Holographic Signatures of Resolved Cosmological Singularities

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work with A. Schäfer, J. Schliemann, F. Mele, J. Münch

[arXiv:1612.06679 , arXiv:1804.01387]

Gauge/Gravity Duality 2018, Würzburg

PHYSICS ADVANCED

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- **Scope of work:**
  - Holography in non-string QG?
  - Focus: loop quantum gravity

Talk in a nutshell

- **Scope of work:**
  - Holography in non-string QG?
  - Focus: loop quantum gravity

- **Content:**
  - Short review of recent work
  - Focus: Implications of bulk singularity resolution

Outline

Loop quantum gravity

- Quantisation of classical gravity in connection variables
- Diffeomorphism-invariant extension of lattice gauge theory

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- Diffeomorphism-invariant extension of lattice gauge theory

Main areas of progress

- 3+0 dimensions (topological), $\Lambda = 0$

  [Ponzano, Regge '68; Turaev, Viro '92; Rovelli '93; Freidel, Louapre '04; Barrett, Naish-Guzman '08; ...]

- State counting / surface entropy

  [Krasnov '96; Rovelli '96; Ashtekar, Baez, Corichi, Kransov '97-; Engle, Noui, Perez '07-; ...]

- Symmetry reduced quantisation $\rightarrow$ quantum cosmology

  [Bojowald '01-; Ashtekar, Bojowald, Lewandowski '03; Ashtekar, Pawlowski, Singh '06; ...]

LQG and Holography (other work)

3+0 dimensions (topological), $\Lambda = 0$

- Partition function can be evaluated exactly
- Various dual statistical models for different boundary states
  [Costantino '11; Dittrich, Hnybida '13; Bonzom, Costantino, Livine '15; Dittrich, Goeller, Livine, Riello '17]

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State counting

- State counting à la black hole entropy for general surfaces
- Augment discrete Ryu-Takayanagi formula for tensor networks by
  [Hayden, Nezami, Qi, Thomas, Walter, Yang '16] to geometric formula [Han, Hung '16]

Outline

Classical limit and singularities

Gravitational bulk singularities at least in classical limit

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Field theory picture:

- Non-perturbative string theory defined via AdS/CFT
- Quantum gravity from field theory

  [Hertog, Horowitz '04, '05; Das, Michelson, Narayan, Trivedi '06; Turok, Craps, Hertog '07; Barbón, Rabinovici '11; Smolkin, Turok '12;]

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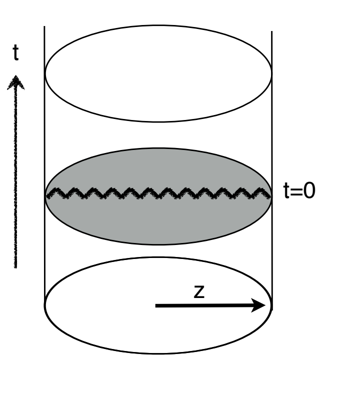
Gravity picture:

- Quantum gravity resolves singularities!?
- Holographic dual of (resolved) singularities

Outline

Two-point correlators in Kasner

[Engelhardt, Horowitz '14; Engelhardt, Horowitz, Hertog '15]



Boundary: $$ds_4^2(t) = -dt^2 + \sum_{i=1}^{3} t^{2p_i} dx_i^2, \quad p_i \in \mathbb{R}$$

Bulk: $$ds_5^2 = \frac{1}{z^2}\left(dz^2 + ds_4^2(t)\right)$$

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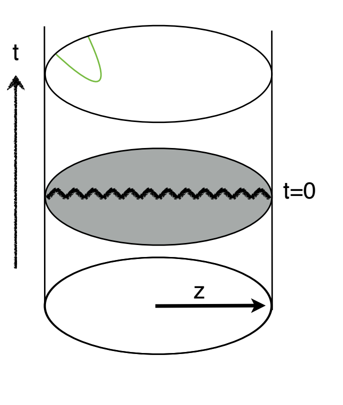

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Geodesic approximation: (heavy scalar operators)

$$\langle \mathcal{O}(x)\mathcal{O}(-x)\rangle \sim \exp(-\Delta L_{\text{ren}})$$

Δ: conformal weight of $\mathcal{O}$
L_{ren} : renormalised geodesic length

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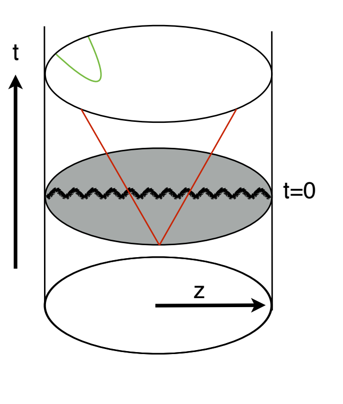

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Main result

Geodesic passing singularity $\leftrightarrow$ finite distance pole in 2-point correlator

