Dynamically Probing Strongly-Coupled Field Theories with Critical Point



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Motivation

in theories with a critical point

* It is interesting to see which universality class the theory belongs to.

* static critical exponent (thermodynamic quantities)

* dynamic critical exponent (transport properties, relaxation times and the response to time-dependent perturbations) Hohenberg, Halperin; 1977

- understanding the evolution of a system near the critical point
- A very interesting example can be QCD theory and heavy ion collision
- out of equilibrium
- strongly coupled
- non-zero chemical potential and temperature



Outline:

- Out-of-equilibrium systems in holography
- Background dual to field theory at non-zero temperature and chemical potential
- Behavior of the equilibration time
- Phase structure and critical exponent
- Conclusion

Holographic Out-of-Equilibrium Systems:

There are two methods to produce an out-of-equilibriums system:

- * injecting energy by turning on a source $H_{\lambda} = H_0 + \lambda \ \delta H$
- * starting from out-of-equilibrium initial states

In the gauge/gravity framework these correspond to:

* deforming the boundary field theory by a time-dependent coupling $\mathcal{L}_{SYM} + \lambda_{\Delta}(t) \mathcal{O}_{\Delta}$ S. Bhattacharyya, S. Minwalla; 2009 P. Chesler, L. Yaffe; 2009

A. Buchel, L. Lehner, R. Myers; 2012

bulk configuration on the initial time-slice

Heller, Mateos, van der Schee, Trancanelli; 2012 Heller, Mateos, van der Schee, Triana; 2013

Holographic model of field theory with non-zero chemical potential and temperature

the gravity background:

 $\Phi(z) = \frac{3}{4}\sqrt{2\gamma(1+\gamma-2\gamma)} \,\ln\Gamma(z)$

$$S = \frac{1}{16\pi} \int d^5 x \sqrt{-g} \left(\mathcal{R} - \frac{4}{3} (\nabla \Phi)^2 - V(\Phi) - e^{-\frac{4\alpha}{3}\Phi} F_{\mu\nu} F^{\mu\nu} \right)$$

$$\begin{split} ds^2 &= -N(z)f(z)dv^2 - \frac{2}{z^2}\sqrt{\frac{N(z)}{1+b^2z^2}}dvdz + \frac{1+b^2z^2}{z^2}g(z)d\bar{x} \\ f(z) &= \frac{1+b^2z^2}{z^2}\Gamma^{2\gamma} - m\frac{z^2}{1+b^2z^2}\Gamma^{1-\gamma} \\ N(z) &= \Gamma^{-\gamma}, \; g(z) = \Gamma^{\gamma}, \; \Gamma(z) = 1 - \frac{b^2z^2}{1+b^2z^2}, \; \gamma = \frac{\alpha^2}{2+\alpha^2} \end{split}$$

For $\alpha = 2$ one recovers the well-known 1-R charged black hole model up to some field redefinitions.

| Gubser; | 1998 |
|-------------------------|------|
| Behrndt, Cvetic, Sabra; | 1998 |
| Kraus, Larsen, Trivedi; | 1998 |
| Cai, Soh; | 1998 |
| Cvetic, Gubser: | 1998 |

$$A_{t} = -\frac{\sqrt{3m}bz^{2}}{\sqrt{2+\alpha^{2}}(1+z^{2}b^{2})} + \frac{\sqrt{3m}bz_{h}^{2}}{\sqrt{2+\alpha^{2}}(1+z_{h}^{2}b^{2})}$$
$$V(\Phi) = \frac{4\Lambda}{(2+\alpha^{2})^{2}}(-\alpha^{2}(1-\alpha^{2})e^{\frac{-8\Phi}{3\alpha}} + (4-\alpha^{2})e^{\frac{4\alpha\Phi}{3}} + 6\alpha^{2}e^{\frac{-2(2-\alpha^{2})\Phi}{3\alpha}})$$

the field theory dual temperature and chemical potential

$$T = \frac{b\Gamma(z_h)^{\frac{3\gamma}{2}-1}}{4\pi\sqrt{1-\Gamma(z_h)}} \left(2(3\gamma-1) - 3(2\gamma-2)\Gamma(z_h)\right)$$

$$=\frac{b\sqrt{3m}}{\sqrt{2\left(\alpha^2+2\right)}\left(b^2+\frac{1}{z_h^2}\right)}$$

 $q = \sqrt{\frac{6m}{2+\alpha^2}}b$

Phase Structure of the Background¹/₇

If we set $\alpha = 2$ and make some field redefinitions we obtain the 1-R charged black hole solution.

