# Chaos, quantum complexity and holographic operator growth

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Based on

D.A., I. Aref'eva, arXiv :1806.05574,

Gauge/Gravity Duality 2018, Wurzburg

# Outlook

- The chaos in quantum field theory and gravity: lightning review
- Qualitative check of correspondence between gravity and chaos: chaos suppression at finite chemical potential in holography
- Complexity of precursor by Complexity=Action

# Chaos in the AdS/CFT framework

- *Scrambling*:*when chaos is in the «on» mode*
- *Quantum chaos: exponential growth(sensitivity)*
- *Quantum complexity: inner measures of the system that follow the chaos*
- Black holes are the fastest scramblers
- Black holes are the fastest quantum computers
- Black holes are-????????????

# New time scales in strongly coupled quantum systems

- **Local thermalization**: (also called diffusion time or collision time)
- **Scrambling:**

- Time of the chaos onset

- Time when all information is governed by higher- and-higher-point correlators and non-local measures. Estimation by vanishing of n-point mutual information.

- **Global thermalization**

# Measures of chaos

- New quantitatives measure of the chaos

1. Commutator square correlator

$$C(t) = -\langle [W(t), V(0)]^2 \rangle$$

2.Spectral form factor

$$\langle \cdot \rangle = Z^{-1} \mathrm{tr}[e^{-\beta H} \cdot]$$

$$\left| \frac{Z(\beta, t)}{Z(\beta)} \right|^2 = \frac{1}{Z(\beta)^2} \sum_{m,n} e^{-\beta(E_m + E_n)} e^{i(E_m - E_n)t}$$

3.Operator size

4.General idea of out-of-time ordered correlators

# Chaos reigns in holographic systems

Bound on chaos

*Maldacena, Stanford, Shenker, 1503.01409*

$$F_d - F(t) = \epsilon \exp \lambda_L t$$

$$\lambda_L \leq \frac{2\pi}{\beta} = 2\pi T$$

- **Examples:**

1. Two dimensional CFT at large central charge (holographic dual of 3-dimensional gravity)

2. Sachdev-Ye-Kitaev model and other melonic models (probable dual of 2-dimensional dilaton gravity)

# Chaos at finite chemical potential

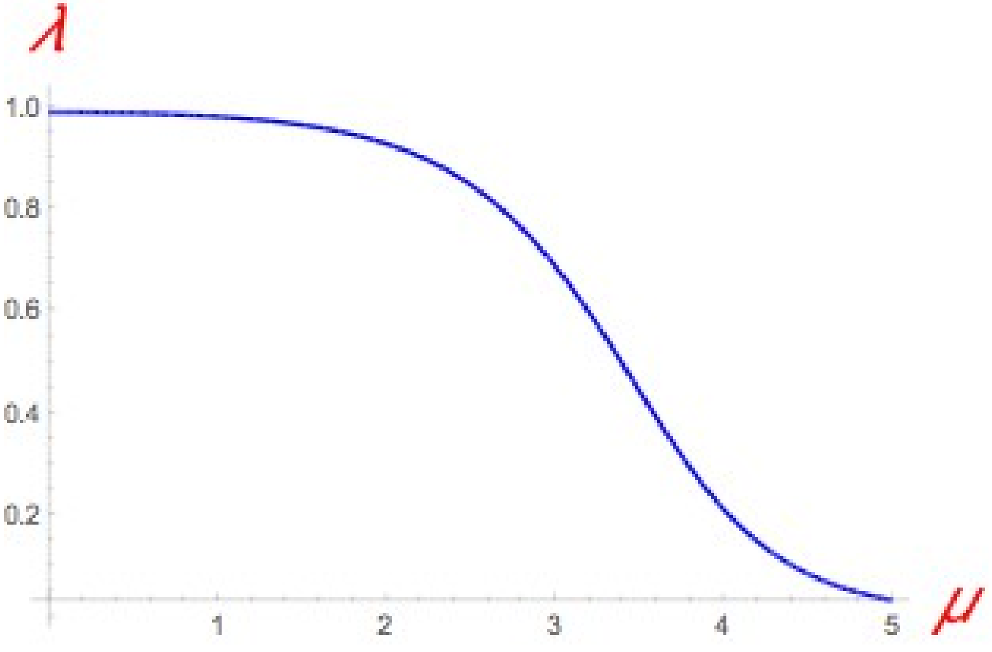
- Recent results from QFT and numerical simulations shows, that finite chemical potential can drive out the system from the chaotic regime
- Our main result is that one can derive this supression from holography.
- This result also supports recent proposal concerning GR=QM by Susskind

