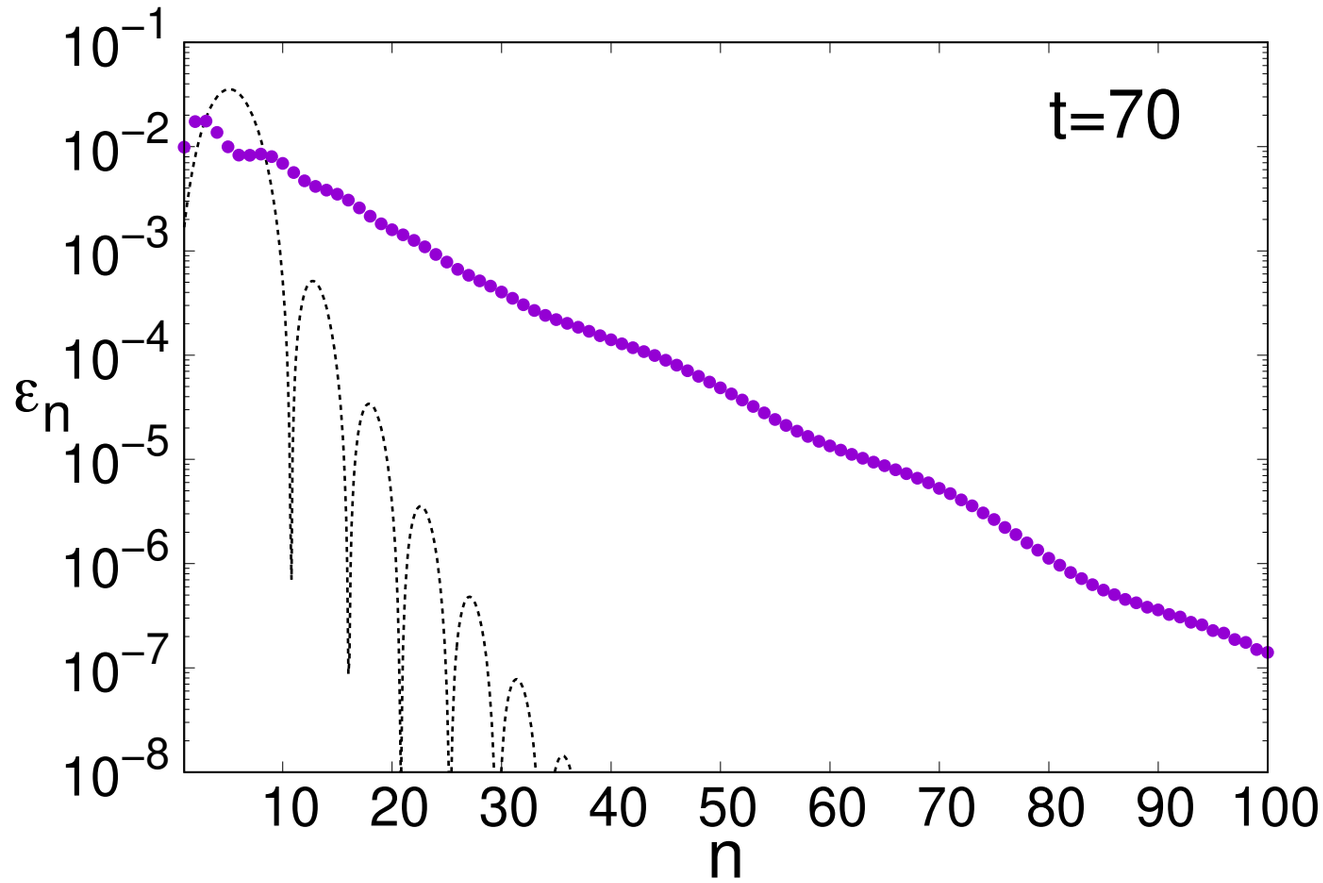
Energy spectrum: $\psi_b=\pi/2$



$\Delta t=2$, $\varepsilon=0.1$

*log plot

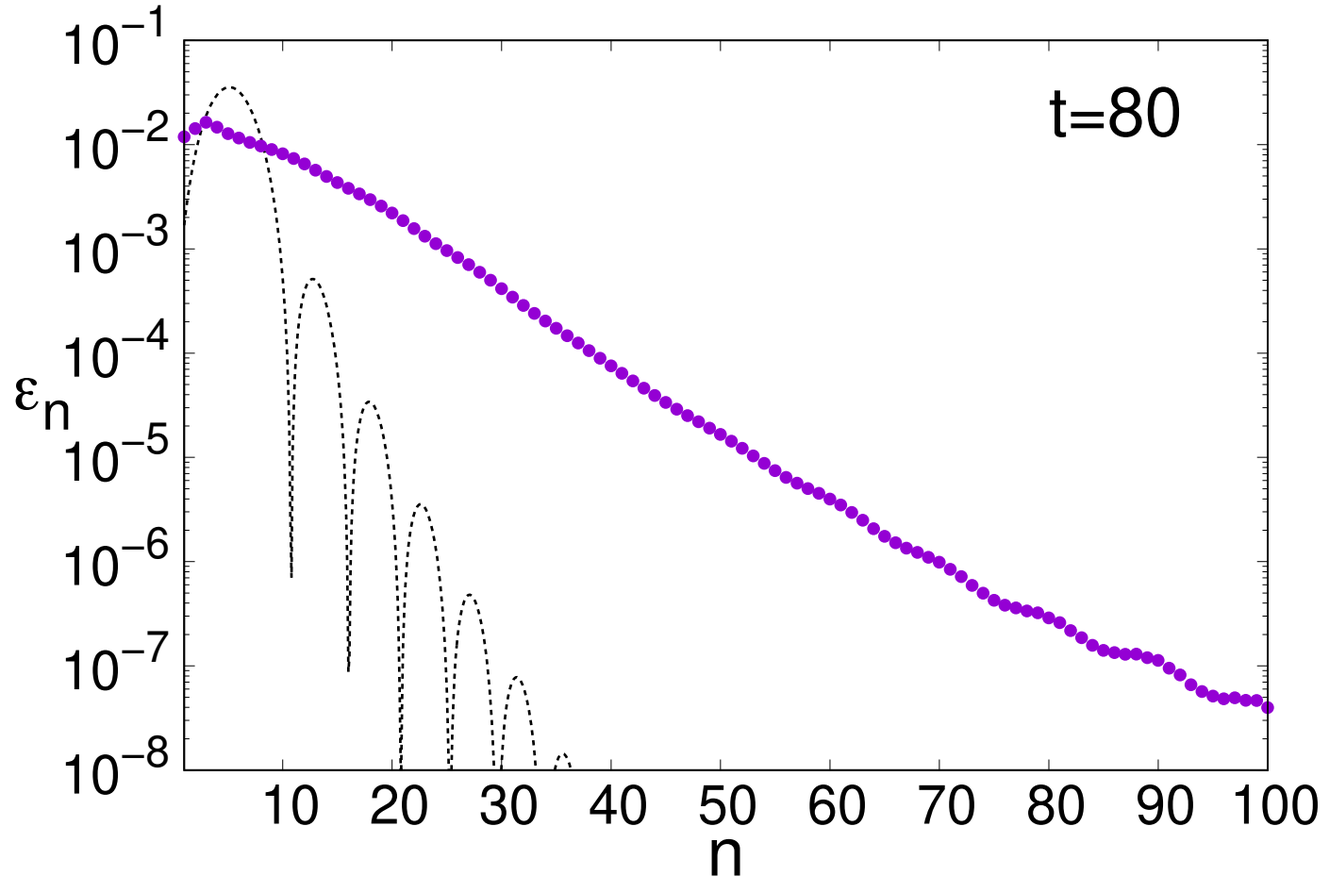
Energy spectrum: $\psi_b=\pi/2$



