Black Hole with Intertwined Order: Holographic Pair Density Wave

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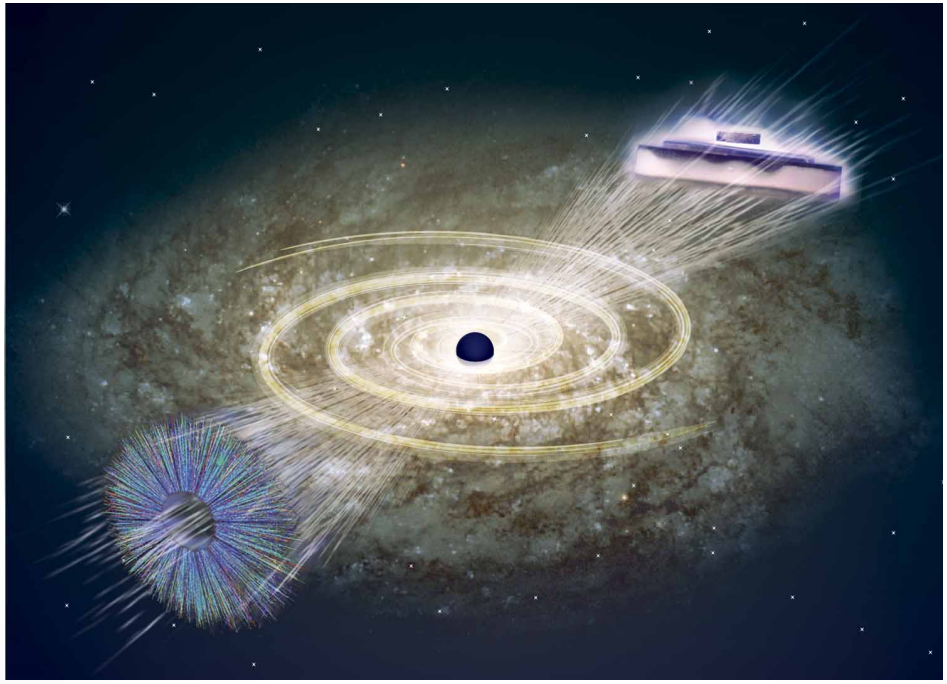
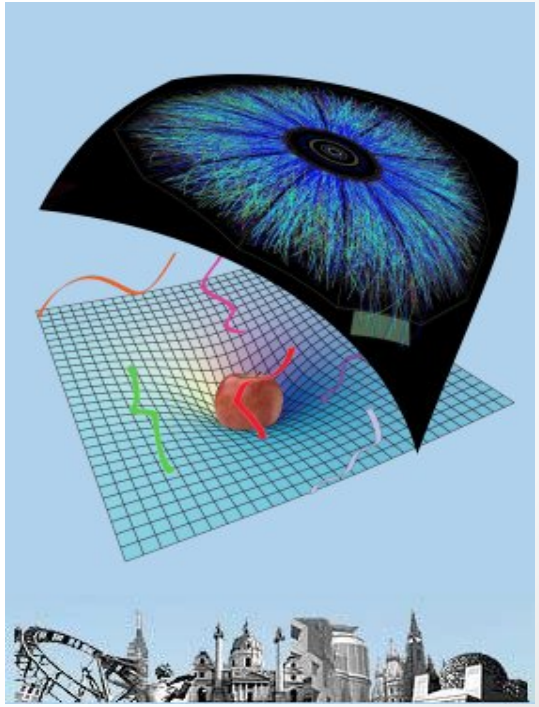


in collaboration with Ren-Gen Cai, Yong-Qiang Wang and Jan Zaanen

Outline

- Introduction
- The Holographic Theory
- Conclusion and Outlook



Introduction

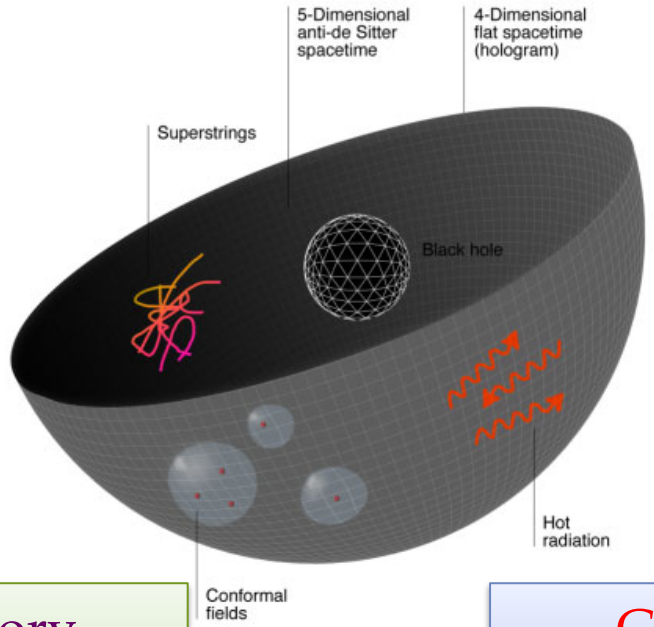
Gauge/Gravity correspondence

Gauge Theory
Quantum Field Theory

⟺

Gravity theory
in high dimensions

Conjecture which follows from a low-energy limit of string theory.



Holography:

Quantum field theory
d dimensional spacetime

⟺

Gravitational theory
d+1 dimensional spacetime

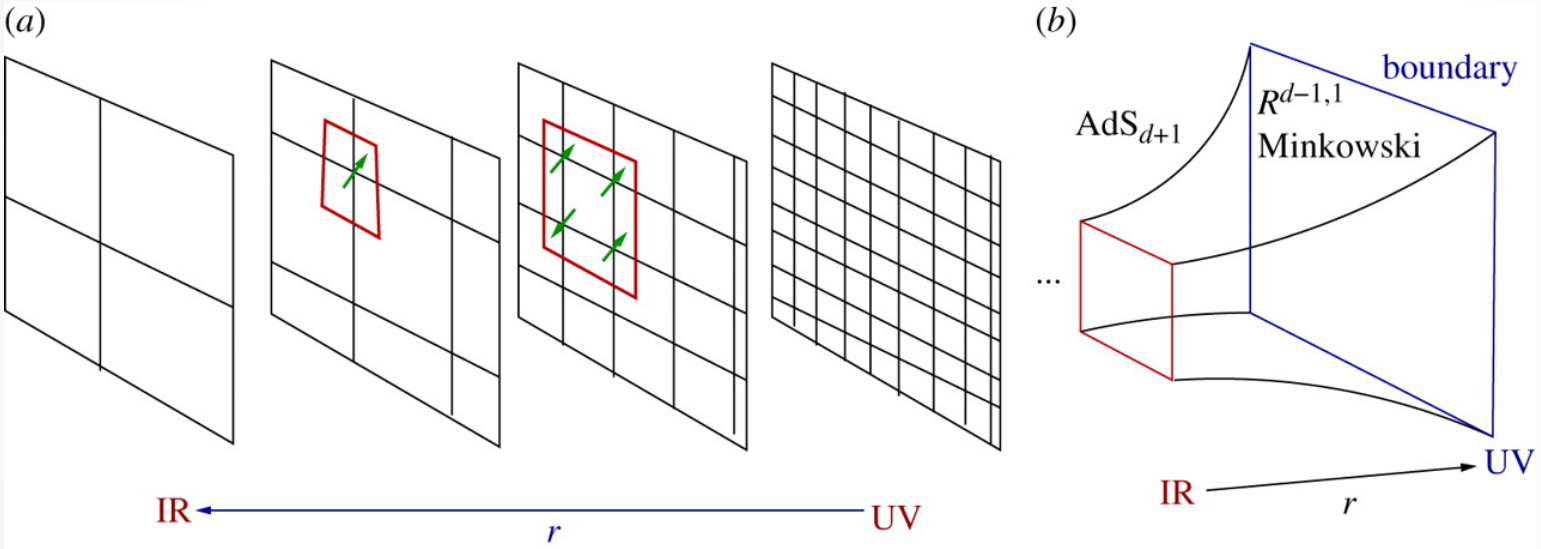
J. Maldacena, arXiv:hep-th/9711200; E. Witten, arXiv:hep-th/9802150; S.S.Gubser et al, arXiv:hep-th/9802109

General Relativity = Renormalization Group

The basic example: AdS=CFT

(dimensions: d+1 d)

$$ds^2 = \frac{r^2}{R^2}(-dt^2 + d\boldsymbol{x}^2) + R^2\frac{dr^2}{r^2}$$



(a): A series of block spin transformations labeled by a parameter r.