# Chaos suppression, 1:SYK

- Sachdev-Ye-Kitaev — randomly all-to-all interacting complex fermions

$$H = \sum_s J_{i_1..i_q} \psi^\dagger_{i_1} ... \psi^\dagger_{i_{q/2}} \psi_{i_{q/2+1}} ... \psi_{i_q}$$

$$G_0 = \frac{1}{i\omega + \mu}$$



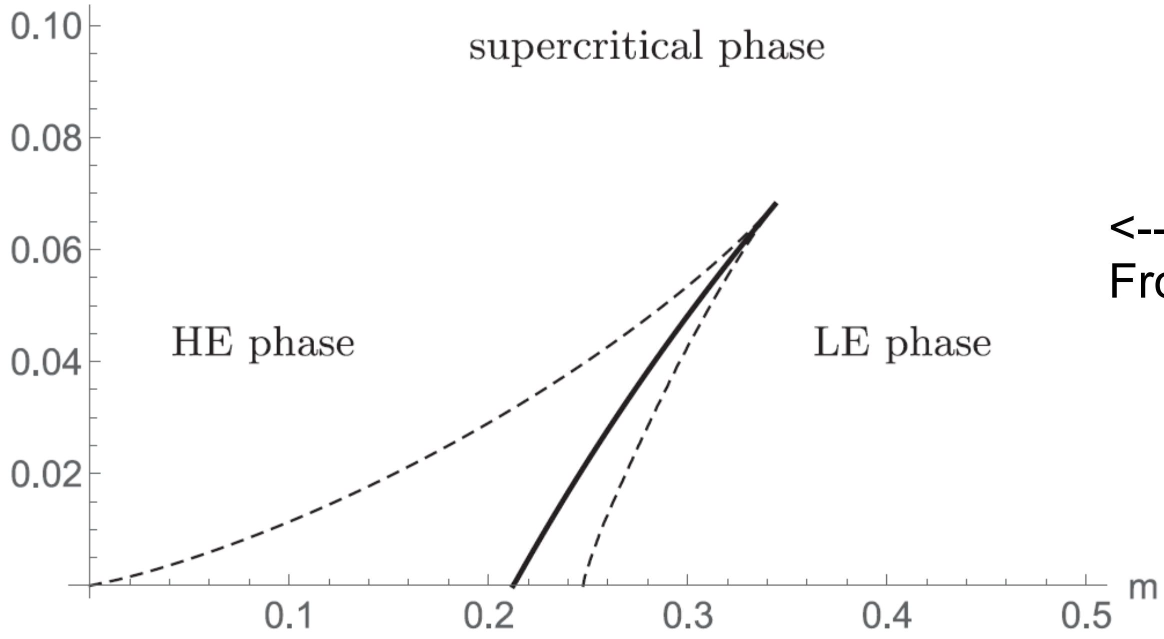

Bhattacharya et.al.,1709.07613

# Chaos supression 2: matrix QM

- Matrix quantum mechanics with the mass term in the special limit (also melon-dominated)

Azeyanagi, et. al., PRL'17

$$H = N D \mathrm{tr}\left( m \psi^\dagger_\mu \psi_\mu + \frac{1}{2} \lambda \sqrt{D} \psi_\mu \psi^\dagger_\nu \psi_\mu \psi^\dagger_\nu \right)$$

