Black holes and scale dependence

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acknowledge:
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Content

- A parable
- Scale dependence
- Black holes, examples
- Conclusion

A parable

A parable

A parable

A parable

A parable

A parable

?

A parable

A parable

A parable

Scale dep.

A parable

Scale dep.

BH solution

A parable

Scale dep.

BH solution

AdS/CFT meaning

A parable

Scale dep.

BH solution

Nothing else

AdS/ CFT meaning

A parable

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BH solution

?

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AdS/ CFT meaning

A parable

Scale dep.

BH solution

You!

Nothing else

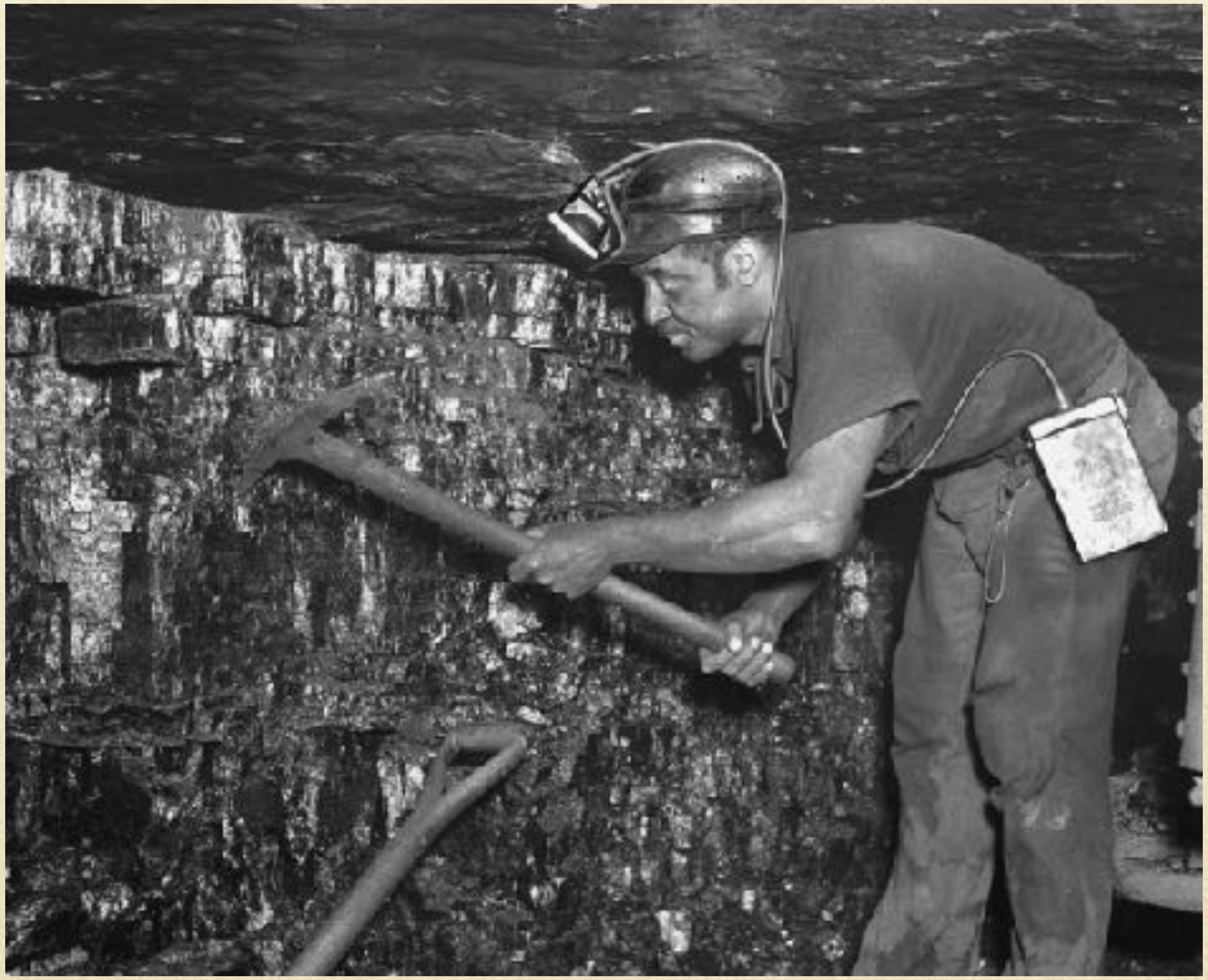
AdS/ CFT meaning

A parable

This is the main
motivation
for this talk
here

?

Scale dependence



Scale dependence

In virtually every QFT

Scale dependence

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S

Classical action

Scale dependence

In virtually every QFT

S $\Rightarrow$

Classical action

Scale dependence

In virtually every QFT

S	$\Rightarrow$	Γ_k
Classical action		Effective action

Scale dependence

In virtually every QFT

$$S \quad \Rightarrow \quad \Gamma_k$$

Classical action   Effective action

Observable prediction?