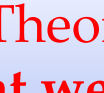
(b): AdS space, which organizes the field theory information in the same way.

Excitations with different wavelength get put in different place in the bulk picture.

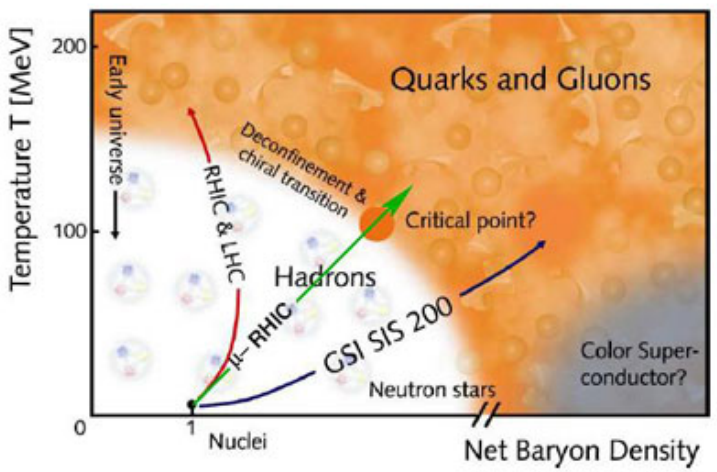
(arXiv: 1101.0597[hep-th])

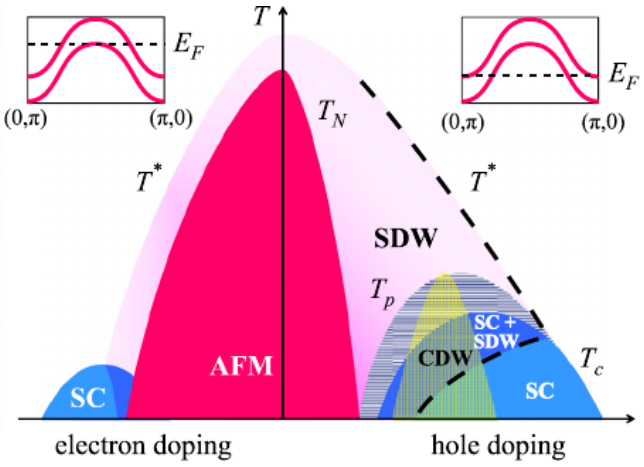
Why holographic duality is important ?

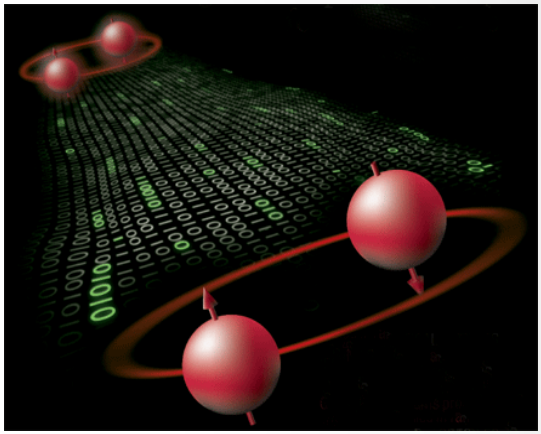
Quantum field theory
at strong coupling



Theory of gravitation
at weak coupling



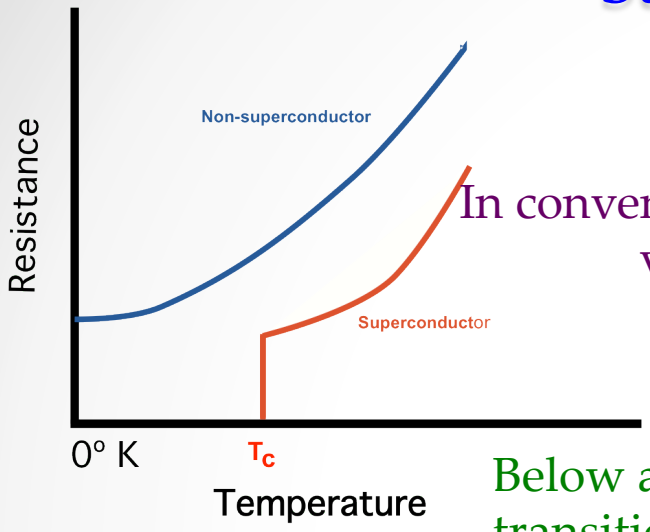




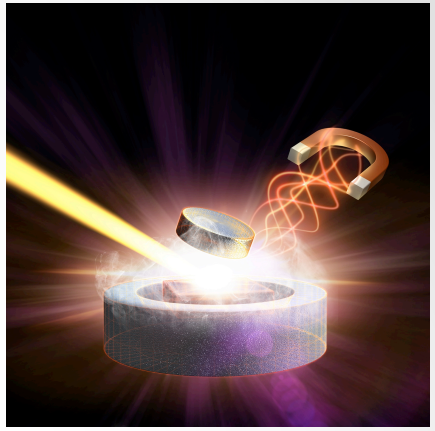
Applied holography:

QGP and QCD (drag force, jet quenching, confinement/deconfinement,…),
Condensed matter (quantum criticality, strange metal, superconductivity,…),
Quantum Entanglement, Non-equilibrium dynamics…

Superconductivity

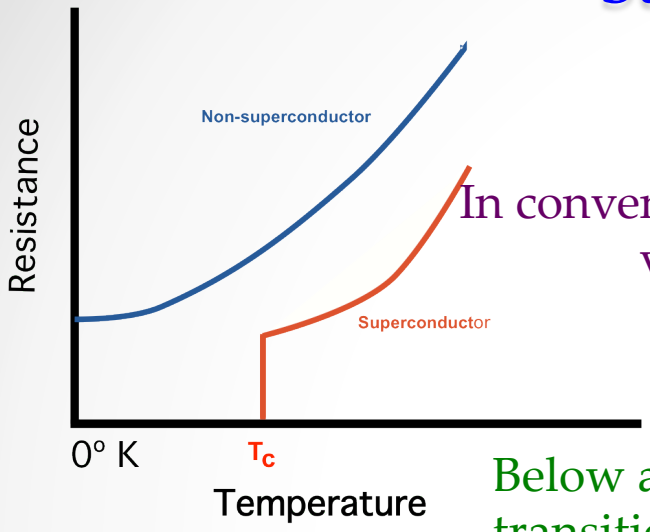


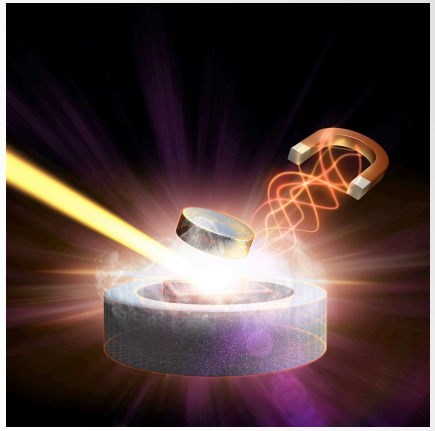
In conventional SC electron-electron pairing via phonon exchange (Cooper pair).



Below a critical temperature Tc, there is a second order phase transition and the DC conductivity becomes infinite.

Superconductivity



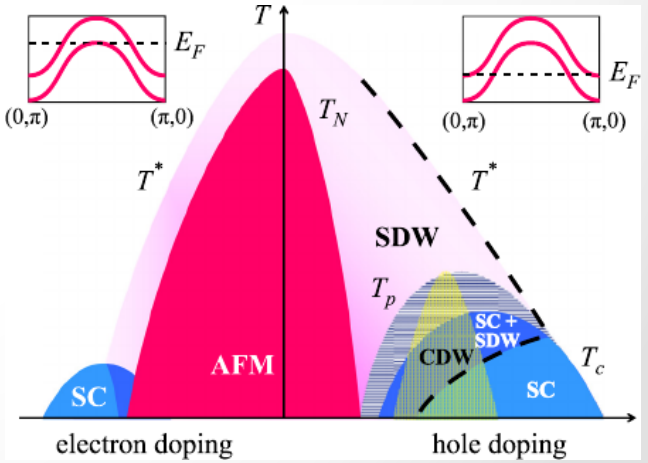


In conventional SC electron-electron pairing via phonon exchange (Cooper pair).

