SECOND SIMPLEST HIGHER SPIN THEORY

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HIGHER SPIN PROBLEM

More symmetric phase of physics with higher spin symmetry?

Construct an interacting theory of massless higher spin fields in $d > 3$

DIFFICULTIES:

No go theorems — clash with locality

[Weinberg'64; Aragone, Deser'79; Berends, Burgers, van Dam'85; …]

Existing results and approaches: no clear structure, no clear connection to lower spin theories

FIRST SIMPLEST HIGHER SPIN THEORY

TOY HS THEORIES IN 3D – CHERN-SIMONS THEORIES

[Blencowe'89]

No go theorems do not apply — absence of bulk degrees of freedom

Straightforwardly generalizes the action of 3d gravity (in AdS):

$$sl(2) \oplus sl(2) \to hs(\lambda) \oplus hs(\lambda)$$

Still have non-trivial physics: e.g. AdS3/CFT2 holography, black hole physics, etc.

CHERN-SIMONS HS THEORIES

ARE THERE ANY OTHER REGIMES WHERE THE HIGHER SPIN PROBLEM GETS SIMPLIFIED?

ANSWER:

Yes, there is a natural higher spin generalization of the self-dual Yang-Mills theory

INTRO TO SELF-DUAL YANG-MILLS

SDYM EQUATIONS OF MOTION

$$F = \pm * F$$

Real in (+,+,+,+) and (+,+, -, -) signatures

PROPERTIES:

Implies $* d * F = 0$, so SDYM eom solve YM eom, YM instantons

Integrability: infinite set of conserved currents, solution-generating techniques, superposition of solutions, etc.

ACTION

Yang-Mills theory in the light-cone gauge (d=4)

$$S = \frac{1}{2}\int d^4 q_i \delta(\sum q_i) q_1^2 \operatorname{Tr}\left[\Phi^+(q_1)\Phi^-(q_2)\right]$$
$$+ \frac{g}{3}\int d^4 q_i \delta(\sum q_i) \frac{\bar{\mathbb{P}}_{12}\beta_3}{\beta_1\beta_2} \operatorname{Tr}\left[\Phi^+(q_1)\Phi^+(q_2)\Phi^-(q_3)\right]$$
$$+ \frac{g}{3}\int d^4 q_i \delta(\sum q_i) \frac{\mathbb{P}_{12}\beta_3}{\beta_1\beta_2} \operatorname{Tr}\left[\Phi^-(q_1)\Phi^-(q_2)\Phi^+(q_3)\right]$$
$$+ \frac{g^2}{12}\int d^4 q_i \delta(\sum q_i) \left(1 - \frac{\beta_3-\beta_1}{\beta_3+\beta_1}\frac{\beta_4-\beta_2}{\beta_4+\beta_2}\right) \operatorname{Tr}\left[\Phi^+(q_1)\Phi^+(q_2)\Phi^-(q_3)\Phi^-(q_4)\right]$$

$$\mathbb{P}_{ij} \equiv q_i\beta_j - q_j\beta_i$$

$$\beta \equiv q^+$$

Self-dual Yang-Mills theory in the light-cone gauge

$$S = \frac{1}{2}\int d^4 q_i \delta(\sum q_i) q_1^2 \operatorname{Tr}\left[\Phi^+(q_1)\Phi^-(q_2)\right]$$
$$+ \frac{g}{3}\int d^4 q_i \delta(\sum q_i) \frac{\bar{\mathbb{P}}_{12}\beta_3}{\beta_1\beta_2} \operatorname{Tr}\left[\Phi^+(q_1)\Phi^+(q_2)\Phi^-(q_3)\right]$$

[Chalmers, Siegel'96]

AMPLITUDES IN SDYM

Only <u>two</u> potentially non-trivial sectors:

TREE-LEVEL (–,+,+,…,+) Vanishes on-shell

1-LOOP (+,+,+,…,+) Non-zero, no cuts

<u>The same as in the Yang-Mills case</u>

GENERALIZATION TO HIGHER SPINS

GENERALIZATION TO HIGHER SPIN CASE

LIGHT-CONE APPROACH

Fix the light-cone gauge and deal with physical degrees of freedom

$$h^{\mu_1 \mu_2 \dots \mu_s} \quad \to \quad \{\Phi^{+s}, \Phi^{-s}\}$$

Lorentz symmetry is not manifest — impose it by hands

[Bengtsson, Bengtsson, Brink, Linden, Metsaev, …]