Scale dependence

In virtually every QFT

$$S \Rightarrow \Gamma_k$$

Classical action → Effective action

Observable prediction?

Scale dependence

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$$S \quad \Rightarrow \quad \Gamma_k$$

Classical action — Effective action

Observable prediction?

Scale setting!

Scale dependence

In virtually every QFT

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Classical action — Effective action

Observable prediction?

Scale setting!

$$k = k(x)$$

Scale dependence

In virtually every QFT

$S \quad \Rightarrow \quad \Gamma_k$

Classical action — Effective action

Observable prediction?

Scale setting!

$k = k(x)$

Want variational meaningful!

Scale dependence

In Gravity (truncated)

$$\Gamma[g_{\mu\nu}, k] = \int d^3x \sqrt{-g} \left[\frac{1}{G_k} \Big(R - 2\Lambda_k \Big) + \mathscr{L}_M \right]$$

Scale dependence

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Selfconsistent backgrounds must solve

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$$G_{\mu\nu} + g_{\mu\nu}\Lambda_k = \kappa_k T^M_{\mu\nu} - \Delta t_{\mu\nu} \qquad \text{where}$$

$$\Delta t_{\mu\nu} \equiv G_k\Big(g_{\mu\nu}\Box - \nabla_\mu \nabla_\nu\Big)G_k^{-1}$$

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Unknown:

Scale dependence

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Unknown: Λ_k, G_k (if answer, depends on whom you ask)

$g_{\mu\nu}(x), k(x)$ Gap equations

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$g_{\mu\nu}(x), k(x)$ Gap equations

Highly symmetric systems simple approach

Black holes

Black holes

Black holes

Line element (e.g. 2+1 dim):

$$ds^2 = -f(r)dt^2 + g(r)dr^2 + r^2\left[d\phi\right]^2$$

Unknown: $\Lambda_k, G_k,$ $f(r), g(r), k(r)$

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Remaining equations:

Black holes

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Remaining equations:

$$G_{\mu\nu} + g_{\mu\nu}\Lambda_k = \kappa_k T^M_{\mu\nu} - \Delta t_{\mu\nu}$$

Can be solved

with only one additional condition / Ansatz!

Black holes

Line element: (Schwarzschild Ansatz)

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The solution is: ….

Black holes

Solution (no charge no rotation)

$$G(r) = \frac{G_0^2}{G_0 + \epsilon r(1 + G_0 M_0)}$$

$$f(r) = f_0(r) + 2M_0 G_0 \left(\frac{G_0}{G(r)} - 1\right)\left[1 + \left(\frac{G_0}{G(r)} - 1\right)\ln\left(1 - \frac{G(r)}{G_0}\right)\right]$$

$$\Lambda(r) = \frac{-G(r)^2}{\ell_0^2 G_0^2}\left[1 + 4\left(\frac{G_0}{G(r)} - 1\right) + \left(5M_0 G_0 \frac{\ell_0^2}{r^2} + 3\right)\left(\frac{G_0}{G(r)} - 1\right)^2 + 6M_0 G_0 \frac{\ell_0^2}{r^2}\left(\frac{G_0}{G(r)} - 1\right)^3\right.$$

$$\left. + 2M_0 G_0 \frac{\ell_0^2}{r^2}\frac{G_0}{G(r)}\left(3\left(\frac{G_0}{G(r)} - 1\right) + 1\right)\left(\frac{G_0}{G(r)} - 1\right)^2 \ln\left(1 - \frac{G(r)}{G_0}\right)\right]$$

with $f_0(r) = -G_0 M_0 + \frac{r^2}{\ell_0^2}$ classical solution

Black holes

Constants of integration:

Black holes

Constants of integration:

$$G_0, M_0, \ell_0, \epsilon$$

Black holes

Constants of integration:

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Black holes

Constants of integration:

$$G_0, M_0, \ell_0, \epsilon$$

Encodes scale dependence:

