Holographic local quench and effective complexity

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based on 1803.11162

with
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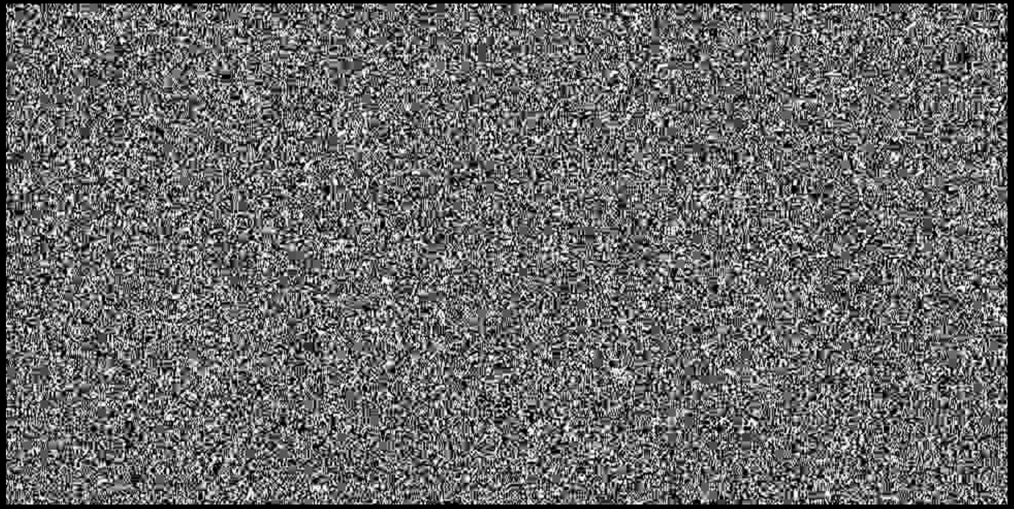
The main motivation

To make use of the concept of holographic complexity to get some intuition of what the physical (non-computational) complexity is

Descriptive (computational) complexity

- The more random – the more complex:



White noise
970 x 485 pixels, gray scale, 253 Kb

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Vermeer "View of Delft"
750 x 624 pixels, colored, 234 Kb

Descriptive (computational) complexity

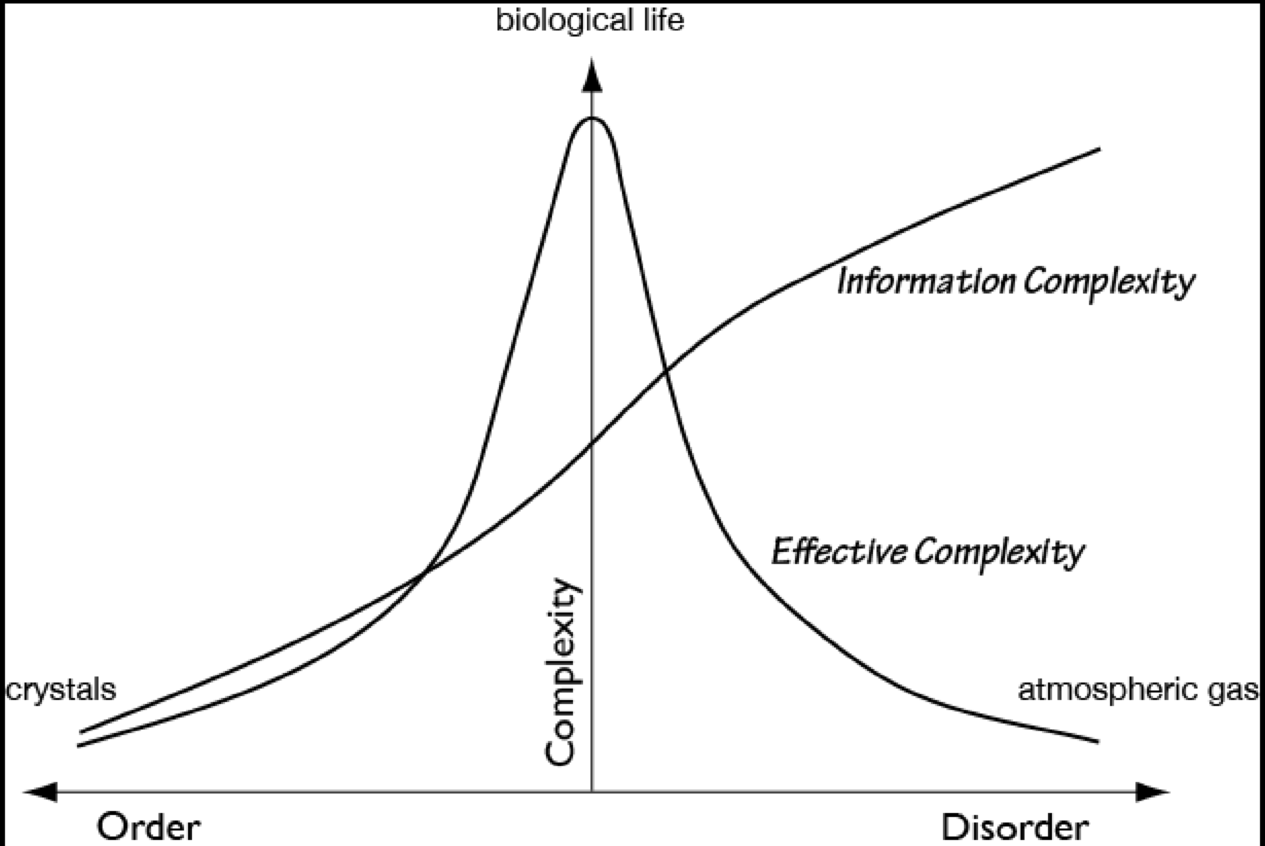
- The more random – the more complex:

≫

Paris japonica - 150 billion base pairs in DNA

Homo sapiens - 3.1 billion base pairs in DNA

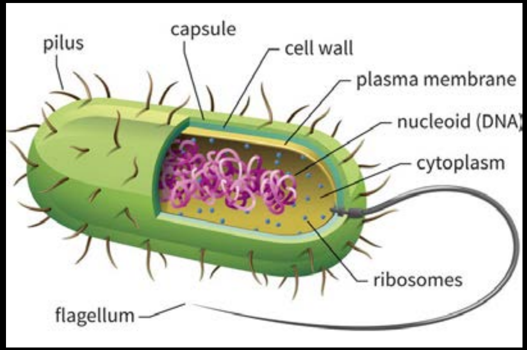
Effective complexity



Can we come up with a quantitative measure?..

Not a mere philosophical question…

- What happens at the major evolutionary transitions?



