Partially Massless Higher Spin Gravity

at One Loop

Thomas Basile

based on [1805.05646] *and* [1805.10092] *in collaboration*

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[Klebanov-Polyakov, 2002][Sezgin-Sundell, 2002]

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AdS$_{d+1}$: Vasiliev's type A Higher Spin Gravity

- Massless fields of *all integer spin $s \geqslant 1$*

$$\varphi_s\,, \qquad \delta_\xi \varphi_s = \nabla\,\xi_{s-1}$$

and a scalar of mass $m^2 = -2(d-2)$.

CFT$_d$: $U(\mathrm{N})$ vector model

- Conserved currents of *all integer spin $s \geqslant 1$*

$$J_s \sim \phi\,\partial^s\,\phi\,, \qquad \partial \cdot J_s \approx 0$$

and scalar operator $J_0 = \bar{\phi}\phi$ with conformal weight $\Delta = d - 2$.

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- $U(\mathbf{N}) \to O(\mathbf{N})$, only even spin operators remain

Higher Spin Holography

[Klebanov-Polyakov, 2002][Sezgin-Sundell, 2002]

Important features of Higher Spin holographic dualities:

- Expected to hold in *arbitrary dimensions*;
- Do not require supersymmetry.

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Several tests passed and evidences in favor of it:

- 3-point functions [Giombi-Yin, 2009], [Giombi-Yin, 2012];
- HS invariants [Didenko-Skvortsov, 2012] [Colombo-Sundell, 2012], [Bonezzi-Boulanger-De Filippi-Sundell, 2017];
- Cubic couplings [Sleight-Taronna, 2016];
- ...

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Focus here: one-loop free energy in $\mathbf{EAdS}_{d+1}$ / $\mathbf{S}^d$.

Introduction

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$$F_{CFT} = \mathbf{N}\, F^{(0)}_{CFT} + F^{(1)}_{CFT} + \frac{1}{\mathbf{N}}\, F^{(2)}_{CFT} + \ldots$$

and

$$\Gamma_{AdS} = \frac{1}{g}\,\Gamma^{(0)}_{AdS} + \Gamma^{(1)}_{AdS} + g\,\Gamma^{(2)}_{AdS} + \ldots$$

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$\rightsquigarrow$ Comparison order by order possible assuming

$$\frac{1}{g} \;\sim\; \mathbf{N}$$

$U(\mathbf{N})$ / $O(\mathbf{N})$ vector model

Free CFT $\Rightarrow$ free energy can be computed *exactly*:

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Complete action *unknown* (only up to cubic order available), and hence

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Possibility: look at the *next order* and check whether $\Gamma_{AdS}^{(1)} = 0$ or not, as the latter one-loop quantity depends only on the *spectrum* of the theory.

One-loop free energy test

Already done up to $d = 15$ for the type-A and type-B theories

[Giombi-Klebanov, 2013] [Giombi-Klebanov-Safdi, 2014] [Giombi-Klebanov-Tan, 2016]

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$\rightsquigarrow$ Extension to (the type-A_ℓ and type-B_ℓ) *partially massless* higher-spin gravities in *arbitrary dimensions.*

Character Integral Representation of the Zeta function

Zeta function in EAdS$_{d+1}$

One-loop free energy in AdS$_{d+1}$

$$\Gamma^{(1)}_{AdS} = -\tfrac{1}{2} \sum_{\varphi \in \mathcal{H}} \ln \det K_\varphi \,, \qquad K_\varphi \ : \ \text{Kinetic operator of } \varphi$$

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Regularization of UV divergences by introducing the *zeta function*

$$\zeta_\varphi(z) := \int_0^\infty \frac{\mathrm{d}t}{\Gamma(z)}\, t^{z-1}\, \mathrm{Tr}(e^{-t\, K_\varphi})$$

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Then

$$\Gamma^{(1)}_{AdS}\Big|_{\text{finite part}} = -\tfrac{1}{2} \sum_{\varphi \in \mathcal{H}} \zeta'_\varphi(0)\,.$$

Zeta function in EAdS_{d+1}: Camporesi-Higuchi

Given a *divergenceless and traceless* tensor $\varphi_{\mathbb{Y}}$ in EAdS_{d+1} subject to

$$\left(\nabla^2 - m_{\mathbb{Y}}^2\right)\varphi_{\mathbb{Y}} = 0\,,$$

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$$\zeta_{[\Delta;\mathbb{Y}]}(z) = C_{d,\mathbb{Y}} \int_0^\infty \mathrm{d}u\, \frac{\mu_{\mathbb{Y}}(u)}{\left[u^2 + (\Delta - \frac{d}{2})^2\right]^z}\,,$$

with

$$\mu_{\mathbb{Y}}(u) = \left(u \tanh(\pi u)\right)^{d-2r} \times \prod_{k=1}^{r}\left(u^2 + [s_k + \tfrac{d}{2} - k]^2\right),$$