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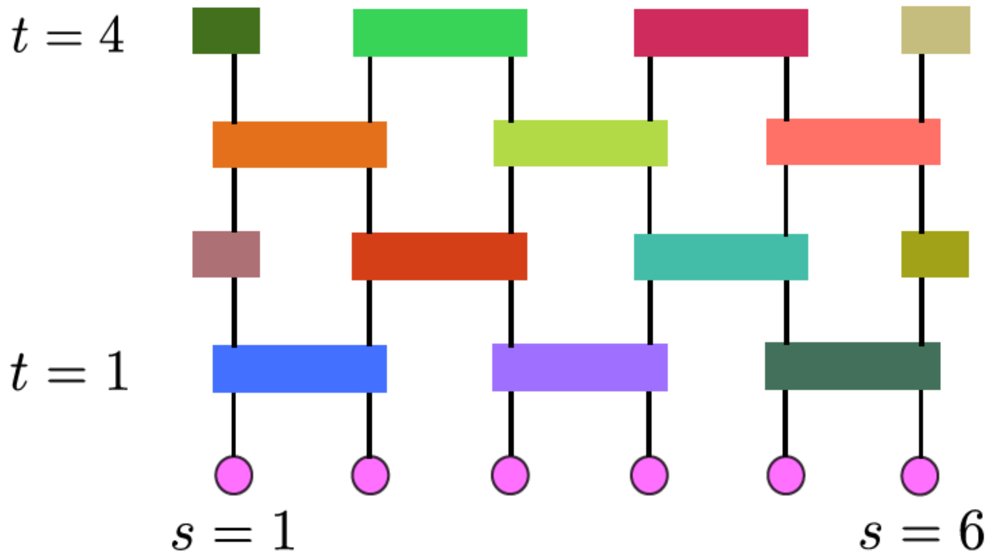
From Phys. Rev. Lett. 120, 061602

# Chaos supression 3: random circuits




Rakovszky, Pollman, von Keyserlingk, 1710.09827;
Khemani, Vishwanath, Huse, 1710.09835

# Can we find the holographic interpretation of chaos supression?

**Yes.**

# Particle~Operator in AdS/CFT

- In the AdS/CFT correspondence the operator in the dual theory is approximately massive point particle in the bulk (semiclassical scalar field with large mass)
- Can we say something about operator characteristics during the evolution?
- We consider the probe limit (different results with backreaction at zero temperature in Andrey Bagrov talk in the similar context)

# Operator size

Roberts, Stanford, Streicher, 1802.02633

$$W(t) = \sum_{s, a_1 < \ldots < a_s} c_{a_1 \ldots a_s}(t) \psi_{a_1} \ldots \psi_{a_s}$$

- «s»-grows while the system evolves
- Characterizes how «complex» becomes the operator during the evolution of the system
- Important quantitative chaotic measure in holographic systems

# «Why things do fall?»

Susskind,1802.01198

Brown, Gharibyan, Streicher, Susskind, Thorlacius, Zhao 1804.04156

- There is recent conjectural correspondence (by L.Susskind) between the particle radial momentum falling in the black hole (i.e. operator evolving at finite temperature) and the operator size.
- It occurs that holographic theories precisely saturate some bound of this growth
- *Gravity makes things more and more complex*

$$\mathbf{Operator\ size} \longleftrightarrow \mathbf{p_z(t)}$$

$$p_z(t) \approx e^{\frac{2\pi}{\beta} t}$$

# «Cold» chaos

- First let us consider the chemical potential effect on neutral operator.
- Nearly extremal black hole dual accelerates the chaos, but the late time growth is still universal

Brown, et.al. 1804.04156

- However T=0 and finite chemical potential late-time size growth has non-universal form (A and s depend on the details like dimension)

$$\text{size} \approx e^{At^s}$$

D.A., I. Aref.eva, work in progress and 1806.05574

# «Why things stop falling?»

D.A., I.Aref'eva, 1806.05574

- We make a quantitative check of this correspondence. We show that finite chemical potential suppresses the chaos both in the holographic model and in the model dual theories.
- Charged operator = charged particle

$$S = -m \int \sqrt{-g_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu}}d\tau + qA_{\mu}\dot{x}^{\mu}d\tau$$

