Quantum Complexity of CFT states dual to bulk cosmological singularities

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[1]with E. Rabinovici (Racah Inst., Hebrew U.) & S. Bolognesi (Pisa U. & INFN, Pisa)

Introduction

- Central dogma of Holography

  Bulk geometry represents an encoding of the entanglement structure of boundary state

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$$C(t) \sim \frac{Vol.(\Sigma_t)}{G_N\, l}$$

  - EAdS-BH: At late times,

$$C \sim \text{"}ERB\ volume\text{"}; \ \frac{dC}{dt} \sim T\,S.$$

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- CV vs CA results: Universal features of singularities

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- Quant. mech., $C_{max} \sim 2^N \times \mathbb{R}!$ (Feynman)

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- no obvious association b/w BH singularities and Complexity?

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- Still CV and CA matches perfectly!

CV and CA

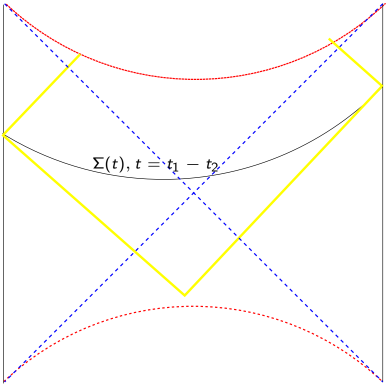

Figure: Eternal SAdS: CV and CA

Cosmological Singularities in the bulk[2]

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- Marginal: Coupling or CFT background metric gains time-dependence

$$ds^2_{CFT} = \frac{L^2}{z^2}\left(-dt^2 + dz^2 + h_{ij}(t, x_i)\, dx_i\, dx_j\right) . \qquad (1)$$

(Kasner $h \sim t^p$, Topological Crunch $h \sim \Omega_{d-1} R \cos t/R$)

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- Relevant: Time dependent Mass scale, $M(t)$ (dS/Crunch) leads to a singular domain wall geometry in bulk.

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- Every case: Complexity decreases as we approach the singularity!

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- Kasner

$$C_{\mathcal{V}} \sim N^2\,\Lambda^{d-1}\,V_x\frac{|t|}{l} + N^2\Lambda^{d-3}\frac{V_x}{tl} + O\left(\Lambda^{d-5}\right)$$

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- dS/Crunch: Subleading terms are also different

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- Complexity Monotonically decreases, these spacelike crunch singularities lack bite!
- Time rate of change of complexity contains a UV divergent time-dependent piece for CFT metric being time-dependent
- Coefficient of the rate of change determined by the subleading term (YGH term for $C \propto \mathcal{A}$)

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- Thanks! [3]

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