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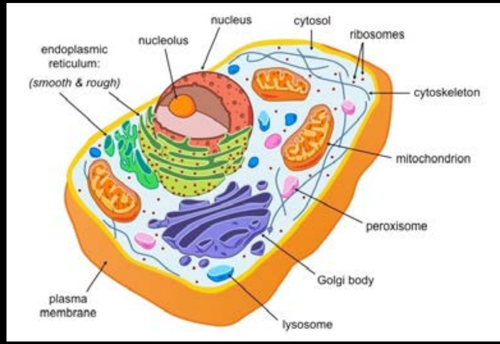
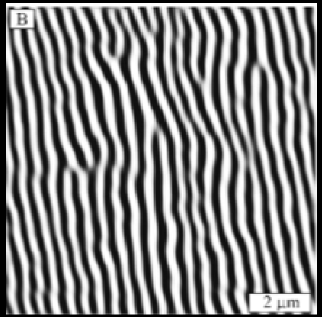


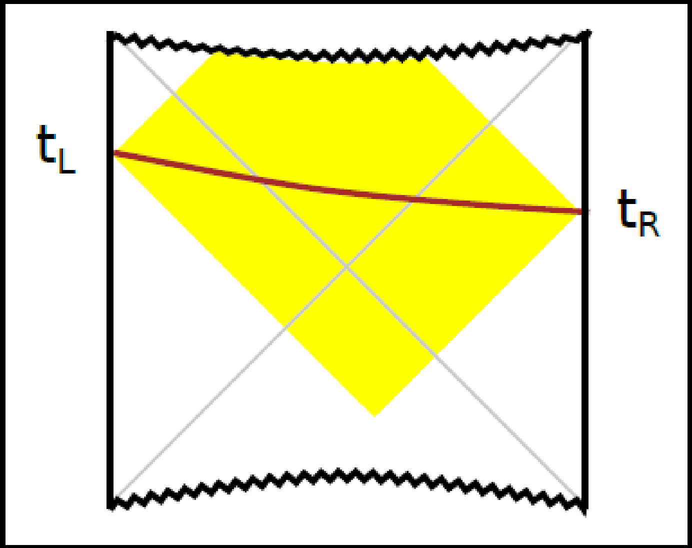
- Why are simple neural algorithms capable of solving complex many-body problems?

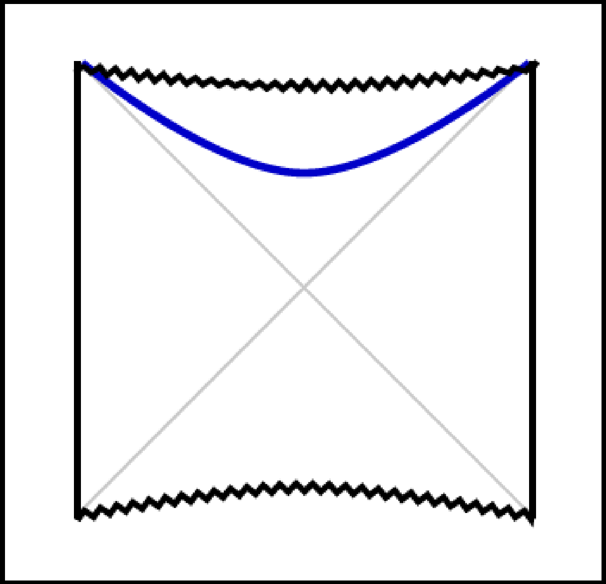
- Why do many natural patterns appear to be universal?



Holographic complexity

- Original motivation – growth of the Einstein-Rosen bridge:





$$ds^2 = -f(r)d\tau^2 + f(r)^{-1}dr^2 + r^2 d\Omega_{D-2}^2$$

$$f(r) = r^2 + 1 - \frac{\mu}{r^{D-3}}$$

$$|\mathrm{TFD}\rangle = \sum_i e^{-\beta E_i/2} |E_i\rangle_L |E_i\rangle_R$$

Eternal AdS-black hole, Wheeler-DeWitt patch

Susskind 1402.5674

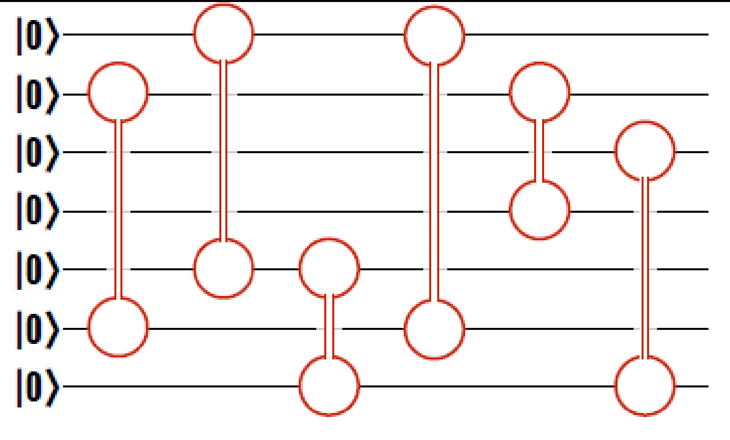
Holographic complexity

- Why should we relate complexity to the size of wormhole?
- Complexity here is the circuit ("computational") complexity:

$$|\mathrm{TFD}\rangle = \sum_i e^{-\beta E_i/2}|E_i\rangle_L|E_i\rangle_R \quad \rightarrow \quad |\psi(t_L,t_R)\rangle = \sum_i e^{-\beta E_i/2+iE_i(t_L+t_R)}|E_i\rangle_L|E_i\rangle_R$$

Reference

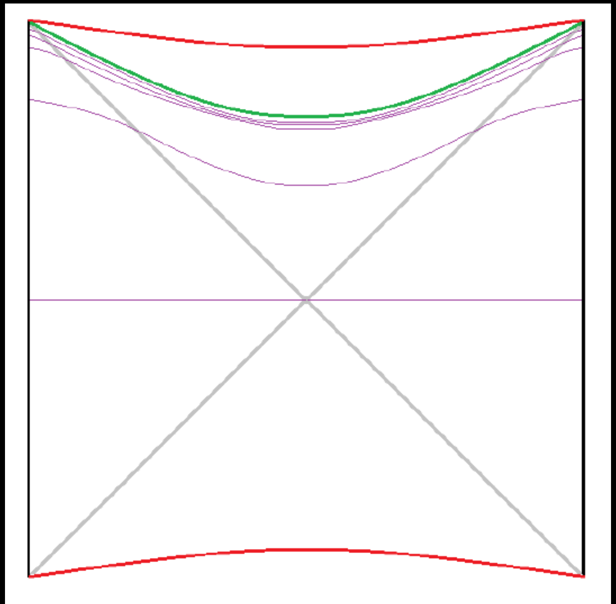
Target

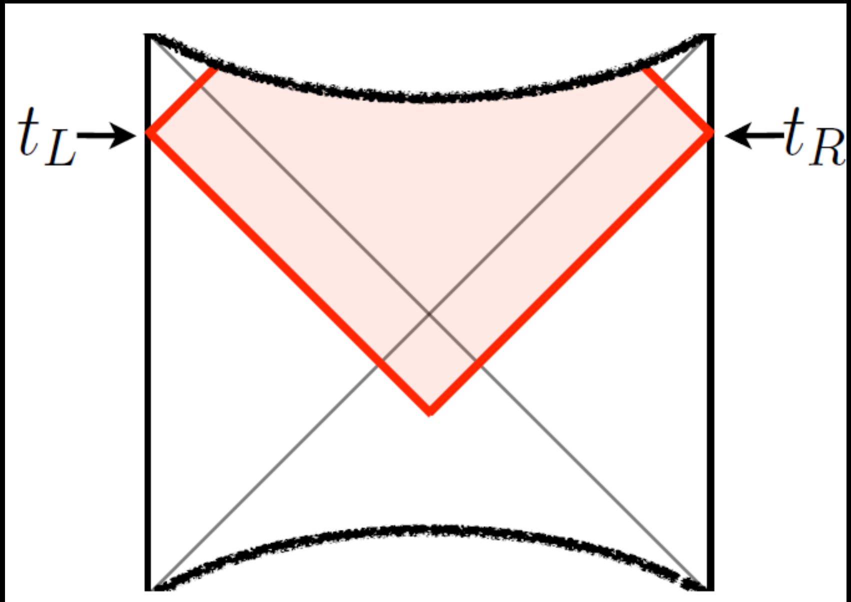


Dowling, Nielsen quant-ph/0701004

Volume and action complexity

- Two alternative ways to measure the size of the bridge:





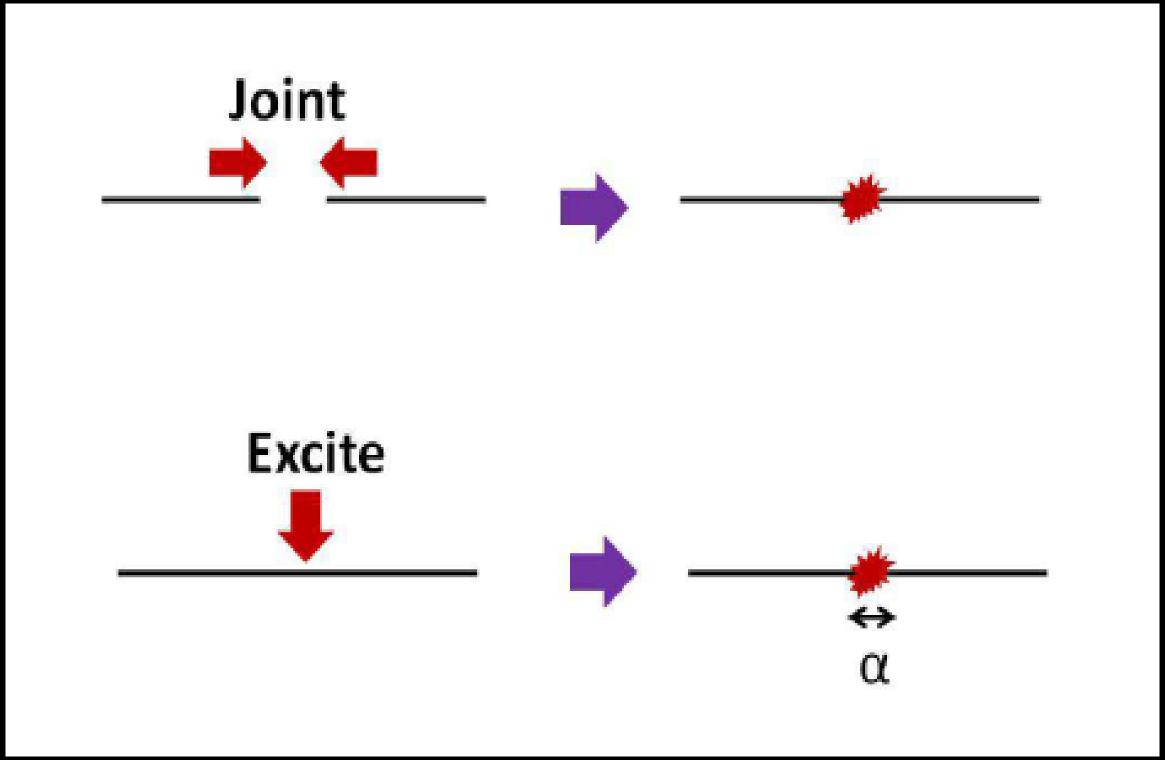
$$\mathcal{C} \sim \frac{V}{G\ell}$$

Susskind 1402.5674

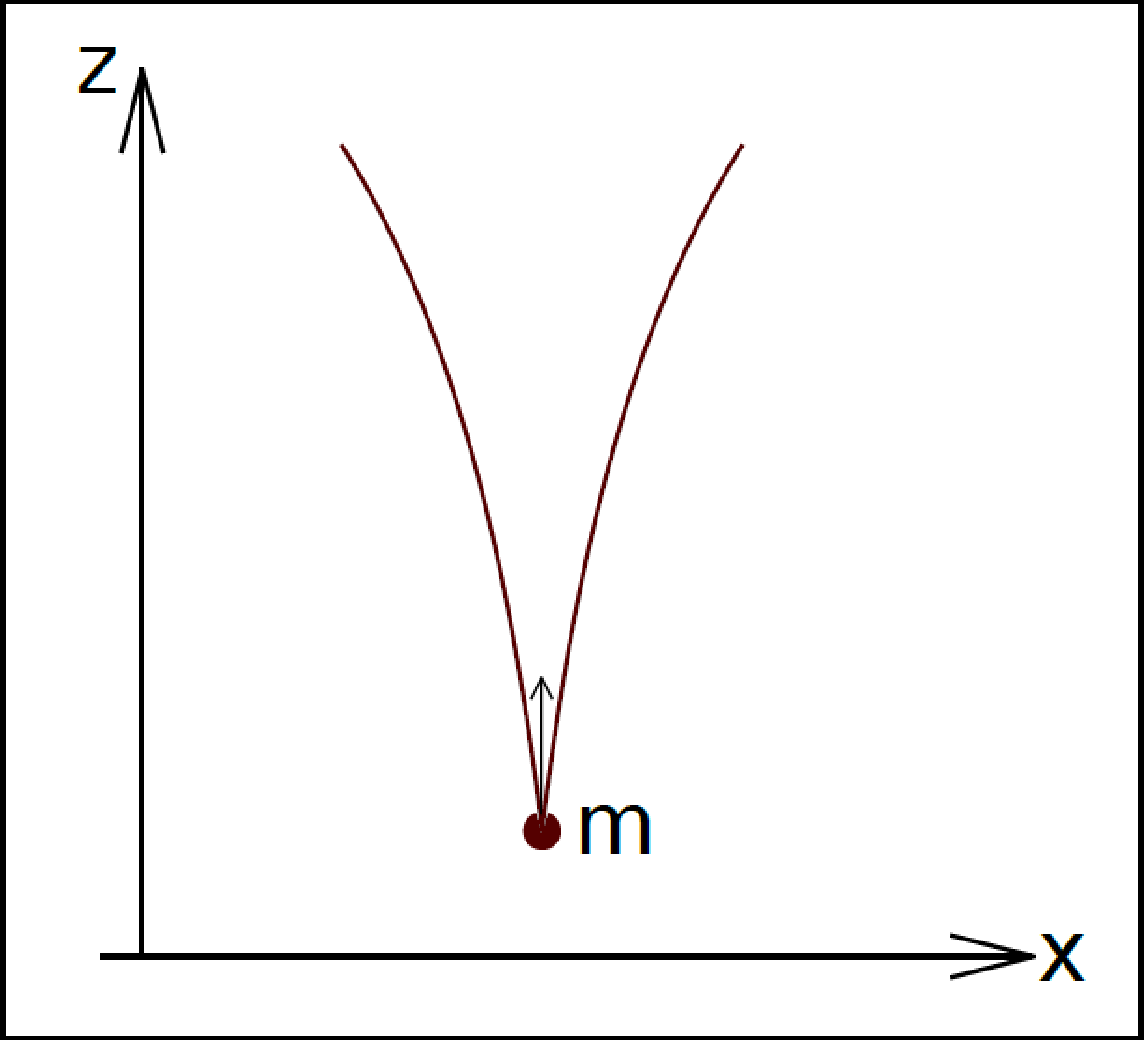
$$\mathcal{C} = \frac{\mathcal{A}}{\pi\hbar}$$