Below a critical temperature Tc, there is a second order phase transition and the DC conductivity becomes infinite.

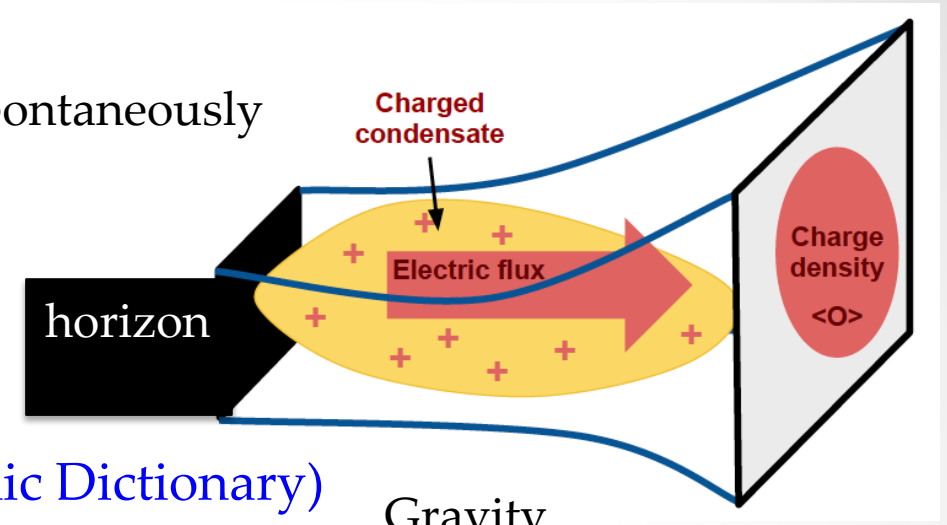
The mechanism is not well understood for HTSC (cuprates, heavy fermions, iron-based SCs, organic SCs…). Unlike BCS theory, it involves the dynamics with strong interaction.

Holographic duality provides a window into the unconventional physics of these systems.



Build a holographic superconductor

The key point: break U(1) symmetry spontaneously



Essential ingredients:

(Holographic Dictionary)

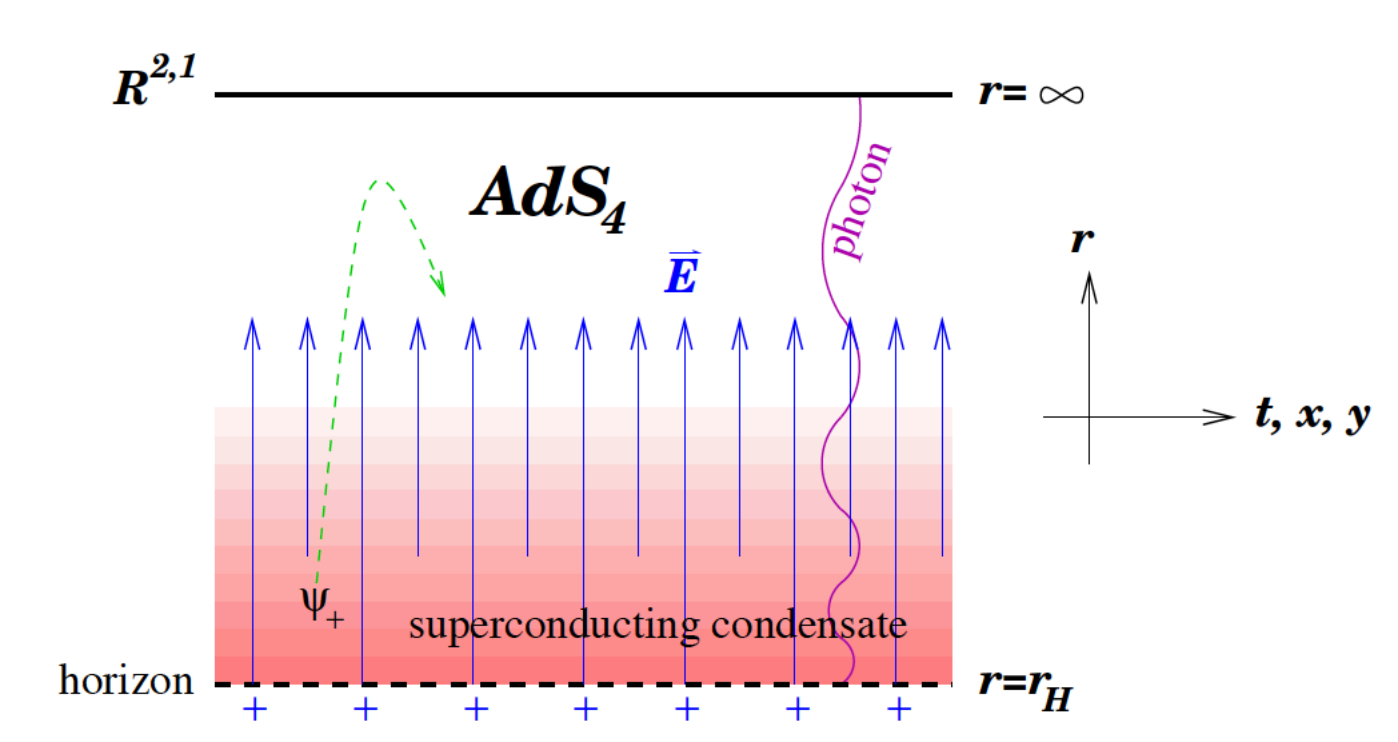
Superconductor		Gravity
temperature	⟺	black hole
chemical potential/ charge density	⟺	U(1) gauge field
condensate/ scalar operator	⟺	charged scalar field

Require a system that admits black holes with scalar hair at low T, but no hair at high T.

(arXiv:0803.3295[hep-th])

Superconducting Condensate:

a charge particle is in balance between **gravitational attraction** and **electric repulsion**.



(arXiv:0805.2960[hep-th])

II Spatially modulated black holes and intertwined orders in superconductivity

We construct stationary inhomogeneous black hole solutions in which both the U(1) symmetry and spatially translational symmetry are spontaneously broken at finite temperature and charge density.

Motivation:

Stationary solutions of the Einstein equation, especially black holes, are the most fundamental of all gravitational objects. The search for new stationary solutions will help to understand general relativity more broadly and deeply.

This novel black hole solution provides a dual description of inhomogeneous superconductor, where various orders appear to be intertwined and have comparable strength. This picture is believed to play an important role in the rich phase structure of high Tc SC.

(see Jan Zaanen's talk)

Gravity Setup:

$$S_0 = \frac{1}{2\kappa_N^2}\int d^4x\sqrt{-g}\left[\mathcal{R} - 2\Lambda + \mathcal{L}_m + \mathcal{L}_{cs}\right] ,$$

$$\mathcal{L}_m = -\frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \mathcal{F}(\chi)(\partial_\mu\theta - qA_\mu)^2 - \frac{Z(\chi)}{4}F_{\mu\nu}F^{\mu\nu} - V(\chi) ,$$

$$\mathcal{L}_{cs} = -\vartheta(\chi)\epsilon^{\mu\nu\lambda\sigma}F_{\mu\nu}F_{\lambda\sigma} . \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

The U(1) gauge field A is dual to conserved current, which provides the charge density.

χ and θ are real scalars. χ is charged under the U(1) gauge field and is the order parameter for U(1)−broken phase.

(a). Normal phase: no symmetry breaking χ=0

(b). Broken phase: break U(1) symmetry and translational invariance spontaneously

$$\chi \sim \chi(x)$$

R.G.Cai, **L.Li**, Y.Q.Wang, J.Zaanen, PRL , 119, 181601 (2017)

Broken phase: unidirectional case

$$ds^2 = \frac{L^2}{z^2}\Big[-H(z)U_1 dt^2 + \frac{U_2}{H(z)}dz^2 + U_3(dx + z^2 U_5 dz)^2$$
$$+U_4(dy + (1-z)U_6 dt)^2\Big],$$
$$A = (1-z)\,\phi\, dt + A_y\, dy, \quad \chi = z\,\psi,$$

The resulted equations reduce to nine PDEs with respect to z and x.

$$U_1(0,x) = U_2(0,x) = U_3(0,x) = U_4(0,x) = 1,$$
$$\psi(0,x) = A_y(0,x) = U_5(0,x) = U_6(0,x) = 0, \quad \phi(0,x) = \mu.$$

spontaneously symmetry breaking

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spontaneously symmetry breaking

Physical quantities can be read off from the boundary data at z=0.