$$\lim_{\epsilon\to 0} G(r) = G_0$$

$$\lim_{\epsilon\to 0} f(r) = -G_0 M_0 + \frac{r^2}{\ell_0^2} = f_0(r)$$

$$\lim_{\epsilon\to 0} \Lambda(r) = -\frac{1}{\ell_0^2}$$

Black holes

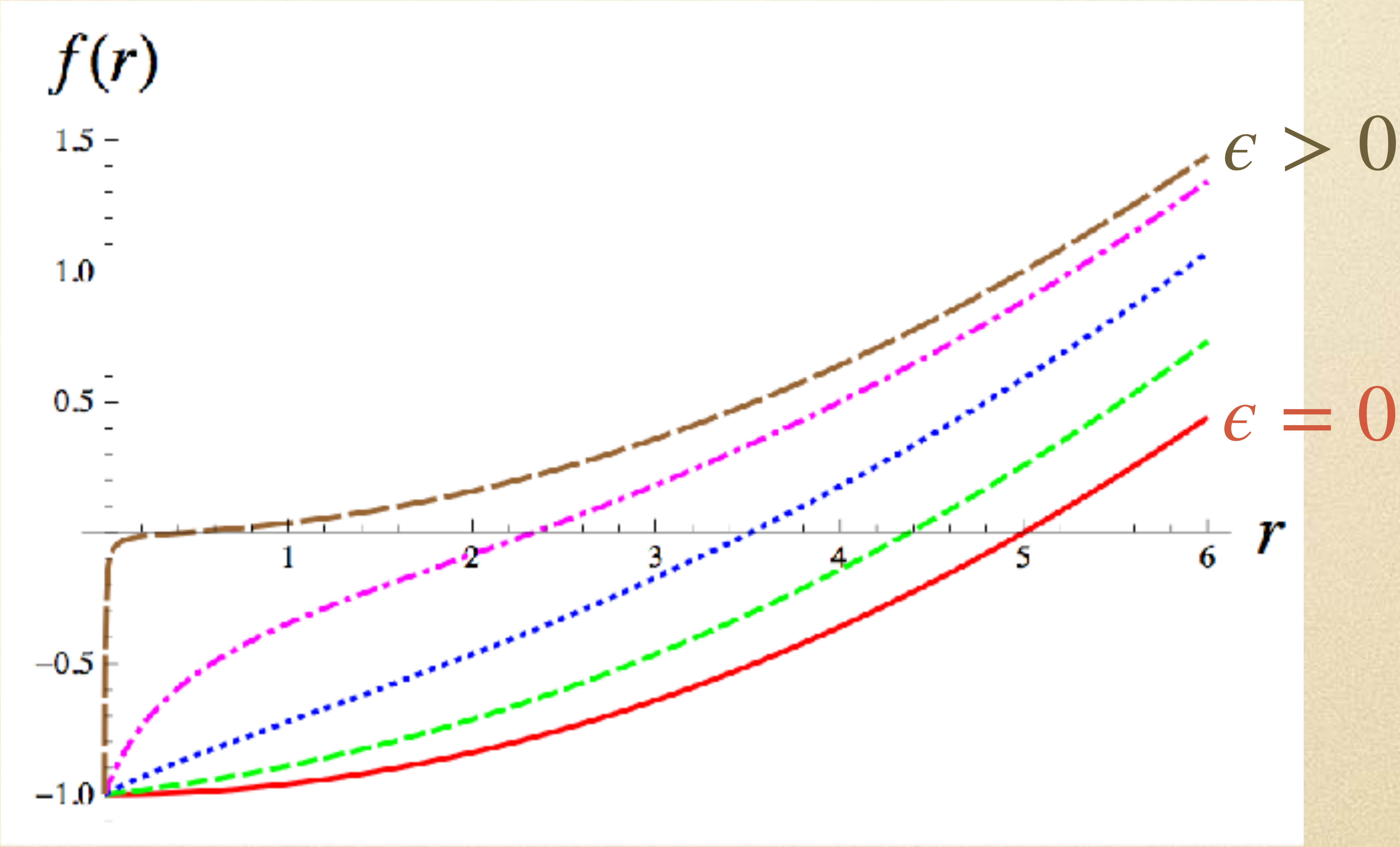
Line element:

$\epsilon > 0$

$\epsilon = 0$

Black holes

Line element:



Black holes

Black holes

Asymptotics: $r \to 0$

$$R = -4M_0\epsilon(1+G_0M_0)\cdot\frac{1}{r} - \left(\frac{6}{\ell_0^2} + 10\frac{M_0}{G_0}(1+G_0M_0)^2\epsilon^2\right) + \mathcal{O}(r^1)$$