Effective bouncing metric

Strategy: modify 4d part, no large curvatures from z-direction

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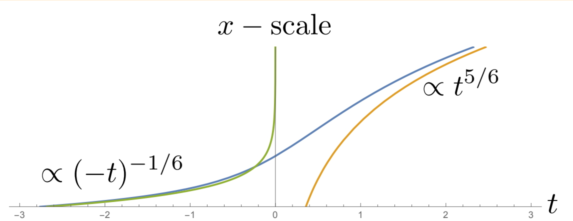
- Quantum bounce interpolates between classical solutions
  [Bojowald '01-; Ashtekar, Bojowald, Lewandowski '03; Ashtekar, Pawlowski, Singh '06; ...]
- Transitions between different Kasner solutions [Gupt, Singh '12]



Improved 2-point correlators

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Possible simplifications

- QG scale is 4d, no Kasner transitions $\rightarrow$ analytic solution
- QG scale is 5d, no Kasner transitions $\rightarrow$ numeric solution

5d scale + Kasner transitions not straight forward
(ansatz too narrow, 5d QG theory required)

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All calculations give qualitatively similar results

Signatures of the resolved singularity

Dual of the resolved singularity

- Finite distance bump instead of pole
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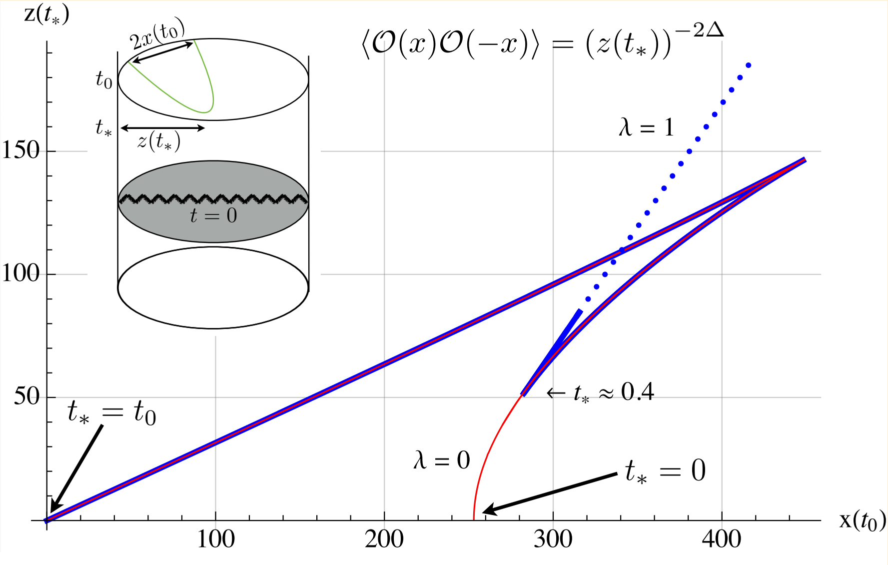
Discussion

- So far: prototype calculation
- Goal: find system where independent field theory computation possible



Analytical result (above): no Kasner transition, 4d Planck scale [NB, Schäfer, Schliemann '16]

Numerics: 5d Planck scale + Kasner transitions qualitatively similar [NB, Mele, Münch '18]

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  - 3d gravity
  - Tensor networks
  - Singularity resolution

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- **Quantum gravity with(out) holographic dual?**

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  - 3d gravity
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Thank you for your attention!

Asymptotic behaviour

Long distance behaviour

- Complex geodesics:

$$\langle \mathcal{O}(x)\mathcal{O}(-x)\rangle \xrightarrow{x\to\infty} \propto (\mathcal{L}_{\text{bdy}})^{-\frac{2\Delta}{1-p}}$$

  - $\neq (\mathcal{L}_{\text{bdy}})^{-2\Delta}$ due to Kasner background breaking conformal symmetry

- Real singularity-free geodesic ($p < 0$):

$$\langle \mathcal{O}(x)\mathcal{O}(-x)\rangle \xrightarrow{x\to\infty} \propto \lambda^{-2p\Delta} (\mathcal{L}_{\text{bdy}})^{-2\Delta}$$

  - Subdominant to complex contribution
  - Vanishes as $\lambda \to 0$

General holography from QG

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Derive dual theory directly from QG partition function!