CHIRAL HIGHER SPIN THEORY

$$S = \frac{1}{2}\sum_{\lambda_i}\int d^4q_i\delta(\sum q_i)\delta_{\lambda_1+\lambda_2,0}\; q_1^2\;\Phi^{\lambda_1}(q_1)\Phi^{\lambda_2}(q_2)$$

$$+\sum_{\lambda_i} C^{\lambda_1\lambda_2\lambda_3}\int d^4q_i\delta(\sum q_i)\frac{\bar{\mathbb{P}}_{12}^{\lambda_1+\lambda_2+\lambda_3}}{\beta_1^{\lambda_1}\beta_2^{\lambda_2}\beta_3^{\lambda_3}}\Phi^{\lambda_1}(q_1)\Phi^{\lambda_2}(q_2)\Phi^{\lambda_3}(q_3)$$

$$+\sum_{\lambda_i} \tilde{C}^{\lambda_1\lambda_2\lambda_3}\int d^4q_i\delta(\sum q_i)\frac{\mathbb{P}_{12}^{-\lambda_1-\lambda_2-\lambda_3}}{\beta_1^{-\lambda_1}\beta_2^{-\lambda_2}\beta_3^{-\lambda_3}}\Phi^{\lambda_1}(q_1)\Phi^{\lambda_2}(q_2)\Phi^{\lambda_3}(q_3)+\dots$$

[Bengtsson, Bengtsson, Brink, Linden '83'86]

$$C^{\lambda_1\lambda_2\lambda_3} = \frac{1}{\Gamma(\lambda_1+\lambda_2+\lambda_3)}$$

[Metsaev '90]

If one sets $\tilde{C}^{\lambda_1\lambda_2\lambda_3} = 0$ no completion with higher vertices is needed

[DP, Skvortsov '16]

CHIRAL HIGHER SPIN THEORY

The complete action

$$S = \frac{1}{2}\sum_{\lambda_i}\int d^4q_i\delta(\sum q_i)\delta_{\lambda_1+\lambda_2,0}\; q_1^2\;\Phi^{\lambda_1}(q_1)\Phi^{\lambda_2}(q_2)$$

$$+\;\sum_{\lambda_i}\frac{1}{\Gamma(\lambda_1+\lambda_2+\lambda_3)}\int d^4q_i\delta(\sum q_i)\frac{\bar{\mathbb{P}}_{12}^{\lambda_1+\lambda_2+\lambda_3}}{\beta_1^{\lambda_1}\beta_2^{\lambda_2}\beta_3^{\lambda_3}}\Phi^{\lambda_1}(q_1)\Phi^{\lambda_2}(q_2)\Phi^{\lambda_3}(q_3)$$

This action is superficially reminiscent of the Chalmers-Siegel action.
Can this connection be made more precise?

CHIRAL HIGHER SPIN THEORY AS SDYM

Equations of motion

$$\Box\Phi - 2g\,\partial^+\left[\frac{\bar\partial}{\partial^+}\Phi, \Phi\right] = 0$$

can be rewritten as

$$\begin{aligned} F^{-x} &= 0 \\ F^{+\bar{x}} &= 0 \\ F^{+-} + F^{\bar{x}x} &= 0 \end{aligned}$$

where

$$A^x \equiv \Phi \qquad\qquad F(x^\mu; z) \equiv \sum_\lambda F(x^\mu) z^\lambda$$

$$[F, G] = \sinh\frac{1}{z}\left(\partial_x^F \partial_-^G - \partial_x^G \partial_-^F\right) FG$$

[DP'17]

CHIRAL HIGHER SPIN THEORY AS SDYM

CONSEQUENCES

Standard techniques allow to construct an infinite family of conserved currents — integrability. The hidden symmetry algebra is explicitly known.

Standard solution generating techniques can also be applied — instantons.

COLOR-KINEMATIC DUALITY

YM:

$$A_4 = \frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}$$

$$c_s + c_t + c_u = 0$$ Jacobi identity

$$n_s + n_t + n_u = 0$$ also true

GRAVITY:

$$M_4 = \frac{n_s n_s}{s} + \frac{n_t n_t}{t} + \frac{n_u n_u}{u}$$

[Bern, Carrasco, Johansson'08]

SD SECTOR: Much more obvious because the action is cubic

[Monteiro, O'Connell'11]

COLOR-KINEMATIC DUALITY

CHS:

$$A_4 = \frac{n_s \tilde{c}_s}{s} + \frac{n_t \tilde{c}_t}{t} + \frac{n_u \tilde{c}_u}{u}$$

$$\tilde{c}_s + \tilde{c}_t + \tilde{c}_u = 0$$ Jacobi identity

$$n_s + n_t + n_u = 0$$ also true

CHS amplitudes can be obtained from those of the Chalmers-Siegel theory by $c \to \tilde{c}$.

Generalized squaring procedures are also available

[DP'17]

CONCLUSIONS

There is a simple higher spin theory, very similar to SDYM

It has similar properties, in particular, infinite-dimensional hidden symmetry algebra

Suitable for constructing instantons — not studied in details yet

Amplitudes — see talk by E. Skvortsov