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where $r := [\frac{d}{2}]$, while $\mathbb{Y} = (s_1, \ldots, s_r)$ is the spin of the field, and

$$m_{\mathbb{Y}}^2 = \Delta(\Delta - d) - \sum_{k=1}^{r} s_k\,.$$

Zeta function in EAdS_{d+1}: alternative representation

Character Integral Representation of the Zeta function

Derived in [Bae-Joung-Lal, 2016] for AdS_4 and AdS_5. For instance,

$$\zeta^{AdS_5}_{\Delta,(s_1,s_2)}(z) = \int_0^\infty \mathrm{d}\beta\, f(z;\beta,\partial_{\alpha_1},\partial_{\alpha_2})\, \chi^{so(2,4)}_{\Delta,(s_1,s_2)}(\beta;\alpha_1,\alpha_2)\Big|_{\alpha_1=\alpha_2=0}$$

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All data of a free field can be encompassed in the *character* $\chi^{so(2,d)}_{\Delta,\mathbb{Y}}(\beta;\vec{\alpha})$ of the corresponding $\mathfrak{so}(2,d)$ module $\mathcal{D}(\Delta;\mathbb{Y})$.

CIRZ: two steps at once

Hilbert space $\mathcal{H}$ of a theory in $\mathrm{AdS}_{d+1}/\mathrm{CFT}_d$ carries a (reducible) representation of $\mathfrak{so}(2,d)$

$$\mathcal{H} = \bigoplus_{\Delta,\mathbb{Y}} N_{\Delta,\mathbb{Y}}\, \mathcal{D}(\Delta;\mathbb{Y}) \;\Rightarrow\; \chi^{\mathfrak{so}(2,d)}_{\mathcal{H}}(\beta;\vec{\alpha}) = \sum_{\Delta,\mathbb{Y}} N_{\Delta,\mathbb{Y}}\, \chi^{\mathfrak{so}(2,d)}_{(\Delta;\mathbb{Y})}(\beta;\vec{\alpha})\,.$$

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By linearity

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Advantages:

- Computation of the one-loop zeta function of a theory in **one shot**;
- Avoid **divergences** due to the summation on an **infinite spectrum** (reduced to a group theoretical statement).

Zeta function in EAdS$_{d+1}$: alternative representation

Extension to *any dimension* $d+1 \geqslant 3$ derived in [TB-Joung-Lal-Li, 2018]

$$\zeta_{[\Delta;\mathbb{Y}]}(z) = \int_0^\infty \mathrm{d}\beta \sum_{k=0}^{r} \phi_k(z;\beta,\boldsymbol{\alpha})\, \chi^{so(2,d)}_{\Delta,\mathbb{Y}}(\beta;\vec{\alpha}_k)\Big|_{\boldsymbol{\alpha}=0}\,.$$

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with

$$\boldsymbol{\alpha} = (\alpha_0,\ldots,\alpha_r) \quad \text{and} \quad \vec{\alpha}_k := (\alpha_0,\ldots,\alpha_{k-1},\alpha_{k+1},\ldots,\alpha_r)$$

some auxiliary variables.

Zeta function in EAdS_{d+1}: CIRZ

Two possible ways to evaluate $\boldsymbol{\alpha} = 0$:

- Contour integral:

$$\zeta_{[\Delta;\mathbb{Y}]}(z) = \int_0^\infty \mathrm{d}\beta \sum_{k=0}^{r} \oint \prod_{j=0}^{r} \frac{\mathrm{d}\alpha_j}{2\,i\,\pi\,\alpha_j}\,\phi_k(z;\beta,\boldsymbol{\alpha})\,\chi_{\Delta,\mathbb{Y}}^{so(2,d)}(\beta;\vec{\alpha}_k)$$

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- Derivative expansion:

$$\zeta_{[\Delta;\mathbb{Y}]}(z) = \int_0^\infty \mathrm{d}\beta \sum_{n=0}^{r} \mathfrak{D}_{(n)}\left[\varphi_n(z;\beta,\vec{\alpha})\,\chi^{so(2,d)}_{\Delta,\mathbb{Y}}(\beta;\vec{\alpha})\right]_{\vec{\alpha}=\vec{0}}\,,$$

where $\mathfrak{D}_{(n)}$ are differential operators whose explicit form are dictated by the *root system of* $so(d+2)$.