Brown et al. 1509.07876

Holographic local quench



Nozaki et al., 1302.5703

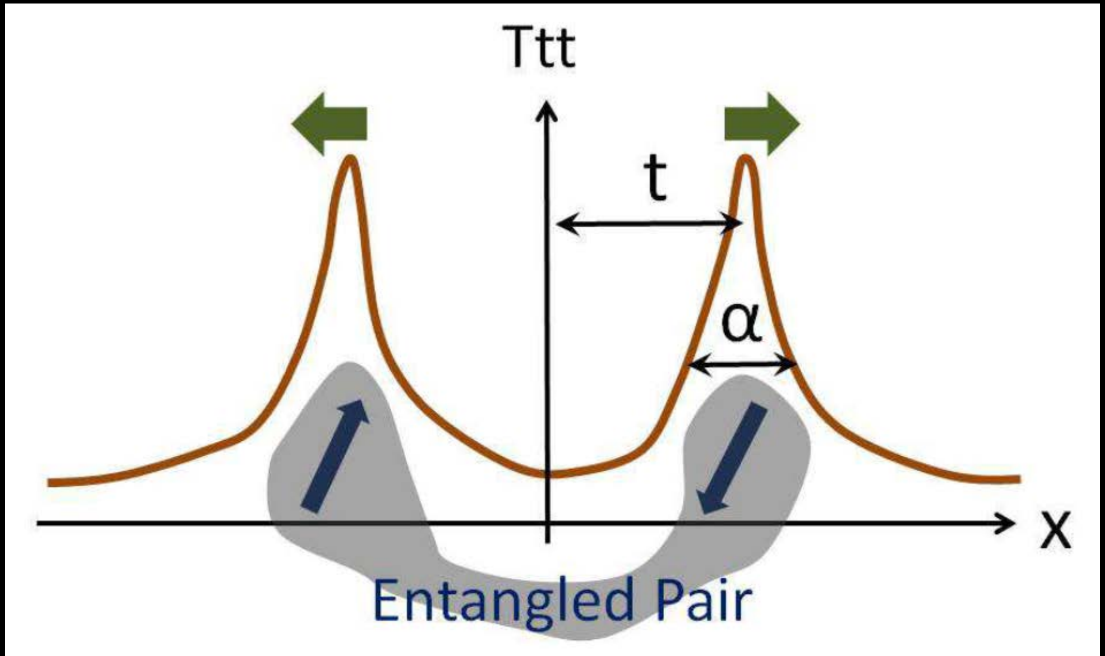


Holographic local quench

- Pair of solitons is formed:

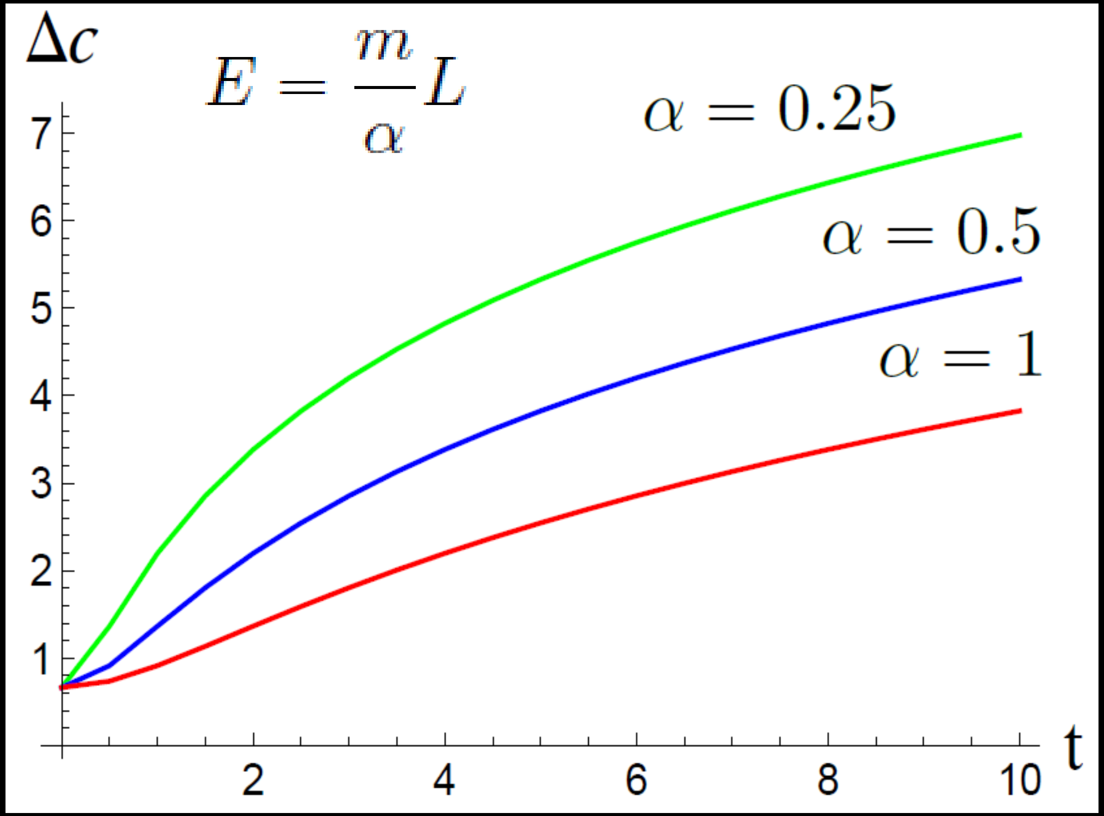
$$g_{ab}(x,z) = \eta_{ab} + t_{ab} z^d + O(z^{d+1})$$