- * dual to strongly-coupled SYM plasma in flat 3+1 dimensions
- * conformal
- * phase diagram is 1-dimensional and a function of a bigging dimension-less ratio $\frac{\mu}{T}$
- The phase diagram has the form of a semi-infinite line, ending on a critical point.

* At the critical point the heat capacity $\frac{\partial s}{\partial T}$ and charge susceptibility $\frac{\partial \rho}{\partial \mu}$ diverge. $s \propto \frac{T^3(1+b^2z_h^2)^2}{(2+b^2z_t^2)^3}$

$$pz_h = \frac{1 \pm \sqrt{1 - \frac{8\mu^2}{\pi^2 T^2}}}{\frac{2\mu}{\pi T}}$$

0.2

0.6

0.0 0.4

0.4

0.0

0.2

$$\rho \propto \frac{\mu}{T} (2 + b^2 z_h^2) \sqrt{1 + b^2 z_h^2}$$

$$bz_{h} = \frac{1 \pm \sqrt{1 - \frac{8\mu^{2}}{\pi^{2}T^{2}}}}{\frac{2\mu}{\pi T}}$$

Each value of $\frac{\mu}{T}$ corresponds to two distinct values of bz_h .

> thermodynamically stable and unstable branches

> > stable

β_φ=0.2, β_{BH}=0.2, c=6

point at

= 1.1107

positive definite Jacobian $\mathcal{J} = \frac{\partial(s,\rho)}{\partial(T,\mu)}$ **Critical exponent**

- static critical exponent $\rho - \rho_c \approx |\frac{\mu}{T} - (\frac{\mu}{T})_c|^{\frac{1}{2}}$ Cai, Soh: 1998 and Cvetic, Gubser: 1999 • conductivity; gauge field fluctuations; Maeda, Natsuume, Okamura: 2008 Buchel: 2010 DeWolfe, Gubser, Rosen: 2011
- quasi-normal modes (wealthy information about the near equilibrium behavior of the theory)
 Finazzo, Rougemont, Zaniboni, Critelli, Noronha; 2016
 - + finite value at the critical

point

the critical point approaches

 the same as static critical exponent (1/2) producing out-of-equilibrium state in two different ways:

- time dependence in background geometry (Vaidya background); time-dependent temperature or chemical potential in field theory mP(v) $q_{\Lambda}/P(v)$ $P(v) = \frac{1}{2} \left(1 + \tanh \frac{v}{\beta_{BH}}\right)$
- dynamical external probe scalar with $m^2 = -3$; scalar operator in field theory with $\Delta = 3$
 - out-of-equilibrium initial conditions
 - time-dependent source (quantum quench)

near boundary expansion of the external probe scalar field: Ebrahim

Ali-Akbari, Charmchi, Ebrahim, Shahkarami; 2016-17

initial condition

 $\tilde{\phi}_r(v_0, z) = z^4$

$$\begin{split} \phi(v,z) &= z\phi_s(v) + z^2\phi'_s(v) + z^3 \left(\frac{1}{2}\log(z)\phi''_s(v) + \phi_r(v)\right) \\ &+ z^4 \left(\frac{6m\phi'_r(v) - 2m\phi'''_s(v) - q^2\phi'_s(v)}{6m} + \frac{1}{2}\log(z)\phi'''_s(v)\right) + \mathcal{O}(z^5) \end{split}$$

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$$\phi(v,z) = z\phi_s(v) + z^2\phi'_s(v) + z^3\frac{1}{2}\log(z)\phi''_s(v) + z\phi_r(v,z).$$

$$\phi_s(v) = \frac{1}{2}\left(1 + \tanh\frac{v}{\beta_\phi}\right)$$
response

0.0

4.8



question: Whether the response in field theory to an external source understands about the phase structure of the theory?



Remarks:

Even though the scalar source is non-zero (quantum quench) the dynamical critical exponent can be obtained from the equilibration time

For fast quenches the dynamical critical exponent matches the static one.

It would be very interesting to check this result for more realistic holographic theories (under investigation).

Thank you