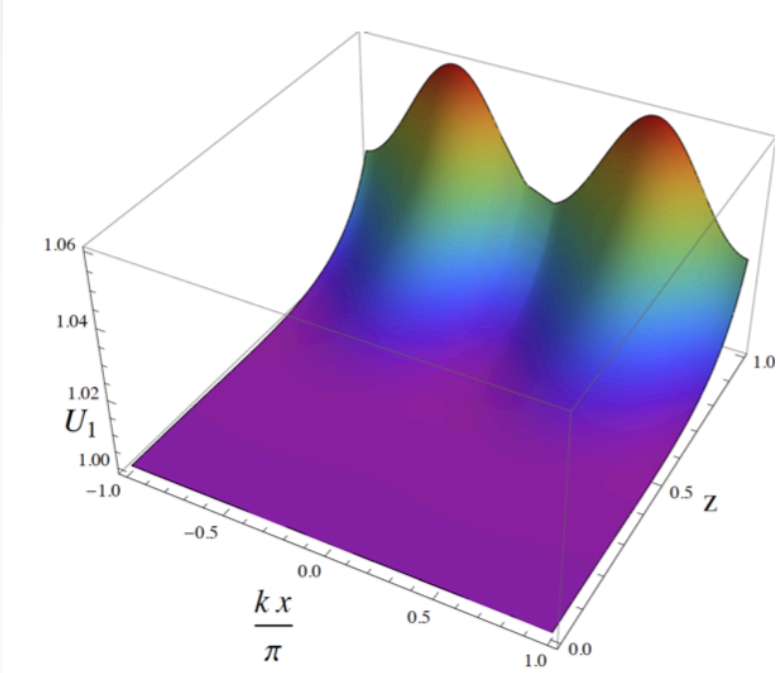
condensate $\quad \psi(z,x) = \langle O_\chi(x)\rangle\, z + \mathcal{O}(z^2)$

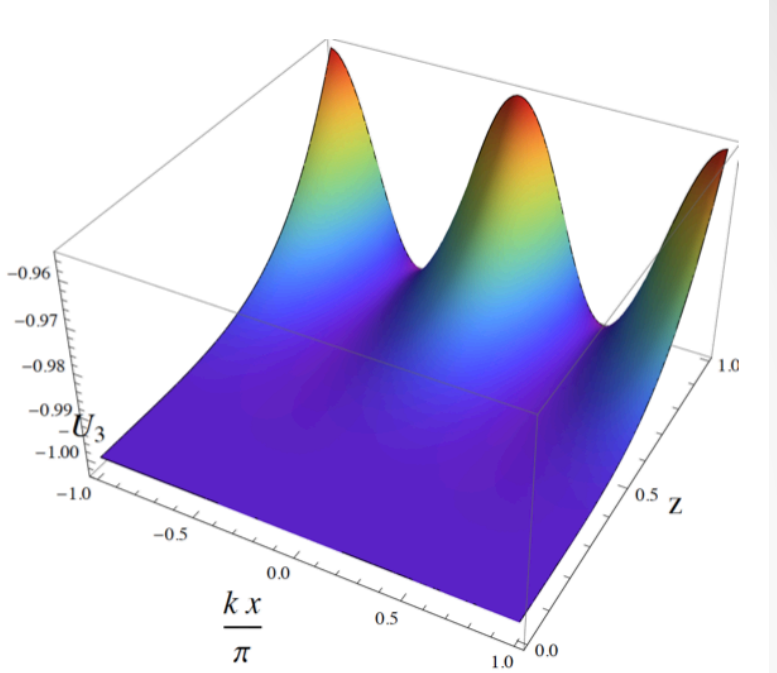
charge density $\quad A_t(z,x) = \mu - \rho(x) z + \mathcal{O}(z^2)$

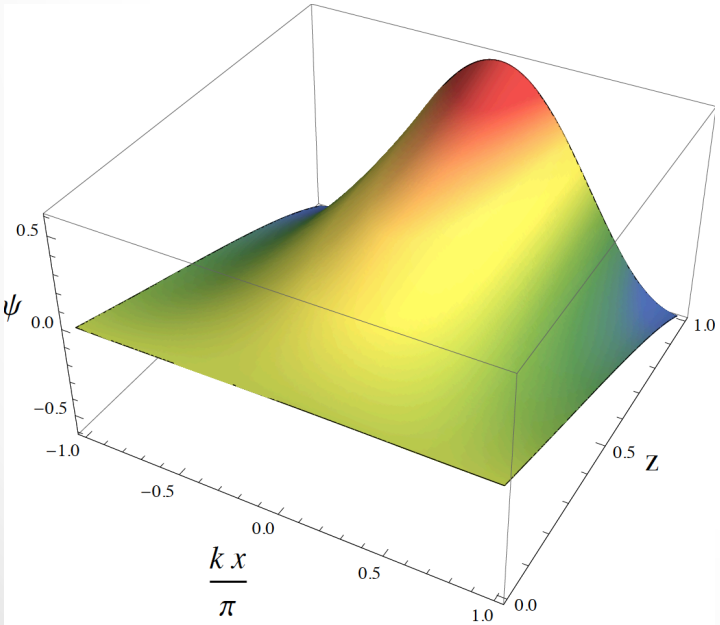
current density $\quad A_y(z,x) = 0 + j_y(x) z + \mathcal{O}(z^2)$

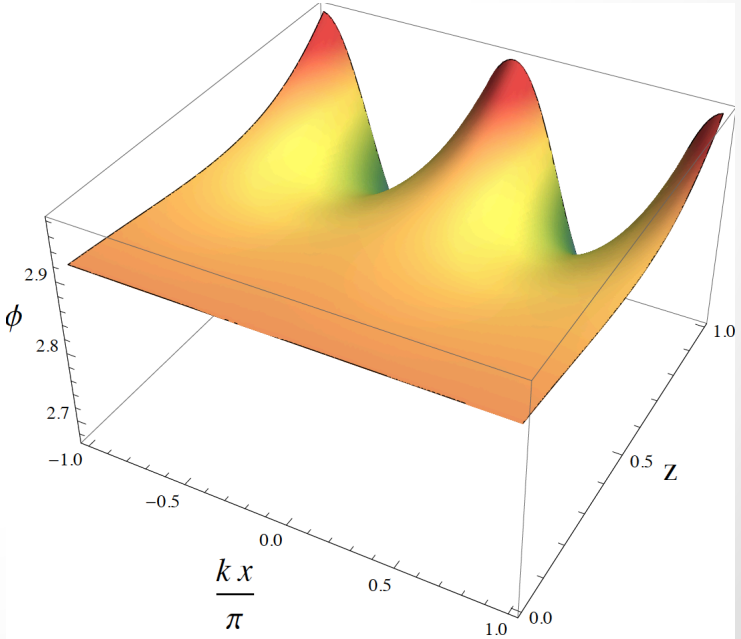
- Numerical Method: pseudo-spectral collocation+DeTurck+Newton-Raphson