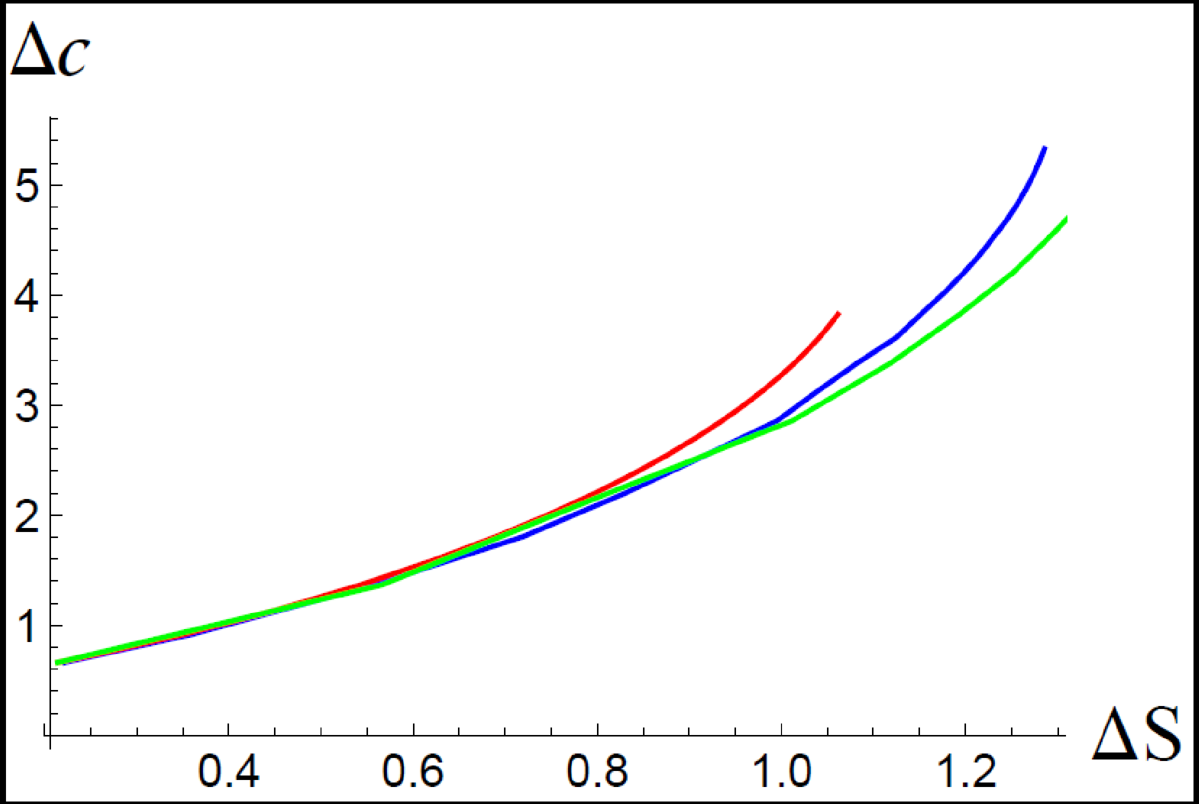
$$T_{ab} = \frac{d \cdot R^{d-1}}{16\pi G_N} \cdot t_{ab}$$