$\Delta t=2$, $\varepsilon=0.1$

*log plot

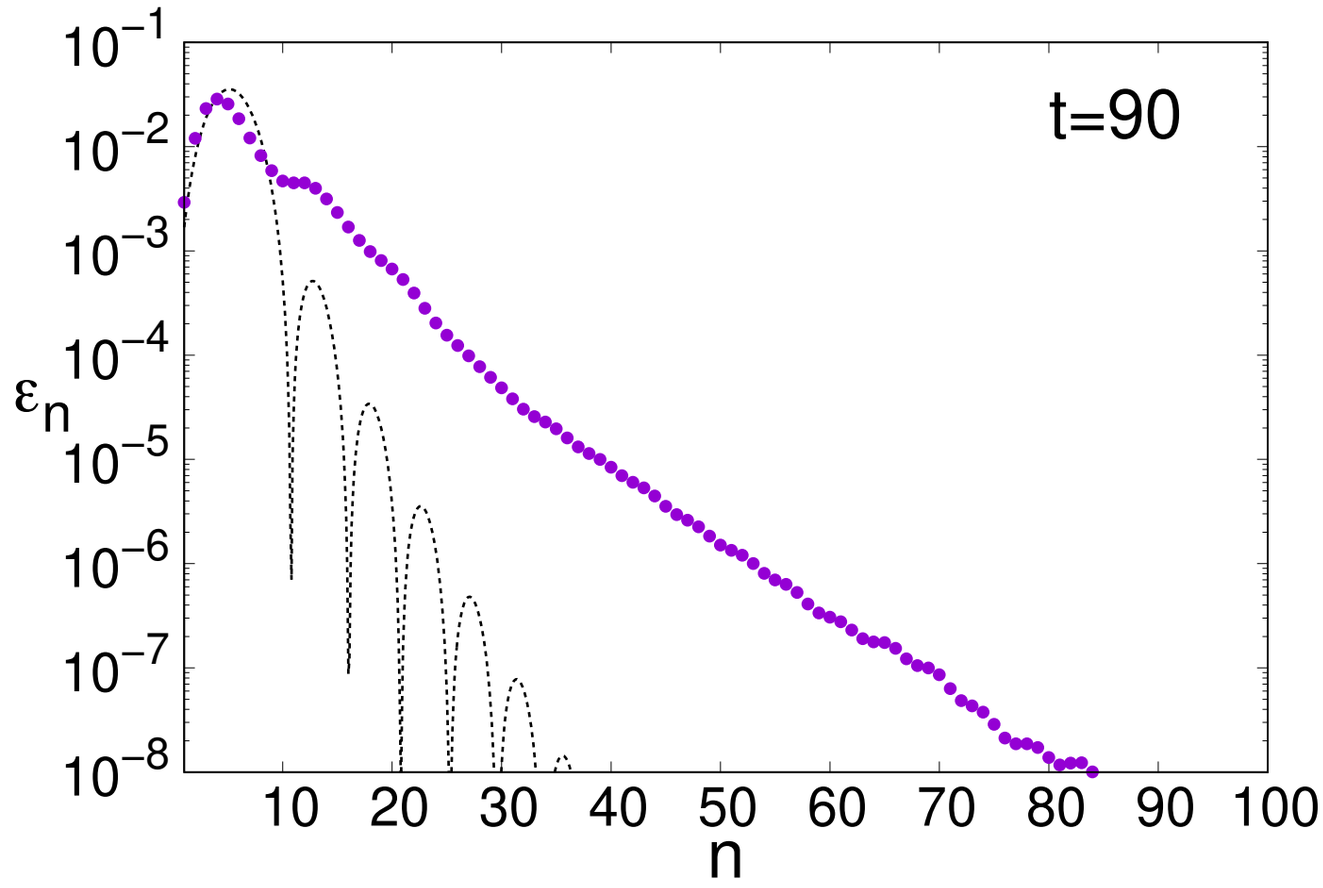
Energy spectrum: $\psi_b=\pi/2$



$\Delta t=2$, $\varepsilon=0.1$

*log plot