Full solutions: novel black hole with scalar, charge and current hairs









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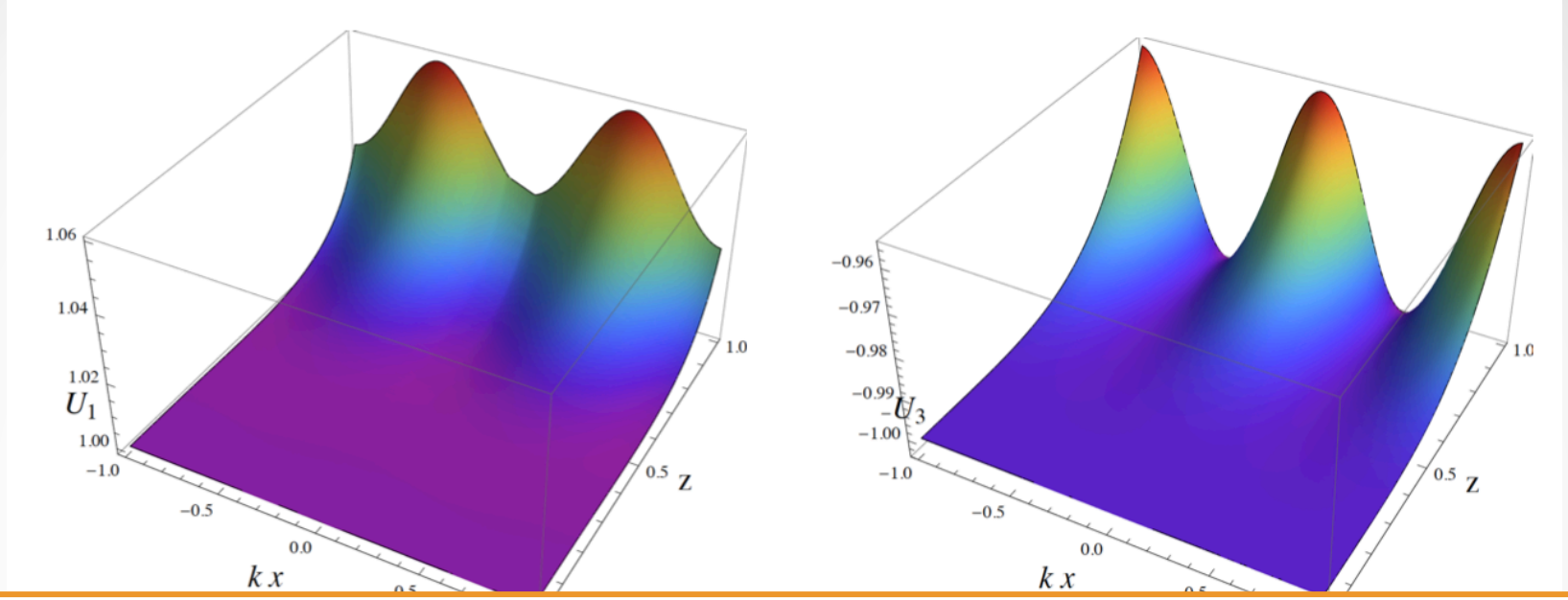
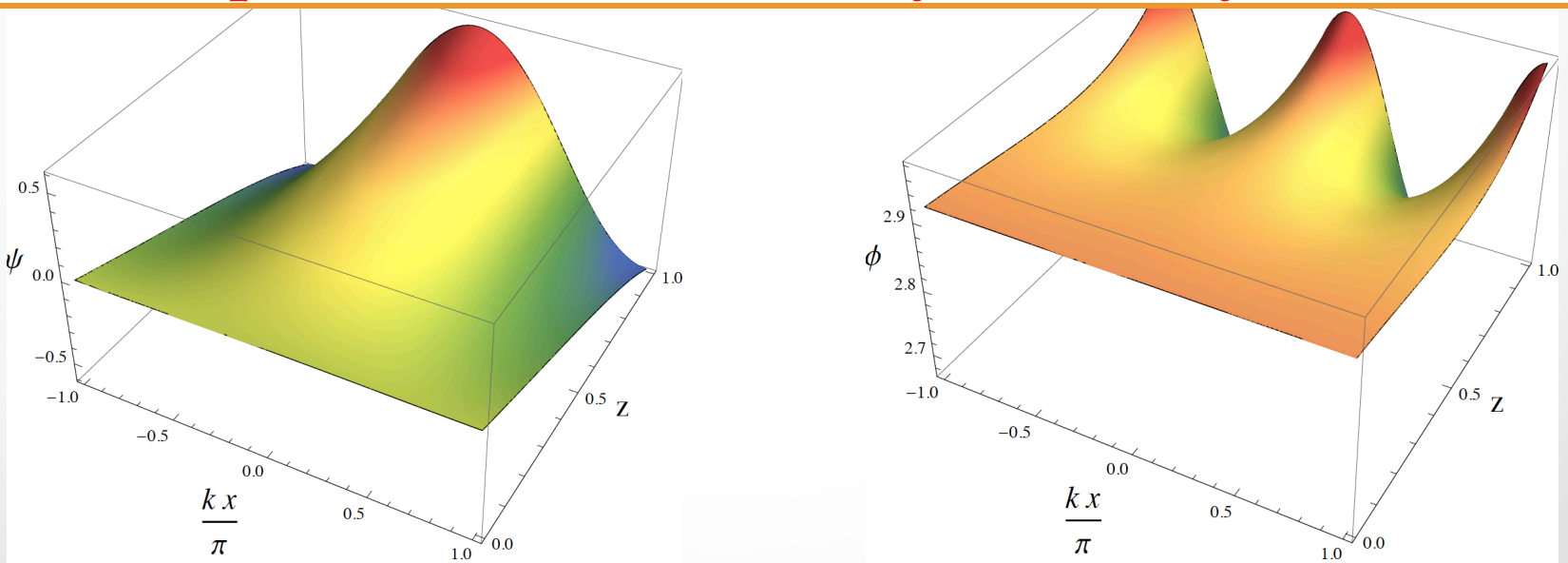
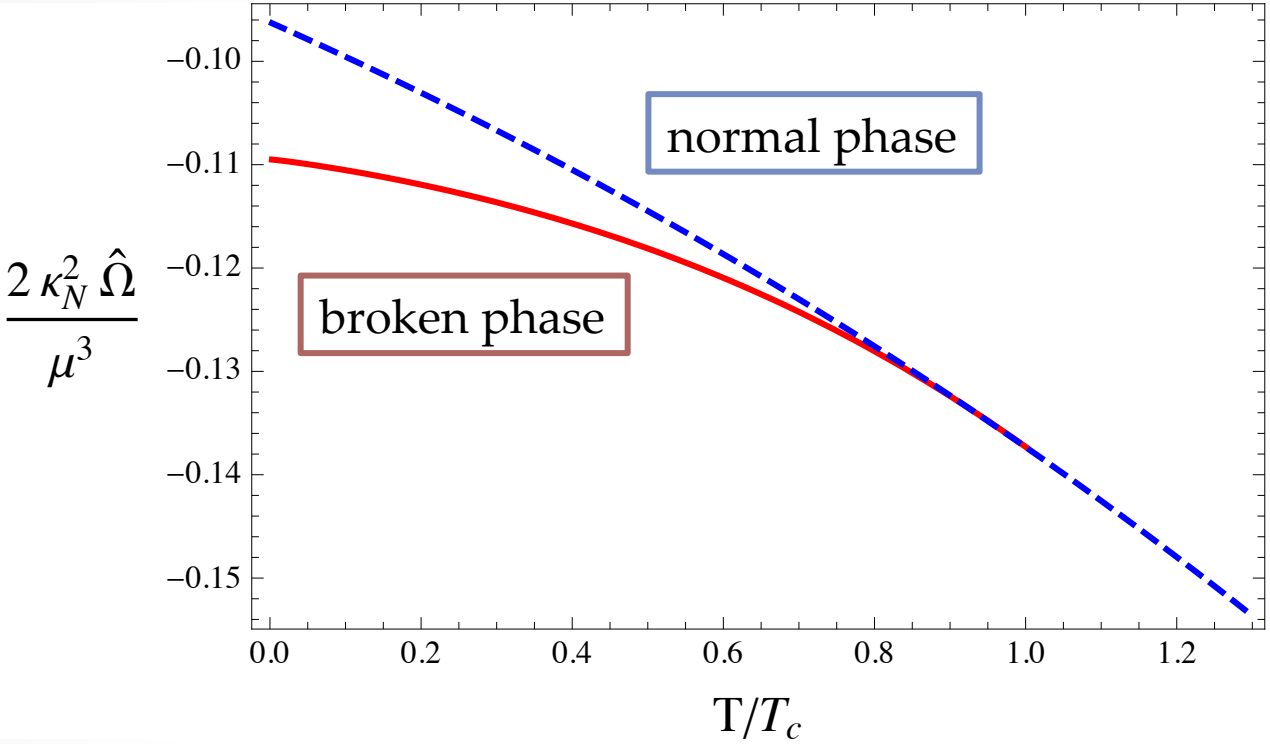


Is the striped solution thermodynamically stable ?