Volume complexity following local quench

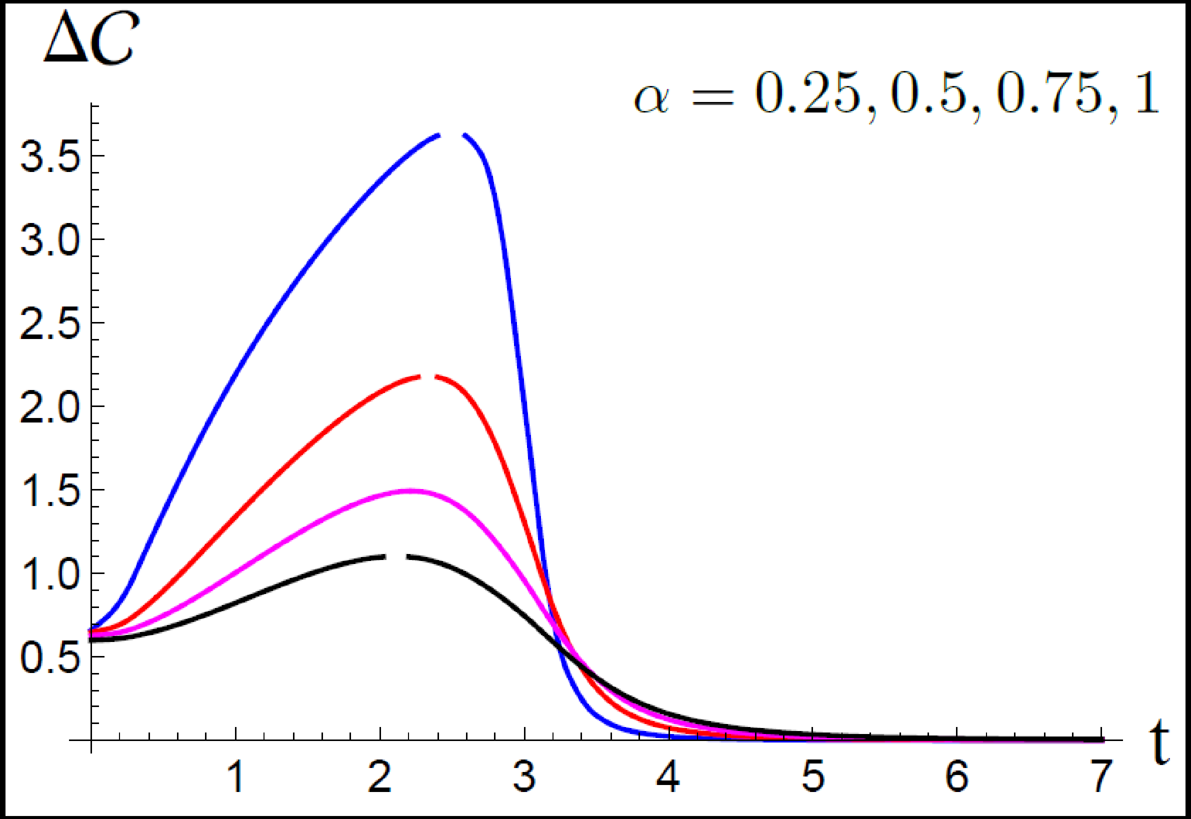
- Complexity of the total system

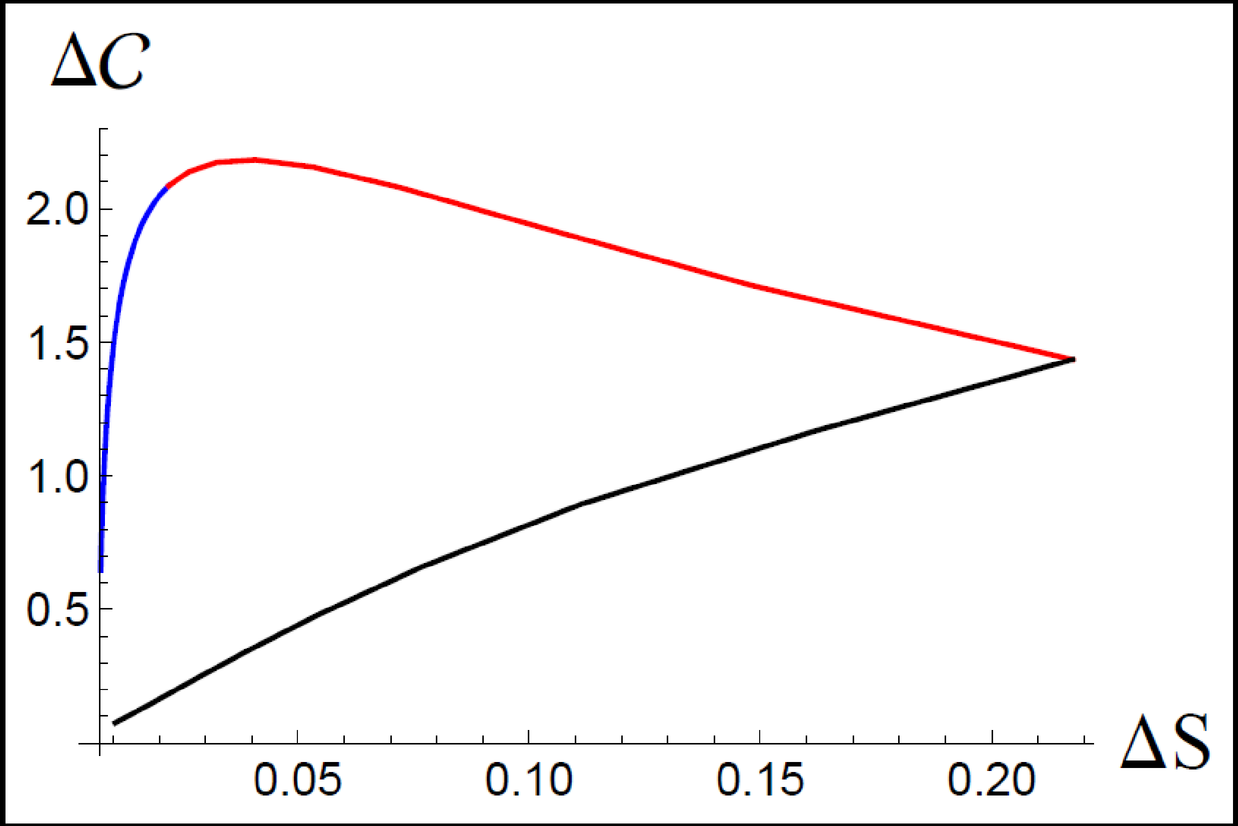




Volume complexity following local quench

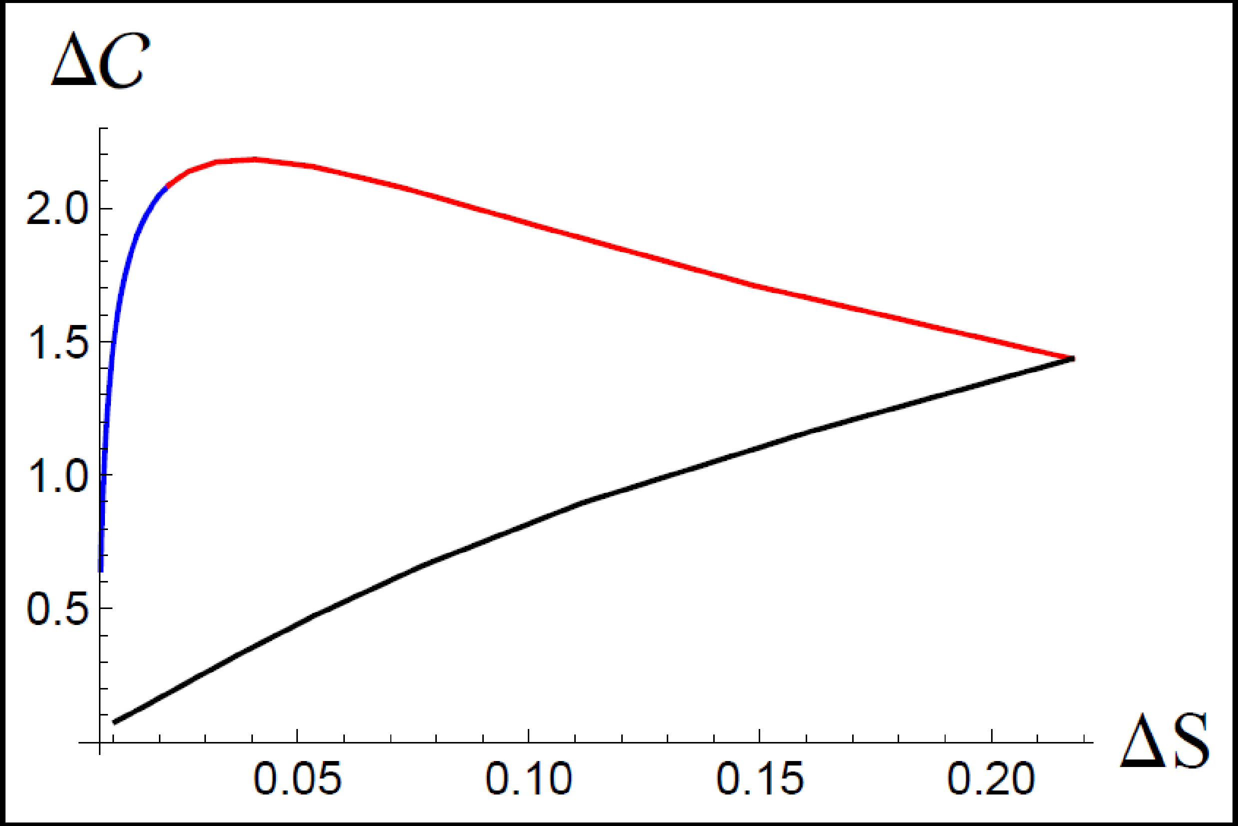
- Complexity of a subregion:





Volume complexity following local quench

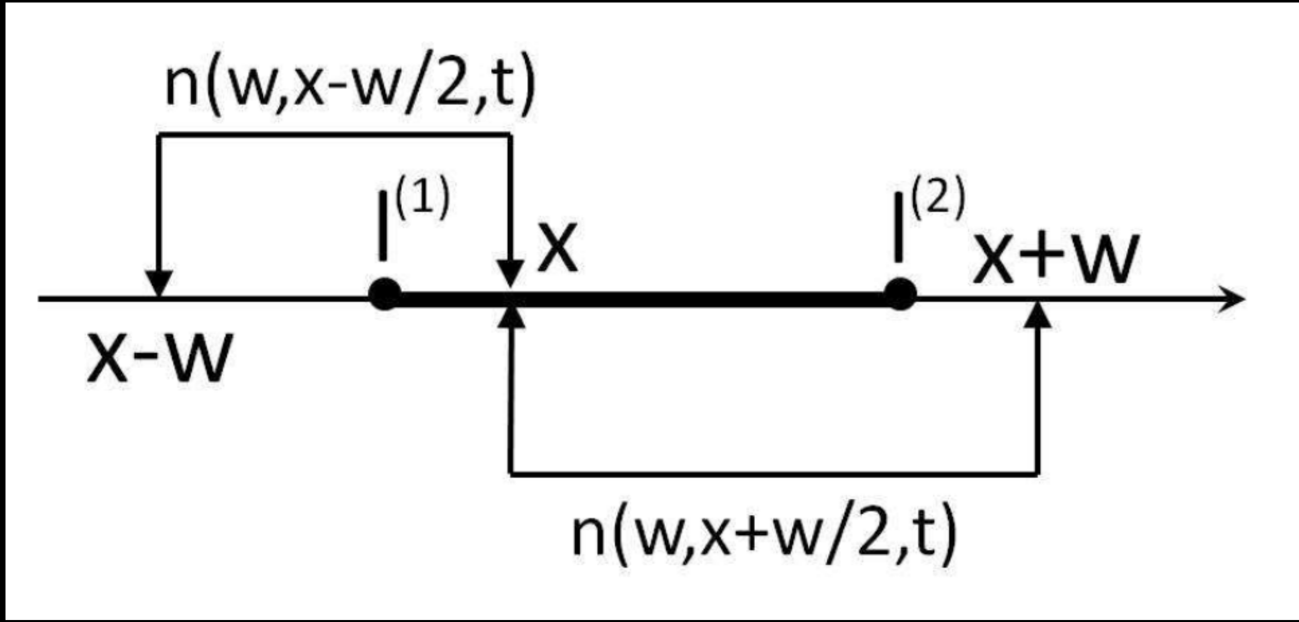
- Complexity of a subregion:



- Complexity is a non-monotonous function of entanglement entropy!
- But the latter is rather a characteristic of interface between the subsystem and the environment than of the subsystem itself…