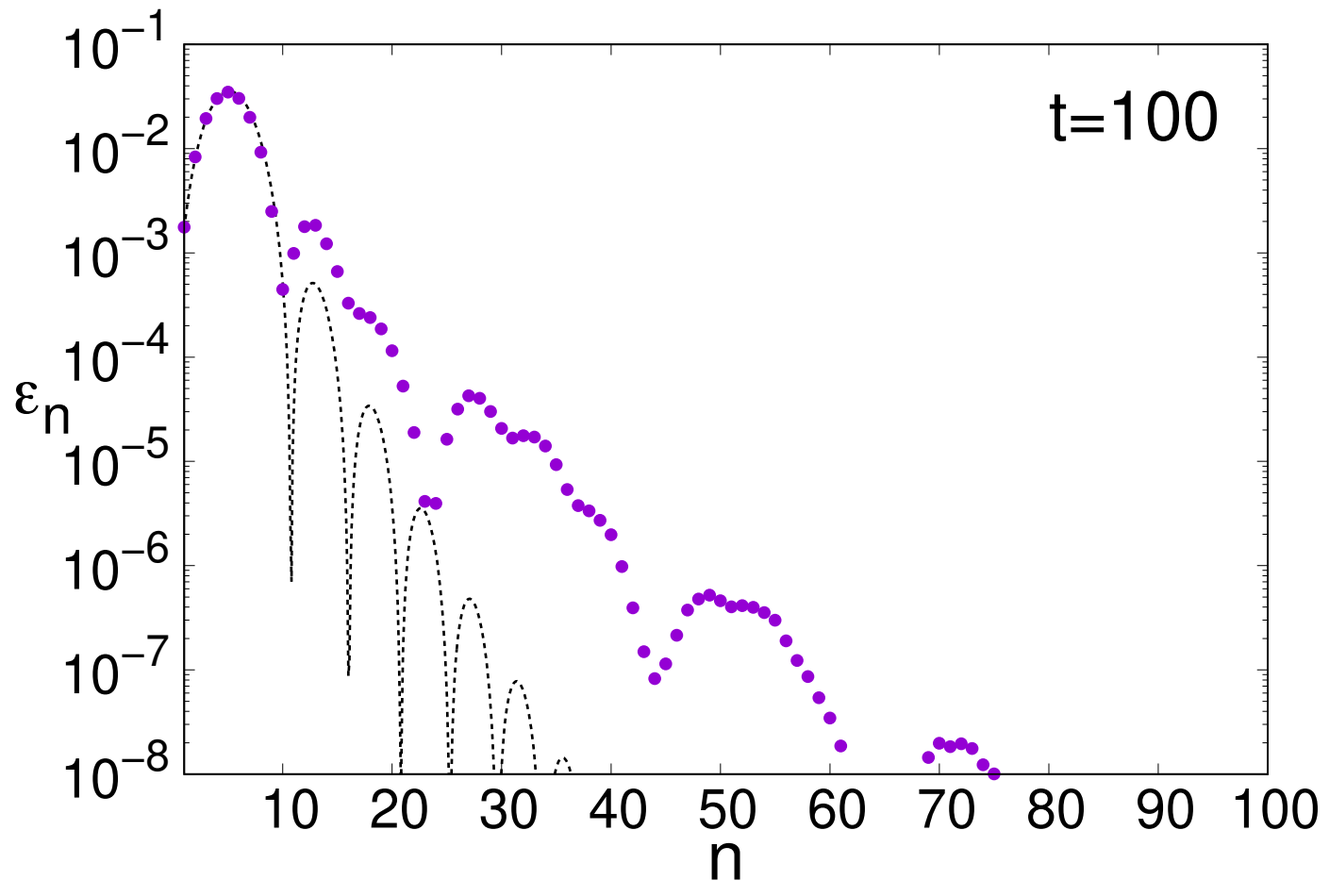
Energy spectrum: $\psi_b=\pi/2$



$\Delta t=2$, $\varepsilon=0.1$

*log plot

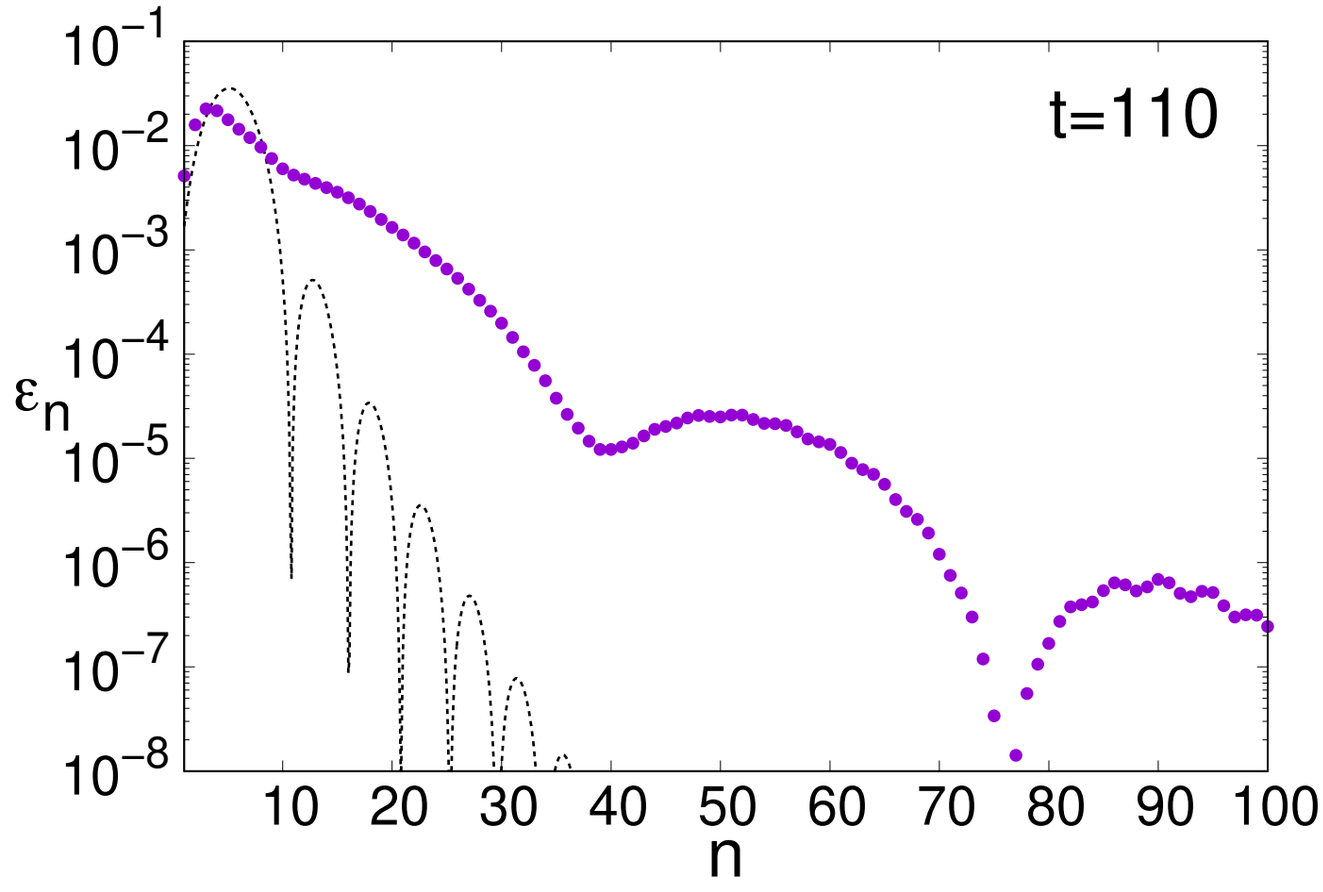
Energy spectrum: $\psi_b=\pi/2$