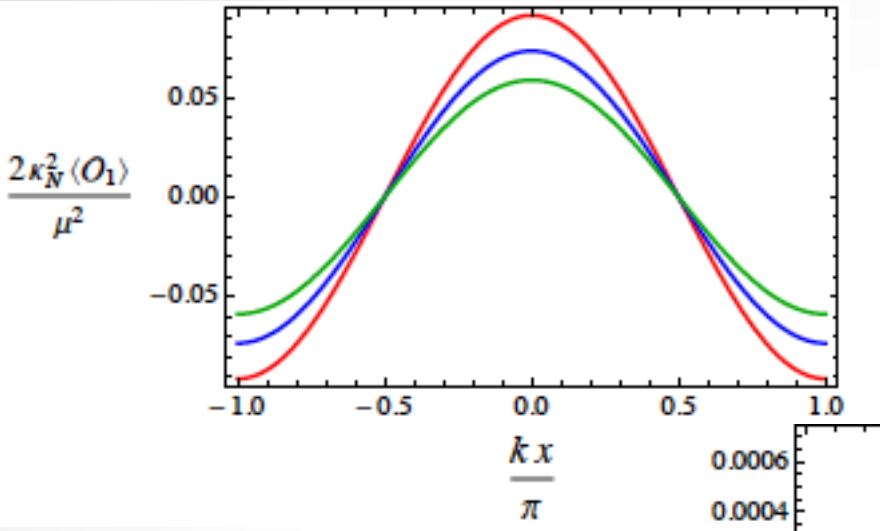


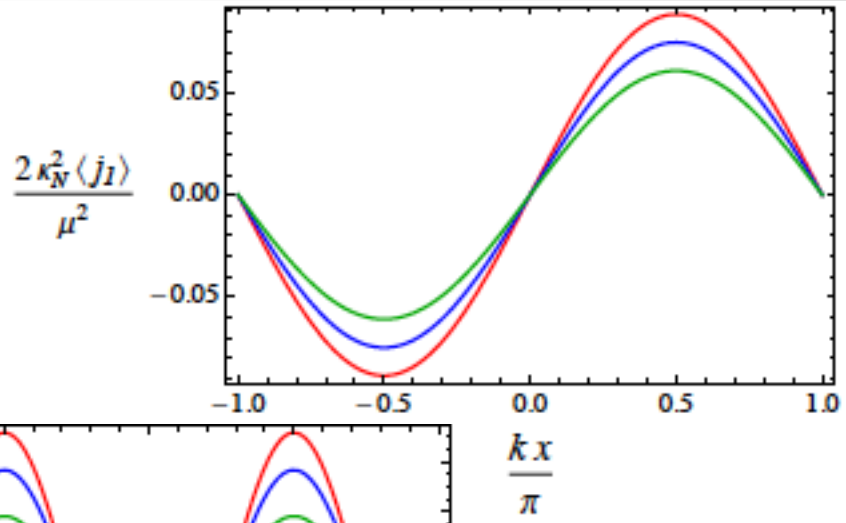
Thermodynamics: the striped phase has a lower free energy!

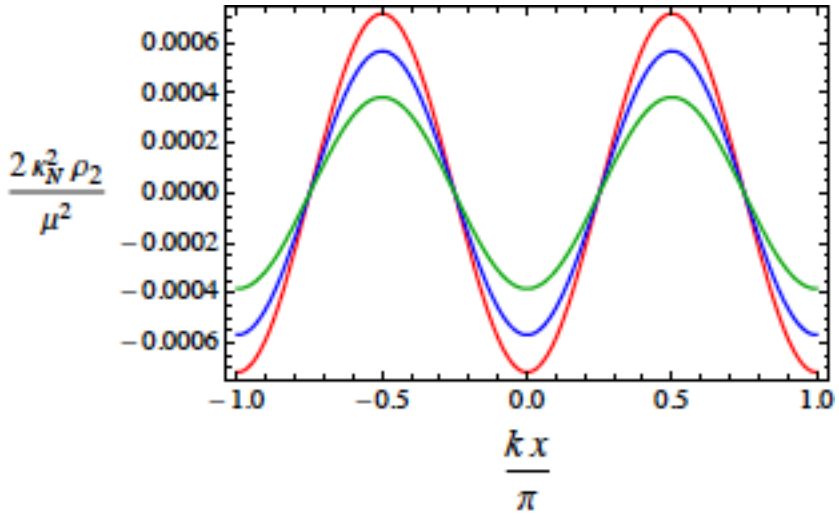


$T_c \approx 0.1284\mu$

second order phase transition

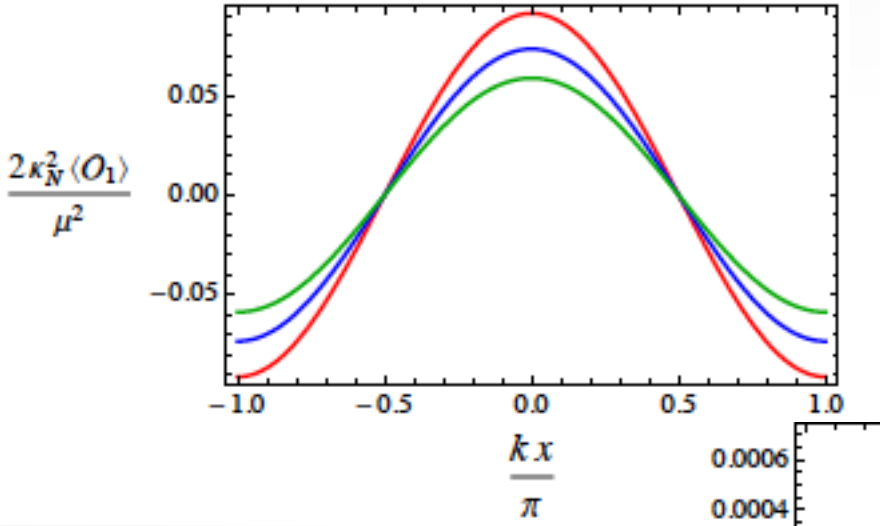


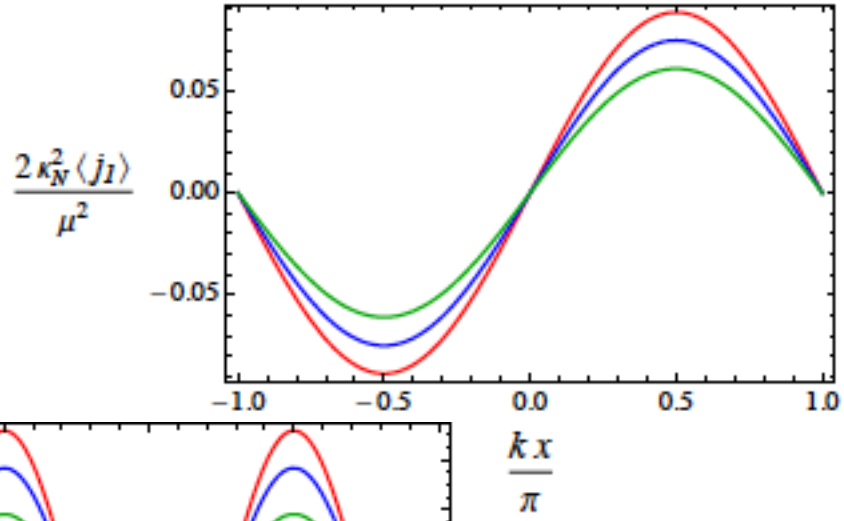


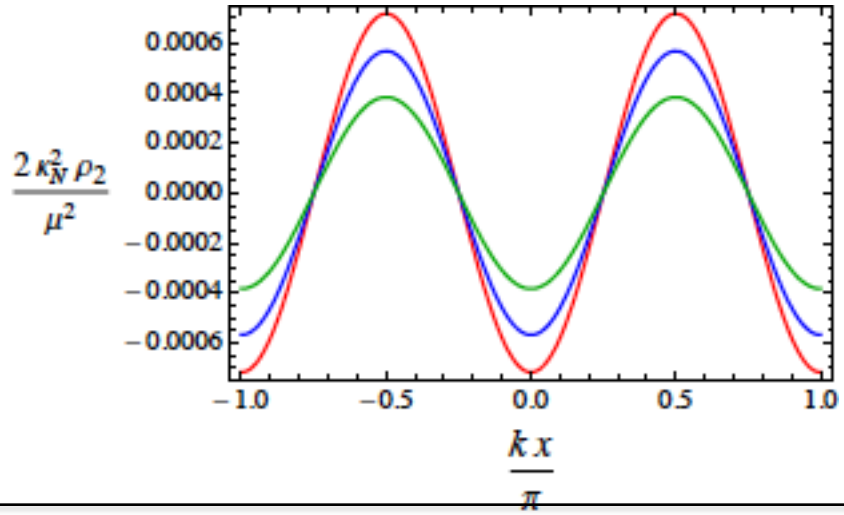


Interesting features

- The SC condensate is spatially modulated in such a way that its uniform component is zero.
- The charge density oscillates at twice the frequency of the current and condensate.
- The current density wave and condensate modulation are precisely out of phase.