Entanglement density

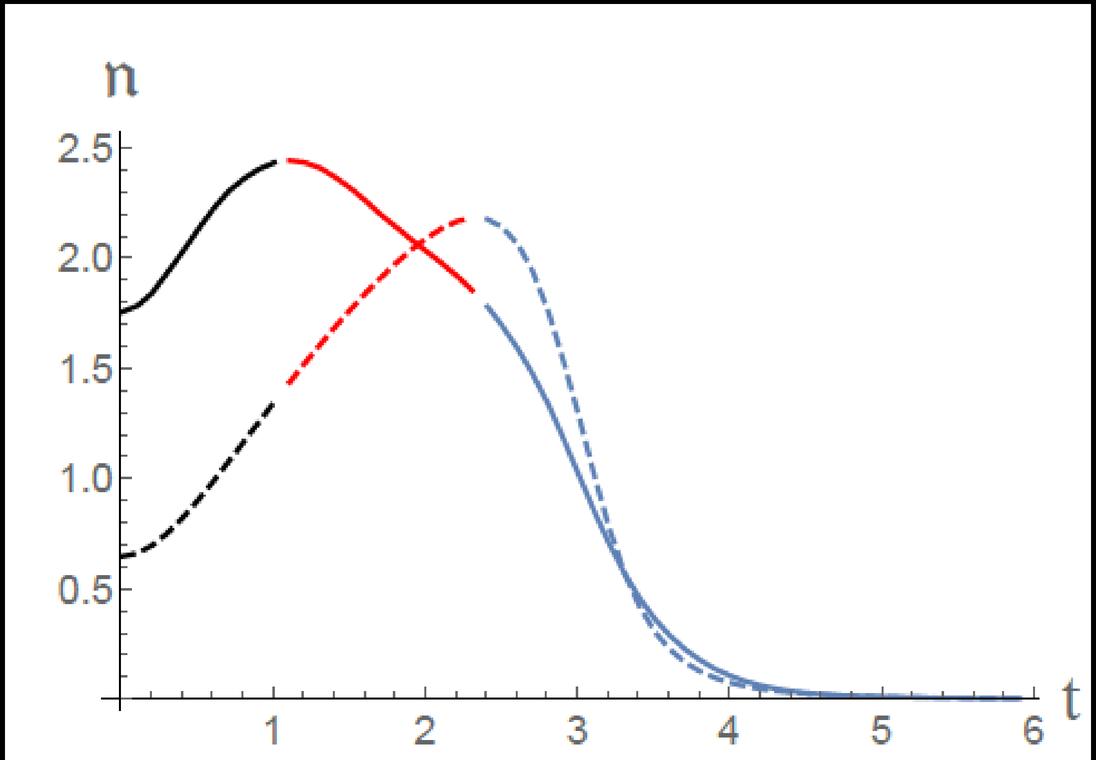
- Intuitive (not rigorous) picture – number of Bell pairs per “bi-local” unit volume

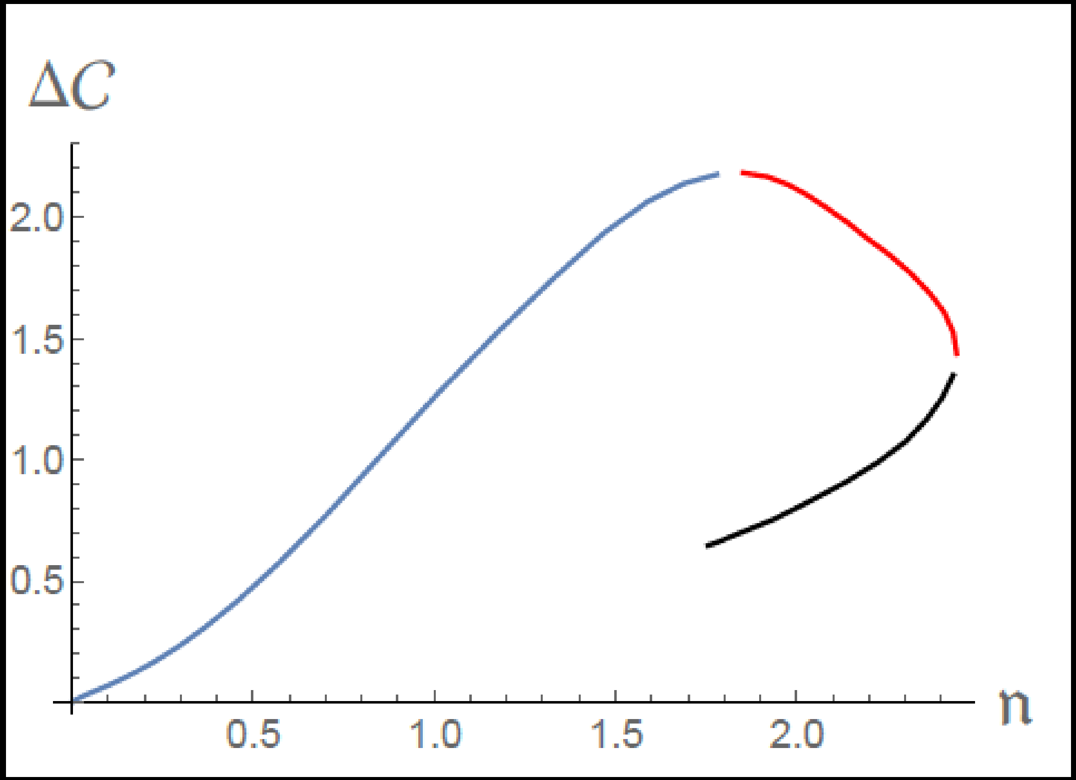


$$n(t,\xi,\ell) = \frac{1}{4}\frac{\partial^2 S(t,\xi,\ell)}{\partial \xi^2} - \frac{\partial^2 S(t,\xi,\ell)}{\partial \ell^2}$$

- Classical Gibbs entropy – 2^N microscopic states contributing to the given macroscopic one
- Integrated entanglement density – 2^N possible outcomes of projective measurement