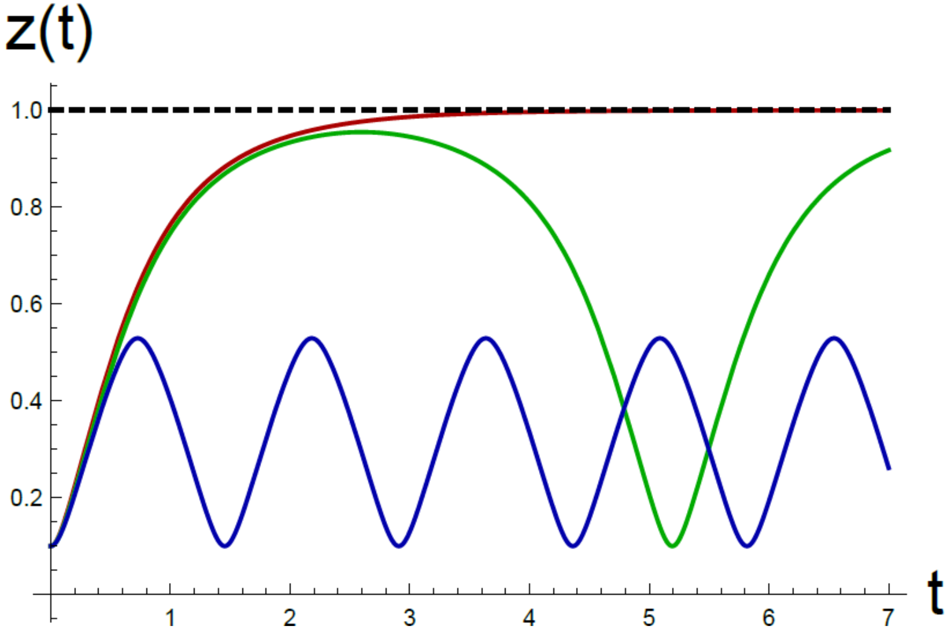
# Reissner-Nordstrom black hole and finite chemical potential

$$ds^2 = \frac{1}{z^2}\left(-f(z)dt^2 + \frac{dz^2}{f(z)} + d\bar{x}^2\right)$$

$$f(z) = 1 - M\left(\frac{z}{z_h}\right)^d + Q\left(\frac{z}{z_h}\right)^{2d-2},$$

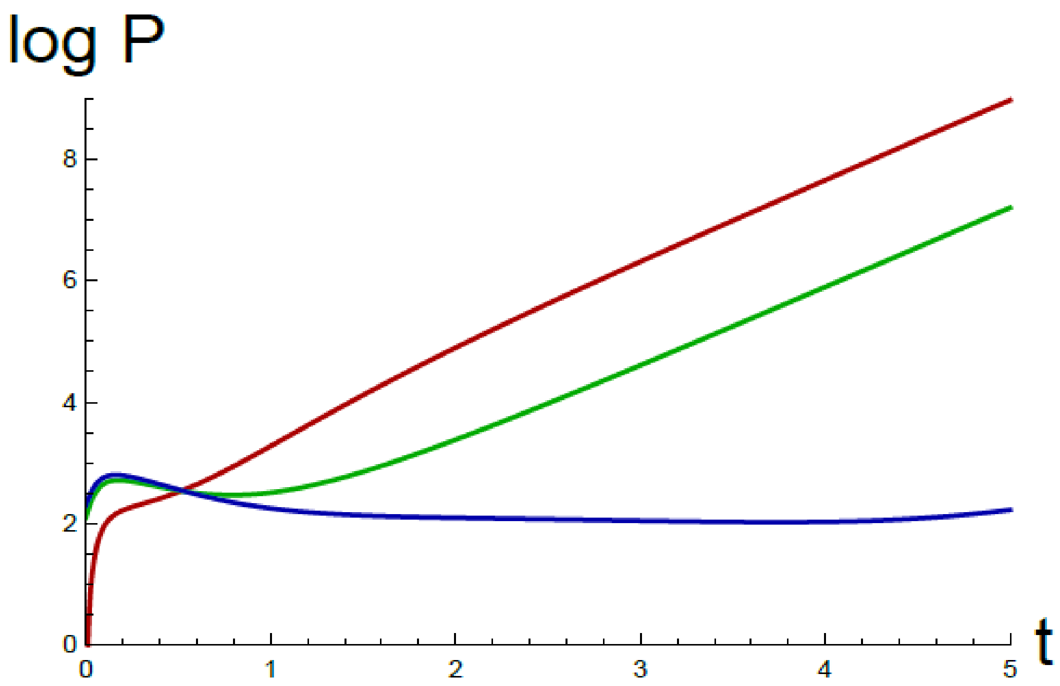
$$A = \mu\left(1 - \left(\frac{z}{z_h}\right)^{d-2}\right)dt$$