- Finite region QG
- Boundary state / condition $\leftrightarrow$ dual theory

$$\langle \ldots \rangle_{\text{Dual theory}(\phi_b^i)} := Z_{\text{QG}} \left[\phi_b^i \right]$$

$\rightarrow$ Euclidean 3d gravity best understood / solvable

[cf. neg. cos. constant: Castro, Gaberdiel, Hartman, Maloney, Volpato '11]

3+0 LQG, $\Lambda = 0$

3-dim. gravity is topological:

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Path integral:

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Discretize on fixed simplicial decomposition:

$$Z_{\text{PR}}(M) = \left(\prod_{\text{links } l} \int_{\text{SU}(2)} dg_l \right) \prod_{\text{faces } f} \delta \left(\overleftarrow{\prod_{l \in f}} g_l^{\epsilon(l,f)} \right)$$

Needs regularization: Gauge fixing / quantum group

Holography from partition functions

Dual 2d Ising model [Costantino '11; Dittrich, Hnybida '13; Bonzom, Costantino, Livine '15]

- Tri-valent boundary graph Γ on 2-sphere

$$\left(Z^{\text{Ising}}(\Gamma)\right)^2 Z^{\text{LQG}}(\Gamma) = \left(\prod_{\text{edges } e} \cosh(y_e)\right)^2 2^{2\#\text{vertices}}$$

- Ising couplings $y_e \leftrightarrow$ QG coherent state parameters

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Dual "twisted" 6-vertex model

[Dittrich, Goeller, Livine, Riello '17]

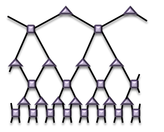
- Four-valent boundary graph Γ on twisted 2-torus
- Only spin 1/2 rep., "fuzzy parallelograms"
- Torus twist + monodromy integration in 6-vertex model:

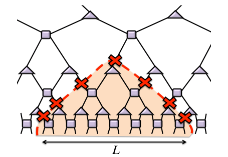
$$Z^{\text{LQG}}(\Gamma) = Z^{\text{6 vertex}}_{\text{twisted}}(\Gamma)$$

- Intertwiners $\leftrightarrow$ vertex parameters

Random tensor networks

- Approximate ground states of interacting many-body Hamiltonians
- Different types, here MERA (gapless systems) [figures from Orús, arXiv:1407.6552]





- $S_{\mathrm{EE}}(L) \sim$ min. # crossed legs

  [Swingle '09; . . . ; Hayden, Nezami, Qi, Thomas, Walter, Yang '16; . . .]

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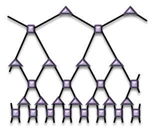

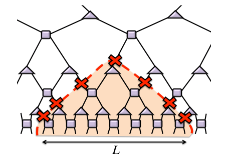

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Compares to

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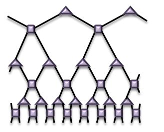

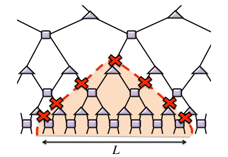

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$\rightarrow$ Model for discrete holography

- How to relate to continuum geometry / continuum RT-formula?

Deriving RT from random tensor networks

[Hayden, Nezami, Qi, Thomas, Walter, Yang '16]

- Average over random tensors $\leftrightarrow$ Ising model $\leftrightarrow$ RT-surface as domain wall
- Discrete RT formula for constant large bond dimension D:

$$S_{\mathrm{EE}}(L) = \log D \;\times\; \text{min. } \# \text{ crossed legs}$$

- Missing input: $\log D \;\leftrightarrow$ geometry

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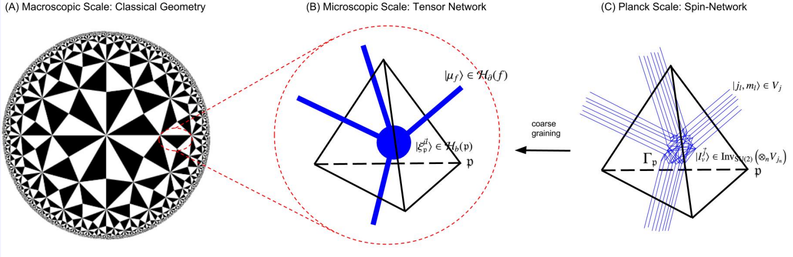
LQG interpretation [Han, Hung '16, figure from Han, Hung: arXiv:1610.02134]



Geometric RT from LQG

- Codim. 2 area from bond dimension $\leftrightarrow$ surface (black hole) entropy
- State counting: [Krasnov '96; Rovelli '96; Ashtekar, Baez, Corichi, Kransov '97-; ...]

$$D \sim \exp(A)$$

- Generic codim. 2 surfaces and dimensions [Husain '98; NB '13,'14]

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Geometric RT from LQG

- Repeat computation for generic large bond dimensions $D \sim \exp(A)$

  $\rightarrow$ discrete Nambu-Goto path integral

  $\rightarrow$ minimal surface [Han, Hung '16]

- Correct entanglement spectrum from Wheeler-de Witt wave function in 3d [Han, Huang '17]

Strategy

- Test hypothesis of singularity resolution for consistency with holography
  [c.f. Engelhardt, Horowitz '16]
- Work with effective bouncing metric in simple models
- Independent of underlying QG approach, e.g.
  - String cosmology
  - Loop quantum cosmology
  - Modified gravity
  - ...
- Compute 2-point boundary correlators in geodesic approximation
  (Neglect possible corrections to geodesic equation)
- Check for consistency with CFT description