#### Fermions in Geodesic Witten Diagrams

[arXiv:1805.00217]

#### Mitsuhiro Nishida

(Gwangju Institute of Science and Technology)

in collaboration with **Kotaro Tamaoka** (Osaka University)

#### Outline

#### 1. Introduction

#### 2. Detail of our result

# Conformal field theory in theoretical physics

e.g. String theory, Critical phenomena, RG flow, …

#### Recent progress of CFT

(They are not directly related to our paper.)

#### Numerical conformal bootstrap

[Rattazzi, Rychkov, Tonni, Vichi, 2008], … [El-Showk, Paulos, Poland, Rychkov, Simmons-Duffin, Vichi, 2012],…

Out of time order correlation function in CFT

[Fitzpatrick, Kaplan, Walters, 2014], [Roberts, Stanford, 2014],…



Conformal partial wave is important for the recent progress of CFT.

### Q What is the gravity dual of conformal partial wave?

#### A 4-point geodesic Witten diagram (GWD)

Diagram for amplitude in AdS spacetime which is integrated over geodesics



[E. Hijano, P. Kraus, E. Perlmutter, R. Sniverly, 2015]

#### Our purpose

#### Generalization to fermion fields

#### Our motivation

- Conformal partial wave for CFT with fermion
- Towards
   super conformal partial wave





#### Our result Geodesic Witten diagrams with spinor fields in odd dimensional AdS

- Embedding formalism for spinor propagators in odd dimensional AdS
- Checking that 4-point GWD with fermion exchange satisfies the conformal Casimir equation for conformal partial wave
- •GWD expansion of fermion exchange 4-point Witten diagram



#### Outline

#### 1. Introduction

#### 2. Detail of our result

#### **Embedding formalism**

[Dirac, 1936], …, [Costa, Penedones, Poland, Rychkov, 2011],…

In the embedding formalism, d+1-dim bulk and d-dim boundary coordinates can be described by d+2-dim Minkowski spacetime.



### AdS spinor propagators in the embedding formalism

We consider odd dim AdS and even dim CFT.

Spinor bulk-boundary  
propagator
$$G_{b\partial}^{\Delta,\frac{1}{2}}(X,\bar{S}_{b};P,S_{\partial}) = \mathcal{C}_{\Delta,\frac{1}{2}} \frac{\langle \bar{S}_{b}\Pi_{-}S_{\partial} \rangle}{(-2X \cdot P)^{\Delta+\frac{1}{2}}}$$

Spinor bulk-bulk propagator

$$G_{bb}^{\Delta,\frac{1}{2}}(X,\bar{S}_b;Y,T_b) = \langle \bar{S}_b \Pi_+ T_b \rangle \left(\frac{d}{du}G_{bb}^{\Delta_+,0}(u)\right) + \langle \bar{S}_b \Pi_- T_b \rangle \left(\frac{d}{du}G_{bb}^{\Delta_-,0}(u)\right)$$

They are consistent with the expressions

without the embedding formalism

in [M. Henningson, K. Sfetsos, 1998], [T. Kawano, K. Okuyama, 1999].

We construct (geodesic) Witten diagram by using these propagators.

#### Conformal Casimir equation and equation of AdS propagator

Conformal partial wave  $W(\Delta, \Delta_i; P_i)$ 

satisfies the conformal Casimir equation.

$$-\frac{1}{2}(L_1 + L_2)^2 W_4(\Delta, \Delta_i; P_i) = C_{\Delta,s} W_4(\Delta, \Delta_i; P_i)$$

In the view point of GWD,

the conformal Casimir equation is related to the equation of the bulk-bulk propagator  $(X \neq Y)$ .

$$\left[ (\Gamma^A \nabla_A)^2 - m^2 \right] \Psi(X) = \left( -\frac{1}{2} L^{AB} L_{AB} - C_{\Delta, \frac{1}{2}} \right) \Psi(X) = 0$$

$$m = \Delta - \frac{d}{2}$$



#### 4-point GWD satisfies the conformal Casimir equation.

Amplitude of 4-point GWD with fermion exchange

 $\mathcal{W}_4(\Delta, \Delta_i) = \int_{-\infty}^{\infty} d\lambda F_{\Delta}[P_1, P_2, Y(\lambda), \bar{S}_{1\partial}, T_b](\overleftarrow{\partial}_{T_b}(1 + Y(\lambda))\overrightarrow{\partial}_{\bar{T}_b}) G_{b\partial}^{\Delta_4, \frac{1}{2}}(Y(\lambda), \bar{T}_b; P_4, S_{4\partial}) G_{b\partial}^{\Delta_3, 0}(Y(\lambda); P_3)$ 

 $F_{\Delta}[P_1, P_2, Y, \bar{S}_{1\partial}, T_b]$ 

 $:= \int_{-\infty}^{\infty} d\lambda \, \bar{G}_{b\partial}^{\Delta_1, \frac{1}{2}}(X(\lambda), S_b; P_1, \bar{S}_{1\partial}) G_{b\partial}^{\Delta_2, 0}(X(\lambda); P_2)(\overleftarrow{\partial}_{S_b}(1 + X(\lambda)) \overrightarrow{\partial}_{\bar{S}_b}) G_{bb}^{\Delta, \frac{1}{2}}(X(\lambda), \bar{S}_b; Y, T_b)$ 



Because of the rotation invariance of  $F_{\Delta}$  and the equation of the bulk-bulk propagator, the 4-point GWD satisfies the conformal Casimir equation.

 $-\frac{1}{2}(L_1+L_2)^2\mathcal{W}_4(\Delta,\Delta_i) = C_{\Delta,\frac{1}{2}}\mathcal{W}_4(\Delta,\Delta_i)$ 

## Ratio between 3-point GWD and Witten diagram

Geodesic Witten diagram  $\mathcal{W}_3$  (Integration over the geodesic)

Witten diagram  $\,A_3$  (Integration over the whole AdS)



The ratio is useful for GWD expansion of the Witten diagram.

$$\frac{\mathcal{A}_3}{\mathcal{W}_3(\gamma_{31})} = \pi^{d/2} \Gamma\left(\frac{1}{2}\left(-d + \Delta_1 + \Delta_2 + \Delta_3 + 1\right)\right) \frac{\Gamma\left(\frac{1}{2}(\Delta_3 + \Delta_1 - \Delta_2)\right)}{\Gamma(\Delta_3)\Gamma(\Delta_1 + 1/2)}$$

### 4-point Witten diagram is expressed as the product of two 3-point diagrams.



#### GWD expansion of fermion exchange 4-point Witten diagram



Poles of  $\frac{(\mathcal{A}_3/\mathcal{W}_3)(\mathcal{A}'_3/\mathcal{W}'_3)}{\nu + i(\Delta_0 - d/2)}$  determine conformal dimensions of intermediate states in GWD (conformal partial wave) expansion.

 $d/2 + i\nu = \Delta_0$ , (Single trace operator)  $d/2 + i\nu = \Delta_1 + \Delta_2 + 2m, \ d/2 + i\nu = \Delta_3 + \Delta_4 + 2m, \ (m = 0, 1, \cdots)$ (Double trace operators)

### Summary

- We study geodesic Witten diagram with spinor fields in odd dimensional AdS.
- We show that 4-point geodesic Witten diagram with fermion exchange satisfies the conformal Casimir equation for conformal partial wave.
- We compute the ratio between
   3-point GWD and Witten diagram and consider GWD expansion of the 4-point fermion exchange Witten diagram.