Application to Partially Massless Higher Spin Gravities

Partially Massless Higher Spin gravity

From Maldacena-Zhiboedov's theorem [Maldacena-Zhiboedov, 2011]

$$\textbf{Free CFT} \quad \leftrightarrow \quad \textbf{HS theory}$$

Extension of HS holography *below* the unitarity bound [Bekaert-Grigoriev, 2013]

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Simplest case: **free scalar**

Unitary ($\ell = 1$)	$\rightarrow$	Non-unitary ($\ell \geqslant 2$)
$\Box\phi = 0\,, \quad \Delta_\phi = \frac{d-2}{2}$		$\Box^{\ell}\phi = 0\,, \quad \Delta_\phi = \frac{d-2\ell}{2}$

Spectrum of single trace operators in CFT_d

Conserved currents	$\rightarrow$	Partially-conserved currents
$\partial \cdot J_s \approx 0$		$\partial^t \cdot J_s^{(t)} \approx 0$

Spectrum of single trace operators in CFT$_d$

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Spectrum of the type-A$_\ell$ HS gravity in AdS$_{d+1}$

Massless fields	$\rightarrow$	Partially-massless fields
$\delta\varphi_s \sim \nabla\xi_{s-1}$		$\delta\varphi_s \sim \nabla^t\xi_{s-t}$

Propagate an intermediate number of helicities between that of a massive and a massless spin-s field.

Type A_ℓ Partially-Massless Higher Spin Gravity

Nonlinear theory for $\ell = 1$ [Vasiliev, 2003] and recently proposed extension for $\ell \geqslant 2$ [Alkalaev-Grigoriev-Skvortsov, 2014] [Brust-Hinterbichler, 2016]

$$\mathrm{Rac}_\ell^{\otimes 2} \cong \bigoplus_{k=1}^{\ell} \left[\bigoplus_{s=0}^{2(k-1)} \mathcal{D}\big(s+d-2k; s\big) \oplus \bigoplus_{s=2k-1}^{\infty} \mathcal{D}\big(s+d-2k; s\big) \right]$$

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Massive fields of spin $s = 0, \ldots, 2k-2$. Only some of the massive scalars are unitary depending on the value of ℓ.

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Partially massless fields of spin $s = 2k-1, 2k, \ldots$ and of depth $t = 2k-1$.

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$\Rightarrow \chi^{\mathfrak{so}(2,d)}_{A_\ell}(\beta; \vec{\alpha}) = \Big(\chi^{\mathfrak{so}(2,d)}_{\mathrm{Rac}_\ell}(\beta; \vec{\alpha})\Big)^2$, Well suited to use the CIRZ method.

PM holography: Type A_ℓ

One-loop computation:

- Odd dimensional AdS ($d = 2r$)

$$\zeta_{A_\ell}(z) = \mathcal{O}(z^2)\,, \qquad \zeta_{A_\ell^{min}}(z) = -2\, a^{(d)}_{\mathrm{Rac}_\ell}\, z + \mathcal{O}(z^2)$$

PM holography: Type $\mathbf{A}_\ell$

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- Even dimensional AdS ($d = 2r + 1$)

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Take away:

Computations in accordance with the holographic duality for the *non-minimal* theory.

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Take away:

Computations in accordance with the holographic duality for the *non-minimal* theory. However for the minimal theory result to agree with holography at one-loop, one needs

$$\frac{1}{g} = \mathbf{N} - 1\,.$$

PM holography: Type B_ℓ

Next to most simple free CFT: **(higher-derivative) free spinor**

$$\partial\!\!\!/^{\,2\ell-1}\,\psi = 0\,,$$

non-unitary for $\ell \geqslant 2$.

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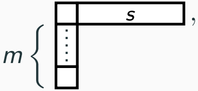
Nonlinear theory recently proposed for $\ell = 1$ [Skvortsov-Grigoriev, 2018] .
Complicated spectrum containing massive and (partially-)massless field of spin

$$(s, 1^m) \qquad \cong$$



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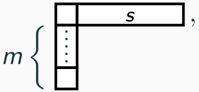

encoded in $\mathrm{Di}_\ell^{\otimes 2} \Rightarrow \chi_{B_\ell}^{\mathfrak{so}(2,d)}(\beta;\vec{\alpha}) = \Big(\chi_{\mathrm{Di}_\ell}^{\mathfrak{so}(2,d)}(\beta;\vec{\alpha})\Big)^2.$

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- Even dimensional AdS ($d = 2r + 1$)

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However

$$-\tfrac{1}{2}\,\zeta'_{B_\ell}(0) \neq F_{\mathrm{Di}_\ell}$$

Already observed for $\ell = 1$, i.e. the massless type-B theory

[Giombi-Klebanov-Tan, 2016] [Gunaydin-Skvortsov-Tran, 2016]

Discussion and questions

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Thank you!