$\Delta t=2$, $\varepsilon=0.1$

*log plot

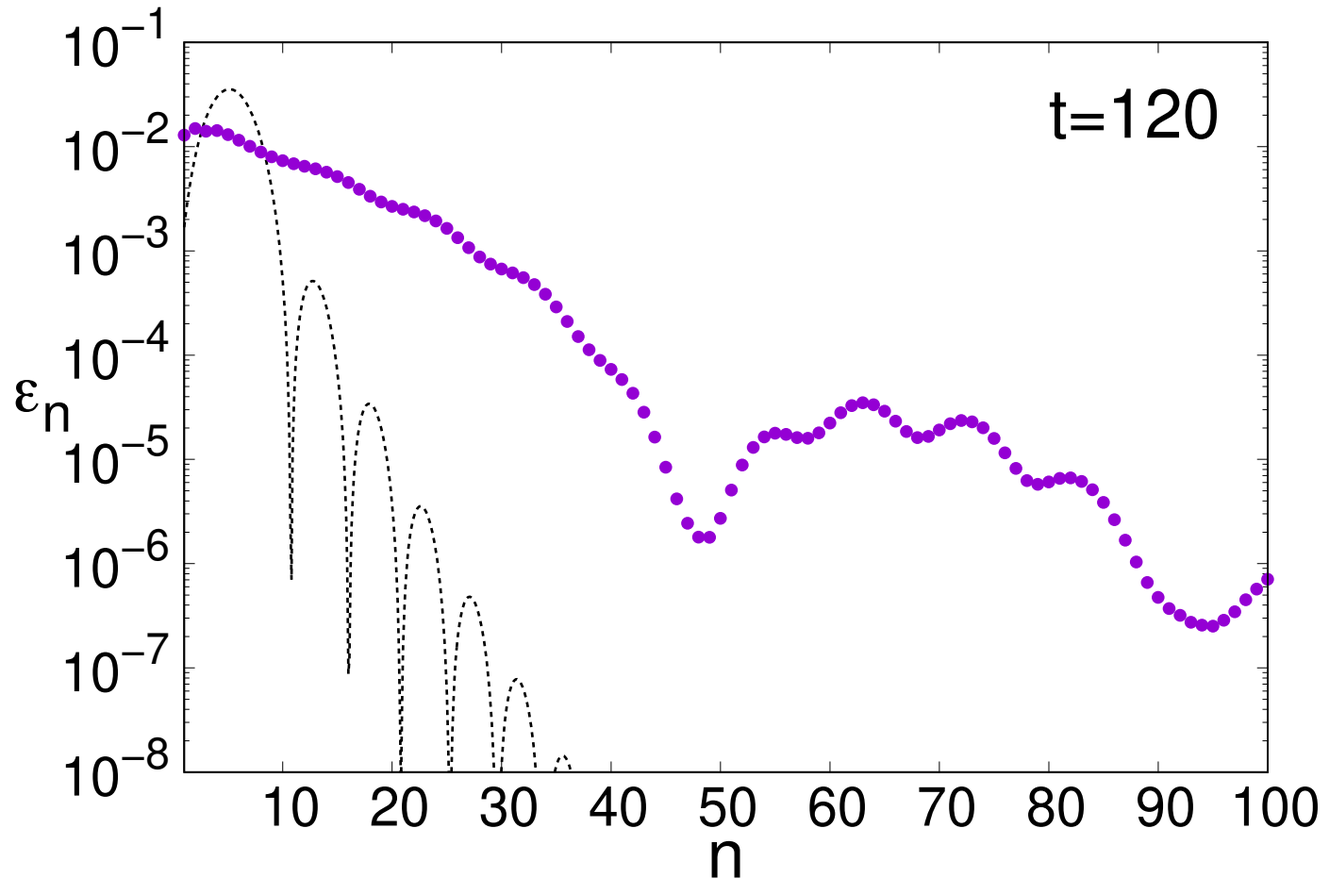
Energy spectrum: $\psi_b=\pi/2$



$\Delta t=2$, $\varepsilon=0.1$

*log plot

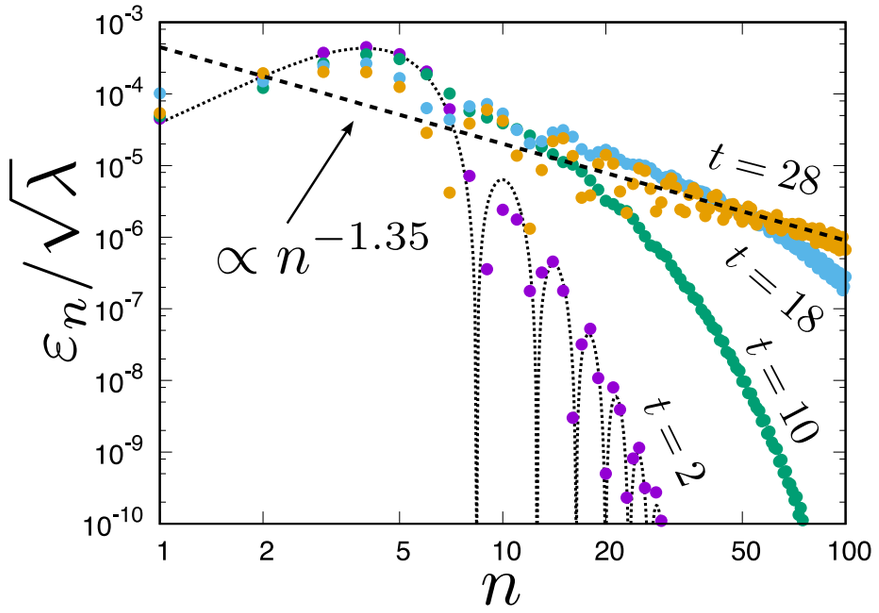