Singularity

Black holes

Asymptotics: $r \to 0$

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Singularity

Asymptotics: $r \to \infty$

$$f(r) \sim \frac{r^2}{\ell_0^2} + \mathcal{O}(r^1)$$

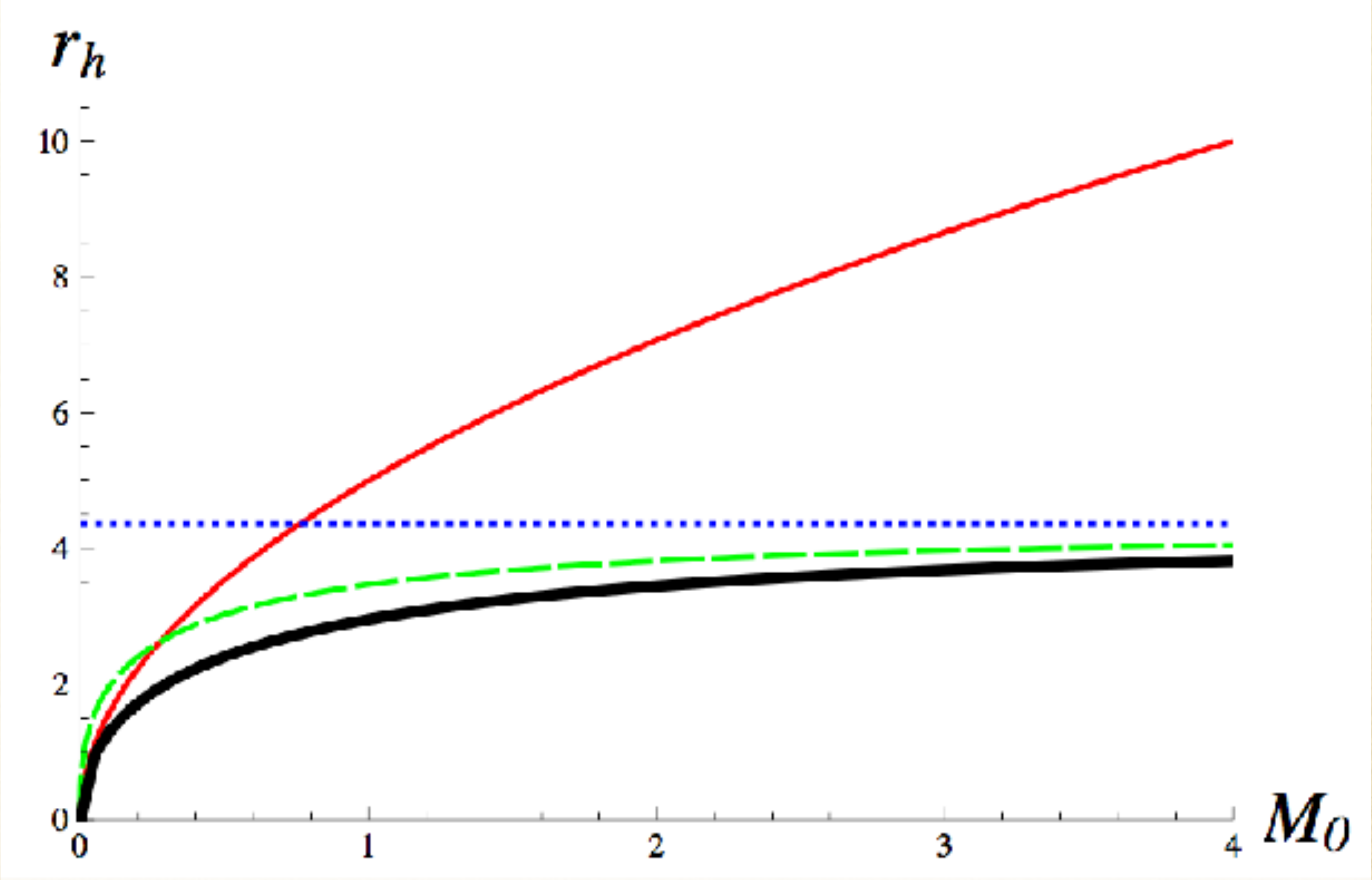
Asymptotically AdS

Black holes

Horizon:

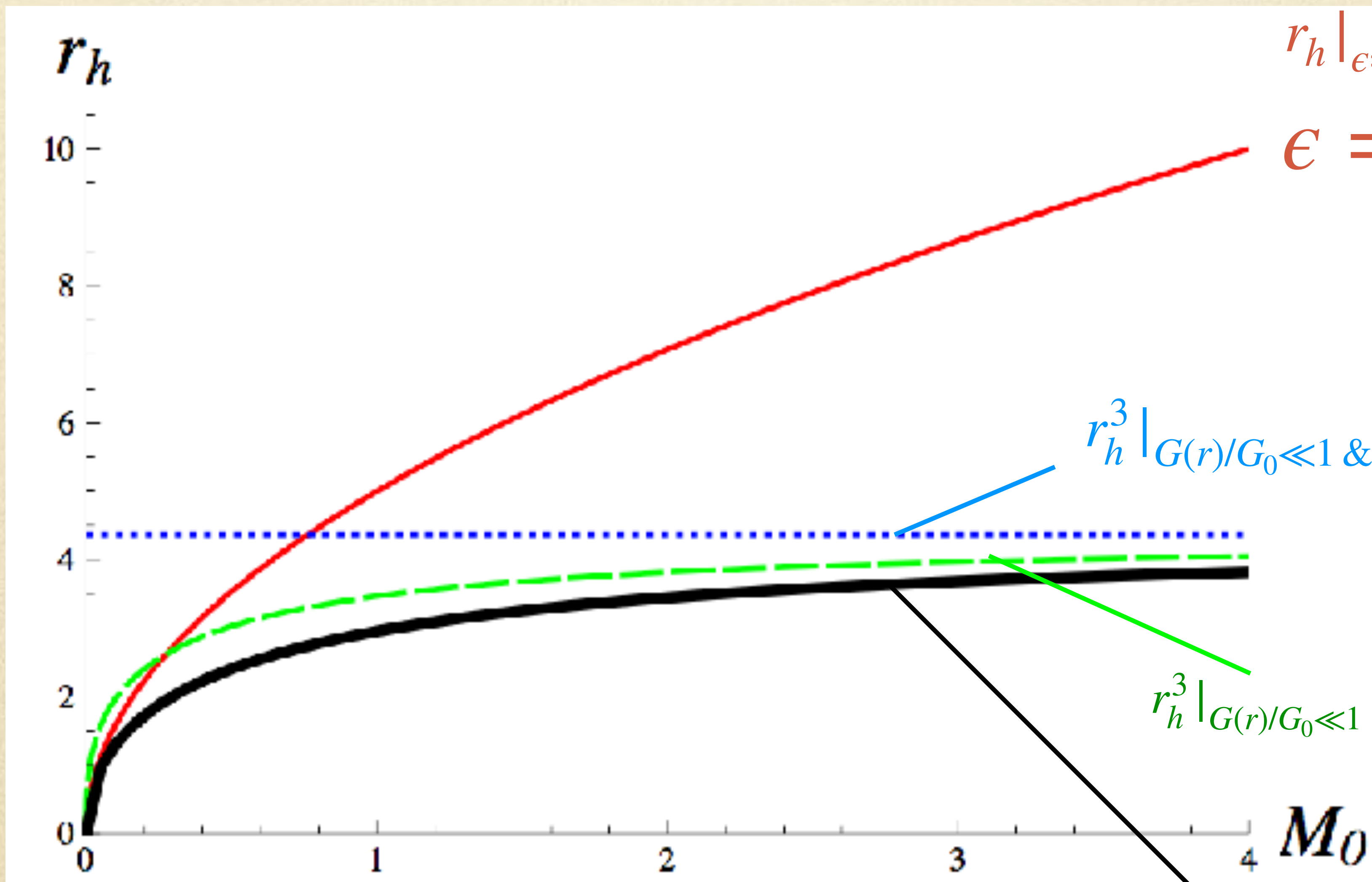
Black holes

Horizon:



Black holes

Horizon:



$r_h|_{\epsilon=0} = \sqrt{G_0 M_0} \ell_0$

$\epsilon = 0$

$r_h^3|_{G(r)/G_0 \ll 1 \,\&\, M_0 G_0 \gg 1} \approx \frac{2}{3} \frac{G_0 \ell_0^2}{\epsilon}$

$r_h^3|_{G(r)/G_0 \ll 1} \approx \frac{2}{3} \frac{M_0 G_0^2 \ell_0^2}{(1 + M_0 G_0)\epsilon}$