# Critical charge




$$q_{crit} = \frac{\sqrt{f(z_*)}}{z_* A(z_*)}$$

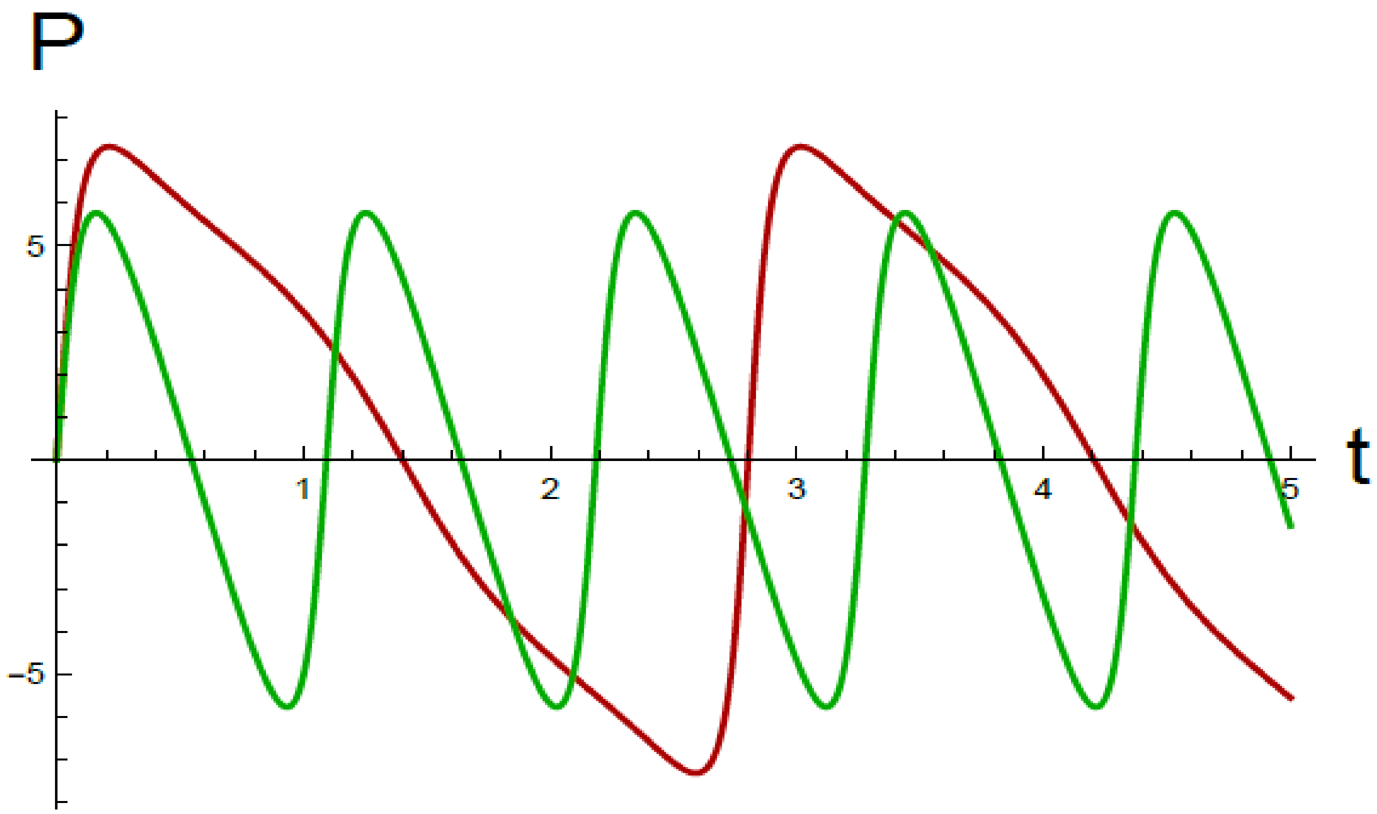
# Momentum stops growing at the critical charge