Interesting features

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- The charge density oscillates at twice the frequency of the current and condensate.
- The current density wave and condensate modulation are precisely out of phase.

(Pair density wave !)

Experimental evidence:

Probing optically silent superfluid stripes in cuprates

S. Rajasekaran[1,*], J. Okamoto[2], L. Mathey[2], M. Fechner[1], V. Thampy[3], G. D. Gu[3], A. Cavalleri[1,4,*]

+ See all authors and affiliations

Science 02 Feb 2018:
Vol. 359, Issue 6375, pp. 575-579

Detection of a Cooper-pair density wave in $Bi_2Sr_2CaCu_2O_{8+x}$

nature
International journal of science

M. H. Hamidian, S. D. Edkins, Sang Hyun Joo, A. Kostin, H. Eisaki, S. Uchida, M. J. Lawler, E.-A. Kim, A. P. Mackenzie, K. Fujita, Jinho Lee & J. C. Séamus Davis

Nature **532**, 343–347 (21 April 2016) | Received: 18 November 2015

Magnetic-field Induced Pair Density Wave State in the Cuprate Vortex Halo

S. D. Edkins,[1,2,3] A. Kostin,[1] K. Fujita,[1,4] A. P. Mackenzie,[2,5] H. Eisaki,[6] S. Uchida,[7] Subir Sachdev,[8] M. J. Lawler,[1,9] E. -A. Kim,[1] J. C. Séamus Davis,[1,2,4] and M. H. Hamidian[1,8]

[1] *LASSP, Department of Physics, Cornell University,*
[2] *School of Physics and Astron., University of St. Andrews, Fife KY16 9SS, Scotland*
[3] *Department of Applied Physics, Stanford University, Stanford, CA 94305, USA*
[4] *CMPMS, Department, Brookhaven National Lab, Upton, NY, USA*

cond-mat > arXiv:1802.04673

Optical Conductivity

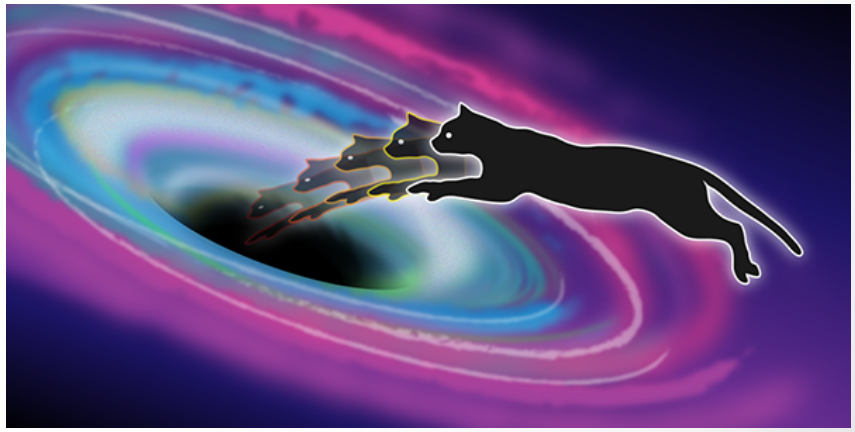
The conductivity matrix is $\begin{pmatrix} J^x \\ J^y \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$

Map to gravity side

$$\frac{1}{2\kappa_N^2} \begin{pmatrix} a_x^v \\ a_y^v \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix} \begin{pmatrix} i\omega a_x^s \\ i\omega a_y^s \end{pmatrix}$$

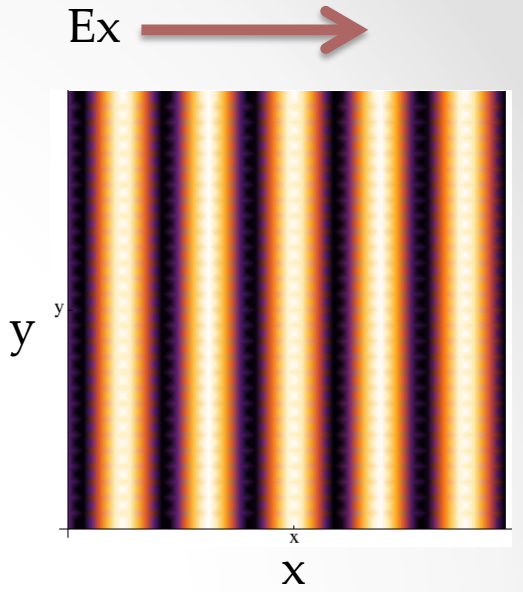
where $a_x = a_x^s + a_x^v(x)\, z + \mathcal{O}(z^2), \quad a_y = a_y^s + a_y^v(x)\, z + \mathcal{O}(z^2)$ at AdS boundary

Impose in-going condition near the black hole horizon



Retarded **two-point Green's function**

Conductivity perpendicular to stripes



Ionic lattice

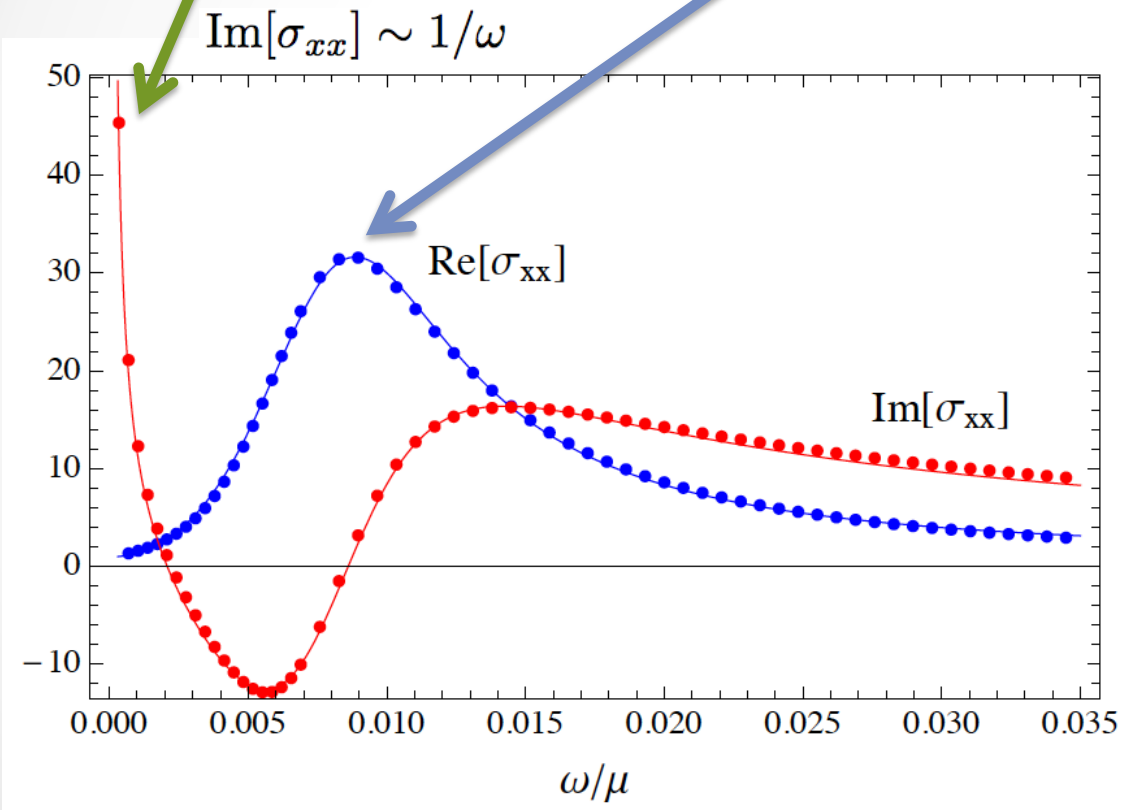
$$\mu(x) = \mu\,[1 + \mathbf{A}\cos(p\,x)]$$

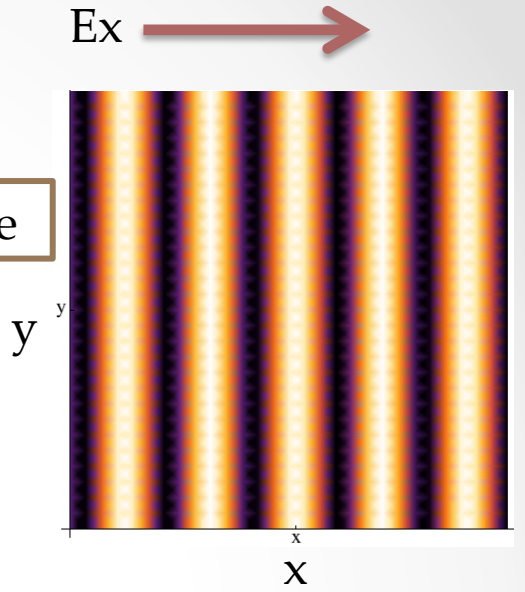
R.G.Cai, **L.Li**, Y.Q.Wang, J.Zaanen, work in progress