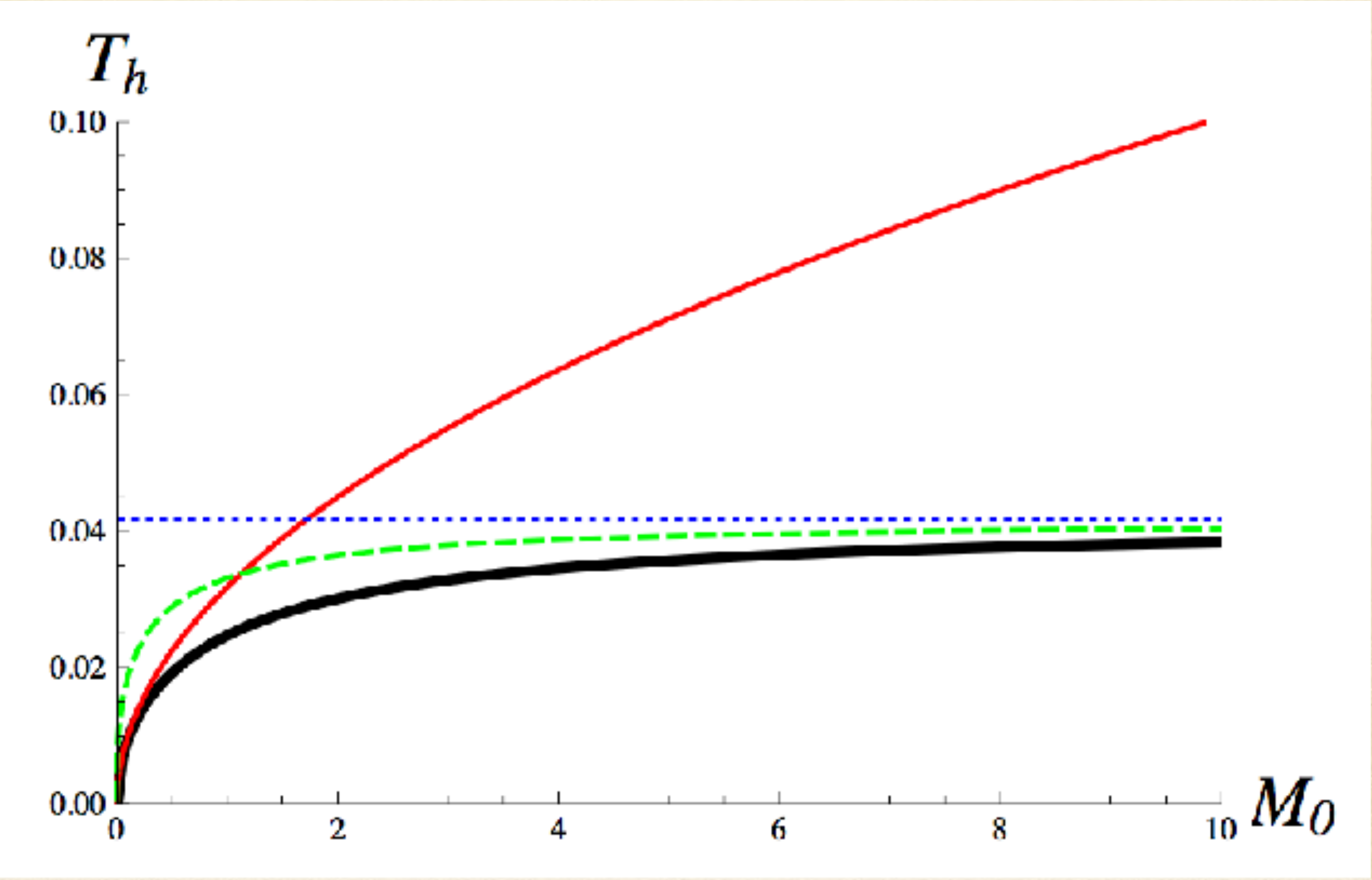
numerical $\epsilon > 0$

Black holes

Temperature:

Black holes

Temperature:



Black holes

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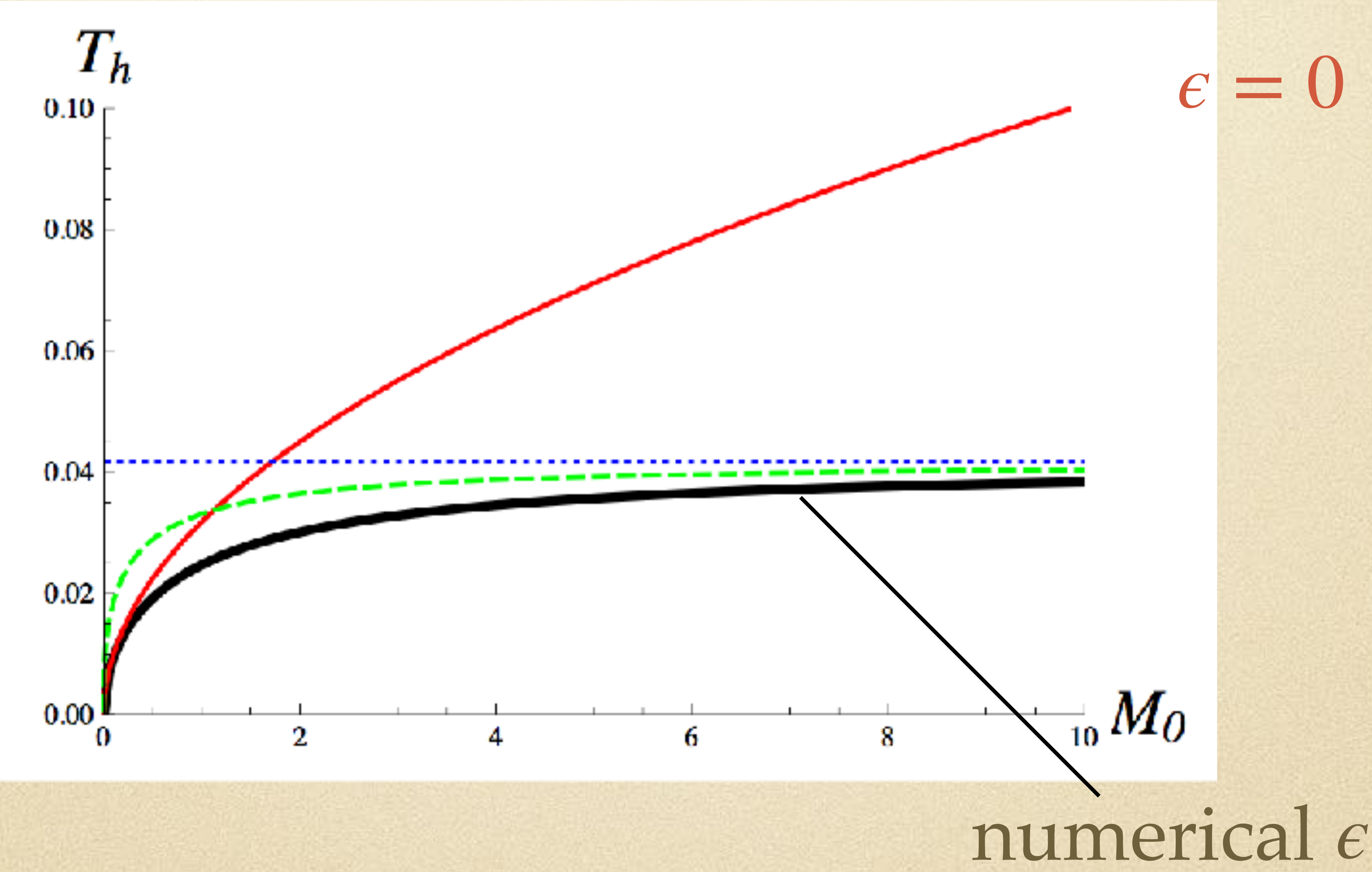