$$\mathbf{Operator\ size} \longleftrightarrow \mathbf{p_z(t)}$$

D.A., I.Aref'eva, 1806.05574

# Oscillations above the critical charge



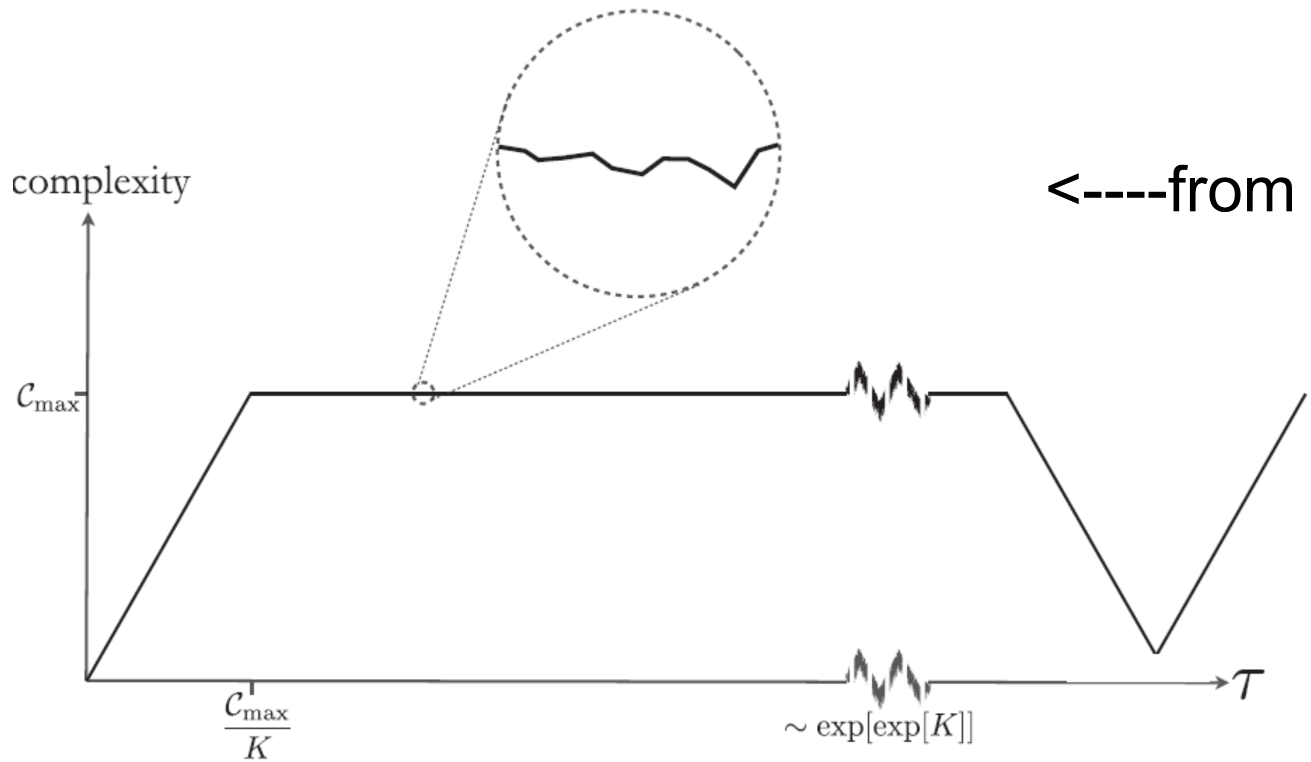

$$\mathbf{Operator\ size} \longleftrightarrow \mathbf{p_z(t)}$$

D.A., I.Aref'eva, 1806.05574

# Complexity vs chaos

Brown, Susskind, 1701.01107

- The conjectured complexity for chaotic precursors increases *linearly* with rate K, and then saturates at a value exponential in K. It fluctuates around this value