Conductivity perpendicular to stripes

Superconducting

phonon collective mode





Ionic lattice

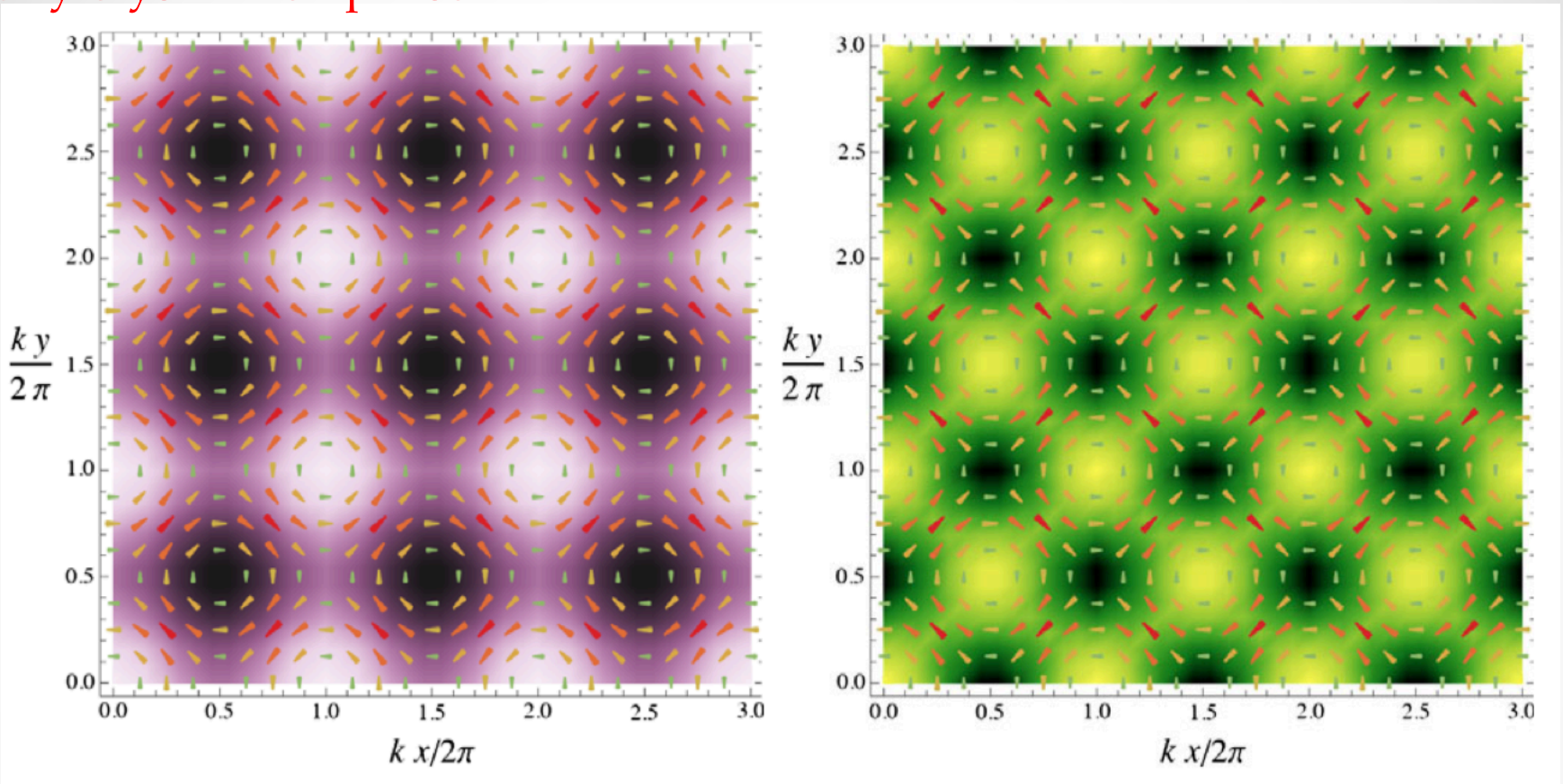
$$\mu(x) = \mu\,[1 + \mathbf{A}\cos(p\,x)]$$

Lorentz formula

$$\sigma(\omega) = i\frac{\rho_s}{\omega} + \sum_{j=1}^{N} \frac{S_j^2\,\tau_j}{1 - i\omega\tau_j(1 - \omega_j^2/\omega^2)}$$

R.G.Cai, **L.Li**, Y.Q.Wang, J.Zaanen, work in progress

Fully crystallized phase: 2D PDW



The rules of striped order repeat themselves in the tetragonally ("checkerboard") ordered case.

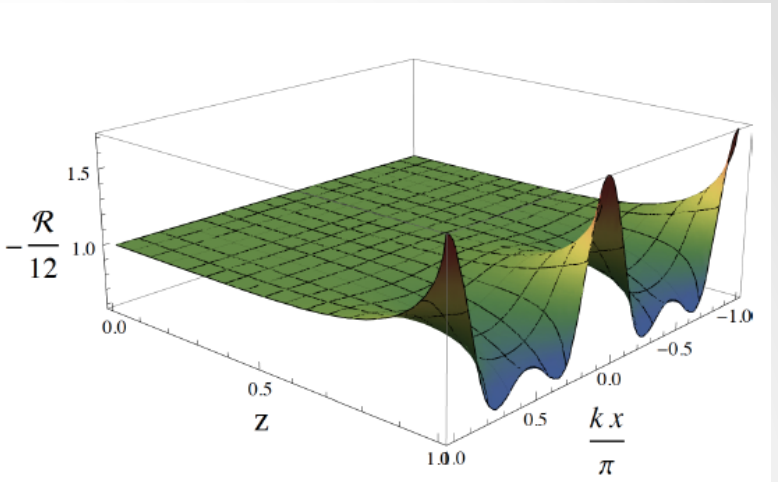
The charge order is now accompanied by spontaneous staggered current patterns similar to the "d-density wave" of condensed matter physics.

Density-wave states of nonzero angular momentum

Chetan Nayak

Phys. Rev. B **62**, 4880 – Published 15 August 2000

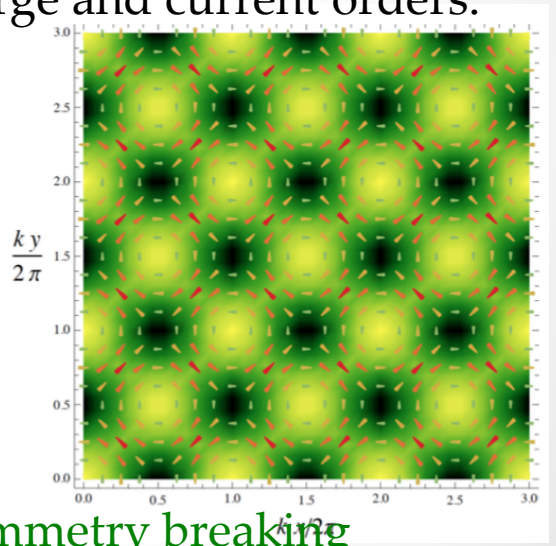
III Conclusion



Holographic duality:

(a). novel inhomogeneous black hole solutions with non-trivial hairs.

(b). PDW: a superconducting phase intertwined with charge and current orders.



A strong sense of non-BCS theory

There are no quasi-particles

Strongly interacting "soup" with spontaneously symmetry breaking

Possible relevant for unconventional superconductivity

Outlook

- The nature of the ground state at T=0
- Transport properties
- Fermion response->ARPES experiments (see talks by S. Cremonini and A. Krikun)
- Generalize the model to cases with different geometries
- The ideas here are also valuable for broader classes of strongly interacting quantum systems with spatially modulated order.

At the present stage the outcomes of the holographic exercise presented in the above offer no more than a rough cartoon. However, the cartoon is suggestive with regard to generalities.

Outlook

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Map the bulk theory to the real word system ?

Thank you !