Black holes

Entropy: $$S = \frac{1}{4}\oint_{r=r_h} d^{D-1}x \frac{\sqrt{h}}{G(x)}$$

$$S = \frac{2\pi r_h}{4G(r_h)} = \frac{2\pi r_h}{4G_0}\left[1 + \frac{(1+G_0M_0)\epsilon r_h}{G_0}\right]$$

Black holes

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Mass:

$$dM = TdS$$

$$M|_{M_0 \gg 1} \sim M_0$$

Black holes

Many more solutions:

- 2+1 dim.& charge
- 2+1 dim. & angular momentum
- Null energy condition (d dim.)
- 3+1 dim.
- 3+1 dim & charge
- Power Maxwell
- Regular black hole
- …

Black holes

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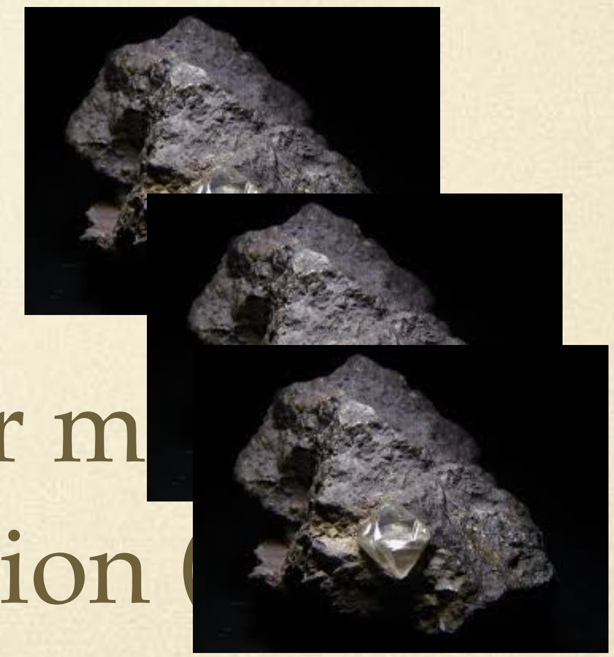

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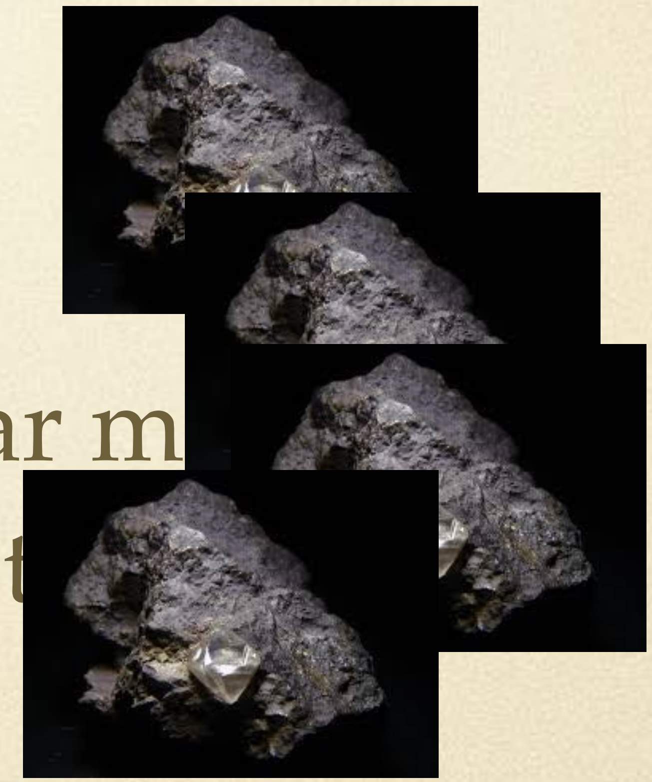