Energy spectrum: $\psi_b=\pi/2$



$\Delta t=2$, $\varepsilon=0.1$

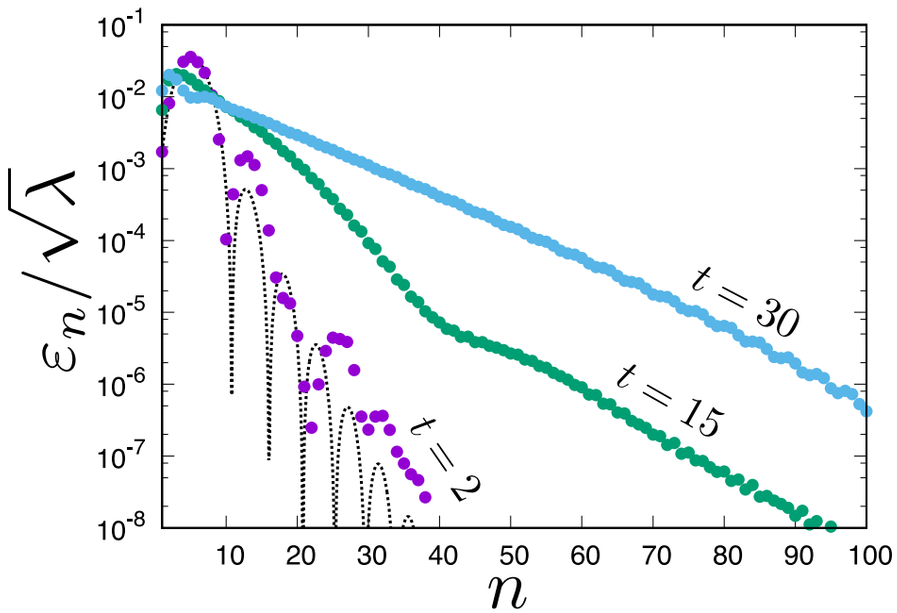
*log plot

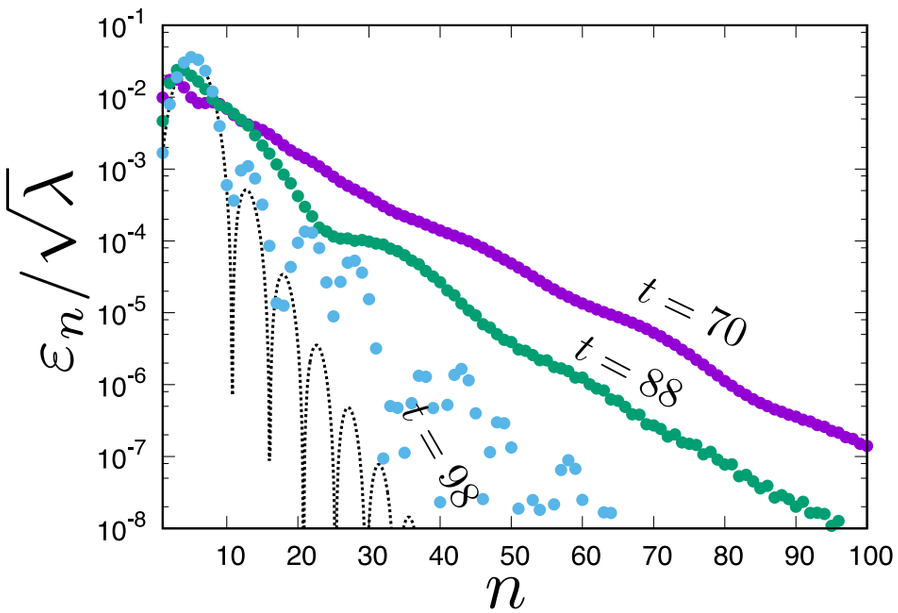
Energy spectrum in global AdS



$\psi_b=\pi/4$: power law

$\psi_b=\pi/2$: no power law but time modulation





Conclusions

Open string in AdS exhibit turbulent behavior in Poincare/global AdS due to nonlinearity.

This results in cusp formation and energy cascades.

The cusp formation is suppressed in the antipodal string embedding in global AdS.

Outlook

Analytic aspects of the turbulent strings