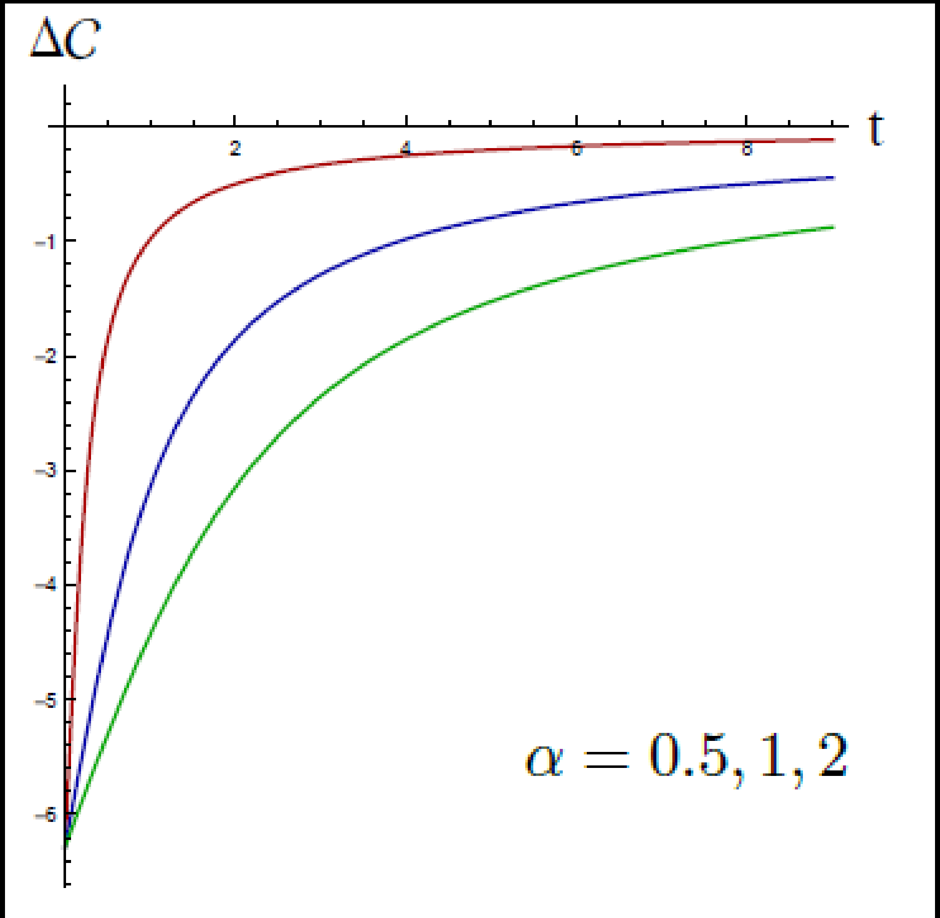
Entanglement density





- State is getting more complex but less random

Action complexity following local quench



A different pattern of complexity thermalization:

The first moment correction is negative and independent of the strength of the quench

Then it returns to the initial value

Lloyd computational bound

- Elementary logical operation takes at least Δt seconds

$$\Delta t \geq \pi\hbar/2E$$

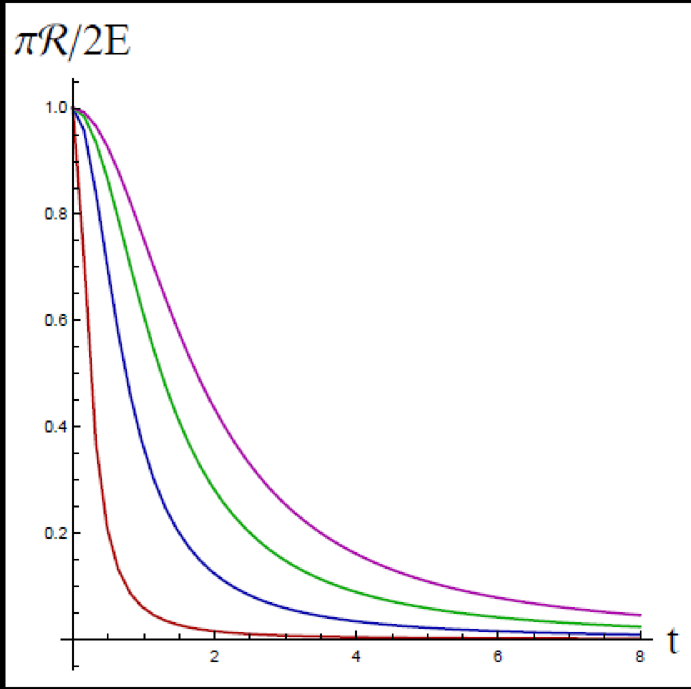
- Relating complexity to the number of elementary gates:

$$R = \frac{dC}{dt}$$

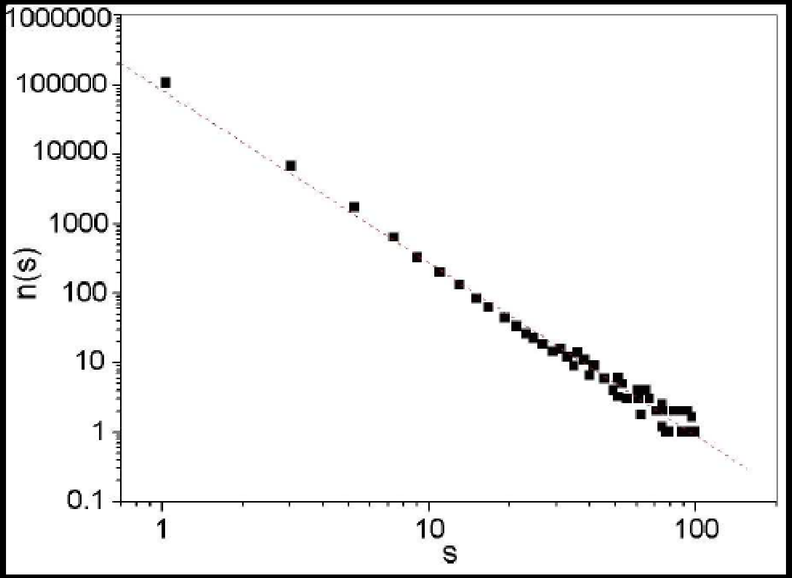
Lloyd, quant-ph/9908043

$$C = \int R dt \leq \frac{2E}{\pi\hbar} t$$

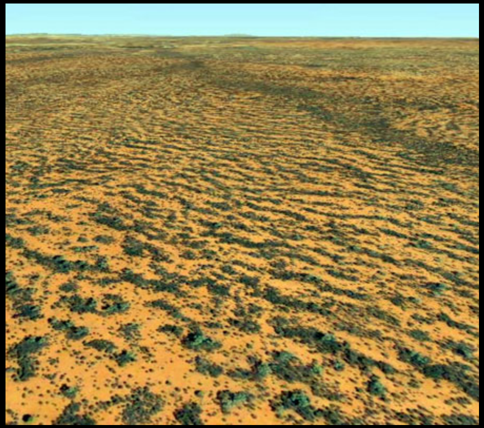
$$R \leq \frac{2E}{\pi\hbar}$$

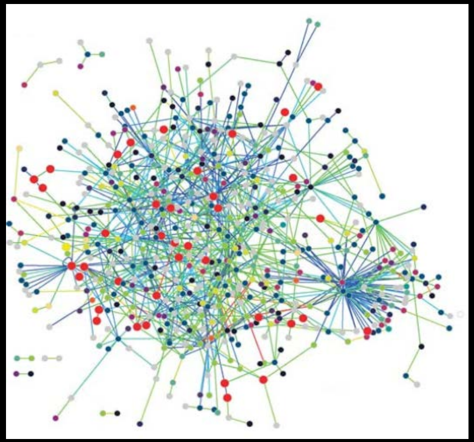


Self-organized natural patterns



Bak, Tang, Wiesenfeld, Phys.Rev.Lett. 59, 381, (1987)





Local quench in a critical system

Local quench → maximally fast growth of complexity??

Conclusions

Though both the volume and the action holographic complexities were conjectured in the context of computational complexity, they seem to exhibit some features of effective complexity

Volume complexity:

Smooth non-monotonous dependence of complexity on the entanglement entropy and integrated entropy density

Action complexity:

Saturation of the Lloyd bound. “Criticality as a seed of complexity”

Outlook

- Entanglement density in 1d can be formally mapped onto 2d patterns (in ξ, l - coordinates).
- Study complexity of different patterns. Arbitrary inhomogeneous quenches.
- Volume/Action complexities are still inspired by the concept of computational complexity. Are there reasonable holographic definitions of physical/interscale complexities?