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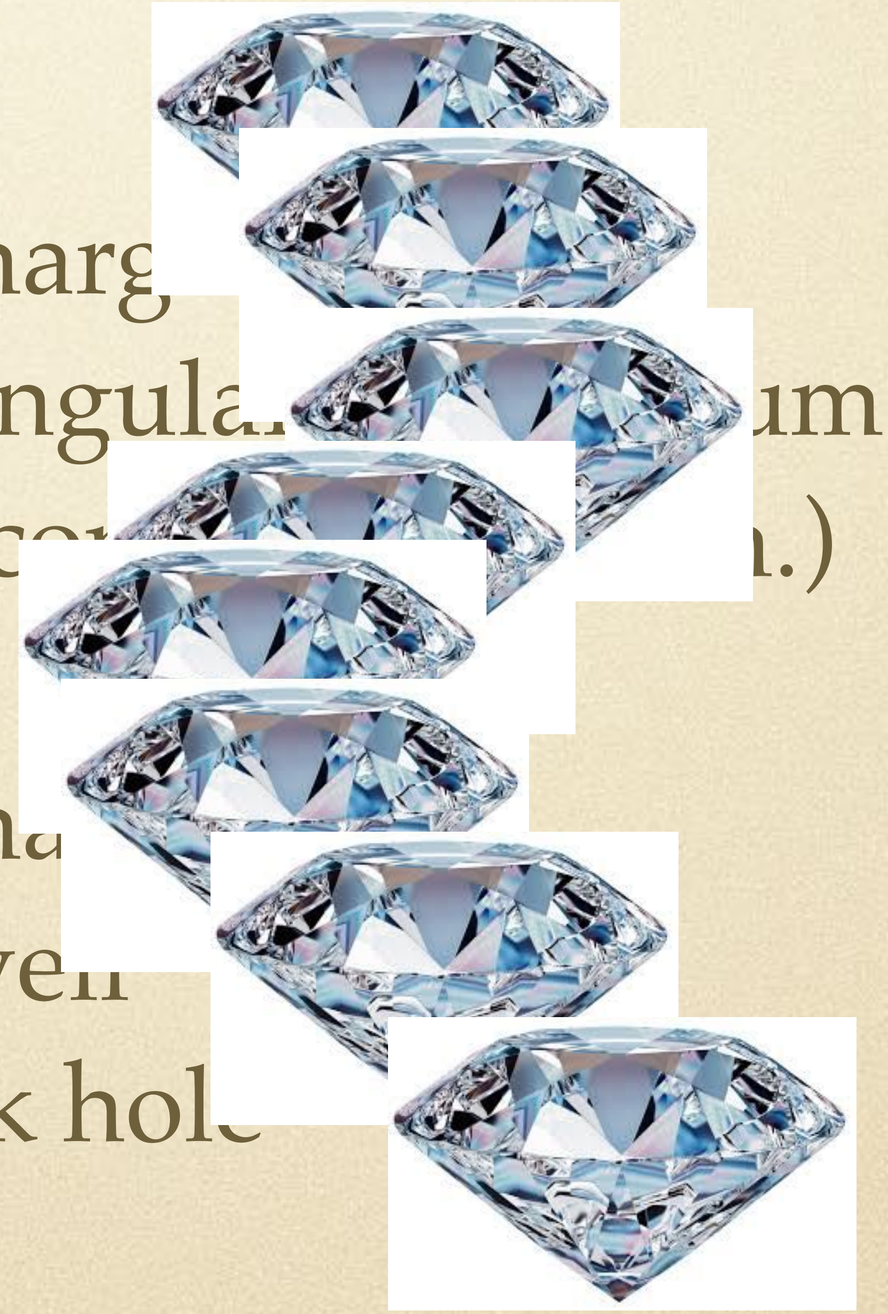

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Concluding Comments

- Scale dependence and setting: variational
- Many BH solutions with simple assumption
- Maybe interesting in AdS/CFT?

Thank You!

Literature

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