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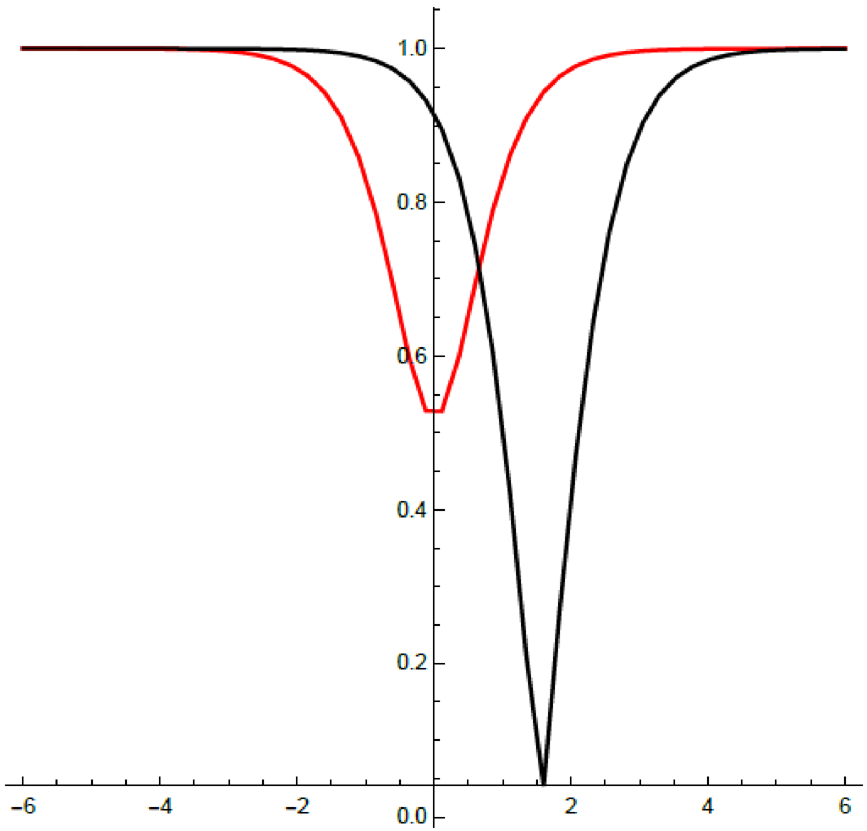
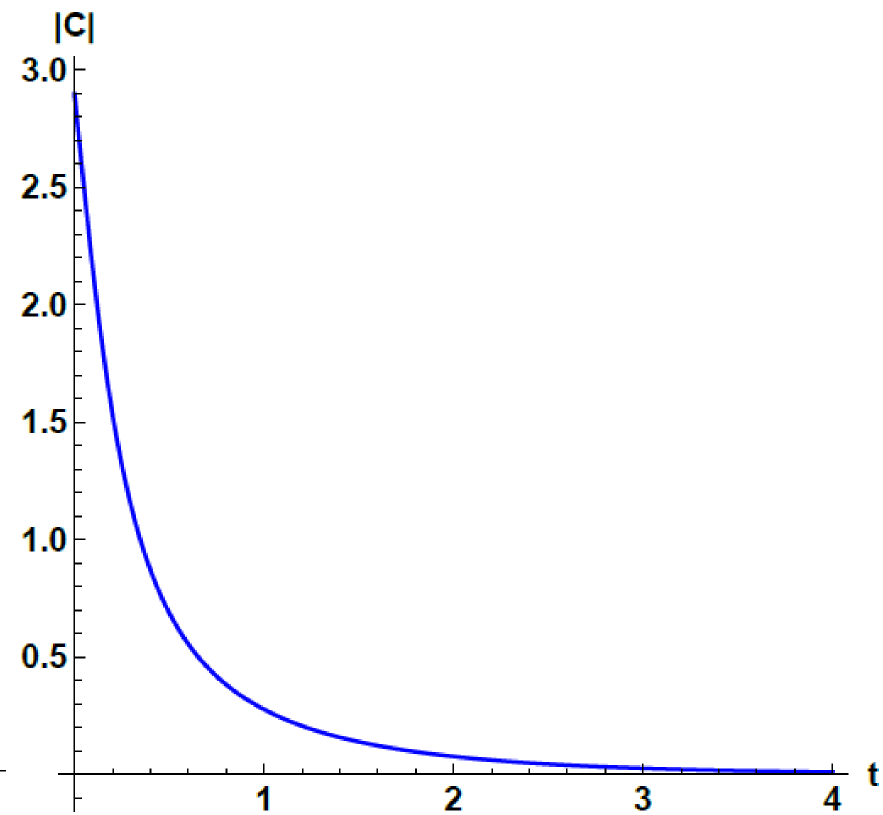
# Massive particle in BTZ black hole model+«Complexity-action»

AD, et. al., work in progress




The WdW patch is black, the particle trajectory is red

# Linear growth coefficient explicitly for d=2 CFT

AD, et. al., work in progress

$$\mathcal{C} \approx \left( E + \sqrt{E^2 + 4\pi^2 T} \right) t$$

- The initial growth is linear in agreement with the conjectural picture
- The saturation is very fast, not exponentially slow.

# Conclusion

- Recent results from chaotic systems show, that the chaos is very sensitive to the chemical potential; these systems include SYK and other melonic-dominated models and random circuits that have holographic interpretation.
- We made the qualitative check of this effect by holographic model of charged operator growth at finite chemical potential
- Our results supports recent Susskind proposal relating momentum and operator size growth